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RATIONAL NUMBERS

Classification of numbers

Numbers are basically of 2 types: Real numbers & Imaginary numbers.

Real Numbers: These are the numbers which can represent actual physical quantities in a meaningful way. These can be represented on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

Natural numbers: Counting numbers are known as natural numbers. N = {1, 2, 3, 4, ...}.

Whole numbers: All natural numbers together with 0 form collection of all whole numbers.

W = {0, 1, 2, 3, 4, ...}.

Prime Numbers: All natural numbers that have one & itself as their only two distinct factors are **prime numbers** i.e. prime numbers are exactly divisible by 1 & themselves. **For example:** 2, 3, 5, 7, 11, 13, 17, 19, 23, etc...

- **Identification of Prime Number:**

Step (i): Find approximate square root of given number.

Step (ii): Divide the given number by prime numbers less than approximate square root of number. If given number is not divisible by any of these prime numbers then the number is prime otherwise not.

Example 1:

Is 571 a prime number?

Solution:

Approximate square root = 24.

Prime number < 24 are 2,3,5,7,11,13,17,19 & 23.

But 571 is not divisible by any of these prime numbers. So, 571 is a prime number.

Composite number: All natural numbers except 1, which are not prime are composite numbers.

For example: 4, 6, 9, 10 etc.

Remark: 1 is neither prime nor composite number.

Integers: All natural numbers, 0 and negative of natural numbers form the collection of all integers.

I or Z = {..., -3, -2, -1, 0, 1, 2, 3, ...}.

Rational numbers

Definition: Numbers that can be expressed in the form $\frac{p}{q}$, where q is a non-zero integer and p is any integer are called **rational numbers.**

Each of the numbers $\frac{2}{3}, \frac{-5}{7}, \frac{-11}{-5}, \frac{7}{-9}$ is a rational number.

Positive Rational Number: A rational number $\frac{p}{q}$ is positive, if p and q are either both positive or both negative.

Each of the rational numbers $\frac{2}{3}, \frac{5}{9}, \frac{-7}{-12}, \frac{-3}{-11}$ is a positive rational number.

Negative Rational Number: A rational number $\frac{p}{q}$ is negative, if p and q are of opposite signs.

$\frac{-3}{7}, \frac{5}{-9}, \frac{-15}{26}$

Decimal Representation of Rational Numbers

Definition: Every rational number can be represented as either a terminating decimal or a non-terminating but repeating decimal. Let us see how to represent them.

Terminating Decimals: Let x be a rational number whose decimal expansion terminates.

Then x can be expressed in the form of $\frac{p}{q}$ and prime factorization of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

For Example: $\frac{1}{2} = 0.5, \frac{3}{20} = 0.15$ etc.

Non-terminating and repeating (Recurring Decimal): Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers. For example $\frac{2}{3} = 0.6666\ldots = 0.\overline{6}, \frac{5}{11} = 0.4545\ldots = 0.\overline{45}$

Lowest form of a rational number

Definition: A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1.

Every rational number can be put in the lowest form using the following steps:

Step I: Obtain the rational number $\frac{p}{q}$.

Step II: Find the HCF of p and q say m.

Step III: If m = 1, then $\frac{p}{q}$ is in lowest form.

Step IV: If $m \neq 1$, then $\frac{p \div m}{q \div m}$ is the lowest form of $\frac{p}{q}$.

Example 2:

Express each of the following rational numbers in the lowest form.

(i) $\frac{12}{16}$ (ii) $\frac{-60}{72}$

Solution:

(i) We have,

$12 = 2 \times 2 \times 2 \times 3$ and $16 = 2 \times 2 \times 2 \times 2$

$\therefore$ HCF of 12 and 16 is $2 \times 2 = 4$.

So, $\frac{12}{16}$ is not in lowest form.

Dividing numerator and denominator by 4, we have

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

$\therefore$ $\frac{3}{4}$ is the lowest form of $\frac{12}{16}$.

(ii) We have

$60 = 2 \times 2 \times 3 \times 5$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

$\therefore$ HCF of 60 and 72 is $2 \times 2 \times 3 = 12$.

Dividing numerator and denominator by 12.

$$\therefore \frac{-60}{72} = \frac{-5}{6}.$$

Note:

(i) Two rational numbers are equal, if they have same standard from.

(ii) If $\frac{x}{y}$ is a rational number and m is any non-zero integer, then $\frac{x}{y} = \frac{x \times m}{y \times m}$.

For example: $\frac{3}{8} = \frac{3 \times 4}{8 \times 4} = \frac{12}{32}$.

(iii) If $\frac{x}{y}$ is a rational number and m is a common divisor of x and y, then

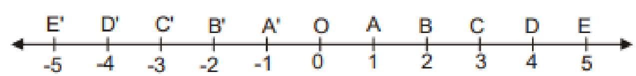
$$\frac{x}{y} = \frac{x \div m}{y \div m}$$

For example:

$$\frac{-27}{45} = \frac{(-27) \div 3}{45 \div 3} = \frac{-9}{15} = \frac{(-9) \div 3}{15 \div 3} = \frac{-3}{5}$$

Representation of rational numbers on real line

Draw any line. Take a point O on it. Call it 0 (zero). Set of equal distances on the right as well as on the left of 0. Such a distance is known as a unit length. Clearly, the points A, B, C, D, E represent the integers 1, 2, 3, 4, 5 respectively and the points A', B', C', D', E', represent the integers – 1, – 2, – 3, – 4, – 5 respectively.



Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of 0 and every negative integer lies to the left of 0. Similarly, we can represent rational numbers.

Example 3:

Represent the following numbers on the number line:

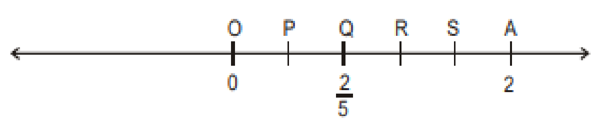
(A) $\frac{2}{5}$ (B) $\frac{-7}{3}$

Solution:

(A) Draw a number line. Mark a point O to represent 0 and another point A to represent the distance 2 units.

