

CONTENTS

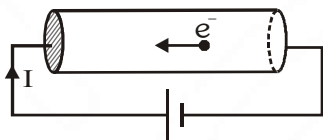
1.	CURRENT ELECTRICITY	01-52
	Theory	01-29
	Objective Exercise-I	30-36
	Objective Exercise-II	37-39
	Subjective Exercise-I	40-43
	Subjective Exercise-II	44-44
	JEE-Main (Previous Year Questions)	45-47
	Jee-Advanced (Previous Year Questions)	48-50
	Answer Key	51-52
2.	SOME BASIC CONCEPTS OF CHEMISTRY	01-27
	Theory	01-16
	Objective Exercise-I	17-18
	Objective Exercise-II	19-20
	Subjective Exercise-I	21-21
	Subjective Exercise-II	22-22
	JEE-Main (Previous Year Questions)	23-24
	Jee-Advanced (Previous Year Questions)	25-27
	Answer Key	27-27
3.	INDEFINITE INTEGRATION	01-32
	Theory	01-13
	Objective Exercise-I	14-17
	Objective Exercise-II	18-20
	Subjective Exercise-I	21-22
	Subjective Exercise-II	23-24
	JEE-Main (Previous Year Questions)	25-28
	Jee-Advanced (Previous Year Questions)	29-29
	Answer Key	30-32

CURRENT ELECTRICITY

In previous chapters we deal largely with electrostatics that is, with charges at rest. With this chapter we begin to focus on electric currents, that is, charges in motion.

Electric Current

Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.



Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors. When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (I). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time as

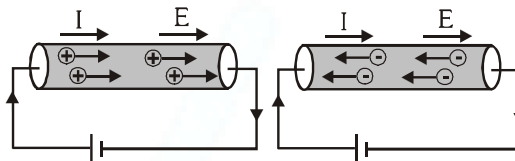
- Average Current $I_{av} = \frac{\Delta Q}{\Delta t}$
- Instantaneous current $I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

KEY POINTS

- Current is a fundamental quantity with dimension $[M^0 L^0 T^0 A^1]$
- Current is a scalar quantity with its SI unit **ampere**.

Ampere: The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross-section of wire.

- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.



- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross-section because current is simply rate of flow of charge.
- If n particles each having a charge q pass per second per unit area then current associated with cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$.
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$.

The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi r}$.

Classification of Materials According to Conductivity

- (i) **Conductor:** In some materials, the outer electrons of each atoms or molecules are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.



(ii) **Insulator:** Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

(iii) **Semiconductor:** In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

Behavior of Conductor in Absence of Applied Potential Difference

In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called

as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron and k is Boltzmann constant.

At room temperature $T = 300\text{ K}$, $v_{rms} = 10^5\text{ m/s}$

• **Mean Free Path λ :** ($\lambda \sim 10\text{ \AA}$),
 $\lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$

• **Relaxation Time:** The time taken by an electron between two successive collisions is called as relaxation time τ : ($\tau \sim 10^{-14}\text{ s}$),

Relaxation time: $\tau = \frac{\text{total time taken}}{\text{number of collisions}}$

Behavior of Conductor in Presence of Applied Potential Difference

When two ends of a conductor are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of

higher to lower potential $E = \frac{V}{L}$ where $V = \text{emf}$

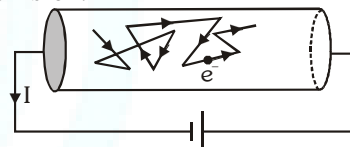
of the battery, $L = \text{length of the conductor}$.

The field exerts an electric force on free electrons causing acceleration of each electron.

$$\text{Acceleration of electron } \vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$$

Drift Velocity

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field :
Random motion of an electron
with superimposed drift

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where \vec{u}_1 = the thermal velocity and $\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field. τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are $\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \dots, \vec{v}_N = \vec{u}_N + \vec{a}\tau_N$.

The average velocity of all the free electrons in the conductor is equal to the drift velocity of the free electrons

$$\begin{aligned} \vec{v}_d &= \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \dots + \vec{v}_N}{N} \\ &= \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \dots + (\vec{u}_N + \vec{a}\tau_N)}{N} \\ &= \frac{(\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N)}{N} + \vec{a} \left(\frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \\ \therefore \frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_N}{N} &= 0 \\ \therefore \vec{v}_d &= \vec{a} \left(\frac{\tau_1 + \tau_2 + \dots + \tau_N}{N} \right) \Rightarrow \vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau \end{aligned}$$

Note: Order of drift velocity is 10^{-4} m/s .



Relation Between Current and Drift Velocity

Let n = number density of free electrons and A = area of cross-section of conductor.

Number of free electrons in conductor of length $L = nAL$. Total charge on these free electron $\Delta q = neAL$

Time taken by drifting electrons to cross conductor $\Delta t = \frac{L}{v_d}$

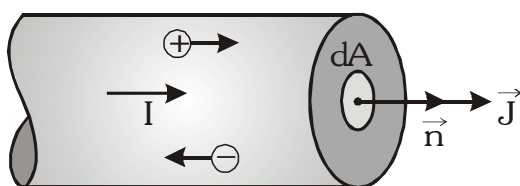
$$\therefore \text{current } I = \frac{\Delta q}{\Delta t} = neAL \left(\frac{v_d}{L} \right) = neAv_d$$

Example:

Find free electrons per unit volume in a metallic wire of density 10^4 kg/m^3 , atomic mass number 100 and number of free electrons per atom is one.

Solution:

Number of free charge particle per unit volume (n) = $\frac{\text{total free charge particle}}{\text{total volume}}$



- If the cross-sectional area is not normal to the current, but makes an angle θ with the direction of current then $J = \frac{dI}{dA \cos \theta} \Rightarrow dI = J dA \cos \theta = \vec{J} \cdot d\vec{A} \Rightarrow I = \int \vec{J} \cdot d\vec{A}$
- Current density \vec{J} is a vector quantity. Its direction is same as that of \vec{E} . Its S.I. unit is ampere/m² and dimension [$L^{-2}A$].

Example:

The current density at a point is $\frac{R}{1 - \frac{1}{4}} = \frac{4R}{3}$.

Find the rate of charge flow through a cross sectional area $\vec{S} = (2\hat{i} + 3\hat{j}) \text{ cm}^2$

Solution:

The rate of flow of charge = current = $I = \int \vec{J} \cdot d\vec{S} \Rightarrow I = \vec{J} \cdot \vec{S} = (2 \times 10^4) [\hat{j} \cdot (2\hat{i} + 3\hat{j})] \times 10^{-4} \text{ A} = 6 \text{ A}$

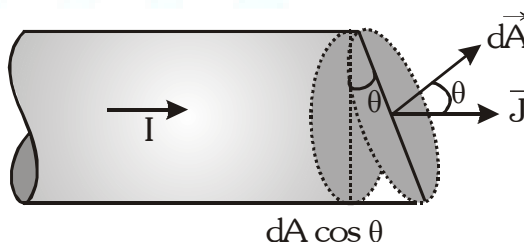
\therefore Total free electrons = number of free electron per atom \times total number of atoms = $\frac{N_A}{M_w} \times M$

$$\text{So } n = \frac{\frac{N_A}{M_w} \times M}{V} = \frac{N_A}{M_w} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}} = 6.023 \times 10^{28}$$

Current Density (J)

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

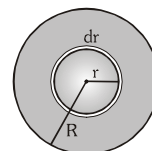
- Current density at point P is given by $\vec{J} = \frac{dI}{dA} \vec{n}$



Example:

A potential difference applied to the ends of a wire made up of an alloy drives a current through it. The current density varies as $J = 3 + 2r$, where r is the distance of the point from the axis. If R be the radius of the wire, then the total current through any cross section of the wire.

Solution:

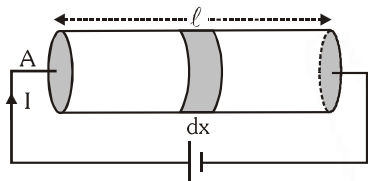


Consider a circular strip of radius r and thickness dr
 $dI = \vec{J} \cdot d\vec{S} = (3 + 2r)(2\pi r dr) \cos 0^\circ = 2\pi(3r + 2r^2) dr$

$$I = \int_0^R 2\pi(3r + 2r^2) dr = 2\pi \left(\frac{3r^2}{2} + \frac{2}{3} r^3 \right)_0^R = 2\pi \left(\frac{3R^2}{2} + \frac{2R^3}{3} \right) \text{ units}$$

Relation Between Current Density, Conductivity and Electric Field

Let the number of free electrons per unit volume in a conductor = n



Total number of electrons in dx distance = $n(Adx)$

Total charge $dQ = n(Adx)e$

Current = $I = \frac{dQ}{dt} = nAe \frac{dx}{dt} = neAv_d$, Current density =

$$J = \frac{I}{A} = nev_d$$

$$J = ne \left(\frac{eE}{m} \right) \tau \because v_d = \left(\frac{eE}{m} \right) \tau \Rightarrow J = \left(\frac{ne^2 \tau}{m} \right) E \Rightarrow J =$$

$$\sigma E, \text{ where conductivity } \sigma = \frac{ne^2 \tau}{m}$$

σ depends only on the material of the conductor and its temperature.

In vector form $\vec{J} = \sigma \vec{E}$ Ohm's law (at microscopic level)

FUNDAMENTAL UNLOCKED- (FU#1) :

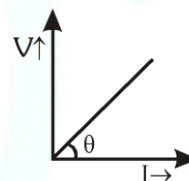
- Q.1** Find free electrons per unit volume in a metallic wire of density 10^4 kg/m^3 , atomic mass number 100 and number of free electron per atom is one.
- Q.2** The current through a wire depends on time as $i = i_0 + \alpha \sin \pi t$, where $i_0 = 10 \text{ A}$ and $\alpha = \frac{\pi}{2} \text{ A}$. Find the charge crossed through a section of the wire in 3 seconds, and average current for that interval.
- Q.3** A current of 5 A exists in a 10Ω resistance for 4 minutes. (Charge of the electron = $1.6 \times 10^{-19} \text{ C}$)
- How many coulombs and
 - How many electrons pass through any cross section of the resistor in this time?
- Q.4** A cylindrical conducting wire of radius 0.2 mm is carrying a current of 20 mA .
- How many electrons are transferred per second between the supply and the wire at one end?
 - Write down the current density in the wire.

Relation Between Potential Difference and Current (Ohm's Law)

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across its two ends i.e. $I \propto V \Rightarrow V = IR$ where R is a proportionality constant, known as electric resistance.

Ohm's law (at macroscopic level)

- Ohm's law is not a universal law.
The substances, which obey ohm's law are known as ohmic.
- Graph between V and I for a metallic conductor is a straight line as shown.



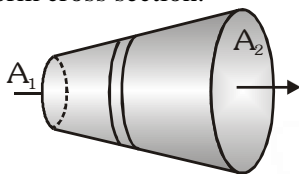
$$\text{Slope of the line} = \tan \theta = \frac{V}{I} = R$$

KEY POINTS

- 1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.
- Current is a scalar quantity but current density is a vector quantity.
- Order of free electron density in conductors = $10^{28} \text{ electrons/m}^3$

Terms	Order
Thermal speed v^T	10^5 m/s
Mean free path λ	10 \AA
Relaxation time τ	10^{-14} s
Drift speed v_d	10^{-14} m/s

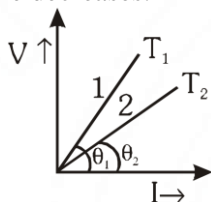
- If a steady current flow in a metallic conductor of non-uniform cross section.



- Along the wire I is same.
- Current density and drift velocity depend on area

$$I_1 = I_2, A_1 < A_2 \Rightarrow J_1 > J_2, E_1 > E_2, v_{d1} > v_{d2}$$

If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.



- At different temperatures $V-I$ curves are different.

$$\text{Here } \tan \theta_1 > \tan \theta_2 \text{ So } R_1 > R_2 \quad \text{i.e. } T_1 > T_2$$

Example:

What will be the number of electrons passing through a heater wire in one minute, if it carries a current of 8 A.

Solution:

$$I = \frac{Ne}{t} \Rightarrow N = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21} \text{ electrons}$$

Example:

An electron moves in a circle of radius 10 cm with a constant speed of 4×10^6 m/s. Find the electric current at a point on the circle.

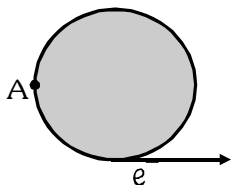
Solution:

Consider a point A on the circle. The electron crosses this point once in every revolution. The number of revolutions made by electron in one second is

$$n = \frac{v}{2\pi r} = \frac{4 \times 10^6}{2\pi \times 10 \times 10^{-2}} = \frac{2}{\pi} \times 10^7 \text{ rot/s}$$

$$\therefore \text{Current } I = \frac{ne}{t} = \frac{2}{\pi} \times 10^7 \times 1.6 \times 10^{-19}$$

$$(\because t = 1 \text{ s.}) = \frac{3.2}{\pi} \times 10^{-12} \approx 1 \times 10^{-12} \text{ A}$$



Example:

A current of 1.34 A exists in a copper wire of cross-section 1.0 mm^2 . Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990 kg/m^3 and atomic mass = 63.50.

Solution:

Mass of 1 m^3 volume of the copper is = $8990 \text{ kg} = 8990 \times 10^3 \text{ g}$

$$\text{Number of moles in } 1 \text{ m}^3 = \frac{8990 \times 10^3}{63.5} = 1.4 \times 10^5$$

Since each mole contains 6×10^{23} atoms therefore number of atoms in 1 m^3

$$n = (1.4 \times 10^5) \times (6 \times 10^{23}) = 8.4 \times 10^{28}$$

$$\therefore I = neAv_d$$

$$\therefore v_d = \frac{I}{neA} = \frac{1.34}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}} = 10^{-4} \text{ m/s} \\ = 0.1 \text{ mm/s } (\because 1 \text{ mm}^2 = 10^{-6} \text{ m}^2)$$

Example:

The current through a wire depends on time as $i = (2 + 3t) \text{ A}$. Calculate the charge crossed through a cross section of the wire in 10 s.

Solution:

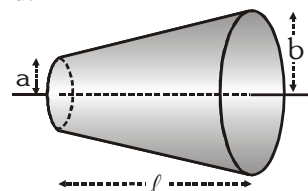
$$I = \frac{dq}{dt} \Rightarrow dq = (2 + 3t)dt \Rightarrow \int_0^{10} dq = \int_0^{10} (2 + 3t) dt \Rightarrow$$

$$q = \left(2t + \frac{3t^2}{2} \right)_0^{10}$$

$$q = 2 \times 10 + \frac{3}{2} \times 100 = 20 + 150 = 170 \text{ C}$$

Example:

Figure shows a conductor of length ℓ carrying current I and having a circular cross-section. The radius of cross section varies linearly from a to b . Assuming that $(b - a) \ll \ell$. Calculate current density at distance x from left end.



Solution:

Since radius at left end is a and that of right end is b , therefore increase in radius over length ℓ is $(b - a)$.

Hence rate of increase of radius per unit length =

$$\left(\frac{b-a}{\ell} \right) \text{ Increase in radius over length } x = \left(\frac{b-a}{\ell} \right) x$$



Since radius at left end is a so radius at distance x , $r =$

$$a + \left(\frac{b-a}{\ell} \right) x$$

Area at this particular section $A = \pi r^2 =$

$$\pi \left[a + \left(\frac{b-a}{\ell} \right) x \right]^2$$

$$\text{Hence current density } J = \frac{I}{A} = \frac{I}{\pi r^2} =$$

$$\frac{I}{\pi \left[a + \frac{x(b-a)}{\ell} \right]^2}$$

Resistance

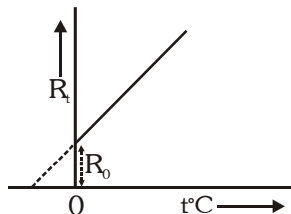
The resistance of a conductor is the opposition which the conductor offers to the flow of charge. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called resistance of the conductor.

Resistance is the property of a conductor by virtue of which it opposes the flow of current in it.

Unit: ohm, volt/ampere, **Dimension** = $M L^2 T^{-3} A^{-2}$

Resistance Depends on:

- Length of the conductor ($R \propto \ell$)
- Area of cross-section of the conductor $R \propto \frac{1}{A}$



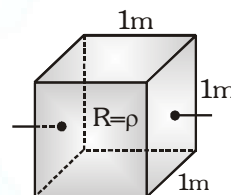
- Nature of material of the conductor $R = \frac{\rho \ell}{A}$
- Temperature $R_t = R_0 (1 + \alpha \Delta t)$
Where R_t = Resistance at $t^\circ C$, R_0 = Resistance at $0^\circ C$
 Δt = Change in temperature, α = Temperature coefficient of resistance

*[For metals: α positive for semiconductors and insulators: α negative]

- Resistance of the conductor decreases linearly with decrease in temperature and becomes zero at a specific temperature. This temperature is called critical temperature. At this temperature conductor becomes a superconductor.

Resistivity

Resistivity: $\rho = RA/\ell$ if $\ell = 1m$, $A = 1m^2$ then $\rho = R$



The specific resistance of a material is equal to the resistance of the wire of that material with unit cross-section area and unit length.

Resistivity Depends On (i) Nature of material (ii) Temperature of material ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.

Specific Use of Conducting Materials

- The **heating element** of devices like heater, geyser, press etc are made of **microhm** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at $800^\circ C$.
- Fuse wire** is made of **tin lead alloy** because it has low melting point and low resistivity. The fuse is used in series, and melts to produce open circuit when current exceeds the safety limit.
- Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of $3300 K$ and gives light at $2400 K$. The bulb is filled with inert gas because at high temperature it reacts with air forming oxide.
- The **connection wires** are made of **copper** because it has low resistance and resistivity.



Colour Code for Carbon Resistors

Colour	Strip A	Strip B	Strip C	Strip D (Tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Grey	8	8	10^8	
White	9	9	10^9	
Gold	-	-	10^{-1}	$\pm 5\%$
Silver	-	-	10^{-2}	$\pm 10\%$
No colour	-	-	-	$\pm 20\%$

May be remembered as
BBROY
Great Britain
Very Good Wife.

Example:

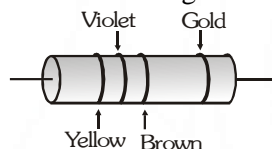
Draw a colour code for $42 \text{ k } \Omega \pm 10\%$ carbon resistance.

Solution:

According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.

Example:

What is resistance of following resistor.



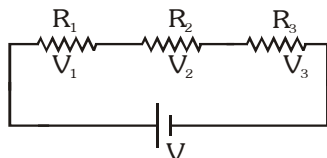
Solution:

Number for yellow is 4, Number of violets is 7
Brown colour gives multiplier 10^1 , Gold gives a tolerance of $\pm 5\%$

So, resistance of resistor is $47 \times 10^1 \Omega \pm 5\% = 470 \pm 5\% \Omega$.

Combination of Resistors

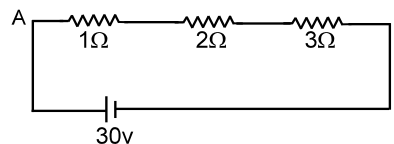
Series Combination



- Same current passes through each resistance
- Voltage across each resistance is directly proportional to its value
 $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$
- Sum of the voltage across resistance is equal to the voltage applied across the circuit.
 $V = V_1 + V_2 + V_3 \Rightarrow IR = IR_1 + IR_2 + IR_3$
 $R = R_1 + R_2 + R_3$
Where R = equivalent resistance

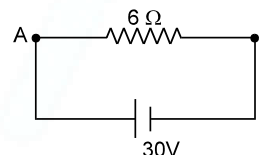
Example:

Find the current in the circuit



Solution:

$R_{eq} = 1 + 2 + 3 = 6 \Omega$ the given circuit is equivalent to



$$\text{current } i = \frac{V}{R_{eq}} = \frac{30}{6} = 5A$$

Example:

The resistance $R, \frac{R}{4}, \frac{R}{16} \dots \infty$ are connected in series.

Find their equivalent resistance.

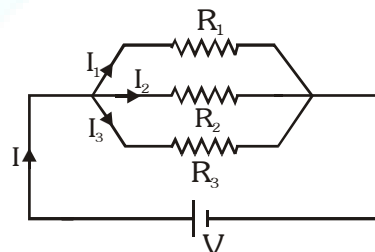
Solution:

Resultant of the given combination $R_{eq} = R + \frac{R}{4} + \frac{R}{16}$

$$\dots \infty = \frac{R}{1 - \frac{1}{4}} = \frac{4R}{3}$$

Parallel Combination

- There is same drop of potential across each resistance.



- Current in each resistance is inversely proportional to the value of resistance.

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

- Current flowing in the circuit is sum of the currents in individual resistance.

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



Example:

Resistance $R, 2R, 4R, 8R, \dots, \infty$ are connected in parallel. What is their resultant resistance?

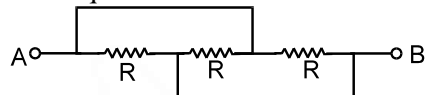
Solution:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R} + \dots \infty$$

$$= \frac{1}{R} \left[1 + \frac{1}{2} + \frac{1}{4} + \dots \infty \right] = \frac{1}{R} \left[\frac{1}{1 - \frac{1}{2}} \right] = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$$

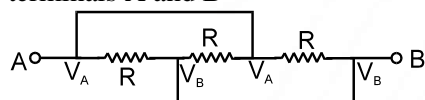
Example:

Find equivalent Resistance

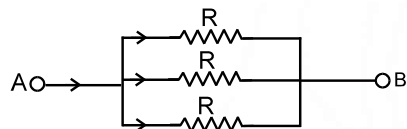


Solution:

Here all the Resistance are connected between the terminals A and B



Modified circuit is So



$$R_{eq} = \frac{R}{3}$$

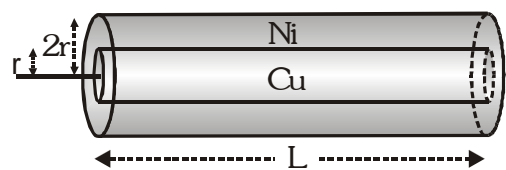
Example:

A copper wire of length ' ℓ ' and radius ' r ' is nickel plated till its final radius is $2r$. If the resistivity of the copper and nickel are ρ_{Cu} and ρ_{Ni} , then find the equivalent resistance of wire?

Solution:

$$R = \rho \frac{\ell}{A}; \text{ Resistance of copper wire } R_{Cu} = \rho_{Cu} \frac{\ell}{\pi r^2}$$

$$(\because A = \pi r^2)$$



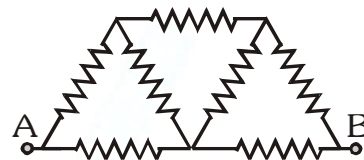
$$\therefore A_{Ni} = \pi(2r)^2 - \pi r^2 = 3\pi r^2 \Rightarrow \text{Resistance of Nickel wire } R_{Ni} = \rho_{Ni} \frac{\ell}{3\pi r^2}$$

Both wire are connected in parallel. So equivalent resistance

$$R = \frac{R_{Cu} R_{Ni}}{R_{Cu} + R_{Ni}} = \left(\frac{\rho_{Cu} \rho_{Ni}}{3\rho_{Cu} + \rho_{Ni}} \right) \frac{\ell}{\pi r^2}$$

Example:

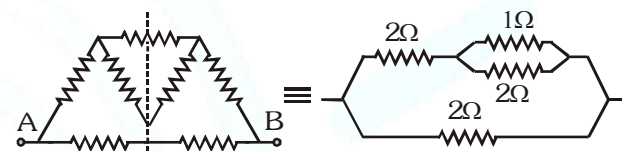
Each resistance is of 1Ω in the circuit diagram shown in figure. Find out equivalent resistance between A and B



Solution:

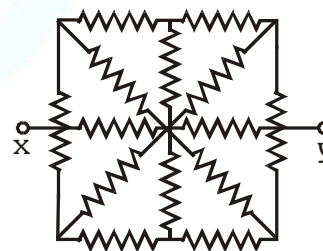
$$\text{By symmetric line method } R_{AB} = (2 + 1 \parallel 2) \parallel 2 = \frac{8}{7}$$

Ω



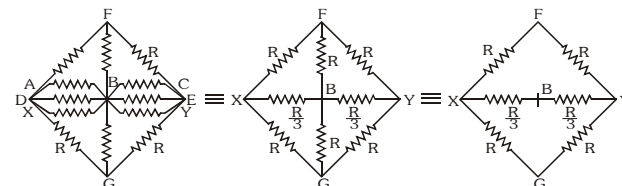
Example:

Identical resistance of resistance R are connected as in figure then find out net resistance between x and y.



Solution:

Given circuit can be modified according to following figures

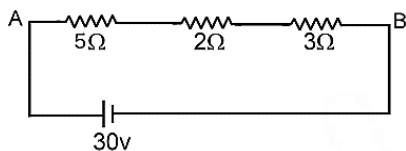


$$\frac{1}{R_{xy}} = \frac{1}{2R} + \frac{3}{2R} + \frac{1}{2R} = \frac{5}{2R} \Rightarrow R_{xy} = \frac{2R}{5}$$

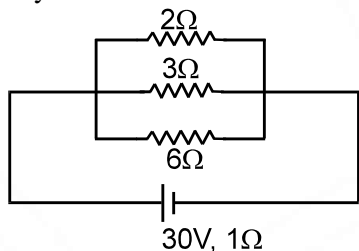


FUNDAMENTAL UNLOCKED- (FU#2) :

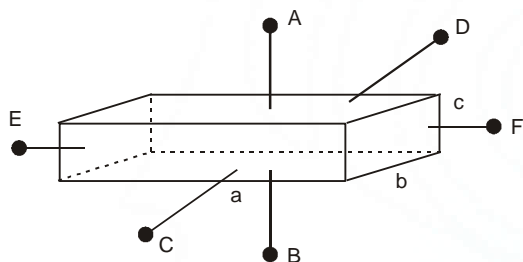
Q.1 Find the current in the circuit



Q.2 Find current which is passing through battery.

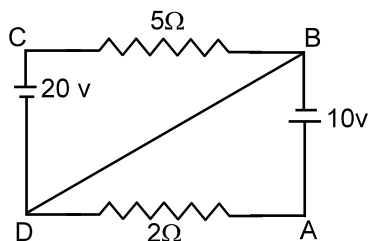


Q.3 The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.

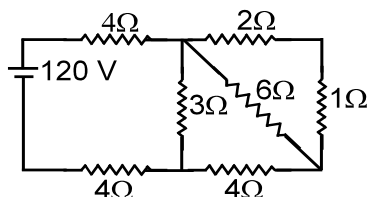


Q.4 If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

Q.5 In the figure given beside find out the current in the wire BD



Q.6 Find the current in 2Ω resistance



Kirchhoff's Law

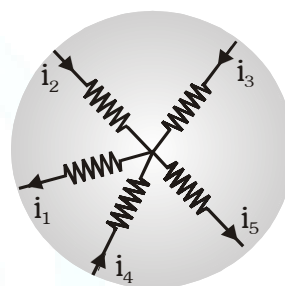
There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

Law of conservation of charge is a consequence of continuity equation

• First law (Junction Law or Current Law)

In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero or Sum of the currents entering the Junction is equal to sum of the current leaving the Junction. $\Sigma I = 0$

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$$



This is based on law of conservation of charge.

• Second Law (Loop Rule or Potential Law)

In any closed circuit the algebraic sum of all potential differences and e.m.f. is zero. $\Sigma E - \Sigma IR = 0$ while moving from negative to positive terminal inside the cell, e.m.f. is taken as positive while moving in the direction of current in a circuit the potential drop (i.e. IR) across resistance is taken as positive.

This law is based on law of conservation of energy.

KEY POINTS

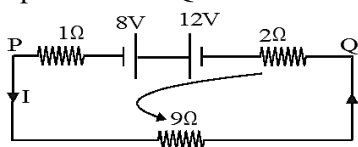
- If a wire is stretched to n times of its original length, its new resistance will be n^2 times.
- If a wire is stretched such that its radius is reduced to n times of its original values, then resistance will increase n^4 times similarly resistance will decrease n^4 times if radius is increased n times by contraction.
- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual.



