1

MATHEMATICAL TOOLS

Introduction

In this unit, we will discuss various identities for multiplication of algebraic expressions and will become familiar with the graphs, a way to represent information. The functions are the important part of our life, we'll see how. We will know what are trigonometric identities and their calculation. We will also discuss the types of physical quantities on the basis of directional properties.

Algebraic Identities and Graphs

An identity is an equality, which is true for all values of the variables. The following three identities are very common and are used often.

Example 1: $(a + b)^2 = a^2 + 2ab + b^2$

Solution: We have : $(a + b)^2 = (a + b)(a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$ [Since ba = ab]

$\therefore (a + b)^2 = a^2 + 2ab + b^2$

Example 2: $(a – b)^2 = a^2 – 2ab + b^2$

Solution: We have :

$(a – b)^2 = (a – b)(a – b)$

$= a(a – b) – b(a – b)$

$= a^2 – ab – ba + b^2$

$= a^2 – ab – ab + b^2$ [Since ba = ab]

$= a^2 – 2ab + b^2$

$\therefore (a – b)^2 = (a^2 – 2ab + b^2)$

Example 3: $(a + b)(a – b) = a^2 – b^2$

Solution: We have :

$(a + b)(a – b) = a(a – b) + b(a – b)$

$= a^2 – ab + ba – b^2$

$= a^2 – b^2$ [Since ba = ab]

$\therefore (a + b)(a–b) = a^2 – b^2$

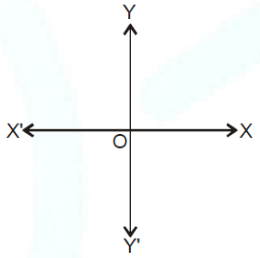
Introduction To Graphs

In our daily routine, we all use, newspapers, TV channels and ratio etc. in order to get information in the form of data and these data can be represented graphically, i.e., on the bar graph, pie chart, histograms etc. But before proceeding, we will discuss how one can locate the position of a point and the plotting of point in the plane.

(a) Co-ordinate System:

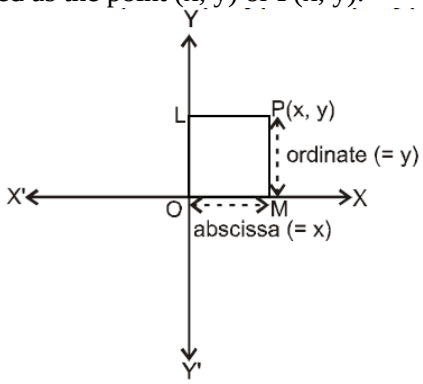
In Cartesian co-ordinate system, we represent any point by ordered pair (x, y), where x and y are called X and Y co-ordinate of that point respectively.

Take two perpendicular lines X'OX and Y'OY intersecting at the point O(known as origin). X'OX and Y'OY are called the co-ordinate axes. X'OX is called the X-axis and Y'OY is called the Y-axis. Lines X'OX and Y'OY are sometimes also called rectangular axes.



(b) Co-ordinates of a point

Let P be any point as shown in figure. Draw PL and PM perpendiculars on Y-axis and X-axis, respectively. The length LP (or OM) is called the x-coordinate or the abscissa of point P and MP is called the y-coordinate or the ordinate of point P. A point whose abscissa is x and ordinate is y named as the point (x, y) or P(x, y).



The two lines X'OX and Y'OY divide the plane into four parts called quadrants. XOY, YOX', X'OY' and Y'OX are, respectively, called the first, second, third and fourth quadrants. The following table shows the signs of the coordinates of points situated in different quadrants:

Quadrant	X-coordinate	Y-coordinate	Point
First quadrant	+	+	(+, +)
Second quadrant	–	+	(–, +)
Third quadrant	–	–	(–, –)
Fourth quadrant	+	–	(+, –)

Note:

(i) Abscissa is the perpendicular distance of a point from y-axis.
(i.e., positive to the right of y-axis or negative to the left of y-axis)

(ii) Ordinate is positive above x-axis or negative below x-axis.

