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MOTION IN A PLANE

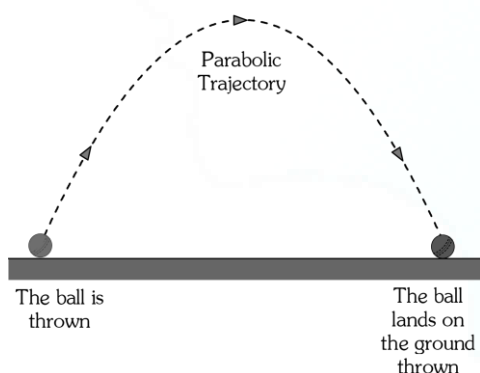
Particle Kinematics

Projectile Motion

An object projected by an external force when continues to move by its own inertia is known as projectile and its motion as projectile motion.

A football kicked by a player, an arrow shot by an archer, water sprinkling out a water-fountain, an athlete in long jump or high jump, a bullet or an artillery shell fired from a gun are some examples of projectile motion.

In simplest case when a projectile does reach great heights above the ground as well as does not cover a very large distance on the ground, acceleration due to gravity can be assumed uniform throughout its motion. Moreover, such a projectile does not spend much time in air not permitting the wind and air resistance to gather appreciable effects. Therefore, while analyzing them, we can assume gravity to be uniform and neglect effects of wind as well as air resistance. Under these circumstances when an object is thrown in a direction other than the vertical, its trajectory assumes shape of a parabola. In the figure, a ball thrown to follow a parabolic trajectory is shown as an example of projectile motion.



At present, we study projectiles moving on parabolic trajectories and by the term projectile motion; we usually refer to this kind of motion.

For a projectile to move on parabolic trajectory, the following conditions must be fulfilled.

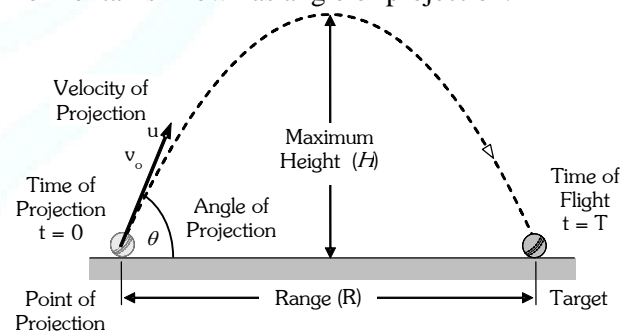
- Acceleration vector must be uniform.
- Velocity vector never coincides with line of acceleration vector.

Analyzing Projectile Motion

Since parabola is a plane curve, projectile motion on parabolic trajectory becomes an example of a two-dimensional motion. It can be conceived as superposition of two simultaneous rectilinear motions in two mutually perpendicular directions, which can be analyzed separately as two Cartesian components of the projectile motion.

Projectile Motion near the Horizontal or Flat Ground using Cartesian Components

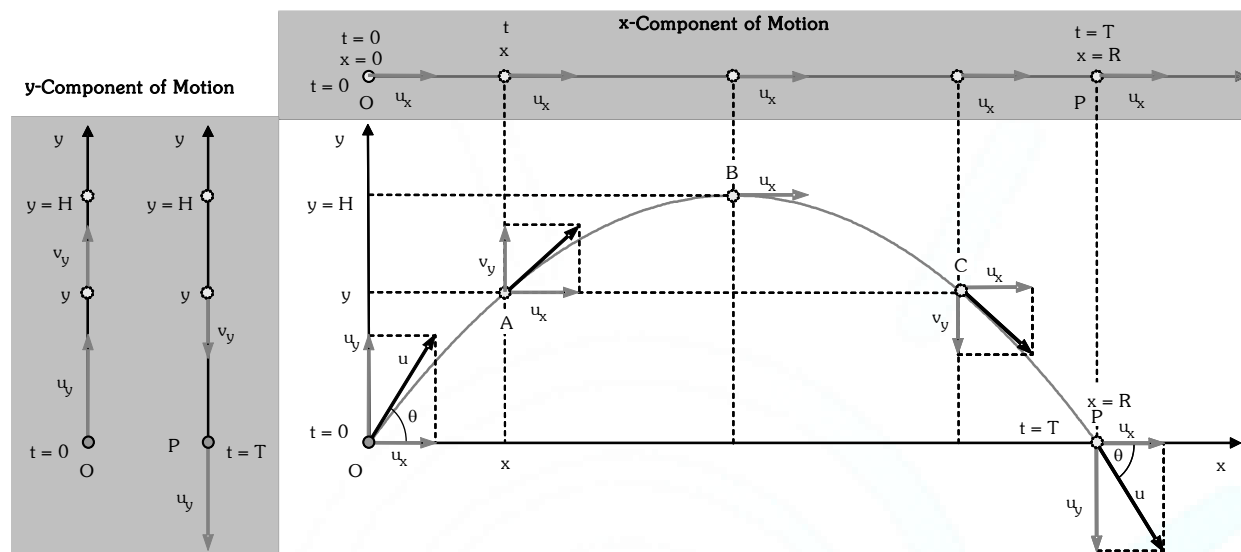
Consider motion of a ball thrown from ground as shown in the figure. The point from where it is projected is known as point of projection, the point where it falls on the ground is known as point of landing or target. The distance between these two points is known as horizontal range or range, the height from the ground of the highest point it reaches during flight is known as maximum height and the duration for which it remain in the air is known as air time or time of flight. The velocity with which it is thrown is known as velocity of projection and angle which velocity of projection makes with the horizontal is known as angle of projection.



A careful observation of this motion reveals that when a ball is thrown its vertical component of velocity decreases in its upward motion, vanishes at the highest point and thereafter increases in its downward motion due to gravity similar to motion of a ball thrown vertically upwards. At the same time, the ball continues to move uniformly in horizontal direction due to inertia. The actual projectile motion on its parabolic trajectory is superposition of these two simultaneous rectilinear motions.



In the following figure, the above ideas are shown representing the vertical by y-axis and the horizontal by x-axis.



Projectile motion resolved into its two Cartesian components.

Projectile motion as superposition of two rectilinear motions one in vertical and other in horizontal direction.

Vertical or y-component of motion.

Component of initial velocity in the vertical direction is u_y . Since forces other than gravitational pull of the earth are negligible, vertical component of acceleration a_y of the ball is g vertically downwards. This component of motion is described by the following three equations. Here v_y denotes y-component of velocity, y denotes position coordinate y at any instant t .

$$v_y = u_y - gt \quad \dots(i) \quad y = u_y t - \frac{1}{2}gt^2 \quad \dots(ii)$$

$$v_y^2 = u_y^2 - 2gy \quad \dots(iii)$$

Horizontal or x-component of motion.

Since effects of wind and air resistance are assumed negligible as compared to effect of gravity, the horizontal component of acceleration of the ball becomes zero and the ball moves with uniform horizontal component of velocity u_x . This component of motion is described by the following equation.

$$x = u_x t \quad \dots(iv)$$

Equation of Trajectory

Equation of the trajectory is relation between the x and the y coordinates of the ball without involvement of time t . To eliminate t , we substitute its expression from equation (iv) into equation (ii).

$$y = x \tan \theta - \frac{g}{2u^2 \cos^2 \theta} x^2 \quad \dots(v)$$

Every projectile motion can be analyzed using the above five equations. In a special case of interest, if the projectile lands the ground again, its time of flight, the maximum height reached and horizontal range are obtained using the above equations.

Time of Flight

At the highest point of trajectory when $t = \frac{1}{2}T$, the vertical component of velocity becomes zero. At the instant $t = T$, the ball strikes the ground with vertical component of velocity $v_y = -u_y$. By substituting either of these conditions in equation (i), we obtain the time of flight.

$$T = \frac{2u_y}{a_y} = \frac{2u_y}{g}$$

Maximum Height

At the highest point of trajectory where $y = H$, the vertical component of velocity becomes zero. By substituting this information in equation (iii), we

obtain the maximum height. $H = \frac{u_y^2}{2g}$



Horizontal Range

The horizontal range or simply the range of the projectile motion of the ball is distance traveled on the ground in its whole time of flight.

$$R = u_x T = \frac{2u_x u_y}{g} = \frac{u^2 \sin 2\theta}{g}$$

Maximum Range

It is the maximum distance traveled by a projectile in the horizontal direction for a certain velocity of projection.

The above expression of range makes obvious that to obtain maximum range the ball must be projected at angle $\theta = 45^\circ$.

Substituting this condition in the expression of range, we obtain the maximum range R_m .

$$R_m = \frac{u^2}{g}$$

Trajectory Equation

If range is known in advance, the equation of trajectory can be written in an alternative form involving horizontal range.

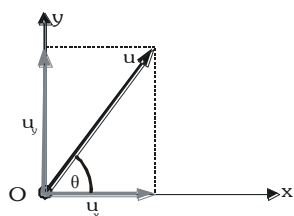
$$y = x \tan \theta \left(1 - \frac{x}{R} \right)$$

Example:

A ball is thrown with 25 m/s at an angle 53° above the horizontal. Find its time of flight, maximum height and range.

Solution:

In the adjoining figure velocity of projection $u = 25$ m/s, angle of projection $\theta = 53^\circ$, the horizontal and vertical components u_x and u_y of velocity of projection are shown. From these information we have $u_x = u \cos 53^\circ = 15$ m/s and $u_y = u \sin 53^\circ = 20$ m/s



Using equations for time of flight T , maximum height H and range R , we have

$$T = \frac{2u_y}{g} = \frac{2 \times 20}{10} = 4 \text{ s}$$

$$H = \frac{u_y^2}{2g} = \frac{20^2}{2 \times 10} = 20 \text{ m}$$

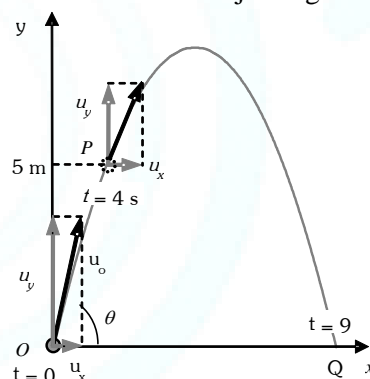
$$R = \frac{2u_x u_y}{g} = \frac{2 \times 15 \times 20}{10} = 60 \text{ m}$$

Example:

A ball 4 s after the instant it was thrown from the ground passes through a point P, and strikes the ground after 5 s from the instant it passes through the point P. Assuming acceleration due to gravity to be 9.8 m/s^2 find height of the point P above the ground.

Solution:

The ball projected with velocity $\vec{u} = u_x \hat{i} + u_y \hat{j}$ from O reaches the point P with velocity $\vec{v} = v_x \hat{i} + v_y \hat{j}$ and hits the ground at point Q at the instant $T = 4 + 5 = 9$ s as shown in the adjoining motion diagram.



From equation of time of flight, we have its initial y-component of velocity u_y

$$T = \frac{2u_y}{g} \rightarrow u_y = \frac{1}{2} gT$$

Substituting above in eq. (ii) and rearranging terms, we have the height y of the point P.

$$y = u_y t - \frac{1}{2} g t^2 \rightarrow y = \frac{1}{2} g t (T - t) = \frac{1}{2} \times 9.8 \times 4 (9 - 4) = 98 \text{ m}$$

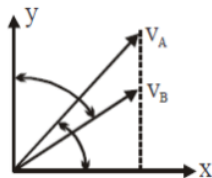
FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 A body is thrown from a point with speed 50 m/s at an angle 37° with horizontal. When it has moved a horizontal distance of 80 m then it's distance from point of projection -

- (A) 40 m (B) $40\sqrt{2}$
(C) $40\sqrt{5}$ (D) none of these



- Q.2** Two projectiles are projected with velocity v_A, v_B at angles θ_A (from horizontal) and θ_B (from vertical) as shown in the figure below, such that $v_A > v_B$ but having same horizontal component of velocity. Which of the following is correct?

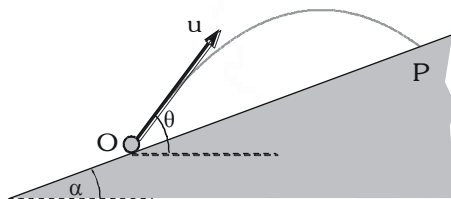


- (A) $T_A > T_B$ (B) $H_A > H_B$
(C) $R_A > R_B$ (D) $R_B > R_A$

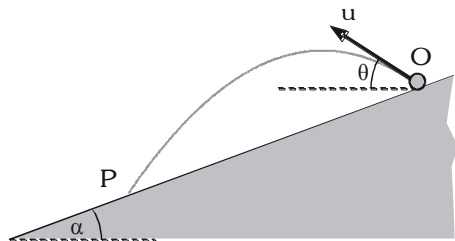
- Q.3** A particle is projected with speed 20m/s at an angle 30° with horizontal. After how much time the angle between velocity and acceleration will be 90° –
(A) 1 sec (B) 2 sec
(C) 1.5 sec (D) Never

Projectile on Inclined Plane

Artillery application often finds target either up a hill or down a hill. These situations can approximately be modeled as projectile motion up or down an inclined plane.



Projectile up an inclined plane



Projectile down an inclined plane

In the above left figure is shown a shell projected from a point O with velocity u at an angle θ to hit a target at point P uphill. This projectile motion is called projectile up a hill or inclined plane. Similarly, in the above right figure is shown a projectile down a hill or inclined plane.

Analyzing Projectile Motion up an Inclined Plane Using Cartesian Components

Consider the projectile motion up an inclined plane described earlier. Assume a Cartesian coordinate system whose x-axis coincides with the line of fire OP and the origin with the point of projection as shown.

Velocity of projection makes angle $\theta - \alpha$ with the positive x-axis, therefore its x and y-components u_x and u_y are

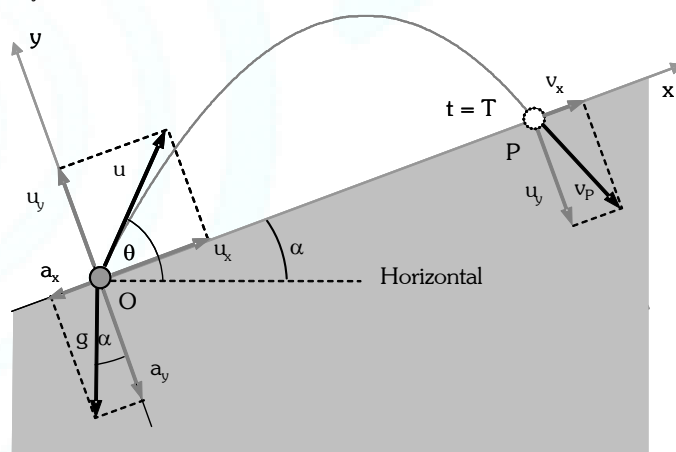
$$u_x = u \cos(\theta - \alpha)$$

$$u_y = u \sin(\theta - \alpha)$$

Acceleration due to gravity g being vertical makes the angle α with the negative y-axis, therefore x and y-components of acceleration vector are

$$a_x = g \sin \alpha$$

$$a_y = g \cos \alpha$$



Projectile motion up an inclined plane resolved into its two cartesian components.

Motion Component Along the y-axis

The projectile starts with initial y-component of velocity u_y in the positive y-direction and has uniform y-component of acceleration $a_y = g \cos \alpha$ in the negative y-direction. This component of motion is described by the following three equations. Here v_y denotes y-component of velocity, y denotes position coordinate y at any instant t .

$$v_y = u_y - a_y t \quad \dots(i)$$

$$y = u_y t - \frac{1}{2} a_y t^2 \quad \dots(ii)$$

$$v_y^2 = u_y^2 - 2a_y y \quad \dots(iii)$$



Motion Component Along the x-axis

The x-component of motion is also uniformly accelerated motion. The projectile starts with initial x-component of velocity u_x in the positive x-direction and has uniform x-component of acceleration a_x in the negative x-direction. This component of motion is described by the following three equations. Here v_x denotes x-component of velocity, x denotes position coordinate x at any instant t .

$$v_x = u_x - a_x t \quad \dots(iv)$$

$$x = u_x t - \frac{1}{2} a_x t^2 \quad \dots(v)$$

$$v_x^2 = u_x^2 - 2a_x x \quad \dots(vi)$$

Every projectile motion up an incline can be analyzed using the above six equations. Quantities of interest in artillery applications and hence in projectile on incline plane are time of flight, range on the incline plane and the angle at which the shell hits the target.

Time of Flight

Moving in air for time interval T the projectile when hits the target P , its y-component of velocity u_y becomes in the negative y-direction. Using this information in equation (i), we obtain the time of flight.

$$T = \frac{2u_y}{a_y} = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

When the projectile hits the target P , its y component of displacement also becomes zero. This information with equation (ii) also yield the time of flight.

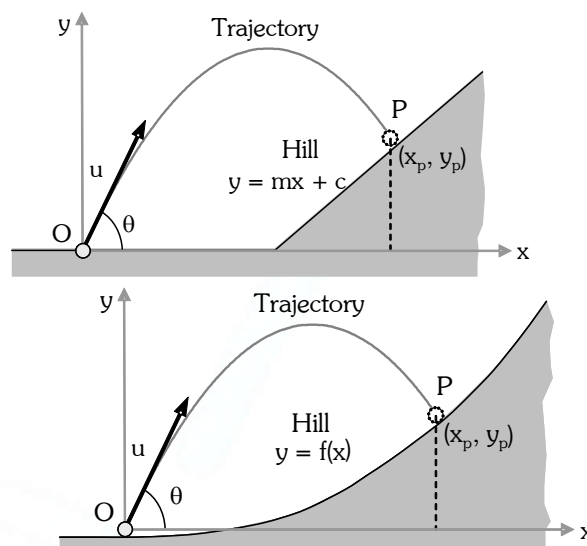
Range on The Plane

The range of a projectile on an incline plane is the distance between the point of projection and the target. It equals to displacement in the x-direction during whole flight. By substituting time of flight in equation (v), we obtain expression for the range R .

$$R = \frac{2u^2 \sin(\theta - \alpha) \cos \theta}{g \cos^2 \alpha}$$

Analysis of Projectile on an Incline Plane using Equation of Trajectory

Sometimes the hill may be away from the point of projection or the hill may not have uniform slope as shown in the following two figures.



In these cases, the shape of the hill can be expressed by a suitable equation of the form $y = mx + c$ for uniform slope hill or $y = f(x)$ for nonuniform slope hill. The target P where the projectile hits the hill is the intersection of trajectory of the projectile and the hill. Therefore, coordinates (x_p, y_p) of the target can be obtained by simultaneously solving equation of the hill and equation of trajectory of the projectile.

Time of Flight

Since a projectile move with uniform horizontal component of the velocity (u_x), its time of flight T can be calculated from the following equation.

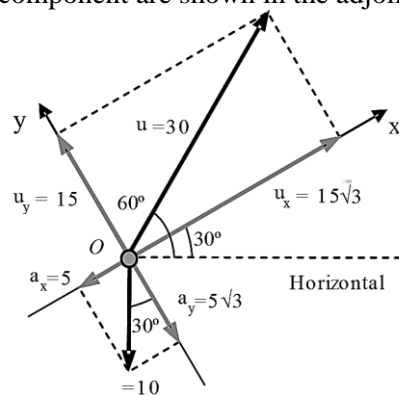
$$T = \frac{x_p}{u_x} = \frac{x_p}{u \cos \theta}$$

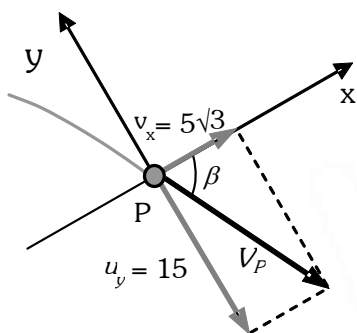
Example:

A particle is projected with a velocity of 30 m/s at an angle 60° above the horizontal on a slope of inclination 30° . Find its range, time of flight and angle of hit.

Solution:

The coordinate system, projection velocity and its component, and acceleration due to gravity and its component are shown in the adjoining figure.





Substituting corresponding values in following equation,

we get the time of flight.

$$T = \frac{2u_y}{a_y} \rightarrow T = \frac{2 \times 15}{5\sqrt{3}} = 2\sqrt{3} \text{ s}$$

Substituting value of time of flight in following equation, we get the range R.

$$R = u_x T - \frac{1}{2} a_x T^2$$

$$\Rightarrow R = 15\sqrt{3} \times 2\sqrt{3} - \frac{1}{2} \times 5 \times (2\sqrt{3})^2 = 60\text{m}$$

In the adjoining figure, components of velocity \vec{v}_p when the projectile hits the slope at point P are shown. The angle β which velocity vector makes with the x-axis is known as angle of hit. The projectile hits the slope with such a velocity \vec{v}_p , whose y-component is equal in magnitude to that of velocity of projection. The x-component of velocity v_x is calculated by substituting value of time of flight in following equation.

$$v_x = u_x - a_x t \rightarrow v_x = 15\sqrt{3} - 5 \times 2\sqrt{3} = 5\sqrt{3}$$

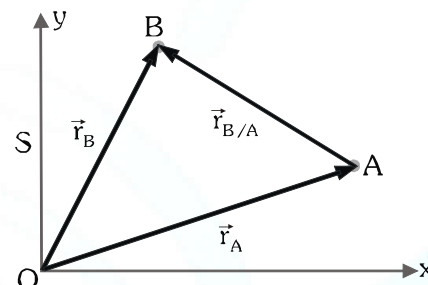
$$\beta = \tan^{-1} \left(\frac{v_y}{v_x} \right) \rightarrow \beta = 60^\circ$$

Relative Motion

Motion of a body can only be observed, when it changes its position with respect to some other body. In this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.

Relative Position, Relative Velocity and Relative Acceleration

Let two bodies represented by particles A and B at positions defined by position vectors \vec{r}_A and \vec{r}_B , moving with velocities \vec{v}_A and \vec{v}_B and accelerations \vec{a}_A and \vec{a}_B with respect to a reference frame S. For analyzing motion of terrestrial bodies, the reference frame S is fixed with the ground.



The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A. Following triangle law of vector addition, we have

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \quad \dots(i)$$

First derivatives of \vec{r}_A and \vec{r}_B with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad \dots(ii)$$

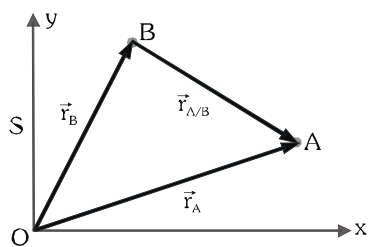
Second derivatives of \vec{r}_A and \vec{r}_B with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \quad \dots(iii)$$

In similar fashion motion of particle A relative to particle B can be analyzed with the help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore

$$\vec{r}_{B/A} = -\vec{r}_{A/B}, \quad \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{B/A} = -\vec{a}_{A/B}.$$





The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.

Example:

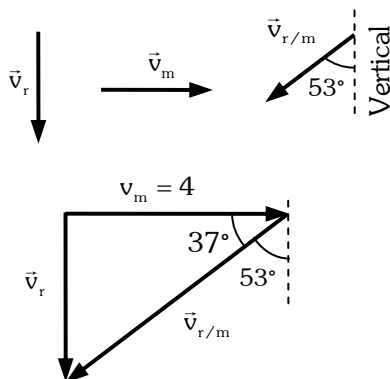
A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.

Solution:

Assigning usual symbols \vec{v}_m , \vec{v}_r and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following eq.

$$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$$

The above equation suggests that a standstill man observes velocity \vec{v}_r of rain relative to the ground and while he is moving with velocity \vec{v}_m , he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.



The addition of velocity vectors is represented according to the above equation is also represented.

From the figure we have

$$v_r = v_m \tan 37^\circ = 3 \text{ km/h}$$

FUNDAMENTAL UNLOCKED- (FU#2) :

- Q.1** Two cars are moving towards each other with velocity 4m/s & 8 m/s respectively from two different points in straight line, 1200 m apart simultaneously, find time after which both cars meet:
 (A) 300 sec (B) 150 sec
 (C) 100 sec (D) None of these
- Q.2** Two cars A & B start from rest (from the same point) in same direction with acceleration 8 m/s^2 & 4 m/s^2 respectively then acceleration of car B in frame of A (Take direction of motion of car is positive):
 (A) 4 m/s^2
 (B) -4 m/s^2
 (C) 12 m/s^2
 (D) None of these
- Q.3** If a lift is moving downwards with constant velocity 5 m/s. A coin is dropped from the lift, find velocity of coin at the same instant when it is dropped in frame of lift and in frame of ground:
 (A) 5 m/s (downward), & 0
 (B) 0 & 5 m/s (downward)
 (C) 5 m/s (upward) & 0
 (D) 0 & 5 m/s (upward)
- Q.4 Statement-1:** For motion in parabolic path $\left| \frac{d\vec{v}}{dt} \right| \neq 0$, but $\frac{d|\vec{v}|}{dt}$ may be zero.
and
Statement-2: Acceleration in a body may be due to change in direction of velocity only.
 (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1.
 (C) Statement-1 is True, Statement-2 is False.
 (D) Statement-1 is False, Statement-2 is True.



Q.5 Statement -1: Two projectiles having same range must have the same time of flight.

Statement -2: Horizontal component of velocity is constant in projectile motion under gravity.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.6 Statement 1: The trajectory of a projectile w.r.t. another projectile is a straight line.

and

Statement 2: The relative velocity of a projectile w.r.t. another projectile is constant.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.7 Statement-1: Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

Statement-2: Relative acceleration between any of the pair of projectiles is zero.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Q.8 Statement-1: The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank. They may reach opposite bank simultaneously moving along different straight-line paths.

Statement-2: For above boats to cross the river in same time, the component of their velocity relative to river in direction normal to flow should be same.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Example:

A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Solution:

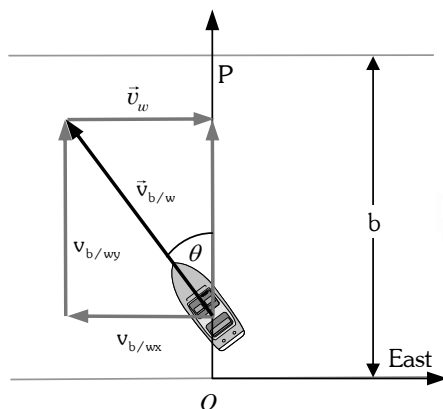
(a) Velocity of a boat on still water is its capacity to move on water surface and equals to its velocity relative to water.

$\vec{v}_{b/w}$ = Velocity of boat relative to water = Velocity of boat on still water

On flowing water, the water carries the boat along with it. Thus velocity \vec{v}_b of the boat relative to the ground equals to vector sum of $\vec{v}_{b/w}$ and \vec{v}_w . The boat crosses the river with the velocity \vec{v}_b .

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$$

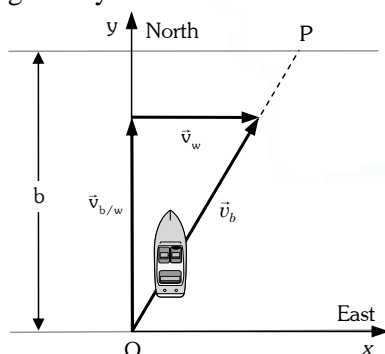




(b) To cross the river perpendicular to current the boat must be steered in a direction so that one of the components of its velocity ($\vec{v}_{b/w}$) relative to water becomes equal and opposite to water flow velocity \vec{v}_w to neutralize its effect. It is possible only when velocity of boat relative to water is greater than water flow velocity. In the adjoining figure it is shown that the boat starts from the point O and moves along the line OP (y-axis) due north relative to ground with velocity \vec{v}_b . To achieve this it is steered at an angle θ with the y-axis.

$$v_{b/w} \sin \theta = v_w \rightarrow 5 \sin \theta = 3 \Rightarrow \theta = 37^\circ$$

(c) The boat will cover river width b with velocity $v_b = v_{b/wy} = v_{b/w} \sin 37^\circ = 4 \text{ m/s}$ in time t , which is given by



$$t = b / v_b \rightarrow t = 50\text{s}$$

(d) To cross the river in minimum time, the component perpendicular to current of its velocity relative to ground must be kept to maximum value. It is achieved by steering the boat always perpendicular to current as shown in the adjoining figure. The boat starts from O at the south bank and reaches point P on the north bank. Time t taken by the boat is given by

$$t = b / v_{b/w} \rightarrow t = 40\text{s}$$

Drift is the displacement along the river current measured from the starting point. Thus, it is given by the following equation. We denote it by x_d .

$$x_d = v_{bx} t$$

Substituting $v_{bx} = v_w = 3 \text{ m/s}$, from the figure, we have

$$x_d = 120 \text{ m}$$

Example :

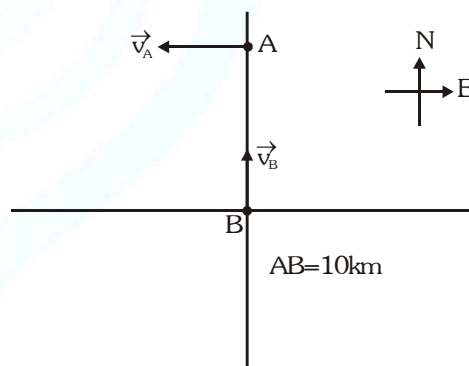
Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 km/h and ship B is streaming north at 20 km/h. What is their distance of closest approach and how long do they take to reach it?

Solution:

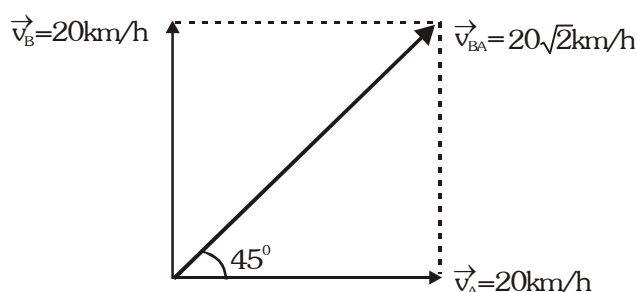
Ships A and B are moving with same speed 20 km/h in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find \vec{v}_{BA}

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$\text{Here, } |\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2} \text{ km/h}$$



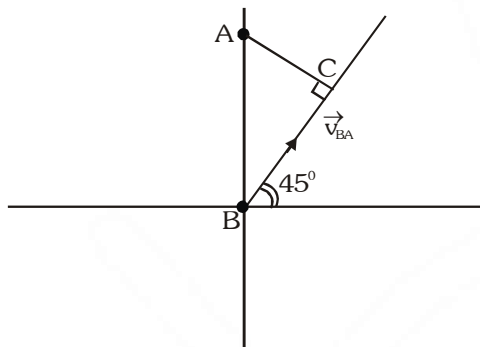
i.e., \vec{v}_{BA} is $20\sqrt{2} \text{ km/h}$ at an angle of 45° from east towards north. Thus, the given problem can be simplified as:





A is at rest and B is moving with \vec{v}_{BA} in the direction shown in figure. Therefore, the minimum distance between the two is

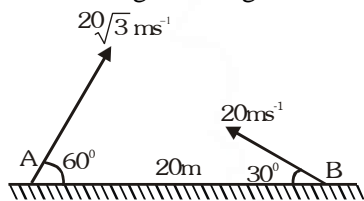
$$s_{\min} = AC = AB \sin 45^\circ = 10 \left(\frac{1}{\sqrt{2}} \right) \text{ km} = 5\sqrt{2} \text{ km}$$



and the desired time is $t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} = \frac{1}{4} \text{ s} = 15 \text{ min}$

Example :

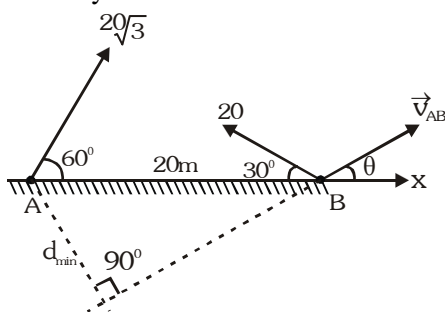
In the figure shown, the two projectile are fired simultaneously. Find the minimum distance between them during their flight.



Solution:

Taking origin at A and x axis along AB

Velocity of A w.r.t B



$$\begin{aligned} &= 20\sqrt{3}(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) - 20(\cos 150^\circ \hat{i} + \sin 150^\circ \hat{j}) \\ &= 20\sqrt{3}\left(\frac{1}{2} \hat{i} + \frac{\sqrt{3}}{2} \hat{j}\right) - 20\left(-\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}\right) = 20\sqrt{3} \hat{i} + 20 \hat{j} \end{aligned}$$

$$\tan \theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

$$\text{So, } \frac{d_{\min}}{20} = \sin \theta = \sin 30^\circ = \frac{1}{2} \Rightarrow d_{\min} = 10 \text{ m}$$

Example :

A particle is dropped from the top of a high building of height 360 m. The distance travelled by the particle in ninth second is ($g = 10 \text{ m/s}^2$)

- (A) 85 m (B) 60 m
(C) 40 m (D) can't be determined

Answer: (C)

Solution:

Total time taken by particle to reach the ground $T =$

$$\sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 360}{10}} = 6\sqrt{2} = 8.484 \text{ s}$$

$$\text{Distance travelled in 8 seconds} = \frac{1}{2} g t^2 = \frac{1}{2} (10) (8)^2$$

$$= 320 \text{ m}$$

Therefore, distance travelled in ninth second

$$= 360 - 320 = 40 \text{ m}$$

Example :

A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is

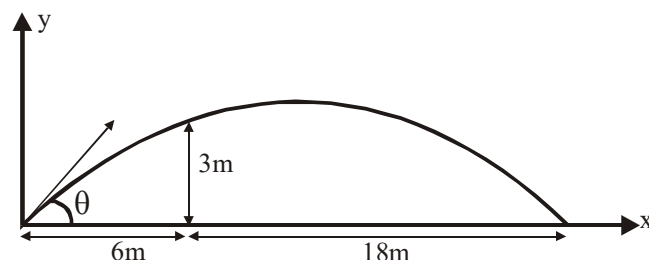
- (A) $\tan^{-1} \left(\frac{3}{2} \right)$ (B) $\tan^{-1} \left(\frac{2}{3} \right)$
(C) $\tan^{-1} \left(\frac{1}{2} \right)$ (D) $\tan^{-1} \left(\frac{3}{4} \right)$

Answer: (B)

Solution:

$$\text{From equation of trajectory } y = x \tan \theta \left[1 - \frac{x}{R} \right]$$

$$\Rightarrow 3 = 6 \tan \theta \left[1 - \frac{1}{4} \right] \Rightarrow \tan \theta = \frac{2}{3}$$



Example :

A particle moves in XY plane such that its position, velocity and acceleration are given by

$$\vec{r} = x\hat{i} + y\hat{j}; \vec{v} = v_x\hat{i} + v_y\hat{j}; \vec{a} = a_x\hat{i} + a_y\hat{j}$$

which of the following condition is correct if the particle is speeding down?

- (A) $xv_x + yv_y < 0$ (B) $xv_x + yv_y > 0$
(C) $a_xv_x + a_yv_y < 0$ (D) $a_xv_x + a_yv_y > 0$

Answer: (C)

Solution:

For speeding down $\vec{a} \cdot \vec{v} < 0 \Rightarrow a_xv_x + a_yv_y < 0$

Example :

A particle is thrown vertically upwards from the surface of the earth. Let T_P be the time taken by the particle to travel from a point P above the earth to its highest point and back to the point P. Similarly, let T_Q be the time taken by the particle to travel from another point Q above the earth to its highest point and back to the same point Q. If the distance between the points P and Q is H, the expression for acceleration due to gravity in terms of T_P , T_Q and H, is:

- (A) $\frac{6H}{T_P^2 + T_Q^2}$ (B) $\frac{8H}{T_P^2 - T_Q^2}$
(C) $\frac{2H}{T_P^2 + T_Q^2}$ (D) $\frac{H}{T_P^2 - T_Q^2}$

Answer: (B)

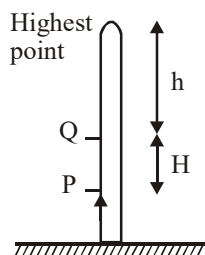
Solution:

$$\text{Time taken from point P to point P } T_P = 2\sqrt{\frac{2(h+H)}{g}}$$

$$\text{Time taken from point Q to point Q } T_Q = 2\sqrt{\frac{2h}{g}}$$

$$\Rightarrow T_P^2 = \frac{8(h+H)}{g}$$

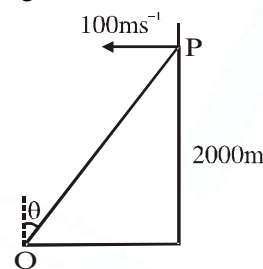
$$\text{and } T_Q^2 = \frac{8h}{g} \Rightarrow T_P^2 = T_Q^2 + \frac{8H}{g} \Rightarrow g = \frac{8H}{T_P^2 - T_Q^2}$$



Example :

An aeroplane is travelling horizontally at a height of 2000 m from the ground. The aeroplane, when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle θ must the line PQ make with the vertical?

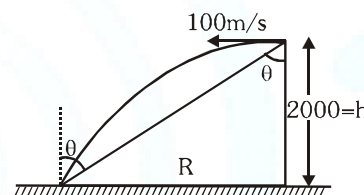
$$[g = 10\text{ms}^{-2}]$$



- (A) 15° (B) 30° (C) 90° (D) 45°

Answer: (D)

Solution:



Let t be the time taken by bomb to hit the target.

$$h = 2000 = \frac{1}{2}gt^2$$

$$\Rightarrow t = 20 \text{ sec}$$

$$R = ut = (100)(20) = 2000 \text{ m}$$

$$\therefore \tan \theta = \frac{R}{h} = \frac{2000}{2000} = 1 \Rightarrow \theta = 45^\circ$$

Example :

Some information's are given for a body moving in a straight line. The body starts its motion at $t = 0$.

Information I: The velocity of a body at the end of 4s is 16 m/s

Information II: The velocity of a body at the end of 12s is 48 m/s

Information III: The velocity of a body at the end of 22s is 88 m/s

The body is certainly moving with

- (A) Uniform velocity
(B) Uniform speed
(C) Uniform acceleration
(D) Data insufficient for generalization

Answer: (D)

Solution:

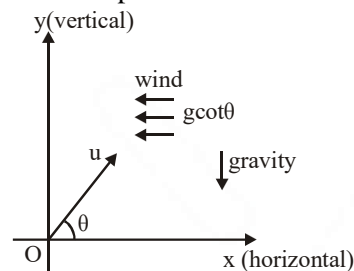
Here average acceleration

$$= \frac{16 - 0}{4 - 0} = \frac{48 - 16}{12 - 4} = \frac{88 - 48}{22 - 12} = 4$$

But we can't say certainly that body have uniform acceleration.

Example :

A ball is projected as shown in figure. The ball will return to point:



- (A) O (B) left to point O
(C) right to point O (D) none of these

Answer: (A)

Solution:

Here $\frac{a_x}{a_y} = \frac{g \cot \theta}{g} = \frac{1}{\tan \theta} = \frac{u_x}{u_y} \Rightarrow$ Initial velocity & acceleration are opposite to each other.
 \Rightarrow Ball will return to point O.

Example :

Throughout a time interval, while the speed of a particle increases as it moves along the x-axis, its velocity and acceleration might be:

- (A) positive and positive respectively.
(B) positive and negative respectively.
(C) negative and negative respectively.
(D) negative and positive respectively.

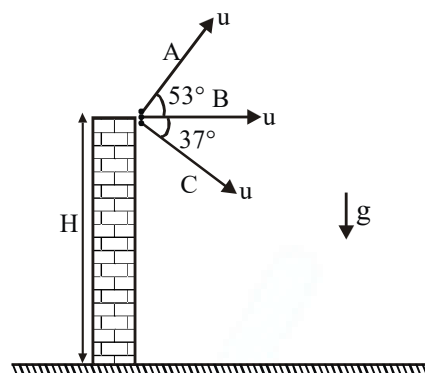
Answer: (AC)

Solution:

Speed increases if both velocity & acceleration have same signs.

Example :

Three point particles A, B and C are projected from same point with same speed at $t=0$ as shown in figure. For this situation select correct statement(s).



- (A) All of them reach the ground at same time.
(B) All of them reach the ground at different time.
(C) All of them reach the ground with same speed.
(D) All of them have same horizontal displacement when they reach the ground.

Answer: (BC)

Solution:

Vertical component of initial velocities are different
 \Rightarrow reach the ground at different time.

Example :

A projectile is thrown with speed u into air from a point on the horizontal ground at an angle θ with horizontal. If the air exerts a constant horizontal resistive force on the projectile then select correct alternative(s).

- (A) At the farthest point, the velocity is horizontal.
(B) The time for ascent equals the time for descent.
(C) The path of the projectile may be parabolic.
(D) The path of the projectile may be a straight line.

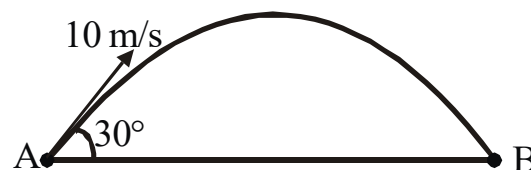
Answer: (CD)

Solution:

Here total acceleration $a = \sqrt{g^2 + a_x^2} = \text{constant}$, so path may be parabolic or straight line.

Example :

As shown in the figure there is a particle of mass $\sqrt{3}$ kg, is projected with speed 10 m/s at an angle 30° with horizontal (take $g = 10 \text{ m/s}^2$) then match the following



Column I

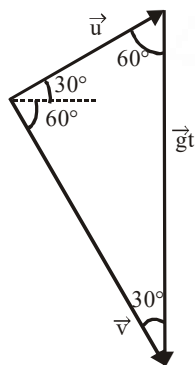
- (A) Average velocity (in m/s) during half of the time of flight, is
 (B) The time (in sec) after which the angle between velocity vector and initial velocity vector becomes $\pi/2$, is
 (C) Horizontal range (in m), is
 (D) Change in linear momentum (in N-s)

Column II

- (P) $\frac{1}{2}$
 (Q) $\frac{5}{2}\sqrt{13}$
 (R) $5\sqrt{3}$
 (S) At an angle of when particle is at $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ highest point, is from horizontal
 (T) 2

Answer: (A)→QS; (B)→T; (C)→R; (D)→R

Solution:



For (A):

$$v_{av} = \sqrt{(v_{avx})^2 + (v_{avy})^2}$$

$$= \sqrt{(10 \cos 30^\circ)^2 + \left(\frac{10 \sin 30^\circ + 0}{2}\right)^2} = \sqrt{75 + \frac{25}{4}} = \frac{5}{2}\sqrt{13} \text{ m/s}$$

Angle with horizontal

$$\theta = \tan^{-1}\left(\frac{v_{avy}}{v_{avx}}\right) = \tan^{-1}\left(\frac{5/2}{5\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

For (B): By using $\vec{v} = \vec{u} + \vec{a}t$ We have

$$\frac{u}{gt} = \sin 30^\circ \Rightarrow t = \frac{10}{(10)(1/2)} = 2$$

For (C): Horizontal range(R)

$$= \frac{u^2 \sin 2\theta}{g} = \frac{100 \times \sqrt{3}/2}{10} = 5\sqrt{3} \text{ m}$$

For (D): Change in linear momentum = mu_y

$$= \sqrt{3} \times 10 \sin 30^\circ = 5\sqrt{3} \text{ N-s}$$

Example :

A particle is moving along a straight line along x-axis with an initial velocity of 2 m/s towards positive x-axis. A constant acceleration of 0.5 m/s^2 towards negative x-axis starts acting on particle at $t=0$. Find velocity (in m/s) of particle at $t = 2\text{s}$.

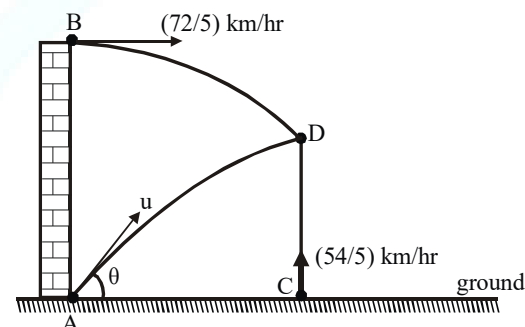
Answer: 1

Solution:

$$v = u + at \Rightarrow v = 2 + (-0.5)(2) = 1 \text{ m/s}$$

Example :

In the given figure points A and C are on the horizontal ground & A and B are in same vertical plane. Simultaneously bullets are fired from A, B and C and they collide at D. The bullet at B is fired horizontally with speed of $\frac{72}{5} \text{ km/hr}$ and the bullet at C is projected vertically upward at velocity of $\frac{54}{5} \text{ km/hr}$. Find velocity of the bullet projected from A in m/s.



Answer: 5

Solution:

For collision

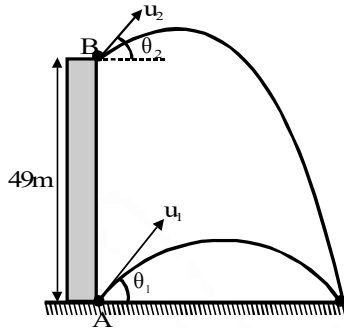
$$u = \sqrt{u_B^2 + u_C^2} = \sqrt{\left(\frac{72}{5} \times \frac{5}{18}\right)^2 + \left(\frac{54}{5} \times \frac{5}{18}\right)^2}$$

$$= \sqrt{4^2 + 3^2} = 5 \text{ m/s}$$



Example :

Two stones A and B are projected simultaneously as shown in figure. It has been observed that both the stones reach the ground at the same place after 7 sec of their projection. Determine difference in their vertical components of initial velocities in m/s. ($g = 9.8 \text{ m/s}^2$)



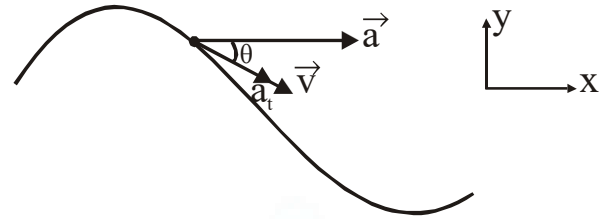
Answer: 7

Solution:

In time of flight i.e. 7 s, the vertical displacement of A is zero and that of B is 49 m so for relative motion of B w.r.t. A ($u_2 \sin \theta_2 - u_1 \sin \theta_1$) $\times 7 = 49$
 $\Rightarrow u_2 \sin \theta_2 - u_1 \sin \theta_1 = 7 \text{ m/s}$

Example :

A particle moves with a tangential acceleration $a_t = \vec{a} \cdot \hat{v}$ where $\vec{a} = (5\hat{i}) \text{ m/s}^2$. If the speed of the particle is zero at $x = 0$, then find v (in m/s) at $x = 4.9 \text{ m}$.