- To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- Ohm's law is not a fundamental law of nature. As it is possible that for an element:
 - V depends on I non-linearly (e.g. vacuum tubes)
 - Relation between V and I depends on the sign of V for the same value
[Forward and reverse Bias in diode]
 - The relation between V and I is non-unique.
That is for the same I there is more than one value of V.
- In General:**
 - Resistivity of alloys is greater than their metals.
 - Temperature coefficient of alloys is lower than pure metals.
 - Resistance of most of non metals decreases with increase in temperature. (e.g. carbon)
 - The resistivity of an insulator (e.g. amber) is greater than the metal by a factor of 10^{22}
- Temperature coefficient (α) of semi conductor including carbon (graphite), insulator and electrolytes is negative.

Example:

In the given circuit calculate potential difference between the points P and Q.



Solution:

Applying Kirchhoff's voltage law (KVL) $12 - 8 = (1)$

$$I + (9) I + (2) I \Rightarrow I = \frac{1}{3} \text{ A}$$

Potential difference between the points P and

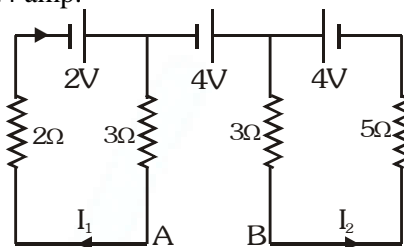
$$Q, V_P - V_Q = 9 \times \frac{1}{3} = 3V$$

Example:

In the given circuit calculate potential difference between A and B.

Solution:

First applying KVL on left mesh $2 - 3 I_1 - 2 I_1 = 0 \Rightarrow I_1 = 0.4 \text{ amp.}$



Now applying KVL on right mesh. $4 - 5 I_2 - 3 I_2 = 0 \Rightarrow I_2 = 0.5 \text{ amp.}$

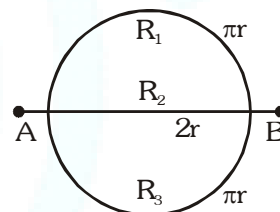
Potential difference between points A and B

$$V_A - V_B = -3 \times 0.4 - 4 + 3 \times 0.5 = -3.7 \text{ volt.}$$

Example:

A wire of resistance per unit length $\rho_L = 10^{-6} \Omega/\text{m}$ is turned in the form of a circle of diameter 2 m. A piece of same material is connected in diameter AB. Then find resistance between A and B.

Solution:



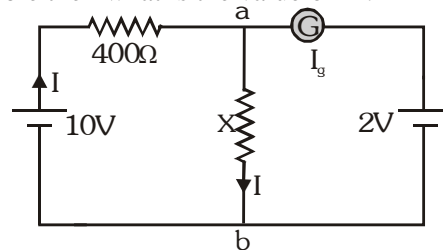
$$\therefore R = \rho_L \times \text{length}$$

$$\therefore R_1 = \pi \times 10^{-6} \Omega, R_2 = 2 \times 10^{-6} \Omega, R_3 = \pi \times 10^{-6} \Omega$$

$$\frac{1}{R_{AB}} = \frac{1}{\pi \times 10^{-6}} + \frac{1}{2 \times 10^{-6}} + \frac{1}{\pi \times 10^{-6}}; R_{AB} = 0.88 \times 10^{-6} \text{ ohm.}$$

Example:

In the following circuit diagram, the galvanometer reading is zero. If the internal resistance of cells are negligible then what is the value of X?



Solution:

$$\therefore I_g = 0$$

$$\therefore I = \frac{10}{400 + X} \text{ also potential difference across X is}$$

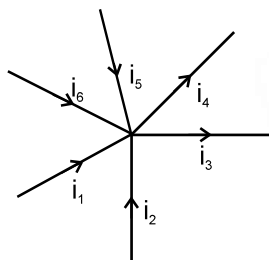
$$2V \Rightarrow I X = 2$$

$$\therefore \frac{10X}{400 + X} = 2 \Rightarrow X = 100\Omega$$

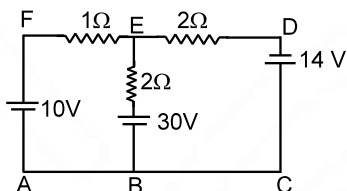


FUNDAMENTAL UNLOCKED- (FU#3) :

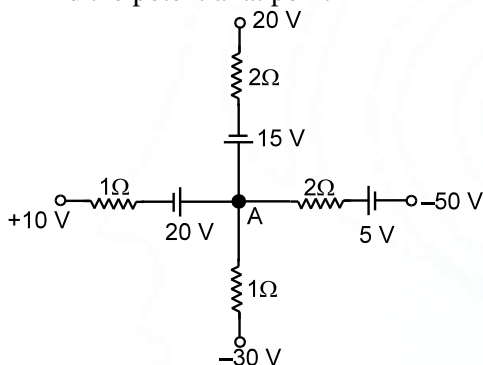
- Q.1** Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6 .



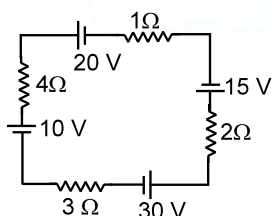
- Q.2** Find the current in each wire



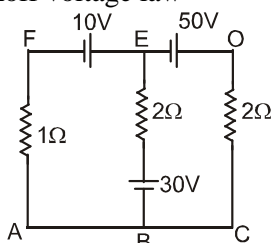
- Q.3** Find the potential at point A



- Q.4** Find current in the circuit



- Q.5** Find the current in each wire applying only Kirchhoff voltage law



Cell

Cell convert chemical energy into electrical energy.

Electro Motive Force (E.M.F.)

The potential difference across the terminals of a cell when it is not giving any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

- emf depends on:
 - nature of electrolyte
 - metal of electrodes
- emf does not depend on:
 - area of plates
 - distance between the electrodes
 - quantity of electrolyte
 - size of cell

Internal Resistance

Offered by the electrolyte of the cell when the electric current flows through it is known as internal resistance.

Distance between two electrodes increases $\Rightarrow r$ increases



Area dipped in electrolyte increases $\Rightarrow r$ decreases

Concentration of electrolyte increases $\Rightarrow r$ increases

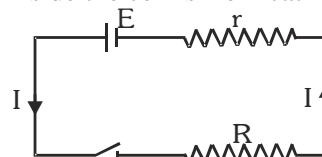
Temperature increases $\Rightarrow r$ decreases

• **Terminal Potential Difference**

The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

(a) When Cell is Discharging

Current inside the cell is from cathode to anode.



$$\text{Current } I = \frac{E}{r + R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V =$$

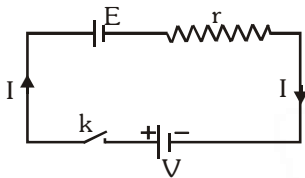
$$E - Ir$$

When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage.



(b) When Cell is Charging

Current inside the cell is from anode to cathode.



$$\text{Current } I = \frac{V - E}{r} \Rightarrow V = E + Ir$$

During charging terminal potential difference is greater than emf of cell.

(c) When Cell is in Open Circuit

$$\text{In open circuit } R = \infty \therefore I = \frac{E}{R + r} = 0 \Rightarrow V = E$$

In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When Cell is Short Circuited

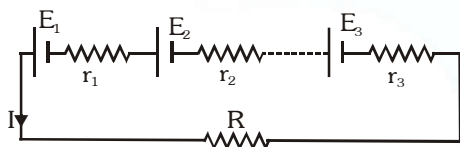
$$\text{In short circuit } R = 0 \Rightarrow I = \frac{E}{R + r} = \frac{E}{r} \text{ and } V = IR = 0$$

In short circuit current from cell is maximum and terminal potential difference is zero.

Combination of Cells

• **Series Combination**

When the cells are connected in series the total e.m.f. of the series combination is equal to the sum of the e.m.f.'s of the individual cells and internal resistance of the cells also come in series.



Equivalent internal resistance $r = r_1 + r_2 + r_3 + \dots$

Equivalent emf $E = E_1 + E_2 + E_3 + \dots$

$$\text{Current } I, = \frac{E_{net}}{r_{net} + R} \text{ If all } n \text{ cell are identical then}$$

$$I = \frac{nE}{nr + R}$$

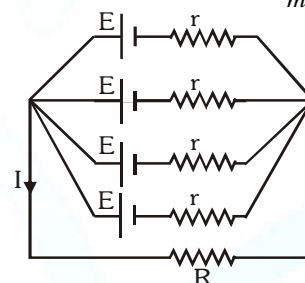
○ If $nr \gg R$, $I = \frac{E}{r}$; current from one cell

○ If $nr \ll R$, $I = \frac{nE}{R}$; $n \times$ current from one cell

• **Parallel Combination**

When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single cell and internal resistance of the cell also come in parallel. If m identical cell connected in parallel then total internal resistance of this combination $r_{net} = \frac{r}{m}$. Total e.m.f. of this combination $= E$

$$\text{Current in the circuit } I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$



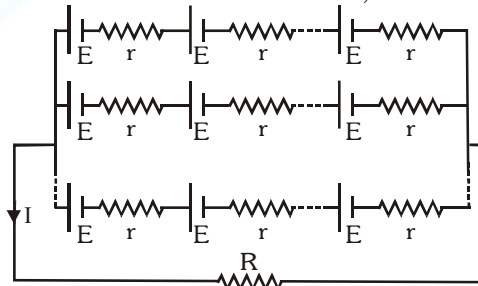
$$\text{If } r \ll mR \quad I = E/R = \text{Current from one cell}$$

$$\text{If } r \gg mR \quad I = \frac{mE}{r} = m \times \text{current from one cell}$$

• **Mixed Combination**

If n identical cells connected in series and there are m such branches connected and parallel in the circuit then total number of identical cell in this circuit is nm . The internal resistance of the cells connected in a row $= nr$.

Since there are such m rows,



$$\text{Total internal resistance of the circuit } r_{net} = \frac{nr}{m}$$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row $E_{net} = nE$

$$\text{Current in the circuit } I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$$

Current in the circuit is maximum when external resistance in the circuit is equal to the total

$$\text{internal resistance of the cells } R = \frac{nr}{m}$$



KEY POINTS

- At the time of charging a cell when current is supplied to the cell, the terminal voltage is greater than the e.m.f. E , $V = E + Ir$
- Series combination is useful when internal resistance is less than external resistance of the cell.
- Parallel combination is useful when internal resistance is greater than external resistance of the cell.
- Power in R (Given Resistance) is maximum, if its value is equal to net resistance of remaining circuit.
- Internal resistance of ideal cell = 0
- If external resistance is zero than current given by circuit is maximum.

Example:

A battery of six cells each of e.m.f. 2 V and internal resistance 0.5Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10Ω . What will be the charging current?

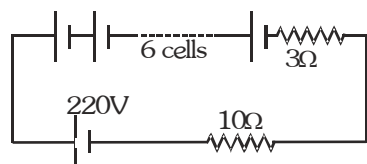
Solution:

Net e.m.f of the battery = 12V and total internal resistance = 3Ω

Total resistance of the circuit = $3 + 10 = 13$

$$\text{Charging current } I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220 - 12}{13} = 16$$

A



Example:

A battery of six cells each of e.m.f. 2 V and internal resistance 0.5Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10Ω . What is the potential difference across the battery?

Solution:

In case of charging of battery, terminal potential $V = E + Ir = 12 + 16 \times 3 = 60$ volt.

Example:

Four identical cells each of e.m.f. 2V are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is 1.6V calculate the internal resistance of each cell.

Solution:

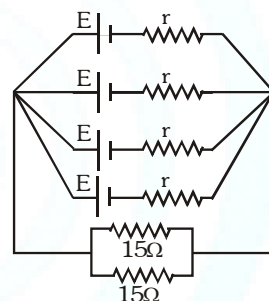
Total internal resistance of the combination $r_{eq} = \frac{r}{4}$

Total e.m.f. $E_{eq} = 2V$

$$\text{Total external resistance } R = \frac{15 \times 15}{15 + 15} = \frac{15}{2} = 7.5 \Omega$$

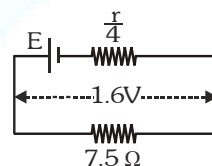
Current drawn from equivalent cell

$$I = \frac{\text{terminal potential}}{\text{external resistance}} = \frac{1.6}{7.5} A$$



$$\therefore E - I \left(\frac{r}{4} \right) = 1.6$$

$$\therefore 2 - \frac{1.6}{7.5} \left(\frac{r}{4} \right) = 1.6 \Rightarrow r = 7.5 \Omega$$



Example:

The e.m.f. of a primary cell is 2 V, when it is shorted then it gives a current of 4 A. Calculate internal resistance of primary cell.

Solution:

$$I = \frac{E}{r + R}, \text{ If cell is shorted then } R = 0, I = \frac{E}{r}$$

$$\therefore r = \frac{E}{I} = \frac{2}{4} = 0.5 \Omega$$



Example:

n rows each containing m cells in series, are joined in parallel. Maximum current is taken from this combination in a 3Ω resistance. If the total number of cells used is 24 and internal resistance of each cell is 0.5Ω , find the value of m and n .

Solution:

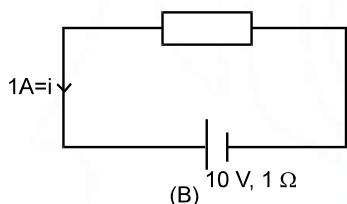
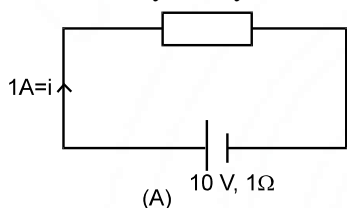
Total number of cell $mn = 24$, For maximum current

$$\frac{mr}{n} = R \Rightarrow 0.5m = 3n, m = \frac{3n}{0.5} = 6n$$

$$\therefore 6n \times n = 24 \Rightarrow n = 2 \text{ and } m \times 2 = 24 \Rightarrow m = 12$$

FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 In following diagram boxes may contain resistor or battery or any other element



then determine in each case

- Emf. of battery
- Battery is acting as a source or load
- Potential difference across each battery
- Power input to the battery or output by the battery
- The rate at which heat is generated inside the battery
- The rate at which the chemical energy of the cell is consumed or increased
- Potential difference across box
- Electric power output across box

Galvanometer

The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

Shunt

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

• **Merits of Shunt**

- To protect the galvanometer coil from burning.
- Any galvanometer can be converted into ammeter of desired range with the help of shunt.
- The range an ammeter can be changed by using shunt resistance of different values.

• **Demerits of Shunt**

Shunt resistance decreases the sensitivity of galvanometer.

Conversion of Galvanometer into Ammeter

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

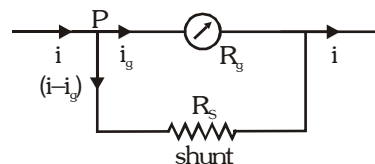
- The value of shunt resistance to be connected in parallel to galvanometer coil is given by

$$R_s = \frac{R_g i_g}{i - i_g}$$

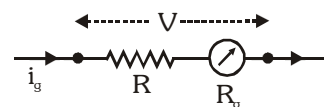
Where i = Range of ammeter

i_g = Current required for full scale deflection of galvanometer.

R_g = Resistance of galvanometer coil.



Conversion of Galvanometer into Voltmeter



- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.
- The high resistance to be connected in series with galvanometer coil is given by $R = \frac{V}{i_g} - R_g$



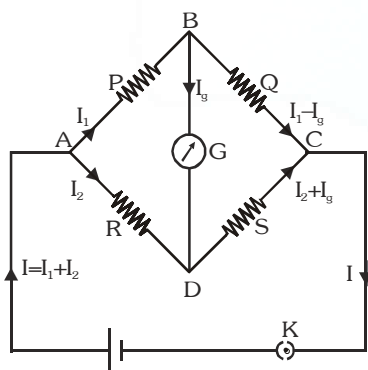
KEY POINTS

- The rate of variation of deflection depends upon the magnitude of deflection itself and so the accuracy of the instrument.
- A suspended coil galvanometer can measure currents of the order of 10^{-9} ampere.
- I_g is the current for full scale deflection. If the current for a deflection, of one division on the galvanometer scale is k and N is the total number of divisions on one side of the zero of galvanometer scale, then $I_g = k \times N$.
- A ballistic galvanometer is a specially designed moving coil galvanometer, used to measure charge flowing through the circuit for small time intervals.

Wheat Stone Bridge

- The configuration in the adjacent figure is called Wheat Stone Bridge.
- If current in galvanometer is zero ($I_g = 0$) then bridge is said to be balanced

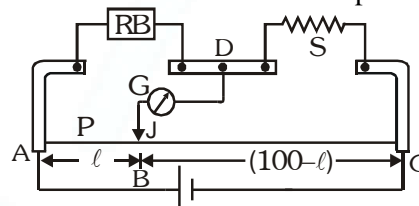
$$V_D = V_B \Rightarrow I_1 P = I_2 R \text{ \& \; } I_1 Q = I_2 S \Rightarrow \frac{P}{Q} = \frac{R}{S}$$



- If $\frac{P}{Q} < \frac{R}{S}$ then $V_B > V_D$ and current will flow from B to D.
- If $\frac{P}{Q} > \frac{R}{S}$ then $V_B < V_D$ and current will flow from D to B.

Metre Bridge

It is based on principle of whetstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1 m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.



$$AB = l \text{ cm} \quad P = r l$$

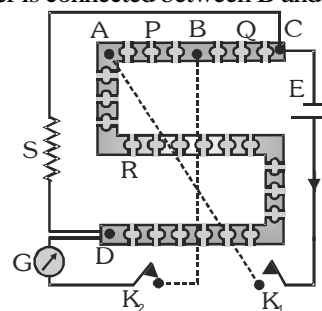
$$BC = (100 - l) \text{ cm} \quad Q = r(100 - l) \text{ where } r = \text{resistance per unit length on wire.}$$

$$\text{At balance condition: } \frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{r l}{r(100 - l)} = \frac{R}{S} \\ \Rightarrow S = \frac{(100 - l)}{l} R$$

Post Office Box

It is also based on wheat stone bridge. The resistance of 10Ω , 100Ω , and 1000Ω are often connected between AB and BC. These are known as ratio arms. Resistance from 1Ω to 5000Ω are connected between A and D, this is known arm. Unknown resistance is connected between C and D.

A cell is connected between A and C with key K_1 and Galvanometer is connected between B and D with key K_2 .



First, we select ratio of resistance Q and P. For given value of S we will take value of resistance from known arm in such a way that Galvanometer show null deflection $S = \frac{Q}{P} R$. On decreasing the

value of $\frac{Q}{P}$ the sensitivity of the box increases. It is used to find out the breakage in telegraph line in post and telegraph offices.

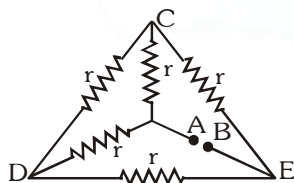


KEY POINTS

- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range I ampere and resistance $R_g \Omega$ into an ammeter of range nI ampere, the value of resistance to be connected in parallel will be $R_g(n - 1)$
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance $R_g \Omega$ and range V volt into a voltmeter of range nV volt, the value of resistance to be connected in series will be $(n - 1)R_g$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.

Example:

In the adjoining network of resistors each is of resistance $r \Omega$. Find the equivalent resistance between point A and B

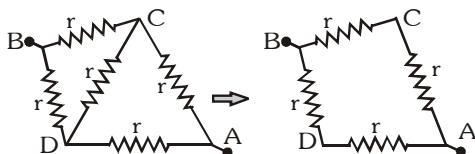


Solution:

Given circuit is balanced Wheat stone Bridge

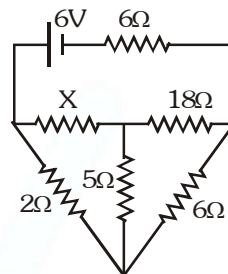
$$\therefore \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r}$$

$$R_{AB} = r$$



Example:

Calculate magnitude of resistance X in the circuit shown in figure when no current flows through the 5Ω resistor?



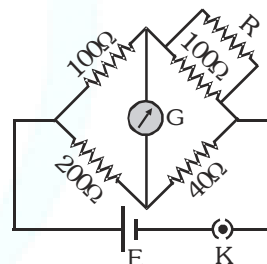
Solution:

Since wheat stone bridge is balanced so $\frac{x}{18} = \frac{2}{6}$ or $x = \frac{18 \times 2}{6} = 6\Omega$

Example:

For the following diagram the galvanometer shows zero deflection then what is the value of R ?

Solution:



For balanced Wheat stone bridge $\frac{100}{200} = \frac{100 + R}{400}$

$$\Rightarrow \frac{100 + R}{R} = 5 \Rightarrow 100 + R = 5R \Rightarrow R = \frac{100}{4} = 25\Omega$$

Example:

A 100 volt voltmeter whose resistance is $20 \text{ k}\Omega$ is connected in series to a very high resistance R . When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R ?

Solution:

When voltmeter connected in 110 volt line, Current through the voltmeter $I = \frac{110}{(20 \times 10^3 + R)}$

The potential difference across the voltmeter $V = IR_V$

$$\Rightarrow 5 = \frac{110 \times 20 \times 10^3}{(20 \times 10^3 + R)}$$

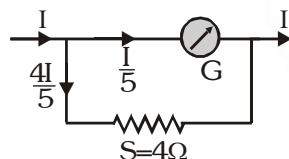
$$\Rightarrow 20 \times 10^3 + R = 440 \times 10^3 \Rightarrow R = 420 \times 10^3 \Omega$$



Example:

When a shunt of 4Ω is attached to a galvanometer, the deflection reduces to $1/5^{\text{th}}$. If an additional shunt of 2Ω is attached what will be the deflection?

Solution:



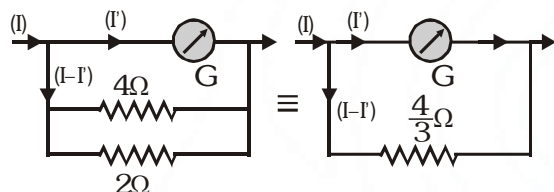
Initial condition: When shunt of 4Ω used

$$\frac{I}{5} \times G = \frac{4}{5} I \times 4 \Rightarrow G = 16\Omega$$

When additional shunt of 2Ω used $I' \times 16 = (I - I') \frac{4}{3}$

$$\Rightarrow I' = \frac{I}{13}$$

\therefore it will reduce to $\frac{I}{13}$ of the initial deflection



Example:

A galvanometer having 30 divisions has current sensitivity of $20\mu\text{A}/\text{division}$. It has a resistance of 25Ω .

- How will you convert it into an ammeter measuring upto 1 ampere.
- How will you convert this ammeter into a voltmeter upto 1 volt.

Solution:

The current required for full scale deflection $I_g = 20\mu\text{A} \times 30 = 600\mu\text{A} = 6 \times 10^{-4}\text{A}$

- To convert it into ammeter, a shunt is required in parallel with it shunt resistance

$$R'_s = \frac{I_g R_g}{(I - I_g)} = \left(\frac{6 \times 10^{-4}}{1 - 6 \times 10^{-4}} \right) 25 = 0.015\Omega$$

- To convert galvanometer into voltmeter, a high resistance in series with it is required series

$$\text{resistance } R = \frac{V}{i_g} - R_g = \frac{1}{6 \times 10^{-4}} - 25 = 1666.67 - 25 = 1641.67\Omega$$

Potentiometer

• **Necessity of Potentiometer**

Practically voltmeter has a finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement it draws no current from the circuit. It means its effective resistance is infinite.

• **Working Principle of Potentiometer**

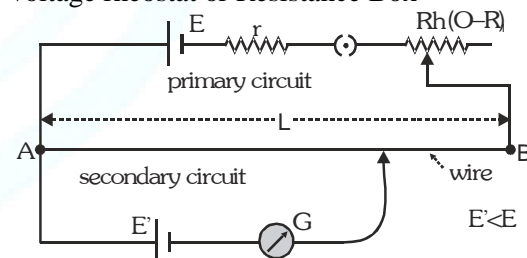
Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.

• **Potentiometer Wire**

Made up of alloys of magnin, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature coefficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

Circuits of Potentiometer

- **Primary circuit** contains constant source of voltage rheostat or Resistance Box



- **Secondary, Unknown or Galvanometer Circuit**

Let ρ = Resistance per unit length of potentiometer wire

- **Potential Gradient (x) (V/m)**

The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}} =$$

$$I \left(\frac{R}{L} \right)$$

The potential gradient depends only on primary circuit and is independent of secondary circuit.



• Applications of Potentiometer

To measure potential difference across a resistance	Connect in series with the given resistance.
To find out emf of a cell	To find out current in a given circuit
Comparison of two emfs E_1/E_2	Calibration of an ammeter or to have a check on reading of (A)
To find out internal resistance of a primary cell	Calibration of a voltmeter or to have a check on reading of (V)
Comparison of two resistance.	To find out thermocouple emf (e_1)
To find out an unknown resistance which is	(mV or mV)

Difference between potentiometer and voltmeter

Potentiometer	Voltmeter
➤ It measures the unknown emf very accurately	It measures the unknown emf approximately.
➤ While measuring emf it does not draw any current from the driving source of known emf	While measuring emf it draws some current from the source of emf
➤ While measuring unknown potential difference the resistance of potentiometer becomes infinite	While measuring unknown potential difference the resistance of voltmeter is high but finite.
➤ It is based on zero deflection method	It is based on deflection method.
➤ It has a high sensitivity.	Its sensitivity is low.
➤ It is used for various applications like measurement of internal resistance of cell, calibration of ammeter and voltmeter, measurement of thermal emf, comparison of emf's etc.	It is only used to measure emf or unknown potential difference.

Example:

There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that first time help each other, and second time they oppose each other. They are balanced on the potentiometer wire at 120 cm and 60 cm length respectively. Compare the electromotive force of the cells.

Solution:

Suppose the potential gradient along the potentiometer wire = x and the emf's of the two cells are E_1 and E_2 .

When the cells help each other, the resultant emf = $(E_1 + E_2)$

$$E_1 + E_2 = x \times 120 \text{ cm} \quad \dots(i)$$

When the cells oppose each other, the resultant emf = $(E_1 - E_2)$

$$E_1 - E_2 = x \times 60 \text{ cm} \quad \dots(ii)$$

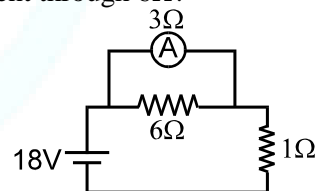
$$\text{From equation (i) and (ii)} \quad \frac{E_1 + E_2}{E_1 - E_2} = \frac{120 \text{ cm}}{60 \text{ cm}} = \frac{2}{1}$$

$$\Rightarrow E_1 + E_2 = 2(E_1 - E_2) \Rightarrow 3E_2 = E_1 \Rightarrow \frac{E_1}{E_2} = \frac{3}{1}$$

FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm?

Q.2 Find the reading of ammeter? Is this the current through 6Ω ?



Q.3 A galvanometer has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

Q.4 In a meter bridge experiment, the value of unknown resistance is 2Ω . To get the balancing point at 40cm distance from the same end, the resistance in the resistance box will be:

- (A) 0.5Ω (B) 3Ω
(C) 20Ω (D) 80Ω

Q.5 In a post office box if the position of the cell and the galvanometer are interchanged, then the:

- (A) Null point will not change
(B) Null point will change
(C) Post office box will not work
(D) Nothing can be said.



Heating Effect of Current

Cause of Heating

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

Joules's Law of Heating

When a current I is made to flow through a passive or ohmic resistance R for time t , heat Q is produced such that

$$Q = I^2 R t = P \times t = V I t = \frac{V^2}{R} t$$

Heat produced in conductor does not depend upon the direction of current.

- **SI unit:** joule; **Practical Units:** 1 kilowatt hour (kWh)
 $1 \text{ kWh} = 3.6 \times 10^6 \text{ joule} = 1 \text{ unit}$ 1 BTU (British Thermal Unit) = 1055 J

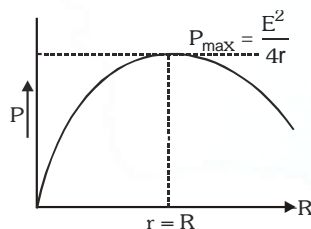
- **Power:** $P = V I = \frac{V^2}{R} = I^2 R$ • **SI unit :**

Watt

The watt-hour meter placed on the premises of every consumer records the electrical energy consumed.