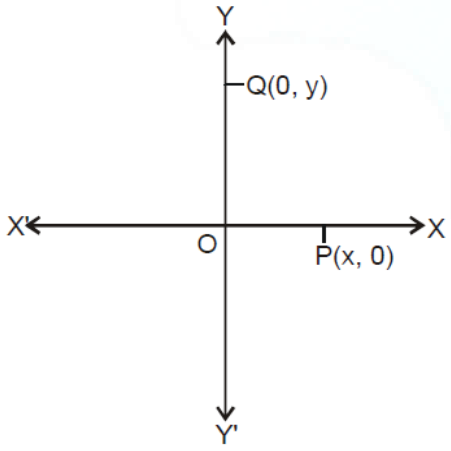
(iii) Abscissa of any point on y-axis is zero.

(iv) Ordinate of any point on x-axis is zero.

(v) Co-ordinates of the origin are (0,0).

(c) Points on Axis:

If point P lies on X-axis then clearly its distance from X-axis will be zero, therefore we can say that it's Y-coordinate will be zero. Similarly if any point Q lies on Y-axis, then its distance from Y-axis will be zero therefore we can say its X-coordinate will be zero.



(d) Plotting the Points

In order to plot the points in a plane, we may use the following algorithm :

Step I: Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

Step II: Mark their intersection point as O (origin).

Step III: Choose a suitable scale on X-axis and Y-axis and mark the points on both the axis.

Step IV: Obtain the coordinates of the point which is to be plotted. Let the point be P(a, b). To plot this point start from the origin and move 'a' units along OX or OX' according as 'a' is positive or negative respectively. Suppose we arrive at point M. From point M, move 'b' units vertically upward or downward accordingly as 'b' is positive or negative, respectively. The point where we arrive finally is the required point P(a, b).

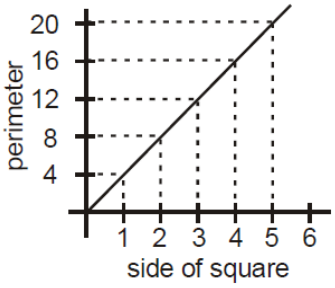
(e) Plotting of Graphs

i. Perimeter vs side of a square

Let us find the perimeter of squares having sides 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, 6 cm and tabulate the result.

Side of square (cm)	Perimeter of the square (4xside) cm	(Side, perimeter)
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)
6	24	(6, 24)

Draw coordinate axes. Take side of the square along x-axis and perimeter of square along y-axis. Then plot the point and join them successively to obtain the required graph given below.

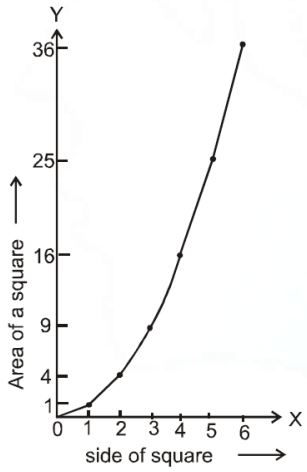


You may observe a straight line after joining all the points together making some angle with x-axis. This line shows that this is a linear graph that means perimeter of square is changing uniformly with side of square.

ii. Area vs side of a square: Let us find the area of squares having sides 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, 6 cm and tabulate the result.

Side of square (cm)	**Area of square (side × side) cm^2**	**(Side, area)**
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)
4	16	(4, 16)
5	25	(5, 25)
6	36	(6, 36)

Draw coordinate axes. Take side of the square along x-axis and area of square along y-axis. Then plot the point and join them successively to obtain the required graph given below.



You may observe a curved line after joining all the points together. This line shows that this is a non-linear graph that means area of square is changing non-uniformly with side of square.

(f) Reading of graphs

So far we have been plotting the points. We have also seen that when these points are joined sometimes we get a line and sometimes we do not get a line. If the points when joined together lie on a straight line, we get a graph called linear graph.

In this section, we shall learn in general the reading of linear graphs and then reading of distance vs time graphs.