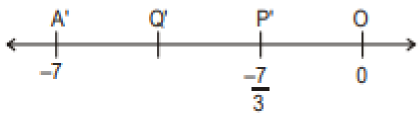
Divide, OA into 5 equal parts (equal to the denominator of $\frac{2}{5}$), at P, Q, R and S.

The point Q represents the rational number $\frac{2}{5}$.



(b) Draw a number line. Mark a point O to represent 0 and a point A' at a distance of 7 units on the left of O to represent – 7. Divide OA' into 3 equal parts at P' and Q'.

The point P' represents $\frac{-7}{3}$



Absolute value of rational number

We have learned in earlier class that the absolute value of a rational number is its numerical value (value without signs).

For example: $\left|-\frac{3}{5}\right| = \frac{3}{5}$ and $\left|\frac{7}{9}\right| = \frac{7}{9}$.

Example 4:

Verify that $|x + y| \leq |x| + |y|$

by taking $x = \frac{3}{5}, y = \frac{-4}{15}$

Solution:

If $x = \frac{3}{5}, y = \frac{-4}{15}$, then

$$|x+y| = \left|\frac{3}{5} + \left(\frac{-4}{15}\right)\right| = \left|\frac{9-4}{15}\right| = \left|\frac{5}{15}\right| = \frac{|5|}{|15|} = \frac{5}{15}$$

$$|x|+|y| = \left|\frac{3}{5}\right| + \left|\frac{-4}{15}\right| = \frac{|3|}{|5|} + \frac{|-4|}{|15|} = \frac{3}{5} + \frac{4}{15} = \frac{9+4}{15} = \frac{13}{15}$$

But $\frac{5}{15} < \frac{13}{15}$

Hence $|x + y| \leq |x| + |y|$ is true in this case.

Example 5:

Verify that $|x \times y| = |x| \times |y|$ by taking $x = \frac{-5}{3}, y = \frac{7}{9}$

Solution: $|x \times y| = \left|\left(\frac{-5}{3} \times \frac{7}{9}\right)\right| = \left|\frac{-35}{27}\right| = \frac{35}{27}$

$$|x| \times |y| = \left|\frac{-5}{3}\right| \times \left|\frac{7}{9}\right| = \frac{5}{3} \times \frac{7}{9} = \frac{35}{27}$$

$\therefore\ |x \times y| = |x| \times |y|$

Comparing two rational numbers

In order to compare any two rational numbers, we can use the following steps:

Step I: Obtain the given rational numbers.

Step II: Write the given rational numbers so that their denominators are positive.

Step III: Find the LCM of the positive denominators of the rational numbers obtained in step II

Step IV: Express each rational number (obtained in step II) with the LCM (obtained in step III) as common denominator.

Step V: Compare the numerators of rational numbers obtained in step IV. The number having greater numerator is the greater rational number.

Example 6:

Which of the two rational numbers $\frac{3}{5}$ and $\frac{-2}{3}$ is greater?

Solution: Clearly, $\frac{3}{5}$ is a positive rational number and $\frac{-2}{3}$ is a negative rational number. We know that every positive rational number is greater than every negative rational number.

$\therefore\ \frac{3}{5} > \frac{-2}{3}$

Example 7:

Which of the two rational numbers $\frac{5}{7}$ and $\frac{3}{5}$ is greater?

Solution:

Clearly denominators of the given rational numbers are positive. The denominators are 7 and 5. The LCM of 7 and 5 is 35. So, we first express each rational number with 35 as common denominator.

$\therefore\ \frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$ and $\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$

Now, we compare the numerators of these rational numbers

$\therefore\ 25 > 21 \Rightarrow \frac{25}{35} > \frac{21}{35} \Rightarrow \frac{5}{7} > \frac{3}{5}$.

Example 8:

Arrange the rational number $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ in ascending order.

Solution:

First write the given rational numbers so that their denominators are positive.

We have,

$$\frac{5}{-8}=\frac{5\times(-1)}{-8\times(-1)}=\frac{-5}{8} \text{ and } \frac{2}{-3}=\frac{2\times(-1)}{-3\times(-1)}=\frac{-2}{3}$$

Thus, the given rational numbers with positive denominators are $\frac{-7}{10},\frac{-5}{8},\frac{-2}{3}$.

Now, LCM of the denominators 10, 8 and 3 is: 2 × 2 × 5 × 2 × 3 = 120

Write the numbers so that they have a common denominator 120 as follows:

$$\frac{-7}{10}=\frac{-7\times12}{10\times12}=\frac{-84}{120},\frac{-5}{8}=\frac{-5\times15}{8\times15}=\frac{-75}{120}$$

and $\frac{-2}{3}=\frac{-2\times40}{3\times40}=\frac{-80}{120}$

Comparing the numerators of these numbers, we get – 84 < – 80 < – 75

$$\therefore \frac{-84}{120}<\frac{-80}{120}<\frac{-75}{120}\Rightarrow\frac{-7}{10}<\frac{-2}{3}<\frac{-5}{8}$$

$$\Rightarrow\frac{-7}{10}<\frac{2}{-3}<\frac{5}{-8}$$

FUNDAMENTAL UNLOCKED- (FU#1)

Q.1 What do we call a number which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$?

Q.2 Does $\frac{3}{40}$ has terminating decimal expansion?

Q.3 Find the greater number in the following pairs of rational numbers:-

(a) $\frac{9}{11}$ and $\frac{10}{12}$ (b) $\frac{-5}{8}$ and $\frac{12}{16}$

Q.4 Show that $\frac{-4}{5}=\frac{32}{-40}$

Addition of Rational Numbers

If two rational numbers are to be added we should convert each of them into a rational number with positive denominator.

Case I: When given numbers have same denominator.

In this case we define $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.