Power Transferred to Load by Cell

$$P = I^2 R = \frac{E^2 R}{(r + R)^2} \Rightarrow P = P_{\max} \text{ if } \frac{dP}{dR} = 0 \Rightarrow r = R$$

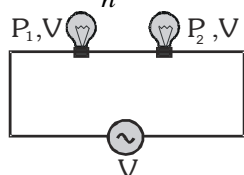


Power transferred by cell to load is maximum when $r = R$ and $P_{\max} = \frac{E^2}{4r} = \frac{E^2}{4R}$

Series Combination of Resistors (Bulbs)

Total power consumed $P_{\text{total}} = \frac{P_1 P_2}{P_1 + P_2}$. If n bulbs

are identical $P_{\text{total}} = \frac{P}{n}$



In series combination of bulbs Brightness \propto

$$\text{Power consumed by bulb} \propto V \propto R \propto \frac{1}{P_{\text{rated}}}$$

Bulb of lesser wattage will shine more. For same current $P = I^2 R$ $P \propto R$ $R \uparrow \Rightarrow P \uparrow$

Parallel Combination of Resistors (Bulbs)

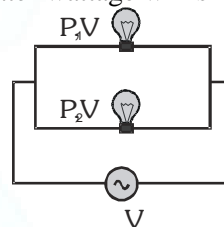
Total power consumed $P_{\text{total}} = P_1 + P_2$

If n bulbs are identical $P_{\text{total}} = nP$

In parallel combination of bulbs

$$\text{Brightness} \propto \text{Power consumed by bulb} \propto I \propto \frac{1}{R}$$

Bulb of greater wattage will shine more.

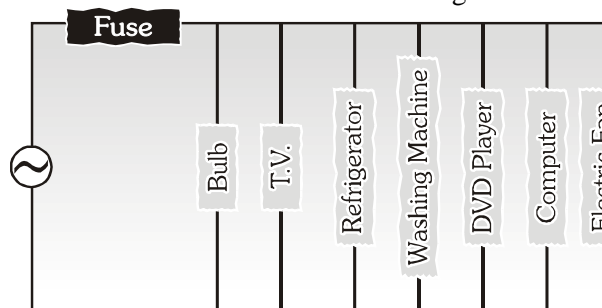


For same V more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

- Two identical heater coils gives total heat H_s when connected in series and H_p when connected in parallel than $\frac{H_p}{H_s} = 4$ [In this, it is assumed that supply voltage is same]
- If a heater boils m kg water in time T_1 and another heater boils the same water in T_2 , then both connected in series will boil the same water in time $T_s = T_1 + T_2$ and in parallel $T_p = \frac{T_1 T_2}{T_1 + T_2}$
 [Use time taken \propto Resistance]
- Instruments based on heating effect of current, works on both A.C. and D.C. Equal value of A.C. (RMS) and D.C. produces, equal heating effect. That why brightness of bulb is same whether it is operated by A.C. or same value D.C.

Fuse Wire

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.





- The fuse wire should have high resistance per unit length and low melting point.
- However, the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit
- A fuse wire is made of alloys of lead (Pb) and tin (Sn).
- Length of fuse wire is immaterial.
- The material of the filament of a heater should have high resistivity and high melting point.
- The temperature of the wire increases to such a value at which, the heat produced per second equals heat lost per second due to radiation from

$$\text{the surface of wire } I^2 \left(\frac{\rho \ell}{\pi r^2} \right) = H \times 2\pi r \ell \quad I^2 \propto r^3$$

H = heat lost per second per unit area due to radiation.

Example:

An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate power consumed by (i) heater (ii) bulb.

Solution:

$$P = \frac{V^2}{R} \text{ or } R = \frac{V^2}{P}, \text{ For heater. Resistance } R_h = \frac{(200)^2}{500} = 96.8 \, \Omega,$$

$$\text{For bulb resistance } R_L = \frac{(220)^2}{100} = 484 \, \Omega$$

$$\text{Current in the circuit when both are connected in series } I = \frac{V}{R_L + R_h} = \frac{220}{484 + 96.8} = 0.38 \, \text{A}$$

$$(i) \text{ Power consumed by heater } = I^2 R_h = (0.38)^2 \times 96.8 = 13.98 \, \text{W}$$

$$(ii) \text{ Power consumed by bulb } = I^2 R_L = (0.38)^2 \times 484 = 69.89 \, \text{W}$$

Example:

A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Solution:

$$\therefore P = \frac{V^2}{R} \therefore R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400 \, \Omega$$

$$\text{Resistance of half piece} = \frac{400}{2} = 200 \, \Omega$$

$$\text{Resistance of pieces connected in parallel} = \frac{200}{2} = 100 \, \Omega$$

$$\text{Energy liberated/second } P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400$$

W

Example:

The power of a heater is 500W at 800°C. What will be its power at 200°C. If $\alpha = 4 \times 10^{-4}$ per °C?

Solution:

$$P = \frac{V^2}{R} \therefore \frac{P_{200}}{P_{800}} = \frac{R_{800}}{R_{200}} = \frac{R_0(1 + 4 \times 10^{-4} \times 800)}{R_0(1 + 4 \times 10^{-4} \times 200)}$$

$$\Rightarrow P_{200} = \frac{500 \times 1.32}{1.08} = 611 \, \text{W}$$

Example:

When a battery sends current through a resistance R_1 for time t , the heat produced in the resistor is Q . When the same battery sends current through another resistance R_2 for time t , the heat produced in R_2 is again Q . Determine the internal resistance of battery.

Solution:

$$\left[\frac{E}{R_1 + r} \right]^2 R_1 = \left[\frac{E}{R_2 + r} \right]^2 R_2 \Rightarrow r = \sqrt{R_1 R_2}$$

Example:

A fuse with a circular cross-sectional radius of 0.15 mm blows at 15A. What is the radius of a fuse, made of the same material which will blow at 120 A?

Solution:

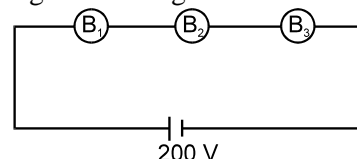
For fuse wire $I \propto r^{3/2}$

So

$$\frac{r_2}{r_1} = \left(\frac{I_2}{I_1} \right)^{2/3} = \left(\frac{120}{15} \right)^{2/3} = (8)^{2/3} = 4 \Rightarrow r_2 = 4r_1 = 0.60 \, \text{mm}$$

FUNDAMENTAL UNLOCKED- (FU#4) :

- Q.1** In the figure shown B_1 , B_2 and B_3 are three bulbs rated as (200V, 50 W), (200V, 100W) and (200V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



- Q.2** Three bulbs, each having a resistance of 180 Ω , are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when (a) all the bulbs are switched on, (b) two of the bulbs are switched on and (c) only one bulb is switched on.




ANSWERS KEY
Fundamental Unlocked- (Fu#1) :

1. $n = 6.023 \times 10^{28} \text{ m}^{-3}$
2. $31 \text{ C}, \frac{31}{3} \text{ A}$
3. (a) $Q = 1200 \text{ C}$ (b) $n = 75 \times 10^{20}$
4. (a) $n = \frac{2}{1.6} \times 10^{17} = 1.25 \times 10^{17}$ (b) $\frac{1}{2\pi} \times 10^6 \text{ A/m}^2$.

Fundamental Unlocked- (Fu#2) :

1. 3 A
2. 15 A
3. $R_{AB} = \frac{\rho c}{ab}, R_{CD} = \frac{\rho b}{ac}, R_{EF} = \frac{\rho a}{bc}$
4. $4R$
5. 1 A from D to B
6. $\frac{8}{3} \text{ A}$

Fundamental Unlocked- (Fu#3) :

1. $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$
2. 1 A from F to E, 10.5 A from B to E, 11.5 A from E to D
3. Potential at A = $\frac{-5}{3} \text{ V}$
4. $i = 2.5 \text{ A}$
5. Current in wire AF = 10 A from A to F
Current in wire EB = 15 A from B to E
Current in wire DE = 25 A from E to D

Fundamental Unlocked- (Fu#4) :

1. (a) $E = 10 \text{ V}$ each
- (b) (A) act as a source and (B) act as load
- (c) $V_A = 9 \text{ V}, V_B = 11 \text{ V}$
- (d) $P_A = 9 \text{ W}, P_B = 11 \text{ W}$
- (e) Heat rate = 1 W each
- (f) 10 W each
- (g) $9 \text{ V}, 11 \text{ V}$
- (h) $-9 \text{ W}, 11 \text{ W}$

Fundamental Unlocked- (Fu#5) :

1. 11Ω .
2. 4 A , No, it is not the current through the 6Ω resistor.
3. $G(n - 1) \text{ ohm}$
4. (B)
5. (A)

Fundamental Unlocked- (Fu#6) :

1. 0.07 A , Bulb B_3 will give more light.
2. (a) 1 A (b) $2/3 \text{ A}$ (c) $1/3 \text{ A}$

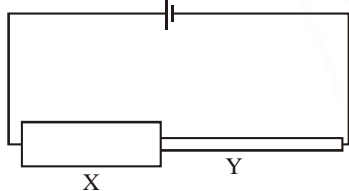




Some Worked Out Examples

Example 1:

Figure shows a thick copper rod X and a thin copper wire Y, joined in series. They carry a current which is sufficient to make Y much hotter than X. Which one of the following is correct?



Density of Conduction Electrons

Mean Time Between Collisions of The Electrons

- | | |
|----------------------|------------------|
| (A) Same in X and Y | Less in X than Y |
| (B) Same in X and Y | Same in X and Y |
| (C) Same in X and Y | More in X than Y |
| (D) More in X than Y | Less in X than Y |

Answer: (A)

Solution:

The number density n of conduction electrons in the copper is a characteristic of the copper and is about 10^{29} at room temperature for both the copper rod X and the thin copper wire Y.

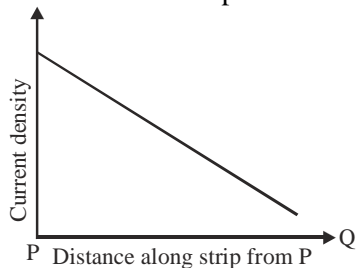
Both X and Y carry the same current I since they are joined in series.

From $I = neAv_d$

We may conclude that rod X has a lower drift velocity of electrons compared to wire Y since rod X has larger cross-sectional area. This is so because the electrons in X collide more often with one another and with the copper ions when drifting towards the positive end. Thus, the mean time between collisions of the electrons is less in X and then in Y.

Example 2:

An electric current flow along an insulated strip PQ of a metallic conductor. The current density in the strip varies as shown in the graph. Which one of the following statements could explain this variation?



- (A) The strip is narrower at P than at Q
 (B) The strip is narrower at Q than at P
 (C) The potential gradient along the strip is uniform
 (D) The resistance per unit length of the strip is constant

Answer: (A)

Solution:

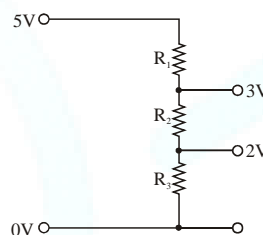
The current density at P is higher than at Q. For the same current flowing through the metallic conductor PQ, the cross-sectional area at P is narrower than at

Q. The resistance per unit length r is given by $r = \frac{\rho}{A}$

where ρ is the resistivity and A is the cross-sectional area of the conductor PQ. Thus, r is inversely proportional to the cross-sectional area A of the conductor.

Example 3:

A potential divider is used to give outputs of 2V and 3V from a 5V source, as shown in figure. Which combination of resistances, R_1 , R_2 and R_3 gives the correct voltages?



R_1	R_2	R_3
(A) 1 k Ω	1 k Ω	2 k Ω
(B) 2 k Ω	1 k Ω	2 k Ω
(C) 3 k Ω	2 k Ω	2 k Ω
(D) 3 k Ω	2 k Ω	3 k Ω

Answer: (B)

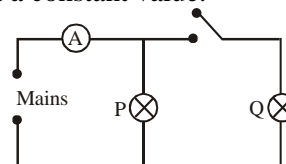
Solution:

For resistors in series connection, current (I) is the same through the resistors. In other words, ratio of the voltage drop across each resistor with its resistance is the same.

That is $I = \frac{5-3}{R_1} = \frac{3-2}{R_2} = \frac{2}{R_3}$ i.e., $R_1 : R_2 : R_3 = 2 : 1 : 2$.

Example 4:

How will the reading in the ammeter A of figure be affected if another identical bulb Q is connected in parallel to P as shown. The voltage in the mains is maintained at a constant value.



- (A) The reading will be reduced to one-half
 (B) The reading will not be affected
 (C) The reading will be doubled of the previous one
 (D) The reading will be increased four-fold



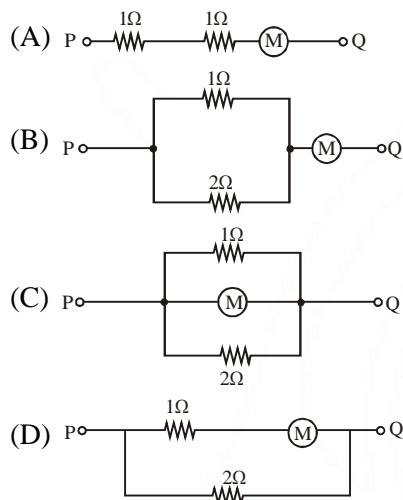
Answer: (C)

Solution:

Resistance is halved. Current is doubled.

Example 5:

In which one of the following arrangements of resistors does the ammeter M, which has a resistance of 2Ω , give the largest reading when the same potential difference is applied between points P and Q?



Answer: (C)

Solution:

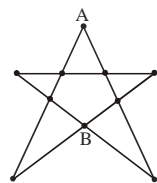
Let $V_{PQ} = E$

For (A) : $I = \frac{E}{4}$ For (B) : $I = \frac{E}{\frac{2}{3} + 2} = \frac{E}{\frac{8}{3}}$ For (C) :

$I = \frac{E}{2}$ For (D) : $I = \frac{E}{3}$

Example 6:

The resistance of all the wires between any two adjacent dots is R . The equivalent resistance between A and B as shown in figure is



(A) $\frac{7}{3} R$

(B) $\frac{7}{6} R$

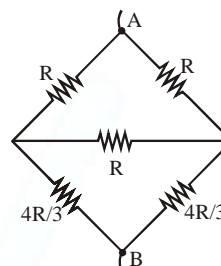
(C) $\frac{14}{8} R$

(D) None of these

Answer: (B)

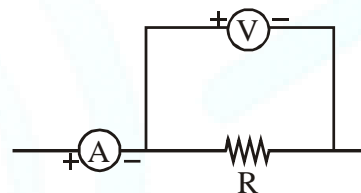
Solution:

Given circuit can be reduce to $R_{AB} = \frac{7}{6} R$



Example 7:

A candidate connects a moving coil voltmeter V, a moving coil ammeter A and a resistance R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4A, R is:



(A) equal to 5Ω

(B) greater than 5Ω

(C) less than 5Ω

(D) greater or less than 5Ω depending upon its material

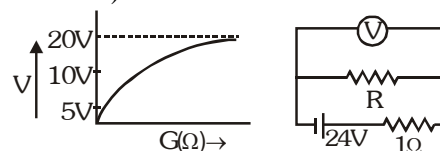
Answer: (B)

Solution:

Let a current of x ampere passes through the voltmeter, then $(4-x)$ ampere passes through the resistance R. Therefore, voltmeter reading $20 = (40 - x) R \therefore R = \frac{20}{4-x}$, i.e., $R > 5\Omega$

Example 8:

A cell of internal resistance 1Ω is connected across a resistor. A voltmeter having variable resistance is used to measure potential difference across resistor. The plot of voltmeter reading V against G is shown. What is value of external resistor R? (G = Resistance of galvanometer)



(A) 5Ω

(B) 4Ω

(C) 3Ω

(D) can't be determined



Answer: (A)

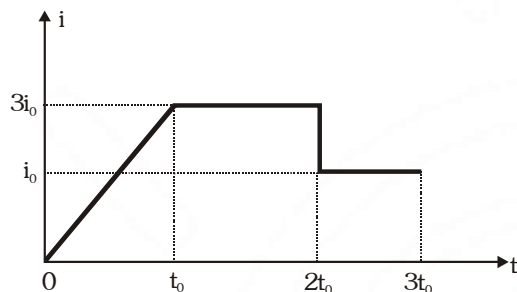
Solution:

When galvanometer resistance tends to infinity $G \rightarrow \infty$,

Potential difference across R is 20V $\Rightarrow 20 = 24 - i \times 1 \Rightarrow i = 4$ A also $20 = 4 \times R \Rightarrow R = 5\Omega$.

Example 9:

A time varying current i is passed through a resistance R as shown in figure. The total heat generated in the resistance is:



- (A) $11i_0^2 R t_0$ (B) $13i_0^2 R t_0$
(C) $17i_0^2 R t_0$ (D) $15i_0^2 R t_0$

Answer: (B)

Solution:

Total heat produced =

$$\int_0^{t_0} \left(\frac{3i_0}{t_0} t \right)^2 R dt + (3i_0)^2 R (2t_0 - t_0) + i_0^2 R (3t_0 - 2t_0) \\ = 3i_0^2 R t_0 + 9i_0^2 R t_0 + i_0^2 R t_0 = 13i_0^2 R t_0$$

Example 10:

The circuit shown in figure, contains a battery, a rheostat and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?

Lamp P

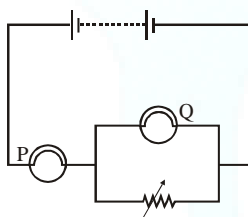
Lamp Q

- (A) Less brighter Brighter
(B) Less brighter Less brighter
(C) Brighter Less brighter
(D) No change Brighter

Answer: (A)

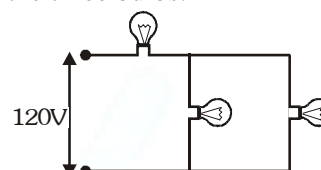
Solution:

Consider two extreme cases. (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P. (ii) When the resistance of the rheostat is very large, almost no current flows through it. So, the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q.



Example 11:

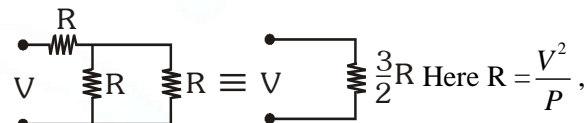
Three 60W, 120V light bulbs are connected across a 120 V power source. If resistance of each bulb does not change with current then find out total power delivered to the three bulbs.



- (A) 180 W (B) 20 W (C) 40 W (D) 60 W

Answer: (C)

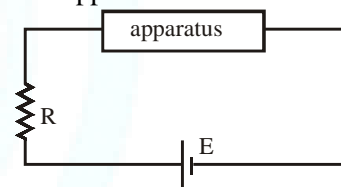
Solution:



$$\text{Total power supplied} = \frac{V^2}{3/2R} = \left(\frac{2}{3} \right) \left(\frac{V^2}{R} \right) = \frac{2}{3} \times 60 = 40 \text{ W}$$

Example 12:

An apparatus is connected to an ideal battery as shown in figure. For what value of current, power delivered to the apparatus will be maximum?



- (A) $\frac{E}{R}$ (B) $\frac{E}{2R}$
(C) $\frac{E}{4R}$ (D) information insufficient

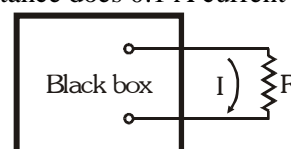
Answer: (B)

Solution:

For maximum power $R_{\text{ext}} = R_{\text{int}} = R \therefore \text{current} = \frac{E}{2R}$

Example 13:

In the given black box unknown emf sources and unknown resistances are connected by an unknown method such that (i) when terminals of 10 ohm resistances are connected to box then 1 ampere current flows and (ii) when 18 ohm resistances are connected then 0.6 A current flows then for what value of resistance does 0.1 A current flow?



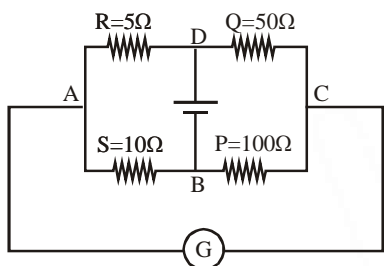
- (A) 118 Ω (B) 27 Ω (C) 18 Ω (D) 58 Ω

Answer: (A)



Example 14:

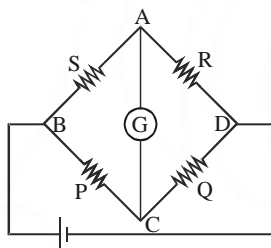
Figure shows a balanced Wheatstone's bridge



- (A) If P is slightly increased, the current in the galvanometer flows from C to A
 (B) If P is slightly increased, the current in the galvanometer flows from A to C
 (C) If Q is slightly increased, the current in the galvanometer flows from C to A
 (D) If Q is slightly increased, the current in the galvanometer flows from A to C

Answer: (BC)

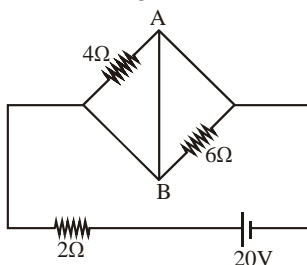
Solution:



If P is slightly increased, potential of C will decrease. Hence current will flow from A to C. If Q is slightly increased, potential of C will increase, hence current will flow from C to A.

Example 15:

In the circuit shown in figure.



- (A) Power supplied by the battery is 200W
 (B) Current flowing in the circuit is 5A
 (C) Potential difference across the 4Ω resistance is equal to the potential difference across the 6Ω resistance
 (D) Current in wire AB is zero

Answer: (AC)

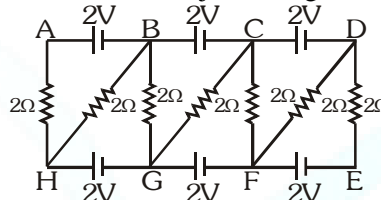
Solution:

4Ω and 6Ω resistor are short-circuited. Therefore, no current will flow through these resistances. Current passing through the battery is $I = (20/2) = 10A$. This is also the current passing in wire AB from B to A. Power supplied by the battery $P = EI = (20)(10) = 200W$

Potential difference across the 4Ω resistance = potential difference across the 6Ω resistance.

Example 16 to 17:

In given circuit, 7 resistors of resistance 2Ω each and 6 batteries of 2V each, are joined together.



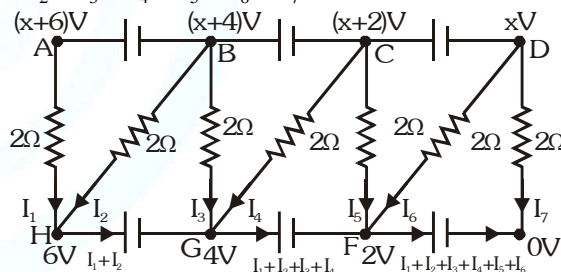
16. The potential difference $V_D - V_E$ is-

- (A) $\frac{5}{6}V$ (B) $\frac{6}{5}V$ (C) $-\frac{5}{6}V$ (D) $-\frac{22}{9}V$

Answer: (B)

Solution:

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = I_7$$



$$\Rightarrow \frac{x}{2} + \frac{x-2}{2} + \frac{x}{2} + \frac{x-2}{2} + \frac{x}{2} + \frac{x-2}{2} = \frac{x}{2}$$

17. The current through branch BG is-

- (A) 1A (B) 0.2A (C) 0.4A (D) 0.6A

Answer: (D)

Solution:

$$\text{Current through branch BG : } I_3 = \frac{x}{2} = \frac{6}{10} = 0.6A$$

18. The current through battery between A & B is-

- (A) 0.6 A (B) 0.8 A (C) 0.4 A (D) 1 A

Answer: (A)

Solution:

$$\text{Current through branch AB} = I_1 = \frac{x}{2} = 0.6 A$$



Example 19 to 21:

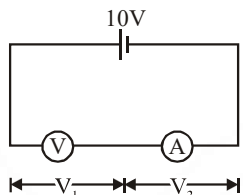
An ammeter and a voltmeter are connected in series to a battery with an emf of 10V. When a certain resistance is connected in parallel with the voltmeter, the reading of the voltmeter decreases three times, whereas the reading of the ammeter increases the two times.

19. Find the voltmeter reading after the connection of the resistance.

- (A) 1V (B) 2V (C) 3V (D) 4V

Answer: (B)

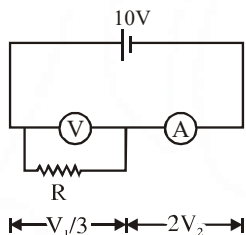
Solution:



Initially $V_1 + V_2 = 10V$... (i)

Finally, $\frac{V_1}{3} + 2V_2 = 10V$... (ii)

From equation (i) & (ii) We get $V_1 = 6$ volt, $V_2 = 4$ volt



\therefore Final reading = $\frac{V_1}{3} = 2$ volt

20. If resistance of the ammeter is 2Ω , then resistance of the voltmeter is:

- (A) 1Ω (B) 2Ω (C) 3Ω (D) 4Ω

Answer: (C)

Solution:

$\frac{V_2}{V_1} = \frac{R_A}{R_V} = \frac{4}{6}$ & $R_A = 2\Omega \Rightarrow R_V = 3\Omega$

21. If resistance of the ammeter is 2Ω , then resistance of the resistor which is added in parallel to the voltmeter is:

- (A) $\frac{3}{5}\Omega$ (B) $\frac{2}{7}\Omega$
(C) $\frac{3}{7}\Omega$ (D) None of these

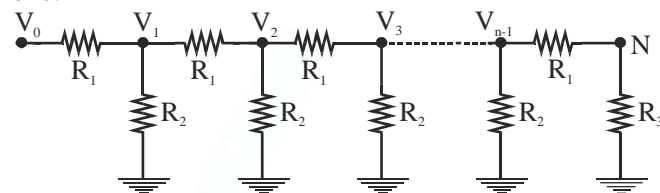
Answer: (A)

Solution:

$\frac{R_V R}{R_V + R} = \frac{V_1}{\frac{3}{2}} = \frac{1}{4} \Rightarrow R = \frac{3}{5}\Omega$

Example 22 to 24:

A network of resistance is constructed with R_1 and R_2 as shown in figure. The potential at the points 1, 2, 3... N are $V_1, V_2, V_3, \dots, V_N$, respectively, each having a potential k times smaller than the previous one.

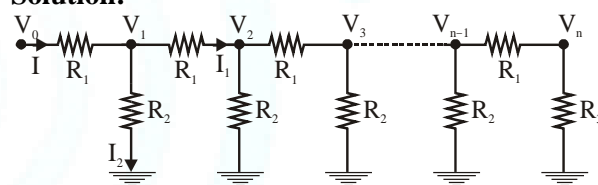


22. The ratio $\frac{R_1}{R_2}$ is

- (A) $k^2 - \frac{1}{k}$ (B) $\frac{k}{k-1}$
(C) $k - \frac{1}{k^2}$ (D) $\frac{(k-1)^2}{k}$

Answer: (D)

Solution:



Given $V_1 = \frac{V_0}{k}, V_2 = \frac{V_1}{k}, V_3 = \frac{V_2}{k}; I = I_1 + I_2$

$\frac{V_0 - V_1}{R_1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - 0}{R_2} \Rightarrow \frac{V_0 - V_1}{R_1} = \frac{V_0/k - V_0/k^2}{R_1} + \frac{V_0/k}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{(k-1)^2}{k}$

23. The ratio $\frac{R_2}{R_3}$ is

- (A) $\frac{(k-1)^2}{k}$ (B) $k^2 - \frac{1}{k}$
(C) $\frac{k}{k-1}$ (D) $k - \frac{1}{k^2}$

Answer: (C)

Solution:

Current in R_1 and R_3 will be same:

$\frac{V_{n-1} - V_n}{R_1} = \frac{V_n}{R_3} \Rightarrow \frac{V_{n-1} - \frac{V_{n-1}}{k}}{R_1} = \frac{V_{n-1}}{kR_3} \Rightarrow R_1 = R_3(k-1)$

Put the value of R_1 in (i) $\frac{R_2}{R_3} = \frac{k}{k-1}$



24. The current that passes through the resistance R_2 nearest to the V_0 is

- (A) $\frac{(k-1)^2 V_0}{k R_3}$ (B) $\frac{(k+1)^2 V_0}{k R_3}$
 (C) $\left(k + \frac{1}{k^2}\right) \frac{V_0}{R_3}$ (D) $\left(k - \frac{1}{k^2}\right) \frac{V_0}{R_3}$

Answer: (D)

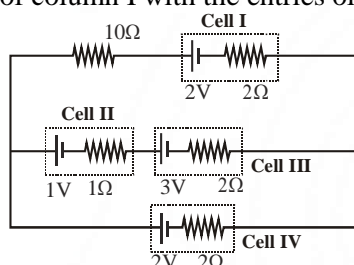
Solution:

Current in R_2 nearest to V_0 :

$$I_2 = \frac{V_1}{R_2} = \frac{V_0/k}{R_3 \left(\frac{k}{k-1} \right)} = \left(\frac{k-1}{k^2} \right) \frac{V_0}{R_3}$$

Example 25:

For the circuit shown in figure, 4 cells are arranged. In column I, the cell number is given while in column II, some statements related to cells are given. Match the entries of column I with the entries of column II.