Answer: 7

Solution:

$$\text{As } v dv = \vec{a} \cdot d\vec{r} = a dx = 5 dx \Rightarrow \int_0^v v dv = 5 \int_0^{4.9} dx$$

$$\Rightarrow \frac{v^2}{2} = 5(4.9)$$

$$\Rightarrow v^2 = 49 \Rightarrow v = 7 \text{ m/s}$$

Example :

A body is thrown up with a speed 49 m/s. It travels 5 m in the last second of its upward journey. If the same body is thrown up with a velocity 98 m/s, how much distance (in m) will it travel in the last second. ($g = 10 \text{ m/s}^2$)

Answer: 5

Solution:

In last second of upward journey, all bodies travel same distance ($= g/2 = 5 \text{ m}$)





ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 (C)

Q.2 (ABC)

Q.3 (A)

FUNDAMENTAL UNLOCKED- (FU#2) :

Q.1 (C)

Q.2 (B)

Q.3 (B)

Q.4 (A)

Q.5 (D)

Q.6 (A)

Q.7 (A)

Q.8 (A)





OBJECTIVE EXERCISE - I

Single Correct Type Questions

1. The coordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by:

(A) $t^2 \sqrt{\alpha^2 + \beta^2}$ (B) $\sqrt{\alpha^2 + \beta^2}$
 (C) $3t \sqrt{\alpha^2 + \beta^2}$ (D) $3t^2 \sqrt{\alpha^2 + \beta^2}$

2. A body has an initial velocity of 3 ms^{-1} and has a constant acceleration of 1 ms^{-2} normal to the direction of the initial velocity. Then its velocity, 4 second after the start is:

(A) 7 ms^{-1} along the direction of initial velocity
 (B) 7 ms^{-1} along the normal to the direction of the initial velocity
 (C) 7 ms^{-1} mid-way between the two directions
 (D) 5 ms^{-1} at an angle of $\tan^{-1} \frac{4}{3}$ with the direction of the initial velocity

3. The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6) \hat{i} + (t^2) \hat{j}$. The time after which the velocity vector and acceleration vector becomes perpendicular to each other is equal to:

(A) 1 sec (B) 2 sec
 (C) 1.5 sec (D) not possible

4. A particle moves in a plane with constant acceleration in a direction different from the initial velocity. The path of the particle will be:

(A) A straight line (B) An arc of a circle
 (C) A parabola (D) An ellipse

Ground to Ground Projectile

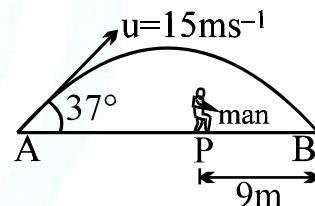
5. The speed at the maximum height of a projectile is half of its initial speed u . Its range on the horizontal plane is:

(A) $\frac{2u^2}{3g}$ (B) $\frac{\sqrt{3} u^2}{2g}$
 (C) $\frac{u^2}{3g}$ (D) $\frac{u^2}{2g}$

6. A projectile is thrown with a speed v at an angle θ with the upward vertical. Its average velocity between the instants at which it crosses half the maximum height is:

(A) $v \sin \theta$, horizontal and in the plane of projection
 (B) $v \cos \theta$, horizontal and in the plane of projection
 (C) $2v \sin \theta$, horizontal and perpendicular to the plane of projection
 (D) $2v \cos \theta$, vertical and in the plane of projection

7. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



(A) 3 ms^{-1} (B) 5 ms^{-1}
 (C) 9 ms^{-1} (D) 12 ms^{-1}

8. A particle is projected from a point O with a velocity u in a direction making an angle α upward with the horizontal. At P it is moving at right angles to its initial direction. Its velocity at P is:

(A) $u \tan \alpha$ (B) $u \cot \alpha$
 (C) $u \operatorname{cosec} \alpha$ (D) $u \sec \alpha$

9. A stone is projected from the ground with velocity 25 m/s . Two seconds later, it just clears a wall 5 m high. The angle of projection of the stone is: ($g = 10 \text{ m/sec}^2$)

(A) 30° (B) 45°
 (C) 50.2° (D) 60°



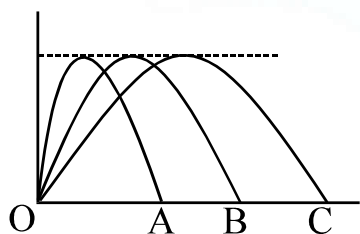
10. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
- (A) The one with the farthest range
(B) The one which reaches maximum height
(C) The one with the greatest initial velocity
(D) The one leaving the bat at 45° with respect to the ground

11. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R , H and T respectively. They are given by,

$$R = \frac{u^2 \sin 2\theta}{g}, H = \frac{u^2 \sin^2 \theta}{2g} \text{ and } T = \frac{2u \sin \theta}{g}$$

Now keeping u as fixed, θ is varied from 30° to 60° . Then,

- (A) R will first increase then decrease, H will increase and T will decrease
(B) R will first increase then decrease while H and T both will increase
(C) R will decrease while H and T will increase
(D) R will increase while H and T will increase
12. Three projectiles A, B and C are thrown simultaneously from the same point in the same vertical plane. Their trajectories are shown in the figure. Then which of the following statement is false.



- (A) The time of flight is the same for all the three
(B) The launch speed is greatest for particle C
(C) The vertical velocity component for particle C is greater than that for the other particles
(D) Y-coordinate of all particles is always same

Horizontal Projectile and Projection from Height

13. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1} . It hits the ground at angle 45° . What is the vertical component of velocity when it strikes the ground?

- (A) $18\sqrt{3} \text{ ms}^{-1}$ (B) 18 ms^{-1}
(C) $9\sqrt{2} \text{ ms}^{-1}$ (D) 9 ms^{-1}

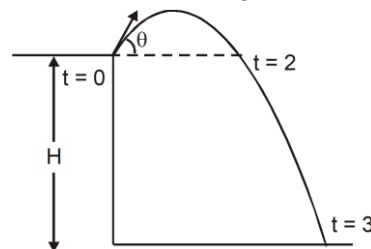
14. An airplane is flying in a horizontal direction with a velocity of 600 km/hour and at a height of 1960 m . When it is vertically above the point A on the ground a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.

- (A) 2.1 km (B) 5.3 km
(C) 4.2 km (D) 3.3 km

15. Particle is dropped from the height of 20 m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 ms^{-2} . Find the horizontal displacement of the particle till it reaches ground.

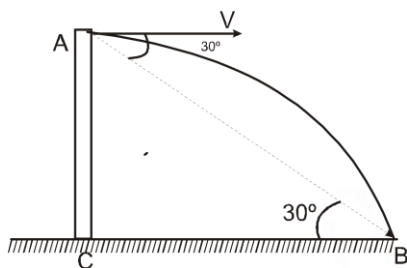
- (A) 6 m (B) 10 m (C) 12 m (D) 24 m

16. A stone projected at angle ' θ ' with horizontal from the roof of a tall building falls on the ground after three second. Two second after the projection it was again at the level of projection. Then the height of the building is:



- (A) 5 m (B) 25 m (C) 20 m (D) 15 m

17. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3 s later at B. The line from 'A' to 'B' makes an angle of 30° with the horizontal. The initial velocity of the object is: (take $g = 10 \text{ m/s}^2$)



- (A) $15\sqrt{3}$ m/s (B) 15 m/s
(C) $10\sqrt{3}$ m/s (D) $25\sqrt{3}$ m/s

Equation of Trajectory

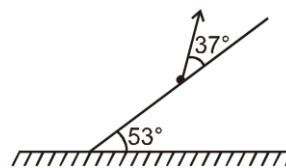
18. A particle moves on the xy-plane such that its position vector is given by $\vec{r} = 2t^2\hat{i} - t^3\hat{j}$. The equation of trajectory of the particle is given by:

- (A) $3x^2 + 16y = 0$ (B) $\left(\frac{3x}{2}\right)^{4/3} + 4y = 0$
(C) $\left(\frac{x}{2}\right)^{3/2} + y = 0$ (D) None of these

19. The equation of the path of the projectile is $y = 0.5x - 0.04x^2$. The initial speed of the projectile is:
(A) 10 m/s (B) 15 m/s
(C) 12.5 m/s (D) None

Projection on Inclined Plane

20. On an inclined plane of inclination 30° , a ball is thrown at an angle of 60° with the horizontal from the foot of the incline with a velocity of $10\sqrt{3} \text{ ms}^{-1}$. If $g = 10 \text{ ms}^{-2}$, then the time in which ball will hit the inclined plane is:
(A) 1 sec. (B) 6 sec.
(C) 2 sec. (D) 4 sec.
21. A particle is projected from the inclined plane at angle 37° with the inclined plane in upward direction with speed 10 m/s. The angle of inclined plane with horizontal is 53° . Then the maximum height attained by the particle from the incline plane from the point of projection will be:

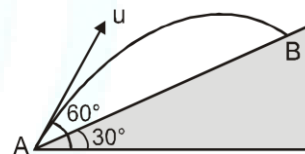


- (A) 3m (B) 4 m
(C) 5 m (D) zero

22. A particle is projected up the inclined such that its component of velocity along the incline is 10 m/s. Time of flight is 2 sec and maximum height above the incline is 5 m. Then velocity of projection will be:

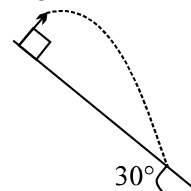
- (A) 10 m/s (B) $10\sqrt{2}$ m/s
(C) $5\sqrt{5}$ m/s (D) none

23. A stone is projected from point A with speed u making an angle 60° with horizontal as shown. The fixed inclined surface makes an angle 30° with horizontal. The stone lands at B after time t . Then the distance AB is equal to



- (A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}ut$ (D) $2ut$

24. A ball is projected from point A with velocity 10m/sec. perpendicular to the inclined plane as in figure. Range of the ball on the incline is:



- (A) $40\sqrt{3}$ m (B) $20/13$ m
(C) $13/20$ m (D) $40/3$ m

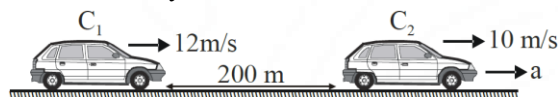
Relative Motion

25. A train is moving slowly on a straight track with a constant speed of 2 ms^{-1} . A passenger in that train starts walking at a steady speed of 2 ms^{-1} with respect to train to the back of the train in the opposite direction of the motion of

the train. So to an observer standing on the platform directly in front of that passenger, the velocity of the passenger appears to be :

- (A) 2 ms^{-1} in the opposite direction of the train
- (B) zero
- (C) 4 ms^{-1}
- (D) 2 ms^{-1}

26. Two cars C_1 and C_2 are moving in the same direction on a straight single lane road with constant velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m , C_2 started accelerating to avoid collision. The minimum acceleration of car C_2 so that they don't collide, is :

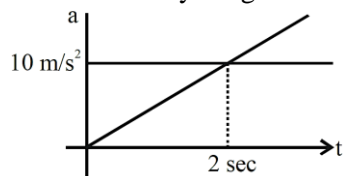


- (A) 1 cm/s^2
- (B) 2 cm/s^2
- (C) 3 cm/s^2
- (D) 4 cm/s^2

27. Two trains A & B, 100 km apart are travelling towards each other on different tracks with starting speed of 50 km/h for both. The train A accelerates at 20 km/h^2 and the train B retards at the rate 20 km/h^2 . The distance covered by the train A when they cross each other is:

- (A) 45 km
- (B) 55 km
- (C) 65 km
- (D) 60 km

28. Two object starts from rest and their acceleration is shown in figure. The time when their relative velocity is again zero is:



- (A) 2 sec
- (B) 4 sec
- (C) 1 sec
- (D) 6 sec

29. Two transparent elevator cars A and B are moving in front of each other. Car A is moving up and retarding at a_1 , while car B is moving down and retarding at a_2 . Person in car A drops a coin inside the car. What is the acceleration observed by person in car B?

- (A) $g + a_2$ downward
- (B) $g - a_1 - a_2$ downward
- (C) $g - a_1 + a_2$ downward
- (D) None of these

30. A lift A is accelerating vertically upwards with acceleration 5 m/s^2 and another lift B is accelerating downwards with acceleration 5 m/s^2 . A coin is released from rest (with respect to lift) from some height in lift A. At that instant velocity of lift A is 8 m/s vertically upwards & velocity of lift B is 12 m/s vertically downwards. Then velocity & acceleration of coin with respect to lift B at that instant will be, respectively:

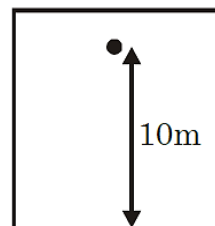
- (A) 20 m/s , 20 m/s^2
- (2) 4 m/s , 15 m/s^2
- (3) 4 m/s , 15 m/s^2
- (4) 20 m/s , 5 m/s^2

31. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be ' t '. The acceleration with which the lift is moving up is:

- (A) $\frac{u - gt}{t}$
- (B) $\frac{2u - gt}{t}$
- (C) $\frac{u + gt}{t}$
- (D) $\frac{2u + gt}{t}$

32. A particle of mass 2 kg in a lift projected vertically upward with a velocity 10 m/s relative to the lift from a height 10 m above the floor of lift. What will be the velocity of the particle just before it strikes the lift?

[At $t = 0$, $V = 10 \text{ m/s}$ & $a = 5 \text{ m/s}^2$ (Both upwards) of lift].



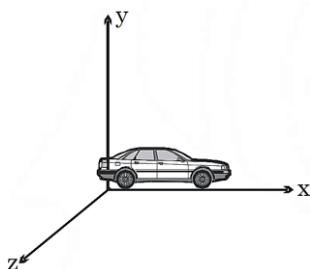
- (A) 0 m/s
- (B) 10 m/s upward
- (C) 10 m/s downward
- (D) 30 m/s downward



33. A particle is projected from the origin with a velocity of 10 m/s at 37° to horizontal in x-y plane. Another particle is simultaneously projected with velocity of 6 m/s in vertical y-direction. The path of 1st particle as seen by second is:

(A) parabola
(B) vertical straight line
(C) horizontal straight line
(D) straight line inclined at an acute angle to the horizontal

34. A car is traveling on a highway at a speed 25 m/s as shown. A passenger in a car throws a ball at an angle 37° with horizontal in a plane perpendicular the motion of the car. The ball is projected with a speed of 10 m/s relative to car. What will be the initial velocity of the ball in unit vector notation ?

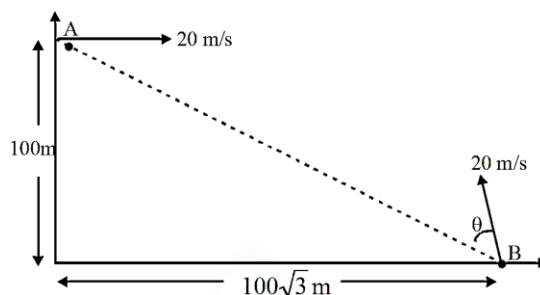


(A) $25\hat{i} + 8\hat{j} + 6\hat{k}$ (B) $10\hat{i} + 8\hat{j} + 6\hat{k}$
(C) $10\hat{i} + 25\hat{j} + 6\hat{k}$ (D) $25\hat{i} + 6\hat{j} + 8\hat{k}$

35. Ram moves with a velocity of 10 m/s in west direction. Shyam moves in direction 23° East of North. Ram is 100 m away from Shyam in direction 53° East of North of him. What should be speed of Shyam so that he collides with Ram?

(A) $4\sqrt{3}$ m/s (B) 10 m/s
(C) 12 m/s (D) None of these

36. Two small balls A and B are launched in the same vertical plane simultaneously, with same speed of 20 m/s at $t = 0$. Ball A has an initial horizontal velocity and ball B has initial velocity at an angle θ above the line joining A and B as shown. If the projectiles collide in mid-air at time t , then:



(A) $\theta = 45^\circ$ (b) $\theta = 60^\circ$
(C) $\theta = 30^\circ$ (D) $\theta = 37^\circ$

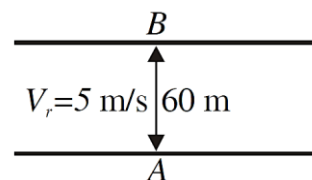
37. A boat can go across a lake and return in time T_0 at a speed V . On a rough day there is uniform current at speed v to help the onward journey and impede the return journey. If the time taken to go across and return on the rough day be T , then $\frac{T}{T_0}$ is:

(A) $1 - \frac{v^2}{V^2}$ (B) $\frac{1}{1 - \frac{v^2}{V^2}}$
(C) $1 + \frac{v^2}{V^2}$ (D) $\frac{1}{1 + \frac{v^2}{V^2}}$

38. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is:

(A) 1 (B) 3
(C) 4 (D) $\sqrt{41}$

39. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across the river at a distance of 60 m in 5 sec. His velocity in still water should be:



(A) 12 m/s (B) 13 m/s
(C) 5 m/s (D) 10 m/s





40. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 sec to cross the river of width 60 m. The velocity of the swimmer w.r.t. water will be :

(A) 10 m/s (B) 12 m/s
(C) $5\sqrt{5}$ m/s (D) $10\sqrt{2}$ m/s

41. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction:

(A) East (B) North-East
(C) South-East (D) South-West

42. A boat is moving towards east with velocity 4 m/s with respect to still water and river is flowing towards north with velocity 2 m/s and the wind is blowing towards north with velocity 6 m/s. The direction of the flag blown over the wind hoisted on the boat is:

(A) north-west
(B) south-east
(C) $\tan^{-1}(1/2)$ with east
(D) north

43. A boat is travelling east at 12 m/s with respect to ground. A flag fixed on boat flutter at 53° N of W with respect to boat. Another flag on shore flutter due north, find the speed (in m/s) of the wind as measured on land.

(A) 16 m/s (B) 20 m/s
(C) 12 m/s (D) 8 m/s

44. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is:

(A) 10 m/s (B) $10\sqrt{3}$ m/s
(C) 20 m/s (D) $20\sqrt{3}$ m/s

45. A car with a vertical wind shield moves along in rain storm at the speed of 40 km/h. The rain drops fall vertically with a terminal speed of 20 m/s. The angle with the vertical at which the rain drop strike the wind shield is :

(A) $\tan^{-1}(5/9)$ (B) $\tan^{-1}(9/5)$
(C) $\tan^{-1}(2)$ (D) $\tan^{-1}(3)$

One or More Than One Correct Type Questions

46. In a projectile motion on horizontal surface:

(A) For same speed of projection, range at 30° is same as range at 60°
(B) Time of flight does not depend on horizontal component of velocity
(C) Maximum height depends on vertical component of velocity
(D) Ratio of range and maximum height depend on angle of projection

47. A particle is projected from a point on the ground with an initial velocity of $u = 50$ m/s at an angle of 53° with the horizontal, then choose the correct statement(s): ($\tan 53^\circ = 4/3$, acceleration due to gravity = 10 m/s^2).

(A) The velocity of the particle will make angle 45° with the horizontal after time 1 s
(B) The velocity of the particle will make angle 45° with the horizontal after time 7 s
(C) The average velocity between the point of projection and the highest point on its path is horizontal
(D) The average velocity between two points on same height will be horizontal

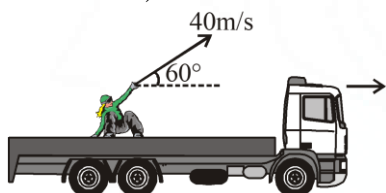
48. A particle moves in the xy plane with a constant acceleration 'g' in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following statements are correct?

(A) The x-component of its velocity is constant
(B) At the origin, the y-component of its velocity is a $\sqrt{\frac{g}{2b}}$
(C) At the origin, its velocity makes an angle $\tan^{-1}(a)$ with the x-axis
(D) The particle moves exactly like a projectile





49. Two particles A & B are projected along different directions from the same point P on the ground with the same speed of 70 m/s in the same vertical plane. They hit the ground at the same point Q such that $PQ = 480$ m. Then: (Use $g = 9.8 \text{ m/s}^2$, $\sin^{-1} 0.96 = 74^\circ$, $\sin^{-1} 0.6 = 37^\circ$)
- (A) ratio of their time of flights is 4 : 5
 (B) ratio of their maximum heights is 9 : 16
 (C) ratio of their minimum speeds during flight is 4 : 3
 (D) the bisector of the angle between their directions of projection makes 45° with horizontal
50. A boy in an open truck throws a ball at 40 m/s at angle 60° from horizontal w.r.t himself. The truck is moving with constant speed 10 m/s and the boy just after throw, jumps off the truck and starts running with constant velocity in same direction to catch the ball. (Assume same horizontal level)



- (A) Speed of the boy must be 10 m/s
 (B) Speed of the boy must be 30 m/s
 (C) Time of flight is $4\sqrt{3}$ s
 (D) Distance at which the boy catch the ball from throwing position is $120\sqrt{3}$ m

Comprehension Type Questions

Paragraph for Q. 51 to Q. 53

Two projectiles are thrown simultaneously in the same vertical plane from the same point. If their velocities of projection are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

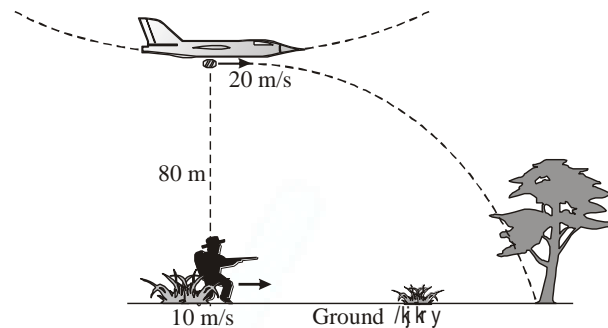
51. The trajectory of particle 1 with respect to particle 2 will be
 (A) a parabola
 (B) a straight line
 (C) a vertical straight line
 (D) a horizontal straight line
52. If $v_1 \cos \theta_1 = v_2 \cos \theta_2$, then choose the incorrect statement
 (A) One particle will remain exactly below or above the other particle
 (B) The trajectory of one with respect to other during the flight will be a vertical straight line
 (C) Both will have the same range
 (D) Both will attain same maximum height
53. If $v_1 \sin \theta_1 = v_2 \sin \theta_2$, then choose the correct statement
 (A) The time of flight of both the particles will be same
 (B) The maximum height attained by the particles will be same
 (C) The trajectory of one with respect to another during the flight will be a horizontal straight line
 (D) None of these



OBJECTIVE EXERCISE - I

Single Correct Type Questions

1. A point moves in x-y plane according to the law $x = 3 \cos 4t$ and $y = 3 (1 - \sin 4t)$. The distance travelled by the particle in 2 sec is (where x and y are in meters)
(A) 48 m (B) 24 m
(C) $48\sqrt{2}$ m (D) $24\sqrt{2}$ m
2. A particle is projected with a velocity u, at an angle α , with the horizontal. At what time, its vertical component of velocity becomes half of its net speed at the highest point?
(A) $\frac{u}{2g}$
(B) $\frac{u}{2g} (\sin \alpha - \cos \alpha)$
(C) $\frac{u}{2g} (2 \cos \alpha - \sin \alpha)$
(D) $\frac{u}{2g} (2 \sin \alpha - \cos \alpha)$
3. A projectile has same range R for two angles of projection. If t_1 & t_2 be the time of flight for the two cases then:
(A) $R = \frac{gt_1 t_2}{2}$ (B) $R = \frac{g(t_1 + t_2)^2}{2}$
(C) $R = g\sqrt{t_1 t_2}$ (D) $R = 2g \frac{t_1 t_2}{t_1 + t_2}$
4. A bomber plane moving at a horizontal speed of 20 m/s releases a bomb at a height of 80 m above ground as shown. At the same instant, a Hunter of negligible height starts running from a point below it, to catch the bomb with speed 10 m/s. After two seconds, he realized that he cannot make it. He stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, the bullet will hit it. What should be the firing speed of bullet. (Take $g = 10 \text{ m/s}^2$)

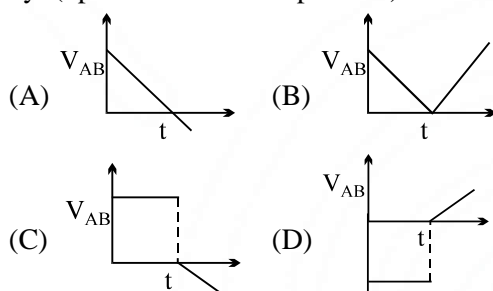


- (A) 10 m/s (B) $20\sqrt{10}$ m/s
(C) $10\sqrt{10}$ m/s (D) None of these
5. A particle is projected horizontally from a tower with velocity V_0 . Wind is blowing in opposite direction and is providing a constant horizontal acceleration a_0 . If particle strikes the ground moving vertically then height of building is
(A) $\frac{V_0^2 g}{a_0^2}$ (B) $\frac{V_0^2 g}{2a_0^2}$
(C) $\frac{2V_0^2 g}{a_0^2}$ (D) $\frac{V_0^2}{g}$
 6. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:
 $x = 10\sqrt{3}t$ and $y = 10t - t^2$
The maximum height attained by the ball is:
(A) 100 m (B) 75 m (C) 50 m (D) 25 m
 7. A particle moves in x-y coordinate system such that its position coordinates are given as $\vec{r} = 2\sin t \hat{i} + 4\sin t \hat{j}$, The path of the particle is:
(A) straight line (B) parabola
(C) circular (D) ellipse

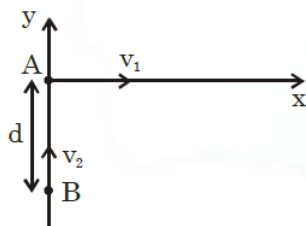


8. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4 m from it. The height of the wall is
(A) $3/4$ m (B) $2/3$ m (C) $4/3$ m (D) $1/3$ m

9. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h . Simultaneously another body B is dropped from height h . It strikes the ground and does not rebound. The velocity of A relative to B v/s time graph is best represented by: (upward direction is positive)



10. Two particles A and B move with velocities v_1 and v_2 respectively along the x and y-axis. The initial separation between them is 'd' as shown in figure. Find the least distance between them during their motion.



- (A) $\frac{dv_1^2}{v_1^2 + v_2^2}$ (B) $\frac{dv_2^2}{v_1^2 + v_2^2}$
(C) $\frac{dv_1}{\sqrt{v_1^2 + v_2^2}}$ (D) $\frac{dv_2}{\sqrt{v_1^2 + v_2^2}}$

11. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5 m/s. The angle 30° is measured from a direction perpendicular to river flow. Velocity of motorboat with respect to water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle:

- (A) 120° with respect to stream direction.
(B) 30° with respect to the perpendicular to the bank.
(C) 30° with respect to the line of destination from starting point.
(D) None of these.

12. A biker at rest on an inclined plane of inclination 30° , finds the rain falling perpendicularly to the plane. Then he starts the bike and moves with a speed of 10 m/s up the incline, he finds rain falling vertically relative to him. The speed of rain is :

- (A) $\frac{10}{\sqrt{3}}$ m/s (B) 10 m/s
(C) $10\sqrt{3}$ m/s (D) 20 m/s

One or More Than One Correct Type Questions

13. A particle leaves the origin with an initial velocity $\vec{v} = 3\hat{i}$ m/s and a constant acceleration $\vec{a} = (-\hat{i} - 0.5\hat{j})$ m/s² in free space. Its velocity \vec{v} and position vector \vec{r} , when it reaches its maximum x-coordinate are:

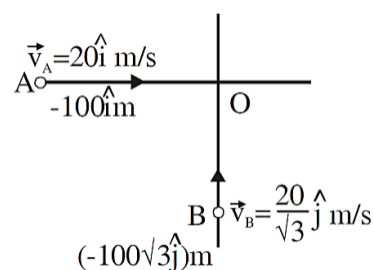
- (A) $\vec{v} = -2\hat{j}$ (B) $\vec{v} = -1.5\hat{j}$
(C) $\vec{r} = (4.5\hat{i} - 2.25\hat{j})$ (D) $\vec{r} = (3\hat{i} - 2\hat{j})$

14. A projectile is projected at an angle α ($> 45^\circ$) with an initial velocity u . The time t at which its horizontal component is equal to the vertical component in magnitude:

- (A) $t = \frac{u}{g} (\cos \alpha - \sin \alpha)$
(B) $t = \frac{u}{g} (\cos \alpha + \sin \alpha)$
(C) $t = \frac{u}{g} (\sin \alpha - \cos \alpha)$
(D) $t = \frac{u}{g} (\sin^2 \alpha - \cos^2 \alpha)$



15. For a particle moving in xy-plane $v_x = 10$ m/s and $y = x - \frac{x^2}{20}$ where x and y are both in meter.
- (A) velocity at $x = 0$ m is $(10\hat{i} + 10\hat{j})$ m/s
 (B) acceleration is $-10\hat{j}$ m/s²
 (C) Maximum value of y is 5 m
 (D) minimum speed is 10 m/s
16. A car traveling on a long straight highway, at a constant speed of 144 km/h passes a police motorcycle moving at 72 km/h. Immediately after the car passes the motorcycle, the police officer accelerates the motorcycle at constant acceleration of 2 m/s² to overtake the speeding car.
- (A) The police officer takes 20 s to overtake the speeding car.
 (B) The police motorcycle travels 800 m to overtake the speeding car.
 (C) The police motorcycle overtakes the car with a speed of 216 km/h.
 (D) Speed of the car relative to the motorcycle 5 seconds after the car passes by the motorcycle is 10 m/s.
17. A long horizontal platform starts rising from ground with acceleration $(2\hat{i} + 2\hat{j})$ m/s². At the same instant, a ball is projected with speed 40 m/s at angle 37° with platform. Neglect the thickness of platform. Then :
- (A) Time after which ball will again hit the platform is 4 sec.
 (B) Range of ball as observed by man on platform is 112 m.
 (C) Range of ball w.r.t. ground is 128 m.
 (D) Height of ball from ground when it strike the platform is 16 m.
18. Positions of two vehicles A and B with reference to origin O and their velocities are as shown.



- (A) they will not collide
 (B) distance of closest approach is 100 m.
 (C) their relative speed is $\frac{40}{\sqrt{3}}$ m/s
 (D) their relative velocity is $\frac{20}{\sqrt{3}}$ m/s
19. Two particles are projected from position 'A' and 'B' separated by a distance of 80 m on horizontal ground. If both the particles collides in mid air. Select the correct statement [Both particles are projected simultaneously]
-
- (A) They will collide if angle of projection for 'A' is 37°
 (B) Particles will collide between the two points 'A' and 'B'.
 (C) Particles will collide at $t = 1$ sec after projection
 (D) They will collide if angle of projection for 'A' is 53°
20. A swimmer wishes to cross a river of width 200 m. He starts swimming perpendicular to river with a constant acceleration 1 m/s². Due to constant river velocity he acquires a drift of 4m along the river, while crossing the river
- (A) River velocity is 2 m/s
 (B) River velocity is 0.2 m/s
 (C) Time taken to cross the river is 20 s
 (D) Time taken to cross the river is 2 s

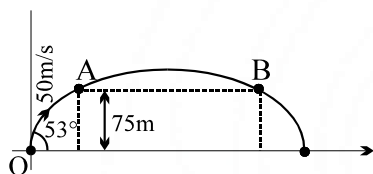


21. River is flowing with a velocity $\vec{v}_R = 4\hat{i} \text{ ms}^{-1}$. A boat is moving with a velocity $(-2\hat{i} + 4\hat{j}) \text{ ms}^{-1}$ relative to river and width of river is 100 m along y direction then,
 (A) The time of crossing is 25 sec
 (B) Absolute velocity of boat is $2\sqrt{5} \text{ ms}^{-1}$
 (C) Drift of boat is 50 m
 (D) The boat can never cross the river

Comprehension Type Questions

Paragraph for Q. 22 to Q. 23

At $t = 0$ a projectile is fired from a point O (taken as origin) on the ground with a speed of 50 m/s at an angle of 53° with the horizontal. It just passes two points A & B each at height 75 m above horizontal as shown.

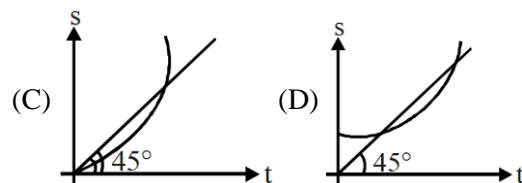
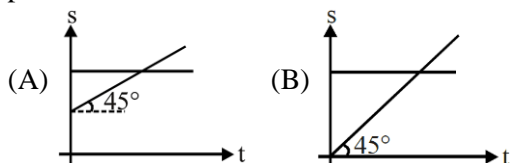


22. The horizontal separation between the points A and B is:
 (A) 30 m (B) 60 m
 (C) 90 m (D) None
23. The distance (in meters) of the particle from origin at $t = 2$ sec is:
 (A) $60\sqrt{2}$ (B) 100
 (C) 60 (D) 120

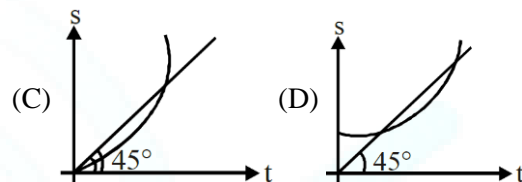
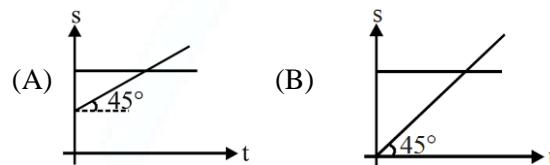
Paragraph for question nos. 24 to 26

Two cars are travelling on a straight road. The car C_1 is going at a constant speed of 9 m/s. 40 m ahead of C_1 is car C_2 at rest, but starts moving with an acceleration of 1 m/s^2 .

24. The graph correctly representing the position (s) of two cars with time (t) w.r.t. initial position of C_1 is:



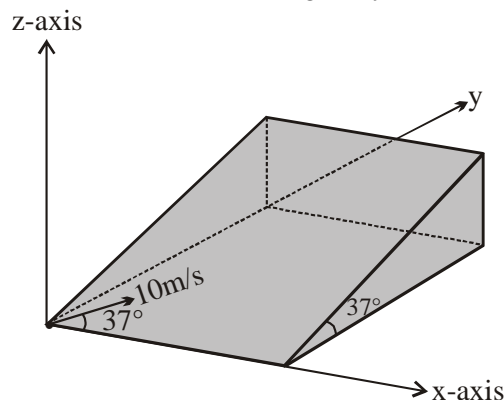
25. The graph correctly representing the speed (s) of the two cars with time (t) is:



26. During their motion, the car C_1 overtakes the car C_2 . But soon C_2 overtakes C_1 so determine the maximum lead that C_1 can have from C_2 .
 (A) 0.5 m (B) 9.5 m
 (C) 81 m (D) 90 m

Matrix Match Type Questions

27. A small ball is projected along the surface of a smooth inclined plane with speed 10 m/s along the direction shown at $t = 0$. The point of projection is origin, z-axis is along vertical. The acceleration due to gravity is 10 m/s^2 .



Column-I lists values of certain parameters related to motion of ball and column-II lists different time instants. Match appropriately.



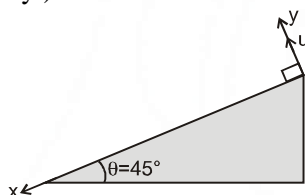
Column-I

- (A) Distance from x-axis is 2.25 m
(B) Speed is minimum
(C) Velocity makes angle 37° with x-axis

Column-II

- (P) 0.5 s
(Q) 1.0 s
(R) 1.5 s
(S) 2.0 s

28. An inclined plane makes an angle $\theta = 45^\circ$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at $t = 0$ sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)



Column I

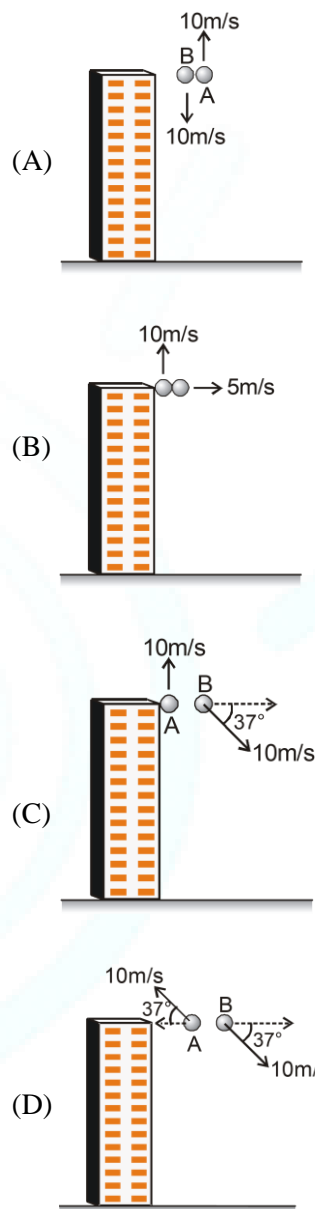
- (A) The instant of time at which velocity of stone is parallel to x-axis
(B) The instant of time at which velocity of stone makes an angle $\theta = 45^\circ$ with positive x-axis, in clockwise direction
(C) The instant of time till which (starting from $t = 0$) component of displacement along x-axis become half the range on inclined plane is
(D) Time of flight on inclined plane is

Column II

- (P) $\frac{2\sqrt{2}u}{g}$
(Q) $\frac{2u}{g}$
(R) $\frac{\sqrt{2}u}{g}$
(S) $\frac{u}{\sqrt{2}g}$

29. Both A & B are thrown simultaneously as shown from a very high tower.

Column I



Column II

- (P) Distance between the two balls at two seconds is $16\sqrt{5}$ m
(Q) Distance between two balls at 2 seconds is 40 m
(R) Distance between two balls at 2 sec is $10\sqrt{5}$ m
(S) Magnitude of relative velocity of B w.r.t A is $5\sqrt{2}$ m/s
(T) magnitude of relative velocity of B with respect to A is $5\sqrt{5}$ m/s



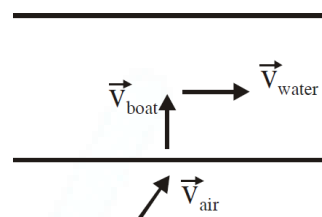
30. Column I

- (A) Time for a boat to cross a river of width ℓ by the shortest distance (\vec{v} -velocity of boat with respect to water; \vec{u} -velocity of water)
- (B) Time for two particles moving with velocities \vec{v} and \vec{u} in opposite directions to meet each other. (initial separation of particles is ℓ)
- (C) Time for a boat to cross a river of width ℓ in the shortest time (\vec{v} -velocity of boat with respect to water; \vec{u} -velocity of water)
- (D) Time for a boat to travel a distance ℓ downstream (\vec{v} -velocity of boat with respect to water; \vec{u} -velocity of water)

Column II

- (P) $\frac{\ell}{|\vec{v} + \vec{u}|}$
- (Q) $\frac{\ell}{\sqrt{v^2 - u^2}}$
- (R) $\frac{\ell}{|\vec{v}| + |\vec{u}|}$
- (S) $\frac{\ell}{|\vec{v}|}$
- (T) $\frac{\ell}{\sqrt{u^2 + v^2}}$

31. A boat is being rowed in a river. Air is also blowing. Direction of velocity vectors of boat, water and air in ground frame are as shown in diagram.



Column-I

- (A) Direction in which boat is being Steered
- (B) Direction in which a flag on the boat may flutter
- (C) Direction of velocity of water relative to boat
- (D) Direction of velocity of air relative to a piece of wood floating on river.

Column-II

- (P)
- (Q)
- (R)
- (S)





SUBJECTIVE EXERCISE - I

Basic 2-D

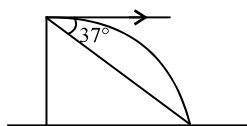
1. A particle of mass 10 kg moves in x-y plane such that its position is given by $(3 \sin t, 4 \cos t)$, where t is the time find
 - (i) its momentum vector at time $t = \pi$ sec.
 - (ii) net force acting on it at this time

Ground to Ground Projectile

2. A particle is projected upwards with a velocity of 100 m/sec at an angle of 60° with the vertical. Find the time of flight, H_{\max} and Range. Taking $g = 10 \text{ m/sec}^2$.
3. A particle is projected in the X-Y plane with y-axis along vertical. Two sec after projection the velocity of the particle makes an angle 45° with the X - axis. Four sec after projection, it moves horizontally. Find the velocity of projection.
4. A bullet is fired with speed 50 m/s at 45° angle find the height of the bullet when its direction of motion makes angle 30° with the horizontal.

Horizontal Projectile and Projection from Height

5. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find: (take $g = 9.8 \text{ m/s}^2$)
 - (i) The time taken to reach the ground
 - (ii) The distance of the target from the foot of hill
 - (iii) The velocity with which the particle hits the ground
6. A ball is thrown horizontally from a cliff such that it strikes ground after 5 sec. The line of sight from the point of projection to the point of hitting makes an angle of 37° with the horizontal. What is the initial velocity of projection.



7. From the top of a tower of height 50 m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. then calculate:

(i) Maximum height from the ground (ii) At what distance from the foot of the tower does the projectile hit the ground. (iii) Time of flight.

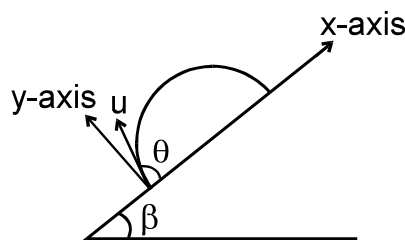
8. A Bomber flying upward at an angle of 53° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 sec after its release. Velocity (in m/s) of the bomber at the time of release of the bomb is: [Given $\sin 53^\circ = 0.8$; $g = 10 \text{ ms}^{-2}$]
9. A person standing on the top of a cliff 30 m high has to throw a packet to his friend standing on the ground 40 m horizontally away. If he throws the packet directly aiming at the friend with a speed of $\frac{125}{3} \text{ m/s}$. Packet falls at a distance $\frac{20}{\alpha} \text{ m}$ from the friend. Here α is an integer. Find α . [Use $g = 10 \text{ m/s}^2$].

Equation of Trajectory

10. The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$, where a and b are positive constants and \hat{i} and \hat{j} are the unit vectors of the x and y axes. Find:
 - (i) The equation of the point's trajectory $y(x)$; plot this function
 - (ii) The time dependence of the velocity v and acceleration a vector as well as of the moduli of these quantities.

Projection on Inclined Plane

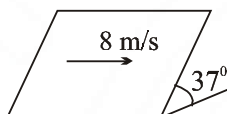
11. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u , after time t find:





- (a) x component of acceleration
- (b) y component of acceleration
- (c) x component of velocity as a function of time
- (d) y component of velocity as a function of time
- (e) x component of displacement
- (f) y component of displacement
- (g) y component of velocity when particle is at maximum distance from the incline plane?

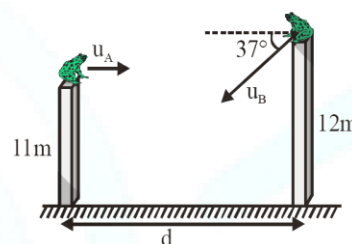
12. A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8 m/s. Find its speed after 1 sec.



Relative Motion

13. A train is moving with a speed of 40 km/h. As soon as another train going in the opposite direction passes by the window, the passenger of the first train starts his stopwatch and notes that other train passes the window in 3 s. Find the speed of the train going in the opposite direction if its length is 75 m.
14. An elevator is moving upward with a constant speed of 10 m/s. A man standing in the elevator drops a coin from a height of 2.5 m from floor of elevator. The coin reaches the floor of the elevator after a time $\frac{t}{\sqrt{2}}$ sec. What is the value of t ?
15. Two perpendicular rail tracks have two trains A & B respectively. Train A moves towards north with a speed of 54 km h⁻¹ and train B moves towards west with a speed of 72 km h⁻¹. Assume that both trains start from same point. Calculate the
- (a) Relative velocity of ground with respect to B.
 - (b) Relative velocity of A with respect to B.
 - (c) Rate of separation of the two trains.

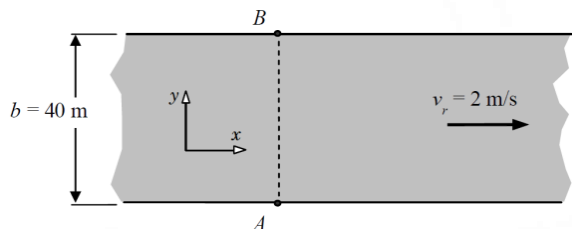
16. A man is swimming in a lake in a direction of 30° East of North with a speed of 5 km/h and a cyclist is going on a road along the lake shore towards East at a speed of 10 km/h. In what direction and with what speed would the man appear to swim to the cyclist.
17. Two frogs A & B jump simultaneously from top of two poles 11 m and 12 m high respectively, towards each other during a fight as shown in figure. They collide in mid air. If the velocities u_A and u_B are 2 m/s and 5 m/s respectively, the value of separation d , in meters is :



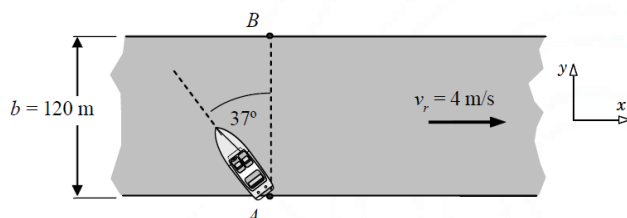
18. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. If the velocity of the swimmer with respect to water is $5\sqrt{n}$ m/s, then find n .
19. A man can swim with a speed of 4 km h⁻¹ in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h⁻¹ and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?
20. A man crosses a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find
- (i) width of the river.
 - (ii) velocity of the boat with respect to water.
 - (iii) speed of the current.
21. Boat moves with velocity 5 m/s on still water. It is steered perpendicular to the river current.