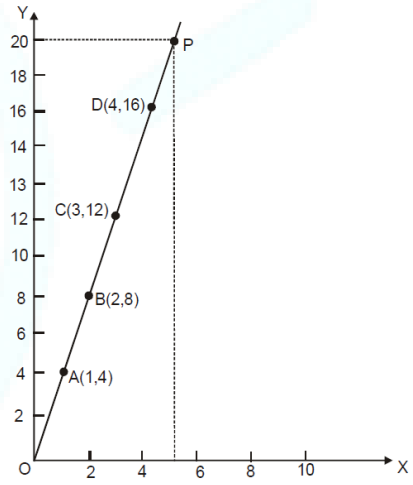
X	1	2	3	4
Y	4	8	12	16

i. Let us plot the following points for the multiples of 4 and join them.

We plot the points (1, 4), (2, 8), (3, 12), (4, 16) and then join them.

Now from the graph we can find 4 × 5.

For that we locate number 5 on the x-axis and from there draw a perpendicular (go parallel to y-axis) which touches the graph at P. From P, we draw a line parallel to x-axis which meets y-axis at the point marked 20.



Thus, 4 × 5 = 20

FUNDAMENTAL UNLOCKED- (FU#1)

Q.1 Find each of the following products:

(i) (3x +2y) (3x +2y)

(ii) $(4x^2 + 5)(4x^2 + 5)$

(iii) $(2x - 5y)^2$

(iv) $(3x^2 + 2y^2)(3x^2 - 2y^2)$

Q.2 Write the quadrants for the following points :

(i) A (3, 4)

(ii) B (– 2, 3)

(iii) C (– 5, – 2)

(iv) D (4, – 3)

(v) E (– 5, – 5)

Trigonometry

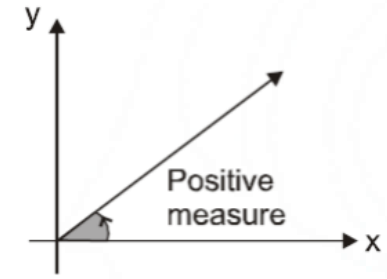
(a) Functions:

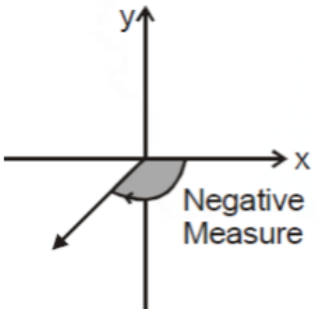
Function is a rule of relationship between two variables in which one is assumed to be dependent and the other independent variable.

Example:

1. The temperature at which water boils depends on the elevation above sea level (the boiling point drops as you ascend). Here elevation above sea level is the independent & temperature is the dependent variable.
2. The interest paid on a cash investment depends on the length of time the investment is held. Here time is the independent and interest is the dependent variable.

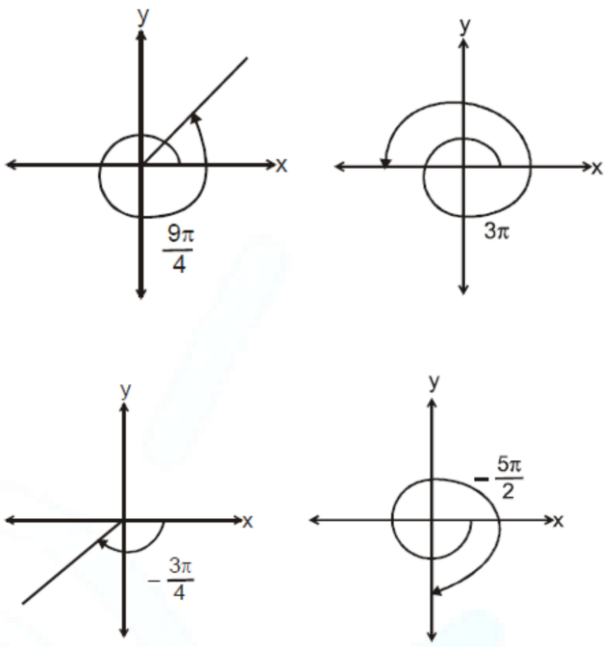
(b) Measurement of Positive and Negative Angles :





Sign of angle

An angle in the x-y plane is said to be in standard position if its vertex lies at the origin and its initial ray lies along the positive x-axis (see figure). Angles measured counterclockwise from the positive x-axis are assigned positive measures and angles measured clockwise are assigned negative measures.