Column I

- (A) Cell I
 (B) Cell II
 (C) Cell III
 (D) Cell IV

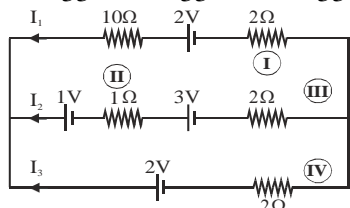
Column II

- (P) Chemical energy of cell is decreasing.
 (Q) Chemical energy of cell is increasing.
 (R) Work done by cell is positive.
 (S) Thermal energy developed in cell is positive.
 (T) None of these

Answer: (A) → QS; (B) → PRS; (C) → PRS; (D) → QS

Solution:

We have, $I_1 = -\frac{2}{33} A$, $I_2 = \frac{14}{33} A$, $I_3 = -\frac{12}{33} A$



In each cell thermal energy will be dissipated due to internal resistance whether the chemical energy of the cell is increasing or decreasing.

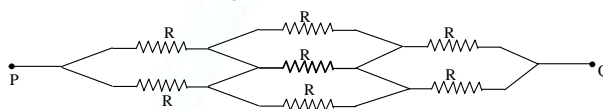
- (i) Cell I is getting charged, hence its chemical energy increases.

- (ii) Cell II and III both are getting discharged; hence their chemical energy is decreasing. So, work done by both of them is positive.

- (iv) Cell IV is getting charged, hence its chemical energy increases.

Example 26:

Equivalent resistance for the given figure between P and Q is $NR/3$. Find value of N.



Answer: 4

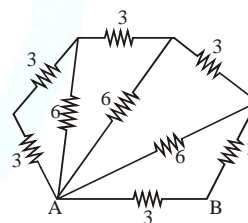
Solution:



$$R_{net} = \frac{4R}{3} \Rightarrow N = 4$$

Example 27:

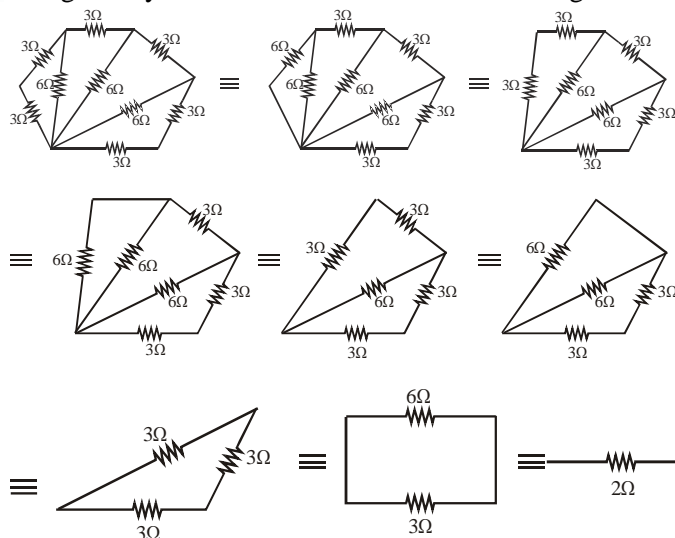
All resistances in the diagram below are in ohms. Find the effective resistance between the point A and B (in Ω).



Answer: 2

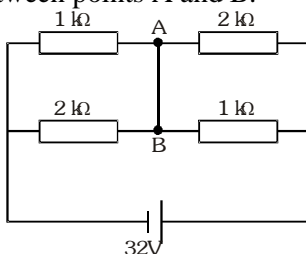
Solution:

The given system can be reduced as shown in figure.



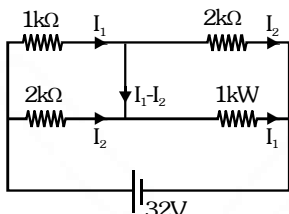
Example 28:

In the given circuit, find the current (in mA) in the wire between points A and B.



Answer: 8

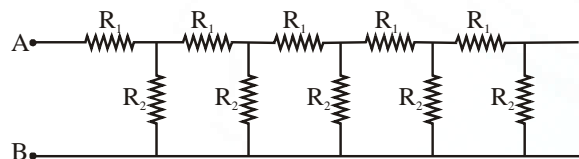
Solution:



$$I_1 = 16 \text{ mA}; I_2 = 8 \text{ mA} \Rightarrow I_1 - I_2 = 8 \text{ mA}$$

Example 29:

Consider an infinite ladder of network shown in figure. A voltage is applied between points A and B. If the voltage is halved after each section, find the ratio of $\frac{R_2}{R_1}$.



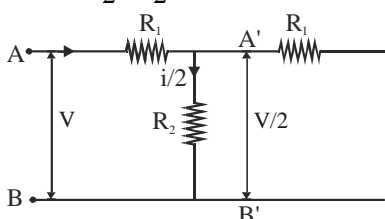
Answer: 2

Solution:

Voltage across AB = V, Voltage across A'B' = $\frac{V}{2}$ i.e.,

$$\text{Voltage across } R_2 = \frac{V}{2}$$

Now from Kirchhoff's law it is obvious that voltage across $R_1 = V - \frac{V}{2} = \frac{V}{2}$

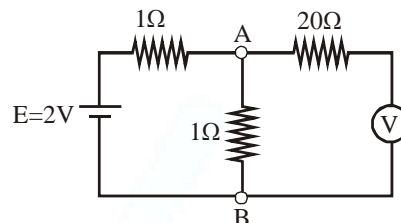


When the voltage is halved, current is also halved, i.e., current in R_2 is half of that in R_1 .

$$\text{So } R_1 i = R_2 \frac{i}{2} \Rightarrow \frac{R_2}{R_1} = 2$$

Example 30:

In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter (in volt)



Answer: 1

Solution:

The electric current through ideal voltmeter is zero.

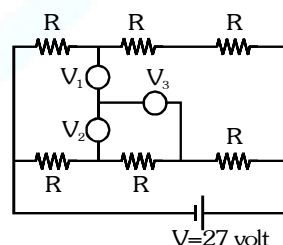
According to loop rule, $E - 1 \times I - 1 \times I = 0$

$$\Rightarrow I = \frac{E}{2} = \frac{2}{2} = 1 \text{ A}$$

$$\text{Reading of the voltmeter} = V_A - V_B = [1 \times 1] = [1 \times 1] = 1 \text{ V}$$

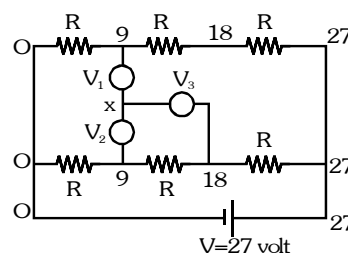
Example 31:

In the circuit shown below, all the voltmeter identical and have very high resistance. Each resistor has the same resistance. The voltage of the ideal battery shown is 27 V. Find the reading of voltmeter V_3 (in volts).



Answer: 6

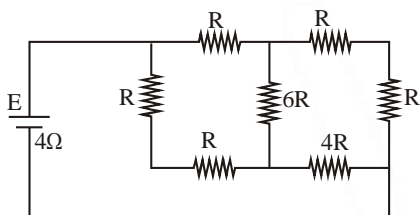
Solution:



$$\frac{x-9}{R_v} + \frac{x-9}{R_v} + \frac{x-18}{R_v} = 0 \Rightarrow x = 12 \therefore V_3 = 6 \text{ volt}$$

Example 32:

A battery of internal resistance 4Ω is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of R in Ω should be



Answer: 2

Solution:

For maximum power, external resistance is equal to internal resistance.

Therefore, $2R = 4$ or $R = 2$

Example 33:

How much time heater will take to increase the temperature of 100 g water by 50°C if resistance of heating coil is 484Ω and supply voltage is 220V a.c.

Solution:

Heat given by heater = heat taken by water $\Rightarrow \frac{V^2}{R} t =$

$$ms \Delta\theta \Rightarrow \frac{220 \times 220}{484}$$

$$t = (100 \times 10^{-3}) (4.2 \times 10^3) (50) \Rightarrow t = 210 \text{ s}$$

Example 34:

A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t . A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t , the value of N is

Answer: 6

Solution:

Let R be the resistance of wire. Let R' be the resistance of another wire so $R' = 2R$ (\because Length is twice)

$$\text{In case (i) Energy released in } t \text{ second} = \frac{(3V)^2}{R} \times t$$

$$\text{In case (ii) } \therefore \text{ Energy released in } t\text{-seconds} = \frac{(N^2 V^2)}{2R} \times t$$

$$\text{But } Q = mc\Delta T \therefore mc\Delta T = \frac{(9V^2)}{R} \times t \quad \dots(i)$$

$$\text{Applying } Q' = m'c\Delta T \Rightarrow 2mc\Delta T = \frac{(N^2 V^2)}{2R} \times t \quad \dots(ii)$$

Dividing equation (ii) by equation (i)

$$\frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times t/R}{N^2 V^2 \times t/2R} \therefore \frac{1}{2} = \frac{9 \times 2}{N^2} \Rightarrow N^2 = 18 \times 2 \therefore N = 6$$

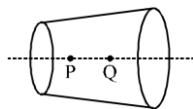


OBJECTIVE EXERCISE – I

Single Correct Type Questions

Microscopic Analysis

1. A wire has a non-uniform cross-section as shown in figure. A steady current flow through it. The drift speed of electrons at points P and Q is v_P and v_Q .



- (A) $v_P = v_Q$ (B) $v_P < v_Q$
(C) $v_P > v_Q$ (D) Data insufficient

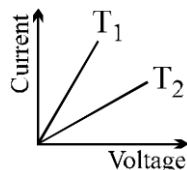
2. Two wires each of radius of cross section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1 : 4, the drift velocity of electrons in the two wires will be in the ratio:

- (A) 1 : 2 (B) 2 : 1 (C) 4 : 1 (D) 1 : 4

3. An insulating pipe of cross-section area 'A' contains an electrolyte which has two types of ions \rightarrow their charges being $-e$ and $+2e$. A potential difference applied between the ends of the pipe result in the drifting of the two types of ions, having drift speed $= v$ ($-ve$ ion) and $v/4$ ($+ve$ ion). Both ions have the same number per unit volume $= n$. The current flowing through the pipe is

- (A) $nev A/2$ (B) $nev A/4$
(C) $5nev A/2$ (D) $3nev A/2$

4. The current in a metallic conductor is plotted against voltage at two different temperatures T_1 and T_2 . Which is correct



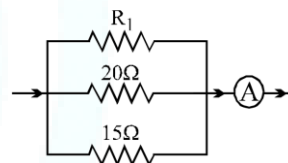
- (A) $T_1 > T_2$ (B) $T_1 < T_2$
(C) $T_1 = T_2$ (D) none

5. A metal rod of length 10 cm and a rectangular cross-section of $1 \text{ cm} \times \frac{1}{2} \text{ cm}$ is connected to a battery across opposite faces. The resistance will be

- (A) maximum when the battery is connected across $1 \text{ cm} \times \frac{1}{2} \text{ cm}$ faces.
(B) maximum when the battery is connected across $10 \text{ cm} \times 1 \text{ cm}$ faces.
(C) maximum when the battery is connected across $10 \text{ cm} \times \frac{1}{2} \text{ cm}$ faces.
(D) same irrespective of the three faces.

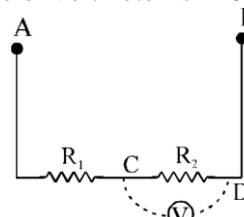
Ohm's law and Circuit Analysis

6. In the given circuit the current flowing through the resistance 20 ohms is 0.3 ampere while the ammeter reads 0.8 ampere . What is the value of R_1 ?



- (A) 30 ohm (B) 40 ohm (C) 50 ohm
(D) 60 ohm

7. Resistances R_1 and R_2 each 60Ω are connected in series as shown in figure. The Potential difference between A and B is kept 120 volt. Then what will be the reading of voltmeter connected between the point C & D if resistance of voltmeter is 120Ω .



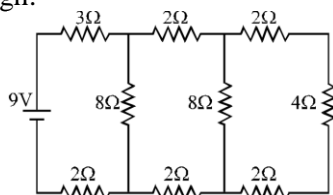
- (A) 48 V (B) 24 V (C) 40 V (D) None

8. A simple circuit contains an ideal battery and a resistance R . If a second resistor is placed in parallel with the first,

- (A) the potential across R will decrease
(B) the current through R will decreased
(C) the current delivered by the battery will increase
(D) the power dissipated by R will increase



9. In the circuit shown in the figure, the current through:



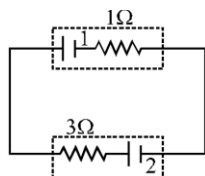
- (A) the 3Ω resistor is 0.50 A
(B) the 3Ω resistor is 0.25 A
(C) 4Ω resistor is 0.50 A
(D) the 4Ω resistor is 0.25 A

10. An energy source will supply a constant current into the load, if its internal resistance is:
(A) equal to the resistance of the load
(B) very large as compared to the load resistance
(C) zero
(D) non-zero but less than the resistance of the load

11. A storage battery is connected to a charger for charging with a voltage of 12.5 Volts. The internal resistance of the storage battery is 1Ω . When the charging current is 0.5 A, the emf of the storage battery is:
(A) 13 Volts (B) 12.5 Volts
(C) 12 Volts (D) 11.5 Volts

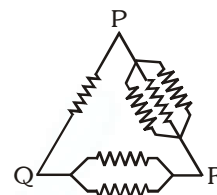
12. When a current of 4 A flows within a battery from its positive to negative terminal, the potential difference across the battery is 12 volts. The potential difference across the battery is 9 volts when a current of 2 A flows within it from its negative to its positive terminal. The internal resistance and the e.m.f. of the battery are:
(A) 0.1Ω , 4V (B) 0.2Ω , 5V
(C) 0.5Ω , 10V (D) 0.7Ω , 10V

13. In the figure shown, battery 1 has emf = 6 V and internal resistance = 1Ω . Battery 2 has emf = 2V and internal resistance = 3Ω . The wires have negligible resistance. What is the potential difference across the terminals of battery 2?



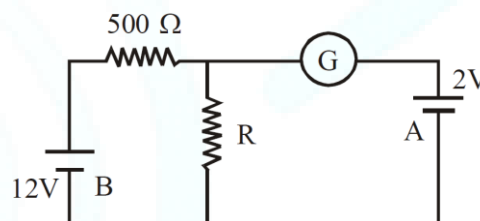
- (A) 4 V (B) 1.5 V (C) 5 V (D) 0.5 V

14. Six equal resistances are connected between points P, Q and R as shown in the figure. Then, the net resistance will be maximum between



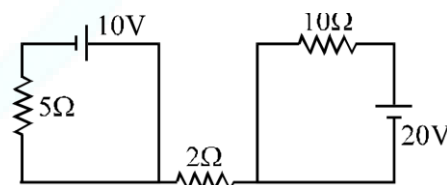
- (A) P and Q (B) Q and R
(C) P and R (D) any two points

15. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be-



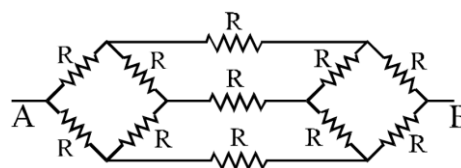
- (A) 200Ω (B) 100Ω (C) 500Ω (D) 1000Ω

16. In the figure shown the current through 2Ω resistor is:



- (A) 2 A (B) 0 A (C) 4 A (D) 6 A

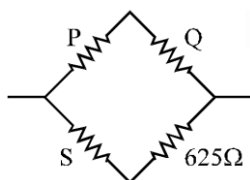
17. The equivalent resistance between the terminal points A and B in the network shown in figure is:



- (A) $\frac{7R}{5}$ (B) $\frac{5R}{6}$ (C) $\frac{7R}{12}$ (D) $\frac{5R}{12}$

18. A Wheatstone's bridge is balanced with a resistance of 625Ω in the third arm, where P,

and S are in the 1st, 2nd and 4th arm respectively. If P and Q are interchanged, the resistance in the third arm has to be increased by 51Ω to secure balance. The unknown resistance in the fourth arm is:



- (A) 625Ω (B) 650Ω (C) 676Ω (D) 600Ω

Electric Power and Joule heating

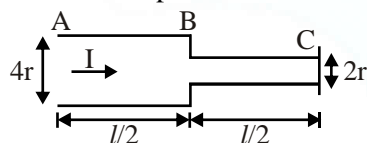
19. Power generated across a uniform wire connected across a supply is H . If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is

- (A) $\frac{H}{n^2}$ (B) n^2H (C) nH (D) $\frac{H}{n}$

20. A rigid container with thermally insulated walls contains a coil of resistance 100Ω , carrying current 1 A . Change in internal energy after 5 min will be

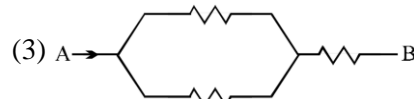
- (A) zero (B) 10 kJ (C) 20 kJ (D) 30 kJ

21. Consider a cylindrical element as shown in the figure. Current flowing through the element is I and resistivity of material of the cylinder is ρ . Choose the correct option out the following.



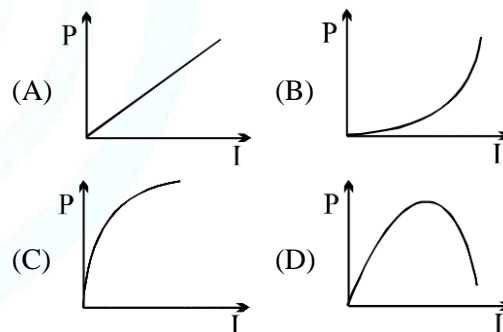
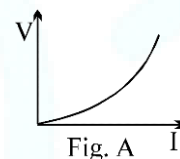
- (A) Power loss in second half is four times the power loss in first half.
 (B) Voltage drop in first half is twice of voltage drop in second half.
 (C) Current density in both halves are equal.
 (D) Electric field in both halves is equal.

22. Arrange the order of power dissipated in the give circuits, if the same current is passing through all circuits and each resistor is ' r '

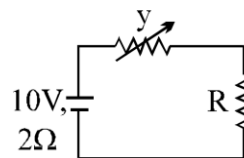


- (A) $P_2 > P_3 > P_4 > P_1$ (B) $P_3 > P_2 > P_4 > P_1$
 (C) $P_4 > P_3 > P_2 > P_1$ (D) $P_1 > P_2 > P_3 > P_4$

23. The variation of current (I) and voltage (V) is as shown in figure A. The variation of power P with current I is best shown by which of the following graph



24. In the figure shown the power generated in y is maximum when $y = 5\Omega$. Then R is:



- (A) 2Ω (B) 6Ω (C) 5Ω (D) 3Ω

25. When electric bulbs of same power, but different marked voltage are connected in series across the power line, their brightness will be:
 (A) proportional to their marked voltage



- (B) inversely proportional to their marked voltage
 (C) proportional to the square of their marked voltage
 (D) inversely proportional to the square of their marked voltage

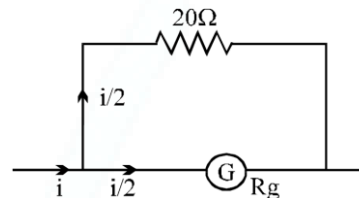
26. Two bulbs rated (25W – 220V) and (100W – 220V) are connected in series to a 440 V line. Which one is likely to fuse?
 (A) 25 W bulb (B) 100 W bulb
 (C) both bulbs (D) none
27. Rate of dissipation of Joule's heat in resistance per unit volume is (symbols have usual meaning)
 (A) σE (B) σj (C) $j E$ (D) None
28. Two bulbs one of 200 volts, 60 watts & the other of 200 volts, 100 watts are connected in series to a 200 volt supply. The power consumed will be:
 (A) 37.5 watt (B) 160 watt
 (C) 62.5 watt (D) 110 watt

Electric instruments

29. A galvanometer has a resistance of 20Ω and reads full-scale when 0.2 V is applied across it. To convert it into a 10 A ammeter, the galvanometer coil should have a
 (A) 0.01Ω resistor connected across it
 (B) 0.02Ω resistor connected across it
 (C) 200Ω resistor connected in series with it
 (D) 2000Ω resistor connected in series with it
30. A galvanometer coil has a resistance 90Ω and full-scale deflection current 10 mA. A 910Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1V, the number of divisions on its scale is
 (A) 90 (B) 91 (C) 100 (D) none
31. A galvanometer has resistance 100Ω and it requires current $100\mu\text{A}$ for full scale deflection. A resistor 0.1Ω is connected to make it an ammeter. The smallest current required in the circuit to produce the full scale deflection is

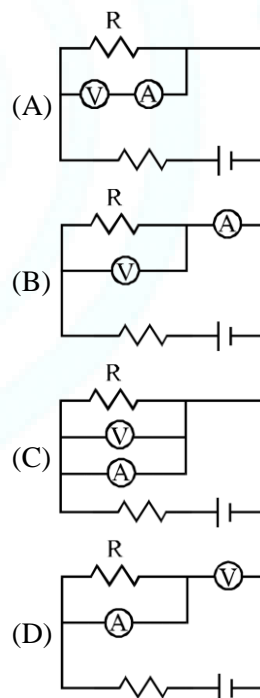
- (A) 1000.1 mA (B) 1.1 mA
 (C) 10.1 mA (D) 100.1 mA

32. In a galvanometer, the deflection become one half when the galvanometer is shunted by a 20Ω resistor. The galvanometer resistance is



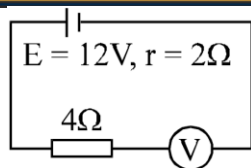
- (A) 5Ω (B) 10Ω (C) 40Ω (D) 20Ω

33. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter.



34. By error, a student places moving-coil voltmeter V (nearly ideal) in series with the resistance in a circuit in order to read the current, as shown. The voltmeter reading will be

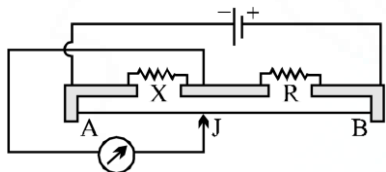




- (A) 0 (B) 4V (C) 6V (D) 12V

35. In a balanced wheat stone bridge, current in the galvanometer is zero. It remains zero when:
 [1] battery emf increased
 [2] all resistances are increased by 10 ohm
 [3] all resistances are made five times
 [4] the battery and the galvanometer are interchanged
 (A) only [1] is correct
 (B) [1], [2] and [3] are correct
 (C) [1], [3] and [4] are correct
 (D) [1] and [3] are correct

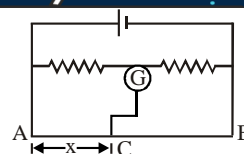
36. The figure shows a metre-bridge circuit, with $AB = 100$ cm, $X = 12\Omega$ and $R = 18\Omega$, and the jockey J in the position of balance. If R is now made 8Ω , through what distance will J have to be moved to obtain balance?



- (A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm

37. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y . If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y ?
 (A) 50 cm (B) 80 cm (C) 40 cm (D) 70 cm

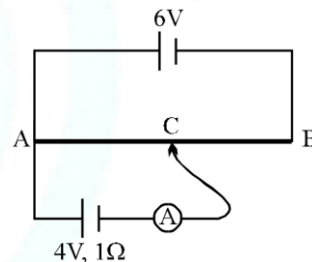
38. In the given circuit, no current is passing through the galvanometer. If the cross-sectional diameter of AB is doubled then for null point of galvanometer the value of AC would be:



- (A) x (B) $x/2$ (C) $2x$ (D) None

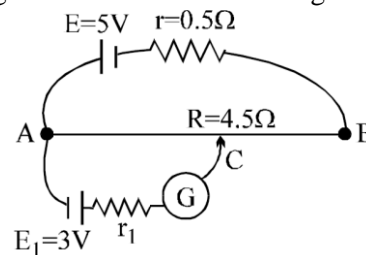
39. A potentiometer wire has length 10 m and resistance 10Ω . It is connected to a battery of EMF 11 volt and internal resistance 1Ω , then the potential gradient in the wire is:
 (A) 10 V/m (B) 1 V/m
 (C) 0.1 V/m (D) none

40. A 6 V battery of negligible internal resistance is connected across a uniform wire of length 1 m. The positive terminal of another battery of emf 4V and internal resistance 1Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. The AC is equal to:



- (A) $2/3$ m (B) $1/3$ m (C) $3/5$ m (D) $1/2$ m

41. In the given potentiometer circuit length of the wire AB is 3 m and resistance is $R = 4.5\Omega$. The length AC for no deflection in galvanometer is:



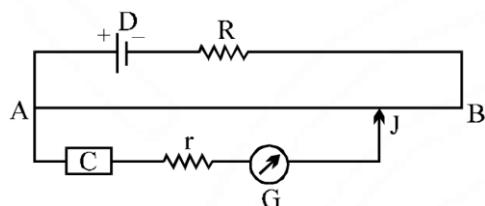
- (A) 2 m (B) 1.8 m
 (C) dependent on r_1 (D) none of these

42. The length of a potentiometer wire is ℓ . A cell of emf E is balanced at a length $\ell/3$ from the positive end of the wire. If the length of the

wire is increased by $\ell/2$. At what distance will the same cell give a balance point.

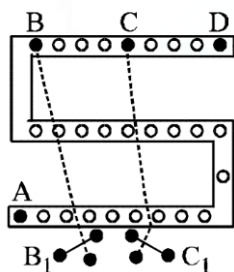
- (A) $\frac{2\ell}{3}$ (B) $\frac{\ell}{2}$ (C) $\frac{\ell}{6}$ (D) $\frac{4\ell}{3}$

43. In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0 . C is a cell of internal resistance r . The galvanometer G does not give zero deflection for any position of the jockey J. Which of the following cannot be a reason for this?



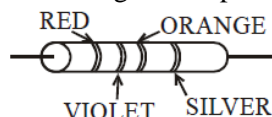
- (A) $r > R_0$
 (B) $R \gg R_0$
 (C) emf of C > emf of D
 (D) The negative terminal of C is connected to A

44. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between



- (A) B and C (B) C and D
 (C) A and D (D) B₁ and C₁

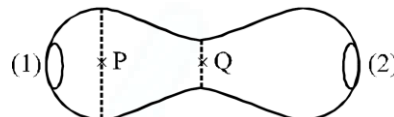
45. A resistance is shown in the figure. Its value and tolerance are given respectively by:



- (A) 27 K Ω , 20% (B) 270 K Ω , 5%
 (C) 270 K Ω , 10% (D) 27 K Ω , 10%

One or More Than One Correct Type Questions Microscopic Analysis

46. A metallic conductor of irregular cross-section is as shown in the figure. A constant potential difference is applied across the ends (1) and (2). Then:



- (A) the current at the cross-section P equals the current at the cross-section Q
 (B) the electric field intensity at P is less than that at Q.
 (C) the rate of heat generated per unit time at Q is greater than that at P
 (D) the number of electrons crossing per unit area of cross-section at P is less than that at Q.
47. A current passes through an ohmic conductor of nonuniform cross section. Which of the following quantities are independent of the cross-section?
 (A) the charge crossing in a given time interval.
 (B) drift speed
 (C) current density
 (D) free-electron density

Ohm's law and Circuit Analysis

48. A battery is of emf E is being charged from a charger such that positive terminal of the battery is connected to terminal A of charger and negative terminal of the battery is connected to terminal B of charger. The internal resistance of the battery is r .
 (A) Potential difference across points A and B must be more than E .
 (B) A must be at higher potential than B
 (C) In battery, current flows from positive terminal to the negative terminal
 (D) No current flows through battery
49. Two identical fuses are rated at 10A. If they are joined
 (A) in parallel, the combination acts as a fuse of rating 20A
 (B) in parallel, the combination acts as a fuse of rating 5A
 (C) in series, the combination acts as a fuse of rating 10A.