- Will it reach point B or somewhere else on the other bank?
- How long will it take to cross the river?
- How far down stream, will it reach the other bank?
- Does it take minimum time in this way?



22. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown. Where will it reach on the opposite bank?



23. An aeroplane has to go from a point A to another point B, 1000 km away due 30° west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. If the angle at which the pilot should head the plane to reach the point B is $\sin^{-1}(1/n)$ west of the line AB, then find n.

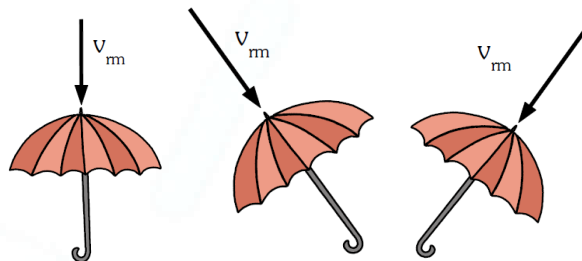
Comprehension Type Questions

Paragraph for Q. 24 to Q. 25

By the term velocity of rain, we mean velocity with which raindrops fall relative to the ground. In absence of wind, raindrops fall vertically and in presence of wind raindrops fall obliquely. Moreover raindrops acquire a constant terminal velocity due air resistance very quickly as they fall toward the earth. A moving man relative to himself observes an altered velocity of raindrops, which is known as velocity of rain relative to the man. It is given by the following equation.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

A standstill man relative to himself observes rain falling with velocity, which is equal to velocity of the raindrops relative to the ground. To protect himself a man should hold his umbrella against velocity of raindrops relative to himself as shown in the following figure.

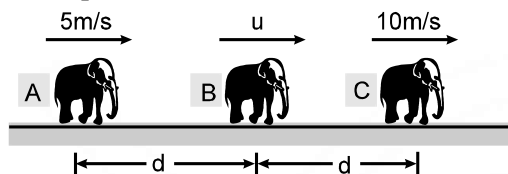


24. Rain is falling vertically with velocity 80 cm/s.
- How should you hold your umbrella?
 - You start walking towards the east with velocity 60 cm/s. How should you hold your umbrella?
 - You are walking towards the south with velocity 80 cm/s. How should you hold your umbrella?
25. When you are standstill in rain, you have to hold your umbrella vertically to protect yourself.
- When you walk with velocity 90 cm/s, you have to hold your umbrella at 53° above the horizontal.
What is velocity of the raindrops relative to the ground and relative to you?
 - If you walk with speed 160 cm/s, how should you hold your umbrella?

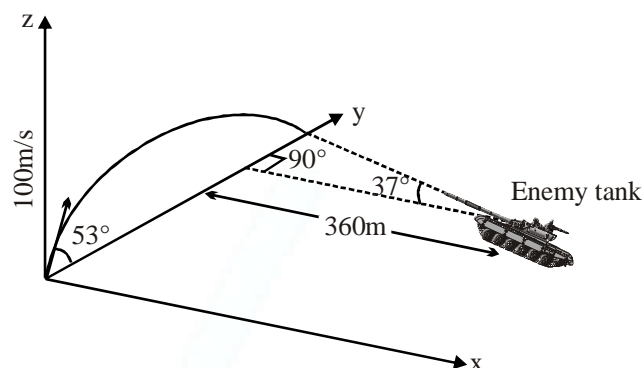


SUBJECTIVE EXERCISE - II

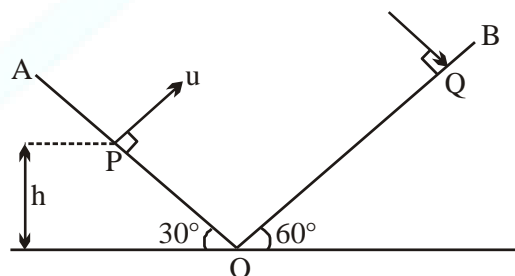
1. Three elephants A, B and C are moving along a straight line with constant speed in same direction as shown in figure. Speed of A is 5 m/s and speed of C is 10 m/s. Initially separation between A & B is 'd' and between B & C is also d. When 'B' catches 'C' separation between A & C becomes 3d. Find the speed of B (in m/s).



2. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined at an angle β . Find the horizontal component of velocity of projection in terms of g , t , α and β . (α and β are positive in anticlockwise direction)
3. A particle is projected from the ground level. It just passes through upper ends of vertical poles A, B, C of height 20 m, 30 m & 20 m respectively. The time taken by the particle to travel from B to C is double of the time taken from A to B. Find the maximum height attained by the particle from the ground level.
4. A man can throw a stone with initial speed of 10 m/s. Find the maximum horizontal distance to which he can throw the stone in a room of height h m for: (i) $h = 2$ m & (ii) $h = 4$ m
5. Figure shows an enemy tank moving with a speed of 9 m/s in direction shown in figure. A gun can fire shells in y - z plane only with a speed 100 m/s at an angle of 53° . Tank is initially at a perpendicular distance of 360 m from the plane of firing. If tank started at $t = 0$ then time interval (in sec) after which shell is to be fired to hit the tank.

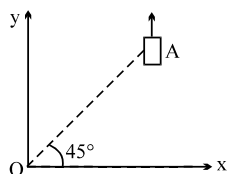


6. Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity $u = 10\sqrt{3} \text{ ms}^{-1}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q. Calculate
- Time of flight,
 - Velocity with which particle strikes the plane OB,
 - Vertical height h of P from O,
 - Maximum height from O attained by the particle and
 - Distance PQ



7. On a frictionless horizontal surface, assumed to be the x - y plane, a small trolley A is moving along a straight line parallel to the y -axis (see figure) with a constant velocity of $(\sqrt{3} - 1) \text{ m/s}$. At a particular instant, when the line OA makes an angle of 45° with the x -axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x -axis and it hits the trolley.

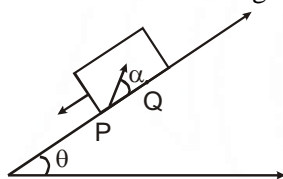




- (a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

- (b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$.

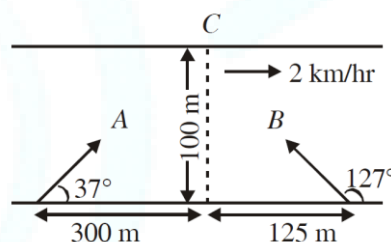
8. A large heavy box is sliding without friction down a smooth plane of inclination θ . From a point P on the bottom of the box, a particle is thrown inside box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure:



- (a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

- (b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

9. Two swimmers start a race. One who reaches the point C first on the other bank wins the race. Boy A makes his strokes in a direction of 37° to the river flow with velocity 5 km/hr relative to water. Boy B makes his strokes in a direction of 127° to the river flow with same relative velocity. River is flowing with speed of 2 km/hr and is 100 m wide. Who will win the race? Compute the time taken by A and B to reach the point C if the speeds of A and B on the ground are 8 km/hr and 6 km/hr respectively.



10. When you walk in rain at 75 cm/s, you have to hold your umbrella vertically and when you double your speed in the same direction, you have to hold your umbrella at 53° above the horizontal. What is the rain velocity?



JEE-MAIN (PREVIOUS YEAR QUESTIONS)

- A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where \hat{i} is along the ground and \hat{j} is along the vertical. If $g = 10 \text{ m/s}^2$, the equation of its trajectory is [JEE(Main)-2013]

(1) $y = 2x - 5x^2$ (2) $4y = 2x - 5x^2$
 (3) $4y = 2x - 25x^2$ (4) $y = x - 5x^2$
- A particle moves such that its position vector $\vec{r}(t) = \cos \omega t \hat{i} + \sin \omega t \hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle:

[JEE(Main)-2020]

(1) \vec{v} is perpendicular to \vec{r} and \vec{a} is directed towards the origin
 (2) \vec{v} and \vec{a} both are parallel to \vec{r}
 (3) \vec{v} and \vec{a} both are perpendicular to \vec{r}
 (4) \vec{v} is perpendicular to \vec{r} and \vec{a} is directed away from the origin
- A particle starts from the origin at $t = 0$ with an initial velocity of $3.0 \hat{i}$ m/s and moves in the x - y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j}) \text{ m/s}^2$. The x -coordinate of the particle at the instant when its y -coordinate is 32 m is D meters. The value of D is:-

[JEE(Main)-2020]

(1) 50 (2) 32
 (3) 60 (4) 40
- When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v , he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45° . The value of β is close to:

[JEE(Main)-2020]

(A) 0.41 (B) 0.50
 (C) 0.37 (D) 0.73
- The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by :- [JEE(Main)-2021]

(1) $\tan^{-1} \alpha, \frac{\alpha^2}{4\beta}$ (2) $\tan^{-1} \beta, \frac{\alpha^2}{2\beta}$
 (3) $\tan^{-1} \alpha, \frac{4\alpha^2}{\beta}$ (4) $\tan^{-1} \left(\frac{\beta}{\alpha} \right), \frac{\alpha^2}{\beta}$
- A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _____°. (Round off to the Nearest Integer) (find the angle in degree)

[JEE(Main)-2021]
- A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is ' x ' m/s. The value of ' x ' to the nearest integer is _____.

[JEE(Main)-2021]
- A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

[JEE(Main)-2021]

(1) 3 m (2) 20 m (3) $12\sqrt{2}$ m (4) 15 m
- A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1 . If he remains stationary on a moving escalator then the escalator takes him up in time t_2 . The time taken by him to walk up on the moving escalator will be :

[JEE(Main)-2021]

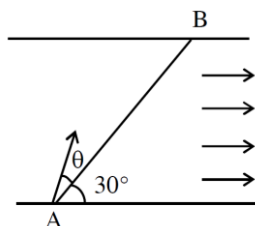
(1) $\frac{t_1 t_2}{t_2 - t_1}$ (2) $\frac{t_1 + t_2}{2}$ (3) $\frac{t_1 t_2}{t_2 + t_1}$ (4) $t_2 - t_1$





10. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ---° , so that the swimmer reaches point B.

[JEE(Main)-2021]



11. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take $g = 10 \text{ ms}^{-2}$)

[JEE(Main)-2021]

- (1) $h_{\text{max}} = 10\text{m}$ $T = 2.5 \text{ s}$
 (2) $h_{\text{max}} = 15.625 \text{ m}$ $T = 3.54 \text{ s}$
 (3) $h_{\text{max}} = 15.625 \text{ m}$ $T = 1.77 \text{ s}$
 (4) $h_{\text{max}} = 3.54 \text{ m}$ $T = 0.125 \text{ s}$

12. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

[JEE(Main)-2021]

- (1) $\sqrt{\frac{2ghv^2 + 1}{h^2}}$ (2) $\sqrt{2ghv^2 + h^2}$
 (3) $\sqrt{\frac{2v^2h}{g} + h^2}$ (4) $\sqrt{\frac{2gh}{v^2} + h^2}$

13. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1 , R_2 and H_1 , H_2 respectively. Choose the correct option :

[JEE(Main)-2021]

- (1) $R_1 > R_2$ and $H_1 = H_2$
 (2) $R_1 = R_2$ and $H_1 < H_2$
 (3) $R_1 < R_2$ and $H_1 < H_2$
 (4) $R_1 = R_2$ and $H_1 = H_2$

14. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at $t = 0\text{s}$. At $t = 2\text{s}$, another ball is projected vertically upward with same velocity. At $t = \text{---}$ s, second ball will meet the first ball ($g = 10 \text{ ms}^{-2}$)

[JEE(Main)-2022]

15. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be : [use $g = 10 \text{ m/s}^2$]

[JEE(Main)-2022]

- (1) $\frac{1}{2} \sin^{-1} \left(\frac{5t^2}{4R} \right)$ (2) $\frac{1}{2} \sin^{-1} \left(\frac{4R}{5t^2} \right)$
 (3) $\tan^{-1} \left(\frac{4t^2}{5R} \right)$ (4) $\cot^{-1} \left(\frac{R}{20t^2} \right)$

16. A projectile is launched at an angle ' α ' with the horizontal with a velocity 20 ms^{-1} . After 10 s , its inclination with horizontal is ' β '. The value of $\tan \beta$ will be: ($g = 10 \text{ ms}^{-2}$)

[JEE(Main)-2022]

- (1) $\tan \alpha + 5 \sec \alpha$ (2) $\tan \alpha - 5 \sec \alpha$
 (3) $2 \tan \alpha - 5 \sec \alpha$ (4) $2 \tan \alpha + 5 \sec \alpha$

17. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2} \text{ kmh}^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is :

[JEE(Main)-2022]

- (1) 30 kmh^{-1} (2) $\frac{25}{\sqrt{2}} \text{ kmh}^{-1}$
 (3) $\frac{30}{\sqrt{2}} \text{ kmh}^{-1}$ (4) 25 kmh^{-1}

18. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their range respectively will be

[JEE(Main)-2022]

- (1) $1 : \sqrt{2}$ (2) $\sqrt{2} : 1$
 (3) $2 : \sqrt{3}$ (4) $\sqrt{3} : 2$





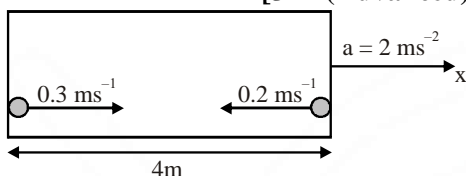
19. A ball is projected from the ground with a speed 15ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then 'tan θ ' will be equal to [JEE(Main)-2022]
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 2 (4) 4
20. Two projective thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is [JEE(Main)-2022]
(1) $1:\sqrt{2}$ (2) $2:1$ (3) $\sqrt{2}:1$ (4) $1:2$
21. A ball is projected with kinetic energy E , at an angle of 60° to the horizontal. The kinetic energy of this ball at the highest point of its flight will become : [JEE(Main)-2022]
(1) zero (2) $\frac{E}{2}$ (3) $\frac{E}{4}$ (4) E
22. The initial speed of a projectile fired from ground is u . At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$. The time of flight of the projectile is: [JEE(Main)-2023]
(1) $\frac{u}{2g}$ (2) $\frac{u}{g}$ (3) $\frac{2u}{g}$ (4) $\frac{\sqrt{3}u}{g}$
23. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5ms^{-1} . Neglecting the air resistance, the speed with which the stone hits the ground will be _____ ms^{-1} (Given, $g = 10\text{ms}^{-2}$). [JEE(Main)-2023]
(1) 20 (2) 15
(3) 30 (4) 25
24. Two bodies are projected from ground with same speeds 40ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60° , with horizontal then sum of the maximum heights, attained by the two projectiles, is _____ m. (Given $g = 10\text{ms}^{-2}$) [JEE(Main)-2023]
25. The speed of a swimmer is 4km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750m down the stream on the opposite bank. The speed of the river water is _____ km h^{-1} . [JEE(Main)-2023]



JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

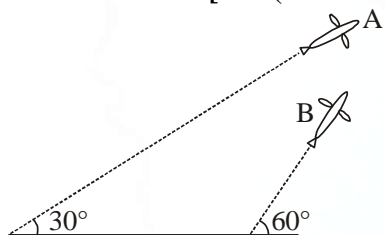
1. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is:

[JEE(Advanced)-2014]



2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3} \text{ ms}^{-1}$. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is

[JEE(Advanced)-2014]



3. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in meters, is _____.
- [JEE(Advanced)-2018]
4. A ball is thrown from ground at an angle θ with the horizontal and with an initial speed u_0 . For the resulting projectile motion, the magnitude of average velocity of the ball upto the point

when it hits the ground for the first time is V_1 . After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α . Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8V_1$, the value of α is _____.

[JEE(Advanced)-2019]



5. Starting at time $t = 0$ from the origin with speed 1 ms^{-1} , a particle follows a two-dimensional trajectory in the x - y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y , respectively. Then
- [JEE(Advanced)-2020]
- (A) $a_x = 1 \text{ ms}^{-2}$ implies that when the particle is at the origin, $a_y = 1 \text{ ms}^{-2}$
- (B) $a_x = 0$ implies $a_y = 1 \text{ ms}^{-2}$ at all times
- (C) at $t = 0$, the particle's velocity points in the x -direction
- (D) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the x axis is 45°
6. A projectile is fired from horizontal ground with speed v and projection angle θ . When the acceleration due to gravity is g , the range of the projectile is d . If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is $d' = nd$. The value of n is _____.
- [JEE(Advanced)-2022]
7. A particle of mass m is moving in the xy -plane such that its velocity at a point (x, y) is given as $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$, where α is a non-zero constant. What is the force \vec{F} acting on the particle?
- [JEE(Advanced)-2023]
- (A) $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$ (B) $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$
- (C) $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$ (D) $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$



ANSWER KEY

OBJECTIVE EXERCISE - I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	D	A	C	B	A	B	B	A	B	B	C	B	D	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	A	C	C	C	A	B	A	D	B	A	D	B	A	D
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	A	C	D	C	C	B	B	B	C	C	A	A	B	A
Que.	46	47	48	49	50	51	52	53							
Ans.	ABCD	ABD	ABCD	BCD	BCD	B	CD	ABC							

OBJECTIVE EXERCISE - II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	D	A	C	B	D	A	C	C	C	C	C	BC	BC	ABCD
Que.	16	17	18	19	20	21	22	23	24	25	26				
Ans.	ABCD	ABCD	AC	CD	BC	ABC	B	A	D	B	A				

27. (A) \rightarrow P, R; (B) \rightarrow Q; (C) \rightarrow S

28. (A) \rightarrow R; (B) \rightarrow S; (C) \rightarrow Q; (D) \rightarrow P

29. (A) \rightarrow Q; (B) \rightarrow R, T; (C) \rightarrow P; (D) \rightarrow Q

30. (A) \rightarrow P, Q; (B) \rightarrow R; (C) \rightarrow S; (D) \rightarrow P, R

31. (A) \rightarrow P; (B) \rightarrow Q, S; (C) \rightarrow S; (D) \rightarrow P, R

SUBJECTIVE EXERCISE - I

1. (i) $\vec{p} = 10(3 \cos \pi \hat{i} - 4 \sin \pi \hat{j}) = -30 \hat{i}$ Ns

(ii) $F = 40 \hat{j}$ N

2. 10 sec, 125 m, $500\sqrt{3}$ m

3. $20\sqrt{5}$ m/s

4. $\frac{125}{3}$ m

5. (i) 10 sec., (ii) 980 m, (iii) $98\sqrt{2}$ m/s

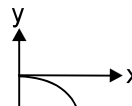
6. 100/3 m/s

7. (i) 61.25 m (ii) $75\sqrt{3}$ m \approx 130 m (iii) 5 sec.

8. 100

9. 3

10. (i) $y = -\frac{bx^2}{a^2}$



(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration = $-2b\hat{j}$,

$$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}, \quad |\text{acceleration}| = 2b$$

11. (a) $-g \sin \beta$,

(b) $-g \cos \beta$,

(c) $u \cos \theta - g \sin \beta \times t$,

(d) $u \sin \theta - g \cos \beta \times t$,

(e) $u \cos \theta \times t - \frac{1}{2}g \sin \beta \times t^2$, (f) $u \sin \theta \times t - \frac{1}{2}g \cos \beta \times t^2$, (g) zero.





12. 10 m/s
14. 1
13. 50 km/h
15. (a) 20 m/s or 72 km/h due east
(b) 25 m/s or 90 km/h at 37° N of E
(c) 25 m/s or 90 km/h
16. 30° N of W at $5\sqrt{3}$ km/h.
17. 2
18. 5
19. $\frac{1}{4}h, \frac{3}{4}km$
20. 200 m, 20 m/min, 12 m/min
21. (a) Somewhere down stream (b) 8 s (c) 16 m (d) Yes
22. 30 m upstream
23. 15
24. (a) Vertically (b) 53° above east (c) 45° above south
25. (a) 120 cm/s vertically 150 cm/s 53° above horizontal (b) 37° above the horizontal.

SUBJECTIVE EXERCISE - II

1. 15
2. $\frac{gt}{\tan \alpha - \tan \beta}$
3. $\frac{125}{4}m$
4. (i) $4\sqrt{6}$, (ii) 10m
5. 34 sec.
6. (a) 2 sec, (b) $10 ms^{-1}$, (c) 5 m, (d) 16.25 m, (e) 20 m
7. (a) 45° , (b) 2 m/sec
8. (a) $PQ = (u^2 \sin 2\alpha) / g \cos \theta$ (b) $v = \frac{u \cos(\alpha + \theta)}{\cos \theta}$
9. B, $t_A = 165$ s, $t_B = 150$ s
10. 125 cm/s at 37° from the vertical

JEE MAIN (PREVIOUS YEAR QUESTIONS)

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	3	4	1	120	5	4	3	30	3	3	2	6	4
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	2	3	3	4	3	3	2	2	80	3					

JEE ADVANCED (PREVIOUS YEAR QUESTIONS)

1. 8 or 2
5. (ABCD)
2. 5
6. 0.95
3. 30 m
7. (A)
4. 4



MOLE CONCEPTS AND CONCENTRATION TERMS

1. Classification of Matter

1.1 Physical classification:

It is based on physical state under ordinary conditions of temperature and pressure, matter is classified into the following three types:

(a) Solid (b) Liquid (c) Gas

(a) Solid:

A substance is said to be solid if it possesses a definite volume and a definite shape.

E.g. sugar, iron, gold, wood etc.

(b) Liquid:

A substance is said to be liquid if it possesses a definite volume but not definite shape.

They take up the shape of the vessel in which they are put.

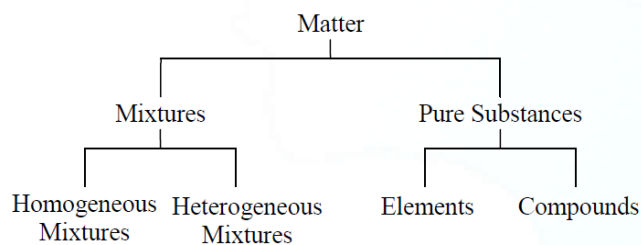
E.g. water, milk, oil, mercury, alcohol etc.

(c) Gas:

A substance is said to be gas if it neither possesses a definite volume nor a definite shape. This is because they fill up the whole vessel in which they are put.

E.g. hydrogen(H_2), oxygen(O_2), carbon dioxide(CO_2), etc.

1.2 Chemical classification:



On the basis of chemical nature matter is classified into the following two types:

(a) Pure Substance (b) Mixture

(a) Pure Substance

When all constituent particles of a substance are same in chemical nature, it is said to be a pure substance. Two type of pure substances:

(i) Element (ii) Compound

(i) Element:

an element consist of only one type of atoms. These particles may exist as Atoms or molecules. E.g. O_2 , P_4 , S_8 , etc.

(ii) Compound:

When two or more atoms of different elements combine together in a definite ratio. E.g. water, ammonia, carbon dioxide, sugar, etc.

(b) Mixture

A mixture contains many types of particles. A mixture contains particles of two or more pure substances which may be present in it in any ratio. Hence, their composition is variable. Pure substances forming mixture are called its components. Many of the substances present around you are mixtures. For example, sugar solution in water, air, tea, etc., are all mixtures.

Two types of mixtures:

(i) Homogeneous mixture

(ii) Heterogeneous mixture

(i) Homogeneous mixture

The components completely mix with each other. This means particles of components of the mixture are uniformly distributed throughout the bulk of the mixture and its composition is uniform throughout. E.g.: Sugar solution, air

(ii) Heterogeneous mixture

In a heterogeneous mixture, the composition is not uniform throughout and sometimes different components are visible. E.g.: mixtures of salt and sugar, grains and pulses along with some dirt (often stone pieces)

2. Introduction

There are a large number of objects around us which we can see and feel. It was John Dalton who firstly developed a theory on the structure of matter, later on which is known as Dalton's atomic theory.

2.1. Dalton's Atomic Theory

(i) Each element is composed of extremely small particles called atoms which can take part in chemical combination.

(ii) All atoms of a given element are identical i.e., atoms of a particular element are all alike but differ from atoms of other elements.



- (iii) Atoms of different elements possess different properties (including different masses).
- (iv) Atoms are indestructible i.e., atoms are neither created nor destroyed in chemical reactions.
- (v) Atoms of elements combine to form molecules and compounds are formed when atoms of more than one element combine.
- (vi) In a given compound, the relative number and kind of atoms is constant.

2.2. Atomic mass

It is the average relative mass of atom of element as compared with times the mass of an atom of carbon-12 isotope.

$$\text{Atomic mass} = \frac{\text{Average mass of an atom}}{1/12 \times \text{Mass of an atom of } C^{12}}$$

2.3. Gram atomic mass (GAM)

Atomic mass of an element expressed in grams is called Gram atomic mass or gram atom or mole atom.

- (i) Number of gram atoms = $\frac{\text{Mass of an element}}{\text{GAM}}$
- (ii) Mass of an element in g = No. of gram atoms \times GAM
- (iii) Number of atoms in 1 GAM = 6.02×10^{23}
Number of atoms in a given substance = No. of gram atoms $\times 6.02 \times 10^{23}$
- (iv) Number of atoms in 1 g of element = $\frac{\text{Mass}}{\text{GAM}}$
- (v) Mass of one atom of the element (in g) = $\frac{\text{GAM}}{6.02 \times 10^{23}}$

2.4. Molecular mass

Molecular mass of a molecule, of an element or a compound may be defined as a number which indicates how many times heavier is a molecule of that element or compound as compared with $\frac{1}{12}$ of the mass of an atom of carbon-12. Molecular mass is also expressed in a.m.u.

Molecular mass

$$\frac{\text{Mass of one molecule of the substance}}{1/12 \times \text{Mass of one atom of } C - 12}$$

$$\text{Actual mass of one molecule} = \text{Mol. mass (in amu)} \times 1.66 \times 10^{-24} \text{ g}$$

Molecular mass of a substance is the additive property and can be calculated by adding the atomic masses of atoms present in one molecule.

2.5. Gram molecular mass (GMM)

Molecular mass of an element or compound when expressed in g is called its gram molecular mass, gram molecule or mole molecule.

$$\text{Number of gram molecules} = \frac{\text{Mass of substance}}{\text{GMM}}$$

$$\text{Mass of substance in g} = \text{No. of gram molecules} \times \text{GMM}$$

Element	R.A.M (Relative Atomic Mass)	Mass of One Atom	Gram Atomic mass/weight
N	14	14 amu	14 gm
He	4	4 amu	4 gm
C	12	12 amu	12 gm

2.6. Mole

One mole of any substance contains a fixed number (6.023×10^{23}) of any type of particles (atoms or molecules or ions) and has a mass equal to the atomic or molecular weight, in grams. Thus it is correct to refer to a mole of helium, a mole of electrons or a mole of any ion, meaning respectively Avogadro's number of atoms, electrons or ions.

Methods of Calculations of mole:

- (1) If no. of some species is given, then no. of moles = $\frac{\text{Given no}}{N_A}$
- (2) If weight of a given species is given, then no. of moles = $\frac{\text{Given wt.}}{\text{Atomic wt.}}$ (for atoms),
or = $\frac{\text{Given wt}}{\text{Molecular wt.}}$ (for molecules)
- (3) If volume of a gas is given along with its temperature (T) and pressure (P).
 $PV = nRT$ (n: Number of moles of gas)





P (Pressure of gas): Pressure of the gas is the force exerted by the gas per unit area of the walls of the container in all directions.

Thus, Pressure (P)

$$= \frac{\text{Force(F)}}{\text{Area(A)}} = \frac{\text{Mass(m)} \times \text{Acceleration(a)}}{\text{Area(A)}}$$

Name	Symbol	Value
Bar	bar	1 bar = 10^5 Pa
Atmosphere	atm	1 atm = 1.01325×10^5
Torr	Torr	1 Torr = $\frac{101325}{760}$ Pa \times 133.322 Pa
millimeter of mercury	mm Hg	1 mm Hg = 133.322 Pa

V (Volume of gas): Volume is expressed in liters (L), milliliters (mL) or cubic centimeters (cm^3), cubic meters (m^3).

$$1 \text{ m} = 10^3 \text{ dm} = 10^6 \text{ cm}^3 = 10^6 \text{ mL} = 10^3 \text{ L}$$

T (Temperature of gas): S.I. unit of temperature is Kelvin (K)

$$T(\text{K}) = t(^{\circ}\text{C}) + 273.15$$

R (Universal gas constant): Values of R = 0.082 $\text{LatmK}^{-1} \text{mol}^{-1}$
= $8.314 \text{ JK}^{-1} \text{mol}^{-1}$
= $1.987 \text{ CalK}^{-1} \text{mol}^{-1}$

Note:

1 mole of atom is also termed as 1 g-atom

1 mole of ions is also termed as 1 g-ion

1 mole of molecule is also termed as 1 g-molecule

Example:

How much time (in years) would it take to distribute one Avogadro number of wheat grains if 1010 grains are distributed each second?

Ans. 1.9×10^6 years (approx)

Solution:

1010 grains are distributed in 1 second

6.02×10^{23} grains are distributed in

$$\frac{6.02 \times 10^{23}}{10^{10}} \text{ sec} = \frac{6.02 \times 10^{23}}{10^{10} \times 60 \times 60 \times 24 \times 365} \text{ years}$$

$$= 1.9 \times 10^6 \text{ years (approx.)}$$

Example:

How many atoms are there in 100 amu of He ?

Ans. 25

Solution:

We know that, $1 \text{ amu} = \frac{1}{12} \times \text{weight of one } ^{12}\text{C atom}$

or weight of one $^{12}\text{C atom} = 12 \text{ amu}$ (at. wt. of C = 12 amu).

Similarly, as the atomic weight of He is 4

weight of one He atom = 4 amu.

Thus, the number of atoms in 100 amu of He =

$$\frac{100}{4} = 25.$$

Example:

The weight of one atom of Uranium is 238 amu. Its actual weight in gm is _____:

Ans. 396.74×10^{-4}

Solution:

Weight of one atom = 238 amu

$$= 238 \times 1.667 \times 10^{-24} = 396.74 \times 10^{-4}$$

Example:

Calculate the number of molecules in a drop of water weighing 0.09 g.

Ans. 3.01×10^{21} molecules of H_2O

Solution:

$$\text{Number of mole} = \frac{0.09}{18}$$

$$\text{So number of molecules} = \frac{0.09}{18} \times N_A = 3.01 \times 10^{21}.$$

Example:

A sample of ethane has the same mass as 10.0 million molecules of methane. How many C_2H_6 molecules does the sample contain?

Ans. 5.34×10^6

Solution:

Let the number of C_2H_6 molecules in the sample be n.

As given, mass of C_2H_6 = mass of 10^7 molecules of CH_4

$$\frac{n}{N_A} \times \text{mol. wt. of } \text{C}_2\text{H}_6 = \frac{10^7}{N_A} \times \text{mol. wt. of } \text{CH}_4$$

$$\frac{n}{N_A} \times 30 = \frac{10^7}{N_A} \times 16$$

$$n = 5.34 \times 10^6.$$



**Example:**

If, from 10 moles NH_3 and 5 moles of H_2SO_4 , all the H-atoms are removed in order to form H_2 gas, then find the number of H_2 molecules formed.

Ans. $20 N_A$

Solution:

10 mole NH_3 have mole of 'H' atom = 10×3

5 mole of H_2SO_4 have mole of 'H' atom = 10

Total mole of 'H' atom = 40

Mole of H_2 = 20

Example:

The weight of 350 mL of a diatomic gas at 0°C and 2 atm pressure is 1 g. The weight of one atom is:

Ans. 16 amu

Solution:

$$PV = nRT; n = \frac{PV}{RT}$$

$$n = \frac{2 \times 0.350}{0.0821 \times 273} =$$

$$n = \frac{\text{Weight}}{\text{Atomic mass}}$$

Atomic mass = 16 amu

Example:

Oxygen is present in a 1-litre flask at a pressure of 7.6×10^{-10} mm of Hg at 0°C . Calculate the number of oxygen molecules in the flask.

Ans. 0.44×10^{-13}

Solution:

Pressure = 7.6×10^{-10} mm = 0.76×10^{-10} cm

= atm (1 atm = 76 cm) = 10^{-12} atm.

Volume = 1 litre, $R = 0.0821$ lit. atm/K/mole, temperature = 273 K.

We know that $PV = nRT$ or $n = \frac{pV}{RT}$

$$n = \frac{10^{-12} \times 1}{0.082 \times 273} = 0.44 \times 10^{-13}$$

Example:

Equal volumes of oxygen gas and a second gas weigh 1.00 and 2.375 grams respectively under the same experimental conditions. Which of the following is the unknown gas?

- (A) NO (B) SO_2
(C) CS_2 (D) CO

Ans. (C)

Solution:

Moles of O_2 = Moles of X(unknown gas)

$$\frac{1}{32} = \frac{2.375}{M_x}; M_x = 76$$

FUNDAMENTAL UNLOCKED- (FU#1) :

- Q.1** The number of molecules in 16 g of methane is:
- Q.2** A sample of aluminium has a mass of 54.0 g. What is the mass of the same number of magnesium atoms? (At. wt. Al = 27, Mg = 24)
(A) 12 g (B) 24 g
(C) 48 g (D) 96 g
- Q.3** Find the total number of H, S and 'O' atoms in the following:
(i) 196 gm H_2SO_4 (ii) 196 amu H_2SO_4
(iii) 5 mole $\text{H}_2\text{S}_2\text{O}_8$
(iv) 3 molecules $\text{H}_2\text{S}_2\text{O}_6$
- Q.4** The volume of a gas at 0°C and 700 mm pressure is 760 cc. The number of molecules present in this volume is:
- Q.5** Four 1-litre flasks are separately filled with the gases H_2 , He, O_2 and O_3 at the same temperature and pressure. The ratio of total number of atoms of these gases present in different flask would be:
(A) 1: 1: 1: 1 (B) 1: 2: 2: 3
(C) 2: 1: 2: 3 (D) 3: 2: 2: 1
- Q.6** The weight of 2.01×10^{23} molecules of CO is—
(A) 9.3 g (B) 7.2 g
(C) 1.2 g (D) 3 g
- Q.7** How many moles of e^- weight one Kg:
(A) 6.023×10^{23} (B) $\frac{1}{9.108} \times 10^{31}$
(C) $\frac{6.023}{9.108} \times 10^{54}$ (D) $\frac{1}{9.108 \times 6.023} \times 10^8$

3. Laws of Chemical Combination:

Antoine Lavoisier, John Dalton and other scientists formulate certain laws concerning the composition of matter and chemical reactions. These laws are known as the law of chemical combination.

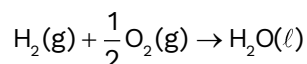
3.1 The Law of Conservation of Mass:

It is given by Antoine Lavoisier.

In a chemical change total mass remains conserved that is mass before reaction is always equal to mass after reaction.



Example:



Before reaction initially 1 mole $\frac{1}{2}$ mole

After the reaction 0 0 1 mole
mass before reaction = mass of 1 mole $\text{H}_2(\text{g})$ + mass
of $\frac{1}{2}$ mole $\text{O}_2(\text{g})$
= 2 + 16 = 18 g

mass after reaction = mass of 1 mole water = 18 g

3.2 Law of Constant or Definite Proportion

It is given by Proust. All chemical compounds are found to have constant composition irrespective of their method of preparation or sources.

Example:

In water (H_2O), Hydrogen and Oxygen combine in 2:1 molar ratio, the ratio remains constant whether it is tap water, river water or sea water or produced by any chemical reaction.

Example:

1.80 g of a certain metal burnt in oxygen gave 3.0 g of its oxide. 1.50 g of the same metal heated in steam gave 2.50 g of its oxide. Show that these results illustrate the law of constant proportion.

Solution:

In the first sample of the oxide,

wt. of metal = 1.80 g.

$$\text{wt. of oxygen} = (3.0 - 1.80) \text{ g} = 1.2 \text{ g}$$

$$\frac{\text{wt. of metal}}{\text{wt. of oxygen}} = \frac{1.80\text{g}}{1.2\text{g}} = 1.5$$

In the second sample of the oxide,

$$\frac{\text{wt. of metal}}{\text{wt. of oxygen}} = \frac{1.50 \text{ g}}{1 \text{ g}} = 1.5$$

Thus, in both samples of the oxide the proportions of the weights of the metal and oxygen are fixed. Hence, the results follow the law of constant proportion.

3.3 The Law of Multiple Proportions:

It is given by Dalton. When one element combines with the other element to form two or more different compounds, the mass of one element, which combines with a constant mass of the other, bear a simple ratio to one another.

Note: Simple ratio here means the ratio between small natural numbers, such as 1: 1, 1: 2, 1: 3, later on this simple ratio becomes the valency and then oxidation state of the element.

Example:

Carbon and Oxygen when combine, can form two oxides, CO (carbon monoxide), CO₂ (Carbon dioxides)

In CO, 12 g carbon combined with 16 g of oxygen.

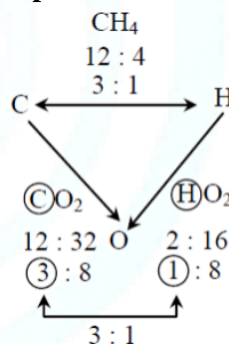
In CO_2 , 12 g carbon combined with 32 g of oxygen.

Thus, we can see the mass of oxygen which combine with a constant mass of carbon (12 g) bear simple ratio of 16: 32 or 1: 2.

3.4 Law of Reciprocal Proportion: It is given by Richter.

The ratio of the weights of two elements A and B which combine separately with a fixed weight of the third element C is either the same or simple ratio of the weights in which A and B combine directly with each other.

Example:



Example:

Ammonia contains 82.35% of nitrogen and 17.65% of hydrogen. Water contains 88.90% of oxygen and 11.10% of hydrogen. Nitrogen trioxide contains 63.15% of oxygen and 36.85% of nitrogen. Show that these data illustrate the law of reciprocal proportions.

Solution:

In NH_3 , 17.65g of H combine with N = 82.35g

$$1 \text{ g of H combine with N } \frac{82}{17.65} \text{ g} = 4.67 \text{ g}$$

In H_2O , 11.10 g of H combine with O = 88.90 g

$$1 \text{ g of H combine with O } \frac{88.90}{11.10} \text{ g} = 8.01 \text{ g}$$

Ratio of the weights of N and O which combine with
fixed weight (=1g) of
= 4.67 : 8.01 = 1 : 1.7



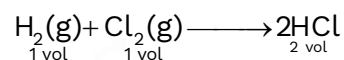
In N_2O_3 , ratio of weights of N and O which combine with each other = 36.85: 63.15

= 1: 1.7

Thus the two ratios are the same. Hence it illustrates the law of reciprocal proportions.

3.5 Gay-Lussac's Law of Combining Volume

According to him elements combine in a simple ratio of atoms, gases combine in a simple ratio of their volumes provided all measurements should be done in the same temperature and pressure.



3.6 Avogadro's Hypothesis

Equal volumes of polyatomic all gases have equal number of molecules (not atoms) at same temperature and pressures conditions.

S.T.P. (Standard Temperature and Pressure)

At S.T.P. condition:

Temperature = 0°C or 273 K

Pressure = 1 bar

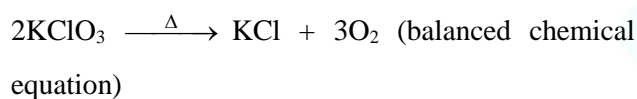
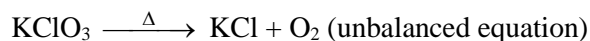
And volume of one mole of gas at STP is found to be equal to 22.7 litres which is known as molar volume.

4. Stoichiometry and Stoichiometric Calculations

The word 'stoichiometry' is derived from two Greek words — stoicheion (meaning, element) and metron (meaning, measure). Stoichiometry, thus, deals with the calculation of masses (sometimes volumes also) of the reactants and the products involved in a chemical reaction

Example:

When potassium chlorate (KClO_3) is heated it gives potassium chloride (KCl) and oxygen (O_2).



Remember a balanced chemical equation is one which contains an equal number atoms of each element on both sides of equation.

4.1 Interpretation of Balanced Chemical Equations:

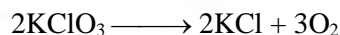
Once we get a balanced chemical equation then we can interpret a chemical equation by following ways

- (a) Mass - mass analysis
- (b) Mass - volume analysis
- (c) Mole - mole analysis

(a) Mole-Mole Analysis:

This analysis is very much important for quantitative analysis point of view.

Now consider again the decomposition of KClO_3 .



In very first step of mole-mole analysis you should read the balanced chemical equation like 2 moles KClO_3 on decomposition gives you 2 moles KCl and 3 moles O_2 and from the stoichiometry of reaction we can write

$$\frac{\text{Moles of } \text{KClO}_3}{2} = \frac{\text{Moles of } \text{KCl}}{2} = \frac{\text{Moles of } \text{O}_2}{3}$$

Now for any general balance chemical equation like $a\text{A} + b\text{B} \longrightarrow c\text{C} + d\text{D}$ you can write.

$$\frac{\text{Moles of A reacted}}{a} = \frac{\text{Moles of B reacted}}{b} = \frac{\text{Moles of C produced}}{c} = \frac{\text{Moles of D produced}}{d}$$

(b) Mass-Mass Analysis:

Consider the reaction $2\text{KClO}_3 \longrightarrow 2\text{KCl} + 3\text{O}_2$ According to stoichiometry of the reaction or

$$\frac{\text{Mass of } \text{KClO}_3}{2 \times 122.5} = \frac{\text{Mass of } \text{KCl}}{2 \times 74.5}$$

$$\frac{\text{Mass of } \text{KClO}_3}{\text{Mass of } \text{O}_2} = \frac{2 \times 122.5}{3 \times 32}$$

Example:

Consider the balanced reaction:



What can be concluded from the coefficients of species in this balanced equation?

- (A) For this reaction, exactly 2 g of Cl_2O_7 must be taken to start the reaction
- (B) For this reaction, exactly 2 mol of Cl_2O_7 must be taken to start the reaction





(C) Mole ratio of Cl_2O_7 , ClO_2 and O_2 during a chemical reaction at any instant are 2, 4 and 3 respectively

(D) The ratio of change in number of moles of Cl_2O_7 , ClO_2 and O_2 is 2: 4: 3

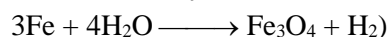
Ans. (D)

Solution:

It follows directly from definition of stoichiometry.

Example:

Calculate the weight of iron which will be converted into its oxide by the action of 36 g of steam. (Given:



Ans. 84 g

Solution:

Mole ratio of reaction suggests,

$$\text{Mole of Fe} = \frac{3}{4} \text{ mol of H}_2\text{O}$$

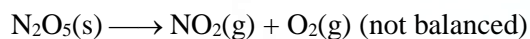
$$= \frac{3}{4} \times \frac{36}{18} = \frac{3}{2}$$

$$\text{Wt. of Fe} = \frac{3}{2} \times 56 = 84 \text{ g}$$

Example:

When Dinitrogen pentaoxide N_2O_5 , a white solid is heated, it decomposes into nitrogen dioxide and oxygen.

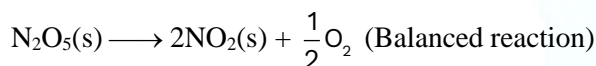
If a sample of N_2O_5 produces 1.6 g O_2 , then how many grams of NO_2 are formed ?



- (A) 9.2 g (B) 4.6 g
(C) 2.3 g (D) 18.4 g

Ans. (A)

Solution:



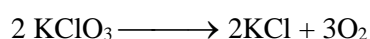
$$\frac{\text{Mole of O}_2}{1/2} = \frac{\text{Mole of NO}_2}{2}$$

$$\frac{1.6}{32} \times 2 \times 2 = \text{Mole of NO}_2 = 0.2$$

$$\text{wt. of NO}_2 = 0.2 \times 46 = 9.2 \text{ g}$$

(c) Mass - volume analysis:

Now again consider decomposition of KClO_3



mass volume ratio: $2 \times 122.5 \text{ g} : 2 \times 74.5 \text{ g} : 3 \times 22.4 \text{ L at STP}$

we can use two relation for volume of oxygen

$$\frac{\text{Mass of KClO}_3}{\text{volume O}_2 \text{ at STP}} = \frac{2 \times 122.5 \text{ g}}{3 \times 22.4 \text{ L}} \dots(i)$$

$$\text{and } \frac{\text{Mass of KCl}}{\text{volume O}_2 \text{ at STP}} = \frac{2 \times 74.5 \text{ g}}{3 \times 22.4 \text{ L}} \dots(ii)$$

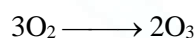
Example:

When oxygen gas is passed through Siemen's ozoniser, it completely gets converted into ozone gas. The volume of ozone gas produced at 1 atm and 273K, if initially 96 g of oxygen gas was taken, is:

- (A) 44.8 L (B) 89.6 L
(C) 67.2 L (D) 22.4 L

Ans. (A)

Solution:

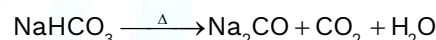


$$\text{Mole} = \frac{96}{32} = 3 \quad \text{mole} = 2$$

$$\text{Volume of O}_3 \text{ gas at 1 atm and 273K} = 2 \times 22.4 = 44.8 \text{ L}$$

FUNDAMENTAL UNLOCKED- (FU#2) :

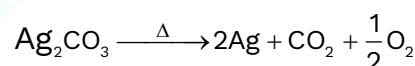
Q.1 Assuming 100% yield of the reaction, how many moles of NaHCO_3 will produce 448 mL of CO_2 gas at STP according to the reaction:



(unbalanced)

- (A) 0.04 (B) 0.4
(C) 4 (D) 40

Q.2 Calculate the residue obtained on strongly heating 2.76 g Ag_2CO_3 .