Measurement of angle

(c) Six Basic Trigonometric Functions:

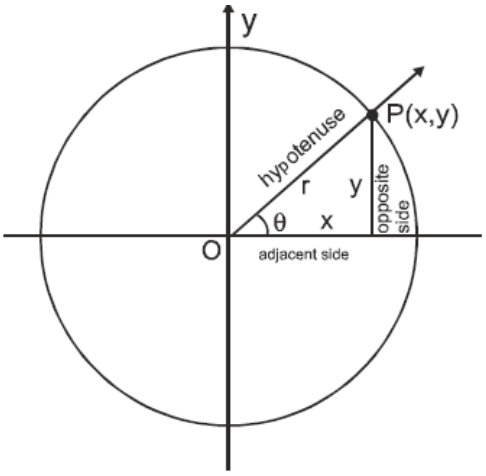
The trigonometric function of a general angle θ are defined in terms of x, y, and r.

Sine: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$

Cosecant: $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y}$

Cosine: $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$

Secant: $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{r}{x}$



Basic trigonometric functions

Tangent : $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$

Cotangent : $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$

Values of sin θ, cos θ and tan θ for some standard angles.

Degree	0	30	37	45	53	60	90
Radians	0	π/6	37π/180	π/4	53 π/180	π/3	π/2
sinθ	0	1/2	3/5	$1/\sqrt{2}$	4/5	$\sqrt{3}/2$	1
cosθ	1	$\sqrt{3}/2$	4/5	$1/\sqrt{2}$	3/5	1/2	0
tanθ	0	$1/\sqrt{3}$	¾	1	4/3	$\sqrt{3}$	∞

FUNDAMENTAL UNLOCKED- (FU#2)

Q.1 Write the formula for sin θ and cos θ.

Q.2 Write general trigonometric formulas.

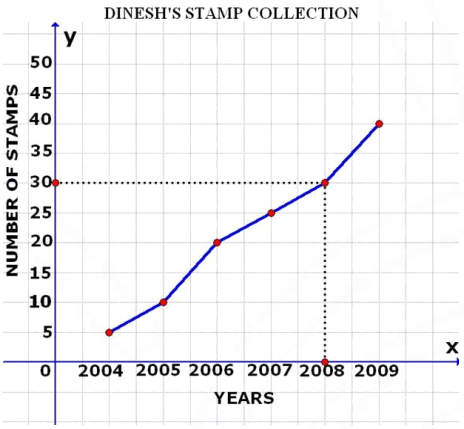
Summary / What we learned so far?

- To locate the position of an object or a point in a plane, we required two perpendicular lines. One of them is horizontal, and the other is vertical.
- The plane is called the Cartesian or coordinate plane and the lines are called the coordinate axes.
- The horizontal line is called the x-axis, and the vertical line is called the y-axis.
- The coordinate axes divided the plane into four parts called quadrants.
- The point of intersection of the axes is called the origin.
- The distance of a point from the y-axis is called its x-coordinate, or abscissa, and the distance of the point from the x-axis is called its y-coordinate, or ordinate.
- If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
- The coordinates of a point on the x-axis are of the form (x, 0) and that of the point on the y-axis are (0, y).
- The coordinates of the origin are (0, 0).
- The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant, where + denotes a positive real number and – denotes a negative real number.
- Function is a rule of relationship between two variables in which one is assumed to be dependent and the other independent variable.
- There are six fundamental trigonometric ratios: sin θ, cos θ, tan θ, cosec θ, sec θ, cot θ

EXERCISE - I

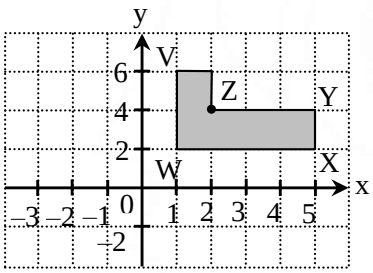
Single Correct Type Questions

1. Dinesh has been collecting stamps from the year 2004 as shown in the graph. The year in which he collected 10 stamps is:



(A) 2005 (B) 2006 (C) 2007 (D) 2008

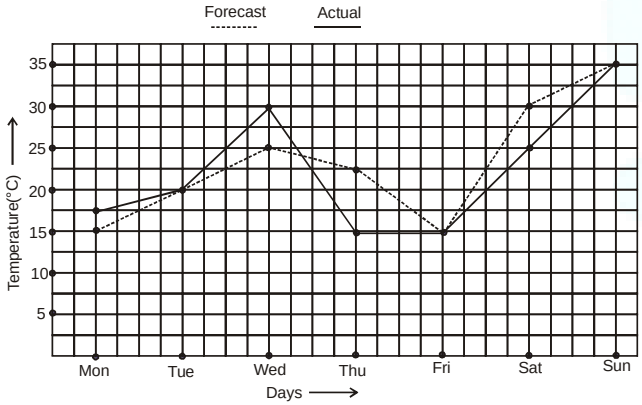
2.



What are the coordinates of the image of point Z with respect to y-axis?

(A) (–2, 4) (B) (2, 4)
(C) (4, –2) (D) (4, 2)

Instructions (3 to 7) : The following graph shows the temperature forecast and the actual temperature for each day of a week.



3. On which days was the forecast temperature the same as the actual temperature?

(A) Tuesday
(B) Friday
(C) Sunday
(D) All of these

4. What was the maximum forecast temperature during the week?

(A) 30°C
(B) 35°C
(C) 40°C
(D) 45°C

5. What was the minimum actual temperature during the week?

(A) 10°C
(B) 15°C
(C) 20°C
(D) 25°C

6. At origin, the x-coordinate & y-coordinate respectively are :

(A) (1, 1)
(B) (0, 0)
(C) (1, 0)
(D) (0, 1)

7. The y-coordinate in (0, 2), (3, 0) are :

(A) 0, 0
(B) 2, 3
(C) 2, 0
(D) 0, 2

8. In which quadrant the sign of x-coordinate and y-coordinate of a point is same?

(A) I
(B) III
(C) (A) and (B) both
(D) None of these

9. The point where the two axes intersect is called:
(A) x – coordinate
(B) Origin
(C) y – coordinate
(D) Vertical point

10. In which of the following quadrants does the point (–3, –2) lie?
(A) I
(B) II
(C) III
(D) IV

Fill In The Blanks

1. Image of point (1, 2) under x-axis is _____ and under y-axis is _____.

2. Two perpendicular lines separate a plane in _____ quadrants.

3. The _____ co-ordinate of a point on x-axis is zero.

4. The _____ co-ordinate of a point on y-axis is zero.

5. A point in _____ quadrant has positive abscissa and negative ordinate.

True / False

1. The perpendicular distance of the point A(4, 5) from x-axis is 5 units.

2. The point (5, 0) lies on x-axis.

3. The point (0, –3) lies in quadrant II.

4. Image of point (1, 2) under y-axis is (–1, 2).

5. Image of point (3, 5) under the line y = 0 is (–3, 5).

EXERCISE - II

VERY SHORT ANSWER TYPE QUESTIONS

1. Locate the following points on the graph paper A(–3, 4), B(4, 0), C(–5, –3), D(–4, –5), E(0, 5), F(0, –4), G(0, 2), H(–3, 0).

2. Mark the following points and join them. Give name.

(i) (–3, 0), (–3, 1), (–3, 2), (–3, 5), (–3, –2)
(ii) (5, 2), (3, 2), (0, 2), (–3, 2), (–1, 2)
(iii) (2, –2), (8, 4), (5, 7), (–1, 1)
(iv) (1, –2), (3, 6), (5, 10), (3, 2)
(v) (3, 0), (6, 4), (–1, 3)

SHORT ANSWER TYPE QUESTIONS

3. Find the distance of the following points from both x and y axis (0, 3), (5, –2), (–3, 4), (7, 3), (8, 17), (15, 3), (7, 7).