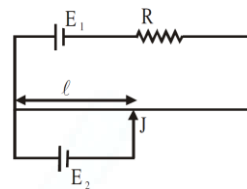
(D) in series, the combination acts as a fuse of rating 20A.

Electric instruments

50. Mark out the correct options.
- (A) An ammeter should have small resistance.
 - (B) An ammeter should have large resistance.
 - (C) A voltmeter should have small resistance.
 - (D) A voltmeter should have large resistance.

Matrix Match Type Questions

51. In the potentiometer arrangement shown in figure, null point is obtained at length ℓ .



Column-I

Column-II

- | | |
|---------------------------|---|
| (A) If E_1 is increased | (P) ℓ should increase |
| (B) If R is increased | (Q) ℓ should decrease |
| (C) If E_2 is increased | (R) ℓ should remain the same to again get the null point |



OBJECTIVE EXERCISE - II

Single Correct Type Questions

- Which of the following quantities do not change when an ohmic resistor connected to a battery is heated due to the current?
(A) drift speed
(B) resistivity
(C) resistance
(D) number of free electrons

- The wire shown in figure has a uniform cross-section A.

$$\overline{\hspace{10em}} \\ x = 0 \hspace{10em} x = L$$

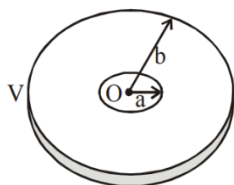
Resistivity of the material of wire is given by

$$\rho = \rho_0 \left(\frac{L}{L+x} \right). \text{ A potential difference } V \text{ is}$$

applied across the wire:

- Resistance of wire is $\frac{\rho_0 L}{A} \ln(2)$
- Current density is variable inside the wire
- Electric field at $x = 0$ is $\frac{2V}{(\ln 2)L}$
- Electric field at $x = L$ is $\frac{V}{(\ln 2)L}$

- A circular portion is cut of a disc of thickness t , its resistivity is U and radii of disc are a and b ($b > a$). A potential difference is maintained between outer and inner cylindrical surfaces of the disc. What is resistance of the disc?

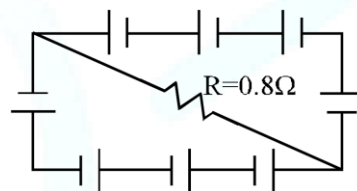


- $\frac{\rho}{2\pi t} \ln\left(\frac{b}{a}\right)$
- $\rho \left(\frac{1}{a} - \frac{1}{b} \right)$
- $2\pi \rho t \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$
- $\frac{\rho}{2\pi t} \left(\frac{b^2 - a^2}{ab} \right)$

- Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2 . The respective temperature coefficients of their series and parallel combinations are nearly:

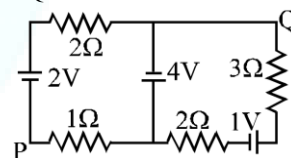
- $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$
- $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$
- $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$
- $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

- A circuit is comprised of eight identical batteries and a resistor $R = 0.8\Omega$. Each battery has an emf of 1.0 V and internal resistance of 0.2Ω . The voltage difference across any of the battery is



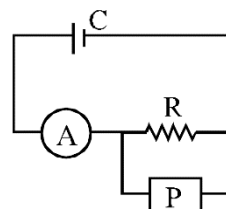
- 0.5V
- 1.0V
- 0V
- 2V

- In the circuit shown, what is the potential difference V_{PQ} ?



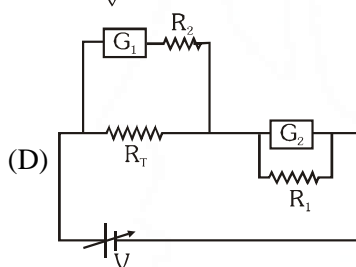
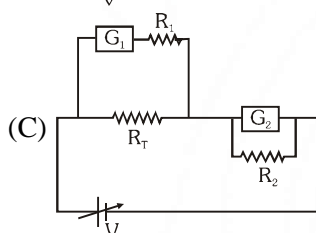
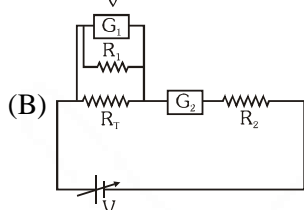
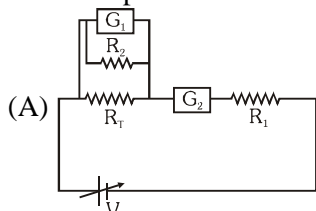
- $+3\text{V}$
- $+2\text{V}$
- -2V
- none

- An ammeter A of finite resistance, and a resistor R are joined in series to an ideal cell C . A potentiometer P is joined in parallel to R . The ammeter reading is I_0 and the potentiometer reading is V_0 . P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V .

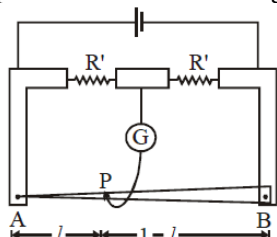


- $I > I_0, V < V_0$
- $I > I_0, V = V_0$
- $I = I_0, V < V_0$
- $I < I_0, V = V_0$

8. To verify Ohm's law, a student is provided with a test resistor R_T , a high resistance R_1 , a small resistance R_2 , two identical galvanometers G_1 and G_2 , and a variable voltage source V . The correct circuit to carry out the experiment is:

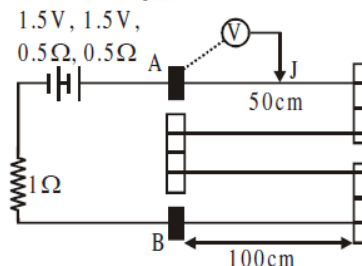


9. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?



- (A) 0.25 m (B) 0.3 m
(C) 0.35 m (D) 0.2 m

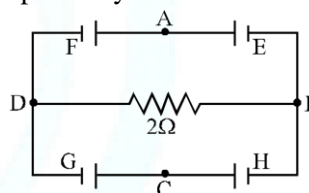
10. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01 \Omega/\text{cm}$. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be:



- (A) 0.20 V (B) 0.25 V (C) 0.75 V (D) 0.50 V

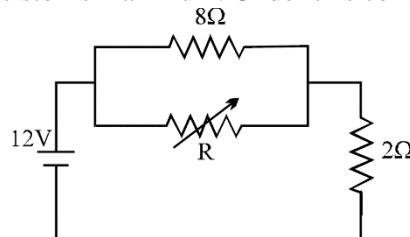
One Or More Than One Correct Type Questions

11. In the circuit shown E, F, G and H are cells of e.m.f. 2V, 1V, 3V and 1V respectively and their internal resistances are 2Ω , 1Ω , 3Ω and 1Ω respectively.



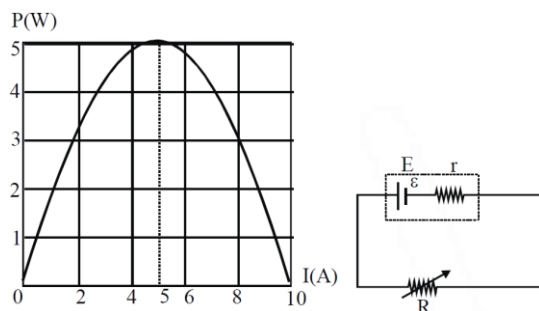
- (A) $V_D - V_B = -2/13 \text{ V}$
(B) $V_D - V_B = 2/13 \text{ V}$
(C) $V_G = 21/13 \text{ V}$ = potential difference across G.
(D) $V_H = 19/13 \text{ V}$ = potential difference across H.

12. The value of the resistance R in figure is adjusted such that power dissipated in the 2Ω resistor is maximum. Under this condition



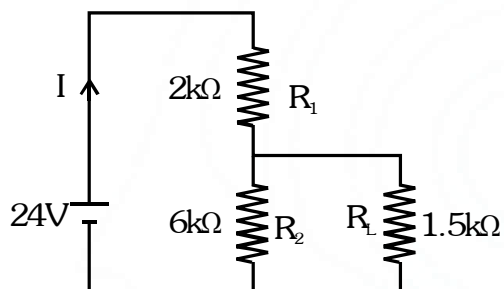
- (A) $R = 0$
(B) $R = 8\Omega$
(C) power dissipated in the 2Ω resistor is 72 W.
(D) power dissipated in the 2Ω resistor is 8 W.

13. Figure shows the net power dissipated in R versus the current in a simple circuit shown.



- (A) The internal resistance of battery is 0.2Ω
 (B) The emf of battery is 2V
 (C) R at which power is 5W is 2.5Ω
 (D) At $i = 2A$, power is 3.2 W

14. For the circuit shown in the figure



- (A) the current I through the battery is 7.5 mA
 (B) the potential difference across R_L is 18 V
 (C) ratio of powers dissipated in R_1 and R_2 is 3
 (D) If R_1 and R_2 are interchanged magnitude of the power dissipated in R_L will decrease by a factor of 9

15. In a potentiometer wire experiment the emf of a battery in the primary circuit is 20V and its internal resistance is 5Ω . There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from 120Ω to 170Ω . Resistance of the potentiometer wire is 75Ω . The following potential differences can be measured using this potentiometer.
 (A) 5V (B) 6V (C) 7V (D) 8V



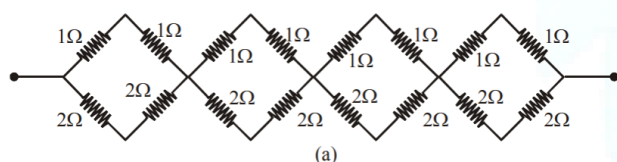
SUBJECTIVE EXERCISE – I

Microscopic Analysis

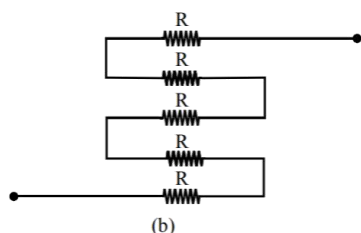
1. A copper wire carries a current density j (= current per unit area). Assuming that n = No. of free electrons per unit volume, e = electronic charge, $\langle v \rangle$ = average speed due to thermal agitation. The distance which will be covered by an electron during its displacement l along the wire _____.
2. The total momentum of electrons in a straight wire of length l carrying a current I is _____ (mass of electron = m_e , charge of electron = e)
3. A copper wire of length L , and cross section area A carries a current I . If the specific resistance of copper is ρ , the electric field in the wire is _____.
4. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outer diameter 2mm and inner diameter 1mm. Find the ratio of resistance R_A to R_B .

Ohm's law and Circuit Analysis

5. (a) Given n resistors each of resistance R , how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?
(b) Given the resistances of 1Ω , 2Ω , 3Ω , how will be combine them to get an equivalent resistance of (i) $(11/3)\Omega$ (ii) $(11/5)\Omega$ (iii) 6Ω (iv) $(6/11)\Omega$?
(c) Determine the equivalent resistance of networks shown in figure

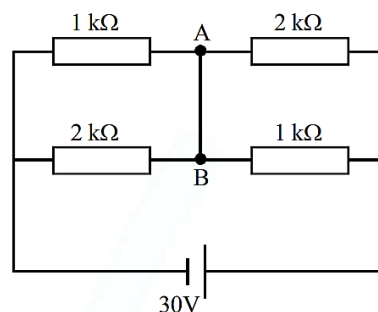


(a)

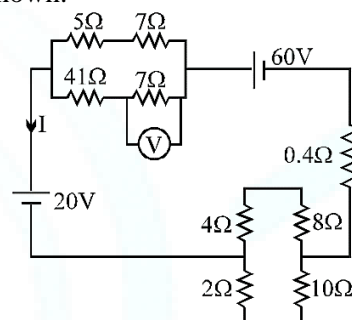


(b)

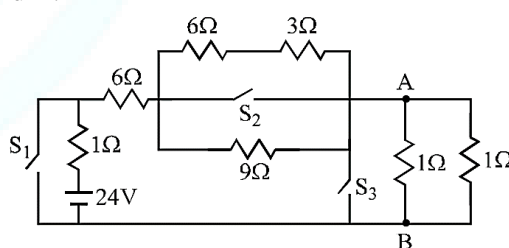
6. Find the current (in mA) in the wire between points A and B.



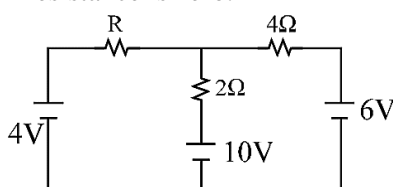
7. Find the current I & voltage V in the circuit shown.



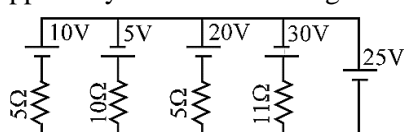
8. If the switches S_1 , S_2 and S_3 in the figure are arranged such that current through the battery is minimum, find the voltage across points A and B.



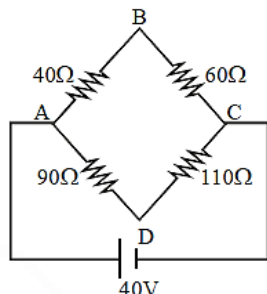
9. For what value of R in circuit, current through 4Ω resistance is zero.



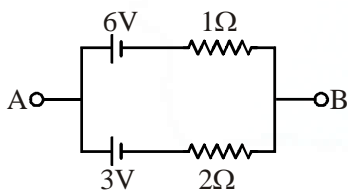
10. Find the current through 25V cell & power supplied by 20V cell in the figure shown.



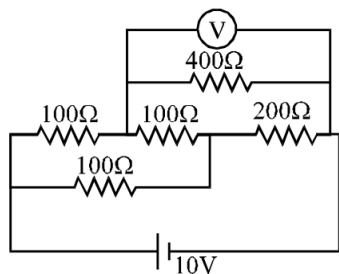
11. Four resistances 40Ω , 60Ω , 90Ω and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf $40V$ and internal resistance negligible. The potential difference across BD in V is _____.



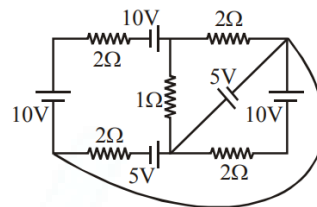
12. The series combination of two batteries, both of the same emf $10V$, but different internal resistance of 20Ω and 5Ω , is connected to the parallel combination of two resistors 30Ω and $R\Omega$. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is: _____.
13. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is:



14. An electrical circuit is shown in the figure. Calculate the potential difference across the resistance of 400Ω , as will be measured by the voltmeter V of resistance 400Ω , either by applying Kirchhoff's rules or otherwise.

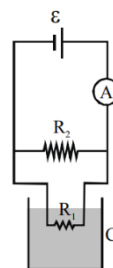


15. In the given circuit diagram, the current through the 1Ω resistor is given by I amp. Fill $2I$ in OMR sheet.



Electric Power and Joule heating

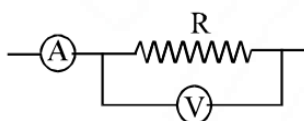
16. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R , the rate of heat produced in R is J_1 . When the same batteries are connected in parallel across R , the rate is J_2 . If $J_1 = 2.25 J_2$ then the value of R in Ω is:
17. A person decides to use his bath tub water to generate electric power to run a 40 watt bulb. The bath tub is located at a height of 10 m from the ground & it holds 200 litres of water. If we install a water driven wheel generator on the ground, at what rate should the water drain from the bath tub to light bulb? How long can we keep the bulb on, if the bath tub was full initially. The efficiency of generator is 90% . ($g = 10\text{m/s}^2$)
18. The coil of a calorimeter C has a resistance of $R_1 = 60\Omega$. The coil R_1 is connected to the circuit as shown in figure. What is the rise in temperature ($^{\circ}\text{C}$) of 240 grams of water poured into the calorimeter when it is heated for 7 minutes during which a current flows through the coil and the ammeter shows $3A$? The resistance $R_2 = 30\Omega$. [Disregard the resistances of the battery and the ammeter, and the heat losses and heat capacity of the calorimeter and the resistor and specific heat of water = $4200 \text{ J/kg}^{\circ}\text{C}$]



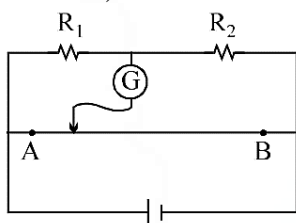
19. An electric kettle has two windings. When one of them is switched on, the water in the kettle begins to boil in 15 minutes, and when the other is switched on it takes 30 minutes for water to boil. If the two windings are joined in series and switched on, water in the kettle begin to boil in $\frac{\alpha}{4}$ hr. Assuming no heat loss to the surrounding fill the value of α in OMR sheet.

Electric instruments

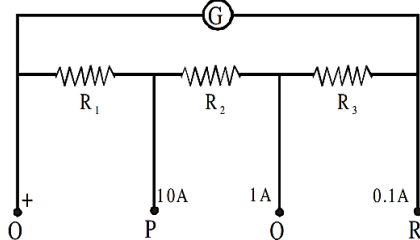
20. A part of a circuit is shown in figure. Here reading of ammeter is 5 ampere and voltmeter is 96V & voltmeter resistance is 480 ohm. Then find the resistance R



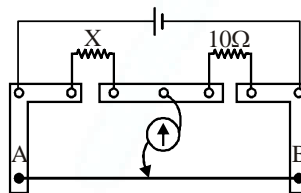
21. In the figure shown for which values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10Ω , balance shifts to 50 cm. Find R_1 and R_2 . ($AB = 1$ m):



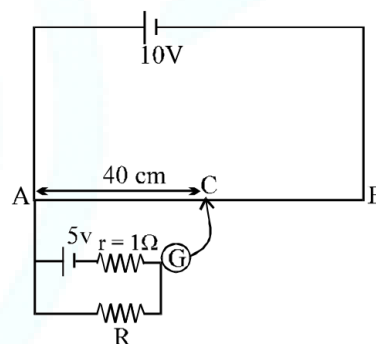
22. The resistance of the galvanometer G in the circuit is 25Ω . The meter deflects full scale for a current of 10 mA. The meter behaves as an ammeter of three different ranges. The range is 0–10A, if the terminals O and P are taken; range is 0–1 A between O and Q; range is 0–0.1 A between O and R. Calculate the resistance R_1 , R_2 and R_3 .



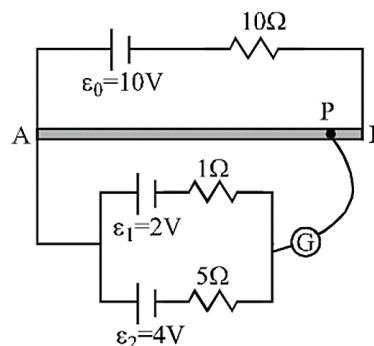
23. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is:



24. A potentiometer wire AB is 100 cm long and has a total resistance of 10 ohm. If the galvanometer shows zero deflection at the position C, then find the value of unknown resistance R.



25. A battery of emf $\epsilon_0 = 10V$ is connected across a 1 m long uniform wire having resistance $10\Omega/m$. Two cells of emf $\epsilon_1 = 2V$ and $\epsilon_2 = 4V$ having internal resistances 1Ω and 5Ω respectively are connected as shown in the figure. If a galvanometer shows no deflection at the point P, find the distance of point P from the point A.

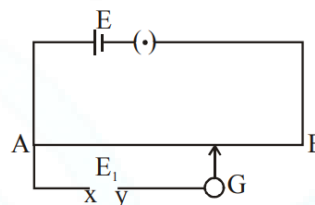




26. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

27. While doing an experiment with potentiometer it was found that the deflection is one sided and two cases are possible

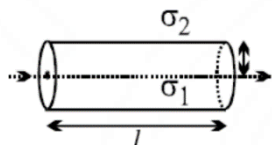
- (i) the deflection decreased while moving from one end A of the wire to the end B;
 - (ii) the deflection increased, while the jockey was moved towards the end B. Then
- (a) Which terminal +ve or -ve of the cell E_1 , is connected at X in case (i) and how is E_1 related to E ?
 - (b) Which terminal of the cell E_1 is connected at X in case (ii)?





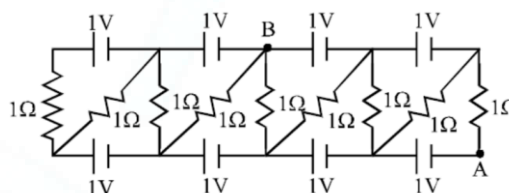
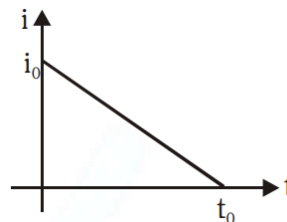
SUBJECTIVE EXERCISE – II

1. A resistance R of thermal coefficient of resistivity $= \alpha$ is connected in parallel with a resistance $= 3R$, having thermal coefficient of resistivity $= 2\alpha$. Find the value of α_{eff} .
2. A long conductor of circular cross-section has radius r and length l as shown in the figure. The conductivity of the material near the axis is σ_1 and increases linearly with the distance from axis and becomes σ_2 near the surface. Find the resistance of the conductor if the current enters from the one end and leaves from the other end.

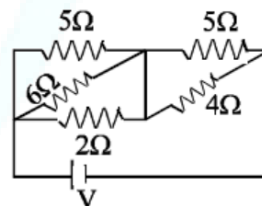


3. A long cylinder with uniformly charged surface and cross-sectional radius $a = 1.0$ cm moves with a constant velocity $v = 10$ m/s, along its axis. An electric field strength at the surface of the cylinder is equal to $E = 0.9$ KV/cm. Find the resulting convection current, that is, the current caused by mechanical transfer of charge.
4. (a) The current density across a cylindrical conductor of radius R varies according to the equation $J = J_0 \left(1 - \frac{r}{R}\right)$, where r is the distance from the axis. Thus the current density is a maximum J_0 at the axis $r = 0$ and decreases linearly to zero at the surface $r = R$. Calculate the current in terms of J_0 and the conductor's cross sectional area is $A = \pi R^2$.
(b) Suppose that instead the current density is a maximum J_0 at the surface and decreases linearly to zero at the axis so that $J = J_0 \frac{r}{R}$. Calculate the current.

5. Find the potential difference $V_A - V_B$ for the circuit shown in the figure.

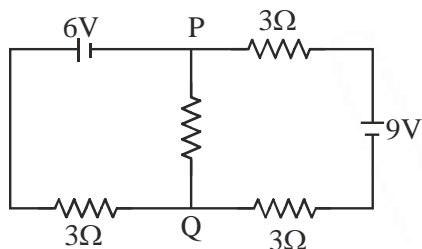


6. Relation between current in conductor and time is shown in figure then determine.
(i) Total charge flow through the conductor
(ii) Write expression of current in terms of time
(iii) If resistance of conductor is R then total heat dissipated across resistance R is
7. Find the resistor in which maximum heat will be produced.



JEE-Main (Previous Year Questions)

1. In the circuit shown, the current in the 1Ω resistor is: [JEE(Main)-2015]



- (1) 0 A
(2) 0.13 A, from Q to P
(3) 0.13 A, from P to Q
(4) 1.3 A, from P to Q
2. A galvanometer having a coil resistance of 100Ω gives a full-scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A is:

[JEE(Main)-2016]

- (1) 2Ω (2) 0.1Ω (3) 3Ω (4) 0.01Ω

3. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter range 0–10 V is: [JEE(Main)-2017]

- (1) $1.985 \times 10^3\Omega$ (2) $2.045 \times 10^3\Omega$
(3) $2.535 \times 10^3\Omega$ (4) $4.005 \times 10^3\Omega$

4. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220V. The minimum capacity of the main fuse of the building will be:

[JEE(Main)-2018]

- (1) 12 A (2) 14 A (3) 8 A (4) 10 A

5. On interchanging the resistances, the balance point of the meter bridge shifts to the left by 10 cm. The resistance of their series combination is $1K\Omega$. How much was the resistance on the left slot before interchanging the resistance.

[JEE(Main)-2018]

- (1) 910Ω (2) 990Ω (3) 505Ω (4) 550Ω

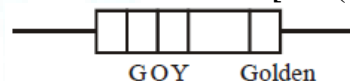
6. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by the resistance of 5Ω , a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

[JEE(Main)-2018]

- (1) 2.5Ω (2) 1Ω (3) 1.5Ω (4) 2Ω

7. A carbon resistance has a following colour code. What is the value of the resistance?

[JEE(Main)-2019]



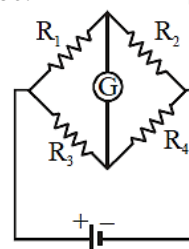
- (1) $1.64 M\Omega \pm 5\%$ (2) $530 k\Omega \pm 5\%$
(3) $64 k\Omega \pm 10\%$ (4) $5.3 M\Omega \pm 5\%$

8. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:

[JEE(Main)-2019]

- (1) 2.5% (2) 0.5%
(3) 1.0% (4) 2.0%

9. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω , respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3 , would be: [JEE(Main)-2019]



- (1) Red, Green, Brown
(2) Brown, Blue, Brown
(3) Grey, Black, Brown
(4) Brown, Blue, Black



10. A current of 5 A passes through a copper conductor (resistivity = $1.7 \times 10^{-8} \Omega\text{m}$) of radius of cross section 5 mm. Find the mobility of the charges if their drift velocity is $1.1 \times 10^{-3} \text{ m/s}$.

[JEE(Main)-2019]

- (1) $1.3 \text{ m}^2/\text{Vs}$ (2) $1.5 \text{ m}^2/\text{Vs}$
(3) $1.8 \text{ m}^2/\text{Vs}$ (4) $1.0 \text{ m}^2/\text{Vs}$

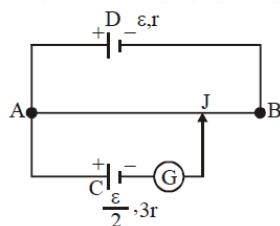
11. Two equal resistance when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be:

[JEE(Main)-2019]

- (1) 60 W (2) 240 W (3) 30 W (4) 120 W

12. A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of emf ε and internal resistance r . A cell C having emf $\varepsilon/2$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is:

[JEE(Main)-2019]



- (1) $\frac{5}{12}L$ (2) $\frac{11}{24}L$ (3) $\frac{11}{12}L$ (4) $\frac{13}{24}L$

13. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6mA it produces a deflection of 2° , its figure of merit is close to:

[JEE(Main)-2020]

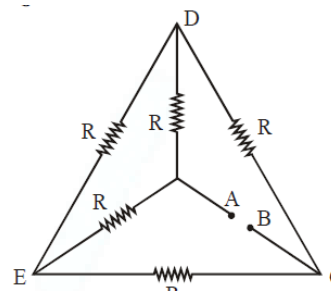
- (1) $3 \times 10^{-3} \text{ A/div.}$ (2) 333° A/div.
(3) $6 \times 10^{-3} \text{ A/div.}$ (4) 666° A/div.

14. A cylindrical wire of radius 0.5 mm and conductivity $5 \times 10^7 \text{ S/m}$ is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi \text{ mA}$. The value of x is ____.

[JEE(Main)-2021]

15. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :

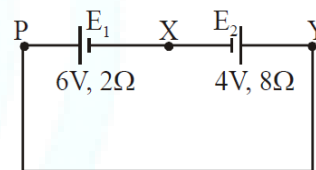
[JEE(Main)-2021]



- (1) $2R$ (2) $\frac{R}{2}$ (3) $\frac{3R}{2}$ (4) R

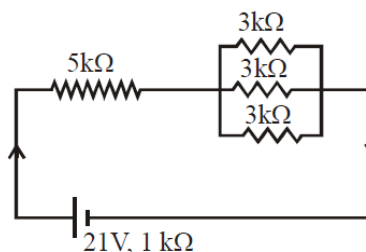
16. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is :

[JEE(Main)-2021]



- (1) 10.0 V (2) 3.6 V (3) 5.6 V (4) 2.0 V

17. In the figure given, the electric current flowing through the $5 \text{ k}\Omega$ resistor is 'x' mA.



The value of x to the nearest integer is ____.