Q.3 For the reaction $2\text{P} + \text{Q} \longrightarrow \text{R}$, 4 mol of P and excess of Q will produce:

- (A) 8 mol of R (B) 5 mol of R
(C) 2 mol of R (D) 1 mol of R

Q.4 If 1.5 moles of oxygen combine with Al to form Al_2O_3 , the weight of Al used in the reaction is:

- (A) 27 g (B) 40.5 g
(C) 54g (D) 81 g

Q.5 How many liters of CO_2 at STP will be formed when 0.01 mol of H_2SO_4 reacts with excess of Na_2CO_3 . $\text{Na}_2\text{CO}_3 + \text{H}_2\text{SO}_4 \longrightarrow \text{Na}_2\text{SO}_4 + \text{CO}_2 + \text{H}_2\text{O}$

- (A) 22.7 L (B) 2.27 L
(C) 0.227 L (D) 1.135 L



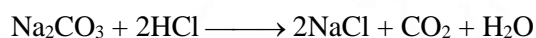


4.2 Limiting Reagent

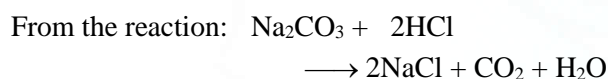
Many a time, reactions are carried out with the amounts of reactants that are different than the amounts as required by a balanced chemical reaction. In such situations, one reactant is in more amount than the amount required by balanced chemical reaction. The reactant which is present in the least amount gets consumed after sometime and after that further reaction does not take place whatever be the amount of the other reactant. Hence, the reactant, which gets consumed first, limits the amount of product formed and is, therefore, called the limiting reagent. The reactant which consumed first into the reaction when we are dealing with balance chemical equation then if number of moles of reactants are not in the ratio of stoichiometric coefficient of balanced chemical equation, then there should be one reactant which should be limiting reactant.

Example:

Six mole of Na_2CO_3 is reacted with 4 moles of HCl solution. Find the volume of CO_2 gas produced at STP. The reaction is:



Solution:



gives moles 3 mol 6 mol

given mole ratio 1 : 2

Stoichiometric coefficient ratio 1 : 2

See here given number of moles of reactants are not in stoichiometric coefficient ratio.

Therefore there should be one reactant which consumed first and becomes limiting reagent. But the question is how to find which reactant is limiting, it is not very difficult you can easily find it. According to the following method.

How to find limiting reagent

Step: I

Divide the given moles of reactant by the respective stoichiometric coefficient of that reactant.

Step: II

See for which reactant this division comes out to be minimum. The reactant having minimum value is limiting reagent.

Step: III

Now once you find limiting reagent then your focus should be on limiting reagent

From Step I & II Na_2CO_3 HCl

$$\frac{6}{1} = 6 \quad \frac{4}{2} = 2 \text{ (Division in minimum)}$$

\therefore HCl is limiting reagent

From Step III

$$\text{From } \frac{\text{Mole of HCl}}{2} = \frac{\text{Mole of CO}_2 \text{ produced}}{1}$$

\therefore Mole of CO_2 produced = 2 moles

\therefore Volume of CO_2 produced at S.T.P. = 2×22.7
= 45.4 L

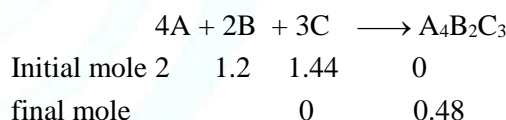
Example:

In the reaction $4\text{A} + 2\text{B} + 3\text{C} \longrightarrow \text{A}_4\text{B}_2\text{C}_3$ what will be the number of moles of product formed, starting from 2 moles of A, 1.2 moles of B & 1.44 moles of C:

- (A) 0.5 (B) 0.6
(C) 0.48 (D) 4.64

Ans. (C)

Solution:



C is limiting reagent

moles of $\text{A}_4\text{B}_2\text{C}_3$ is 0.48

Example:

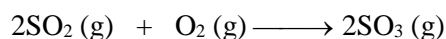
A 5 g mixture of SO_2 and O_2 gases is reacted to form SO_3 gas. What should be the mass ratio of SO_2 and O_2 gases in mixture to obtain maximum amount of SO_3 gas:

- (A) 4: 1 (B) 3: 2
(C) 2: 3 (D) 1: 4

Ans. (A)

Solution:

For maximum amount of product, the reactants should be present in their stoichiometric ratio.



Mass x 5 - x

Mole $\frac{x}{64}$ $\frac{5-x}{32}$



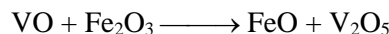
$$\text{So, } \frac{\left(\frac{x}{64}\right)}{\left(\frac{5-x}{32}\right)} = 2 : 1$$

Therefore, $x = 4$

$$m_{\text{SO}_2} : m_{\text{O}_2} = 4 : 1$$

Example:

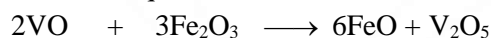
Calculate the weight of FeO from 4 g VO and 5.75 g of Fe_2O_3 . Also report the limiting reactant.



Ans. Weight of FeO formed = 5.17 g

Solution:

Balanced equation



Moles before reaction

$$\begin{array}{ccccccc} \frac{4}{67} & \frac{5.75}{160} & 0 & & 0 & & \\ = & 0.05970 & 0.03590 & & & & \\ \text{Moles after reaction} & (0.05970 - 0.0359) & 0 & & & & \\ \left(\frac{6}{5} \times 0.0359\right) & \left(\frac{1}{3} \times 0.0359\right) & & & & & \end{array}$$

As 2 moles of VO react with 3 mole of Fe_2O_3

$$\therefore 0.05970 \text{ g mole of VO} = \frac{3}{2} 0.05970 = 0.08955$$

moles of Fe_2O_3

Mole of Fe_2O_3 available = 0.0359 only

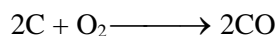
$$\text{Mole of FeO formed} = \frac{6}{3} 0.0359$$

$$\therefore \text{Weight of FeO formed} = 0.0359 \times 2 \times 72 = 5.17 \text{ g}$$

$$\left(\frac{n_{\text{FeO}}}{n_{\text{Fe}_2\text{O}_3}} = \frac{6}{3}\right) \Rightarrow n_{\text{FeO}} = \frac{6}{3} \times n_{\text{Fe}_2\text{O}_3}$$

FUNDAMENTAL UNLOCKED- (FU#3) :

Q.1 The reaction



is carried out by taking 24 g of carbon and 128 g of O_2 .

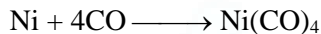
Find out:

- Which reactant is left in excess?
- How much of it is left?
- How many moles of CO are formed?
- How many grams of other reactant should be taken so that nothing is left at the end of reaction?

Q.2 How many mole of $\text{Zn}(\text{FeS}_2)$ can be made from 2 mole zinc, 3 mole iron and 5 mole sulphur.

- (A) 2 mole (B) 3 mole
(C) 4 mole (D) 5 mole

Q.3 Calculate the amount of Ni needed in the Mond's process given below



If CO used in this process is obtained through a process, in which 6 g of carbon is mixed with 44 g CO_2 ($\text{Ni} = 59 \text{ u}$)

- (A) 14.675 g (B) 29 g
(C) 58 g (D) 28 g

Q.4 0.5 mole of H_2SO_4 is mixed with 0.2 mole of $\text{Ca}(\text{OH})_2$. The maximum number of moles of CaSO_4 formed is:

- (A) 0.2 (B) 0.5
(C) 0.4 (D) 1.5

Q.5 The mass of Na_2SO_4 produced from 196 gram of H_2SO_4 and 1 mole of NaOH.

- (A) 49 g (B) 98 g
(C) 71 g (D) 34.3 g

4.3 Principle of Atom Conservation (POAC)

Infect POAC is nothing but the conservation of mass, expressed before in the concepts of atomic theory. And if atoms are conserved, moles of atoms shall also be conserved.

The principle is fruitful for the students when they don't get the idea of balanced chemical equation in the problem.

This principle can be under stand by the following example.

Consider the decomposition of



(unbalanced chemical react ion)

Apply the principle of atom conservation (POAC) for K atoms.

Moles of K atoms in reactant = moles of K atoms in products

or moles of K atoms in KClO_3 = moles of K atoms in KCl

Now, since 1 molecule of KClO_3 contains 1 atom of K

or 1 mole of KClO_3 contains 1 mole of K, similarly 1 mole of KCl contains 1 mole of K



Thus, moles of K atoms in $\text{KClO}_3 = 1 \times \text{moles of KClO}_3$

and moles of K atoms in $\text{KCl} = 1 \times \text{moles of KCl}$

$\therefore \text{moles of KClO}_3 = \text{moles of KCl}$

$$\text{or } \frac{\text{wt. of KClO}_3 \text{ in g}}{\text{mol. wt. of KClO}_3} = \frac{\text{wt. of KCl in g}}{\text{mol. wt. of KCl}} *$$

The above equation gives the mass-mass relationship between KClO_3 and KCl which is important in stoichiometric calculations.

Again, applying the principle of atom conservation for O atoms,

moles of O in $\text{KClO}_3 = 3 \times \text{moles of KClO}_3$

moles of O in $\text{O}_2 = 2 \times \text{moles of O}_2$

$\therefore 3 \times \text{moles of KClO}_3 = 2 \times \text{moles of O}_2$

$$\text{or } \frac{\text{wt. of KClO}_3}{\text{mol. wt. of KClO}_3} = 2 \times \frac{\text{vol. of O}_2 \text{ at NTP}}{\text{standard molar vol. (22.4 lt)}}$$

The above equations thus give the mass-volume relationship of reactants and products.

Example:

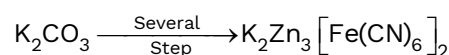
27.6g K_2CO_3 was treated by a series of reagents so as to convert all of its carbon to $\text{K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2$. Calculate the weight of the product.

[mol. wt. of $\text{K}_2\text{CO}_3 = 138$ and mol. wt. of $\text{K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2 = 698$]

Ans. 11.6 g

Solution:

Here we have not knowledge about series of chemical reactions but we known about initial reactant and final product accordingly



Since C atoms are conserved, applying POAC for C atoms,

Moles of C in $\text{K}_2\text{CO}_3 = \text{moles of C in K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2$

$1 \times \text{moles of K}_2\text{CO}_3 = 12 \times \text{moles of K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2$

(\because 1 mole of K_2CO_3 contains 1 moles of C)

$$\frac{\text{wt. of K}_2\text{CO}_3}{\text{mol. wt. of K}_2\text{CO}_3} = 12 \frac{\text{wt. of the product}}{\text{mol. wt. of product}}$$

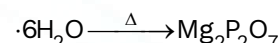
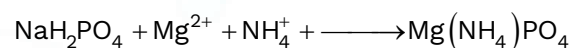
$$\text{of K}_2\text{Zn}_3[\text{Fe}(\text{CN})_6]_2 = \frac{27.6}{138} \frac{698}{12} = 11.6 \text{ g}$$

Example:

In a gravimetric determination of P of an aqueous solution of dihydrogen phosphate in H_2PO_4^- is treated with a mixture of ammonium and magnesium ions to precipitate magnesium ammonium phosphate, $\text{Mg}(\text{NH}_4)\text{PO}_4 \cdot 6\text{H}_2\text{O}$. This is heated and decomposed to magnesium pyrophosphate, $\text{Mg}_2\text{P}_2\text{O}_7$. A solution of H_2PO_4^- yielded 2.054 g of $(\text{Mg}_2\text{P}_2\text{O}_7)$. What weight of NaH_2PO_4 was present originally?

Ans. 2.22g

Solution:



As P atoms are conserved, applying POAC for P atoms, moles of P in $\text{NaH}_2\text{PO}_4 = \text{Moles of P in Mg}_2\text{P}_2\text{O}_7$

$1 \times \text{Moles of NaH}_2\text{PO}_4 = 2 \times \text{Moles of Mg}_2\text{P}_2\text{O}_7$

$$\therefore \frac{W_{\text{NaH}_2\text{PO}_4}}{M_{\text{NaH}_2\text{PO}_4}} = 2 \times \frac{W_{\text{Mg}_2\text{P}_2\text{O}_7}}{M_{\text{Mg}_2\text{P}_2\text{O}_7}} \Rightarrow \frac{W_{\text{NaH}_2\text{PO}_4}}{120} = 2 \times \frac{2.054}{222}$$

$$\therefore W_{\text{NaH}_2\text{PO}_4} = 2.22 \text{ g}$$

4.4 Percentage Yield

In general, in any chemical reaction, the amount of product formed is always less than the calculated amount due to reversibility in the chemical reaction. Therefore, yield of a chemical reaction (Y) comes into picture and is given by:

The percentage yield of product =

$$\frac{\text{actual yield}}{\text{the theoretical maximum yield}} \times 100$$

Example:

In a certain operation 358 g of TiCl_4 is reacted with 96 g of Mg. Calculate % yield of Ti if 32 g of Ti is actually obtained [At. wt. Ti = 48, Mg = 24]

(A) 35.38 %

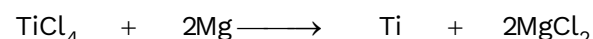
(B) 66.6 %

(C) 100 %

(D) 60 %

Ans. (A)

Solution:



$$\text{Initial mole } \frac{358}{190} = 1.88 \quad \frac{96}{24} = 4$$

Final mole

$$0 \quad 4 - 2 \times 1.88 \quad 1.88 \quad 2 \times 1.88$$



$$\text{Wt. of Ti obtained} = \frac{358}{190} \times 48$$

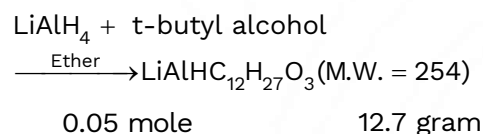
$$\% \text{ yield} = \frac{32 \times 100}{\frac{358 \times 48}{190}} = 35.38\%$$

Example:

0.05 mole of LiAlH_4 in ether solution was placed in a flask containing 74g (1 mole) of t-butyl alcohol. The product $\text{LiAlHC}_{12}\text{H}_{27}\text{O}_3$ weighed 12.7 g. If Li atoms are conserved, the percentage yield is: (Li = 7, Al = 27, H = 1, C = 12, O = 16).

- (A) 25% (B) 75%
(C) 100% (D) 15%

Ans. (C)

Solution:


$$= \frac{12.7}{254} = 0.05 \text{ mole}$$

Li atom remain conserved so

Number of mole of LiAlH_4 = Number of mole of $\text{LiAlHC}_{12}\text{H}_{27}\text{O}_3$

So Number of mole of $\text{LiAlHC}_{12}\text{H}_{27}\text{O}_3 = 0.05$

$$\% \text{ yield} = \frac{0.05}{0.05} \times 100 = 100\%$$

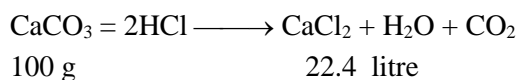
4.5 Percentage Purity

Percentage purity equals the mass of the pure chemical divided by the mass of the sample times 100 percent. Using this equation, we can see that a pure sample has a percentage purity of 100 percent, while an impure sample has a percentage purity of less than 100 percent.

Example:

How much marble of 90.5 % purity would be required to prepare 10 litres of CO_2 at 1 atm, 0°C when the marble is acted upon by dilute HCl ?

Ans. 49.326 g

Solution:


22.4 L of CO_2 at STP will be obtained from 100 g of CaCO_3

\therefore 10 L of CO_2 will be obtained from pure

$$\text{CaCO}_3 = \frac{100}{22.4} \times 10 = 44.64 \text{ g}$$

\therefore Impure marble required

$$= \frac{100}{90.5} \times 44.64 = 49.326 \text{ g}$$

4.6 Sequential Reactions

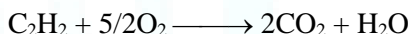
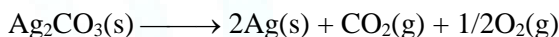
This reaction is defined as that reaction which proceeds from reactants to final products through one or more intermediate stages. The overall reaction is a result of several successive or consecutive steps.

Example: $\text{A} \rightarrow \text{B} \rightarrow \text{C}$

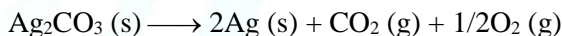
Example:

Minimum amount of Ag_2CO_3 (s) required to produce sufficient oxygen for the complete combustion of C_2H_2 which produces 11.2 L of CO_2 at 1 atm and 273K after combustion is:

[Ag = 108]



Ans. 345 g

Solution:


By Stoichiometry of reaction

$$\text{Moles of } \text{CO}_2 \text{ formed} = \frac{11.2}{22.4} = \frac{1}{2}$$

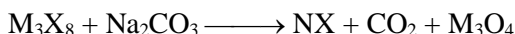
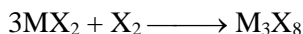
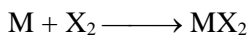
$$\text{Moles of } \text{O}_2 \text{ required} = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$$

$$\text{Moles of } \text{Ag}_2\text{CO}_3 \text{ required} = 2 \times \frac{5}{8} = \frac{5}{4}$$

$$\text{Mass of } \text{Ag}_2\text{CO}_3 \text{ required} = \frac{5}{4} \times 276 = 345 \text{ g}$$

Example:

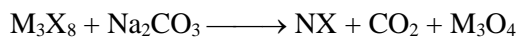
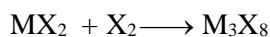
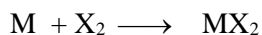
NX is produced by the following step of reactions:



How much M (metal) is consumed to produce 206 g of NX . (Take at wt. of M = 56, N=23, X = 80)

- (A) 42 g (B) 56 g
(C) 52g (D) 64 g

Ans. (A)

**Solution:**

$$\text{Mole of NX} = \frac{206}{103} = 2$$

POAC for X Atom:

No. of X atom in M_3X_8 = No. of X Atom in NX

$$8 [\text{No. of mole of } M_3X_8] = 1 [\text{No. of mole of NX}]$$

$$\text{Number of mole of } M_3X_8 = \left[\frac{2}{8} \right] = \frac{1}{4} \text{ mole}$$

Now POAC for M Atom

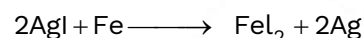
$$3 [\text{No. of mole of } M_3X_8] = 1 \times [\text{No. of Mole of M}]$$

$$\therefore 3 \times \frac{1}{4} \text{ Number of mole of M}$$

$$\text{weight of M atom} = \frac{3}{4} \times 56 = 42 \text{ gram}$$

Example:

The following process has been used to obtain iodine from oil-field brines in California.



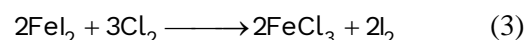
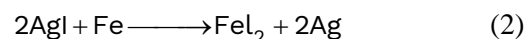
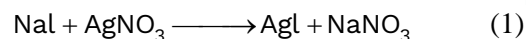
How many grams of $AgNO_3$ are required in the first step for every 254 kg I_2 produced in the third step.

- (A) 340 kg (B) 85 kg
(C) 68 kg (D) 380 kg

Ans. (A)

Solution:

Balanced equation:



From (3)

$$\frac{\text{mole of } I_2}{2} = \frac{\text{mole of } FeI_2}{2} ;$$

$$\frac{\text{mole of } FeI_2}{1} = \frac{\text{mole of } AgI}{2}$$

$$\frac{\text{mole of } AgI}{1} = \frac{\text{mole of } AgNO_3}{1}$$

$$\therefore \text{mole of } I_2 = (\text{mole of } FeI_2)$$

$$= \left(\frac{\text{mole of AgI}}{2} \right) = \left(\frac{\text{mole of AgNO}_3}{2} \right)$$

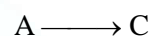
$$\frac{254 \times 10^3}{254} = \frac{\text{mole of AgNO}_3}{2}$$

$$2 \times 10^3 = \text{mole of AgNO}_3 = \frac{\text{mass of AgNO}_3}{\text{molar mass of AgNO}_3}$$

$$\text{mass of AgNO}_3 = 170 \times (2 \times 10^3) \text{ g} = 340 \times 10^3 \text{ g} = 340 \text{ kg}$$

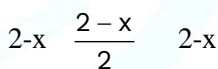
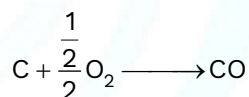
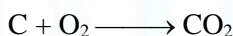
4.7 Parallel Reactions

The reactions in which a substance reacts or decomposes in more than one way are called parallel or side reactions.

**Example:**

Find out moles of CO_2 & CO produced by combustion of 2 mol carbon with 1.25 O_2 leaving number residue:

Ans. $CO_2 = 0.5 \text{ mol}$, $CO_2 = 1.5 \text{ mol}$

Solution:

$$x + 1 - \frac{x}{2} = 1.25$$

$$\frac{x}{2} = \frac{125}{100}$$

$$X = 0.5 \text{ mol}, \quad CO_2 = 0.5 \text{ mol}, \quad CO_2 = 1.5 \text{ mol}$$

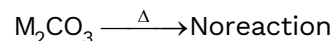
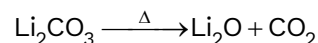
4.8 Mixture Analysis

The analysis of a chemical reaction is generally carried out in the form of mass of reacting species taking part in a given reaction (**gravimetric analysis**) or in terms of concentrations of reacting species taking part in a given reaction (**volumetric analysis**). In Gravimetric Analysis, we generally analyse reactions such as: decomposition of compounds under heat to produce a residue and a gas, or displacement reactions, action of acids on metals, or simple balanced chemical equations involving Weight (solid) – Volume (gas) relationships. In Volumetric Analysis, we generally analyse



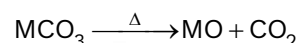
Neutralisation and Redox Titrations involving aqueous solutions in general.

(i) Except Li carbonates of all the alkali metals are thermally stable and does not decompose on heating.

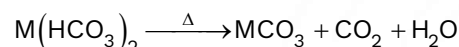
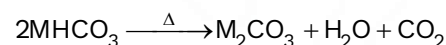


(M = Na, K, Rb, Cs)

(ii) All the carbonates of alkaline earth metals are thermally unstable and decompose on heating as follow.



(iii) Bicarbonates of both alkali metals and alkaline earth metals are decomposed at relatively low temperature as follow.



Example:

A sample of 3g containing Na_2CO_3 and NaHCO_3 loses 0.248 g when heated to 300°C , the temperature at which NaHCO_3 decomposes to Na_2CO_3 , CO_2 and H_2O . What is the percentage of Na_2CO_3 in the given mixture?

Ans. 77.48%

Solution:

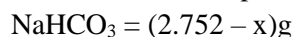
The loss in weight is due to removal of CO_2 and H_2O which escape out on heating. wt. of Na_2CO_3 in the product = $3.00 - 0.248 = 2.752$ g

Let wt. of Na_2CO_3 in the mixture be x g

\therefore wt. of $\text{NaHCO}_3 = (3.00 - x)$ g

Since Na_2CO_3 in the products contains x g of unchanged reactant Na_2CO_3 and rest produced from NaHCO_3 .

The wt. of Na_2CO_3 produced by



Applying POAC for Na atom

$1 \times \text{moles of NaHCO}_3 = 2 \times \text{moles of Na}_2\text{CO}_3$

$$\Rightarrow \frac{(3 - x)}{84} = 2 \times \frac{(2.752 - x)}{106}$$

$$\therefore x = 2.3244 \text{ g}$$

$$\therefore \% \text{ of Na}_2\text{CO}_3 = \frac{2.3244}{3} \times 100 = 77.48\%$$

Example:

10 g of a sample of a mixture of CaCl_2 and NaCl is treated to precipitate all the calcium as CaCO_3 . This CaCO_3 is heated to convert all the Ca to CaO and the final mass of CaO is 1.62 g. The percent by mass of CaCl_2 in the original mixture is.

(A) 32.1 % (B) 16.2 %

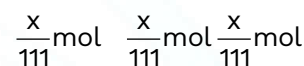
(C) 21.8 % (D) 11.0 %

Ans. (A)

Solution:



Let weight of $\text{CaCl}_2 = x$ g



$$\text{Mole of CaO} = \frac{1.62}{56}$$

$$\therefore \frac{x}{111} = \frac{1.62}{56}$$

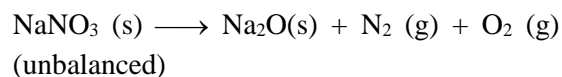
$$x = 3.21 \text{ g}$$

$$\% \text{ of CaCl}_2 = \frac{3.21}{10} \times 100 = 32.1\%$$

FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 3.0 g an impure sample of sodium sulphate dissolved in water was treated with excess of barium chloride solution when 1.74 g of BaSO_4 was obtained as dry precipitate. Calculate the percentage purity of sample.

Q.2 If the percentage yield of given reaction is 30%, how many total moles of the gases will be produced, if 8 moles of NaNO_3 are taken initially:



(A) 4.2 mole (B) 2.4 mole

(C) 4.8 mole (D) 2.1 mole

Q.3 A 5 g mixture of SO_2 and O_2 gases is reacted to form SO_3 gas. What should be the mass ratio of SO_2 and O_2 gases in mixture to obtain maximum amount of SO_3 gas:

(A) 4: 1 (B) 3: 2

(C) 2: 3 (D) 1: 4

$$1 \text{ H}_2\text{O molecule weigh} \dots \frac{18}{6.022 \times 10^{23}} = 3 \times 10^{-23} \text{ g}$$

$$d = \frac{\text{mass}}{\text{volume}}, \text{ So, volume} = \frac{3 \times 10^{-23} \text{ g}}{1 \text{ (g/mL)}} = 3 \times 10^{-23} \text{ mL}$$

6. Average/ Mean Atomic Mass

The weighted average of the isotopic masses of the element's naturally occurring isotopes. Mathematically, average atomic mass of X (A_x) =

$$\frac{a_1x_1 + a_2x_2 + \dots + a_nx_n}{100}$$

Where:

a_1, a_2, a_3 atomic mass of isotopes.

and x_1, x_2, x_3 mole % of isotopes.

Example:

Naturally occurring chlorine is 75.53% Cl^{35} which has an atomic mass of 34.969 amu and 24.47% Cl^{37} which has a mass of 36.966 amu. Calculate the average atomic mass of chlorine-

- (A) 35.5 amu (B) 36.5 amu
(C) 71 amu (D) 72 amu

Ans. (A)

Solution:

$$\begin{aligned} \text{Average atomic mass} &= (\% \text{ of I isotope} \times \text{Its atoms mass}) + (\% \text{ II isotope} \times \text{its atomic mass})/100 \\ &= \frac{75.53 \times 34.969 + 24.47 \times 36.96}{100} = 35.5 \text{ amu.} \end{aligned}$$

6.1 Average Molar Mass or Average Gram Molecular Mass

The average molar mass of the different substance present in the container

$$= \frac{n_1M_1 + n_2M_2 + \dots + n_nM_n}{n_1 + n_2 + \dots + n_n}$$

Where: M_1, M_2, M_3 are molar masses.
 n_1, n_2, n_3 moles of substances.

$$\text{Average molecule wt.} = \frac{\sum n_i M_i}{\sum n_i} \text{ where } n_i = \text{no. of}$$

moles of compound, m_i = molecular mass of compound

Example:

The molar composition of polluted air is as follows:

Gas	mole percentage composition
Oxygen	16%
Nitrogen	80%
Carbon dioxide	03%
Sulphurdioxide	01%

What is the average molecular weight of the given polluted air?

Solution:

$$\begin{aligned} M_{\text{avg}} &= \frac{16 \times 32 + 80 \times 28 + 44 \times 3 + 64 \times 1}{100} \\ &= \frac{512 + 2240 + 132 + 64}{100} = \frac{2948}{100} = 29.48 \end{aligned}$$

6.2 Degree of Dissociation (α)

Degree of dissociation represents the fraction of one mole dissociated into the products.

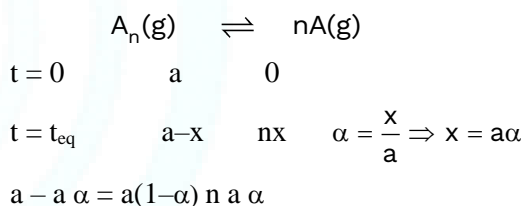
(Defined for one mole of substance) So, α = no. of moles dissociated / initial no. of moles taken = fraction of moles dissociated out of 1 mole.

Note: % dissociation = $\alpha \times 100$

Suppose 5 moles of PCl_5 is taken and if 2 moles of PCl_5 dissociated then $\alpha = \frac{2}{5} = 0.4$

6.3 Relationship Between Average Molar Mass & Degree of Dissociation (α)

Let a gas a dissociates to give n moles of a as follows-



Total no. of moles = $a - a\alpha + n a \alpha = [1 + (n-1)\alpha]a$

Average molecular weight of mixture(g) =

$$\frac{\text{molecular weight of } A_n(g)}{\text{total no. of moles at equilibrium}}$$

$$= \frac{a \cdot M_{\text{th}}}{a(1 + (n-1)\alpha)} M$$

$$M_{\text{avg}} = \frac{M_{\text{th}}}{[1 + (n-1)\alpha]} ; \text{ where } M_{\text{th}} = \text{theoretical molecular weight (n = atomicity)}$$

$$M_{\text{mixture}} = \frac{M_{A_n}}{[1 + (n-1)\alpha]}, M_{A_n} = \text{Molar mass of gas } A_n$$

Vapour density (V.D):

Density of the gas divided by density of hydrogen under same temperature & pressure is called vapour density.

$$D = \text{vapour density without dissociation} = \frac{M_{A_n}}{2}$$

d = vapour density of mixture = average vapour density



$$\frac{D}{d} = 1 + (n-1)\alpha$$

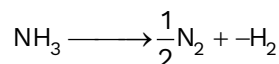
$$\alpha = \frac{D-d}{(n-1) \times d} = \frac{M - M_0}{(n-1)M_0}$$

Example:

NH₃ decomposes into N₂ & H₂. If average molar mass of reaction mixture is 10 then, find α ?

Ans. 0.7

Solution:



$$n_1 \quad 1 \quad 0 \quad 0$$

$$1 - \alpha \quad \frac{\alpha}{2} \quad \frac{3}{2}$$

$$10 = \frac{17}{1 - \alpha + \frac{\alpha}{2} + \frac{3\alpha}{2}}$$

$$10 = \frac{17}{1 + \alpha}$$

$$1 + \alpha = 1.7$$

$$\alpha = 0.7$$

7. Percentage Composition and Molecular Formula

Here we are going to find out the percentage of each element in the compound by knowing the molecular formula of compound. We know that according to law of definite proportion any sample of a pure compound always possess constant ratio with their combining elements.

Example:

Every molecule of ammonia always has formula NH₃ irrespective of method of preparation or sources. i.e. 1 mole of ammonia always contains 1 mol of N and 3 mole of H. In other words 17g of NH₃ always contains 14 g of N and 3 g of H. Now find out % of each element in the compound.

Mass % of N in

$$\text{NH}_3 = \frac{\text{Mass of N in 1molNH}_3}{\text{Mass of 1mol of NH}_3} \times 100 = \frac{14\text{g}}{17} \times 100 = 82.35\%$$

Mass % of H in

$$\text{NH}_3 = \frac{\text{Mass of N in mol NH}_3}{\text{Mass of 1 mol of NH}_3} \times 100 = \frac{3}{17} \times 100 = 17.65\%$$

7.1. Empirical And Molecular Formula

We have just seen that knowing the molecular formula of the compound we can calculate percentage composition of the elements. Conversely if we know the percentage composition of the elements initially, we can calculate the relative number of atoms of each element in the molecules of the compound. This gives as the empirical formula of the compound. Further if the molecular mass is known then the molecular formula can be easily determined.

Thus, the empirical formula of a compound is a chemical formula showing the relative number of atoms in the simplest ratio, the molecular formula gives the actual number of atoms of each element in a molecule.

Empirical formula:

An empirical formula represents the simple whole number ratio of various atoms present in a compound,

Molecular formula:

whereas, the molecular formula shows the exact number of different types of atoms present in a molecule of a compound. The molecular formula is generally an integral multiple of the empirical formula.

That is:

$$\text{molecular formula} = \text{empirical formula} \times n$$

$$\text{where; } n = \frac{\text{molecular formula mass}}{\text{empirical formula mass}}$$

Example:

An organic substance containing carbon, hydrogen and oxygen gave the following percentage composition. C = 40.687 % ; H = 5.085 % and O = 54.228%. The molecular weight of the compound is 118. Calculate the molecular formula of the compound.

Ans. C₄H₆O₄



Solution:

Step -1

To calculate the empirical formula of the compound.

Element	Symbol	Percentage of element	At. mass of element	Relative no. of atoms = $\frac{\text{Percentage}}{\text{At. mass}}$	St. mass atomic ratio = $\frac{\text{Relative no. of atoms}}{\text{Smallest value}}$	Simplest whole no. atomic ratio
Carbon	C	40.687	12	$\frac{40.687}{12} = 3.390$	$\frac{3.390}{3.389} = 1$	2
Hydrogen	H	5.085	1	$\frac{5.085}{1} = 5.035$	$\frac{5.085}{3.389} = 1.5$	3
Oxygen	O	54.228	16	$\frac{54.228}{16} = 3.389$	$\frac{3.389}{3.389} = 1$	2

Step-2

To calculate the empirical formula mass.

The empirical formula of the compound is $C_2H_3O_2$.

\therefore Empirical formula mass

$$= (2 \times 12) + (3 \times 1) + (2 \times 16) = 59.$$

Step-3

To calculate the value of 'n'

$$n = \frac{\text{molecular mass}}{\text{Empirical formula mass}} = \frac{118}{59} = 2$$

Step-4

To calculate the molecular formula of the salt

$$\text{Molecular formula} = n \times (\text{Empirical formula})$$

$$= 2 \times C_2H_3O_2 = C_4H_6O_4$$

Thus, the molecular formula is $C_4H_6O_4$.

Example:

Chlorophyll the green colouring material of plants contains 3.68 % of magnesium by mass. Calculate the number of magnesium atom in 5.00 g of the complex.

Solution:

$$\text{Mass of magnesium in 5.0 g of complex} = \frac{3.68}{100} \times 5.00$$

$$= 0.184 \text{ g}$$

$$\text{Atomic mass of magnesium} = 24$$

$$24 \text{ g of magnesium contain} = 6.023 \times 10^{23} \text{ atoms}$$

$$0.184 \text{ g of magnesium would contain} = \frac{6.023 \times 10^{23}}{24}$$

$$0.184 = 4.617 \times 10^{21} \text{ atom}$$

Therefore, 5.00 g of the given complex would contain 4.617×10^{21} atoms of magnesium.

Example:

A sample of $CaCO_3$ has Ca = 40%, C = 12% and O = 48% by mass. If the law of constant proportions is true, then the mass of Ca in 5 g of $CaCO_3$ obtained from another source will be:

- (A) 0.2 g (B) 2 g
(C) 0.6 g (D) Cannot be determined

Solution:

$$\text{Mass of Ca} = 5 \times \frac{40}{100} = 2 \text{ g}$$

8. Experimental Methods to Determine Atomic & Molecular Masses

8.1 For determination of atomic mass

(a) Dulong's & Pettit's law

In case of metals, it is observed that product of atomic weight and specific heat capacity is constant. The equation connecting the two parameters was given by Dulong's and Petit's Law.

$$\text{Atomic weight of metal} \times \text{specific heat capacity (cal/gm}^\circ\text{C)} = 6.4.$$

It should be remembered that this law is an empirical observation and this gives an approximate value of atomic weight. Also this law can be applied only to metals at high temperature conditions only.



8.2 Experimental methods for molecular mass determination.

(a) Victor Meyer's Method

(b) Silver Salt Method

(c) Chloroplatinate Salt Method

(a) Victor Meyer's Method: (Applicable for volatile substance)

A known mass of the volatile substance taken in the Hoffmann's bottle and is vapourised by throwing the Hoffmann's bottle into the Victor Meyer's tube. The vapour displace an equal volume of the moist air. Which vapours is measured at the room temperature and atmospheric pressure. The barometric pressure and the room temperature is recorded. Following diagram gives the experimental set-up for the Victor-Meyer's process.

Calculation involved

Let the mass of the substance taken by = W g

Volume of moist vapours collected = V cm³

Room temperature = T K

Barometric pressure = P mm

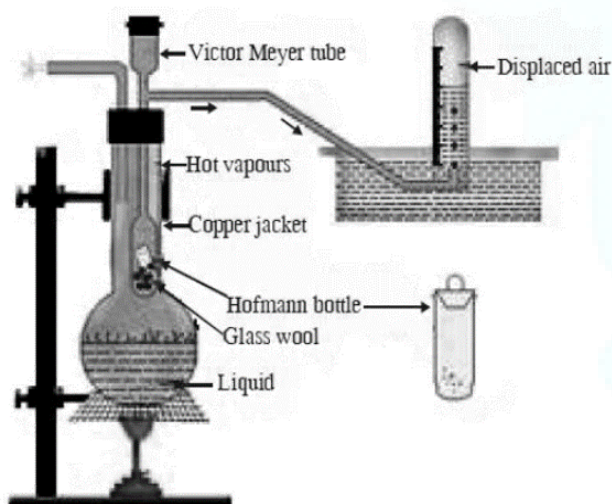
Aqueous tension at T K = p mm

Pressure of dry vapour = $(P - p)$ mm

Calculation of molecular mass (M)

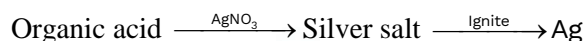
$$\frac{(P - p)}{760} \times \frac{V}{1000} = \frac{W}{M} \times RT$$

$$\Rightarrow M = \frac{W \times RT \times 760 \times 1000}{(P - p) \times V}$$



(b) Silver salt Method: (A used for organic acids)

A known mass of the acid is dissolved in water followed by the subsequent addition of silver nitrate solution till the precipitation of silver salt is complete. The precipitate is separated, dried, weighed



And ignited till decomposition is complete. The residue of pure silver left behind is weighed.

Calculations involved

Let the mass of the silver salt formed = W g

The mass of Ag formed = x g

Let us understand to calculations by considering the monobasic acid MX .



Mass of the salt that gives x gm of Ag = W g

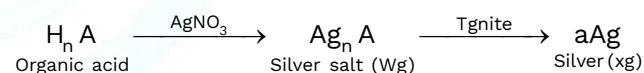
Mass of the salt gives 108g (1g-atom) of Ag $\frac{108W}{x}$ g

$$\text{Molar mass of salt} = \frac{108W}{x} \text{ g}$$

Molar mass of acid = (Molar mass of salt) - (Atomic mass of Ag) + (Atomic mass of H)

$$= \frac{108W}{x} - 108 + 1 = \left(\frac{108W}{x} - 107 \right) \text{ g mol}^{-1}$$

For polybasic acid of the type H_nX (n is basicity)



Mass of the silver that gives x g of Ag = W g

Mass of the silver that gives $(108n)$ g of

$$Ag = \frac{108nw}{x} \text{ g}$$

Molar mass of acid = (molar mass of salt)

$$= \frac{108 \times nW}{x} - n \times 108 - n \times 1$$

$$= n \left(\frac{108W}{x} - 107 \right) \text{ g mol}^{-1}$$

(c) Platinic chloride Method:

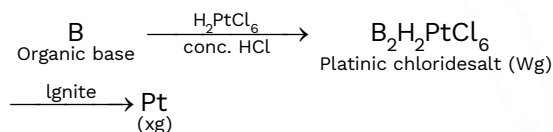
(Applicable for finding the molecular masses of organic bases).

A known mass of organic base is allowed to react with chloroplatinic acid (H_2PtCl_6) in conc. HCl to form insoluble platinic chloride. The precipitate of platinic chloride is separated, dried, weighed and





subsequently ignited till decomposition is complete. The residue left is platinum which is again weighed. The molecular mass is then calculated by knowing the mass of the platinum chloride salt and that of platinum left. If B represents the molecule of monoacidic organic base, then the formula of platinum chloride salt is $B_2H_2PtCl_6$.



Let the mass of platinum chloride salt =

Wg, The mass of the platinum residue left = xg

It may be noted that salt formed with diacidic base would be $B_2(H_2PtCl_6)_2$ with triacidic base it would be $B_2(H_2PtCl_6)_3$ and with polyacidic base would be $B_2(H_2PtCl_6)_n$.

Mass of salt which gives 195 g (1 g-atom) of Pt

$$\frac{W \times 195}{x} \text{ g}$$

$$\text{Molar mass of salt} = \frac{W \times 195}{x} \text{ g mol}^{-1}$$

Now from the formula $B_2(H_2PtCl_6)$

Molar mass of salt = (2 × molar mass of base) + (Molar mass of H_2PtCl_6)

Molar mass of base = $\frac{1}{2}$ (molar mass of salt – Molar mass of H_2PtCl_6)

$$= \frac{1}{2} \left(\frac{W \times 195 \times n}{x} - n \times 410 \right) = \frac{n}{2} \left(\frac{W \times 195}{x} - 410 \right) \text{ g mol}^{-1}$$

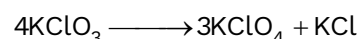
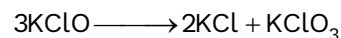
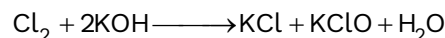
FUNDAMENTAL UNLOCKED- (FU#5) :

Q.1 1. 120 g Mg is burnt in air to give a mixture of MgO and Mg_3N_2 . The mixture is now dissolved in HCl to form $MgCl_2$ and NH_4Cl . If 107 g NH_4Cl is produced, then determine the moles of $MgCl_2$ formed:

- (A) 2.5 (B) 4 (C) 2 (D) 5

Q.2 Penicillin V was treated chemically to convert the sulphur present to barium sulphate, $BaSO_4$. A 9.6 mg sample of penicillin V gave 4.66 mg $BaSO_4$. The percentage of sulphur in Penicillin V is x %. If there is one sulphur atom in the molecule, the molecular weight of Penicillin V is y amu. Report your answer as y/x.

Q.3 From the following reaction sequence:



Calculate the mass of chlorine needed to produce 138.5 g of $KClO_4$:

- (A) 142 g (B) 284 g
(C) 432 g (D) None of these

Q.4 The density of air at STP is 0.0013 g mL^{-1} . Its vapour density is:

- (A) 0.015 (B) 15
(C) 1.5 (D) Data insufficient

Q.5 $SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$

If observed vapour density of mixture at equilibrium is 35 then find out value of α :

- (A) 0.28 (B) 0.38
(C) 0.48 (D) 0.58

Q.6 A sample of a compound contains 9.75 g Zn, 9×10^{22} atoms of Cr and 0.6 gram-atoms of O. What is empirical formula of compound? (Atomic Mass Zn = 65)

- (A) $ZnCrO_4$ (B) $ZnCr_2O_4$
(C) Zn_2CrO_4 (D) None of these

Q.7 An organic compound on analysis was found to contain 0.032% of sulphur by mass. The molecular mass of the compound, if it's one molecule contains two sulphur atoms, is:

- (A) 100000 u (B) 10000 u
(C) 20000 u (D) 200000 u

Q.8 In an organic compound of molar mass 108 g mol^{-1} C, H and N atoms are present in 9: 1: 3.5 by weight. Molecular formula can be:

- (A) $C_6H_8N_2$ (B) $C_7H_{10}N$
(C) $C_5H_6N_3$ (D) $C_4H_{18}N_3$

Q.9 At 100°C and 1 atm, if the density of liquid water is 1.0 g cm^{-3} and that of water vapour is 0.0006 g cm^{-3} , then the volume occupied by water molecules in 1 L of steam at that temperature is:

- (A) 6 cm^3 (B) 60 cm^3
(C) 0.6 cm^3 (D) 0.06 cm^3



**Solutions**

A mixture of two or more substances can be a solution. We can also say that a solution is a homogeneous mixture of two or more substances. 'Homogeneous' means 'uniform throughout'. Thus a homogeneous mixture, i.e., a solution, will have uniform composition throughout.

1. Concentration Terms

The following concentration terms are used to express the concentration of a solution. These are:

1. Molarity (M)
2. Molality (m)
3. Mole fraction (x)
4. % calculation
5. ppm

Remember that all of these concentration terms are related to one another. By knowing one concentration term you can also find the other concentration terms. Let us discuss all of them one by one.

1.1. Molarity (M)

The number of moles of a solute dissolved in 1 L (1000 ml) of the solution is known as the molarity of the solution.

That is, Molarity of solution

$$= \frac{\text{number of moles}}{\text{volume of solution in litre}}$$

Let a solution is prepared by dissolving w g of solute of mol. wt. M in V mL water.

$$\therefore \text{Number of moles of solute dissolved} = \frac{w}{M}$$

$$\therefore V \text{ mL water have } \frac{w}{M} \text{ mole of solute}$$

$$\therefore 1000 \text{ mL water have}$$

$$\frac{w \times 1000}{M \times V (\text{in mL})} \Rightarrow \therefore \text{Molarity (M)} = \frac{w \times 1000}{(\text{Mol. wt of solute}) \times V (\text{in mL})}$$

Some other relations may also useful.

$$\text{Number of millimoles} = \frac{\text{mass of solute}}{(\text{Mol. wt. of solute})} \times 1000$$

$$1000 = (\text{Molarity of solution} \times V \text{ml})$$

Molarity of solution may be also given as:

$$\frac{\text{Number of millimole of solute}}{\text{Total volume of solution in ml}}$$

Molarity is a unit that depends upon temperature. It decreases as temperature increases.

Example:

Find the mass of solute and solvent in 100 mL, 1 M NaOH solution having density 1.5 g/mL.

- (A) 40 g, 110 g (B) 4 g, 150 g
(C) 4 g, 146 g (D) 40 g, 150 g

Ans. (C)

Solution:

Mole of NaOH = molarity \times volume (l) = $1 \times 0.1 = 0.1$
Mass of NaOH = $0.1 \times 40 = 4$ gm
Mass of solution = volume \times density = $100 \times 1.5 = 150$ gm
Hence: mass of solvent = $150 - 4 = 146$ g

Example:

Molality of pure water if its density is 0.936 gm/ml

- (A) 50 (B) 55.56 (C) 57.56 (D) 56.56

Ans. (B)

Solution:

$$m = \frac{1000}{\text{M.W.}} = \frac{1000}{18} = 55.56$$

1.2 Molality (m):

The number of moles of solute dissolved in 1000 g (1 kg) of a solvent is known as the molality of the solution.