Example 9:

Add $\frac{7}{5}$ and $\frac{9}{5}$.

Solution:

$$\frac{7}{5}+\frac{9}{5}=\frac{7+9}{5}=\frac{16}{5}$$

Case II: When denominators of given numbers are unequal. In this case we take the LCM of their denominators and express each of the given numbers with this LCM as the common denominator. Now we add these numbers as shown above.

Example 10:

Add $\frac{3}{8}$ and $\frac{5}{6}$

Solution:

The denominators of the given rational numbers are 8 and 6 respectively.

LCM of 8 and 6 is 24

Now, $\frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24};\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$

$$\therefore \frac{3}{8}+\frac{5}{6}=\frac{9}{24}+\frac{20}{24}=\frac{9+20}{24}=\frac{29}{24}.$$

Short cut method

Example 11:

Find the sum: $\frac{-7}{5}+\frac{2}{3}$.

Solution:

LCM of 5 and 3 = (5 × 3) = 15.

$$\therefore \frac{-7}{5}+\frac{2}{3}=\frac{3\times(-7)+5\times2}{15}=\frac{-21+10}{15}=\frac{-11}{15}.$$

Properties of addition of rational numbers

Property 1. Closure Property:

The sum of two rational numbers is always a rational number. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

For examples: Consider the rational number $\frac{1}{3}$ and $\frac{3}{4}$. Then, $\left(\frac{1}{3}+\frac{3}{4}\right)=\frac{(4+9)}{12}=\frac{13}{12}$, which is a rational number.

Property 2. Commutative Law:
Two rational numbers can be added in any order.

Thus, for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have $\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right)$

Example:

$$\left(\frac{1}{2}+\frac{3}{4}\right)=\frac{(2+3)}{4}=\frac{5}{4} \text{ and } \left(\frac{3}{4}+\frac{1}{2}\right)=\frac{(3+2)}{4}=\frac{5}{4}.$$

So, $\left(\frac{1}{2}+\frac{3}{4}\right)=\left(\frac{3}{4}+\frac{1}{2}\right)$.

Property 3. Associative Law:
While adding three rational numbers, they can be grouped in any order.

Thus, for any three rational numbers $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$, we have $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$.

For example, consider three rationals $\frac{-2}{3},\frac{5}{7}$ and $\frac{1}{6}$.

Then, $\left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{(-14+15)}{21}+\frac{1}{6}\right\}$

$=\left(\frac{1}{21}+\frac{1}{6}\right)=\frac{(2+7)}{42}=\frac{9}{42}=\frac{3}{14}$

and $\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}=\left\{\frac{-2}{3}+\frac{(30+7)}{42}\right\}$

$=\left(\frac{-2}{3}+\frac{37}{42}\right)=\frac{(-28+37)}{42}=\frac{9}{42}=\frac{3}{14}$.

$\therefore\left\{\left(\frac{-2}{3}+\frac{5}{6}\right)+\frac{1}{6}\right\}=\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}$.

Property 4. Existence of Additive identity:
0 is a rational number such that the sum of any rational number 0 is the rational number itself.

Thus, $\left(\frac{a}{b}+0\right)=\left(0+\frac{a}{b}\right)=\frac{a}{b}$, for every rational number $\frac{a}{b}$

0 is called the additive identity for rationals.

For example, $\left(\frac{3}{5}+0\right)=\left(\frac{3}{5}+\frac{0}{5}\right)\Rightarrow\frac{(3+0)}{5}=\frac{3}{5}$ and similarly, $\left(0,\frac{3}{5}\right)=\frac{3}{5}\ \therefore\ \left(\frac{3}{5}+0\right)=\left(0+\frac{3}{5}\right)=\frac{3}{5}$

Property 5. Existence of Additive Inverse:

For every rational number $\frac{a}{b}$, there exists a rational number $\frac{-a}{b}$ such that $\left(\frac{a}{b}+\frac{-a}{b}\right)=\frac{\{a+(-a)\}}{b}$ $=\frac{0}{b}=0$ and similarly, $\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

Thus, $\left(\frac{a}{b}+\frac{-a}{b}\right)=\left(\frac{-a}{b}+\frac{a}{b}\right)=0$

$\frac{-a}{b}$ is called the additive inverse of $\frac{a}{b}$.

For example: $\left(\frac{4}{7}+\frac{-4}{7}\right)=\frac{\{4+(-4)\}}{7}=\frac{0}{7}=0$ and similarly, $\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

$\therefore\left(\frac{4}{7}+\frac{-4}{7}\right)=\left(\frac{4}{7}+\frac{-4}{7}\right)=0$.

Thus, $\frac{4}{7}$ and $\frac{-4}{7}$ are additive inverse of each other.

Subtraction of Rational Numbers

For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$\left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\frac{-c}{d}\right)=\frac{a}{b}+\left(\text{additive inverse of }\frac{c}{d}\right)$

Example 12:
Find the additive inverse of:

(i) $\frac{5}{9}$ (ii) $\frac{9}{-11}$

Solution:

(i) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(ii) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$.

Hence, its additive inverse is $\frac{9}{11}$.

Example 13:

Subtract:

(i) $\frac{-5}{7}$ from $\frac{-2}{5}$ (ii) $\frac{9}{16}$ from $\frac{7}{24}$

Solution:

(i) $\left\{\frac{-2}{5}-\left(\frac{-5}{7}\right)\right\}=\left(\frac{-2}{5}+\frac{5}{7}\right)=\frac{(-14+25)}{35}=\frac{11}{35}$

(ii) $\frac{7}{24}-\frac{9}{16}=\frac{14-27}{48}=\frac{-13}{48}$

Example 14:

What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?

Solution:

Let the required number to be added be x.

Then, $\frac{-7}{8}+x=\frac{4}{9}$

$x=\left(\frac{4}{9}+\frac{7}{8}\right)=\frac{32+63}{72}=\frac{95}{72}$

Hence, the required number is $\frac{95}{72}$.