[JEE(Main)-2021]

18. A wire of 1Ω has a length of 1m. It is stretched till its length increases by 25%. The percentage change in resistance to the nearest integer is :-

[JEE(Main)-2021]

- (1) 56% (2) 25 % (3) 12.5 % (4) 76 %



19. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is _____J.

[JEE(Main)-2021]

20. A resistor develops 500 J of thermal energy in 20s when a current of 1.5 A is passed through it.

If the current is increased from 1.5 A to 3A, what will be the energy developed in 20 s.

[JEE(Main)-2021]

(1) 1500 J (2) 1000 J (3) 500 J (4) 2000 J

21. Two identical cells each of emf 1.65 V are connected in parallel across a parallel combination of two resistor each of resistance 20Ω . A voltmeter connected in the circuit measures 1.2 V.

The internal resistance of each cell is

[JEE(Main)-2022]

(1) 2.5Ω (2) 4Ω (3) 5Ω (4) 10Ω

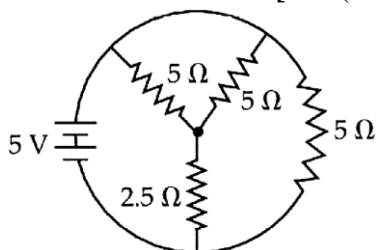
22. What will be the most suitable combination of three resistors $A = 2\Omega$, $B = 4\Omega$, $C = 6\Omega$ so that $\left(\frac{22}{3}\right)\Omega$ is equivalent resistance of combination?

[JEE(Main)-2022]

- (1) Parallel combination of A and C connected in series with B.
- (2) parallel combination of A and B connected in series with C
- (3) Series combination of A and C connected in parallel with b
- (4) Series combination of b and C connected in parallel with A.

23. The total current supplied to the circuit as shown in figure by the 5V battery is _____A.

[JEE(Main)-2022]



24. A resistor develops 300 J of thermal energy in 15s, when a current of 2A is passed through it. If the current increases to 3A, the energy developed in 10s _____J.

[JEE(Main)-2022]

25. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be _____cm.

[JEE(Main)-2022]

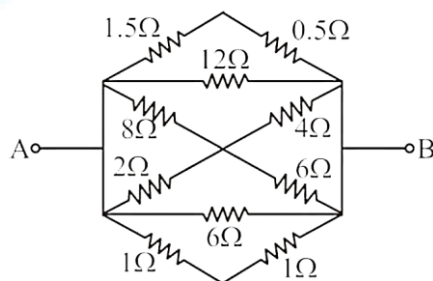
26. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm, respectively. The resistance of the conductor is $n \times 10^{-3}\Omega$. If the resistivity of the material is $2.4 \times 10^{-8}\Omega\text{m}$.

The value of n is _____.

[JEE(Main)-2023]

27. The equivalent resistance between A and B is

[JEE(Main)-2023]



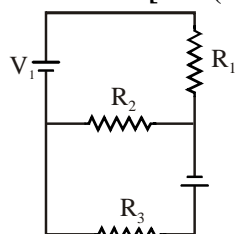
- | | |
|-------------------------|-------------------------|
| (1) $\frac{2}{3}\Omega$ | (2) $\frac{1}{2}\Omega$ |
| (3) $\frac{3}{2}\Omega$ | (4) $\frac{1}{3}\Omega$ |



JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

1. Two ideal batteries of emf V_1 and V_2 and three resistances R_1 , R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if:

[JEE(Advanced)-2014]



- (A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
 (B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
 (C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
 (D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

2. Heater of an electric kettle is made of a wire of length L and diameter d . It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?

[JEE(Advanced)-2014]

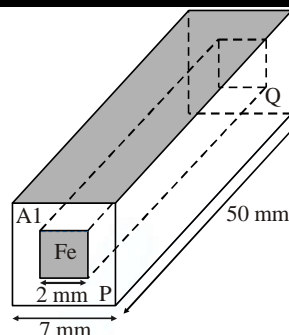
- (A) 4 if wires are in parallel
 (B) 2 if wires are in series
 (C) 1 if wires are in series
 (D) 0.5 if wires are in parallel.

3. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 – 1.5 A. The value of n is:

[JEE(Advanced)-2014]

4. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8} \Omega\text{m}$ and $1.0 \times 10^{-7} \Omega\text{m}$, respectively. The electrical resistance between the two faces P and Q of the composite bar is

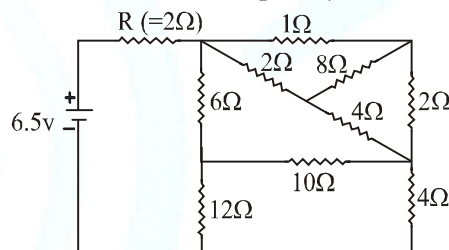
[JEE(Advanced)-2015]



- (A) $\frac{2475}{64}\Omega$ (B) $\frac{1875}{64}\mu\Omega$
 (C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

5. In the following circuit, the current through the resistor $R = (2\Omega)$ is I Amperes. The value of I is:

[JEE(Advanced)-2015]

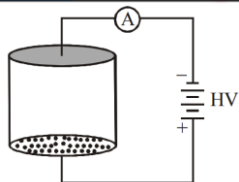


Paragraph for Questions No. 6 and 7

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r \ll h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled

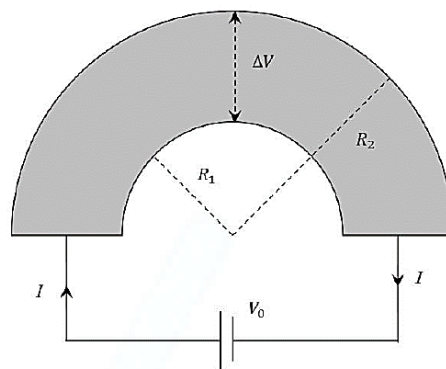
by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collision between the balls and the interaction between them is negligible. (Ignore gravity) [JEE(Advanced)-2016]





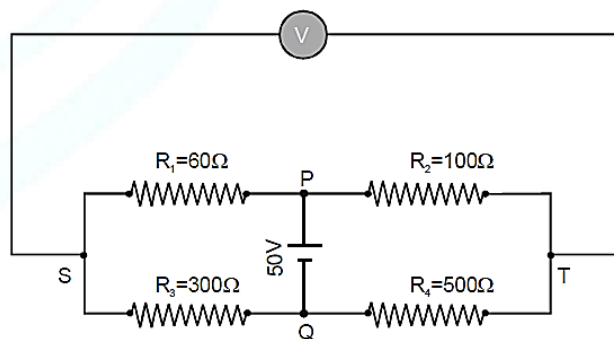
6. Which of the following statements is correct?
 (A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
 (B) The balls will execute simple harmonic motion between the two plates
 (C) The balls will bounce back to the bottom plate carrying the same charge they went up with
 (D) The balls will stick to the top plate and remain there
7. The average current in the steady state registered by the ammeter in the circuit will be:
 (A) Proportional to $V_0^{1/2}$
 (B) Proportional to V_0^2
 (C) Proportional to the potential V_0
 (D) Zero
8. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? **[JEE(Advanced)-2016]**
 (A) The temperature distribution over the filament is uniform
 (B) The resistance over small sections of the filament decreases with time
 (C) The filament emits more light at higher band of frequencies before it breaks up
 (D) The filament consumes less electrical power towards the end of the life of the bulb
9. Shown in the figure is a semicircular metallic strip that has thickness t and resistivity ρ . Its inner radius is R_1 and outer radius is R_2 . If a voltage V_0 is applied between its two ends, a current I flows in it. In addition, it is observed that a transverse voltage ΔV develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale)-

[JEE(Advanced)-2020]



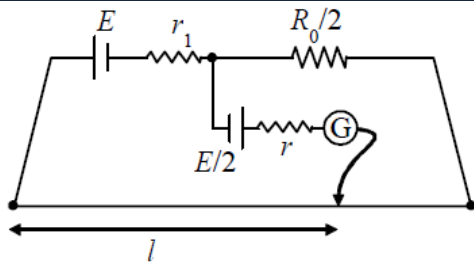
- (A) $I = \frac{V_0 t}{\pi \rho} \ln \left(\frac{R_2}{R_1} \right)$
 (B) the outer surface is at a higher voltage than the inner surface
 (C) the outer surface is at a lower voltage than the inner surface
 (D) $\Delta V \propto I^2$
10. In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient $0.0004^\circ\text{C}^{-1}$. If the temperature of R_3 is increased by 100°C , the voltage developed between S and T will be _____ volt.

[JEE(Advanced)-2020]



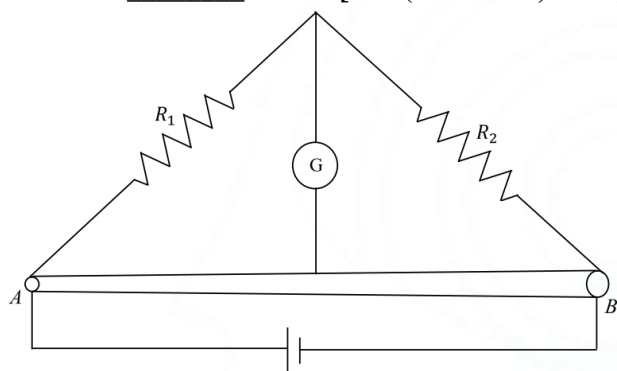
11. In order to measure the internal resistance r_1 of a cell of emf E , a meter bridge of wire resistance $R_0 = 50\Omega$, a resistance $R_0/2$, another cell of emf $E/2$ (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at $\ell = 72$ cm, then the value of $r_1 =$ _____ Ω .

[JEE(Advanced)-2021]



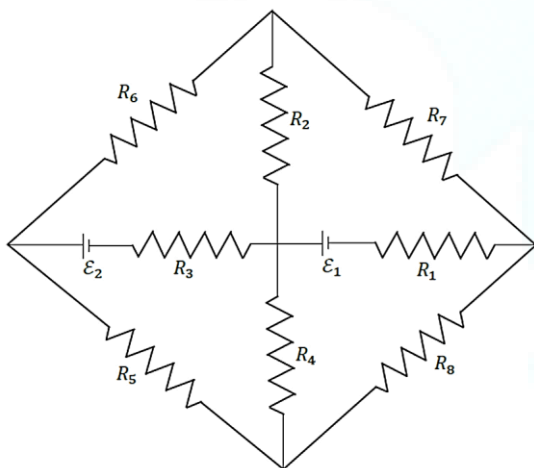
12. Two resistances $R_1 = X \Omega$ and $R_2 = 1 \Omega$ are connected to a wire AB of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at A to 1 mm at B. A galvanometer (G) connected to the center of the wire, 50 cm from each end along its axis, shown zero deflection when A and B are connected to a battery. The value of X is _____.

[JEE(Advanced)-2022]



13. The figure shows a circuit having eight resistances of 1Ω each, labelled R_1 to R_8 , and two ideal batteries with voltages $\varepsilon_1 = 12V$ and $\varepsilon_2 = 6V$.

[JEE(Advanced)-2022]

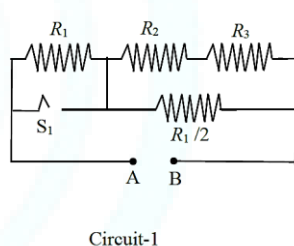


Which of the following statement(s) is(are) correct?

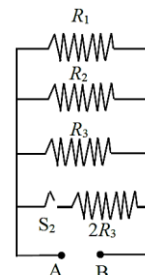
- (A) The magnitude of current flowing through R_1 is 7.2 A.
 (B) The magnitude of current flowing through R_2 is 1.2 A.
 (C) The magnitude of current flowing through R_3 is 4.8 A.
 (D) The magnitude of current flowing through R_5 is 2.4 A.

14. In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1 \Omega$, $R_2 = 2 \Omega$ and $R_3 = 3 \Omega$. P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively. Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.

[JEE(Advanced)-2022]



Circuit-1



Circuit-2

Which of the following statement(s) is(are) correct?

- (A) When a voltage source of 6 V is connected across A and B in both circuits, $P_1 < P_2$.
 (B) When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$.
 (C) When a voltage source of 9 V is connected across A and B in Circuit-1, $Q_1 > P_1$.
 (D) When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$.




ANSWER KEY
OBJECTIVE EXERCISE-I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	C	D	B	A	D	A	C	D	B	C	C	C	A	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	A	B	B	D	A	A	B	D	C	A	C	A	B	C
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	D	D	B	D	C	B	A	A	B	A	D	B	A	C	D
Que.	46	47	48	49	50										
Ans.	ABCD	AD	ABC	AC	AD										

51. (A) \rightarrow (Q); (B) \rightarrow (P); (C) \rightarrow (P)

OBJECTIVE EXERCISE-II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	A	A	A	C	B	A	C	A	B	ACD	AC	ABD	AD	ABC

SUBJECTIVE EXERCISE-I

- $S = e n \ell \langle v \rangle / j$
- $p = I m_e \ell / e$
- I_p / A
- 3 : 1
- (i) in series, (ii) all in parallel: n^2 .
 - Join 1Ω , 2Ω in parallel and the combination in series with 3Ω ,
 - parallel combination of 2Ω and 3Ω in series with 1Ω ,
 - all in series, (iv) all in parallel
 - (i) $(16/3)\Omega$, (ii) $5R$.
- 7.5 mA
- $I = 2.5\text{ A}$, $V = 3.5\text{ Volts}$
- 1 V
- 1Ω
- 12A, -20 W
- 2
- 30
- 5
- $20/3\text{ V}$
- 5
- 4
- $4/9\text{ kg/sec.}$, 450 sec
- 25
- 3
- 20 ohm
- $\frac{10}{3}\Omega$, 5Ω
- $R_1 = 0.0278\Omega$, $R_2 = 0.25\Omega$, $R_3 = 2.5\Omega$
- 10.6
- 4 ohm
- 46.67 cm
- 2.25 V
- +ve, $E_\ell > E$ (b) -ve





SUBJECTIVE EXERCISE-II

1. $\alpha_{eff} = \frac{5}{4} \alpha$

3. $5 \times 10^{-7} \text{ A}$

5. $-\frac{22}{9} \text{ V}$

7. 4Ω

2. $\frac{1}{R} = \frac{\pi r^2}{3l} (2\sigma_2 + \sigma_1)$

4. (a) $J_0 A/3$; (b) $2J_0 A/3$

6. (i) $\frac{1}{2} i_0 t_0$ (ii) $i = i_0 \left(1 - \frac{t}{t_0} \right)$ (iii) $\frac{R t_0 i_0^2}{3}$

JEE-MAIN (PREVIOUS YEAR QUESTIONS)

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	4	1	1	4	3	2	3	2	4	2	4	1	5	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27			
Ans.	3	3	1	300	4	3	2	2	450	25	2	1			

JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

1. (ABD) 2. (BD) 3. 5 4. (B) 5. 1 6. (A) 7. (B)
 8. (CD) 9. (ACD) 10. 0.26 to 0.28 11. 3 12. 5 13. (ABCD)
 14. (ABC)

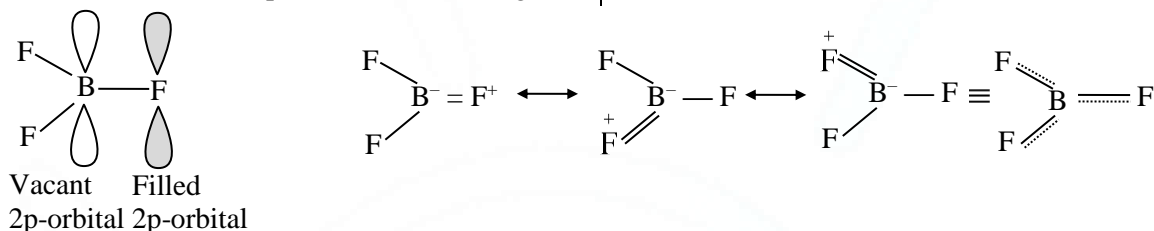




CHEMICAL BONDING

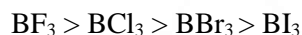
1. Back Bonding

Back bonding generally takes place when out of two bonded atoms one of the atom has vacant orbitals (generally this atom is from second or third period) and the other bonded atom is having some non-bonded electron pair (generally this atom is from the second period). Back bonding



Decrease in B – F bond length is due to delocalised $p\pi-p\pi$ back bonding between filled p-orbital of F atom and vacant p-orbital of B atom.

The extent of back bonding is much larger if the orbitals involved in the back bonding are of same size, for example the extent of back bonding in boron trihalides is as follows:



There is $p\pi-p\pi$ back bonding in boron trihalide. The extent of back bonding decreases from BF_3 to BI_3 because of increasing size of p-orbitals participating in back bonding that is from 2p(in F) to 5p(in I).

Molecules	Type of back bonding	Shape
BF_3	$2p\pi-2p\pi$	Planar
$\text{N}(\text{CH}_3)_3$	no back bonding	Pyramidal (w.r.t. Nitrogen)
$\text{N}(\text{SiH}_3)_3$	$2p\pi-3d\pi$	Planar (w.r.t. Nitrogen)
$\text{P}(\text{SiH}_3)_3$	No back bonding	Pyramidal (w.r.t. Phosphors)
CH_3NCS	No back bonding	Bent (w.r.t. Nitrogen)
SiH_3NCS	$2p\pi-3d\pi$	Linear (w.r.t. Nitrogen)
$\text{N}(\text{GeH}_3)_3$	$2p\pi-4d\pi$	Planar (w.r.t. Nitrogen)
GeH_3NCS	No back bonding	Bent (w.r.t. Nitrogen)

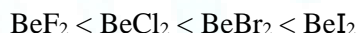
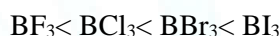
increases the bond strength and decreases the bond length.

For example, in BF_3 the boron atom completes its octet by accepting 2p-electrons of fluorine into 2p empty orbital.

• Lewis Basic Order:



• Lewis Acid Order:



Que. Which is better proton donor acid- Between $(\text{CH}_3)_3\text{C}-\text{OH}$ & $(\text{CH}_3)_3\text{Si}-\text{OH}$?

Ans. $(\text{CH}_3)_3\text{Si}-\text{OH}$, because $(\text{CH}_3)_3\text{Si}-\text{O}^-$ is stabilized by $2p\pi-3d\pi$ back bonding.

Que. Which is better proton donor acid between CHF_3 and CHCl_3 ?

Ans. CHCl_3 , because CCl_3^- is stabilized by $2p\pi-3d\pi$ back bonding.

Effect of Back Bonding

- Bond Length must be decrease.
- Bond angle may be change
- Hybridization may be change

- When lone pair of central atom involved in back bonding, bond angle increases.
- When lone pair of surrounding atom involved in back bonding, bond angle increases due to bond pair- bond pair repulsion.





Comparison of Bond Length

d_{B-O} in $H_3BO_3 < d_{B-O}$ in $[B(OH)_4]^-$

d_{B-F} bond in $BF_3 < d_{B-F}$ in BF_4^-

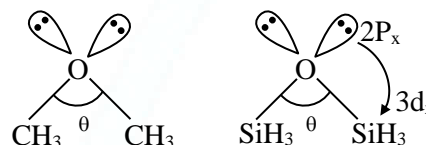
Hint: No back bonding involved in $[B(OH)_4]^-$ and BF_4^-

Some Important Order:

- Extent of Back Bonding:** $OCl_2 < O(SiH_3)_2$
- Order of Bond Angle:** $x < y < z$
where $x = \angle BOH$ in $B(OH)_3$, $y = \angle BOC$ in $B(OCH_3)_3$ and $z = \angle BOS$ in $B(OSiH_3)_3$
- Metaboric Acid (HBO_2):** $HBO_2 \longrightarrow (BO_2^-)_n$
 $B_3O_6^{3-}$ is aromatic, planar, O and B are sp^2 Hybridized

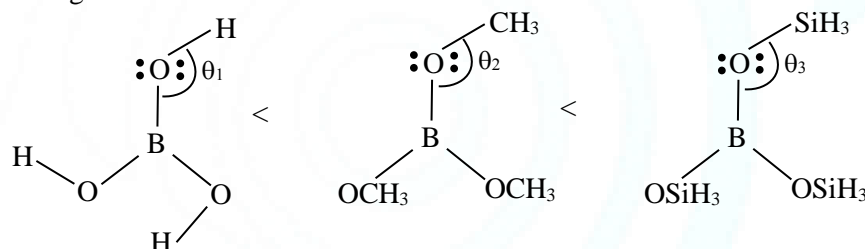
- $B_3N_3H_6 + 3 HCl \longrightarrow B_3N_3H_9Cl_3$
- Inorganic Graphite $\xrightarrow{3000^\circ C} (BN)_x$ 3 D network like structure
 $2p_\pi-2p_\pi$ back bonding involved in inorganic Graphite

Que. Order of bond angle

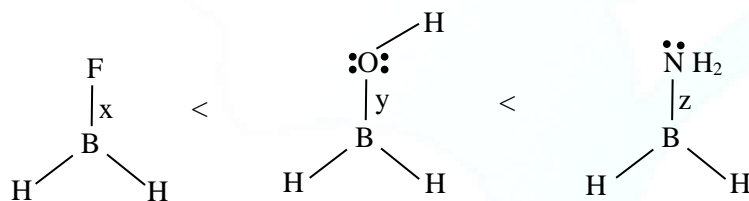


$112^\circ < 144^\circ$
Close to sp^3 sp^2

Que. Order of bond angle:-

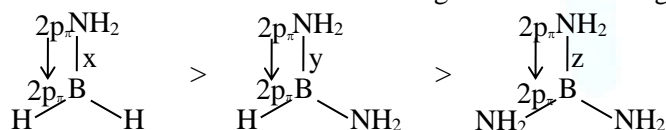


Que. Order of Strength Back Bonding:



Que. Order of Strength Back Bonding:

\Rightarrow number of donar atom \downarrow strength of back bonding \uparrow

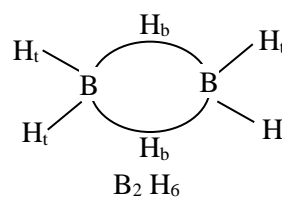
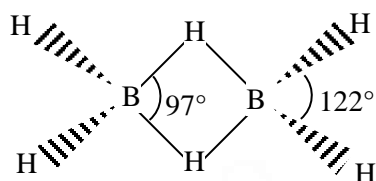


2 Bridge Bonding

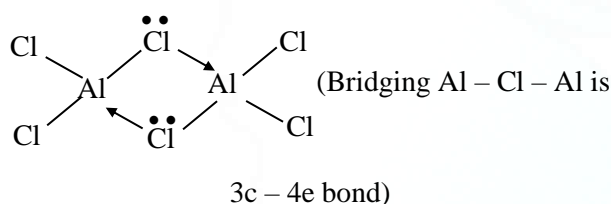
There are many compounds in which some electron deficient bonds are present apart from normal covalent bonds or coordinate bonds which are $2c-2e^-$ bonds (two centre two electron bonds).

These electron deficient bonds have less number of electrons than the expected such as three centre-two electron bonds ($3c-2e$) which are present in diborane B_2H_6 , $Al_2(CH_3)_6$, $BeH_2(s)$ etc.





The structure of diborane containing four terminal(t) and two bridging(b) hydrogen atoms. The model determined by molecular orbital theory indicates that the bonds between boron and the terminal hydrogen atoms are conventional $2c-2e^-$ covalent bonds. The bonding between the boron atoms and the bridging hydrogen atoms is, however different from that in molecules such as hydrocarbons. Having used two electrons in bonding to the terminal hydrogen atoms, each boron has one valence electron remaining for additional bonding. The bridging hydrogen atoms provide one electron each. Thus the B_2H_2 ring is held together by four electrons, an example of $3c-2e^-$ bonding. This type of bond is sometimes called as 'banana bond'. Group 13, gallium is known to form a similar compound, digallane, Ga_2H_6 . But Al_2Cl_6 have covalent bond only and there is no electron deficient bonding as depicted in the given structure.



Key points:

- Two Types:
 - $3c-4e^-$ bond (surrounding atom has lone pair)
 - $3c-2e^-$ bond (surrounding atom has no lone pair)
- $3c-4e^-$ bond remove more deficiency as compared to $3c-2e^-$.
- Vacant orbital takes part in hybridisation.

(A) Al_2Cl_6

Information:

- Number of $3c-4e^-$ bond = 2,
- Number of $2c-2e^-$ bond = 4
- Hybridisation of central atom = sp^3 .
- Octet: complete
- Octet complete

(B) I_2Cl_6

Information:

- Planarity:- planar
- Hybridisation of central atom = sp^3d^2
- Maximum number of atom in a plane = 8
- Number of $3c-4e^-$ bond = 2,
- Number of $2c-2e^-$ bond = 4
- Octet complete

(C) B_2H_6

Information

- Planarity:- Non- planar
- Maximum number of atom in a plane = 6
- Octet: incomplete, Lewis Acid
- $3c-2e^-$ bond = 2
- $2c-2e^-$ bond = 4
- Bridge bonds are stronger and longer than the terminal bonds.
- 2 bridging 'H' are perpendicular to the plane.

(D) $Al_2(CH_3)_6$

Information

- Hybridisation of central atom = sp^3
- Planarity:- non- planar
- Number of $3c-2e^-$ bond = 2
- Number of $2c-2e^-$ bond = 22
- Maximum number of atoms in one plane = 10

(E) Be_2Cl_4

Information

- Hybridisation of central atom = sp^2
- Planarity:-Planar (all atoms are in one plane)
- e^- Deficient
- Number of $3c-4e^-$ bond = 2
- Number of $2c-2e^-$ bond = 2



(F) Be_2H_4

Information

- Hybridisation of central atom = sp^2
- Planarity:-Planar (all atoms are in one plane)
- Number of $3\text{c}-2\text{e}^-$ bond = 2
- Number of $2\text{c}-2\text{e}^-$ bond = 2

(G) $\text{BeH}_{2(s)}$ (Polymer)

Information

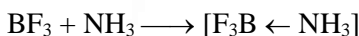
- Hybridisation of central atom = sp^3
- Planarity:-Non- Planar
- Borane**

FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 $\text{N}(\text{SiH}_3)_3$ has -

- (A) sp^3 hybridisation, pyramidal shape
 (B) sp^2 hybridisation, planar shape
 (C) sp^3 hybridisation, tetrahedral shape
 (D) dsp^2 hybridisation, square planar shape

Q.2 Correct statement regarding this reaction



- (A) Hybridisation of N is changed
 (B) Hybridisation of B is changed
 (C) NH_3 act as a lewis base
 (D) (B) & (C) both

Q.3 Shape of $\text{N}(\text{CH}_3)_3$ is

- (A) pyramidal (B) Linear
 (C) Tetrahedral (D) None of these

Q.4 Which of the following molecule is stable by back bonding

- (A) AlCl_3 (B) AlH_3
 (C) BCl_3 (D) ICl_3

Q.5 Which of the following represents $(3\text{C}-4\text{e}^-)$ Bridge bonding

- (A) Al_2Cl_6 (B) BH_3
 (C) B_2H_6 (D) none of these

Odd Electron Molecules

- If number of electron present in molecule are in odd number.

Example: Odd electron molecule NO_2 ClO_3
 OF O_2^- ClO_2

Number of electron 23 41 17 17 33

Information:

- (i) All odd electron molecules are paramagnetic in nature.
 (ii) Bond Order of odd electron molecule is 0.5.
 (iii) Bond Order of simple covalent bond is 1.

Hybridisation of odd electron molecule:

According to fact orbital having odd electron present at high energy level so it doesn't involve in hybridisation but if surrounding atom is more electronegative than central atom, then it develop partial positive charge on central atom so orbital contracts towards central atom and involve in hybridisation.