That is, molality

$$= \frac{\text{number of moles of solute}}{\text{mass of solvent in gram}} \times 1000$$

Let y g of a solute is dissolved in x g of a solvent. The molecular mass of the solute is m . Then y/m mole of the solute are dissolved in x g of the solvent. Hence

$$\text{Molality} = \frac{y}{m \times x} \times 1000$$

Example:

225 gm of an aqueous solution contains 5 gm of urea.

What is the concentration of the solution in terms of molarity? (Mol. wt. of urea = 60)

Ans. 0.332

Solution:

Mass of urea = 5 gm Molecular mass of urea = 60

$$\text{Number of moles of urea} = \frac{5}{60} = 0.083$$

Mass of solvent = $(225 - 5) = 220$ gm

\therefore Molality of the solution

$$= \frac{\text{Number of moles of solute}}{\text{Mass of solvent in gram}} \times 1000$$

$$= \frac{0.083}{220} \times 1000 = 0.332$$



Example:

A solution is made by dissolving CaBr_2 in water (solvent) such that mass fraction of solute and solvent is same in the solution. The molality of solution is –

- (A) 2.5 m (B) 55.55 m
(C) 2 m (D) 5 m

Ans. (D)

Solution:

$$m = \frac{w / 200 \times 1000}{w} = 5 \quad m = \frac{w / 200 \times 1000}{w} = 5$$

1.3 Mole fraction (x):

The ratio of number of moles of the solute or solvent present in the solution and the total number of moles present in the solution is known as the mole fraction of substances concerned. Let number of moles of solute in solution = n Number of moles of solvent in solution = N

$$\therefore \text{Mole fraction of solute } (x_1) = \frac{n}{n + N}$$

$$\therefore \text{Mole fraction of solvent } (x_2) = \frac{N}{n + N}$$

$$\Rightarrow \text{also } x_1 + x_2 = 1$$

1.4 % Calculation

The concentration of a solution may also express in terms of percentage in the following ways.

(i) % weight by weight (%w/w):

It is given as mass of solute present in per 100 g of solution.

$$\text{i.e. \% w/w} = \frac{\text{mass of solute in g}}{\text{mass of solution in g}} \times 100$$

[X % by mass means 100 g solution contains X g solute;

\therefore (100 - X) g solvent]

(ii) % weight by volume (%w/V)

It is given as mass of solute present in per 100 mL of solution.

$$\text{i.e. \% w/v} = \frac{\text{mass of solute in g}}{\text{volume of solution in mL}} \times 100$$

[X% $\left(\frac{w}{v}\right)$ means 100 mL solution contains X g solute]

(iii) % volume by volume (%V/V)

It is given as volume of solute present in per 100 mL solution.

$$\text{i.e. \% v/v} = \frac{\text{volume of solute in g}}{\text{volume of solution in mL}} \times 100$$

Example:

0.25 of a substance is dissolved in 6.25 g of a solvent. Calculate the percentage amount of the substance in the solution.

Ans. 3.8%.

Solution:

$$\text{wt. of solution} = 0.25 + 6.25 = 6.50.$$

$$\text{so \% (w/w)} = \frac{0.25}{6.50} \times 100 = 3.8\%$$

Example:

0.5 g of a substance is dissolved in 25 g of a solvent. Calculate the percentage amount of the substance in the solution.

Ans. 1.96

Solution:

$$\text{Mass of substance} = 0.5 \text{ g} \quad \text{Mass of solvent} = 25 \text{ g}$$

$$\therefore \text{Percentage of the substance (w/w)} =$$

$$\frac{0.5}{0.5 + 25} \times 100 = 1.96$$

1.5 Parts per million (ppm):

$$\frac{\text{Mass of solute}}{\text{Mass of solvent}} \times 10^6 \cong \frac{\text{Mass of solute}}{\text{Mass of solution}} \times 10^6$$

FUNDAMENTAL UNLOCKED- (FU#6) :

Q.1 Calculate the molarity when:

(a) 4.9 g H_2SO_4 acid dissolved in water to result 500 mL solution.

(b) 2 gram-molecules of KOH dissolved in water to result 500 mL solution.

(A) (a) 0.1 M (b) 0.07 M

(B) (a) 0.4 M (b) 4 M

(C) (a) 0.4 M (b) 0.07 M

(D) (a) 0.1 M (b) 4 M2.

Q.2 Calculate the volume in litre of 0.1 M solution of HCl which contains 0.365 g HCl?

(A) 10^{-2} L (B) 0.1 L

(C) 1 L (D) 10 L3

Q.3 What volume of a 0.8 M solution contains 100 millimoles of the solute?

(A) 80 mL (B) 125 mL

(C) 125 L (D) 80 L



Q.4 Which of the following methods of expressing concentration of a solution is/are independent of temperature ?

- (A) Molality
(B) % w/w
(C) Mole fraction of solute
(D) All of these

Q.5 20 cm³ of an alcohol is dissolved in 80 cm³ of water. Calculate the percentage of alcohol in solution.

Q.6 Calculate the amount of 75% pure NaI required to prepare 5 litre of 0.5 M solution.

- (A) 281.25 g (B) 500 g
(C) 923.33 g (D) 519.375 g

2. Dilution And Intermixing of Solutions

Dilution: Whenever a given solution of known concentration i.e. normality and molarity (known as standard solution) is diluted (adding solvent), the number of millimoles (or milliequivalents) of solute remain unchanged. The concentration of solution however changes. In such a case if: M_1 = Molarity of original solution; V_1 = volume of original solution and M_2 = normality of diluted solution; V_2 = total volume of diluted solution. Since the number of millimoles remains same,

$$\Rightarrow M_1 V_1 = M_2 V_2$$

Example:

Calculate the resultant molarity of following:

- (a) 200 ml 1M HCl + 300 ml water
(b) 1500 ml 1M HCl + 18.25 g HCl
(c) 200 ml 1M HCl + 100 ml 0.5 M H₂SO₄
(d) 200 ml 1M HCl + 100 ml 0.5 M HCl
(A) 0.4 M (B) 1.33 M
(C) 1 M (D) 0.83 M.

Solution:

(a) Final molarity = $\frac{200 \times 1 + 0}{200 + 300} = 0.4 \text{ M}$

(b) Final molarity = $\frac{1500 \times 1 + \frac{18.25 \times 1000}{36.5}}{1500} = 1.33 \text{ M}$

(c) Final molarity of H⁺ = $\frac{200 \times 1 + 100 \times 0.5 \times 2}{200 + 100} = 1 \text{ M}$

(d) Final molarity = $\frac{200 \times 1 + 100 \times 0.5}{200 + 100} = 0.83 \text{ M}$

Example:

The molarity of Cl⁻ in an aqueous solution which was (w/V) 2% NaCl, 4% CaCl₂ and 6% NH₄Cl will be:

- (A) 0.342 (B) 0.721
(C) 1.12 (D) 2.18

Ans. (D)

Solution:

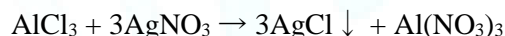
Moles of Cl⁻ in 100 ml of solution =

$$\frac{2}{58.5} + \frac{4}{111} \times 2 + \frac{6}{53.5} = 0.2184$$

Molarity of Cl⁻ = $\frac{0.2184}{100} \times 1000 = 2.184$

Example:

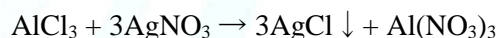
How many milli-litres of 0.2 M AlCl₃ solution is required to precipitate all the Ag⁺ from 45 ml of a 0.2 M AgNO₃ solution:



- (A) 15 ml (B) 30 ml
(C) 45 ml (D) 60 ml

Ans. (A)

Solution:



45 × 0.2 milli moles

$$\frac{1}{3} \times 45 \times 0.2 \text{ milli mol}$$

$$\frac{1}{3} \times 0.45 \times 0.2 = 0.2 \times V$$

$$\Rightarrow V = 15 \text{ ml}$$

Example:

The specific gravity of a solution is 1.8, having 62% by weight of acid. It is to be diluted to specific gravity of 1.2. What volume of water should be added to 100 ml of this solution?

Ans. 300 mL

Solution:

Let, to 100 ml of given acid solution (sp. gr 1.8) x ml. of water is added.

∴ The total volume of resulting solution

$$= (100 + x) \text{ ml}$$

∴ The total weight of resulting solution

$$= (100 + x) \times 1.2 \text{ gm.}$$



Weight of acid present in the given acid solution (per 100 ml) = $100 \times 1.8 \times 0.62$

\therefore The amount of water present in 100 ml of given acid solution = $1.8 \times 100 \times 0.38$

\therefore Total wt. of acid present in the diluted solution = $(100 + x) 1.2 - x - 180 \times 0.38$
 $= 1.8 \times 100 \times 0.62$

$\therefore 120 + 0.2x = 180$ or $x = 300$

\therefore To lower sp. gravity of the given acid solution to 1.2, we are to add 300 ml of water per 100 ml of acid solution (sp gr. 1.2).

Example:

How would you prepare exactly 3 L of 1 M NaOH solution by mixing proportions of stock solutions of 2.5 M NaOH and 0.4 M NaOH, if no water is to be used? Find the ratio of the volume (V_1/V_2).

(A) 1: 3

(B) 3: 7

(C) 2: 5

(D) Data insufficient

Ans. (C)

Solution:

$$M_1V_1 + M_2V_2 = M_TV_T$$

$$2.5 V_1 + 0.4 V_2 = 3 \times 1$$

$$2.5 V_1 + 0.4 (3 - V_1) = 3$$

$$\Rightarrow 2.5 V_1 + 1.2 - 0.4 V_1$$

$$= 32.1 V_1 = 1.8$$

$$V_1 = \frac{1.8}{2.1} = \frac{6}{7}$$

$$V_2 = 3 - \frac{6}{7} = \frac{15}{7}$$

$$\frac{V_1}{V_2} = \frac{6}{7} \times \frac{7}{15} = 2 : 5$$

3. Interconversion Of Concentration Terms

Comprehension # (Q.51 to Q.52)

Molarity(mol/L)	Molality(mol/Kg)	Density (g/mL)	Gram molecular	
mass of solute				
Solution-1	a	-	d_1	P
Solution-2	-	b	d_2	Q
Solution-3	1	-	1.060	60

Now answer the following questions:

Example:

What is molality of solution-1:

(A) $\frac{(1000 \times a)}{(1000 \times d_1) - aP}$ (B) $\frac{1000d_1}{1000a - P}$

(C) $\frac{a}{1000d_1 - aP}$ (D) None of these

Ans. (A)

Solution:

For solution 1 'a' moles of solute are present in 1000 ml of solution.

wt. of solution = $1000 \times d_1$ g

wt. of solute = aP g

$$\text{So, Molality} = \left[\frac{a \times 1000}{1000 \times d_1 - aP} \right]$$

Example:

What is the molarity of solution 2:

(A) $\frac{b \times d_2}{1000 + bQ}$ (B) $\frac{b \times 1000 \times d_2}{1000 + bQ}$

(C) $\frac{1000 \times bQ}{1000 + bd_2}$ (D) None of these

Ans. (B)

Solution:

For solution 2 'b' moles of solute are present in 1000 g of solvent.

wt. of solution = $1000 + bQ$

$$\text{vol. of solution} = \frac{1000 + bQ}{d_2}$$

$$\text{Molality} = \frac{b \times 1000}{1000 + bQ} = \frac{b \times 1000 \times d_2}{1000 + bQ}$$

**Example:**

The molarity of the solution containing 2.8% (mass/volume) solution of KOH is:

(Given atomic mass of K = 39)

- (A) 0.1 M (B) 0.5 M
(C) 0.2 M (D) 1 M

Solution:

Weight of KOH = 2.8 gram Volume of solution = 100 ml

$$M = \frac{2.8 \times 1000}{56 \times 100} = \frac{5}{49} = 0.5 \text{ M}$$

Example:

What is the mole fraction of ethanol in 20% by weight solution in water?

- (A) 0.095 (B) 0.089
(C) 0.9 (D) 1.2

Ans. (B)

Solution:

100 gm of solution contain 20 gm $\text{C}_2\text{H}_5\text{OH}$ and 80 gm of water.

\therefore moles of ethanol present = $\frac{20}{46} = 0.435$ (mol. wt. of ethanol = 46)

\therefore moles of water present = $\frac{80}{18}$

= 4.444 Total no. of moles

= $0.435 + 4.444 = 4.879$

\therefore mole fraction of $\text{C}_2\text{H}_5\text{OH} = \frac{0.435}{4.879} = 0.089$

FUNDAMENTAL UNLOCKED- (FU#7) :

Q.1 What volume of water is required to make 0.2 M solution from 16 mL of a 0.5 M solution?

- (A) 24 mL (B) 40 mL
(C) 6.4 mL (D) 20 mL

Q.2 What approximate volume of 0.40 M $\text{Ba}(\text{OH})_2$ solution must be added to 50.0 mL of 0.30M NaOH solution to get a solution in which the molarity of the OH^- ions is 0.50 M ?

- (A) 33 mL (B) 66 mL
(C) 133 mL (D) 100 mL

Q.3 100 mL 30% (w/v) NaOH solution is mixed with 100 ml 90% (w/v) NaOH solution. The molarity of final solution is-

- (A) 30M (B) 15M
(C) 7.5M (D) 2M

Q.4 Molality (m) of a sulphuric acid solution in which the mol fraction of water is 0.85 is:

- (A) 4.9 (B) 9.8
(C) 19.6 (D) Can't be determined

Q.5 The molality of a sulphuric acid solution is 0.2. Total weight of the solution having 100 g of solvent is about:

- (A) 119.6 g (B) 109.8 g
(C) 104.9 g (D) 102 g

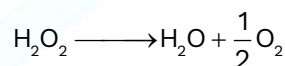
Q.6 For a mixture of 100 mL of 0.3 M CaCl_2 solution and 400 mL of 0.1 M HCl solution, select the correct option:

- (A) Total concentration of cations = 0.14 M
(B) $[\text{Cl}^-] = 0.2 \text{ M}$
(C) Both (A) and (B)
(D) None of these

Q.7 What volume of water should be added to 50 ml of HNO_3 having density 1.5 g ml^{-1} and 63.0% by weight to have one molar solution.

3. Some Special Concentration Terms**3.1 VOLUME STRENGTH OF H_2O_2 :**

Strength of H_2O_2 is represented as 10V, 20 V, 30 V etc. 20V H_2O_2 means one litre of this sample of H_2O_2 on decomposition gives 20L of O_2 gas at STP. Decomposition of H_2O_2 is given as:



1 mole $\frac{1}{2} \times 22.7 \text{ L O}_2$ at STP

= 34g = 11.35 L O_2 at STP

Molarity of H_2O_2 (M) = $\frac{\text{Volume strength of H}_2\text{O}_2}{11.35}$

Strength (in g/L) = Molarity \times Mol. Wt.

= Molarity \times 34

Example:

A fresh H_2O_2 solution is labeled 11.35 V at STP. This solution has the same concentration as a solution which is:

- (A) 3.4% (w/w) (B) 3.4% (v/v)
(C) 3.4% (w/v) (D) None of these

Ans. (C)



Solution:

$$\text{Molarity of H}_2\text{O}_2 = \frac{\text{vol. strength}}{11.2} = \frac{11.35}{11.35} = 1$$

$$\text{Now, \% (w/v)} = \frac{\text{wt. of solute in g}}{\text{wt. of solution in mL}} \times 100$$

$$= \text{Molarity} \times \text{Mol. wt. of solute} \times \frac{1}{10}$$

$$= 1 \times 34 \times \frac{1}{10} = 3.4\%$$

3.2. Percentage Labeling of Oleum:

Oleum is SO_3 dissolved in 100% H_2SO_4 . Sometimes, oleum is reported as more than 100% by weight, say $y\%$ (where $y > 100$). This means that $(y - 100)$ grams of water, when added to 100 g of given oleum sample, will combine with all the free SO_3 in the oleum to give 100% sulphuric acid.

Hence, weight % of free SO_3 in oleum
 $= 80(y - 100)/18$

Example:

What volume of water is required (in mL) to prepare 1 L of 1 M solution of H_2SO_4
 (density = 1.5 g/mL) by using 109% oleum and water only (Take density of pure water = 1 g/mL).

Ans. 1410.09 mL

Solution:

1 mole H_2SO_4 in 1L solution = 98 g H_2SO_4 in 1500 g solution = 98 g H_2SO_4 in 1402 g water.

Also, in 109% oleum, 9 g H_2O is required to form 109 g pure H_2SO_4 & so, to prepare 98 g H_2SO_4 , water needed is $9/109 \times 98 = 8.09$ g.

Total water needed = $1402 + 8.09 = 1410.09$ g
 $= 1410.09$ mL

Example:

A 50 gm oleum sample contains $\left(\frac{400}{90}\right)$ gm of combined

SO_3 . Find percent label of the oleum sample.

Ans. (118)

Solution:

Combined $\text{SO}_3 = \left(\frac{400}{49}\right)$ g is present in H_2SO_4

$$\text{mole of SO}_3 = \frac{\frac{400}{49}}{80} = \frac{5}{49}$$

$$\text{mole of H}_2\text{SO}_4 \text{ in oleum} = \frac{5}{49}$$

$$\text{In 50 g oleum mass of H}_2\text{SO}_4 = \frac{5}{49} \times 98 = 10 \text{ g}$$

$$100 \text{ g oleum mass of H}_2\text{SO}_4 = 20 \text{ g}$$

$$\text{Mass of SO}_3 = 100 - 20 = 80 \text{ g}$$

$$\text{SO}_3 + \text{H}_2\text{O} \text{ mass } 80 \text{ g}$$

$$\text{Mol} = \frac{80}{80} = 1 \text{ mol} = 18 \text{ g}$$

$$\% \text{ labeling} = (100 + 18) \% = 118 \%$$

4. Eudiometry or Gas Mixture Analysis:

Gaseous reactions are carried out in a special type of tube known as eudiometer tube. The tube is graduated in millimeters for volume measurement. The reacting gases taken in the eudiometer tube are exploded by sparks. The volume s of the product of gases are determined by absorbing them in suitable reagents.

Example:

Solvent	gas(es) absorb
KOH	$\text{CO}_2, \text{SO}_2, \text{Cl}_2$
Ammonical	Cu_2Cl_2 CO
Turpentine oil	O_3
Alkaline pyrogallol	O_2
Water	NH_3, HCl
CuSO_4	H_2O

Eudiometry is mainly bases on Avogadro's law i.e. $V \propto n$ at the same temperature and pressure.

\therefore The mole concept may be applied in solving the problems, keeping in mind that in a gaseous reaction the relative volumes (measured under identical conditions) of each reactant and product represent their relative numbers of moles.

eg: $\text{A(g)} + \text{B(g)} \rightarrow \text{C(g)} + \text{D(g)}$

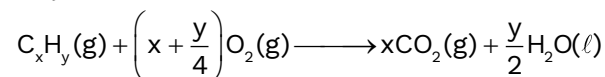
a volumes b volumes c volumes d volumes

a moles b moles c moles d moles

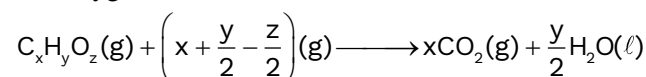
Generally, explosions are carried out at STP and H_2O is assumed to be in liquid state, means its volume is negligible as compared to product gases.

Burning of hydrocarbon:

1. Hydrocarbon containing carbon and hydrogen only.



2. Hydrocarbon containing carbon and hydrogen and oxygen.



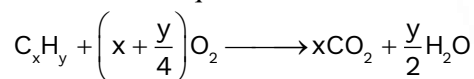
**Example:**

A gaseous hydrocarbon requires 6 times its own volume of O_2 for complete oxidation and produces 4 times its volume of CO_2 . What is its formula?

Ans. C_4H_8

Solution:

The balanced equation for combustion



$$1 \text{ volume} \left(x + \frac{y}{4}\right) \text{ volume}$$

$$\therefore x + \frac{y}{4} = 6 \text{ (by equation) or } 4x + y = 24 \quad \dots\dots(1)$$

Again $x = 4$ since evolved CO_2 is 4 times that of hydrocarbon

$$\therefore 16 + y = 24 \text{ or } y = 8$$

\therefore formula of hydrocarbon C_4H_8

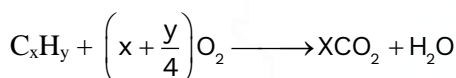
Example:

7.5 ml of a gaseous hydrocarbon was exploded with 36 ml of oxygen. The volume of gases on cooling was found to be 28.5 ml, 15 ml of which was absorbed by KOH and the rest was absorbed in a solution of alkaline pyrogallol. If all volumes are measured under same conditions, the formula of hydrocarbon is:

(A) C_3H_4 (B) C_2H_4

(C) C_2H_6 (D) C_3H_6

Ans. (B)

Solution:

7.5 ml 36 ml

$$36 - 7.5 \left(15 + \frac{y}{4}\right) + 7.5x = 28.5$$

$$36 - 7.5 \left(15 + \frac{y}{4}\right) + 7.5x = 28.5$$

$$y = 4$$

$$x = 2$$

So, formula = C_2H_4

Example:

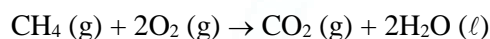
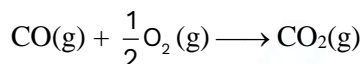
A 30 mL mixture of CO, CH_4 and He gases is exploded by an electric discharge at room temperature with excess of oxygen. The decrease in volume is found to be 13 mL. A further contraction of 14 mL occurs, when the residual gas is treated with KOH solution. Find out the composition of the gaseous mixture in terms of volume percentage.

Ans. Percentage composition of CO = 33.33 %;
 CH_4 = 13.33 %; He = 53.33%

Solution:

Let the volume of CO be 'a' mL and CH_4 be 'b' mL \therefore

Volume of He = (30 - a - b) on explosion with oxygen



'a' mL of CO give 'a' mL of CO_2 and 'b' mL of CH_4 gives 'b' mL of CO_2 .

Therefore the volume decrease is due to the consumption of O_2 . O_2 consumed for 'a' mL of CO is

$\frac{a}{2}$ mL and O_2 consumed for 'b' mL of CH_4 is '2b' mL

$$\therefore \frac{a}{2} + 2b = 13$$

The further contraction occurs because of the absorption of CO_2 by KOH, $a + b = 14$

$$\therefore b = 4 \text{ mL}$$

$$\therefore a = 10 \text{ mL}$$

$$\therefore \text{Percentage composition of CO} = \frac{10}{30} \times 100$$

$$= 33.33 \%$$

$$\text{Percentage composition of } CH_4 = \frac{4}{30} \times 100 = 13.33$$

$$\text{Percentage composition of He} = \frac{(30 - 10 - 4)}{30} \times 100$$

$$= 53.33 \%$$

FUNDAMENTAL UNLOCKED- (FU#8) :

Q.1 A gaseous alkane is exploded with oxygen. The volume of O_2 for complete combustion to CO_2 formed is in the ratio 7/4. The molecular formula of alkane is:

(A) C_2H_4

(B) C_2H_6

(C) CH_4

(D) C_4H_{12}

Q.2 10 ml of gaseous hydrocarbon is exploded with 100 ml O_2 . The residual gas on cooling is found to measure 95 ml of which 20 ml is absorbed by KOH and the reminder by alkaline pyrogallol. The formula of the hydrocarbon is:

(A) CH_4

(B) C_2H_6

(C) C_2H_4

(D) C_2H_2





ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1) :

- Q.1** N_A
Q.2 (C)
Q.3 (i) $H = 4N_A$, $S = 2N_A$, $O = 8N_A$ atoms (ii) $H = 4$ atoms, $S = 2$ atoms, $O = 8$ atoms.
 (iii) $H = 10N_A$, $S = 10N_A$, $O = 40 N_A$ atoms (iv) $H = 6$ atoms, $S = 6$ atoms, $O = 18$ atoms.
Q.4 1.88×10^{22}
Q.5 (C) **Q.6** (A) **Q.7** (B)

FUNDAMENTAL UNLOCKED- (FU#2) :

- Q.1** (A) **Q.2** 2.16 g **Q.3** (C) **Q.3** (C) **Q.4** (C)

FUNDAMENTAL UNLOCKED- (FU#3) :

- Q.1** (i) O_2 is left in excess.
 (ii) 3 moles of O_2 or 96 g of O_2 is left.
 (iii) 2 moles of CO or 56 g of CO is formed.
 (iv) To use O_2 completely, total 8 moles of carbon or 96 g of carbon is needed.
Q.2 (A)
Q.3 (A) **Q.4** (A) **Q.5** (C)

FUNDAMENTAL UNLOCKED- (FU#4) :

- Q.1** 35.33% **Q.2** (D) **Q.3** (A) **Q.4** (A) **Q.5** 11
Q.6 0.95 g

FUNDAMENTAL UNLOCKED- (FU#5) :

- Q.1** (D) **Q.2** (72) **Q.3** (B) **Q.4** (B) **Q.5** (A)
Q.6 (A) **Q.7** (D) **Q.8** (A) **Q.9** (C)

FUNDAMENTAL UNLOCKED- (FU#6) :

- Q.1** (D) **Q.2** (B) **Q.3** (B) **Q.4** (D) **Q.5** 20%
Q.6 (B)

FUNDAMENTAL UNLOCKED- (FU#7) :

- Q.1** (A) **Q.2** (A) **Q.3** (B) **Q.4** (B) **Q.5** (D)
Q.6 (D) **Q.7** 700 ml.

FUNDAMENTAL UNLOCKED- (FU#8) :

- Q.1** (B) **Q.2** (D)





OBJECTIVE EXERCISE - I

Single Correct Type Question

Problems Related with different types of Atomic Masses & Basic Concept of Mole

- Which of the following has the Maximum mass?
(A) 1 g-atom of C
(B) $\frac{1}{2}$ mole of CH_4
(C) 10 mL of water
(D) 3.011×10^{23} atoms of oxygen
- The number of molecules of CO_2 present in 44 g of CO_2 is:
(A) 6.0×10^{23} (B) 3×10^{23}
(C) 12×10^{23} (D) 3×10^{10}
- The number of mole of ammonia in 4.25 g of ammonia is:
(A) 0.425 (B) 0.25
(C) 0.236 (D) 0.2125
- The charge on 1 gram ions of Al^{3+} is: (N_A = Avogadro number, e = charge on one electron)
(A) $\frac{1}{27} N_{Ae}$ coulomb
(B) $\frac{1}{3} \times N_{Ae}$ coulomb
(C) $\frac{1}{9} \times N_{Ae}$ coulomb
(D) $3 \times N_{Ae}$ coulomb
- The atomic weights of two elements A and B are 40u and 80u respectively. If x g of A contains y atoms, how many atoms are present in 2x g of B?
(A) $\frac{y}{2}$ (B) $\frac{y}{4}$ (C) y (D) 2y
- A sample of aluminium has a mass of 54.0 g. What is the mass of the same number of magnesium atoms?
(At. wt. Al = 27, Mg = 24)
(A) 12 g (B) 24 g (C) 48 g (D) 96 g.
- The weight of a molecule of the compound $\text{C}_{60}\text{H}_{22}$ is:
(A) 1.09×10^{-21} g (B) 1.24×10^{-21} g
(C) 5.025×10^{-23} g (D) 16.023×10^{-23} g
- The number of electron in 3.1 mg NO_3^- is -
(A) 32 (B) 1.6×10^{-3}
(C) 9.6×10^{20} (D) 9.6×10^{23}
- A gaseous mixture contains CO_2 (g) and N_2O (g) in a 2: 5 ratio by mass. The ratio of the number of molecules of CO_2 (g) and N_2O (g) is:
(A) 5: 2 (B) 2: 5 (C) 1: 2 (D) 5: 4
- Which of the following contain largest number of carbon atoms?
(A) 15 gm ethane, C_2H_6
(B) 40.2 gm sodium oxalate, $\text{Na}_2\text{C}_2\text{O}_4$
(C) 72 gm glucose, $\text{C}_6\text{H}_{12}\text{O}_6$
(D) 35 gm pentene, C_5H_{10}
- The number of hydrogen atoms in 0.9 gm glucose, $\text{C}_6\text{H}_{12}\text{O}_6$, is same as
(A) 0.048 gm hydrazine, N_2H_4
(B) 0.17 gm ammonia, NH_3
(C) 0.30 gm ethane, C_2H_6
(D) 0.03 gm hydrogen, H_2
- The weight of 1×10^{22} molecules of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ is:
(A) 41.59 g (B) 415.9 g
(C) 4.159 g (D) 2.38 g
- The number of carbon atoms present in a signature, if a signature written by carbon pencil weights 1.2×10^{-3} g is:
(A) 12.04×10^{20} (B) 6.02×10^{19}
(C) 3.01×10^{19} (D) 6.02×10^{20}
- Ethanol, $\text{C}_2\text{H}_5\text{OH}$, is the substance commonly called alcohol. The density of liquid alcohol is 0.8 g/ml at 293 K. If 1.2 mole of ethanol are needed for a particular experiment, what volume of ethanol should be measured out?
(A) 55 ml (B) 58 ml
(C) 69 ml (D) 79 ml
- 112.0 ml of NO_2 at 1atm & 273 K was liquefied, the density of the liquid being 1.15 gm/ml. Calculate the volume of and the number of molecules in the liquid NO_2 .



- (A) 0.10 ml and 3.01×10^{22}
 (B) 0.20 ml and 3.01×10^{21}
 (C) 0.20 ml and 6.02×10^{23}
 (D) 0.40 ml and 6.02×10^{21}
16. X gm A atoms on combining with Y atoms of B form 5 molecules of a compound containing A & B. Find the molecular weight of compound formed. (Atomic weight of B = M)
- (A) $\frac{(XN_A + MY)}{5}$ (B) $\frac{X + M}{5}$
 (C) $\frac{X + MY}{5}$ (D) $\left(\frac{X + MYN_A}{5}\right)$
17. At same temperature and pressure, two gases have the same number of molecules. They must
- (A) have same mass
 (B) have equal volumes
 (C) have a volume of 22.7 dm^3 each
 (D) have an equal number of atoms
18. An iodized salt contains 0.5 % of NaI. A person consumes 3 gm of salt everyday. The number of iodide ions going into his body everyday is:
- (A) 10^{-4} (B) 6.02×10^{-4}
 (C) 6.02×10^{19} (D) 6.02×10^{23}
19. Equal volumes of oxygen gas and a second gas weigh 1.00 and 2.375 grams respectively under the same experimental conditions. Which of the following is the unknown gas?
- (A) NO (B) SO_2
 (C) CS_2 (D) CO

Stoichiometry

20. For the reaction $2P + Q \rightarrow R$, 8 mol of P and excess of Q will produce:
- (A) 8 mol of R (B) 5 mol of R
 (C) 4 mol of R (D) 13 mol of R
21. If 1.5 moles of oxygen combine with Al to form Al_2O_3 , the weight of Al used in the reaction is:
- (A) 27 g (B) 40.5 g
 (C) 54g (D) 81 g

22. 74 gm of a sample on complete combustion gives 132 gm CO_2 and 54 gm of H_2O . The molecular formula of the compound may be:
- (A) C_5H_{12} (B) $\text{C}_4\text{H}_{10}\text{O}$
 (C) $\text{C}_3\text{H}_6\text{O}_2$ (D) $\text{C}_3\text{H}_7\text{O}_2$
23. The mass of CO_2 produced from 620 gm mixture of $\text{C}_2\text{H}_4\text{O}_2$ & O_2 , prepared to produce maximum energy is (Combustion reaction is exothermic)
- (A) 413.33 gm (B) 593.04 gm
 (C) 440 gm (D) 320 gm
24. The minimum mass of mixture of A_2 and B_4 required to produce at least 1 kg of each product is:
- (Given At. mass of 'A' = 10 ; At. mass of 'B' = 120)
- $$5\text{A}_2 + 2\text{B}_4 \longrightarrow 2\text{AB}_2 + 4\text{A}_2\text{B}$$
- (A) 2120 gm (B) 1060 gm
 (C) 560 gm (D) 1660 gm

Limiting Reagent

25. The mass of Mg_3N_2 produced if 48 gm of Mg metal is reacted with 34 gm NH_3 gas is
- $$\text{Mg} + \text{NH}_3 \longrightarrow \text{Mg}_3\text{N}_2 + \text{H}_2$$
- (A) $\frac{200}{3}$ gm (B) $\frac{100}{3}$ gm
 (C) $\frac{400}{3}$ gm (D) $\frac{150}{3}$ gm
26. The mass of P_4O_{10} produced if 440 gm of P_4S_3 is mixed with 384 gm of O_2 is
- $$\text{P}_4\text{S}_3 + \text{O}_2 \longrightarrow \text{P}_4\text{O}_{10} + \text{SO}_2$$
- (A) 568 gm (B) 426 gm
 (C) 284 gm (D) 396 gm
27. Mass of sucrose $\text{C}_{12}\text{H}_{22}\text{O}_{11}$ produced by mixing 84 gm of carbon, 12 gm of hydrogen and 56 lit. O_2 at 1 atm & 273 K according to given reaction, is
- $$\text{C(s)} + \text{H}_2\text{(g)} + \text{O}_2\text{(g)} \longrightarrow \text{C}_{12}\text{H}_{22}\text{O}_{11}\text{(s)}$$
- (A) 138.5 (B) 155.5 (C) 172.5 (D) 199.5
28. 0.5 mole of H_2SO_4 is mixed with 0.2 mole of Ca(OH)_2 . The maximum number of moles of CaSO_4 formed is:
- (A) 0.2 (B) 0.5 (C) 0.4 (D) 1.5

**Sequential & Parallel Reactions**

29. 25.4 g of iodine and 14.2g of chlorine are made to react completely to yield a mixture of ICl and ICl₃. Calculate the number of moles of ICl and ICl₃ formed.
 (A) 0.1 mole, 0.1 mole
 (B) 0.1 mole, 0.2 mole
 (C) 0.5 mole, 0.5 mole
 (D) 0.2 mole, 0.2 mole
30. What weights of P₄O₆ and P₄O₁₀ will be produced by the combustion of 31g of P₄ in 32g of oxygen leaving no P₄ and O₂.
 (A) 2.75 g, 219.5 g
 (B) 27.5 g, 35.5 g
 (C) 55 g, 71 g
 (D) 17.5 g, 190.5 g
31. What weight of CaCO₃ must be decomposed to produce the sufficient quantity of carbon dioxide to convert 21.2 kg of Na₂CO₃ completely in to NaHCO₃.
 [Atomic mass Na = 23, Ca = 40]
 $\text{CaCO}_3 \longrightarrow \text{CaO} + \text{CO}_2$
 $\text{Na}_2\text{CO}_3 + \text{CO}_2 + \text{H}_2\text{O} \longrightarrow 2\text{NaHCO}_3$
 (A) 100 Kg (B) 20 Kg
 (C) 120 Kg (D) 30 Kg
32. The following process has been used to obtain iodine from oil-field brines in California.
 $\text{NaI} + \text{AgNO}_3 \longrightarrow \text{AgI} + \text{NaNO}_3$; $2\text{AgI} + \text{Fe} \longrightarrow \text{FeI}_2 + 2\text{Ag}$
 $2\text{FeI}_2 + 3\text{Cl}_2 \longrightarrow 2\text{FeCl}_3 + 2\text{I}_2$
 How many grams of AgNO₃ are required in the first step for every 254 kg I₂ produced in the third step.
 (A) 340 kg (B) 85 kg
 (C) 68 kg (D) 380 kg
33. 10 g of a sample of a mixture of CaCl₂ and NaCl is treated to precipitate all the calcium as CaCO₃. This Ca CO₃ is heated to convert all the Ca to CaO and the final mass of CaO is 1.62 g. The percent by mass of CaCl₂ in the original mixture is.
 (A) 32.1 % (B) 16.2 %
 (C) 21.8 % (D) 11.0 %

Miscellaneous Problem

34. 40 gm of a carbonate of an **alkali metal** or **alkaline earth metal** containing some inert impurities was made to react with excess HCl solution. The liberated CO₂ occupied 12.315 lit. at 1 atm & 300 K. The correct option is—
 (A) Mass of impurity is 1 gm and metal is Be
 (B) Mass of impurity is 3 gm and metal is Li
 (C) Mass of impurity is 5 gm and metal is Be
 (D) Mass of impurity is 2 gm and metal is Mg
35. In chemical scale, the relative mass of the isotopic mixture of X atoms (X²⁰, X²¹, X²²) is approximately equal to: (X²⁰ has 99 percent abundance)
 (A) 20.002 (B) 21.00
 (C) 22.00 (D) 20.00
36. Calculate percentage change in M_{avg} of the mixture, if PCl₅ undergo 50% decomposition in a closed vessel: $\text{PCl}_5 \longrightarrow \text{PCl}_3 + \text{Cl}_2$
 (A) 50% (B) 66.66 %
 (C) 33.33 % (D) Zero
37. A compound possess 8% sulphur by mass. The least molecular mass is:
 (A) 200 (B) 400
 (C) 155 (D) 355
38. The empirical formula of a compound of molecular mass 120 is CH₂O. The molecular formula of the compound is:
 (A) C₂H₄O₂ (B) C₄H₈O₄
 (C) C₃H₆O₃ (D) all of these
39. Calculate the molecular formula of compound which contains 20% Ca and 80% Br (by wt.) if molecular weight of compound is 200. (Atomic wt. Ca = 40, Br = 80)
 (A) Ca_{1/2}Br (B) CaBr₂
 (C) CaBr (D) Ca₂Br
40. Cortisone is a molecular substance containing 21 atoms of carbon per molecule. The mass percentage of carbon in cortisone is 69.98%. Its molar mass is:
 (A) 176.5 (B) 252.2
 (C) 287.6 (D) 360.1



41. The percentage by mole of NO_2 in a mixture of $\text{NO}_2(\text{g})$ and $\text{NO}(\text{g})$ having average molecular mass 34 is:
(A) 25% (B) 20% (C) 40% (D) 75%

Analysis of Gaseous Mixture (Eudiometry)

42. A definite amount of gaseous hydrocarbon was burnt with just sufficient amount of O_2 . The volume of all reactants was 600 ml, after the explosion the volume of the products [$\text{CO}_2(\text{g})$ and $\text{H}_2\text{O}(\text{g})$] was found to be 700 ml under the similar conditions. The molecular formula of the compound is:
(A) C_3H_8 (B) C_3H_6 (C) C_3H_4 (D) C_4H_{10}
43. $\text{C}_6\text{H}_5\text{OH}(\text{g}) + \text{O}_2(\text{g}) \longrightarrow \text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{l})$
Magnitude of volume change if 30 ml of $\text{C}_6\text{H}_5\text{OH}(\text{g})$ is burnt with excess amount of oxygen, is:
(A) 30 ml (B) 60 ml (C) 20 ml (D) 10 ml
44. 10 ml of a compound containing 'N' and 'O' is mixed with 30 ml of H_2 to produce $\text{H}_2\text{O}(\text{l})$ and 10 ml of $\text{N}_2(\text{g})$. Molecular formula of compound if both reactants react completely, is:
(A) N_2O (B) NO_2 (C) N_2O_3 (D) N_2O_5
45. When 20 ml of mixture of O_2 and O_3 is heated, the volume becomes 29 ml and disappears in alkaline pyrogallol solution. What is the volume percent of O_2 in the original mixture?
(A) 90% (B) 10%
(C) 18% (D) 2%
46. The % by volume of C_4H_{10} in a gaseous mixture of C_4H_{10} , CH_4 and CO is 40. When 200 ml of the mixture is burnt in excess of O_2 . Find volume (in ml) of CO_2 produced.
(A) 220 (B) 340
(C) 440 (D) 560
47. The percentage by volume of C_3H_8 in a mixture of C_3H_8 , CH_4 and CO is 36.5. Calculate the volume of CO_2 produced when 100 mL of the mixture is burnt in excess of O_2 .
(A) 173 mL (B) 106.5 mL
(C) 206.5 mL (D) 156.5 mL
48. 4 gm of C_3H_8 and 14 gm of O_2 are allowed to react maximum possible extent to forms only CO & H_2O . In final gaseous mixture which of the given relation is incorrect-
(A) $\frac{n_{\text{CO}}}{n_{\text{O}_2}} = \frac{16}{7}$ (B) $\% w_{\text{CO}} = \frac{200}{3}$
(C) $W_{\text{CO}} = 7.636 \text{ gm}$ (D) $W_{\text{CO}} = 14 \text{ gm}$
49. One litre of CO_2 passed over hot coke the volume becomes 1.4 litres then the composition of products will not be (At STP):
(A) $V_{\text{CO}_2} : V_{\text{CO}} = 3 : 4$ (B) $V_{\text{CO}_2} = 1.6 \text{ ltr.}$
(C) $n_{\text{CO}_2} : n_{\text{CO}} = 3 : 4$ (D) $\% V \text{ of CO} = \frac{400}{7}$
50. 25 moles of mixture of SO_2 & O_2 was passed over a catalyst 8 moles of SO_3 was formed. After reaction the final mixture composition is/are -
(A) 19 mole of O_2 , 8 mole of SO_3
(B) 13 mole of SO_2 , 8 mole of O_2
(C) 9 mole of O_2 , 12 mole of SO_3
(D) 15 mole of O_2 , 10 mole of SO_3
51. For a chemical reaction occurring at constant pressure and temperature.
 $2\text{A}(\text{g}) + 5\text{B}(\text{g}) \longrightarrow \text{C}(\text{g}) + 2\text{D}(\text{g})$
(A) Contraction in volume is double the volume of A taken if B is taken in excess.
(B) Contraction in volume is more than the volume of B taken if A is in excess.
(C) Volume contracts by 20 mL if 10 mL A is reacted with 20 mL B.
(D) No change in volume due to reaction

Concentration Terms

52. 8 g NaOH is dissolved in one litre of solution, its molarity is:
(A) 0.8 M (B) 0.4 M
(C) 0.2 M (D) 0.1 M
53. The molarity of a solution of sodium chloride (mole wt. = 58.5) in water containing 5.85 gm of sodium chloride in 500 ml of solution is:
(A) 0.25 (B) 2.0
(C) 1.0 (D) 0.2



54. For preparing 0.1 M solution of H_2SO_4 in one litre, we need H_2SO_4 :
(A) 0.98 g (B) 4.9 g
(C) 49.0 g (D) 9.8 g
55. H_2O_2 solution used for hair bleaching is sold as a solution of approximately 5.0 g H_2O_2 per 100 mL of the solution. The molecular mass of H_2O_2 is 34. The molarity of this solution is approximately:
(A) 0.15 M (B) 1.5 M
(C) 3.0 M (D) 3.4 M
56. 171 g of cane sugar ($\text{C}_{12}\text{H}_{22}\text{O}_{11}$) is dissolved in 1 litre of water. The molarity of the solution is:
(A) 2.0 M (B) 1.0 M
(C) 0.5 M (D) 0.25 M
57. How much grams of CH_3OH should be dissolved in water for preparing 150 ml. of 2.0 M CH_3OH solution:
(A) 9.6 (B) 2.4
(C) 9.6×10^3 (D) 4.3×10^2
58. Equal weight of NaCl and KCl are dissolved separately in equal volumes of solutions molarity of the two solutions will be—
(A) Equal
(B) That of NaCl will be less than that of KCl
(C) That of NaCl will be more than that of KCl
(D) That of NaCl will be half of that of KCl
59. The molarity of pure water is:
(A) 100 M (B) 55.5 M
(C) 50 M (D) 18M
60. Molarity of liquid HCl if density of solution is 1.17 g/cc.:
(A) 36.5 (B) 18.25
(C) 32.05 (D) 42.10
61. If 18 g of glucose is present in 1000 g of solvent, the solution is said to be:
(A) 1 molar (B) 0.1 molar
(C) 0.5 molar (D) 0.1 molal
62. A molal solution is one that contains one mole of a solute in
(A) 1000 g of the solvent
(B) one litre of the solution
(C) one litre of the solvent
(D) 22.4 litres of the solution
63. Which of the following solution has maximum mass of pure NaOH ?
(I) 50 g of 40% (W/W) NaOH
(II) 50 mL of 40% (W/V) NaOH ($d_{\text{sol}} = 1.2$ g/ml).
(III) 50 g of 12 M NaOH ($d_{\text{sol}} = 1$ g/ml).
(A) I (B) II
(C) III (D) III = II = I.
64. Mole fraction of $\text{C}_3\text{H}_5(\text{OH})_3$ (glycerine) in a solution of 36 g of water and 46 g of glycerine is:
(A) 0.46 (B) 0.36
(C) 0.20 (D) 0.40
65. The mole fraction of oxygen in a mixture of 7g of nitrogen and 8g of oxygen is:
(A) $\frac{8}{15}$ (B) 0.5
(C) 0.25 (D) 1.0
66. 1000 g aqueous solution of CaCO_3 contains 10 g of calcium carbonate concentration of the solution is:
(A) 10 ppm (B) 100 ppm
(C) 1000 ppm (D) 10,000 ppm
67. Which one of the following modes of expressing concentration of solution is independent of temperature—
(A) Molarity (B) Molality
(C) % w/v (D) Grams per litre
68. One mole mixture of CH_4 & air (containing 80% N_2 20% O_2 by volume) of a composition such that when underwent combustion gave maximum heat (assume combustion of only CH_4). Then which of the statements are correct, regarding composition of initial mixture. (X presents mole fraction)



(A) $X_{\text{CH}_4} = \frac{1}{11}$, $X_{\text{O}_2} = \frac{2}{11}$, $X_{\text{N}_2} = \frac{8}{11}$

(B) $X_{\text{CH}_4} = \frac{3}{8}$, $X_{\text{O}_2} = \frac{1}{8}$, $X_{\text{N}_2} = \frac{1}{2}$

(C) $X_{\text{CH}_4} = \frac{1}{6}$, $X_{\text{O}_2} = \frac{1}{6}$, $X_{\text{N}_2} = \frac{2}{3}$

(D) Data insufficient

Interconversion of Concentration Terms

69. The molarity of 98% by wt. H_2SO_4 ($d = 1.8$ g/ml) is

(A) 6 M (B) 18 M (C) 10 M (D) 4 M

70. Mole fraction of A in H_2O is 0.2. The molality of A in H_2O is:

(A) 13.9 (B) 15.5 (C) 14.5 (D) 16.8

71. The molarity of the solution containing 2.8% (mass/volume) solution of KOH is:

(Given atomic mass of K = 39) is:

(A) 0.1 M (B) 0.5 M
(C) 0.2 M (D) 1 M

72. Calculate the mass percent (w/w) of sulphuric acid in a solution prepared by dissolving 4 g of sulphur trioxide in a 100 ml sulphuric acid solution containing 80 mass percent (w/w) of H_2SO_4 and having a density of 1.96 g/ml. (molecular weight of $\text{H}_2\text{SO}_4 = 98$).