Multiplication of Rational Numbers

For any two rationals $\frac{a}{b}$ and we define:

$\left(\frac{a}{b}\times\frac{c}{d}\right)=\frac{(a\times c)}{(b\times d)}$

Example 15: Find each of the following products:

(i) $\frac{-15}{4}\times\frac{-3}{8}$ (ii) $\frac{3}{7}\times\frac{-5}{8}$

Solution: We have

(i) $\frac{-15}{4}\times\frac{-3}{8}=\frac{(-15)\times(-3)}{4\times 8}=\frac{45}{32}$

(ii) $\frac{3}{7}\times\frac{-5}{8}=\frac{3\times(-5)}{7\times 8}=\frac{-15}{56}$

Properties of multiplication of national numbers

Property 1. Closure Property:

The product of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

Example:

Consider the rational numbers $\frac{1}{2}$ and $\frac{5}{7}$.

Then, $\left(\frac{1}{2}\times\frac{5}{7}\right)=\frac{(1\times 5)}{(2\times 7)}=\frac{5}{14}$, which is a rational number.

Property 2. Commutative Law:

Two rational numbers can be multiplied in any order.

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have:

$\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$.

Example:

Let us consider the rational numbers $\frac{3}{4}$ and $\frac{5}{7}$. Then,

$\left(\frac{3}{4}\times\frac{5}{7}\right)=\frac{(3\times 5)}{(4\times 7)}=\frac{15}{28}$ and $\left(\frac{5}{7}\times\frac{3}{4}\right)=\frac{(5\times 3)}{(7\times 4)}=\frac{15}{28}$.

$\therefore\left(\frac{3}{4}\times\frac{5}{7}\right)=\left(\frac{5}{7}\times\frac{3}{4}\right)$.

Property 3. Associative Law:

While multiplying three rational numbers, they can be grouped in any order.

Thus, for any rationals $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ we have

$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$

Example:

Consider the rationals $\frac{-5}{2},\frac{-7}{4}$ and $\frac{1}{3}$ We have

$\left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\left\{\frac{(-5)\times(-7)}{2\times 4}\times\frac{1}{3}\right\}$

$=\left(\frac{35}{8}\times\frac{1}{3}\right)=\frac{(35\times 1)}{(8\times 3)}=\frac{35}{24}$

and $\frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right)=\frac{-5}{2}\times\frac{(-7)\times 1}{4\times 3}$

$=\left(\frac{-5}{2}\times\frac{-7}{12}\right)=\frac{(-5)\times(-7)}{(2\times 12)}=\frac{35}{24}$

$\therefore\left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right)$.

Property 4. Existence of Multiplicative Identity:

For any rational number $\frac{a}{b}$, we have

$$\left(\frac{a}{b}\times 1\right)=\left(1\times\frac{a}{b}\right)=\frac{a}{b}.$$

1 is called the multiplicative identity for rationals.

Example:

Consider the rational number $\frac{3}{4}$. Then we have

$$\left(\frac{3}{4}\times 1\right)=\left(\frac{3}{4}\times\frac{1}{1}\right)=\frac{(3\times 1)}{(4\times 1)}=\frac{3}{4}$$

and $\left(1\times\frac{3}{4}\right)=\left(\frac{1}{1}\times\frac{3}{4}\right)=\frac{(1\times 3)}{(1\times 4)}=\frac{3}{4}.$

$$\therefore\left(\frac{3}{4}\times 1\right)=\left(1\times\frac{3}{4}\right)=\frac{3}{4}.$$

Property 5. Existence of Multiplicative Inverse:

Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1$

$\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$. Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (–1) is (–1).

Example:

Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1$

Property 6. Distributive Law of Multiplication Over Addition:

For any three rational numbers $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ and we have: $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right).$

Example:

Consider the rational numbers $\frac{-3}{4},\frac{2}{3}$ and $\frac{-5}{6}$.

We have

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\right)\times\left\{\frac{4+(-5)}{6}\right\}$$

$$=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{6}\right)=\frac{(-3)\times(-1)}{4\times 6}=\frac{3}{24}=\frac{1}{8}$$

Again, $\left(\frac{-3}{4}\right)\times\frac{2}{3}=\frac{(-3)\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$

and $\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)=\frac{(-3)\times(-5)}{4\times 6}=\frac{15}{24}=\frac{5}{8}.$

$$\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}.$$

$$=\left(\frac{-1}{2}+\frac{5}{8}\right)=\frac{(-4+5)}{8}=\frac{1}{8}$$

Hence,

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}.$$

Property 7. Multiplicative Property of 0:

Every rational multiplied with 0 gives 0.

Thus, for any rational number $\frac{a}{b}$, we have:

$$\left(\frac{a}{b}\times 0\right)=\left(0\times\frac{a}{b}\right)=0.$$

Example: $\left(\frac{5}{18}\times 0\right)=\left(\frac{5}{18}\times\frac{0}{1}\right)=\frac{(5\times 0)}{(18\times 1)}=\frac{0}{18}=0.$.

Similarly, $\left(0\times\frac{5}{18}\right)=0$

$$\therefore\left(\frac{5}{18}\times 0\right)=\left(0\times\frac{5}{18}\right)=0.$$

Example 16:

Find the reciprocal of each of the following:

(i) –8 (ii) $\frac{5}{16}$

Solution:

(i) Reciprocal of –8 is $\frac{1}{-8}$, i.e., $\frac{-1}{8}$

(ii) Reciprocal of $\frac{5}{16}$ is $\frac{16}{5}$

Division of Rational Numbers

When $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is called dividend; $\frac{c}{d}$ is called the divisor and the result is known as quotient.

Properties of division of rational numbers

Property 1. Closure Property:

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers such that $\frac{c}{d} \neq 0$, then $\left(\frac{a}{b} \div \frac{c}{d}\right)$ is also a rational number.