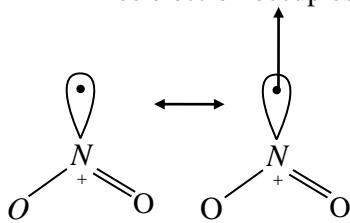
Odd Electron Molecule	Hybridisation of central atom	odd electron present in	formation of dimer
NO_2	sp^2	sp^2 hybrid orbital	Yes
ClO_3	sp^3	sp^3 hybrid orbital	Yes
CH_3	sp^2	p- orbital	Yes
CF_3	sp^3	sp^3 hybrid orbital	Yes
ClO_2	sp^2	3 d- orbital	No



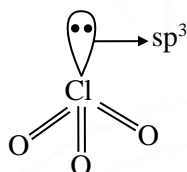
Structure of odd e^- molecules

(1) NO_2 Structure:

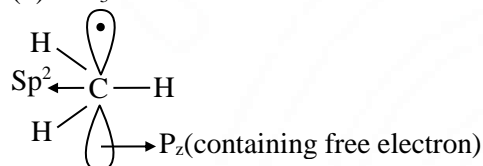
Free electron occupies the one sp^3 orbital.



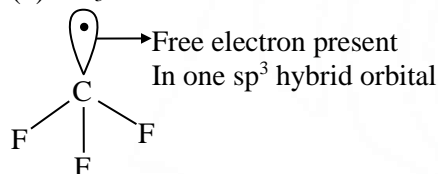
(2) ClO_3^- Structure:-



(3) CH_3^- Structure:-



(4) CF_3^- Structure:



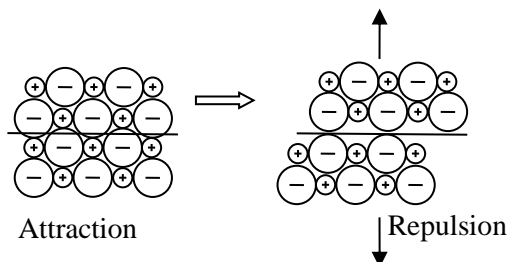
Key point

Species	Type of orbital (odd e^-)
NO_2	sp^2
CF_3	sp^3
ClO_3	sp^3
CH_3	Pure 'P'
ClO_2	d-orbital

Ionic Compound

Properties of ionic compound

(a) **Physical state:** Ionic compounds are hard, crystalline and brittle due to strong electrostatic force of attraction. Brittleness \rightarrow {Same charged ions comes nearer. So they repel each other}



(b) **Isomorphism:** The phenomenon of different ionic compounds, having same crystal arrangement of ions is termed as isomorphism

Condition of Isomorphism:

- Same charge on cation & anion between isomorphs
- Same radius ratio range of cation & anion between isomorphs
- Same number of water of crystallization between isomorphs

Example:

- $\text{ZnSO}_4 \cdot 7\text{H}_2\text{O}$, $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$ are isomorphous
- All alums are isomorphous

(c) **Boiling point and melting point:** Ionic compounds have high boiling point and melting point due to strong electrostatic force of attraction among oppositely charged ions.

(d) **Conductivity:** It depends on ionic mobility.

- In solid state – (No free ions) - Bad conductor of electricity.
- In fused state or aqueous solution Due to free ions - Good conductor of electricity.

Conductivity order: Solid state < fused state < Aqueous solution

(e) **Solubility:** Ionic compounds are more soluble in polar solvents and less soluble in non polar solvents. Solubility of ionic compounds in water mainly depends upon hydration energy & lattice energy.

Que. Why does the solubility of alkaline earth metal hydroxides in water increase down the group?

Ans. Among alkaline earth metal hydroxides, the anion being common, the cationic radius will influence the lattice enthalpy. Since lattice enthalpy decreases much more than the hydration enthalpy with increasing ionic size, the solubility increases as we go down the group.

Que. Why does the solubility of alkaline earth metal carbonates and sulphates in water decrease down the group?



Ans. The size of anions being much larger compared to cations, the lattice enthalpy will remain almost constant within a particular group. Since the hydration enthalpies decrease down the group, solubility will decrease as found for alkaline earth metal carbonates and sulphates.

Fajan's Rule

- **Just as all the covalent bonds have some partial ionic character, the ionic bonds also have partial covalent character. The partial covalent character of ionic bonds was discussed by Fajans in terms of the following rules:**
- The smaller the size of the cation and the larger the size of the anion, the greater the covalent character of an ionic bond.
- The greater the charge on the cation, the greater the covalent character of the ionic bond.
- For cations of the same size and charge, the one, with electronic configuration $(n-1)d^x ns^0$, typical of transition metals, is more polarising than the one with a noble gas configuration, $ns^2 np^6$, typical of alkali and alkaline earth metal cations.
- The cation polarises the anion, pulling the electronic charge toward itself and thereby increasing the electronic charge between the two. This is precisely what happens in a covalent bond, i.e., buildup of electron charge density between the nuclei. The polarising power of the cation, the polarisability of the anion and the extent of distortion (polarisation) of anion are the factors, which determine the per cent covalent character of the ionic bond.

Polarisation power of a cation is usually called ionic potential or charge density.

$$\text{Ionic potential } \phi (\text{phi}) = \frac{\text{Charge on cation}}{\text{Size of cation}}$$

Application of The Concept of Polarisation

- To compare the covalent and ionic character of molecule
- To compare the nature of oxide
- To compare the electrical conductivity of ionic compounds
- Tendency of the formation of complex compounds

- To compare the thermal stability of metal salts
- To compare the intensity of colour of compounds
- To compare the solubility of heavier metal halide in water.

Factors Affecting polarization

(1) Charge on cation

charge \uparrow , ϕ \uparrow , polarisation \uparrow , covalent character \uparrow , ionic character \downarrow

Order of covalent character: $\text{NaCl} < \text{MgCl}_2 < \text{AlCl}_3 < \text{SiCl}_4$

(2) Size of cation

size \downarrow , ϕ \uparrow , polarization \uparrow , covalent character \uparrow , ionic character \downarrow

Order of covalent character: $\text{LiCl} > \text{NaCl} > \text{KCl} > \text{RbCl} > \text{CsCl}$

(3) Size of anion

size \uparrow , polarization \uparrow , covalent character \uparrow , ionic character \downarrow

Order of covalent character: $\text{NaF} < \text{NaCl} < \text{NaBr} < \text{NaI}$

(4) Charge on anion

charge \uparrow , polarization \uparrow , covalent character \uparrow , ionic character \downarrow

Order of covalent character: $\text{LiF} < \text{Li}_2\text{O} < \text{Li}_3\text{N}$

(5) Type of cation

(i) Inert gas configuration cation: $ns^2 np^6$ ($8e^-$)

Ex.: $\text{Na}^+ = 1s^2 2s^2 2p^6$

(ii) Pseudo inert gas configuration cation: $ns^2 np^6 nd^{10}$ ($18e^-$)

$\text{Cu}^+ = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10}$

Some other examples are: Zn^{+2} , Ga^{+3} , Ge^{+4} , Ag^+ , Cd^{+2} , Au^+ , Hg^{+2}

(iii) Non inert gas configuration: $ns^2 np^6 nd^{10}(n+1)s^2$ ($18 + 2e^-$)

$\text{Tl}^+ = [\text{Xe}] 6s^2 4f^{14} 5d^{10}$

Note: order of polarizing power of cation

Non inert gas configuration > Pseudo inert gas configuration > inert gas Configuration



Example

Order of covalent character:

- (a) $\text{NaCl} < \text{CuCl}$ (b) $\text{KCl} < \text{AgCl}$
 (c) $\text{CaCl}_2 < \text{CdCl}_2$ (d) $\text{CaCl}_2 < \text{PbCl}_2$
 (e) $\text{CdCl}_2 < \text{PbCl}_2$

Inert Pair Effect

In p-block elements the stability of the lower oxidation state increases on descending the group. Because increased effective nuclear charge holds ns electrons tightly due to poor shielding effect of inner d & f orbitals and thereby, restrict their (ns electrons) participation in bonding only np electrons take part in bond formation. As a result of this, +1 oxidation state of Tl is more stable than its +3 oxidation state. Pb shows +2 stable oxidation state and Bi shows +3 stable oxidation state.

For example:

Group 13

- B (+3)
 Al (+3)
 Ga (+3), (+1)
 In (+3), (+1)
 Tl (+3), (+1)

Group 14

- C (+4)
 Si (+4)
 Ge (+4), (+2)
 Sn (+4), (+2)
 Pb (+4), (+2)

Order of stability:

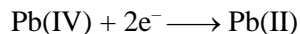
- $\text{Tl}^{+1} > \text{In}^{+1} > \text{Ga}^{+1}$ (due to inert pair effect)
 $\text{Pb}^{+2} > \text{Sn}^{+2} > \text{Ge}^{+2}$ (due to inert pair effect)

Example:

PbCl_4 is stable at room temperature whereas PbI_4 doesn't exist.

Solution:

Due to inert pair effect Pb(+4) is less stable than Pb(+2). Hence it is very good oxidant.



Reducing abilities of halides follows the sequence $\text{I}^- > \text{Br}^- > \text{Cl}^-$

Inert pair effect: Order of Stability

Due to inert pair effect

- (a) $\text{Tl}^{+1} > \text{Tl}^{+3}$ (b) $\text{Bi}^{+3} > \text{Bi}^{+5}$
 (c) $\text{Pb}^{+2} > \text{Pb}^{+4}$ (d) $\text{Hg} > \text{Hg}^{+2}$
 (e) $\text{Pb}^{+4} + 2e^- \rightarrow \text{Pb}^{+2}$ (f) $\text{Tl}^{+3} + 2e^- \rightarrow \text{Tl}^{+1}$
 (Oxidizing agent) (Oxidizing agent)
 (g) $\text{Bi}^{+5} + 2e^- \rightarrow \text{Bi}^{+3}$
 (Oxidizing agent)

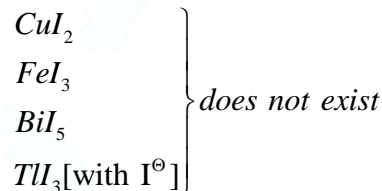
Note: PbO_2 , Pb_2O_3 ($\text{PbO} + \text{PbO}_2$) are oxidizing agents because Pb^{+2} is more stable than Pb^{+4}

Reducing power order $\text{I}^- > \text{Br}^- > \text{Cl}^- > \text{F}^-$

- (a) $\text{PbI}_4 \xrightarrow{\text{Spontaneous}} \text{PbI}_2 + \text{I}_2$
 PbI_4 does not exist because Pb^{+4} act as oxidizing agent and I^- act as reducing agent.
 $\text{Pb}^{+4} + \text{I}^- \longrightarrow \text{Pb}^{+2} + \text{I}_2$

- (b) $\text{PbBr}_4 \xrightarrow{\text{Room Temp.}} \text{PbBr}_2 + \text{Br}_2$

- (c) $\text{PbCl}_4 \xrightarrow{\text{Heating}} \text{PbCl}_2 + \text{Cl}_2$



Note: Only pentahalide of Bi is BiF_5 is possible. $[\text{F}^\ominus]$ is weak reducing agent]

- (a) $\text{Cu}^{+2} + \text{I}^\ominus \longrightarrow \text{Cu}_2\text{I}_2 + \text{I}_2$
 (O.A.) (R.A.)
 (b) $\text{Fe}^{+3} + \text{I}^\ominus \longrightarrow \text{Fe}^{+2} + \text{I}_2$
 (O.A.) (R.A.)
 (c) $\text{Bi}^{+5} + \text{I}^\ominus \longrightarrow \text{Bi}^{+3} + \text{I}_2$
 (O.A.) (R.A.)
 (d) $\text{Tl}^{+3} + 3\text{I}^\ominus \longrightarrow \text{Tl}^{+1} + \text{I}_2$
 (O.A.) (R.A.)

But TlI_3 [with I_3^\ominus] exists, $\text{TlI}_3 \rightarrow \text{Tl}^{+1} + \text{I}_3^\ominus$

Stability of higher oxidation state ↓ down the group

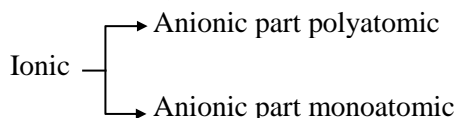
$\text{CX}_4 > \text{SiX}_4 > \text{GeX}_4 > \text{SnX}_4 > \text{PbX}_4$ (Where X is F, Cl, Br, I)

Stability of lower oxidation state ↑ down the group.

$\text{CX}_2 < \text{SiX}_2 < \text{GeX}_2 < \text{SnX}_2 < \text{PbX}_2$ (Where X is F, Cl, Br, I)

Application of Fajan's Rule

Thermal stability





Thermal stability of Ionic compound having polyatomic anion

Order of Thermal stability

- (a) $\text{Li}_2\text{CO}_3 < \text{Na}_2\text{CO}_3 < \text{K}_2\text{CO}_3 < \text{Rb}_2\text{CO}_3 < \text{Cs}_2\text{CO}_3$
 (b) $\text{BeCO}_3 < \text{MgCO}_3 < \text{CaCO}_3 < \text{SrCO}_3 < \text{BaCO}_3$
 (c) $\text{Li}_2\text{SO}_4 < \text{Na}_2\text{SO}_4 < \text{K}_2\text{SO}_4 < \text{Rb}_2\text{SO}_4 < \text{Cs}_2\text{SO}_4$
 (d) $\text{BeSO}_4 < \text{MgSO}_4 < \text{CaSO}_4 < \text{SrSO}_4 < \text{BaSO}_4$
 (e) $\text{LiNO}_3 < \text{NaNO}_3 < \text{KNO}_3 < \text{RbNO}_3 < \text{CsNO}_3$
 (f) $\text{Be}(\text{NO}_3)_2 < \text{Mg}(\text{NO}_3)_2 < \text{Ca}(\text{NO}_3)_2 < \text{Sr}(\text{NO}_3)_2 < \text{Ba}(\text{NO}_3)_2$
 (g) $\text{NaHCO}_3 < \text{KHCO}_3 < \text{RbHCO}_3 < \text{CsHCO}_3$
 (h) $\text{Be}(\text{HCO}_3)_2 < \text{Mg}(\text{HCO}_3)_2 < \text{Ca}(\text{HCO}_3)_2 < \text{Sr}(\text{HCO}_3)_2 < \text{Ba}(\text{HCO}_3)_2$

Thermal stability of ionic compound having monoatomic anion

$\text{MX} \rightarrow \text{M}^+ + \text{X}^-$ [Thermal Stability decided by Lattice Energy]

$$\text{L.E} = Kq_1q_2/r^2(r_c + r_a)$$

$$\Rightarrow \text{L.E.} \propto \text{charge} \quad \text{L.E.} \propto \frac{1}{\text{size}}$$

\Rightarrow Charge \uparrow , L.E. $\uparrow \Rightarrow$ size \uparrow , L.E. \downarrow
 Thermal stability \propto Lattice energy

Example:

- (a) $\text{LiCl} > \text{NaCl} > \text{KCl} > \text{RbCl} > \text{CsCl}$
 (b) $\text{LiBr} > \text{NaBr} > \text{KBr} > \text{RbBr} > \text{CsBr}$
 (c) $\text{Li}_2\text{O} > \text{Na}_2\text{O} > \text{K}_2\text{O} > \text{Rb}_2\text{O} > \text{Cs}_2\text{O}$
 (d) $\text{Li}_3\text{N} > \text{K}_3\text{N} > \text{Rb}_3\text{N}$
 (e) $\text{KF} > \text{KCl} > \text{KBr} > \text{KI}$

Colour of Ionic compound

Higher the polarization of ionic compound greater will be the colour intensity of ionic compound.



Que. Predict the colour of CuCl and CuBr if colour of CuI is white:

Ans. White

Ionic Mobility

$\phi \uparrow$, Hydrate size \uparrow , Ionic mobility \downarrow

Example:

- (i) Hydrate Size $\text{Li}^+_{(\text{aq})} > \text{Na}^+_{(\text{aq})} > \text{K}^+_{(\text{aq})} > \text{Rb}^+_{(\text{aq})} > \text{Cs}^+_{(\text{aq})}$
 Ionic Mobility $\text{Li}^+_{(\text{aq})} < \text{Na}^+_{(\text{aq})} < \text{K}^+_{(\text{aq})} < \text{Rb}^+_{(\text{aq})} < \text{Cs}^+_{(\text{aq})}$
 (ii) Hydrate Size $\text{Be}^{+2}_{(\text{aq.})} > \text{Mg}^{+2}_{(\text{aq.})} > \text{Ca}^{+2}_{(\text{aq.})} > \text{Sr}^{+2}_{(\text{aq.})} > \text{Ba}^{+2}_{(\text{aq.})}$
 Ionic Mobility $\text{Be}^{+2}_{(\text{aq.})} < \text{Mg}^{+2}_{(\text{aq.})} < \text{Ca}^{+2}_{(\text{aq.})} < \text{Sr}^{+2}_{(\text{aq.})} < \text{Ba}^{+2}_{(\text{aq.})}$
 (iii) Hydrate Size $\text{F}^-_{(\text{aq.})} > \text{Cl}^-_{(\text{aq.})} > \text{Br}^-_{(\text{aq.})} > \text{I}^-_{(\text{aq.})}$
 Ionic Mobility $\text{F}^-_{(\text{aq.})} < \text{Cl}^-_{(\text{aq.})} < \text{Br}^-_{(\text{aq.})} < \text{I}^-_{(\text{aq.})}$

Exception Case

- (iv) Note: ϕ of: $\text{Be}^{+2} > \text{Al}^{+3}$
 6.4 6

Hydrate Size $\text{Be}^{+2}_{(\text{aq.})} > \text{Al}^{+3}_{(\text{aq.})}$

Ionic Mobility $\text{Be}^{+2}_{(\text{aq.})} < \text{Al}^{+3}_{(\text{aq.})}$

Solubility

Case-I: Down the group, H.E. \downarrow , L.E. \downarrow , solubility \downarrow

- (a) $\text{LiF} < \text{NaF} < \text{KF} < \text{RbF} < \text{CsF}$
 (b) $\text{BeO} < \text{MgO} < \text{CaO} < \text{SrO} < \text{BaO}$
 (c) $\text{BeS} < \text{MgS} < \text{CaS} < \text{SrS} < \text{BaS}$
 (d) $\text{LiOH} < \text{NaOH} < \text{KOH} < \text{RbOH} < \text{CsOH}$

Case-II: Down the group, solubility \downarrow

- (a) $\text{LiClO}_3 > \text{NaClO}_3 > \text{KClO}_3 > \text{RbClO}_3 > \text{CsClO}_3$
 (chlorate)
 (b) $\text{LiClO}_4 > \text{NaClO}_4 > \text{KClO}_4 > \text{RbClO}_4 > \text{CsClO}_4$
 (perchlorate)
 (c) $\text{BeSO}_3 > \text{MgSO}_3 > \text{CaSO}_3 > \text{SrSO}_3 > \text{BaSO}_3$
 (sulphite)
 (d) $\text{BeSO}_4 > \text{MgSO}_4 > \text{CaSO}_4 > \text{SrSO}_4 > \text{BaSO}_4$
 (sulphate)
 (e) $\text{BeS}_2\text{O}_3 > \text{MgS}_2\text{O}_3 > \text{CaS}_2\text{O}_3 > \text{SrS}_2\text{O}_3 > \text{BaS}_2\text{O}_3$
 (thiosulphate)
 (f) $\text{BeCrO}_4 > \text{MgCrO}_4 > \text{CaCrO}_4 > \text{SrCrO}_4 > \text{BaCrO}_4$
 (chromate)
 (g) $\text{BeCO}_3 > \text{MgCO}_3 > \text{CaCO}_3 > \text{SrCO}_3 > \text{BaCO}_3$
 (carbonate)

Note: Br^-/I^- : They follow trend of polyatomic

- (a) $\text{LiBr} > \text{NaBr} > \text{KBr} > \text{RbBr} > \text{CsBr}$

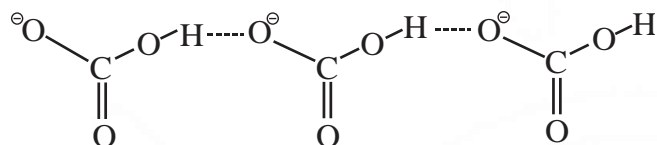
(b) $\text{NaI} > \text{LiI} > \text{KI} > \text{RbI} > \text{CsI}$

Case –III: If number of cations > number of anions,
then solubility \uparrow down the group.

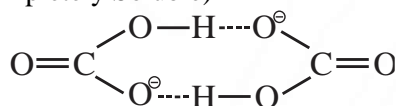
- (a) $\text{Li}_2\text{O} < \text{Na}_2\text{O} < \text{K}_2\text{O} < \text{Rb}_2\text{O} < \text{Cs}_2\text{O}$
 (b) $\text{Li}_2\text{CO}_3 < \text{Na}_2\text{CO}_3 < \text{K}_2\text{CO}_3 < \text{Rb}_2\text{CO}_3 < \text{Cs}_2\text{CO}_3$
 (c) $\text{NaHCO}_3 < \text{KHCO}_3 < \text{RbHCO}_3 < \text{CsHCO}_3$
 (Sparingly soluble)

Reason:

Massive H bonding in NaHCO_3 (Sparingly Soluble)



Compact H-bonding in KHCO_3 , RbHCO_3 , CsHCO_3
 (Completely Soluble)



Que. Find order of K_{sp} .

Solubility \uparrow , $K_{sp} \uparrow$

$\text{NaHCO}_3 < \text{KHCO}_3 < \text{RbHCO}_3 < \text{CsHCO}_3$

Case (IV): Number of monoatomic anion > Number
of cation, then solubility \uparrow down the
group.

$\text{Be}(\text{OH})_2 < \text{Mg}(\text{OH})_2 < \text{Ca}(\text{OH})_2 < \text{Sr}(\text{OH})_2$

But if number of polyatomic anion > number of
cation, then solubility \downarrow down the group.

$\text{Be}(\text{NO}_3)_2 > \text{Mg}(\text{NO}_3)_2 > \text{Ca}(\text{NO}_3)_2 > \text{Sr}(\text{NO}_3)_2 > \text{Ba}(\text{NO}_3)_2$

$\text{BeX}_2 > \text{MgX}_2 > \text{CaX}_2 > \text{SrX}_2 > \text{BaX}_2$ ($\text{X} = \text{Cl}, \text{Br}, \text{I}$) – Exception to key point

Key point (For Cases I to IV)

If number of polyatomic anion \geq Number of cation
then solubility \downarrow down the group, otherwise solubility
increases down the group.

Alkaline earth metals

- (a) $\text{BeS}_2\text{O}_3 > \text{MgS}_2\text{O}_3 > \text{CaS}_2\text{O}_3 > \text{SrS}_2\text{O}_3 > \text{BaS}_2\text{O}_3$
 (Thiosulphate)
 (b) $\text{BeCrO}_4 > \text{MgCrO}_4 > \text{CaCrO}_4 > \text{SrCrO}_4 > \text{BaCrO}_4$
 (Chromate)
 (c) $\text{BeSO}_3 > \text{MgSO}_3 > \text{CaSO}_3 > \text{SrSO}_3 > \text{BaSO}_3$

(Sulphite)

Hydroxide

- (a) $\text{Be}(\text{OH})_2 < \text{Mg}(\text{OH})_2 < \text{Ca}(\text{OH})_2 < \text{Sr}(\text{OH})_2$
 (b) $\text{LiOH} < \text{NaOH} < \text{KOH} < \text{RbOH} < \text{CsOH}$

Carbonate

- (a) $\text{BeCO}_3 > \text{MgCO}_3 > \text{CaCO}_3 > \text{SrCO}_3 > \text{BaCO}_3$
 (b) $\text{Li}_2\text{CO}_3 < \text{Na}_2\text{CO}_3 < \text{K}_2\text{CO}_3 < \text{Rb}_2\text{CO}_3 < \text{Cs}_2\text{CO}_3$

Nitrate

- (a) $\text{LiNO}_3 > \text{NaNO}_3 > \text{KNO}_3 > \text{RbNO}_3 > \text{CsNO}_3$
 (b) $\text{Be}(\text{NO}_3)_2 > \text{Mg}(\text{NO}_3)_2 > \text{Ca}(\text{NO}_3)_2 > \text{Sr}(\text{NO}_3)_2 > \text{Ba}(\text{NO}_3)_2$

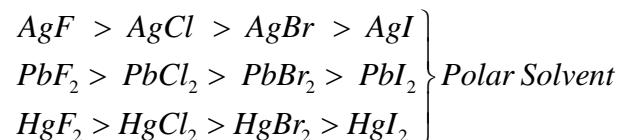
Other example

- (a) $\text{BeS} < \text{MgS} < \text{CaS} < \text{SrS} < \text{BaS}$
 (b) $\text{LiF} < \text{NaF} < \text{KF} < \text{RbF} < \text{CsF}$
 (c) $\text{BeO} < \text{MgO} < \text{CaO} < \text{SrO} < \text{BaO}$
 (d) $\text{LiClO}_4 > \text{NaClO}_4 > \text{KClO}_4 > \text{RbClO}_4 > \text{CsClO}_4$
 (e) $\text{LiClO}_3 > \text{NaClO}_3 > \text{KClO}_3 > \text{RbClO}_3 > \text{CsClO}_3$
 (f) $\text{BeSO}_4 > \text{MgSO}_4 > \text{CaSO}_4 > \text{SrSO}_4 > \text{BaSO}_4$

Exception in solubility

- $\text{LiCl} > \text{NaCl} > \text{KCl} < \text{RbCl} < \text{CsCl}$
or
 $\text{LiCl} > \text{CsCl} > \text{RbCl} > \text{NaCl} > \text{KCl}$
- $\text{MgF}_2 < \text{CaF}_2 < \text{SrF}_2 < \text{BaF}_2 < \text{BeF}_2$
- Oxalate $\text{MgC}_2\text{O}_4 < \text{CaC}_2\text{O}_4 < \text{SrC}_2\text{O}_4 < \text{BaC}_2\text{O}_4 < \text{BeC}_2\text{O}_4$

Solubility order of heavier metal halides (in polar solvent)



Nature of oxide

Order of acidic character:

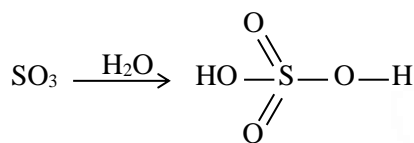
- (a) $\text{Li}_2\text{O} > \text{Na}_2\text{O} > \text{K}_2\text{O} > \text{Rb}_2\text{O} > \text{Cs}_2\text{O}$
 (b) $\text{MgO} > \text{CaO} > \text{SrO} > \text{BaO}$
 (c) $\text{ZnO} > \text{MgO} > \text{CaO} > \text{SrO} > \text{BaO}$
 (d) $\text{NiO} > \text{CaO} > \text{SrO} > \text{BaO}$
 (e) $\text{PbO} > \text{CaO}$
 (f) $\text{PbO} < \text{PbO}_2$



(g) $\text{SnO} < \text{SnO}_2$

s-block-metal oxide are basic except BeO

Nature of non-metallic oxides:



E.N of non-metal \uparrow , acidic nature \uparrow

(a) $\text{CO}_2 > \text{SiO}_2$

(b) $\text{CO} < \text{CO}_2$ positive charge \uparrow , EN \uparrow , acidic nature \uparrow

(c) $\text{SO}_2 < \text{SO}_3$

(d) $\text{N}_2\text{O} < \text{NO} < \text{N}_2\text{O}_3 < \text{N}_2\text{O}_4 < \text{N}_2\text{O}_5$

(e) $\text{H}_2\text{SO}_3 < \text{H}_2\text{SO}_4$

(f) $\text{HClO} < \text{HClO}_2 < \text{HClO}_3 < \text{HClO}_4$

(g) $\text{HNO}_2 < \text{HNO}_3$

(h) $\text{Na}_2\text{O} < \text{MgO} < \text{Al}_2\text{O}_3 < \text{SiO}_2$
Strong Basic Basic Amphoteric weak acid

$< \text{P}_2\text{O}_5 < \text{SO}_3 < \text{Cl}_2\text{O}_7$
Acidic Strong acid Strong acid

(i) MnO MnO_2 Mn_2O_7
Basic Amphoteric acidic

(j) CrO Cr_2O_3 CrO_3
Basic Amphoteric Acidic

Amphoteric oxides

(a) Oxides of Pb, Zn, Be, Al, Ga, Sn, Cr^{+3}

(b) As_2O_3 , Sb_2O_3 , V_2O_5

Melting Point

Melting point \propto Lattice energy

Lattice energy $\propto \frac{1}{\text{size}}$, Lattice energy \propto charge

(a) $\text{LiF} > \text{LiCl} > \text{LiBr} > \text{LiI}$

(b) $\text{CaF}_2 > \text{CaCl}_2 > \text{CaBr}_2 > \text{CaI}_2$

(c) $\text{NaF} < \text{AlF}_3$

(d) $\text{ZnF}_2 > \text{ZnCl}_2$

Exception:

1. $\text{NaCl} > \text{KCl} > \text{RbCl} > \text{CsCl} > \text{LiCl}$
2. $\text{KI} > \text{NaI} > \text{RbI} > \text{CsI} > \text{LiI}$
3. $\text{NaF} > \text{KF} > \text{LiF} > \text{RbF} > \text{CsF}$
4. $\text{CaF}_2 > \text{MgF}_2 > \text{SrF}_2 > \text{BaF}_2 > \text{BeF}_2$
5. $\text{BeX}_2 < \dots < \text{BaX}_2$ X = (Cl, Br, I)
6. $\text{MgO} > \text{CaO} > \text{BeO} > \text{SrO} > \text{BaO}$

FUNDAMENTAL UNLOCKED- (FU#2) :

Q.1 In which of the following species the bonds are non-directional ?