Take reaction: $\text{SO}_3 + \text{H}_2\text{O} \rightarrow \text{H}_2\text{SO}_4$

(A) 80.8% (B) 84%
(C) 41.65% (D) 20%

Problems Related with Mixing & Dilution

73. How much volume of 3.0 M H_2SO_4 is required for the preparation of 1.0 litre of 1.0 M solution?

(A) 300 ml (B) 320 ml
(C) 333.3 ml (D) 350.0 ml

74. How much water should be added to 200 cc of semimolar solution of NaOH to make it exactly decimolar:

(A) 1000 cc (B) 400 cc
(C) 800 cc (D) 600 cc

75. The molarity of a solution made by mixing 50 ml of conc. H_2SO_4 (18 M) with 50 ml. of water, is:

(A) 36 M (B) 18 M (C) 9 M (D) 6M

76. 100 ml of 0.3 M HCl solution is mixed with 200 ml of 0.3 M H_2SO_4 solution what is the molarity of H^+ in resultant solution.

(A) 0.9 (B) 0.6 (C) 0.4 (D) 0.5

77. 60 g of solution containing 40% by mass of NaCl are mixed with 100 g of a solution containing 15% by mass NaCl. Determine the mass percent of sodium chloride in the final solution.

(A) 24.4% (B) 78% (C) 48.8% (D) 19.68%

78. 125 ml of 8% w/w NaOH solution (sp. gravity 1) is added to 125 ml of 10% w/v HCl solution. The nature of resultant solution would be

(A) basic (B) neutral
(C) acidic (D) can't be predicted.

79. Equal volumes of 10% (v/v) of HCl is mixed with 10% (v/v) NaOH solution. If density of pure NaOH is 1.5 times that of pure HCl then the resultant solution be.

(A) basic (B) neutral
(C) acidic (D) can't be predicted.

80. What volumes should you mix of 0.2 M NaCl and 0.1 M CaCl_2 solution so that in resulting solution the concentration of positive ion is 40% lesser than concentration of negative ion. Assuming total volume of solution 1000 ml.

(A) 400 ml NaCl, 600 ml CaCl_2
(B) 600 ml NaCl, 400 ml CaCl_2
(C) 800 ml NaCl, 200 ml CaCl_2
(D) None of these

81. Assuming complete precipitation of AgCl, calculate the sum of the molar concentration of all the ions if 2 lit of 2M Ag_2SO_4 is mixed with 4 lit of 1 M NaCl solution is:

(A) 4M (B) 2M (C) 3 M (D) 2.5 M

Some Typical Concentration Terms

82. A fresh H_2O_2 solution is labeled as 11.35 V. Calculate its concentration in %w/v ?

(A) 2.5% (B) 3.4% (C) 4.2% (D) 5.4%



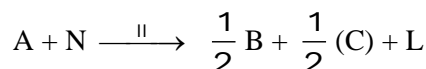
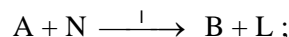
83. 100 ml each of 2M H_2O_2 and 11.35 V H_2O_2 solution are mixed then find the strength of final solution in g/L.
(A) 25 (B) 51 (C) 42 (D) 54
84. If a piece of iron gains 10% of its weight due to partial rusting into Fe_2O_3 the percentage of total iron that has rusted is:
(A) 23 (B) 13
(C) 23.3 (D) 25.67
85. 1 mol of iron (Fe) reacts completely with 0.65 mol O_2 to give a mixture of only FeO and Fe_2O_3 . Mole ratio of ferrous oxide to ferric oxide is:
(A) 3: 2 (B) 4: 3
(C) 20: 13 (D) none of these
86. 500 mL of a glucose solution contains 6.02×10^{22} molecules. The concentration of the solution is:
(A) 0.1 M (B) 1.0 M (C) 0.2 M (D) 2.0 M
87. Equal moles of H_2O and NaCl are present in a solution. Hence, molality of NaCl solution is:
(A) 0.55 (B) 55.5 (C) 1.00 (D) 0.18
88. Decreasing order of mass of pure NaOH in each of the aqueous solution.
(I) 50 g of 40% (W/W) NaOH
(II) 50 ml of 50% (W/V) NaOH ($d_{\text{sol}} = 1.2$ g/ml).
(III) 50 g of 15 M NaOH ($d_{\text{sol}} = 1$ g/ml).
(A) I, II, III (B) III, II, I
(C) II, III, I (D) III = II = I.
89. Mole fraction of A in H_2O is 0.2. The molality of A in H_2O is:
(A) 13.9 (B) 15.5
(C) 14.5 (D) 16.8
90. What is the molarity of H_2SO_4 solution that has a density of 1.84 g/cc and contains 98% by mass of H_2SO_4 ? (Given atomic mass of S = 32)
(A) 4.18 M (B) 8.14 M
(C) 18.4 M (D) 18 M
91. The molarity of the solution containing 2.8% (mass/volume) solution of KOH is:
(Given atomic mass of K = 39)
(A) 0.1 M (B) 0.5 M (C) 0.2 M (D) 1 M
92. A solution of FeCl_3 is $\frac{M}{30}$ its molarity for Cl^- ion will be:
(A) $\frac{M}{90}$ (B) $\frac{M}{30}$ (C) $\frac{M}{10}$ (D) $\frac{M}{5}$
93. If 500 ml of 1 M solution of glucose is mixed with 500 ml of 1 M solution of glucose final molarity of solution will be:
(A) 1 M (B) 0.5 M (C) 2 M (D) 1.5 M
94. The volume of water that must be added to a mixture of 250 ml of 0.6 M HCl and 750 ml of 0.2 M HCl to obtain 0.25 M solution of HCl is:
(A) 750 ml (B) 100 ml
(C) 200 ml (D) 300 ml
95. What volume of a 0.8 M solution contains 100 milli moles of the solute?
(A) 100 mL (B) 125 mL
(C) 500 mL (D) 62.5 mL
96. The molarity of Cl^- in an aqueous solution which was (w/V) 2% NaCl, 4% CaCl_2 and 6% NH_4Cl will be:
(A) 0.342 (B) 0.721
(C) 1.12 (D) 2.18
97. 2M of 100 ml Na_2SO_4 is mixed with 3M of 100 ml NaCl solution and 1M of 200 ml CaCl_2 solution. Then the ratio of the concentration of cation and anion.
(A) 1/2 (B) 2 (C) 1.5 (D) 1
98. What volume (in ml) of 0.2 M H_2SO_4 solution should be mixed with the 40 ml of 0.1 M NaOH solution such that the resulting solution has the concentration of H_2SO_4 as $\frac{6}{55}$ M.
(A) 70 (B) 45 (C) 30 (D) 58



OBJECTIVE EXERCISE - II

One Or More Than One Correct Type Question

- Select the correct statement(s) for $(\text{NH}_4)_3\text{PO}_4$.
 (A) Ratio of number of oxygen atom to number of hydrogen atom is 1: 3
 (B) Ratio of number of cation to number of anion is 3: 1
 (C) Ratio of number of gm-atom of nitrogen to gm-atoms of oxygen is 3: 2
 (D) Total number of atoms in one mole of $(\text{NH}_4)_3\text{PO}_4$ is 20.
- 12 g of Mg was burnt in a closed vessel containing 32 g oxygen. Which of the following is not correct.
 (A) 2 gm of Mg will be left unburnt.
 (B) 0.75 gm-molecule of O_2 will be left unreacted.
 (C) 20 gm of MgO will be formed.
 (D) The mixture at the end will weight 44 g.
- 50 gm of CaCO_3 is allowed to react with 68.6 gm of H_3PO_4 then select the correct option(s)-
 $3\text{CaCO}_3 + 2\text{H}_3\text{PO}_4 \rightarrow \text{Ca}_3(\text{PO}_4)_2 + 3\text{H}_2\text{O} + 3\text{CO}_2$
 (A) 51.67 gm salt is formed
 (B) Amount of unreacted reagent = 35.93 gm
 (C) $n_{\text{CO}_2} = 0.5$ moles
 (D) 0.7 mole CO_2 is evolved
- Industrially TNT ($\text{C}_7\text{H}_5\text{N}_3\text{O}_6$, explosive material) is synthesized by reacting toluene (C_7H_8) with nitric acid in presence of sulphuric acid. Calculate the maximum weight of $\text{C}_7\text{H}_5\text{N}_3\text{O}_6$ which can be produced by 140.5 gm of a mixture of C_7H_8 and HNO_3 .
 $\text{C}_7\text{H}_8 + 3\text{HNO}_3 \longrightarrow \text{C}_7\text{H}_5\text{N}_3\text{O}_6 + 3\text{H}_2\text{O}$
 (A) 140.5 (B) 113.5
 (C) $\frac{140.5}{2}$ (D) $140.5 - (3 \times 18)$
- 'A' reacts by following two parallel reaction to give B & C If half of 'A' goes into reaction I and other half goes to reaction-II. Then , select the correct statement(s)



- (A) B will be always greater than C
 (B) If 2 mole of C are formed then total 2 mole of B are also formed
 (C) If 2 mole of C are formed then total 4 mole of B are also formed
 (D) If 2 mole of C are formed then total 6 mole of B are also formed
- NaBr, used to produce AgBr for use in photography can be self prepared as follows:
 $\text{Fe} + \text{Br}_2 \longrightarrow \text{FeBr}_2 \dots(\text{i})$
 $\text{FeBr}_2 + \text{Br}_2 \longrightarrow \text{Fe}_3\text{Br}_8 \dots(\text{ii})$ (not balanced)
 $\text{Fe}_3\text{Br}_8 + \text{Na}_2\text{CO}_3 \longrightarrow \text{NaBr} + \text{CO}_2 + \text{Fe}_3\text{O}_4 \dots(\text{iii})$ (not balanced)
 - Mass of iron required to produce 2.06×10^3 kg NaBr
 (A) 420 gm (B) 420 kg
 (C) 4.2×10^5 kg (D) 4.2×10^8 gm
 - If the yield of (ii) is 60% & (iii) reaction is 70% then mass of iron required to produce 2.06×10^3 kg NaBr
 (A) 10^5 kg (B) 10^5 gm
 (C) 10^3 kg (D) None
 - If yield of (iii) reaction is 90% then mole of CO_2 formed when 2.06×10^3 gm NaBr is formed
 (A) 20 (B) 10
 (C) 40 (D) None
- A 10 ml mixture of N_2 , Alkane & O_2 undergo combustion in Eudiometry tube. There was contraction of 2 ml, when residual gases are passed through KOH. To the remaining mixture comprising of only one gas excess H_2 was added & after reaction the gas produced is absorbed by water, causing a reduction in volume of 8 ml.



- (a) Gas produced after introduction of H_2 in the mixture?
 (A) H_2O (B) CH_4
 (C) CO_2 (D) NH_3
- (b) Volume of N_2 present in the mixture?
 (A) 2 ml (B) 4 ml
 (C) 6 ml (D) 8 ml
- (c) Volume of O_2 remained after the first combustion?
 (A) 4 ml (B) 2 ml
 (C) 0 (D) 8 ml
- (d) Identify the hydrocarbon.
 (A) CH_4 (B) C_2H_6
 (C) C_3H_8 (D) C_4H_{10}
8. Solution(s) containing 40 gm NaOH is/are
 (A) 50 gm of 80% (w/w) NaOH
 (B) 50 gm of 80% (w/v) NaOH [$d_{soln.} = 1.2$ gm/ml]
 (C) 50 gm of 20 M NaOH [$d_{soln.} = 1$ gm/ml]
 (D) 50 gm of 5m NaOH
9. The incorrect statement(s) regarding 2M $MgCl_2$ aqueous solution is/are: ($d_{solution} = 1.09$ gm/ml)
 (A) Molality of Cl^- is 4.44 m
 (B) Mole fraction of $MgCl_2$ is exactly 0.035
 (C) The conc. of $MgCl_2$ is 19% w/v
 (D) The conc. of $MgCl_2$ is 19×10^4 ppm
10. A sample of H_2O_2 solution labelled as 56 volume has density of 530 gm/L. Mark the correct option(s) representing concentration of same solution in other units. (Solution contains only H_2O and H_2O_2)
 (A) $M_{H_2O_2} = 6$
 (B) $\frac{w}{v} \% = 17$
 (C) Mole fraction of $H_2O_2 = 0.25$
 (D) $M_{H_2O_2} = \frac{1000}{72}$
11. 100 mL of 0.06 M $Ca(NO_3)_2$ is added to 50 mL of 0.06 M $Na_2C_2O_4$. After the reaction is complete (CaC_2O_4 is precipitated)

- (A) 0.003 moles of calcium oxalate will get precipitated
 (B) 0.003 M Ca^{2+} will remain in excess
 (C) $Na_2C_2O_4$ is the limiting reagent
 (D) Oxalate ion ($C_2O_4^{2-}$) concentration in final solution is 0.003 M

12. Solutions containing 23 g $HCOOH$ is/are:

- (A) 46 g of 70% $\left(\frac{w}{v}\right)$ $HCOOH$ ($d_{solution} = 1.40$ g/mL)
 (B) 50 g of 10 M $HCOOH$ ($d_{solution} = 1$ g/mL)
 (C) 50 g of 25% $\left(\frac{w}{w}\right)$ $HCOOH$
 (D) 46 g of 5 M $HCOOH$ ($d_{solution} = 1$ g/mL)

13. **Statement -1:** $2A + 3B \longrightarrow C$

$4/3$ moles of 'C' are always produced when 3 moles of 'A' & 4 moles of 'B' are added.

Statement -2: 'B' is the limiting reactant for the given data.

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
 (C) Statement-1 is false, statement-2 is true.
 (D) Statement-1 is true, statement-2 is false.

14. **Statement-1:** Molality of pure ethanol is lesser than pure water.

Statement-2: As density of ethanol is lesser than density of water.

[Given: $d_{ethanol} = 0.789$ gm/ml; $d_{water} = 1$ gm/ml]

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
 (C) Statement-1 is false, statement-2 is true.
 (D) Statement-1 is true, statement-2 is false.



15. **Statement -1:** Mass of a solution of 1 litre of 2M H_2SO_4 [$d_{\text{solution}} = 1.5 \text{ gm/ml}$] is greater than the mass of solution containing 400 gm MgO which is labelled as 40% (w/w) MgO.

Statement -2: Mass of H_2SO_4 in 1 litre 2M H_2SO_4 [$d_{\text{solution}} = 1.5 \text{ gm/ml}$] is greater than the mass of MgO in 1 litre 40% (w/w) MgO [$d_{\text{solution}} = 2 \text{ gm/ml}$] solution.

- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
 (C) Statement-1 is false, statement-2 is true.
 (D) Statement-1 is true, statement-2 is false.

Comprehension Type Question

Comprehension 16 and 17 (2 questions)

2 litre of 9.8 % w/w H_2SO_4 ($d = 1.5 \text{ gm/ml}$) solution is mixed with 3 litre of 1 M KOH solution.

16. The number of moles H_2SO_4 added are
 (A) 1 (B) 2 (C) 3 (D) 0.5
17. The concentration of H^+ if solution is acidic or concentration of OH^- if solution is basic in the final solution is
 (A) 0 (B) $\frac{3}{10}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

Comprehension 18 and 19 (2 questions)

Estimation of halogens:

Carius method: A known mass of compound is heated with conc. HNO_3 in the presence of AgNO_3 contained in a hard glass tube known as carius tube in a furnace. C and H are oxidised to CO_2 and H_2O . The halogen forms the corresponding AgX . It is filtered, dried, and weighed.

Estimation of sulphur: A known mass of compound is heated with fuming HNO_3 or sodium peroxide (Na_2O_2) in the presence of BaCl_2 solution in Carius tube. Sulphur is oxidised to H_2SO_4 and precipitated as BaSO_4 . It is filtered, dried and weighed.

18. 0.15gm of an organic compound gave 0.12 gm of silver bromide by the Carius method. Find the percentage of bromine in the compound.
 (A) 34.0 (B) 40 (C) 17 (D) 68

19. 0.2595 gm of an organic substance when treated by Carius method gave 0.35gm of BaSO_4 . Calculate the percentage of sulphur in the compound.
 (A) 9 (B) 30.4 (C) 18.52 (D) 40.52

Comprehension 20 and 21 (2 questions)

Estimation of phosphorous:

A known mass of compound is heated with fuming HNO_3 or sodium peroxide (Na_2O_2) in Carius tube which converts phosphorous to H_3PO_4 . Magnesia mixture ($\text{MgCl}_2 + \text{NH}_4\text{Cl}$) is then added, which gives the precipitate of magnesium ammonium phosphate (MgNH_4PO_4) which on heating gives magnesium pyrophosphate ($\text{Mg}_2\text{P}_2\text{O}_7$), which is weighed.

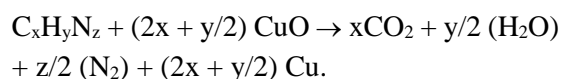
20. 0.12 gm of an organic compound containing phosphorus gave 0.22 gm of $\text{Mg}_2\text{P}_2\text{O}_7$ by the usual analysis. Calculate the percentage of phosphorous in the compound.
 (A) 25 (B) 9.25 (C) 80.1 (D) 51.20
21. An organic compound has 6.2% of phosphorus. On sequence of reaction the phosphorous present in the 10gm of organic compound is converted to $\text{Mg}_2\text{P}_2\text{O}_7$. Find wt. of $\text{Mg}_2\text{P}_2\text{O}_7$ formed.
 (A) 2.22 (B) 10.2 (C) 15 (D) 20

Comprehension 22 and 25 (4 questions)

Estimation of nitrogen: There are two methods for the estimation of nitrogen (i) Dumas method and (ii) Kjeldahl's method.

Dumas method:

A known mass of compound is heated with copper oxide (CuO) in an atmosphere of CO_2 , which gives free nitrogen along with CO_2 and H_2O .

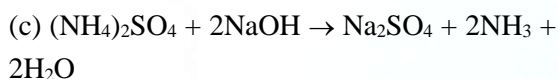
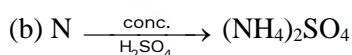
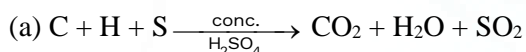




The gaseous mixture is passed over a heated copper gauze which converts traces of nitrogen oxides formed to N_2 . The gaseous mixture is collected over an aqueous solution of KOH which absorbs CO_2 , and nitrogen is collected in the upper part of the graduated tube.

Kjeldahl's method:

A known mass of organic compound (0.5 gm) is mixed with K_2SO_4 (10 gm) and $CuSO_4$ (1.0 gm) or a drop of mercury (Hg) and conc. H_2SO_4 (25 ml), and heated in Kjeldahl's flask. $CuSO_4$ or Hg acts as a catalyst, while K_2SO_4 raises the boiling point of H_2SO_4 . The nitrogen in the organic compound is quantitatively converted to ammonium sulphate. The resulting mixture is then distilled with excess of NaOH solution and the NH_3 evolved is passed into a known but excess volume of standard HCl or H_2SO_4 . The acid left unused is estimated by titration with some standard alkali. The amount of acid used against NH_3 can thus be known and from this the percentage of nitrogen is calculated.



This method is not applicable to compounds containing N in nitro and azo groups, and N present in the ring (e.g., pyridine) as N of these compounds does not change to $(NH_4)_2SO_4$ (ammonium sulphate) under these reaction conditions.

22. 0.30 gm of an organic compound gave 50 ml of nitrogen collected at 300K and 715 mm pressure in Dumas method. Calculate the percentage of nitrogen in the compound. (Vapour pressure of water or aqueous tension of water at 300K is 15 mm.)
(A) 10.2 (B) 17.46 (C) 24 (D) 34

23. 0.50 gm of an organic compound was treated according to Kjeldahl's method. The ammonia evolved was absorbed in 50 ml of 0.5M H_2SO_4 . The residual acid required 60 ml of M/2 NaOH solution. Find the percentage of nitrogen in the compound.
(A) 50 (B) 56.0 (C) 66 (D) 40
24. 0.4 gm of an organic compound was treated according to Kjeldahl's method. The ammonia evolved was absorbed in 50 ml of 0.5M H_3PO_3 . The residual acid required 30 ml of 0.5M $Ca(OH)_2$. Find the percentage of N_2 in the compound.
(A) 20 (B) 50 (C) 70 (D) 90
25. 0.002 gm of an organic compound was treated according to Kjeldahl's method. 0.2×10^{-4} mol of H_2SO_4 was required to neutralise NH_3 . Calculate the percentage of N_2 .
(A) 50 (B) 28 (C) 70 (D) 18

Matrix Match Type Question

26. One type of artificial diamond (commonly called YAG for yttrium aluminium garnet) can be represented by the formula $Y_3Al_5O_{12}$. [Y = 89, Al = 27]

Column-I

Element

- (P) Y
(Q) Al
(R) O

Column-II

Weight percentage

- (1) 22.73%
(2) 32.32%
(3) 44.95%

27. The recommended daily dose is 17.6 milligrams of vitamin C (ascorbic acid) having formula $C_6H_8O_6$. Match the following. Given: $N_A = 6 \times 10^{23}$

Column I

- (A) O-atoms present
(B) Moles of vitamin C in 1 gm of vitamin C
(C) Moles of vitamin C that should be consumed daily

Column II

- (P) 10^{-4} mole
(Q) 5.68×10^{-3}
(R) 3.6×10^{20}





- 28. Column-I**
(mass of product)
- (P) $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$ 1g 1g
- (Q) $3\text{H}_2 + \text{N}_2 \rightarrow 2\text{NH}_3$ 1g 1g
- (R) $\text{H}_2 + \text{Cl}_2 \rightarrow 2\text{HCl}$ 1g 1g
- (S) $2\text{H}_2 + \text{C} \rightarrow \text{CH}_4$ 1g 1g

- Column-II**
- (1) 1.028 g
- (2) 1.333 g
- (3) 1.125 g
- (4) 1.214 g

Code:

	P	Q	R	S
(A)	3	4	1	2
(B)	2	4	1	3
(C)	4	3	1	2
(D)	2	3	1	4

- 29.** Gaseous alkane ($\text{C}_n\text{H}_{2n+2}$) exploded with oxygen. Ratio of the mol of O_2 for complete combustion to the mole of CO_2 formed is given in column-I & in column II formula is given.

Column-I	Column-II
(P) 7: 4	(1) C_3H_8
(Q) 2: 1	(2) C_4H_{10}
(R) 5: 3	(3) C_2H_6
(S) 13: 8	(4) CH_4

Code:

P	Q	R	S
(A) 3	2	1	2
(B) 2	4	1	3
(C) 3	4	1	2
(D) 2	3	1	4

- 30. Column-I**
(solvent)
- (P) Turpentine oil
- (Q) $\text{CuSO}_4/\text{CaCl}_2$
- (R) KOH
- (S) Alkaline pyrogallol
- Column-II**
(gases absorbed)
- (1) H_2O
- (2) O_2
- (3) $\text{CO}_2, \text{SO}_2, \text{Cl}_2$
- (4) O_3

Code:

P	Q	R	S
(A) 3	2	1	2
(B) 2	4	1	3
(C) 4	1	3	2
(D) 2	3	1	4

- 31. Column-I**
- (A) 10 M MgO ($d_{\text{solution}} = 1.20 \text{ gm/ml}$)
- Solute: MgO
- Solvent: H_2O
- (B) 40% w/v NaOH ($d_{\text{solution}} = 1.6 \text{ gm/ml}$)
- Solute: NaOH
- Solvent: H_2O
- (C) 8 m CaCO_3
- Solute: CaCO_3
- Solvent: H_2O
- (D) 0.6 mol fraction of 'X' (molecular mass = 20) in 'Y' (molecular mass 25)

Solute: X

Solvent: Y

Column-II

- (P) $W_{\text{solvent}} = 120 \text{ gm per 100 ml of solution}$
- (Q) $W_{\text{solution}} = 150 \text{ gm per 100 gm solvent}$
- (R) $W_{\text{solute}} = 120 \text{ gm per 100 gm of solvent}$
- (S) $W_{\text{solvent}} = 125 \text{ gm per 100 gm of solute}$

- 32. Column-I**
- (A) 20 V H_2O_2
- (B) 24.5 % w/v H_2SO_4
- (C) Pure water
- (D) 5% w/w NaOH ($d_{\text{solution}} = 1.2 \text{ gm/ml}$)
- Column-II**
- (P) 2.5 M
- (Q) 1.78 M
- (R) 1.5 M
- (S) 55.5 M

- 33. Column-I**
- (A) 120 g CH_3COOH in 1 L solution ($d_{\text{sol}} = 1.2 \text{ g/mL}$)
- (B) 120 g glucose dissolved in 1 L solution ($d_{\text{sol}} = 1.2 \text{ g/mL}$)
- (C) $X_{\text{NH}_2\text{CONH}_2} = 1/31$ (aqueous solution)
- (D) 19.6% (w/v) H_2SO_4 solution \rightarrow ($d_{\text{solution}} = 1.2 \text{ g/mL}$)
- Column-II**
- (P) $M = 2$
- (Q) 10% w/w solution
- (R) 12% w/v solution
- (S) $m = 1.85$
- (T) $m = 0.617$





SUBJECTIVE EXERCISE - I

Problems Related with Different Types of Atomic Masses & Basic Concept of Mole

- How much time (in seconds) would it take to distribute one Avogadro number of wheat grains if 10^{10} grains are distributed each second?
- What is the mass of one ^{12}C atom in gram?
- Calculate the weight of 12.046×10^{23} atoms of carbon.
- Find:
 - No. of moles of Cu atom in 10^{20} atoms of Cu.
 - Mass of 200 atoms of $^{16}_8\text{O}$ in amu
 - Mass of 100 atoms of $^{14}_7\text{N}$ in gm.
 - No. of molecules & atoms in 54 gm H_2O .
 - No. of atoms in 88 gm CO_2 .
- Calculate mass of O atoms in 6 gm CH_3COOH ?
- Calculate mass of water present in 499 gm $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$?
(Atomic mass – Cu = 63.5, S = 32, O = 16, H = 1)
- What mass of $\text{Na}_2\text{SO}_4 \cdot 7\text{H}_2\text{O}$ contains exactly 6.023×10^{22} atoms of oxygen?
- What is the volume of following at STP (i) 2 g of H_2 (ii) 16 g of O_3 .
- Calculate the number of electrons, protons and neutrons in 1 mole of $^{16}\text{O}^{2-}$ ions.
- The density of liquid mercury is 13.6 g/cm^3 . How many moles of mercury are there in 1 litre of the metal? (Atomic mass of Hg = 200.)
- A sample of ethane has the same mass as 10.0 million molecules of methane. How many C_2H_6 molecules does the sample contain?
- If, from 10 moles NH_3 and 5 moles of H_2SO_4 , all the H-atoms are removed in order to form H_2 gas, then find the number of H_2 molecules formed.

Stoichiometry

- Chlorine can be prepared by reacting HCl with MnO_2 . The reaction is represented by the equation: $\text{MnO}_2(\text{s}) + 4\text{HCl} \longrightarrow \text{Cl}_2(\text{g}) + \text{MnCl}_2(\text{aq}) + 2\text{H}_2\text{O}(\text{l})$
Assuming the reaction goes to completion, what mass of HCl solution is needed to produce 142 g of Cl_2 .
- Calculate the residue obtained on strongly heating 2.76 g Ag_2CO_3 .
$$\text{Ag}_2\text{CO}_3 \xrightarrow{\Delta} 2\text{Ag} + \text{CO}_2 + \frac{1}{2} \text{O}_2$$
- Nitric acid is manufactured by the Ostwald process, in which nitrogen dioxide reacts with water.
 $3\text{NO}_2(\text{g}) + \text{H}_2\text{O}(\text{l}) \rightarrow 2\text{HNO}_3(\text{aq}) + \text{NO}(\text{g})$
How many grams of nitrogen dioxide are required in this reaction to produce 25.2 gm HNO_3 ?
- Flourine reacts with uranium to produce uranium hexafluoride, UF_6 , as represented by this equation
 $\text{U}(\text{s}) + 3\text{F}_2(\text{g}) \rightarrow \text{UF}_6(\text{g})$
How many fluorine molecules are required to produce 2.0 mg of uranium hexafluoride, UF_6 , from an excess of uranium? The molar mass of UF_6 is 352 gm/mol.

Limiting Reactant

- 50 g of CaCO_3 is allowed to react with 73.5 g of H_3PO_4 . Calculate:
 - Amount of $\text{Ca}_3(\text{PO}_4)_2$ formed (in moles)
 - Amount of unreacted reagent (in moles)
- Reaction $4\text{A} + 2\text{B} + 3\text{C} \longrightarrow \text{A}_4\text{B}_2\text{C}_3$, is started from 2 moles of A, 1.2 moles of B & 1.44 moles of C. find number of moles of product formed.
- A chemist wants to prepare diborane by the reaction
 $6\text{LiH} + 8\text{BF}_3 \longrightarrow 6\text{LiBF}_4 + \text{B}_2\text{H}_6$
If he starts with 2.0 moles each of LiH & BF_3 . How many moles of B_2H_6 can be prepared.



20. Carbon reacts with chlorine to form CCl_4 . 36 gm of carbon was mixed with 142 g of Cl_2 . Calculate mass of CCl_4 produced and the remaining mass of reactant.
21. 50 g of CaCO_3 is allowed to react with 73.5 g of H_3PO_4 . Calculate:
- Amount of $\text{Ca}_3(\text{PO}_4)_2$ formed (in moles)
 - Amount of unreacted reagent (in moles)

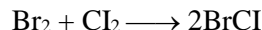
Problems Related with Mixture

22. One gram of an alloy of aluminium and magnesium when heated with excess of dil. HCl forms magnesium chloride, aluminium chloride and hydrogen. The evolved hydrogen collected at 0°C has a volume of 1.12 litres at 1 atm pressure. Calculate the composition of (% by mass) of the alloy.
23. A sample containing only CaCO_3 and MgCO_3 is ignited to CaO and MgO . The mixture of oxides produced weight exactly half as much as the original sample. Calculate the percentages of CaCO_3 and MgCO_3 (by mass) in the sample.
24. A 2g sample containing Na_2CO_3 and NaHCO_3 losses 0.248 g when heated to 300°C , the temperature at which NaHCO_3 decomposes to Na_2CO_3 , CO_2 and H_2O . What is the percentage of Na_2CO_3 in the given mixture ?
25. 92 g mixture of CaCO_3 , and MgCO_3 heated strongly in an open vessel. After complete decomposition of the carbonates it was found that the weight of residue left behind is 48 g. Find the mass of MgCO_3 in grams in the mixture.
26. When 4 gm of a mixture of NaHCO_3 and NaCl is heated, 0.66 gm CO_2 gas is evolved. Determine the percentage composition (by mass) of the original mixture.

Percentage Yield and Percentage Purity

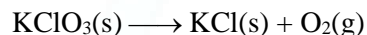
27. 200 g impure CaCO_3 on heating gives 11.35 L CO_2 gas at STP. Find the percentage of calcium in the lime stone sample.

28. The percent yield for the following reaction carried out in carbon tetrachloride (CCl_4) solution is 80%



- What amount of BrCl would be formed from the reaction of 0.025 mol Br_2 and 0.025 mol Cl_2 ?
- What amount of Br_2 is left unchanged?

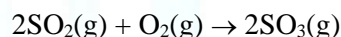
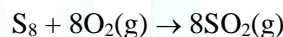
29. Calculate the percent loss in weight after complete decomposition of a pure sample of potassium chlorate.



30. A sample of calcium carbonate is 80% pure, 25 gm of this sample is treated with excess of HCl . How much volume of CO_2 will be obtained at 1 atm & 273 K?

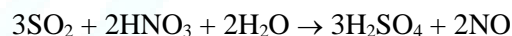
Sequential & Parallel Reactions

31. Sulphur trioxide may be prepared by the following two reactions:



How many grams of SO_3 will be produced from 1 mol of S_8 ?

32. $2\text{PbS} + 3\text{O}_2 \rightarrow 2\text{PbO} + 2\text{SO}_2$



According to the above sequence of reactions, how much H_2SO_4 will 1075.5 gm of PbS produce?

Miscellaneous Problem

33. In a determination of P an aqueous solution of NaH_2PO_4 is treated with a mixture of ammonium and magnesium ions to precipitate magnesium ammonium phosphate $\text{Mg}(\text{NH}_4)\text{PO}_4 \cdot 6\text{H}_2\text{O}$. This is heated and decomposed to magnesium pyrophosphate, $\text{Mg}_2\text{P}_2\text{O}_7$ which is weighed. A solution of NaH_2PO_4 yielded 1.054 g of $\text{Mg}_2\text{P}_2\text{O}_7$. What weight of NaH_2PO_4 was present originally?
34. Calculate the amount of H_2SO_4 produced (in gm) when 40 ml H_2O ($d = 0.9 \text{ gm/ml}$) reacts with 50 litre SO_3 at 1 atm. and 300 K, according to the following reaction ?
- $$\text{H}_2\text{O} + \text{SO}_3 \rightarrow \text{H}_2\text{SO}_4$$



35. Average atomic mass of Magnesium is 24.31 amu. This magnesium is composed of 79 mole % of ^{24}Mg and remaining 21 mole % of ^{25}Mg and ^{26}Mg . Calculate mole % of ^{26}Mg .
36. A moth repellent has the composition 49% C, 2.7% H and 48.3% Cl. Its molecular weight is 147 gm. Determine its molecular formula
37. Haemoglobin contains 0.25% iron by mass. The molecular mass of Haemoglobin is 89600 then the number of iron atoms per molecule of Haemoglobin
(Atomic mass of Fe = 56)

Analysis of Gaseous Mixture

38. When 100 ml of $\text{O}_2 - \text{O}_3$ mixture was passed through turpentine, there was reduction of volume by 20 ml. If 100 ml of such a mixture is heated, what will be the increase in volume?
39. 60 ml of a mixture of nitrous oxide and nitric oxide was exploded with excess of hydrogen. If 38 ml of N_2 was formed, calculate the volume of NO gas in the mixture.
40. 20 ml of a mixture of C_2H_2 and CO was exploded with 30 ml of oxygen. The gases after the reaction had a volume of 34 ml. On treatment with KOH, 8 ml of oxygen remained. Calculate the volume of C_2H_2 in the mixture.
41. 10 ml of CO is mixed with 25 ml air having 20% O_2 by volume. What would be the final volume if none of CO and O_2 is left after the reaction?
42. Calculate the volume of CO_2 evolved by the combustion of 50 ml of a mixture containing 40% C_2H_4 and 60% CH_4 (by volume).
43. 10 ml of a mixture of CH_4 , C_2H_4 and CO_2 were exploded with excess of air. After explosion and further cooling, there was contraction of 17 ml and after treatment with KOH, there was further reduction of 14 ml. What is the composition of the mixture?
44. Find the hydrocarbon for which volume of oxygen required is 1.5 times volume of carbon dioxide produced.
45. 10 moles of a mixture of CO (g) and CH_4 (g) was mixed with 22 moles of O_2 gas and subjected to sparking. Find the moles of gas absorbed when the residual gases are passed through alc. KOH.

Concentration Term

46. Calculate the molarity of the following solutions:
(a) 4g of caustic soda is dissolved in 200 mL of the solution.
(b) 5.3 g of anhydrous sodium carbonate is dissolved in 100 mL of solution.
(c) 0.365 g of pure HCl gas is dissolved in 50 mL of solution.
47. 0.115 gm of sodium metal was dissolved in 500 ml of the solution in distilled water. Calculate the molarity of the solution?
48. The average concentration of Na^+ ion in human body is 3 to 4 gm per litre. The molarity of Na^+ ion is about.
49. What is the concentration of chloride ion, in molarity, in a solution containing 10.56 gm $\text{BaCl}_2 \cdot 8\text{H}_2\text{O}$ per litre of solution ? (Ba = 137)
50. How much BaCl_2 (in gm) would be needed to make 250 ml of a solution having the same concentration of Cl^- as one containing 1.825 gm HCl per 100 ml ? (Ba = 137)
51. Equal moles of H_2O and NaCl are present in a solution. Find molality of solution?
52. What is the quantity of water (in g) that should be added to 16 g. methanol to make the mole fraction of methanol as 0.25:

Interconversion of Concentration Terms

53. Density of a solution containing 13% by mass of sulphuric acid is 0.98 g/mL. Then molarity of solution will be
54. The density of a solution containing 40% by mass of HCl is 1.2 g/mL. Calculate the molarity of the solution.



55. 15 g of methyl alcohol is present in 100 mL of solution. If density of solution is 0.90 g mL^{-1} . Calculate the mass percentage of methyl alcohol in solution
56. A 6.90 M solution of KOH in water contains 30% by mass of KOH. What is density of solution in gm/ml.
57. The concentration of a solution of NaOH is 8% (w/w) and 10% (w/v). Calculate density (in gm/mL) of solution?
58. The mole fraction of solute in aqueous urea solution is 0.2. Calculate the mass percent of solute ?
59. Calculate molality (m) of each ion present in the aqueous solution of 2M NH_4Cl assuming 100% dissociation according to reaction.
 $\text{NH}_4\text{Cl (aq)} \longrightarrow \text{NH}_4^+ \text{ (aq)} + \text{Cl}^- \text{ (aq)}$
Given: Density of solution = 3.107 gm / mL .
60. The concentration of $\text{Ca(HCO}_3)_2$ in a sample of hard water is 405 ppm. The density of water sample is 1.0 gm/mL . Calculate the molarity of solution ?
61. Units of parts per million (ppm) or per billion (ppb) are often used to describe the concentrations of solutes in very dilute solutions. The units are defined as the number of grams of solute per million or per billion grams of solvent. Bay of Bengal has 1.9 ppm of lithium ions. What is the molality of Li^+ in this water ?

Problems Related with Mixing & Dilution

62. Find molarity of Na^+ ions if 500 mL of 0.2 M $\text{NaCl}_{(\text{aq})}$ solution is mixed with 500 mL 0.5 M $\text{Na}_2\text{SO}_4_{(\text{aq})}$ solution?
63. Find out the volume of 98% w/w H_2SO_4 (density = 1.8 gm/ mL), must be diluted to prepare 12.5 litres of 2.5 M sulphuric acid solution:

64. Determine the volume (in mL) of diluted nitric acid 20% w/v HNO_3 that can be prepared by diluting 50 mL of conc. HNO_3 with water 69.8% w/v.
65. When V ml of 2.2 M H_2SO_4 solution is mixed with 10 V ml of water, the volume contraction of 2% take place. Calculate the molarity of diluted solution ?
66. 500 ml of 2 M NaCl solution was mixed with 200 ml of 2 M NaCl solution. Calculate the final volume and molarity of NaCl in final solution if final solution has density 1.5 gm/mL .
67. What volume (in mL) of 0.8 M AlCl_3 solution should be mixed with 50 ml of 0.2M CaCl_2 solution to get solution of chloride ion concentration equal to 0.6 M ?
68. A mixture containing equimolar amounts of Ca(OH)_2 and Al(OH)_3 requires 0.5 L of 4.0 M HCl to react with it completely. Total moles of the mixture are:
69. How would you prepare exactly 3.0 litre of 1.0 M NaOH by mixing proportions of stock solution of 2.50 M NaOH and 0.40 M NaOH. No water is to be used. Find the ratio of the volume (v_1/v_2).
70. 20 mL of 0.2M $\text{Al}_2(\text{SO}_4)_3$ is mixed with 30 mL of 0.6 M BaCl_2 . Calculate the mass of BaSO_4 formed in solution.
 $\text{BaCl}_2 + \text{Al}_2(\text{SO}_4)_3 \longrightarrow \text{BaSO}_4 + \text{AlCl}_3$

Some Typical Concentration Terms

71. 50 ml of '20V' H_2O_2 is mixed with 200 ml, '10V' H_2O_2 . Find the volume strength of resulting solution ?
72. 500 ml of a H_2O_2 solution on complete decomposition produces 2 moles of H_2O . Calculate the volume strength of H_2O_2 solution?
[Given: Volume of O_2 is measured at 1atm and 273 K]



SUBJECTIVE EXERCISE - II

- $$\text{Cl}_2 + \text{KOH} \xrightarrow{60\%} \text{KCl} + \text{KClO} + \text{H}_2\text{O}$$

$$\text{KClO} \xrightarrow{50\%} \text{KCl} + \text{KClO}_3$$

$$\text{KClO}_3 \xrightarrow{80\%} \text{KClO}_4 + \text{KCl}$$

112 L Cl_2 gas at STP is passed in 10 L KOH solution, containing 1 mole of potassium hydroxide per liter. Calculate the total moles of KCl produced, rounding it off to nearest whole number. (Yield of chemical reactions are written above the arrow (\rightarrow) of respective reaction)
- Two substance P_4 & O_2 are allowed to react completely to form mixture of P_4O_6 & P_4O_{10} leaving none of the reactants. Using this information calculate the composition of final mixture when mentioned amount of P_4 & O_2 are taken.

$$\text{P}_4 + 3\text{O}_2 \longrightarrow \text{P}_4\text{O}_6$$

$$\text{P}_4 + 5\text{O}_2 \longrightarrow \text{P}_4\text{O}_{10}$$

(i) If 1 mole P_4 & 4 mole of O_2
 (ii) If 3 mole P_4 & 11 mole of O_2
 (iii) If 3 mole P_4 & 13 mole of O_2
- A 10 g sample of a mixture of calcium chloride and sodium chloride is treated with Na_2CO_3 to precipitate calcium as calcium carbonate. This CaCO_3 is heated to convert all the calcium to CaO and the final mass of CaO is 1.12 gm. Calculate % by mass of NaCl in the original mixture.
- A mixture of Ferric oxide (Fe_2O_3) and Al is used as a solid rocket fuel which reacts to give Al_2O_3 and Fe. No other reactants and products are involved. On complete reaction of 1 mole of Fe_2O_3 , 200 units of energy is released.

(a) Write a balance reaction representing the above change.
 (b) What should be the ratio of masses of Fe_2O_3 and Al taken so that maximum energy per unit mass of fuel is released.
 (c) What would be energy released if 16 kg of Fe_2O_3 reacts with 2.7 kg of Al.
- 5.33 mg of salt $[\text{Cr}(\text{H}_2\text{O})_5\text{Cl}]\cdot\text{Cl}_2 \cdot \text{H}_2\text{O}$ is treated with excess of $\text{AgNO}_3(\text{aq.})$ then mass of AgCl ppt. obtained will be:
 Given: $[\text{Cr} = 52, \text{Cl} = 35.5]$
- If mass % of oxygen in monovalent metal carbonate is 48%. Then find the number of atoms of metal present in 5mg of this metal carbonate sample is ($N_A = 6.0 \times 10^{23}$)
- To find formula of compound composed of A & B which is given by A_xB_y , it is strongly heated in oxygen as per reaction-
 $\text{A}_x\text{B}_y + \text{O}_2 \rightarrow \text{AO} + \text{Oxide of B}$
 If 2.5gm of A_xB_y on oxidation gives 3gm oxide of A, Find empirical formula of A_xB_y ,
 [Take atomic mass of A = 24 & B = 14]
- Calculate maximum mass of CaCl_2 produced when 2.4×10^{24} atoms of calcium is taken with 96 litre of Cl_2 gas at 380 mm pressure and at 27°C . [R: 0.08 atm L/mole-K & $N_A = 6 \times 10^{23}$]
- Consider the given reaction $\text{H}_4\text{P}_2\text{O}_7 + 2\text{NaOH} \rightarrow \text{Na}_2\text{H}_2\text{P}_2\text{O}_7 + 2\text{H}_2\text{O}$
 If 534 gm of $\text{H}_4\text{P}_2\text{O}_7$ is reacted with 30×10^{23} molecules of NaOH then total number of molecules produced in the product is
- How much minimum volume (in ml) of 0.1 M aluminium sulphate solution should be added to excess calcium nitrate to obtain atleast 1 gm of each salt in the reaction.

$$\text{Al}_2(\text{SO}_4)_3 + 3\text{Ca}(\text{NO}_3)_2 \longrightarrow 2\text{Al}(\text{NO}_3)_3 + 3\text{CaSO}_4$$
- 100 ml of 0.1 M solution of AB ($d = 1.5 \text{ gm/ml}$) is mixed with 100 ml of 0.2 M solution of CB_2 ($d = 2.5 \text{ gm/ml}$). Calculate the molarity of B^- in final solution if the density of final solution is 4 gm/ml. Assuming AB and CB_2 are non reacting & dissociates completely into A^+ , B^- , C^{+2} .
- 60 ml of a "x" % w/w alcohol by weight ($d = 0.6 \text{ g/cm}^3$) must be used to prepare 200 cm^3 of 12% alcohol by weight ($d = 0.90 \text{ g/cm}^3$). Calculate the value of "x"?
- 500 ml of 2M CH_3COOH solution is mixed with 600 ml 12% w/v CH_3COOH solution then calculate the final molarity of solution.