4. Find the distance of the following points from origin (3, 4), (1, 1), (–5, –12), (9, 40).

5. Find the distance between

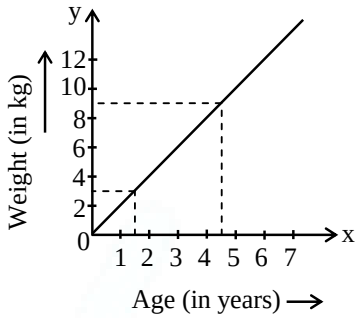
(i) A(–5, 0) and B(7, 0)
(ii) P(0, 3) and Q(0, –2)

6. Locate the following points on graph paper by taking suitable scale.

(i) A(15, 25) (ii) B(15, –30)
(iii) C(–35, 25) (iv) D(–15, –20)
(v) E(–15, 35) (vi) F(25, 15)
(vii) G(–5, 0) (viii) H(0, 10)

7. In the given line graph, weight of a child according to age is given. Give answers of the following questions.

(i) What is the weight of child when age is 3 years?
(ii) What is the weight of child when age is 1 year 6 months and 4 years 6 months?
(iii) What is the age of child when weight is (1) 6 kg (2) 12 kg ?
(iv) The increase in weight between 6 years to 7 years.



Scale 1 cm = 1 year (x-axis)
1 cm = 2 kg (y-axis)

LONG ANSWER TYPE QUESTIONS

8. The temperature for 6 months of a town as follows :

Month	Jan.	Feb.	Mar.	April	May	June
Temp. (in °C)	15	18	24	32	39	40

Draw a line graph to represent the above information.

9. Draw the line graph to represent the given information.

Age (in years)	10	12	14	16	18	20
Weight (in kg)	25	28	33	41	45	56

10. The temperature of a patient as follows.

Time	8am	10am	12noon	2pm	8pm	9pm
Temp. in °C	37°	37.2°	37.8°	38°	39.2°	38.5°

Draw line graph for the above information and find the temperature at 9 am and 2 pm.

11. The given graph describes the distances of a car from a city P at different times when it is travelling from City P to City Q, which are 350 km apart. Study the graph and answer the following :

(i) What information is given on the two axes?
(ii) From where and when did the car begin its journey?

(iii) How far did the car go in the first hour?

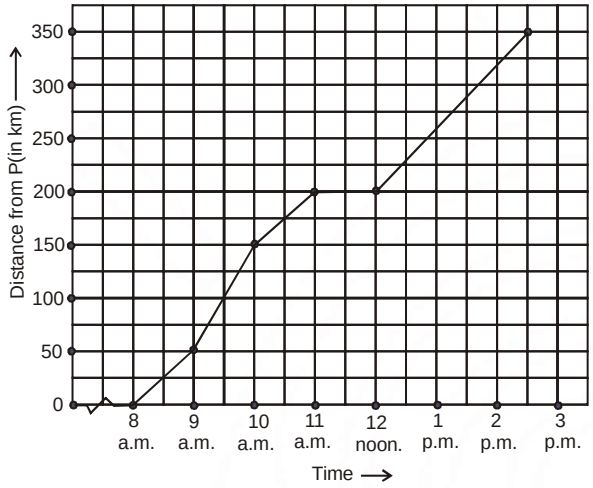
(iv) How far did the car go during :

(a) 2nd hour? (b) the 3rd hour?

(v) Was the speed same during the first three hours? How do you know it?

(vi) Did the car stop for some duration at any place? Justify your answer.

(vii) When did the car reach City Q?



12. The following table gives the body temperature in °F corresponding to °C. Draw graph using this table and answer questions that follow :

Temperature (°C)	0	10	20
Temperature (°F)	32	50	68

(a) What will be the temperature in °F when it is 15°C?

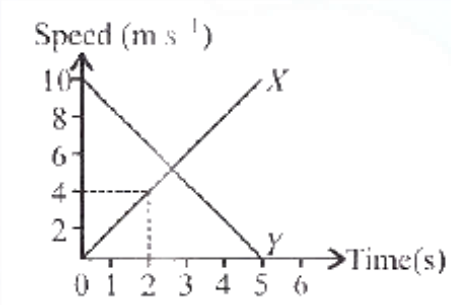
(b) What will be the temperature in °C when it is 86°F?