Property 2. For every rational number $\frac{a}{b}$, we have:

$$\left(\frac{a}{b} \div 1\right) = \frac{a}{b}$$

Property 3. For every non-zero rational number $\frac{a}{b}$, we have $\left(\frac{a}{b} \div \frac{a}{b}\right) = 1$

Example 17: Divide $\frac{4}{7}$ by $\frac{-3}{8}$

Solution: $\frac{4}{7} \div \frac{-3}{8} = \frac{4}{7} \times \frac{-8}{3} = \frac{-32}{21}$

FUNDAMENTAL UNLOCKED- (FU#2)

Q.1 Does closure property is applicable on subtraction of two rational numbers?

Q.2 The division of two rational numbers is always a rational number. True or False? Justify your answer.

Q.3 Find the additive inverse of $\frac{3}{7} \times \frac{-5}{8}$.

Rational Numbers Between Two Rational Numbers

If a and b be two rational number such that a < b, then $\frac{1}{2}(a+b)$ is a rational number between a and b.

Example 18:

Find 3 rational numbers between $\frac{1}{3}$ & $\frac{1}{2}$.

Solution:

A rational number between

$$\frac{1}{3} \& \frac{1}{2} = \frac{\frac{1}{3} + \frac{1}{2}}{2} = \frac{\frac{2+3}{6}}{2} = \frac{5}{12}\left(\therefore \frac{1}{3}, \frac{5}{12}, \frac{1}{2}\right)$$

A rational number between

$$\frac{1}{3} \text{ and } \frac{5}{12} = \frac{\frac{1}{3} + \frac{5}{12}}{2} = \frac{\frac{4+5}{12}}{2} = \frac{9}{24}.$$

A rational number between

$$\frac{5}{12} \text{ and } \frac{1}{2} = \frac{\frac{5}{12} + \frac{1}{2}}{2} = \frac{\frac{5}{12} + \frac{6}{12}}{2} = \frac{11}{24}$$

$$\left(\therefore \frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{11}{24}, \frac{1}{2}\right)$$

$\therefore$ Three rational number between $\frac{1}{3}$ & $\frac{1}{2}$ are $\frac{5}{12}, \frac{9}{24}, \frac{11}{24}$.

Example 19:

Find 5 rational numbers between $\frac{-3}{5}$ and $\frac{1}{4}$.

Solution:

Convert to equivalent rational numbers having same denominators

$$\frac{-3}{5} = \frac{-3 \times 4}{5 \times 4} = \frac{-12}{20} \text{ and } \frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

The integer between –12 and 5 are – 11, –10, –9, ……, 3, 4.

The corresponding rational number are $\frac{-11}{20}, \frac{-10}{20}, \frac{-9}{20}, \ldots\ldots, \frac{2}{20}, \frac{3}{20}, \frac{4}{20}$.

Selecting any five of them, we get $\frac{-11}{20}, \frac{-10}{20}, \frac{-9}{20}, \frac{-8}{20}, \frac{-7}{20}$ are five rational numbers between $\frac{-3}{5}$ and $\frac{1}{4}$.

FUNDAMENTAL UNLOCKED- (FU#3)

Q.1 What should be subtracted from $\frac{-5}{9}$ so as to get $\frac{6}{7}$?

Q.2 The product of two rational number is $\frac{-64}{81}$. If one of the numbers is $\frac{8}{9}$, find the other.

Q.3 Divide the sum of $\frac{-7}{6}$ and $\frac{4}{5}$ by their product.

Q.4 Write any 6 rational numbers between $\frac{1}{6}$ and $\frac{5}{9}$

ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1)

Q.1 Rational number **Q.2** Yes, 0.075 **Q.3** (a)$\frac{10}{12}$;(b)$\frac{12}{16}$

FUNDAMENTAL UNLOCKED- (FU#2)

Q.1 Yes **Q.2** False **Q.3** $\frac{15}{56}$

FUNDAMENTAL UNLOCKED- (FU#3)

Q.1 $\frac{-89}{63}$ **Q.2** $\frac{-8}{9}$ **Q.3** $\frac{11}{28}$

Q.4 $\frac{4}{18},\frac{5}{18},\frac{6}{18},\frac{7}{18},\frac{8}{18},\frac{9}{18}$

EXERCISE - I

Single Correct Type Questions

1. The reciprocal of a negative rational number:
(A) is a positive rational number
(B) is a negative rational number
(C) can be either a positive or a negative rational number
(D) does not exist

2. |-138|-|-243|=?
(A) 105 (B) 381 (C) -381 (D) -105

3. Multiplicative inverse of $\frac{3}{5}$ is:
(A) 1 (B) 0 (C) $\frac{-3}{5}$ (D) $\frac{5}{3}$

4. If $\frac{x}{y}=\frac{6}{5}$ then $\frac{x^2+y^2}{x^2-y^2}$ is:
(A) $\frac{36}{25}$ (B) $\frac{25}{36}$
(C) $\frac{11}{61}$ (D) $\frac{61}{11}$

5. How many rational numbers exist between any two distinct rational numbers?
(A) 2 (B) 3
(C) 11 (D) Infinite

6. The product of a non - zero rational number with an irrational number is:
(A) Irrational number (B) Rational number
(C) Whole number (D) Natural number

7. If $\frac{3}{11}$ of a number is 22, what is $\frac{6}{11}$ of that number?
(A) 6 (B) 11
(C) 12 (D) 44

8. Which of the following is (are) greater than x when $x=\frac{9}{11}$?
I. $\frac{1}{x}$ II. $\frac{x+1}{x}$ III. $\frac{x+1}{x-1}$
(A) I only (B) I and II only
(C) I and III only (D) II and III only

9. Arrange the following fractions in ascending order $\frac{3}{7},\frac{4}{5},\frac{7}{9},\frac{1}{2}$.
(A) $\frac{4}{5},\frac{7}{9},\frac{3}{7},\frac{1}{2}$ (B) $\frac{3}{7},\frac{1}{2},\frac{7}{9},\frac{4}{5}$
(C) $\frac{4}{5},\frac{7}{9},\frac{1}{2},\frac{3}{7}$ (D) $\frac{1}{2},\frac{3}{7},\frac{7}{9},\frac{4}{5}$