JEE-MAIN (PREVIOUS YEAR QUESTIONS)

1. 5 g of zinc is treated separately with an excess of
(a) dilute hydrochloric acid and
(b) aqueous sodium hydroxide.
The ratio of the volumes of H_2 evolved in these two reactions is: [JEE(Main)-2020]
(1) 1: 4 (2) 1: 2
(3) 2: 1 (4) 1: 1
2. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y . The value of y is. [JEE(Main)-2021]
3. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O . The percentage of oxygen in the organic compound is: [JEE(Main)-2021]
(1) 51.63 (2) 63.53
(3) 53.33 (4) 50.33
4. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O . The percentage composition of carbon and hydrogen in organic compound is 15.3 and _____ respectively. (Round off to the Nearest Integer) [JEE(Main)-2021]
5. The number of chlorine atoms in 20 mL of chlorine gas at STP is _____ 10^{21} . (Round off to the Nearest Integer). [Assume chlorine is an ideal gas at STP
 $R = 0.083 \text{ L bar mol}^{-1} \text{ K}^{-1}$, $N_A = 6.0^{23} \times 10^{23}$] [JEE(Main)-2021]
6. Complete combustion of 3 g of ethane gives $x \times 10^{22}$ molecules of water. The value of x is _____.
(Round off to the Nearest Integer).
[Use: $N_A = 6.0^{23} \times 10^{23}$; Atomic masses in u: C: 12.0 ; O: 16.0 ; H: 1.0] [JEE(Main)-2021]
7. 4g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g of Na_2CO_3 . The value of x is _____. (Nearest integer) [JEE(Main)-2021]
8. When 0.15 g of an organic compound was analyzed using Carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is _____. (Nearest integer) [Atomic mass: Silver = 108, Bromine = 80] [JEE(Main)-2021]
9. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1}$ g. (in nearest integer) [JEE(Main)-2021]
10. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are: [JEE(Main)-2022]
(Given: density of the fuel is 0.756 g/mL)
(1) 1188 g and 1296 g
(2) 2376 g and 2592 g
(3) 2592g and 2376 g
(4) 3429 g and 3142 g
11. A 0.166 g sample of an organic compound was digested with cone. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4 . The used acid required 30.0 mL of 0.25 N NaOH for complete neutralization. The mass percentage of nitrogen in the organic compound is _____. [JEE(Main)-2022]
12. 120 of an organic compound that contains only carbon and hydrogen gives 330g of CO_2 and 270g of water on complete combustion. The percentage of carbon and hydrogen, respectively are. [JEE(Main)-2022]
(1) 25 and 75 (2) 40 and 60
(3) 60 and 40 (4) 75 and 25



13. 0.2 g of an organic compound was subjected to estimation of nitrogen by Dumas method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is _____. [nearest integer]

(Given: Molar mass of N_2 is 28 mol^{-1} . Molar volume of N_2 at STP: 22.4 L)

[JEE(Main)-2022]

14. The number of N atoms is 681 g of $C_7H_5N_3O_6$ is $x \times 10^{21}$. The value of x is _____

($N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$) (Nearest Integer)

[JEE(Main)-2022]

15. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4 . 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is _____ $\times 10^3$ m mols.

[JEE(Main)-2022]

16. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is _____ $\times 10^3$ g mol^{-1} [nearest integer]

[JEE(Main)-2022]

17. The neutralization occurs when 10 mL of 0.1 M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is _____. [M is a metal]

[JEE(Main)-2022]

18. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL, the molarity of this solution is: (Atomic mass: Cl = 35.5 amu, H = 1 amu)

[JEE(Main)-2022]

- (1) 10.2 M (2) 12.5 M
(3) 14.0 M (4) 18.2 M

19. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is _____ (Nearest integer).

[JEE(Main)-2022]

20. CNG is an important transportation fuel. When 100 g CNG is mixed with 208 oxygen in vehicles, it leads to the formation of CO_2 and H_2O and produces large quantity of heat during this combustion, then the amount of carbon dioxide, produced in grams is _____. [nearest integer] [Assume CNG to be methane]

[JEE(Main)-2022]

21. The moles of methane required to produce 81 g of water after complete combustion is _____ $\times 10^{-2}$ mol. [nearest integer]

[JEE(Main)-2022]

22. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R)

[JEE(Main)-2022]

Assertion (A): At 10°C , the density of a 5M solution of KCl [atomic masses of K and Cl are 39 & 35.5 g mol^{-1}]. The solution is cooled to -21°C . The molality of the solution will remain unchanged.

Reason (R): The molality of a solution does not change with temperature as mass remains unaffected with temperature.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both (A) and (R) are true and (R) is the correct explanation of (A)
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

23. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is. Times of atomic weight of B.

[JEE(Main)-2022]

24. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12 respectively. The data agrees with how many formulae of the following?

[JEE(Main)-2022]

- (1) CH_3COOH (2) $HCHO$
(3) CH_3OOCH_3 (4) CH_3CHO



25. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is _____. [in nearest integer] (Given: Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1})

[JEE(Main)-2022]

26. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide. The percentage of bromine in the given compound is _____% (nearest integer) (Relative atomic masses of Ag and Br are 108u and 80u, respectively).

[JEE(Main)-2022]

27. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is. (Given: Atomic masses of C, H and N are 12, 1 and 14 amu respectively)

The molar mass of the compound A is 162 g mol^{-1} .

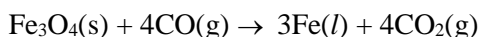
[JEE(Main)-2022]

- (1) $\text{C}_4\text{H}_6\text{N}_2$ (2) $\text{C}_2\text{H}_3\text{N}$
(3) $\text{C}_5\text{H}_7\text{N}$ (4) $\text{C}_{10}\text{H}_{14}\text{N}_2$

28. The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442 g of H_2O . The percentage of oxygen composition in the organic compound is _____. (nearest integer)

[JEE(Main)-2022]

29. Production of iron in blast furnace follows the following equation



when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is: [Given: Molar Atomic mass (g mol^{-1}): Fe = 56

Molar Atomic mass (g mol^{-1}): O = 16

Molar Atomic mass (g mol^{-1}): C = 12

[JEE(Main)-2022]

- (1) 1400 (2) 2200 (3) 3360 (4) 4200

30. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H_2SO_4 solution. The percentage of nitrogen in the compound is _____. (Nearest integer)

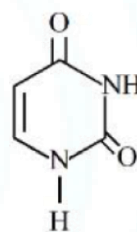
[JEE(Main)-2022]

31. The strength of 50 volume solution of hydrogen peroxide is _____ g/L (Nearest integer).

Given: Molar mass of H_2O_2 is 34 g mol^{-1} ; Molar volume of gas at STP = 22.7 L .

[JEE(Main)-2023]

32. Uracil is base present in RNA with the following structure. % of N in uracil is _____.



Given: Molar mass N = 14 g mol^{-1} ; O = 16 g mol^{-1} ; C = 12 g mol^{-1} ; H = 1 g mol^{-1} ;

[JEE(Main)-2023]

33. The number of units, which are used to express concentration of solutions from the following is _____.

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality.

[JEE(Main)-2023]

34. '25 volume' hydrogen peroxide means

[JEE(Main)-2023]

- (1) 1 L marketed solution contains 250 g of H_2O_2 .
(2) 1 L marketed solution contains 75 g of H_2O_2 .
(3) 100 mL marketed solution contains 25 g of H_2O_2 .
(4) 1 L marketed solution contains 25 g of H_2O_2 .



35. In sulphur estimation. 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is _____ (Nearest Integer)

(Given: Atomic mass Ba:137u; S:32 u, O:16 u)

[JEE(Main)-2023]

36. What is the mass ratio of ethylene glycol ($\text{C}_2\text{H}_6\text{O}_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molar aqueous solution?

(1) 1 : 2 (2) 1 : 1 (3) 3 : 1 (4) 2 : 1

[JEE(Main)-2023]

37. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is _____

(Given: Molar mass of A = 84 g mol⁻¹)

[JEE(Main)-2023]

38. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _____ g mol⁻¹ (Nearest integer)

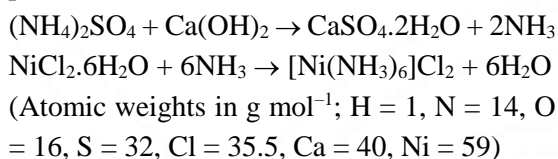
[JEE(Main)-2023]



JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

1. The ammonia prepared by treating ammonium sulphate with calcium hydroxide is completely used by $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ to form a stable coordination compound. Assume that both the reactions are 100% complete. If 1584 g of ammonium sulphate and 952 g of $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ are used in the preparation, the combined weight (in kg) of gypsum and the nickel-ammonia coordination compound thus produced is ____.

[JEE(Advanced)-2018]



2. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnace such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is ____.

[JEE(Advanced)-2018]



(Atomic weights in g mol^{-1} : O = 16, S = 32, Pb = 207)

3. To check the principle of multiple proportions, a series of pure binary compounds ($\text{P}_m \text{Q}_n$) were analyzed and their composition is tabulated below. The correct option(s) is (are)

[JEE(Advanced)-2022]

Compound	Weight % of P	Weight % of Q
1	50	50
2	44.4	55.6
3	40	60

- (A) If empirical formula of compound 3 is P_3Q_4 , then the empirical formula of compound 2 is P_3Q_5 .
- (B) If empirical formula of compound 3 is P_3Q_2 and atomic weight of element P is 20, then the atomic weight of Q is 45.
- (C) If empirical formula of compound 2 is PQ, then the empirical formula of the compound 1 is P_5Q_4 .
- (D) If atomic weight of P and Q are 70 and 35, respectively, then the empirical formula of compound 1 is P_2Q .

4. The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product **X** in 75% yield. The weight (in g) of **X** obtained is:

[Use, molar mass

(g mol^{-1}): H = 1, C = 12, O = 16, Si = 28, Cl = 35.5]

[JEE(Advanced)-2023]

8. To form a complete monolayer of acetic acid on 1 g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $\text{P} \times 10^{-23} \text{ m}^2$ surface area on charcoal, the value of P is ____.

[JEE(Advanced)-2024]

[Use given data: Surface area of charcoal = $1.5 \times 10^2 \text{ m}^2 \text{ g}^{-1}$; Avogadro's number (N_A) = $6.0 \times 10^{23} \text{ mol}^{-1}$]

Ans. (2500)



ANSWER KEY

OBJECTIVE EXERCISE - I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	A	B	D	C	C	B	C	B	D	C	C	B	C	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	C	C	C	C	C	C	A	A	B	B	A	A	B
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	A	A	B	A	C	B	B	B	D	A	A	B	C	B
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	A	D	B	A	A	C	D	D	B	C	A	C	B	C
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	D	A	C	C	B	D	B	A	B	A	B	A	C	C	C
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	D	A	C	A	D	B	B	B	C	B	C	B	C	A	C
Que.	91	92	93	94	95	96	97	98							
Ans.	B	C	A	C	B	D	D	A							

OBJECTIVE EXERCISE - II

Que.	1	2	3	4	5	6(a)	6(b)	6(c)	7(a)	7(b)	7(c)	7(d)	8	9	10
Ans.	AB	A	ABC	B	AD	B	C	B	D	B	C	A	AC	BD	BD
Que.	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	AC	AB	C	B	D	C	C	34	C	D	A	B	B	C	B
Que.	28	29	30												
Ans.	A	C	C												

26. (A) → R; (B) → P; (C) → Q

27. (A) → R; (B) → Q; (C) → P

31. (A) → Q; (B) → P; (C) → S; (D) → R

32. (A) → Q; (B) → P; (C) → S; (D) → R

33. (A) → P, Q, R, S; (B) → Q, R, T; (C) → Q, S; (D) → P

SUBJECTIVE EXERCISE - I

- 6.023×10^{13}
- $1.99 \times 10^{-23} \text{ g}$
- 24g
- (i) $\frac{10^{20}}{N_A}$ moles, (ii) 3200 amu, (iii) $14 \times 1.66 \times 10^{-24} \text{ g}$, (iv) $3N_A$, $9N_A$, (v) $6N_A$
- 3.2 g
- 180 g
- 2.5 g
- 22.7 L
- $(10 \times 6.023 \times 10^{23}, 8 \times 6.023 \times 10^{23}, 8 \times 6.023 \times 10^{23})$
- (68 mole)
- (5.34×10^6)
- (20 N_A)
- (292 g)
- (2.16 g)
- (27.6 gm)
- (1.0×10^{19})
- (i) 1/6 mole (ii) 5/12 mole
- (0.48)
- (0.25 mole)
- ($w_c = 24 \text{ gm}$; = 154 gm)
- (i) 1/6 (ii) 5/12
- (Al = 60%; Mg = 40%)
- ($\text{CaCO}_3 = 28.4\%$; $\text{MgCO}_3 = 71.6\%$)
- (63%, 37%)
- (66.4%)
- (42 g)
-





- | | | |
|--|--|------------------------------|
| 27. (10%) | 28. (a) 0.050 mol, (b) 0.050 mol | 29. (39.18) |
| 30. (4.48 litre) | 31. (640.0) | 32. (441 gm) |
| 33. (1.14 gm) | 34. (0196) | 35. (10) |
| 36. (C ₆ H ₄ C ₁₂) | 37. (4) | 38. (10 ml) |
| 39. (NO = 44 ml; N ₂ O = 16 ml) | 40. (C ₂ H ₂ = 6 ml, CO = 14 ml) | 41. (30 ml) |
| 42. (70 ml) | 43. (CH ₄ = 4.5 ml, CO ₂ = 1.5 ml) | 44. (alkene) |
| 45. (10) | 46. (a) 0.5M, (b) 0.5M, (c) 0.2M | 47. (0.01 M) |
| 48. (0.15 M) | 49. (0.06 M) | 50. (13 gm) |
| 51. (55.55 m) | 52. (27) | 53. (13 × 10 ⁻¹) |
| 54. (13.15) | 55. (16.66%) | 56. (1.2888) |
| 57. (1.25 gm/mL) | 58. (45.45%) | 59. (0.6667, 0.6667) |
| 60. (2.5 × 10 ⁻³ M) | 61. (2.7 × 10 ⁻⁴) | 62. (0.6 M) |
| 63. (1.736 litre) | 64. (174.5 mL) | 65. (0.204 M) |
| 66. (2 M) | 67. (5.56 ml) | 68. (0.8) |
| 69. (0.4) | 70. (2.796) | 71. (12 V) |
| 72. (44.8 V) | | |

SUBJECTIVE EXERCISE - II

- | | | |
|--|---------------------------------------|--------------------|
| 1. (4) | 2. (i) 0.5, 0.5; (ii) 2, 1 (iii) 1, 2 | 3. (%NaCl = 77.8%) |
| 4. ((i) Fe ₂ O ₃ + 2 Al → Al ₂ O ₃ + 2Fe; (ii) 80: 27; (iii) 10,000 units) | | |
| 5. 5.74 gm | | |
| 6. Ans. 6 × 10 ²⁰ | | |
| 7. (A ₃ B ₂) | | |
| 8. (222 gm) | 9. (7.5 × Na) | 10. (24.51 ml) |
| 12. (60) | 13. (2) | 11. (0.5) |

JEE MAIN (PREVIOUS YEAR QUESTIONS)

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	4	8	3	3	1	18	1	68	464	3	63	4	14	5418	0
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	25	3	3	18	143	225	1	2	2	40	34	4	46	3	64
Que.	31	32	33	34	35	36	37	38							
Ans.	150	25	5	2	42	4	12	200							

JEE ADVANCED (PREVIOUS YEAR QUESTIONS)

- | | | | |
|-----------|-------------------|---------|----------|
| 1. (2.99) | 2. (6.46 or 6.47) | 3. (BC) | 4. (222) |
|-----------|-------------------|---------|----------|





BINOMIAL THEOREM

Binomial Expression

Any algebraic expression which contains two dissimilar terms is called binomial expression. For

example: $x - y$, $xy + \frac{1}{x}$, $\frac{1}{z} - 1$, $\frac{1}{(x-y)^{1/3}} + 3$ etc.

Binomial Theorem

The formula by which any positive integral power of a binomial expression can be expanded in the form of a series is known as **BINOMIAL THEOREM**.

If $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$, then:

$$(x + y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_r x^{n-r} y^r + \dots + {}^nC_n y^n$$

$$= \sum_{r=0}^n {}^nC_r x^{n-r} y^r$$

This theorem can be proved by induction.

Observations

- The number of terms in the expansion is $(n + 1)$ i.e. one more than the index.
- The sum of the indices of x and y in each term is n .
- The binomial coefficients of the terms (${}^nC_0, {}^nC_1, \dots$) equidistant from the beginning and the end are equal. i.e. ${}^nC_r = {}^nC_{n-r}$
- Symbol nC_r can also be denoted by $\binom{n}{r}$, $C(n, r)$ or C_r^n .

Some important expansions

- $(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$.
- $(1 - x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - \dots + (-1)^n \cdot {}^nC_n x^n$.

Note:

The coefficient of x^r in $(1 + x)^n = {}^nC_r$ and that in $(1 - x)^n = (-1)^r \cdot {}^nC_r$

Example:

Expand: $(y + 2)^6$.

Solution:

$$\begin{aligned} & {}^6C_0 y^6 + {}^6C_1 y^5 \cdot 2 + {}^6C_2 y^4 \cdot 2^2 + {}^6C_3 y^3 \cdot 2^3 + {}^6C_4 y^2 \cdot 2^4 + {}^6C_5 y^1 \cdot 2^5 + {}^6C_6 \cdot 2^6 \\ &= y^6 + 12y^5 + 60y^4 + 160y^3 + 240y^2 + 192y + 64. \end{aligned}$$

Example:

Write first 4 terms of $\left(1 - \frac{2y^2}{5}\right)^7$

Solution:

$${}^7C_0, {}^7C_1 \left(-\frac{2y^2}{5}\right), {}^7C_2 \left(-\frac{2y^2}{5}\right)^2, {}^7C_3 \left(-\frac{2y^2}{5}\right)^3$$

Example:

If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and -6 respectively then m is -

- (A) 6 (B) 9 (C) 12 (D) 24

Solution:

$$\begin{aligned} (1 + x)^m (1 - x)^n &= \left[1 + mx + \frac{(m)(m-1)}{2} x^2 + \dots\right] \\ &\quad \left[1 - nx + \frac{n(n-1)}{2} x^2 + \dots\right] \end{aligned}$$

Coefficient of $x = m - n = 3$ (i)

Coefficient of $x^2 = -mn + \frac{n(n-1)}{2} + \frac{m(m-1)}{2} = -6$ (ii)

Solving (i) and (ii), we get

$m = 12$ and $n = 9$.

FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 Expand $\left(3x^2 - \frac{x}{2}\right)^5$

Q.2 Expand $(y + x)^n$

Pascal's triangle

$(x + y)^0$	1	1
$(x + y)^1$	$x + y$	1 1
$(x + y)^2$	$x^2 + 2xy + y^2$	1 2 1
$(x + y)^3$	$x^3 + 3x^2y + 3xy^2 + y^3$	1 3 3 1
$(x + y)^4$	$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$	1 4 6 4 1

Pascal's triangle





- (i) **Pascal's triangle** - A triangular arrangement of numbers as shown. The numbers give the binomial coefficients for the expansion of $(x + y)^n$. The first row is for $n = 0$, the second for $n = 1$, etc. Each row has 1 as its first and last number. Other numbers are generated by adding the two numbers immediately to the left and right in the row above.
- (ii) Pascal triangle is formed by binomial coefficient.
- (iii) The number of terms in the expansion of $(x + y)^n$ is $(n + 1)$ i.e. one more than the index.
- (iv) The sum of the indices of x and y in each term is n .
- (v) Power of first variable (x) decreases while of second variable (y) increases.
- (vi) Binomial coefficients are also called **combinatorial coefficients**.
- (vii) Binomial coefficients of the terms equidistant from the beginning and end are equal.
- (viii) r^{th} term from the beginning in the expansion of $(x + y)^n$ is same as r^{th} term from end in the expansion of $(y + x)^n$.
- (ix) r^{th} term from the end in $(x + y)^n$ is **$(n - r + 2)^{\text{th}}$ term from the beginning**.

Important Terms in the Binomial Expansion

(a) General term:

The general term or the $(r+1)^{\text{th}}$ term in the expansion of $(x + y)^n$ is given by $T_{r+1} = {}^nC_r x^{n-r} y^r$.

Example:

Find:

- (a) The coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$
- (b) The coefficient of x^{-7} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$

Also, find the relation between a and b , so that these coefficients are equal.

Solution:

- (a) In the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$, the general term is :

$$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r = {}^{11}C_r \cdot \frac{a^{11-r}}{b^r} \cdot x^{22-3r}$$

putting $22-3r = 7$

$$\therefore 3r = 15 \Rightarrow r = 5$$

$$\therefore t_6 = {}^{11}C_5 \frac{a^6}{b^5} \cdot x^7$$

Hence the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is ${}^{11}C_5 a^6 b^{-5}$.

Note that binomial coefficient of sixth term is ${}^{11}C_5$.

- (b) In the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$, general term is :

$$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r = (-1)^r {}^{11}C_r \frac{a^{11-r}}{b^r} \cdot x^{11-3r}$$

putting $11-3r = -7$

$$\therefore 3r = 18 \Rightarrow r = 6$$

$$\therefore T_7 = (-1)^6 \cdot {}^{11}C_6 \frac{a^5}{b^6} \cdot x^{-7}$$

Hence the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6 a^5 b^{-6}$.

Also given:

Coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ = coefficient of

$$x^{-7} \text{ in } \left(ax - \frac{1}{bx^2}\right)^{11}$$

$$\Rightarrow {}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6}$$

$$\Rightarrow ab = 1 \quad (\because {}^{11}C_5 = {}^{11}C_6)$$

which is the required relation between a and b .

Example:

Find the number of rational terms in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$.




Solution:

The general term in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$ is

$$T_{r+1} = {}^{1000}C_r \left(9^{1/4}\right)^{1000-r} \left(8^{1/6}\right)^r = {}^{1000}C_r 3^{\frac{1000-r}{2}} 2^{\frac{r}{2}}$$

The above term will be rational if exponents of 3 and 2 are integers

It means $\frac{1000-r}{2}$ and $\frac{r}{2}$ must be integers

The possible set of values of r is $\{0, 2, 4, \dots, 1000\}$

Hence, number of rational terms is 501

(b) Middle term :

The middle term(s) in the expansion of $(x + y)^n$ is (are) :

(i) If n is even, there is only one middle term which is given by $T_{(n+2)/2} = {}^nC_{n/2} \cdot x^{n/2} \cdot y^{n/2}$

(ii) If n is odd, there are two middle terms which are $T_{(n+1)/2}$ and $T_{[(n+1)/2]+1}$

Important Note:

Middle term has greatest binomial coefficient and if there are 2 middle terms their coefficients will be equal.

$\Rightarrow {}^nC_r$ will be maximum

$$\left\{ \begin{array}{l} \text{When } r = \frac{n}{2} \text{ if } n \text{ is even} \\ \text{When } r = \frac{n-1}{2} \text{ or } \frac{n+1}{2} \text{ if } n \text{ is odd} \end{array} \right.$$

\Rightarrow The term containing greatest binomial coefficient will be middle term in the expansion of $(1 + x)^n$

Example:

Find the middle term in the expansion of

$$\left(3x - \frac{x^3}{6}\right)^9$$

Solution:

The number of terms in the expansion of

$$\left(3x - \frac{x^3}{6}\right)^9 \text{ is } 10(\text{even}). \text{ So there are two middle terms.}$$

i.e. $\left(\frac{9+1}{2}\right)^{\text{th}}$ and $\left(\frac{9+3}{2}\right)^{\text{th}}$ are two middle terms.

They are given by T_5 and T_6

$$\therefore T_5 = T_{4+1} = {}^9C_4 (3x)^5 \left(-\frac{x^3}{6}\right)^4 = {}^9C_4 3^5 x^5 \cdot \frac{x^{12}}{6^4} =$$

$$\frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{3^5}{2^4 \cdot 3^4} x^{17} = \frac{189}{8} x^{17}$$

$$\text{and } T_6 = T_{5+1} = {}^9C_5 (3x)^4 \left(-\frac{x^3}{6}\right)^5 = -{}^9C_4 3^4 \cdot x^4 \cdot \frac{x^{15}}{6^5}$$

$$= -\frac{9 \cdot 8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{3^4}{2^5 \cdot 3^5} x^{19} = -\frac{21}{16} x^{19}$$

(c) Term independent of x :

Term independent of x does not contain x ; Hence find the value of r for which the exponent of x is zero.

Example:

The term independent of x in $\left[\sqrt{\frac{x}{3}} + \sqrt{\frac{3}{2x^2}}\right]^{10}$ is—

- (A) 1 (B) $\frac{5}{12}$
(C) ${}^{10}C_1$ (D) none of these

Solution:

General term in the expansion is

$${}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^r \left(\sqrt{\frac{3}{2x^2}}\right)^{10-r} = {}^{10}C_r x^{\frac{3r}{2}-10} \cdot \frac{3^{5-r}}{2^{\frac{10-r}{2}}} \text{ for}$$

$$\text{constant term, } \frac{3r}{2} = 10 \Rightarrow r = \frac{20}{3}$$

which is not an integer. Therefore, there will be no constant term.

FUNDAMENTAL UNLOCKED- (FU#2) :

Q.1 Find the 7th term of $\left(3x^2 - \frac{1}{3}\right)^{10}$

Q.2 Find the term independent of x in the expansion : $\left(2x^2 - \frac{3}{x^3}\right)^{25}$





Q.3 Find the middle term in the expansion of : (a)

$$\left(\frac{2x}{3} - \frac{3}{2x}\right)^6 \quad (b) \left(2x^2 - \frac{1}{x}\right)^7$$

(d) Numerically greatest term :

Let numerically greatest term in the expansion of $(a + b)^n$ be T_{r+1} .

$$\Rightarrow \begin{cases} |T_{r+1}| \geq |T_r| \\ |T_{r+1}| \geq |T_{r+2}| \end{cases} \text{ where } T_{r+1} = {}^nC_r a^{n-r} b^r$$

Solving above inequalities we get $\frac{n+1}{1 + \left|\frac{a}{b}\right|} - 1 \leq r$

$$\leq \frac{n+1}{1 + \left|\frac{a}{b}\right|}$$

Case I:

When $\frac{n+1}{1 + \left|\frac{a}{b}\right|}$ is an integer equal to m , then T_m

and T_{m+1} will be numerically greatest term.

Case II:

When $\frac{n+1}{1 + \left|\frac{a}{b}\right|}$ is not an integer and its integral part

is m , then T_{m+1} will be the numerically greatest term.

Example:

Find numerically greatest term in the expansion of $(3 - 5x)^{11}$ when $x = \frac{1}{5}$

Solution:

$$\text{Using } \frac{n+1}{1 + \left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1 + \left|\frac{a}{b}\right|}$$

$$\frac{11+1}{1 + \left|\frac{3}{-5x}\right|} - 1 \leq r \leq \frac{11+1}{1 + \left|\frac{3}{-5x}\right|}$$

solving we get $2 \leq r \leq 3$

$\therefore r = 2, 3$

so, the greatest terms are T_{2+1} and T_{3+1} .

\therefore Greatest term (when $r = 2$)

$$T_3 = {}^{11}C_2 \cdot 3^9 (-5x)^2 = 55 \cdot 3^9 = T_4$$

From above we say that the value of both greatest terms are equal.

Example:

Given T_3 in the expansion of $(1 - 3x)^6$ has maximum numerical value. Find the range of 'x'.

Solution:

$$\text{Using } \frac{n+1}{1 + \left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1 + \left|\frac{a}{b}\right|}$$

$$\frac{6+1}{1 + \left|\frac{1}{-3x}\right|} - 1 \leq 2 \leq \frac{6+1}{1 + \left|\frac{1}{-3x}\right|}$$

Let $|x| = t$

$$\frac{21t}{3t+1} - 1 \leq 2 \leq \frac{21t}{3t+1}$$

$$\begin{cases} \frac{21t}{3t+1} \leq 3 \\ \frac{21t}{3t+1} \geq 2 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{4t-1}{3t+1} \leq 0 \Rightarrow t \in \left[-\frac{1}{3}, \frac{1}{4}\right] \\ \frac{15t-2}{3t+1} \geq 0 \Rightarrow t \in \left(-\infty, -\frac{1}{3}\right) \cup \left[\frac{2}{15}, \infty\right) \end{cases}$$

$$\text{Common solution } t \in \left[\frac{2}{15}, \frac{1}{4}\right] \Rightarrow x \in \left[-\frac{1}{4}, -\frac{2}{15}\right] \cup$$

$$\left[\frac{2}{15}, \frac{1}{4}\right]$$

FUNDAMENTAL UNLOCKED- (FU#3) :

- (i) Find the numerically greatest term in the expansion of $(3 - 2x)^9$, when $x = 1$.
- (ii) In the expansion of $\left(\frac{1}{2} + \frac{2x}{3}\right)^n$ when $x = -\frac{1}{2}$, it is known that 3rd terms is the greatest term. Find the possible integral values of n .

Properties of Binomial Coefficients

$$(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots + C_nx^n =$$

$$\sum_{r=0}^n {}^nC_r x^r; n \in \mathbb{N} \quad \dots\dots\dots(i)$$





where $C_0, C_1, C_2, \dots, C_n$ are called combinatorial (binomial) coefficients.

(a) The sum of all the binomial coefficients is 2^n .

Put $x = 1$, in (i) we get

$$C_0 + C_1 + C_2 + \dots + C_n = 2^n \Rightarrow \sum_{r=0}^n {}^nC_r = 2^n$$

0(ii)

(b) Put $x = -1$ in (i) we get

$$C_0 - C_1 + C_2 - C_3 + \dots + C_n = 0 \Rightarrow \sum_{r=0}^n (-1)^r {}^nC_r = 0$$

0(iii)

(c) The sum of the binomial coefficients at odd position is equal to the sum of the binomial coefficient set even position and each is equal to 2^{n-1} .

$$\text{From (ii) and (iii), } C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$

(d) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$$(e) \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$$(f) {}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1} = \frac{n}{r} \cdot \frac{n-1}{r-1} {}^{n-2}C_{r-2} = \dots = \frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 1}$$

$$(g) {}^nC_r = \frac{r+1}{n+1} \cdot {}^{n+1}C_{r+1}$$

Example:

Prove that : ${}^{25}C_{10} + {}^{24}C_{10} + \dots + {}^{10}C_{10} = {}^{26}C_{11}$

Solution:

$$\text{LHS} = {}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + \dots + {}^{25}C_{10}$$

$$= {}^{11}C_{11} + {}^{11}C_{10} + {}^{12}C_{10} + \dots + {}^{25}C_{10}$$

$$= {}^{12}C_{11} + {}^{12}C_{10} + \dots + {}^{25}C_{10}$$

$$= {}^{13}C_{11} + {}^{13}C_{10} + \dots + {}^{25}C_{10}$$

$$\text{and so on. } \therefore \text{LHS} = {}^{26}C_{11}$$

Aliter:

$$\text{LHS} = \text{coefficient of } x^{10} \text{ in } \{(1+x)^{10} + (1+x)^{11} + \dots + (1+x)^{25}\}$$

$$\Rightarrow \text{coefficient of } x^{10} \text{ in } \left[(1+x)^{10} \frac{(1+x)^{16} - 1}{1+x-1} \right]$$

$$\Rightarrow \text{coefficient of } x^{10} \text{ in } \frac{(1+x)^{26} - (1+x)^{10}}{x}$$

$$\Rightarrow \text{coefficient of } x^{11} \text{ in } [(1+x)^{26} - (1+x)^{10}] = {}^{26}C_{11} - 0 = {}^{26}C_{11}$$

Example:

A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select books is 63, find the value of n .

Solution:

Given student selects at most n books from a collection of $(2n+1)$ books. It means that he selects one book or two books or three books or or n books. Hence, by the given condition-

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \dots + {}^{2n+1}C_n = 63 \dots (i)$$

But we know that

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \dots + {}^{2n+1}C_{2n+1} = 2^{2n+1} \dots (ii)$$

Since ${}^{2n+1}C_0 = {}^{2n+1}C_{2n+1} = 1$, equation (ii) can also be written as

$$2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \dots + {}^{2n+1}C_n) + ({}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + {}^{2n+1}C_{n+3} + \dots + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n}) = 2^{2n+1}$$

$$\Rightarrow 2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \dots + {}^{2n+1}C_n) + ({}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \dots + {}^{2n+1}C_2 + {}^{2n+1}C_1) = 2^{2n+1}$$

$$(\because {}^{2n+1}C_r = {}^{2n+1}C_{2n+1-r})$$

$$\Rightarrow 2 + 2 ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \dots + {}^{2n+1}C_n) = 2^{2n+1} \quad [\text{from (i)}]$$

$$\Rightarrow 2 + 2 \cdot 63 = 2^{2n+1} \Rightarrow 1 + 63 = 2^{2n}$$

$$\Rightarrow 64 = 2^{2n} \Rightarrow 2^6 = 2^{2n} \therefore 2n = 6$$

Hence, $n = 3$.

Example:

Prove that :

$$(i) C_1 + 2C_2 + 3C_3 + \dots + nC_n = n \cdot 2^{n-1}$$

$$(ii) C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$$





Solution:

$$\begin{aligned} \text{(i) L.H.S.} &= \sum_{r=1}^n r \cdot {}^nC_r = \sum_{r=1}^n r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} \\ &= n \sum_{r=1}^n {}^{n-1}C_{r-1} = n \cdot [{}^{n-1}C_0 + {}^{n-1}C_1 + \dots + {}^{n-1}C_{n-1}] = \\ &= n \cdot 2^{n-1} \end{aligned}$$

Aliter:

(Using method of differentiation)

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n \dots \dots \dots \text{(A)}$$

Differentiating (A), we get

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + \dots + n.C_nx^{n-1}.$$

Put $x = 1$,

$$C_1 + 2C_2 + 3C_3 + \dots + n.C_n = n.2^{n-1}$$

$$\text{(ii) L.H.S.} = \sum_{r=0}^n \frac{C_r}{r+1} = \frac{1}{n+1} \sum_{r=0}^n \frac{n+1}{r+1} {}^nC_r$$

$$= \frac{1}{n+1} \sum_{r=0}^n {}^{n+1}C_{r+1} = \frac{1}{n+1}$$

$$[{}^{n+1}C_1 + {}^{n+1}C_2 + \dots + {}^{n+1}C_{n+1}] = \frac{1}{n+1} [2^{n+1} - 1]$$

Aliter:

(Using method of integration)

Integrating (A), we get

$$\frac{(1+x)^{n+1}}{n+1} + C = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \dots + \frac{C_nx^{n+1}}{n+1}$$

(where C is a constant)

$$\text{Put } x = 0, \text{ we get, } C = -\frac{1}{n+1}$$

$$\therefore \frac{(1+x)^{n+1}}{n+1} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \dots + \frac{C_nx^{n+1}}{n+1}$$

$$\text{Put } x = 1, \text{ we get } C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$$

$$\text{Put } x = -1, \text{ we get } C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \dots + \frac{1}{n+1}$$

Example:

$$\text{If } (1+x)^n = \sum_{r=0}^n {}^nC_r x^r, \text{ then prove that } C_1^2 + 2.C_2^2 +$$

$$3.C_3^2 + \dots + n.C_n^2 = \frac{(2n-1)!}{((n-1)!)^2}$$

Solution:

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots + C_nx^n \dots \dots \dots \text{(i)}$$

Differentiating both the sides, w.r.t. x, we get

$$n(1+x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + \dots + n.C_nx^{n-1} \dots \dots \dots \text{(ii)}$$

also, we have

$$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \dots + C_n \dots \dots \dots \text{(iii)}$$

Multiplying (ii) and (iii), we get

$$(C_1 + 2C_2x + 3C_3x^2 + \dots + C_nx^{n-1})(C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \dots + C_n) = n(1+x)^{2n-1}$$

Equating the coefficients of x^{n-1} , we get

$$C_1^2 + 2C_2^2 + 3C_3^2 + \dots + n.C_n^2 = n.2^{n-1}C_{n-1} = \frac{(2n-1)!}{((n-1)!)^2}$$

Example:

$$\text{Prove that : } C_0 - 3C_1 + 5C_2 - \dots + (-1)^n(2n+1)C_n = 0$$

$$\text{Solution : } T_r = (-1)^r(2r+1) {}^nC_r = 2(-1)^r r \cdot {}^nC_r + (-1)^r {}^nC_r$$

$$\Sigma T_r = 2 \sum_{r=1}^n (-1)^r \cdot r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + \sum_{r=0}^n (-1)^r {}^nC_r = 2n$$

$$\begin{aligned} &\sum_{r=1}^n (-1)^r \cdot {}^{n-1}C_{r-1} + \sum_{r=0}^n (-1)^r \cdot {}^nC_r \\ &= 2n [{}^{n-1}C_0 - {}^{n-1}C_1 + \dots] + [{}^nC_0 - {}^nC_1 + \dots] = 0 \end{aligned}$$

Example:

$$\text{Prove that } ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n \cdot {}^{2n}C_n$$

Solution:

$$(1-x)^{2n} = {}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 - \dots + (-1)^n {}^{2n}C_{2n}x^{2n} \dots \dots \dots \text{(i)}$$

$$\text{and } (x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + \dots + {}^{2n}C_{2n} \dots \dots \dots \text{(ii)}$$

Multiplying (i) and (ii), we get





$$(x^2 - 1)^{2n} = ({}^{2n}C_0 - {}^{2n}C_1 x + \dots + (-1)^n {}^{2n}C_{2n} x^{2n}) \times ({}^{2n}C_0 x^{2n} + {}^{2n}C_1 x^{2n-1} + \dots + {}^{2n}C_{2n}) \dots (iii)$$

Now, coefficient of x^{2n} in R.H.S.

$$= ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2$$

$$\therefore \text{General term in L.H.S., } T_{r+1} = {}^{2n}C_r (x^2)^{2n-r} (-1)^r$$

$$\text{Putting } 2(2n - r) = 2n$$

$$\therefore r = n$$

$$\therefore T_{n+1} = {}^{2n}C_n x^{2n} (-1)^n$$

$$\text{Hence coefficient of } x^{2n} \text{ in L.H.S.} = (-1)^n {}^{2n}C_n$$

But (iii) is an identity, therefore coefficient of x^{2n} in

R.H.S. = coefficient of x^{2n} in L.H.S.

$$\Rightarrow ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n {}^{2n}C_n$$

Example:

$$\text{Prove that : } {}^nC_0 \cdot {}^{2n}C_n - {}^nC_1 \cdot {}^{2n-2}C_{n-2} + {}^nC_2 \cdot {}^{2n-4}C_{n-4} + \dots = 2^n$$

Solution:

$$\text{L.H.S.} = \text{Coefficient of } x^n \text{ in } [{}^nC_0(1+x)^{2n} - {}^nC_1(1+x)^{2n-2} \dots]$$

$$= \text{Coefficient of } x^n \text{ in } [(1+x)^2 - 1]^n$$

$$= \text{Coefficient of } x^n \text{ in } x^n(x+2)^n = 2^n$$

Example:

If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$ then show that the sum of the products of the C_i 's taken two at a

time represented by : $\sum_{0 \leq i < j \leq n} C_i C_j$ is equal to $2^{2n-1} -$

$$\frac{2n!}{2.n!n!}$$

Solution:

$$\text{Since } (C_0 + C_1 + C_2 + \dots + C_{n-1} + C_n)^2$$

$$= C_0^2 + C_1^2 + C_2^2 + \dots + C_{n-1}^2 + C_n^2 + 2(C_0 C_1 + C_0 C_2 + C_0 C_3 + \dots + C_0 C_n + C_1 C_2 + C_1 C_3 + \dots + C_1 C_n + C_2 C_3 + C_2 C_4 + \dots + C_2 C_n + \dots + C_{n-1} C_n)$$

$$(2^n)^2 = {}^{2n}C_n + 2 \sum_{0 \leq i < j \leq n} C_i C_j$$

$$\text{Hence } \sum_{0 \leq i < j \leq n} C_i C_j = 2^{2n-1} - \frac{2n!}{2.n!n!}$$

Example:

If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$ then prove that

$$\sum_{0 \leq i < j \leq n} (C_i + C_j)^2 = (n-1) {}^{2n}C_n + 2^{2n}$$

Solution:

$$\text{L.H.S. } \sum_{0 \leq i < j \leq n} (C_i + C_j)^2$$

$$= (C_0 + C_1)^2 + (C_0 + C_2)^2 + \dots + (C_0 + C_n)^2 + (C_1 + C_2)^2 + (C_1 + C_3)^2 + \dots + (C_1 + C_n)^2 + (C_2 + C_3)^2 + (C_2 + C_4)^2 + \dots + (C_2 + C_n)^2 + \dots + (C_{n-1} + C_n)^2$$

$$= n(C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2) + 2 \sum_{0 \leq i < j \leq n} C_i C_j$$

$$= n {}^{2n}C_n + 2 \cdot \left\{ 2^{2n-1} - \frac{2n!}{2.n!n!} \right\}$$

{from Example 17}

$$= n {}^{2n}C_n + 2^{2n} - {}^{2n}C_n = (n-1) {}^{2n}C_n + 2^{2n} = \text{R.H.S.}$$

FUNDAMENTAL UNLOCKED- (FU#4) :

$$\text{Q.1 } \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} =$$

$$(A) 2^{n-1}$$

$$(B) {}^{2n}C_n$$

$$(C) 2^n$$

$$(D) 2^{n+1}$$

Q.2 If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$, $n \in \mathbb{N}$. Prove that

(a) $3C_0 - 8C_1 + 13C_2 - 18C_3 + \dots$ upto $(n+1)$ terms = 0, if $n \geq 2$.

$$(b) 2C_0 + 2^2 \frac{C_1}{2} + 2^3 \frac{C_2}{3} + 2^4 \frac{C_3}{4} + \dots + 2^{n+1} \frac{C_n}{n+1} = \frac{3^{n+1} - 1}{n+1}$$

$$(c) C_0^2 + \frac{C_1^2}{2} + \frac{C_2^2}{3} + \dots + \frac{C_n^2}{n+1} = \frac{(2n+1)!}{((n+1)!)^2}$$





Multinomial Theorem

Using binomial theorem, we have $(x + a)^n =$

$$\sum_{r=0}^n {}^nC_r x^{n-r} a^r, \quad n \in \mathbb{N}$$

$$= \sum_{r=0}^n \frac{n!}{(n-r)!r!} x^{n-r} a^r = \sum_{r+s=n} \frac{n!}{r!s!} x^s a^r, \quad \text{where } s+r=n$$

This result can be generalized in the following form.

$$(x_1 + x_2 + \dots + x_k)^n =$$

$$\sum_{r_1+r_2+\dots+r_k=n} \frac{n!}{r_1!r_2!\dots r_k!} x_1^{r_1} x_2^{r_2} \dots x_k^{r_k}$$

The general term in the above expansion $\frac{n!}{r_1!r_2!\dots r_k!}$

$$\cdot x_1^{r_1} x_2^{r_2} x_3^{r_3} \dots x_k^{r_k}$$

The number of terms in the above expansion is equal to the number of non-negative integral solution of the equation $r_1 + r_2 + \dots + r_k = n$ because each solution of this equation gives a term in the above expansion.

The number of such solutions is ${}^{n+k-1}C_{k-1}$

Particular Cases

$$(i) (x + y + z)^n = \sum_{r+s+t=n} \frac{n!}{r!s!t!} x^r y^s z^t$$

The above expansion has ${}^{n+3-1}C_{3-1} = {}^{n+2}C_2$ terms

$$(ii) (x + y + z + u)^n = \sum_{p+q+r+s=n} \frac{n!}{p!q!r!s!} x^p y^q z^r u^s$$

There are ${}^{n+4-1}C_{4-1} = {}^{n+3}C_3$ terms in the above expansion.

Example:

Find the coefficient of $x^2 y^3 z^4 w$ in the expansion of $(x - y - z + w)^{10}$

Solution:

$$(x - y - z + w)^{10} = \sum_{p+q+r+s=10} \frac{10!}{p!q!r!s!} (x)^p (-y)^q (-z)^r (w)^s$$

We want to get $x^2 y^3 z^4 w$ this implies that $p = 2, q = 3, r = 4, s = 1$

$$\therefore \text{Coefficient of } x^2 y^3 z^4 w \text{ is } \frac{10!}{2!3!4!1!} (-1)^3 (-1)^4 = -12600$$

Example:

Find the total number of terms in the expansion of $(1 + x + y)^{10}$ and coefficient of $x^2 y^3$.

Solution:

$$\text{Total number of terms} = {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$$

$$\text{Coefficient of } x^2 y^3 = \frac{10!}{2! \times 3! \times 5!} = 2520$$

Example:

Find the coefficient of x^5 in the expansion of $(2 - x + 3x^2)^6$.

Solution:

The general term in the expansion of $(2 - x + 3x^2)^6 =$

$$\frac{6!}{r!s!t!} 2^r (-x)^s (3x^2)^t$$

where $r + s + t = 6$.

$$= \frac{6!}{r!s!t!} 2^r (-1)^s (3)^t x^{s+2t}$$

For the coefficient of x^5 , we must have $s + 2t = 5$.

But, $r + s + t = 6$,

$\therefore s = 5 - 2t$ and $r = 1 + t$, where $0 \leq r, s, t \leq 6$.

Now $t = 0 \Rightarrow r = 1, s = 5$.

$t = 1 \Rightarrow r = 2, s = 3$.

$t = 2 \Rightarrow r = 3, s = 1$.

Thus, there are three terms containing x^5 and coefficient of x^5

$$= \frac{6!}{1!5!0!} 2^1 \times (-1)^5 \times 3^0 + \frac{6!}{2!3!1!} \times 2^2 \times (-1)^3 \times 3^1 +$$

$$\frac{6!}{3!1!2!} \times 2^3 \times (-1)^1 \times 3^2$$

$$= -12 - 720 - 4320 = -5052.$$

Example:

If $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then prove that (a) $a_r =$

$$a_{2n-r} \quad (b) \sum_{r=0}^{n-1} a_r = \frac{1}{2} (3^n - a_n)$$

Solution:

(a) We have

$$(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r \quad \dots (A)$$





Replace x by $\frac{1}{x}$

$$\therefore \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^n = \sum_{r=0}^{2n} a_r \left(\frac{1}{x}\right)^r$$

$$\Rightarrow (x^2 + x + 1)^n = \sum_{r=0}^{2n} a_r x^{2n-r}$$

$$\Rightarrow \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{2n} a_r x^{2n-r} \quad \{\text{Using (A)}\}$$

Equating the coefficient of x^{2n-r} on both sides, we get $a_{2n-r} = a_r$ for $0 \leq r \leq 2n$.

Hence $a_r = a_{2n-r}$.