(c) How many °F is equivalent to 5°C?

EXERCISE - III

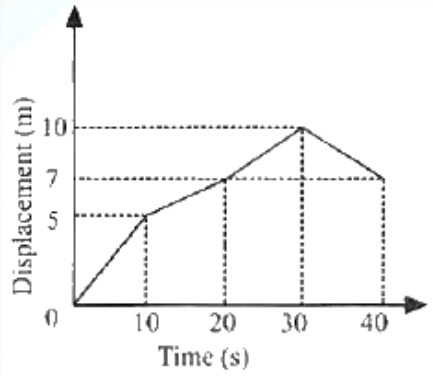
PREVIOUS YEAR QUESTIONS

1. The speed-time graphs of motion of two bicycles X and Y are shown here. If at t = 0 s, both X and Y are at same position, then at what time will they be at same position again?



(A) 2 s (B) 2.5 s (C) 5 s (D) After 6 s

2. Study the displacement time graph of a toy car given here and choose the correct statement(s) from the following.



(A) The toy car is slowest during first 10 seconds.

(B) The toy car is slowest in between 10 to 20 seconds.

(C) The toy car is fastest during first 10 seconds.

(D) Both (B) and (C)

ANSWER KEY

EXERCISE-I

SINGLE CORRECT TYPE QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	D	B	B	B	C	C	B	C

FILL IN THE BLANKS

1. (1, –2), (–1, 2) **2.** 4 **3.** y **4.** x **5.** IV

TRUE / FALSE

1. True **2.** True **3.** False **4.** True **5.** False

EXERCISE-II

SHORT ANSWER TYPE QUESTIONS

3. 3, 0 ; 2, 5 ; 4, 3 ; 3, 7 ; 17, 8 ; 3, 15 ; 7, 7

4. $5, \sqrt{2}, 13, 41$ units

5. (i) 12 (ii) 5

7. (i) 6 kg (ii) 3 kg, 9 kg (iii) 3 years, 6 years (iv) 2 kg

LONG ANSWER TYPE QUESTIONS

10. 37.1°C & 38°C

11. (i) The horizontal (x) axis shows the time. The vertical (y) axis shows the distance of the car from City P.

(ii) Started from City P at 8 a.m. (iii) 50 km (iv) (a) 100 km (b) 50 km

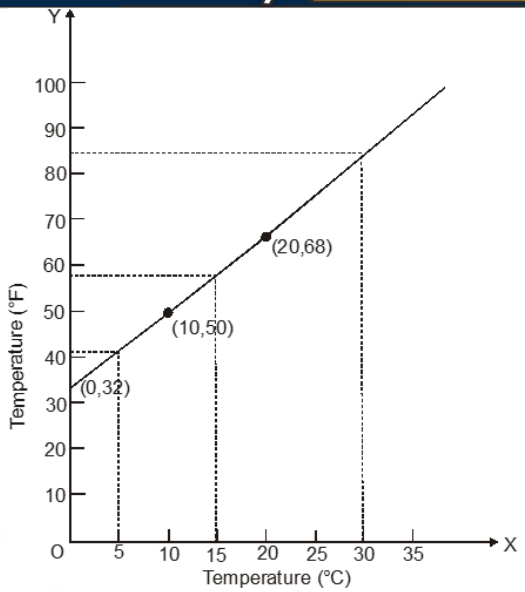
(v) The speed of the car was not the same all the time.

(In fact the graph illustrate how the speed varied)

(vi) Car did not travel during the interval 11 am to 12 noon. The horizontal line segment representing "travel" during this period is illustrative of this fact.

(vii) 2.30 pm.

12.



Plot the point (0, 32) (10, 50) and (20, 68) and join them. Produce the line.

(a) 59°F (b) 30°C (c) 5°C = 41°F

EXERCISE-III

PREVIOUS YEAR QUESTIONS

Que.	1	2
Ans.	C	D