10. What number should be subtracted from –5 to get $\frac{8}{9}$.
(A) $\frac{-53}{9}$ (B) $\frac{37}{9}$ (C) $\frac{9}{37}$ (D) $\frac{-9}{37}$

11. Which of the following statements is true?
(A) $\frac{10}{14}<\frac{14}{18}<\frac{18}{22}<\frac{22}{26}$
(B) $\frac{22}{26}<\frac{18}{22}<\frac{14}{18}<\frac{10}{14}$
(C) $\frac{10}{14}<\frac{22}{26}<\frac{14}{18}<\frac{18}{22}$
(D) $\frac{10}{14}<\frac{18}{22}<\frac{22}{26}<\frac{14}{18}$

12. A rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ is:
(A) $\frac{4}{24}$ (B) $-0.208\overline{3}$
(C) $-\frac{5}{24}$ (D) All of these

13. If $2=x+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$, then the value of x is:
(A) $\frac{12}{17}$ (B) $\frac{13}{17}$ (C) $\frac{18}{17}$ (D) $\frac{21}{17}$

14. If R: every fraction is rational number and T: Every rational number is a fraction, then which of the following is correct?
(A) R is True and T is False
(B) R is False and T is true
(C) Both R and T are True
(D) Both R and T are false

16. Which of the following rational numbers is in the standard form?

(A) $\frac{6}{-72}$ (B) $\frac{-19}{37}$

(C) $\frac{3}{-4}$ (D) None of these

17. If $\frac{-4}{x}=\frac{x}{25}$, then the value of 'x' is:

(A) a rational number
(B) not a rational number
(C) an integer
(D) a natural number

18. The given rational number are $\frac{1}{2},\frac{4}{-5},\frac{-7}{8}$. If these numbers are arranged in the ascending order or descending order, then the middle number is:

(A) $\frac{1}{2}$ (B) $\frac{-7}{8}$

(C) $\frac{4}{-5}$ (D) None of these

19. The average of the middle two rational numbers, if $\frac{4}{7},\frac{1}{3},\frac{2}{5},\frac{5}{9}$ are arranged in ascending order is

(A) $\frac{86}{90}$ (B) $\frac{86}{45}$

(C) $\frac{43}{45}$ (D) $\frac{43}{90}$

20. If P: The quotient of two integers is always a rational numbers and Q: $\frac{4}{0}$ is not rational, then which of the following statement is true?

(A) P is true and Q is correct explanation of P
(B) P is false and Q is the correct explanation of P
(C) P is true and Q is false
(D) Both P and Q are false.

Very Short Answer Type Questions

1. Write the smallest prime number.

2. Write the smallest composite number.

3. Find the sum of even prime number and smallest composite number.

4. Represent $\frac{15}{7}$ and $\frac{-15}{7}$ on the number line.

5. Find the additive inverse of the following:

(a) $\frac{2}{5}$ (b) 6

(c) 0 (d) $\frac{7}{-9}$

6. Find the multiplicative inverse of the following:

(a) $\frac{3}{7}$ (b) –9

(c) 0 (d) $\frac{13}{-2}$

7. Find the sum of the square of additive inverse and multiplicative inverse of $-\frac{1}{3}$.

8. What number should be subtracted from –1 to get $\frac{1}{3}$?

9. Find the lowest form of $\frac{-219}{365}$.

10. If $\frac{2}{11}$ of a number is 22, what is $\frac{4}{11}$ of the number?

Short Answer Type Questions

1. Arrange the following rational number in ascending order: $\frac{1}{2},\frac{1}{3},\frac{5}{3},\frac{23}{25},\frac{5}{7}$.

2. If the additive inverse and multiplicative inverse of $\left(-\frac{1}{2}\right)$ is x and y, then find the value of $\frac{x+y}{1+xy}$

3. The product of two rational numbers is $\frac{-7}{9}$, if one of the number is $\frac{4}{3}$, find the other.

4. Find the number divided by $\frac{7}{9}$ gives 2.

5. Represent –17.5 on the number line.

6. Give three rational numbers between 3 and 4.

7. Find three different rational numbers between $\frac{5}{7}$ and $\frac{9}{11}$.

8. Answer the following questions:
(i) Which rational number is its own additive inverse?
(ii) Is the difference of two rational numbers a rational number?
(iii) What is the negative of negative rational number?

9. Find the five rational numbers between $\frac{2}{7}$ and $\frac{5}{13}$

10. Find the value of $\frac{x^2+y^2}{x^2-y^2}$ if $\frac{x}{y}=\frac{5}{4}$

Long Answer Type Questions

1. (i) Verify that $|x+y|\le|x|+|y|$ by taking $x=\frac{-5}{12}, y=\frac{-7}{18}$.
(ii) Verify that $|x\times y|=|x|\times|y|$ by taking $x=\frac{-2}{3}, y=\frac{-9}{8}$.

2. The cost of $3\frac{7}{9}$ m cloth is Rs.$212\frac{4}{5}$. Find the cost of $7\frac{2}{3}$ m cloth.

3. Simplify: $2\frac{3}{4}\times1\frac{2}{3}+9\frac{11}{12}-1\frac{5}{6}$.

4. Simplify: $5-\left[\frac{3}{4}+\left\{2\frac{1}{2}-\left(0.5+\overline{\frac{1}{6}-\frac{1}{7}}\right)\right\}\right]$

5. Find the product of $\left[\frac{46}{56}\times\frac{3}{\left(\frac{4}{7}\right)}+\frac{6}{16}\times\frac{-512}{81}\right]$ and the reciprocal of $\left(\frac{35}{21}+\frac{7}{9}-\frac{24}{16}\right)$.