(b) Putting $x=1$ in given series, then

$$a_0 + a_1 + a_2 + \dots + a_{2n} = (1+1+1)^n$$

$$a_0 + a_1 + a_2 + \dots + a_{2n} = 3^n \quad \dots(1)$$

But $a_r = a_{2n-r}$ for $0 \leq r \leq 2n$

\therefore series (1) reduces to

$$2(a_0 + a_1 + a_2 + \dots + a_{n-1}) + a_n = 3^n.$$

$$\therefore a_0 + a_1 + a_2 + \dots + a_{n-1} = \frac{1}{2}(3^n - a_n)$$

FUNDAMENTAL UNLOCKED- (FU#5) :

Q.1 Find the coefficient of x^2y^5 in the expansion of $(3 + 2x - y)^{10}$.

Application of Binomial Theorem

Example:

If $(6\sqrt{6} + 14)^{2n+1} = [N] + F$ and $F = N - [N]$; where $[.]$ denotes greatest integer function, then NF is equal to:

- (A) 20^{2n+1} (B) an even integer
(C) odd integer (D) 40^{2n+1}

Solution:

$$\text{Since } (6\sqrt{6} + 14)^{2n+1} = [N] + F$$

Let us assume that $f = (6\sqrt{6} - 14)^{2n+1}$; where $0 \leq f < 1$.

$$\text{Now, } [N] + F - f = (6\sqrt{6} + 14)^{2n+1} - (6\sqrt{6} - 14)^{2n+1}$$

$$= 2 \left[{}^{2n+1}C_1 (6\sqrt{6})^{2n} (14) + {}^{2n+1}C_3 (6\sqrt{6})^{2n-2} (14)^3 + \dots \right]$$

$$\Rightarrow [N] + F - f = \text{even integer.}$$

$$\text{Now } 0 < F < 1 \text{ and } 0 < f < 1$$

so $-1 < F - f < 1$ and $F - f$ is an integer so it can only

be zero

$$\text{Thus NF} = (6\sqrt{6} + 14)^{2n+1} (6\sqrt{6} - 14)^{2n+1} = 20^{2n+1}$$

Example:

Find the last three digits in 11^{50} .

Solution:

$$\begin{aligned} \text{Expansion of } (10 + 1)^{50} &= {}^{50}C_0 10^{50} + {}^{50}C_1 10^{49} + \dots \\ &+ {}^{50}C_{48} 10^2 + {}^{50}C_{49} 10 + {}^{50}C_{50} \\ &= \underbrace{{}^{50}C_0 10^{50} + {}^{50}C_1 10^{49} + \dots + {}^{50}C_{47} 10^3}_{1000 \text{ k}} + 49 \times 25 \times \end{aligned}$$

$$100 + 500 + 1$$

$$\Rightarrow 1000 \text{ K} + 123001 \Rightarrow \text{Last 3 digits are 001.}$$

Example:

Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.

Solution:

When 2222 is divided by 7 it leaves a remainder 3.

So adding and subtracting 3^{5555} , we get :

$$E = \underbrace{2222^{5555} - 3^{5555}}_{E_1} + \underbrace{3^{5555} + 5555^{2222}}_{E_2}$$

For E_1 : Now since $2222-3 = 2219$ is divisible by 7, therefore E_1 is divisible by 7

($\because x^n - a^n$ is divisible by $x - a$)

For E_2 : 5555 when divided by 7 leaves remainder 4.

So adding and subtracting 4^{2222} , we get :

$$\begin{aligned} E_2 &= 3^{5555} + 4^{2222} + 5555^{2222} - 4^{2222} \\ &= (243)^{1111} + (16)^{1111} + (5555)^{2222} - 4^{2222} \end{aligned}$$

Again $(243)^{1111} + 16^{1111}$ and $(5555)^{2222} - 4^{2222}$ are divisible by 7

($\because x^n + a^n$ is divisible by $x + a$ when n is odd)

Hence $2222^{1111} + 5555^{2222}$ is divisible by 7.

FUNDAMENTAL UNLOCKED- (FU#6) :

Q.1 Prove that $5^{25} - 3^{25}$ is divisible by 2.

Q.2 Find the remainder when the number 9^{100} is divided by 8.

Q.3 Find last three digits in 19^{100} .

Q.4 Let $R = (8 + 3\sqrt{7})^{20}$ and $[.]$ denotes greatest integer function, then prove that :

(a) $[R]$ is odd

$$(b) R - [R] = 1 - \frac{1}{(8 + 3\sqrt{7})^{20}}$$





Q.5 Find the digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$.

Binomial Theorem For Negative or Fractional Indices

If $n \in \mathbb{Q}$, then $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \infty$ provided $|x| < 1$.

Note:

- (i) When the index n is a positive integer the number of terms in the expansion of $(1+x)^n$ is finite i.e. $(n+1)$ and the coefficient of successive terms are : ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$
- (ii) When the index is other than a positive integer such as negative integer or fraction, the number of terms in the expansion of $(1+x)^n$ is infinite and the symbol nC_r cannot be used to denote the coefficient of the general term.
- (iii) Following expansion should be remembered ($|x| < 1$).
 - (a) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots \infty$
 - (b) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots \infty$
 - (c) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \infty$
 - (d) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots \infty$
 - (e) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \dots + \frac{(-1)^r(r+1)(r+2)}{2!}x^r + \dots$
 - (f) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(r+1)(r+2)}{2!}x^r + \dots$
- (iv) The expansions in ascending powers of x are only valid if x is 'small'. If x is large, i.e., $|x| > 1$ then we may find it convenient to expand in powers of $1/x$, which then will be small.

Approximations

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + \dots$$

If $x < 1$, the terms of the above expansion go on

decreasing and if x be very small, a stage may be reached when we may neglect the terms containing higher powers of x in the expansion. Thus, if x be so small that its square and higher powers may be neglected then $(1+x)^n = 1 + nx$, approximately. This is an approximate value of $(1+x)^n$.

Example:

If x is so small such that its square and higher powers may be neglected then find the approximate value of

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{(4+x)^{1/2}}$$

Solution:

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{(4+x)^{1/2}} = \frac{1 - \frac{3}{2}x + 1 - \frac{5x}{3}}{2\left(1 + \frac{x}{4}\right)^{1/2}} = \frac{1}{2}$$

$$\begin{aligned} & \left(2 - \frac{19}{6}x\right) \left(1 + \frac{x}{4}\right)^{-1/2} \\ &= \frac{1}{2} \left(2 - \frac{19}{6}x\right) \left(1 - \frac{x}{8}\right) = \frac{1}{2} \left(2 - \frac{x}{4} - \frac{19}{6}x\right) = 1 - \frac{x}{8} - \frac{19}{12}x = 1 - \frac{41}{24}x \end{aligned}$$

Example:

The value of cube root of 1001 up to five decimal places is –

- (A) 10.03333 (B) 10.00333
(C) 10.00033 (D) none of these

Solution:

$$\begin{aligned} (1001)^{1/3} &= (1000 + 1)^{1/3} = 10 \left(1 + \frac{1}{1000}\right)^{1/3} = 10 \left\{1 + \frac{1}{3} \cdot \frac{1}{1000} + \frac{1/3(1/3-1)}{2!} \cdot \frac{1}{1000^2} + \dots\right\} \\ &= 10\{1 + 0.0003333 - 0.00000011 + \dots\} = 10.00333 \end{aligned}$$

Example:

The sum of $1 + \frac{1}{4} + \frac{1.3}{4.8} + \frac{1.3.5}{4.8.12} + \dots \infty$ is –

- (A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$





(C) $\sqrt{3}$

(D) $2^{3/2}$

Solution:

Comparing with $1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + nx^n = 1/4$

.....(i)

and $\frac{n(n-1)x^2}{2!} = \frac{1.3}{4.8}$ or $\frac{nx(nx-x)}{2!} = \frac{3}{32}$

$\Rightarrow \frac{1}{4} \left(\frac{1}{4} - x \right) = \frac{3}{16}$ (by (i))

$\Rightarrow \left(\frac{1}{4} - x \right) = \frac{3}{4}$

$\Rightarrow x = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$ (ii)

putting the value of x in (i)

$n(-1/2) = 1/4 \Rightarrow n = -1/2$

\therefore sum of series $= (1+x)^n = (1-1/2)^{-1/2} = (1/2)^{-1/2} = \sqrt{2}$

Exponential Series

(a) e is an irrational number lying between 2.7 and 2.8. Its value correct upto 10 places of decimal is 2.7182818284.

(b) Logarithms to the base 'e' are known as the **Napier an** system, so named after Napier, their inventor. They are also called **Natural**

Logarithm.

(c) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$; where x may be any

real or complex number and $e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$.

(d) $a^x = 1 + \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a + \frac{x^3}{3!} \ln^3 a + \dots$,

where $a > 0$

(e) $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \infty$

Logarithmic Series

(a) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$, where $-1 < x \leq 1$

(b) $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$, where $-1 \leq x < 1$

Remember:

(i) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty = \ln 2$

(ii) $e^{\ln x} = x$; for all $x > 0$

(iii) $\ln 2 = 0.693$

(iv) $\ln 10 = 2.303$





ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1) :

$$\begin{aligned} \text{Q.1} \quad & {}^5C_0 x(3x^2)^5 + {}^5C_1 (3x^2)^4 \left(-\frac{x}{2}\right) + {}^5C_2 (3x^2)^3 \left(-\frac{x}{2}\right)^2 \\ & + {}^5C_3 (3x^2)^2 \left(-\frac{x}{2}\right)^3 + {}^5C_4 (3x^2)^1 \left(-\frac{x}{2}\right)^4 + {}^5C_5 \left(-\frac{x}{2}\right)^5 \end{aligned}$$

$$\text{Q.2} \quad {}^nC_0 y^n + {}^nC_1 y^{n-1} \cdot x + {}^nC_2 y^{n-2} \cdot x^2 + \dots + {}^nC_n \cdot x^n$$

FUNDAMENTAL UNLOCKED- (FU#2) :

$$\text{Q.1} \quad \frac{70}{3} x^8;$$

$$\text{Q.2} \quad \frac{25!}{10!5!} 2^{15} 3^{10};$$

$$\text{Q.3} \quad (\text{a}) -20; (\text{b}) -560x^5, 280x^2$$

FUNDAMENTAL UNLOCKED- (FU#3) :

$$\text{Q.1} \quad 4^{\text{th}} \text{ and } 5^{\text{th}} \text{ i.e. } 489888$$

$$\text{Q.2} \quad n = 4, 5, 6$$

FUNDAMENTAL UNLOCKED- (FU#4) :

$$\text{Q.1} \quad C$$

FUNDAMENTAL UNLOCKED- (FU#5) :

$$\text{Q.1} \quad -272160 \text{ or } -{}^{10}C_5 \times {}^5C_2 \times 108$$

FUNDAMENTAL UNLOCKED- (FU#6) :

$$\text{Q.2} \quad 1$$

$$\text{Q.3} \quad 001$$

$$\text{Q.5} \quad 1$$





OBJECTIVE EXERCISE - I

General Term & Coefficient of x^k in $(ax + b)^n$

1. If the coefficients of x^7 and x^8 in the expansion

of $\left[2 + \frac{x}{3}\right]^n$ are equal, then the value of n is :

(A) 15 (B) 45 (C) 55 (D) 56

2. If the constant term of the binomial expansion

$\left(2x - \frac{1}{x}\right)^n$ is -160 , then n is equal to -

(A) 4 (B) 6 (C) 8 (D) 10

3. The coefficient of x^{49} in the expansion of $(x - 1) \left(x - \frac{1}{2}\right) \left(x - \frac{1}{2^2}\right) \dots \left(x - \frac{1}{2^{49}}\right)$ is equal to -

(A) $-2 \left(1 - \frac{1}{2^{50}}\right)$

(B) +ve coefficient of x

(C) -ve coefficient of x

(D) $-2 \left(1 - \frac{1}{2^{49}}\right)$

4. Number of rational terms in the expansion of $(\sqrt{2} + \sqrt[4]{3})^{100}$ is :

(A) 25 (B) 26 (C) 27 (D) 28

5. The largest real value for x such that

$$\sum_{k=0}^4 \left(\frac{5^{4-k}}{(4-k)!} \right) \left(\frac{x^k}{k!} \right) = \frac{8}{3} \text{ is -}$$

(A) $2\sqrt{2} - 5$ (B) $2\sqrt{2} + 5$

(C) $-2\sqrt{2} - 5$ (D) $-2\sqrt{2} + 5$

6. The expression $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree:

(A) 5 (B) 6 (C) 7 (D) 8

7. Given $(1 - 2x + 5x^2 - 10x^3)(1 + x)^n = 1 + a_1x + a_2x^2 + \dots$ and that $a_1^2 = 2a_2$ then the value of n is-

(A) 6 (B) 2 (C) 5 (D) 3

8. The sum of the coefficients of all the even powers of x in the expansion of $(2x^2 - 3x + 1)^{11}$ is -

(A) 2.6^{10} (B) 3.6^{10} (C) 6^{11} (D) none

9. Co-efficient of α^t in the expansion of ,

$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + \dots + (\alpha + q)^{m-1}$ where $\alpha \neq -q$ and $p \neq q$ is:

(A) $\frac{{}^m C_t (p^t - q^t)}{p - q}$ (B) $\frac{{}^m C_t (p^{m-t} - q^{m-t})}{p - q}$

(C) $\frac{{}^m C_t (p^t + q^t)}{p - q}$ (D) $\frac{{}^m C_t (p^{m-t} + q^{m-t})}{p - q}$

Properties of Binomial Coefficients

10. Set of value of r for which, ${}^{18}C_{r-2} + 2 \cdot {}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ contains :

(A) 4 element

(B) 5 elements

(C) 7 elements

(D) 10 elements

11. If $n \in \mathbb{N}$ and n is even, then

$$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \dots +$$

$$\frac{1}{(n-1)!1!} =$$

(A) 2^n

(B) $\frac{2^{n-1}}{n!}$

(C) $2^n n!$

(D) none of these

12. Let $\binom{n}{k}$ represents the combination of 'n' things

taken 'k' at a time, then the value of the sum

$$\binom{99}{97} + \binom{98}{96} + \binom{97}{95} + \dots + \binom{3}{1} + \binom{2}{0} \text{ equals -}$$

(A) $\binom{99}{97}$

(B) $\binom{100}{98}$

(C) $\binom{99}{98}$

(D) $\binom{100}{97}$





13. The value of $\frac{C_0}{1.3} - \frac{C_1}{2.3} + \frac{C_2}{3.3} - \frac{C_3}{4.3} + \dots +$

$$(-1)^n \frac{C_n}{(n+1).3} \text{ is:}$$

- (A) $\frac{3}{n+1}$ (B) $\frac{n+1}{3}$
(C) $\frac{1}{3(n+1)}$ (D) none of these

14. The value of the expression ${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$

is equal to:

- (A) ${}^{47}C_5$ (B) ${}^{52}C_5$ (C) ${}^{52}C_4$ (D) ${}^{49}C_4$

15. The value of $\binom{50}{0}\binom{50}{1} + \binom{50}{1}\binom{50}{2} + \dots + \binom{50}{49}\binom{50}{50}$ is, where ${}^nC_r = \binom{n}{r}$

- (A) $\binom{100}{50}$ (B) $\binom{100}{51}$ (C) $\binom{50}{25}$ (D) $\binom{50}{25}^2$

Multinomial Theorem

16. If $|x| < 1$, then the co-efficient of x^n in the expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is:

- (A) n (B) $n-1$ (C) $n+2$ (D) $n+1$

17. The co-efficient of x^4 in the expansion of $(1 - x + 2x^2)^{12}$ is:

- (A) ${}^{12}C_3$ (B) ${}^{13}C_3$
(C) ${}^{14}C_4$ (D) ${}^{12}C_3 + 3 {}^{13}C_3 + {}^{14}C_4$

Application of Binomial Theorem

18. Let $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If

$[t]$ denotes the greatest integer $\leq t$, then

- (A) $[x] + [y]$ is even
(B) $[x]$ is even but $[y]$ is odd
(C) $[x]$ and $[y]$ are both odd
(D) $[x]$ is odd but $[y]$ is even

19. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is _____.

- (A) 21 (B) 22 (C) 23 (D) 28

20. The remainder on dividing 5^{99} by 11 is _____.

- (A) 9 (B) 11 (C) 13 (D) 15

21. Among the statements:

(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8.

(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

- (A) both (S1) and (S2) are incorrect
(B) only (S2) is correct
(C) both (S1) and (S2) are correct
(D) only (S1) is correct

22. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

- (A) 34 but not by 14 (B) both 14 and 34
(C) neither 14 nor 34 (D) 14 but not by 34

23. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

- (A) 10 (B) 5 (C) 20 (D) 13





OBJECTIVE EXERCISE - II

Single Correct Type Questions

1. Let $(5 + 2\sqrt{6})^n = p + f$ where $n, p \in \mathbb{N}$ and $0 < f < 1$ then the value of $f^2 - f + pf - p$ is –
 (A) a natural number
 (B) a negative integer
 (C) a prime number
 (D) are irrational number

2. Greatest term in the binomial expansion of $(a + 2x)^9$ when $a = 1$ and $x = \frac{1}{3}$ is :
 (A) 3rd and 4th (B) 4th and 5th
 (C) only 4th (D) only 5th

3. If $\sum_{r=1}^{10} r(r-1) {}^{10}C_r = k \cdot 2^9$, then k is equal to-
 (A) 10 (B) 45 (C) 90 (D) 100

4. The sum $\frac{\binom{11}{0}}{1} + \frac{\binom{11}{1}}{2} + \frac{\binom{11}{2}}{3} + \dots + \frac{\binom{11}{11}}{12}$ equals
 (where $\binom{n}{r}$ denotes nC_r):
 (A) $\frac{2^{11}}{12}$ (B) $\frac{2^{12}}{12}$ (C) $\frac{2^{11}-1}{12}$ (D) $\frac{2^{12}-1}{12}$

5. **Statement-1** : The sum of the series ${}^nC_0 \cdot {}^mC_r + {}^nC_1 \cdot {}^mC_{r-1} + {}^nC_2 \cdot {}^mC_{r-2} + \dots + {}^nC_r \cdot {}^mC_0$ is equal to ${}^{n+m}C_r$, where nC_r 's and mC_r 's denotes the combinatorial coefficients in the expansion of $(1+x)^n$ and $(1+x)^m$ respectively.

Statement-2 : Number of ways in which r children can be selected out of (n + m) children consisting of n boys and m girls if each selection may consist of any number of boys and girls is equal to ${}^{n+m}C_r$.

- (A) Statement-1 is true, statement-2 is true ; statement-2 is a correct explanation for statement-1.

- (B) Statement-1 is true, statement-2 is true ; statement-2 is NOT a correct explanation for statement-1.
 (C) Statement-1 is true, statement-2 is false.
 (D) Statement-1 is false, statement-2 is true.

One More than One Correct Type Questions

6. In the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$, the term which does contain x^0 is -
 (A) ${}^{11}C_4 - {}^{10}C_3$ (B) ${}^{10}C_7$
 (C) ${}^{10}C_4$ (D) ${}^{11}C_5 - {}^{10}C_5$

7. In the expansion of $\left(x^3 + 3 \cdot 2^{-\log \sqrt{x}} \sqrt{x^3} \right)^{11}$
 (A) there appears a term with the power x^2
 (B) there does not appear a term with the power x^2
 (C) there appears a term with the power x^{-3}
 (D) the ratio of the co-efficient of x^3 to that of x^{-3} is 1/3

8. If it is known that the third term of the binomial expansion $\left(x + x^{\log_{10} x} \right)^5$ is 10^6 then x is equal to-
 (A) 10 (B) $10^{-5/2}$ (C) 100 (D) 5

9. Which of the following statement(s) is/are correct ?
 (A) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \dots + \infty = 4$
 (B) Integral part of $(9 + 4\sqrt{5})^n$, $n \in \mathbb{N}$ is even.
 (C) $({}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n)^2 = 1 + {}^{2n}C_1 + {}^{2n}C_2 + \dots + {}^{2n}C_{2n}$
 (D) $\frac{1}{(3+2x)^2}$ can be expanded as infinite series in ascending powers of x only if $|x| < \frac{2}{3}$.





10. If $(9 + \sqrt{80})^n = I + f$ where I, n are integers and $0 < f < 1$, then -
 (A) I is an odd integer
 (B) I is an even integer
 (C) $(I + f)(1 - f) = 1$
 (D) $1 - f = (9 - \sqrt{80})^n$

11. If for $n \in \mathbb{I}, n > 10; 1 + (1 + x) + (1 + x)^2 + \dots + (1 + x)^n = \sum_{k=0}^n a_k \cdot x^k, x \neq 0$ then

(A) $\sum_{k=0}^n a_k = 2^{n+1}$

(B) $a_{n-2} = \frac{n(n+1)}{2}$

(C) $a_p > a_{p-1}$ for $p < \frac{n}{2}, p \in \mathbb{N}$

(D) $(a_9)^2 - (a_8)^2 = {}^{n+2}C_{10} ({}^{n+1}C_{10} - {}^{n+1}C_9)$

12. Let $P(n) = \sum_{r=0}^n \frac{(-1)^r r}{r+1} {}^nC_r$. Now which of the following holds good ?

(A) $|P_{10}|$ is harmonic mean of $|P_9|$ and $|P_{11}|$

(B) $\sum_{r=5}^{10} P(r) P(r-1) = -\frac{6}{55}$

(C) $|P_{10}|$ is arithmetic mean of $|P_9|$ and $|P_{11}|$

(D) $\sum_{r=5}^{10} P(r) P(r-1) = \frac{6}{55}$

13. Let $(1 + x)^m = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots + C_mx^m$, where $C_r = {}^mC_r$ and $A = C_1C_3 + C_2C_4 + C_3C_5 + C_4C_6 + \dots + C_{m-2}C_m$, then -

(A) $A \geq {}^{2m}C_{m-2}$

(B) $A < {}^{2m}C_{m-2}$

(C) $A > C_0^2 + C_1^2 + C_2^2 + \dots + C_m^2$

(D) $A < C_0^2 + C_1^2 + C_2^2 + \dots + C_m^2$

14. Consider $E = (\sqrt[3]{x} + \sqrt[5]{y})^z = I + f, 0 \leq f < 1$

(A) If $x = 5, y = 2, z = 100$, then number of irrational terms in expansion of E is 98

(B) If $x = 5, y = 2, z = 100$, then number of rational terms in expansion of E is 4

(C) If $x = 16, y = 1$ and $z = 6$, then $I = 197$

(D) If $x = 16, y = 1$ and $z = 6$, then $f = (\sqrt{2} - 1)^6$




SUBJECTIVE EXERCISE - I
General Term & Coefficient of x^k in $(ax + b)^n$

- If the coefficients of $(2r + 4)^{\text{th}}$, $(r - 2)^{\text{th}}$ terms in the expansion of $(1 + x)^{18}$ are equal, find r .
 - If the coefficients of the r^{th} , $(r + 1)^{\text{th}}$ and $(r + 2)^{\text{th}}$ terms in the expansion of $(1 + x)^{14}$ are in AP, find r .
 - If the coefficients of 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in AP, show that $2n^2 - 9n + 7 = 0$.
- Find the term independent of x in the expansion of
 (i) $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{2x^2}\right]^{10}$ (ii) $\left[\frac{1}{2}x^{1/3} + x^{-1/5}\right]^8$
- Prove that the ratio of the coefficient of x^{10} in $(1 - x^2)^{10}$ and the term independent of x in $\left(x - \frac{2}{x}\right)^{10}$ is 1 : 32.
- Find the sum of the series $\sum_{r=0}^n (-1)^r \cdot {}^nC_r$
 $\left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \dots \text{up to } m \text{ terms}\right]$
- Find the term independent of x in the expansion of $(1 + x + 2x^3) \left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$.
- Let $(1 + x^2)^2 \cdot (1 + x)^n = \sum_{k=0}^{n+4} a_k \cdot x^k$. If a_1, a_2 and a_3 are in AP, find n .
- Let $f(x) = 1 - x + x^2 - x^3 + \dots + x^{16} - x^{17} = a_0 + a_1(1 + x) + a_2(1 + x)^2 + \dots + a_{17}(1 + x)^{17}$, find the value of a_2 .
- Find the coefficient of x^r in the expression :
 $(x + 3)^{n-1} + (x + 3)^{n-2}(x + 2) + (x + 3)^{n-3}(x + 2)^2 + \dots + (x + 2)^{n-1}$

Properties of Binomial Coefficients

- Let $N = {}^{2000}C_1 + 2 \cdot {}^{2000}C_2 + 3 \cdot {}^{2000}C_3 + \dots + 2000 \cdot {}^{2000}C_{2000}$. Prove that N is divisible by 2^{2003} .
- Prove the following identities using the theory of permutation where $C_0, C_1, C_2, \dots, C_n$ are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in \mathbb{N}$:
 (a) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{(2n)!}{n!n!}$
 (b) $C_0 C_1 + C_1 C_2 + C_2 C_3 + \dots + C_{n-1} C_n = \frac{(2n)!}{(n+1)!(n-1)!}$
 (c) $C_0 C_r + C_1 C_{r+1} + C_2 C_{r+2} + \dots + C_{n-r} C_n = \frac{2n!}{(n-r)!(n+r)!}$
 (d) $\sum_{r=0}^{n-2} ({}^nC_r \cdot {}^nC_{r+2}) = \frac{(2n)!}{(n-2)!(n+2)!}$
 (e) ${}^{100}C_{10} + 5 \cdot {}^{100}C_{11} + 10 \cdot {}^{100}C_{12} + 10 \cdot {}^{100}C_{13} + 5 \cdot {}^{100}C_{14} + {}^{100}C_{15} = {}^{105}C_{90}$
- If $C_0, C_1, C_2, \dots, C_n$ are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in \mathbb{N}$, then prove the following :
 (a) $C_1 + 2C_2 + 3C_3 + \dots + n \cdot C_n = n \cdot 2^{n-1}$
 (b) $C_0 + 2C_1 + 3C_2 + \dots + (n+1)C_n = (n+2)2^{n-1}$
 (c) $C_0 + 3C_1 + 5C_2 + \dots + (2n+1)C_n = (n+1)2^n$
 (d) $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \dots (C_{n-1} + C_n) = \frac{C_0 \cdot C_1 \cdot C_2 \dots C_{n-1} (n+1)^n}{n!}$
 (e) $1 \cdot C_0^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \dots + (2n+1) C_n^2 = \frac{(n+1)(2n)!}{n!n!}$
- Prove that
 (a) $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \dots + \frac{n \cdot C_n}{C_{n-1}} = \frac{n(n+1)}{2}$
 (b) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$





$$(c) 2 \cdot C_0 + \frac{2^2 \cdot C_1}{2} + \frac{2^3 \cdot C_2}{3} + \frac{2^4 \cdot C_3}{4} + \dots$$

$$\frac{2^{n+1} \cdot C_n}{n+1} = \frac{3^{n+1} - 1}{n+1}$$

$$(d) C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \dots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$$

13. If $\binom{n}{r}$ denotes nC_r , then

$$(a) \text{ Evaluate : } 2^{15} \binom{30}{0} \binom{30}{15} - 2^{14} \binom{30}{1} \binom{29}{14} +$$

$$2^{13} \binom{30}{2} \binom{28}{13} - \dots - \binom{30}{15} \binom{15}{0}$$

$$(b) \text{ Prove that : } \sum_{r=1}^n \binom{n-1}{n-r} \binom{n}{r} = \binom{2n-1}{n-1}$$

$$(c) \text{ Prove that : } \binom{n}{r} \binom{r}{k} = \binom{n}{k} \binom{n-k}{r-k}$$

Multinomial Theorem

14. Find the coefficient of:

$$(a) x^2 y^3 z^4 \text{ in the expansion of } (ax - by + cz)^9.$$

$$(b) a^2 b^3 c^4 d \text{ in the expansion of } (a - b - c + d)^{10}.$$

15. Find the coefficient of:

$$(a) x^4 \text{ in the expansion of } (1 + x + x^2 + x^3)^{11}$$

$$(b) x^4 \text{ in the expansion of } (2 - x + 3x^2)^6$$

16. Given that $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{2n} x^{2n}$, find the values of :

$$(i) a_0 + a_1 + a_2 + \dots + a_{2n}$$

$$(ii) a_0 - a_1 + a_2 - a_3 + \dots + a_{2n}$$

$$(iii) a_0^2 - a_1^2 + a_2^2 - a_3^2 + \dots + a_{2n}^2$$

Numerically/Algebraically Greatest terms, Application of Binomial Theorem

17. Find numerically greatest term in the expansion of:

$$(i) (2 + 3x)^9 \text{ when } x = \frac{3}{2}$$

$$(ii) (3 - 5x)^{15} \text{ when } x = \frac{1}{5}$$

18. (a) Show that the integral part in each of the following is odd. $n \in \mathbb{N}$

$$(A) (5 + 2\sqrt{6})^n \quad (B) (8 + 3\sqrt{7})^n$$

(b) Show that the integral part in each of the following is even. $n \in \mathbb{N}$

$$(A) (3\sqrt{3} + 5)^{2n+1} \quad (B) (5\sqrt{5} + 11)^{2n+1}$$




SUBJECTIVE EXERCISE - II

1. If $(7 + 4\sqrt{3})^n = p + \beta$ where n and p are positive integers and β is a proper fraction and $(1 - \beta)(p + \beta) = k$, find k

2. Let $P = (2 + \sqrt{3})^5$ and $f = P - [P]$, where $[P]$ denotes the greatest integer function. Find the value of $\left(\frac{f^2}{1-f}\right)$.

3. Let $a = (4^{1/401} - 1)$ and let $b_n = {}^nC_1 + {}^nC_2 \cdot a + {}^nC_3 \cdot a^2 + \dots + {}^nC_n \cdot a^{n-1}$. Find the value of $(b_{2006} - b_{2005})$

4. Let a and b be the coefficient of x^3 in $(1 + x + 2x^2 + 3x^3)^4$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^4$ respectively. Find the value of $(a - b)$.

5. Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$.

6. Find the index n of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the 9^{th} term of the expansion has numerically the

greatest coefficient ($n \in \mathbb{N}$).

7. Find the absolute value of coefficient of x^{49} in the polynomial

$$\left(x - \frac{C_1}{C_0}\right) \left(x - 2^2 \cdot \frac{C_2}{C_1}\right) \left(x - 3^2 \cdot \frac{C_3}{C_2}\right) \dots \left(x - 50^2 \cdot \frac{C_{50}}{C_{49}}\right), \text{ where } C_r = {}^{50}C_r.$$

Paragraph for question nos. 8 to 10

If $n \in \mathbb{N}$ and if $(1 + 4x + 4x^2)^n = \sum_{r=0}^{2n} a_r x^r$, where $a_0, a_1, a_2, \dots, a_{2n}$ are real numbers.

8. The value of $2 \sum_{r=0}^n a_{2r}$, is:

(A) $9^n - 1$ (B) $9^n + 1$ (C) $9^n - 2$ (D) $9^n + 2$

9. The value of $2 \sum_{r=1}^n a_{2r-1}$, is -

(A) $9^n - 1$ (B) $9^n + 1$ (C) $9^n - 2$ (D) $9^n + 2$

10. The value of a_{2n-1} is -

(A) 2^{2n} (B) $n \cdot 2^{2n}$
(C) $(n - 1)2^{2n}$ (D) $(n + 1)2^{2n}$





JEE MAIN (PREVIOUS YEAR QUESTIONS)

1. The value of ${}^{21}C_1 - {}^{10}C_1 + {}^{21}C_2 - {}^{10}C_2 + {}^{21}C_3 - {}^{10}C_3 + {}^{21}C_4 - {}^{10}C_4 + \dots + {}^{21}C_{10} - {}^{10}C_{10}$ is :
 (1) $2^{20} - 2^{10}$ (2) $2^{21} - 2^{11}$
 (3) $2^{21} - 2^{10}$ (4) $2^{20} - 2^9$

2. The sum of the co-efficient of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1)$ is : [JEE(Main)-2018]
 (1) 2 (2) -1 (3) 0 (4) 1

3. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: [JEE(Main)-2019]
 (1) 14 (2) 6 (3) 4 (4) 8

4. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is : [JEE(Main)-2019]
 (1) 12 (2) 10 (3) 15 (4) 14

5. If the third term in the binomial expansion of $(1 + x^{\log_2 x})^5$ equals 2560, then a possible value of x is: [JEE(Main)-2019]
 (1) $2\sqrt{2}$ (2) $4\sqrt{2}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

6. If $\sum_{i=1}^{20} \left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \frac{k}{21}$, then k equals: [JEE(Main)-2019]
 (1) 200 (2) 100 (3) 50 (4) 400

7. The positive value of λ for which the co-efficient of x^2 in the expression $x^2 \left(\sqrt{x} + \frac{\lambda}{x^2} \right)^{10}$ is 720, is: [JEE(Main)-2019]
 (1) 3 (2) $2\sqrt{2}$ (3) $\sqrt{5}$ (4) 4

8. If $\sum_{r=0}^{25} \{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \} = K({}^{50}C_{25})$, then K is equal to: [JEE(Main)-2019]
 (1) 2^{25} (2) $2^{25} - 1$ (3) 2^{24} (4) $(25)^2$

9. The value of r for which ${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$ is maximum, is: [JEE(Main)-2019]
 (1) 15 (2) 10 (3) 20 (4) 11

10. The sum of the real values of x for which the middle term in binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is: [JEE(Main)-2019]
 (1) 8 (2) 0 (3) 4 (4) 6

11. Let $S_n = 1 + q + q^2 + \dots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \dots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to: [JEE(Main)-2019]
 (1) 200 (2) 2^{99} (3) 202 (4) 2^{100}

12. A ratio of the 5th term from the beginning to the 5th term the end in the binomial expansion of $\left(2^{1/3} + \frac{1}{2(3)^{1/3}}\right)^{10}$ is: [JEE(Main)-2019]
 (1) $2(36)^{\frac{1}{3}} : 1$ (2) $1 : 4(16)^{\frac{1}{3}}$
 (3) $1 : 2(16)^{\frac{1}{3}}$ (4) $4(36)^{\frac{1}{3}} : 1$

13. If ${}^nC_4, {}^nC_5$ and nC_6 are in A.P., the n can be: [JEE(Main)-2019]
 (1) 11 (2) 12 (3) 9 (4) 14

14. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is: [JEE(Main)-2019]
 (1) 232 (2) 625 (3) 964 (4) 227

15. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to: [JEE(Main)-2019]
 (1) (28, 315) (2) (-21, 714)
 (3) (-54, 315) (4) (28, 861)

16. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is: [JEE(Main)-2019]
 (1) 38 (2) 35 (3) 58 (4) 23





17. If ${}^{20}C_1 + (2^2) {}^{20}C_2 + (3^2) {}^{20}C_3 + \dots + (20^2) {}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to: **[JEE(Main)-2019]**

(1) (380, 18) (2) (380, 19)
(3) (420, 18) (4) (420, 19)

18. The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right) \cdot \left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to:

[JEE(Main)-2019]

(1) -108 (2) -72 (3) -36 (4) 36

19. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \dots + x^{2n})(1 - x + x^2 - x^3 + \dots + x^{2n})$ is 61, then n is equal to _____. **[JEE(Main)-2020]**

20. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer is:

[JEE(Main)-2020]

(1) 2 (2) 4 (3) 3 (4) 6

21. If a, b and c are the greatest values of ${}^{19}C_p, {}^{20}C_q$ and ${}^{21}C_r$ respectively, then :

[JEE(Main)-2020]

(1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (2) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$
(3) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$ (4) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

22. In the expression of $\left(\frac{x}{\cos \theta} + \frac{1}{x \sin \theta}\right)^{16}$, if l_1 is the least value of term independent of x when $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$ and l_2 is the least value of the term

independent of x when $\frac{\pi}{16} \leq \theta \leq \frac{\pi}{8}$, then the

ratio $l_2 : l_1$ is equal to: **[JEE(Main)-2020]**

(1) 1 : 8 (2) 1 : 16 (3) 16 : 1 (4) 8 : 1

23. If $C_r = {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \dots + (101) \cdot C_{25} = 2^{25} \cdot k$, then k is equal to _____. **[JEE(Main)-2020]**

24. Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $(\alpha x^{1/9} + \beta x^{-1/6})^{10}$

is $10k$, then k is equal to: **[JEE(Main)-2020]**

(1) 352 (2) 336 (3) 176 (4) 84

25. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is: **[JEE(Main)-2020]**

(1) 128 (2) 264 (3) 256 (4) 248

26. The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51^{th} term) $+ (1! - 2! + 3! - \dots$ up to 51^{th} term) is equal to: **[JEE(Main)-2020]**

(1) $1 - 51(51)!$ (2) 1
(3) $1 + (51)!$ (4) $1 + (52)!$

27. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

[JEE(Main)-2020]

(1) ${}^{50}C_6 - {}^{30}C_6$ (2) ${}^{51}C_7 + {}^{30}C_7$
(3) ${}^{51}C_7 - {}^{30}C_7$ (4) ${}^{50}C_7 - {}^{30}C_7$

28. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is

equal to _____. **[JEE(Main)-2020]**

29. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x , is _____. **[JEE(Main)-2020]**

30. If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals:

[JEE(Main)-2020]

(1) 2 (2) 1 (3) 3 (4) 9

31. If the coefficient of $a^7 b^8$ in the expansion of $(a + 2b + 4ab)^{10}$ is $K \cdot 2^{16}$, then K is equal to. **[JEE(Main)-2021]**

32. Let $\binom{n}{k}$ denotes nC_k and

$$\left[\begin{matrix} n \\ k \end{matrix} \right] = \begin{cases} \binom{n}{k}, & \text{if } 0 \leq k \leq n \\ 0, & \text{Otherwise} \end{cases}$$

If $A_k =$

$$\sum_{i=0}^9 \binom{9}{i} \left[\begin{matrix} 12 \\ 12-k+i \end{matrix} \right] + \sum_{i=0}^8 \binom{8}{i} \left[\begin{matrix} 13 \\ 13-k+i \end{matrix} \right]$$

And $A_4 - A_3 = 190p$, then p is equal to:

[JEE(Main)-2021]





33. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is:

[JEE(Main)-2021]

- (1) $\frac{n(n+1)^2(n+2)}{12}$
 (2) $\frac{n(n-1)(2n+1)}{6}$
 (3) $\frac{n(n+1)(2n+1)}{6}$
 (4) $\frac{n(2n+1)(3n+1)}{6}$

34. Let $m, n \in \mathbb{N}$ and $\gcd(2, n) = 1$. If $30 \binom{30}{0} + 29 \binom{30}{1} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29} = n \cdot 2^m$, then $n + m$ is equal to:

[JEE(Main)-2021]

35. If b is very small as compared to the value of a , so that the cube and other higher powers of $\frac{b}{a}$

can be neglected in the identity

$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$$

, then the value of γ is:

[JEE(Main)-2021]

- (1) $\frac{a^2 + b}{3a^3}$ (2) $\frac{a + b}{3a^2}$ (3) $\frac{b^2}{3a^3}$ (4) $\frac{a + b^2}{3a^3}$

36. If the greatest value of the term independent of 'x'

in the expansion of $\left(x \sin \alpha + a \frac{\cos \alpha}{x}\right)^{10}$ is

$\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:

[JEE(Main)-2021]

- (1) -1 (2) 1 (3) -2 (4) 2

37. Let $[x]$ denote greatest integer less than or equal to x . If for

$n \in \mathbb{N}$, $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} + 1$ is equal to:

[JEE(Main)-2021]

- (1) 2 (2) 2^{n-1} (3) 1 (4) n

38. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _____.

[JEE(Main)-2021]

39. Let nC_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k) {}^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$, $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to _____.

[JEE(Main)-2021]

40. If the maximum value of the term independent of t in the expansion of

$\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K , then $8K$ is

equal to _____. [JEE(Main)-2022]

41. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is:

[JEE(Main)-2022]

- (1) 1 (2) 4 (3) 6 (4) 8

42. Let the coefficients of the middle terms in the

expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4$, $(1 - 3\beta x)^2$ and

$\left(1 - \frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first

three terms of an A.P. If d is the common difference of this A.P., then $50 - \frac{2d}{\beta^2}$ is equal

to _____. [JEE(Main)-2022]

43. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \dots + {}^{49}C_{49}) ({}^{50}C_2 + {}^{50}C_4 + \dots + {}^{50}C_{50})$ is equal to 2^n , m , where m is odd, then $n + m$ is equal to _____.

[JEE(Main)-2022]

44. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in \mathbb{R}$.

$C_1 + 3.2C_2 + 5.3C_3 + \dots$ upto 10 terms =

$$\frac{\alpha \times 2^{11}}{2^\beta - 1} \left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots \text{upto 10 terms} \right)$$

then the value of $\alpha + \beta$ is equal to:

[JEE(Main)-2022]





45. If the sum of the coefficients of all the positive even powers of x in the expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, then β is equal to _____.

[JEE(Main)-2022]

46. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^4} + \frac{\sqrt{5}}{x^3}\right)^{60}$ is $5^k l$, where $l, k \in \mathbb{N}$ and l is co-prime to 5, then k is equal to _____.

[JEE(Main)-2022]

47. If $\sum_{k=1}^{31} \binom{31}{k} \binom{31}{k-1} - \sum_{k=1}^{30} \binom{30}{k} \binom{30}{k-1} = \frac{\alpha(60!)}{(30!)(31!)}$, where $\alpha \in \mathbb{R}$, then the value of 16α is equal to:

[JEE(Main)-2022]

(1) 1411 (2) 1320 (3) 1615 (4) 1855

48. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is _____.

[JEE(Main)-2023]

49. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is _____.

[JEE(Main)-2023]

50. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to:

[JEE(Main)-2023]

(1) 5445 (2) 4895 (3) 3025 (4) 1210

51. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is _____.

[JEE(Main)-2023]

52. The remainder when $(2023)^{2023}$ is divided by 35 is _____.

[JEE(Main)-2023]

53. Let the coefficient of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _____.

[JEE(Main)-2023]

54. Let K be the sum of the coefficient of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99} K}{a} = \frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to:

[JEE(Main)-2023]

(1) (51, 99) (2) (51, 101)
(3) (50, 51) (4) (50, 101)





JEE ADVANCED (PREVIOUS YEAR QUESTIONS)

1. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then $n =$
[JEE(Advanced)-2013]

2. Coefficient of x^{11} in the expansion of $(1+x^2)^4 (1+x^3)^7 (1+x^4)^{12}$ is -
[JEE(Advanced)-2014]
(A) 1051 (B) 1106 (C) 1113 (D) 1120

3. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is:
[JEE(Advanced)-2015]

4. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integer n . Then the value of n is
[JEE(Advanced)-2016]

5. Let
$$X = \binom{10}{0}C_1^2 + 2\binom{10}{1}C_2^2 + 3\binom{10}{2}C_3^2 + \dots + 10\binom{10}{10}C_{10}^2$$

where $\binom{10}{r}C_r$, $r \in \{1, 2, \dots, 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430} X$ is _____.
[JEE(Advanced)-2018]

6. Suppose $\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^nC_k k^2 \\ \sum_{k=0}^n {}^nC_k k & \sum_{k=0}^n {}^nC_k 3^k \end{bmatrix} = 0$
holds for some positive integer n . Then $\sum_{k=0}^n \frac{{}^nC_k}{k+1}$ equals _____.
[JEE(Advanced)-2019]

7. For non-negative integers s and r , let

[JEE(Advanced)-2020]

$$\binom{s}{r} = \begin{cases} \frac{s!}{r!(s-r)!} & \text{if } r \leq s, \\ 0 & \text{if } r > s. \end{cases}$$

For positive integers m and n , let

$$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m, n, p)}{\binom{n+p}{p}}$$

where for any non-negative integer p ,

$$f(m, n, p) = \sum_{i=0}^p \binom{m}{i} \binom{n+i}{p} \binom{p+n}{p-i}$$

Then which of the following statements is/are TRUE?

- (A) $g(m, n) = g(n, m)$ for all positive integers m, n
(B) $g(m, n+1) = g(m+1, n)$ for all positive integers m, n
(C) $g(2m, 2n) = 2g(m, n)$ for all positive integer m, n
(D) $g(2m, 2n) = (g(m, n))^2$ for all positive integers m, n

8. Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2 + \frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^7$, then the value of $2b$ is

[JEE(Advanced)-2023]





ANSWER KEY

OBJECTIVE EXERCISE - I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	A	B	A	C	A	B	B	C	B	D	C	C	B	D	D	A	C	A
Que.	21	22	23																	
Ans.	C	A	B																	

OBJECTIVE EXERCISE - II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14						
Ans.	B	B	B	D	A	ACD	BCD	AB	ABC	ACD	BCD	AD	BD	AC						

SUBJECTIVE EXERCISE - I

1. (a) $r = 6$ (b) $r = 5$ or 9 2. (i) $\frac{5}{12}$ (ii) $T_6 = 7$
4. $\frac{(2^{mn} - 1)}{(2^n - 1)(2^{mn})}$ 5. $\frac{17}{54}$ 6. $n = 2$ or 3 or 4 7. 816
8. ${}^nC_r(3^{n-r} - 2^{n-r})$ 13. (a) $\binom{30}{15}$ 14. (a) $-1260 \cdot a^2b^3c^4$; (b) -12600
15. (a) 990 (b) 3660 16. (i) 3^n , (ii) 1 , (iii) a_n
17. (i) $T_7 = \frac{7 \cdot 3^{13}}{2}$ (ii) 455×3^{12}

SUBJECTIVE EXERCISE - II

1. 1 2. 722 3. 2^{10} 4. 0 5. 500
6. $n = 12$ 7. 22100 8. (B) 9. (A) 10. (B)

JEE MAIN (PREVIOUS YEAR QUESTIONS)

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	4	3	4	2	4	1	3	2	4	4	4	1	1	1	3	3	30.00	2
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	3	51.00	2	3	4	3	8.00	120.00	3	315	49	3	45	3	4	3	4	19	6006
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54						
Ans.	4	57	99	286	83	5	1	29	04.00	4	1080	7	1120	4						

JEE ADVANCED (PREVIOUS YEAR QUESTIONS)

1. 6 2. (C) 3. 8 4. 5 5. 646.00 6. 6.20
7. (ABD) 8. 3