6. Given $x=\frac{2}{5}, y=\frac{-7}{6}$ and $z=\frac{-1}{4}$, then verify that $x-(y-z)\neq(x-y)-z$.

7. If the additive inverse of $-\frac{1}{2}$ is A and the multiplicative inverse is M, find the value of
$$AM\left[(A-M)\left\{\frac{(A+M)\left(A^2-AM+M^2\right)}{\left(A^2+M^2\right)}\right\}\right]$$

8. Ramesh, Suman, Suraj, Varun and Sandip went to a restaurant for dinner. Ramesh paid $\frac{1}{3}$ of the bill, Suman paid $\frac{1}{5}$ of the bill and rest of the bill was shared equally by Suraj, Varun and Sandip. What fraction of the bill was paid by Sandip?

Case Based Question

1. Vijay who is working in a multinational company earns Rs.150000 per month. Out of his earnings he spend $\frac{1}{10}^{th}$ on food items, $\frac{1}{4}^{th}$ on shopping with family, $\frac{1}{5}^{th}$ of remaining on education of his two kids and rest of money he puts on his savings.
Based on the above information, answer the following questions:

(I) How much money did he spend on the food items?

(II) How much money did he spend on the shopping?

(III) Calculate the amount spend by Vijay on education of children.

(IV) How much money did he save?

2. Three people A, B and C started walking. A walked to a distance of $\frac{11}{3}$ km, B walked to a distance of $\frac{13}{4}$ km and C walked to a distance of $\frac{23}{6}$ km.

(I) Who covers the maximum distance?

(A) A (B) B (C) C (D) None

(II) Who covers the minimum distance?

(A) A (B) B (C) C (D) None

(III) What is the maximum distance between any two people?

(A) $\frac{5}{12}$ km (B) $\frac{12}{7}$ km

(C) $\frac{14}{7}$ km (D) $\frac{7}{12}$ km

(IV) What is the minimum distance between any two people?

(A) $\frac{5}{7}$ km (B) $\frac{1}{6}$ km

(C) $\frac{3}{7}$ km (D) $\frac{8}{5}$ km

(V) Total distance covered by all three of them is

(A) $\frac{129}{12}$ km (B) $\frac{131}{7}$ km

(C) $\frac{156}{12}$ km (D) $\frac{129}{7}$ km

EXERCISE - II

HOTS

1. Let p, q and r be distinct integers where p and q are odd and positive, and r is even and positive. Which one of the following statements cannot be true?

(A) $(p-r)^2q$ is even

(B) $(p-r)q^2$ is odd

(C) $(p-r)q$ is odd

(D) $(p-q)^2r$ is even

2. Choose the rational number which does not lie between -3 and -2.

(A) $\frac{-5}{2}$ (B) $\frac{-7}{3}$ (C) $\frac{-5}{3}$ (D) $\frac{-8}{3}$

3. If $y=\frac{p+1}{p-1}, x=\frac{p+2}{p-1}$ and $\frac{y+x}{y-x}=7$. Find the value of $\frac{24\left(y^2+x^2\right)}{pxy}$.

(A) –8 (B) 10 (C) –10 (D) –12

4. Given that the smallest odd prime is x, smallest composite number is y, even prime number is z, smallest prime is p and q is the largest four digit number divisible by 3. Then the number $\frac{2xyq}{zp}$ is divisible by

(A) 11 (B) 9

(C) 18 (D) All of these

5. Which of the following rational numbers $\frac{1}{3}, \frac{5}{8}, \frac{23}{24}, \frac{5}{6}$ is having the maximum absolute value of their additive inverse?

(A) $\frac{1}{3}$ (B) $\frac{5}{8}$ (C) $\frac{23}{24}$ (D) $\frac{5}{6}$

6. $\left[\frac{9}{4}\times\frac{3}{5}\div\frac{12}{5}+\frac{7}{8}\div\frac{5}{4}+\frac{3}{5}\right]$ is equal to

(A) $1\frac{69}{80}$ (B) $1\frac{41}{80}$ (C) $2\frac{2}{9}$ (D) $20\frac{7}{9}$

7. The product $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\ldots\left(2-\frac{97}{99}\right)$ is equal to:

(A) $\frac{5}{99}$ (B) $\frac{101}{99}$ (C) $\frac{101}{3}$ (D) $\frac{97}{99}$

8. The product of the following fractions

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}\times\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{5}-\frac{1}{6}}\times\frac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{7}-\frac{1}{8}}\times\ldots\ldots\times\times\frac{\frac{1}{98}-\frac{1}{99}}{\frac{1}{99}-\frac{1}{100}}$$, is:

(A) 2 (B) 50 (C) 100 (D) $\frac{1}{100}$

9. 0.018 can be expressed in the rational form as:

(A) $\frac{18}{1000}$ (B) $\frac{18}{990}$

(C) $\frac{18}{9900}$ (D) $\frac{18}{999}$

10. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is:

(A) 364724

(B) 365387

(C) 365737

(D) 366757

11. The number 3.46464646 ….. can be written as a fraction, when reduced to the lowest term, the sum of numerator and denominator is:

(A) 5 (B) 39 (C) 442 (D) 241

12. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then which of the following rational number lies exactly between $\frac{a}{b}$ and $\frac{c}{d}$?

(A) $\frac{a\times b}{c\times d}$ (B) $\frac{ac+bd}{2ab}$

(C) $\frac{ab+cd}{2bc}$ (D) $\frac{ad+bc}{2bd}$

13. Find the sum of numerator and denominator of a number expressed in the lowest form if $\frac{4}{7}$ of that number exceeds its $\frac{2}{5}$ by 2.

(A) $\frac{35}{3}$ (B) 35

(C) 38 (D) None of these

14. In $\frac{5}{6}$ of his field, a farmer grows tomatoes, in $\frac{1}{9}$ he grows onions and in the rest of the field he grows potatoes. In what part of the field does he grow potatoes?

(A) $\frac{17}{18}$ (B) $\frac{1}{18}$

(C) $\frac{3}{4}$ (D) $\frac{1}{6}$

15. A rod of length 25.5 m is broken into equal parts of length 85 cm. Find the number of parts into which the rod is broken.

(A) 30 (B) 20 (C) 3 (D) 130

Assertion and Reason

1. **Assertion:** Zero is a rational number.

Reason: Each rational number is a quotient of any two integers, while its divisor should not be zero. Thus, a number of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is a rational number.

(A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(C) Assertion is correct but Reason is incorrect.

(D) Assertion is incorrect but Reason is correct.

2. **Assertion:** 5 is a rational number.

Reason: The square roots of all positive integers are irrational.

(A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(C) Assertion is correct but Reason is incorrect.

(D) Assertion is incorrect but Reason is correct.

3. **Assertion:** Rational number lying between two rational numbers x and y is $\frac{1}{2}(x+y)$.

Reason: There is one rational number lying between any two rational numbers.

(A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(C) Assertion is true but Reason is false.

(D) Assertion is false but Reason is true.

4. **Assertion:** Natural numbers are closed under subtraction

Reason: A rational number is a number that is in the form of p/q, where p and q are integers, and q is not equal to 0.

(A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(C) Assertion is correct but Reason is incorrect.

(D) Assertion is incorrect but Reason is correct.

5. **Assertion:** Whole numbers are not closed under subtraction

Reason: A rational number is a number that is in the form of p/q, where p and q are integers, and q is not equal to 0.

(A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(C) Assertion is correct but Reason is incorrect.

(D) Assertion is incorrect but Reason is correct.

Numerical Type Questions

1. What should be subtracted from $-\frac{7}{8}$ so as to get $-\frac{47}{8}$

2. If

$$x=\left(\left(\frac{13}{9}\right)\times\left(\frac{-15}{2}\right)\right)+\left(\left(\frac{7}{3}\right)\times\left(\frac{8}{5}\right)+\left(\frac{3}{5}\right)\times\left(\frac{1}{2}\right)\right)$$

then find the value of $\frac{-30}{51}x$.

3. If $x=\frac{3}{7}, y=\frac{12}{13}$ and $z=\frac{-5}{6}$ then find the value of $130x(y+z)$.

4. The product of two rational number is $\frac{-8}{9}$. If one of the numbers is $\frac{-4}{15}$ and the other is p. Find the value of 0.9 p.

5. The cost of $7\frac{2}{3}$ meters of rope is Rs. $12\frac{3}{4}$. Find the cost of 92 meter rope.

6. If $x=\frac{10}{3}+\frac{5}{6}-\frac{5}{12}$, then find the value of $\frac{144}{9}x$.

7. If $p=\left\{\frac{6}{5}\times\frac{-3}{5}\right\}+\left\{\frac{7}{15}\times\frac{6}{5}\right\}$ and $q=\left\{\frac{9}{16}\times\frac{4}{15}\right\}+\left\{\frac{9}{4}\times\frac{-1}{15}\right\}$ then find $\frac{3p+q}{p+3q}$.

8. If $x=\frac{1}{6}$, find the value of

$$35\left\{\frac{\frac{(1+x)^2}{(1-x)^2}+\frac{(1+x)^4}{(1-x)^4}}{\frac{(1+x)^3}{(1-x)^3}}\right\}.$$

9. Reciprocal of $-\frac{1}{6},\frac{1}{8},\frac{1}{2},2$ is given as x_1, x_2, x_3 and x_4, respectively. If $\left[\left(\frac{x_1^2+x_3^2-x_2^2}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}}\right)(x_1x_2x_3x_4)\right]$ is equal to $\frac{1152D}{d}$ where D, d are co-prime. Find the value of (D + d).

10. The product of $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{10}\right)$ is equal to $\frac{p}{q}$, where p and q are coprime. Find p^2+q^2.

ANSWER KEY

EXERCISE - I

Single Correct Type Questions

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	D	D	D	A	D	B	B	A	A	D	D	B	A
Que.	16	17	18	19	20										
Ans.	B	B	C	D	B										

Very Short Answer Type Questions

1. 2 **2.** 4 **3.** 6 **5.** **(a)** $\frac{-2}{5}$ **(b)** –6 **(c)** 0 **(d)** $\frac{7}{9}$

6. **(a)** $\frac{7}{3}$ **(b)** $\frac{-1}{9}$ **(c)** Not defined **(d)** $\frac{-2}{13}$ **7.** $\frac{82}{9}$ **8.** $\frac{-4}{3}$ **9.** $\frac{-3}{5}$

10. 44

Short Answer Type Questions

1. $\frac{1}{3},\frac{1}{2},\frac{5}{7},\frac{23}{25},\frac{5}{3}$ **2.** Not defined **3.** $\frac{-7}{12}$ **4.** $\frac{14}{9}$

6. $\frac{13}{4},\frac{14}{4},\frac{15}{4}$ **7.** $\frac{56}{77},\frac{57}{77},\frac{58}{77}$ **8.** **(i)** 0, **(ii)** yes, **(iii)** positive rational number

9. $\frac{27}{91},\frac{28}{91},\frac{29}{91},\frac{30}{91},\frac{31}{91}$ **10.** $\frac{41}{9}$

Long Answer Type Questions

2. Rs.$431\frac{73}{85}$ **3.** $12\frac{2}{3}$ **4.** $2\frac{23}{84}$ **5.** $\frac{839}{408}$ **7.** $\frac{315}{68}$ **8.** $\frac{7}{45}$

Case Based Question

Case-1					
Que.	1	2	3	4	
Ans.	15000	37500	19500	78000	
Case-2					
Que.	1	2	3	4	5
Ans.	C	B	D	B	A

EXERCISE - II

HOTS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	C	D	C	A	C	B	A	C	C	D	C	B	A

Assertion and Reason

Que.	1	2	3	4	5
Ans.	A	C	C	D	B

Numerical Type Questions

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	5	4	5	3	153	60	3	74	83	26