- (A) NCl_3 (B) RbCl
(C) BeCl_2 (D) BCl_3

Q.2 Out of following which one has least value of melting point

- (A) LiCl (B) BeCl_2
(C) MgCl_2 (D) CaCl_2

Q.3 Out of following which one has maximum ionic character -

- (A) NaCl (B) KCl
(C) CaCl_2 (D) MgCl_2

Q.4 Which of the following has highest melting point -

- (A) NaCl (B) NaI
(C) NaBr (D) NaF

Q.5 Among LiCl , BeCl_2 , BCl_3 and CCl_4 , the covalent bond character follows the order -

- (A) $\text{LiCl} < \text{BeCl}_2 > \text{BCl}_3 > \text{CCl}_4$
(B) $\text{LiCl} > \text{BeCl}_2 < \text{BCl}_3 < \text{CCl}_4$
(C) $\text{LiCl} < \text{BeCl}_2 < \text{BCl}_3 < \text{CCl}_4$
(D) $\text{LiCl} > \text{BeCl}_2 > \text{BCl}_3 > \text{CCl}_4$

Molecular orbital theory (MOT)

Given by Hunds & Mulliken

1. Two atomic orbital come nearer & then overlap each other to form two molecular orbitals
2. Combination of atomic orbital (AO) forms molecular orbital (MO)

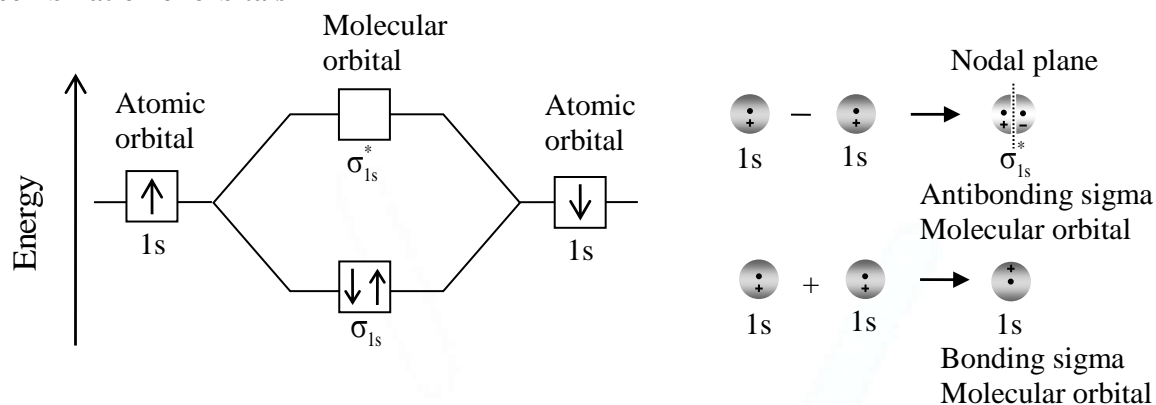
Types of molecular orbitals

Molecular orbitals of diatomic molecules are designated as σ (sigma), π (pie), δ (delta) etc.

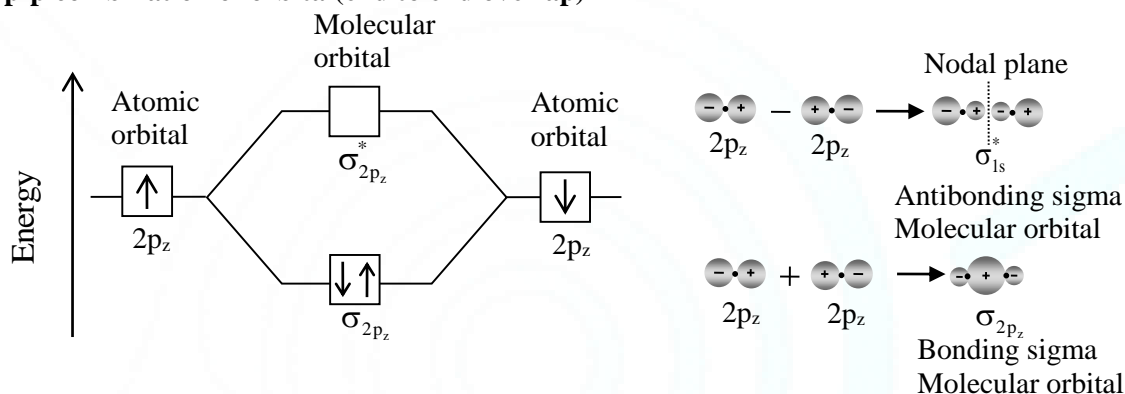
In this nomenclature, the sigma (σ) molecular orbitals are symmetrical around the inter molecular axis (assumed to be z-axis) while pi (π) molecular orbitals are not symmetrical.



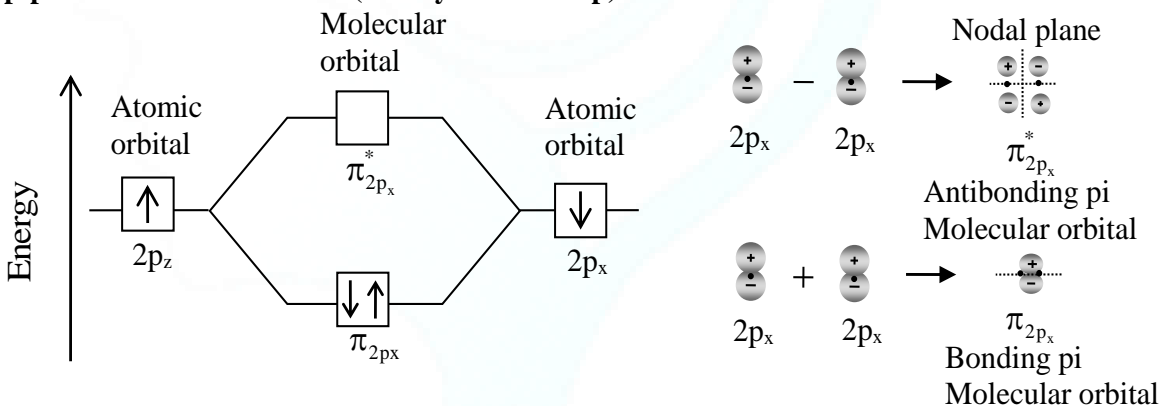
(a) s-s combination of orbitals



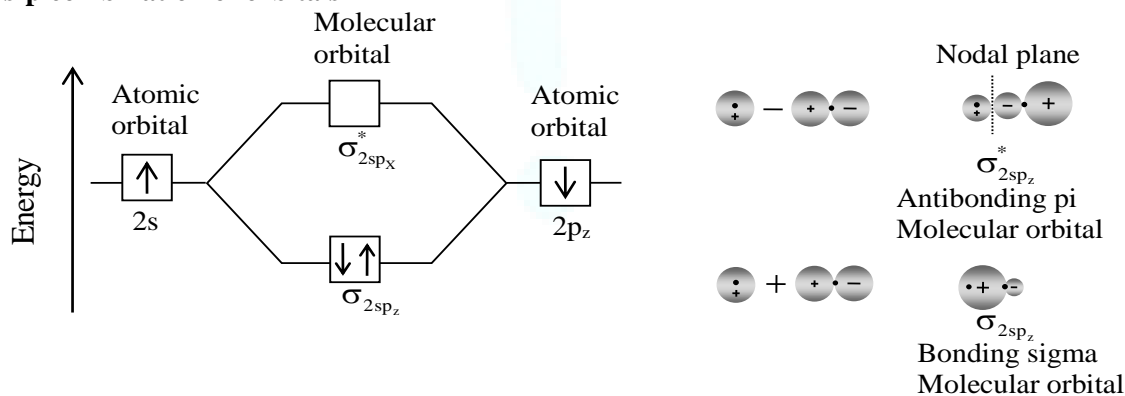
(b) p-p combination of orbital(end to end overlap)



(c) p-p combination of orbitals (side by side overlap)



(d) s-p combination of orbitals





3. Energy of BMO < Energy of ABMO.
4. Molecular orbitals can be filled by electrons according to Aufbau, Hund's, Pauli's principle.
5. Energy order of the molecular orbitals of homonuclear di-atomic molecules.

Note: Molecular orbital energy order for up to N_2 (molecule having ≤ 14 electrons)

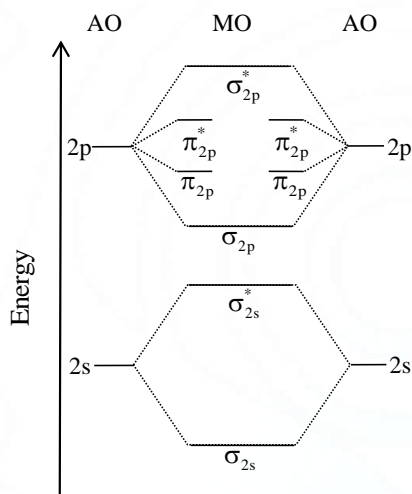
$$\sigma_{1s} < \sigma_{1s}^* < \sigma_{2s} < \sigma_{2s}^* < \pi_{2p_x}$$

Example:

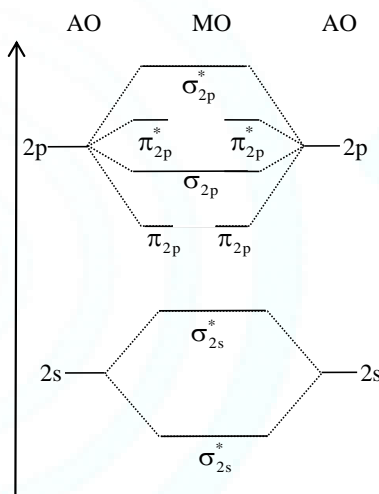
Why molecular orbitals have different order of energy in N_2 & O_2 ?

Sol. s-p mixing

Hint



The correct MO energy-level diagram When s-p mixing is not allowed.



The correct MO energy-level diagram When s-p mixing is allowed, the energies of the σ_{sp} and π_{2p} orbitals are reversed.

Bond Order

Bond order can be defined as:

$$\text{Bond order} = \frac{N_b - N_a}{2}$$

N_b = No. of electron in bonding MO's

N_a = No. of electron in antibonding MO's

If bond order = 0, it means species does not exist.

Bond order of 1, 2 & 3 corresponds to a single bond, double & triple bond respectively.

Bond order \uparrow stability of molecule \uparrow bond length \downarrow

Magnetic behaviour

If the molecule has one or more unpaired electron, it will be paramagnetic,

If all the electrons are paired it will be diamagnetic.

Magnetic strength can be calculated by using spin only formula of magnetic moment (μ).

$$= \pi_{2p_y} < \sigma_{2p_z} < \pi_{2p_x}^* = \pi_{2p_y}^* < \sigma_{2p_z}^*$$

Note: Molecular orbital energy order for O_2 and F_2 (molecule having > 14 electrons)

$$\sigma_{1s} < \sigma_{1s}^* < \sigma_{2s} < \sigma_{2s}^* < \sigma_{2p_z} < \pi_{2p_x} = \pi_{2p_y} < \pi_{2p_x}^* = \pi_{2p_y}^* < \sigma_{2p_z}^*$$

σ^*, π^* = antibonding molecular orbital

σ, π = bonding molecular orbital

$$\mu = \sqrt{n(n+2)} \text{ B.M. (where } n = \text{number of unpaired electron)}$$

Example:

$$H_2 = \text{Configuration: } \sigma_{(1s)}^2, \sigma_{(1s)}^{*0}$$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{2 - 0}{2} = 1, \text{ Hence H - H}$$

(diamagnetic)

Order of Ionisation Energy

(a) $N_2 > O_2$ (b) $N_2 > N$ (c) $O_2 < O$

(d) $N_2 > O$ (e) $F_2 > F$

For Axial

After two rotation from 180° , If sign is changed then orbital is ungerade, if not then gerade.

Gerade molecular orbital have even no. of nodal



plane and ungerade molecular orbital have odd no. of nodal plane.

Que. Halogen are diamagnetic but colourful explain:

Colours: F_2 : Pale yellow
 Cl_2 : Greenish yellow
 Br_2 : Reddish-Brown
 I_2 : Violet

Sol. Halogen are colourful due to HOMO-LUMO transition.

HOMO: Highest occupied molecular orbital

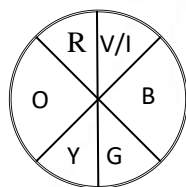
LUMO: Lowest unoccupied molecular orbital

In F_2 molecule HOMO is ABMO

In O_2 molecule HOMO is ABMO

In N_2 molecule HOMO is BMO

In CO molecule HOMO is NBMO

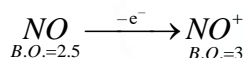


VIBGYOR

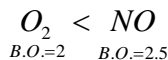


Note:

- If bond order is fractional, then it wants to make it integer greater than previous



- If bond order is greater, then molecule is more stable.



- CO CO⁺

Bond order: 3 > 3

Bond length: 1.128 Å 1.115 Å

Electron is removed from NBMO having slight anti-bonding character, so bond order slightly increase

FUNDAMENTAL UNLOCKED- (FU#3) :

- Q.1** In the conversion of N_2 into N_2^{2+} the electron will be lost from which of the following molecular orbitals ?

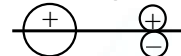
(A) $\sigma_{2p_z}^*$ (B) σ_{2p_z} (C) π_{2p_x} (D) $\pi_{2p_x}^*$

- Q.2** The bond orders in BN, BO and CO respectively are -

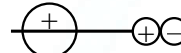
(A) 2, 3, 5/2 (B) 2, 5/2, 2
 (C) 2, 5/2, 3 (D) 5/2, 2, 3

- Q.3** Which of the following leads to bonding ?

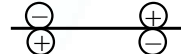
(A) s-orbital p-orbital



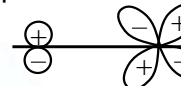
(B) s-orbital p-orbital



(C) p-orbital p-orbital



(D) p-orbital d-orbital



- Q.4** In an antibonding molecular orbital, electron density is minimum -

(A) Around one atom of the molecule
 (B) Between the two nuclei of the molecule
 (C) At the region away from the nuclei of the molecule
 (D) at no place

- Q.5** Which have odd bond order -

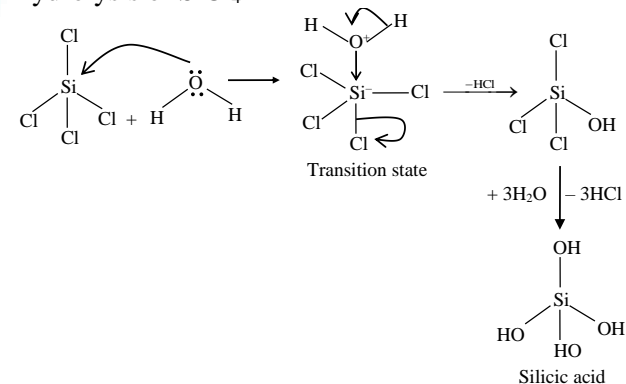
(A) O_2^+ (B) O_2^- (C) NO (D) All

Hydrolysis

In hydrolysis of covalent molecules the nucleophilic centre of molecule is replaced by OH^- group of water generally through nucleophilic substitution reaction.

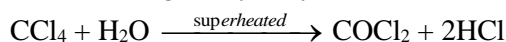
Example:

Hydrolysis of $SiCl_4$



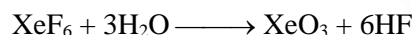
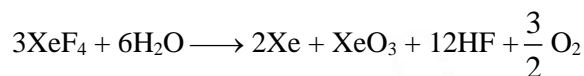
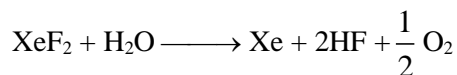
Note:

CCl_4 , NF_3 , is inert towards hydrolysis due to the absence of d orbital, but under drastic condition these molecules under goes hydrolysis.





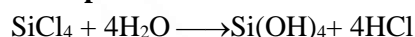
Note: Hydrolysis of XeF_2 & XeF_4 takes place through with redox reaction.



Example:

$\text{CCl}_4 + \text{H}_2\text{O} \longrightarrow$ No hydrolysis due to absence of vacant d-orbital (possible through SN^1 mechanism)

Example:



Explanation:

- Product is oxyacid as well as hydracids.
- Hybridisation of reactant is sp^3 but that of transition state is sp^3d .
- Generally the oxyacid form of central atom and same oxidation state.
- Generally the hydracid form by surrounding atom.

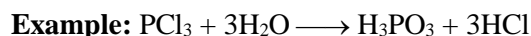
SOLVED EXAMPLE

Example:



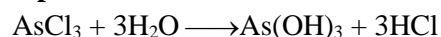
Explanation:

- Product is base and oxyacid.
- Hybridisation of central atom doesn't change in transition state.
- Electronegativity of nitrogen and chlorine is almost equal.



Note: Basicity of $\text{H}_3\text{PO}_3 = 2$

Example:



Note: Basicity of $\text{H}_3\text{AsO}_3 = 3$

Q.1 Compare rate of hydrolysis?

- (A) MgCl_2 (B) AlCl_3
(C) CCl_4 (D) SiCl_4
(E) PCl_5

Ans.

Correct order $\text{CCl}_4 < \text{MgCl}_2 < \text{AlCl}_3 < \text{SiCl}_4 < \text{PCl}_5$

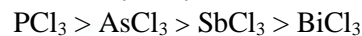
- (A) SnCl_2 (B) SnCl_4

Ans. $\text{SnCl}_2 < \text{SnCl}_4$

- (A) SnCl_4 (B) SnMe_4

Ans. $\text{SnCl}_4 > \text{SnMe}_4$

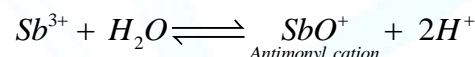
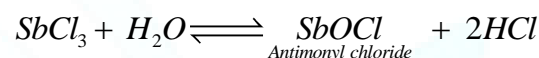
Rate of hydrolysis order



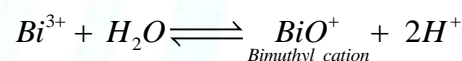
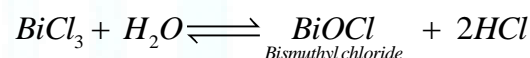
Note:

- BeX_2 – Predominantly covalent ($\text{X} = \text{F}, \text{Cl}, \text{Br}, \text{I}$)
- MgX_2 – Predominantly covalent ($\text{X} = \text{Cl}, \text{Br}, \text{I}$)
- AlX_3 – Predominantly covalent ($\text{X} = \text{Cl}, \text{Br}, \text{I}$)
- $\text{LiF}/\text{MgF}_2/\text{AlF}_3$ – Predominantly ionic

Example:



Example:



Q.2 Which of the following ions are present in BiOCl ?

- (A) Bi^+ (B) BiO^+ (C) OCl^- (D) Cl^-

Ans. (BD)

Remember:

Parent Oxyacid of following atoms:

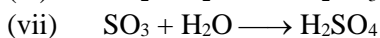
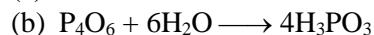
- | | | |
|---------------------------------|--|--|
| (a) $\text{P} \longrightarrow$ | (i) H_3PO_3 | (ii) H_3PO_4 |
| (b) $\text{S} \longrightarrow$ | (i) H_2SO_3 | (ii) H_2SO_4 |
| (c) $\text{N} \longrightarrow$ | (i) HNO_2 | (ii) HNO_3 |
| (d) $\text{Cl} \longrightarrow$ | (i) HClO | (ii) HClO_2 |
| | (iii) HClO_3 | (iv) HClO_4 |
| (e) $\text{Se} \longrightarrow$ | H_2SeO_4 , | $\text{Te} \longrightarrow \text{H}_6\text{TeO}_6$, |
| | $\text{B} \longrightarrow \text{H}_3\text{BO}_3$, | $\text{Si} \longrightarrow \text{H}_4\text{SiO}_4$ |

Q.3 Identify the product obtained on hydrolysis?

Solution:

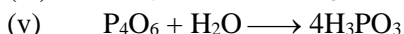
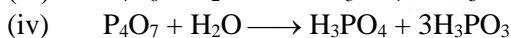
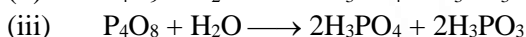
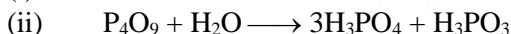
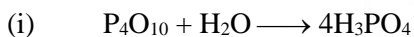
- $\text{Cl}_2\text{O} + \text{H}_2\text{O} \longrightarrow \text{HClO}$
- $\text{SF}_4 + \text{H}_2\text{O} \longrightarrow \text{H}_2\text{SO}_3 + \text{HF}$
- $\text{SO}_2\text{Cl}_2 + \text{H}_2\text{O} \longrightarrow \text{H}_2\text{SO}_4 + \text{HCl}$
- $\text{N}_2\text{O}_3 + \text{H}_2\text{O} \longrightarrow \text{HNO}_2$
- $\text{N}_2\text{O}_5 + \text{H}_2\text{O} \longrightarrow \text{HNO}_3$



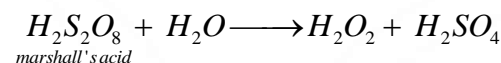

Addition and addition elimination reaction


Q.4 Identify the product obtained on hydrolysis?

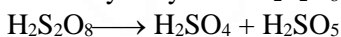
Solution:



Example:



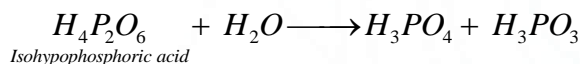
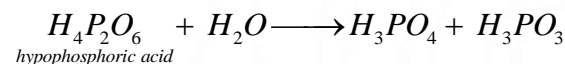
(i) Partial hydrolysis of $H_2S_2O_8$



(ii) Complete hydrolysis of $H_2S_2O_8$



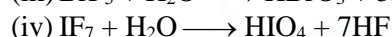
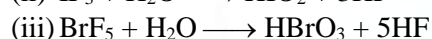
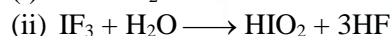
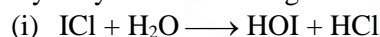
Example:



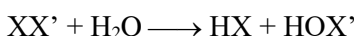
Note: Product obtained after hydrolysis of both given compound is same.

Important

Hydrolysis of Interhalogen compound



Q.5 Which of the following statement is correct about X and X'?



(A) X is more electronegative than X'

(B) X' has larger size than X

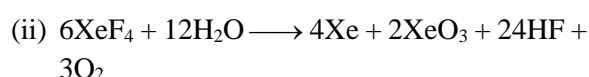
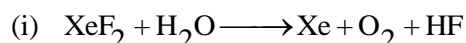
(C) Both (A) and (B)

(D) None of these

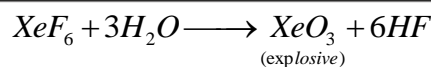
Ans. (C)

Important

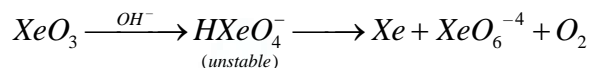
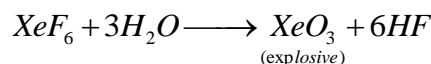
Hydrolysis of noble gases compound



(iii) Hydrolysis of XeF_6

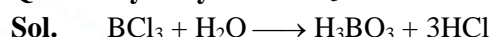


(iv) Alkaline hydrolysis of XeF_6



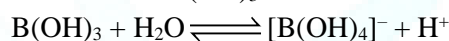
Two gaseous product Xe and O_2 are obtained during alkaline hydrolysis.

Q.6 **Hydrolysis of BCl_3**

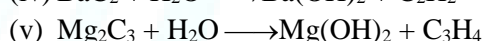


Product of hydrolysis of BCl_3 is H_3BO_3 and HCl .

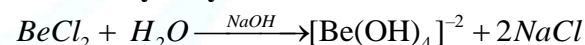
Ionisation of $B(OH)_3$:



Hydrolysis of Carbide



Alkaline hydrolysis of $BeCl_2$



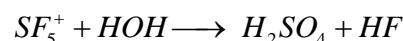
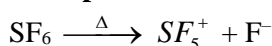
(vii) Rate of hydrolysis

Order: $SF_6 < SeF_6 < TeF_6$

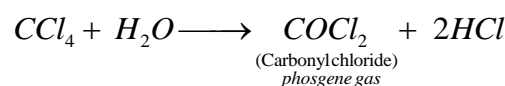
Conclusion: Size \uparrow , steric crowding \downarrow , rate of hydrolysis \uparrow

SN^1 /Drastic condition

Example:



Example:



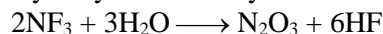
Note: CCl_4 is a fire extinguisher but it is not used as fire extinguisher at high temperature because it forms $COCl_2$





Example:

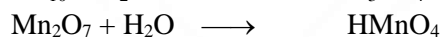
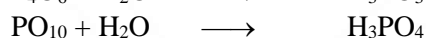
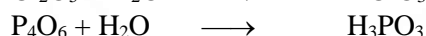
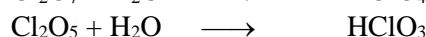
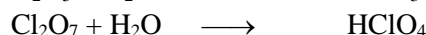
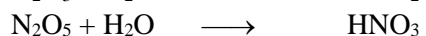
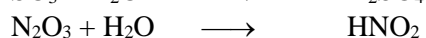
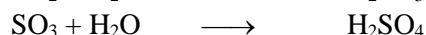
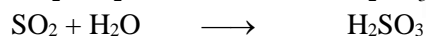
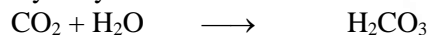
Hydrolysis of NF_3 by SN^1 mechanism:



Note: Via SN^1 , hydrolysis of SF_6 , CCl_4 , NF_3 Can be done.

Other example

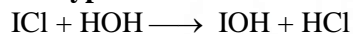
Hydrolysis of oxide



An example of disproportionation reaction required bond

Combinatic of group number & oxidation state must be in odd & even.

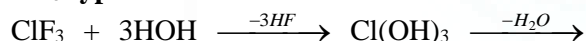
1. AX type



Hypo Halous HXO Halous HXO_2

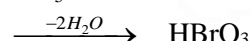
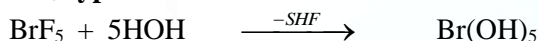
Hallic HXO_3 Per halic HXO_4

2. AX₃ type

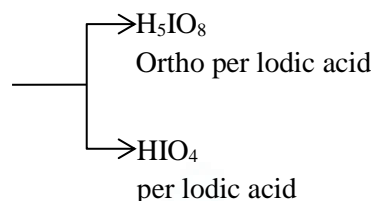
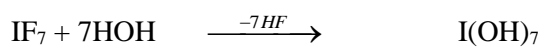


HClO_2 Chlonis

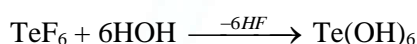
3. AX₅ type



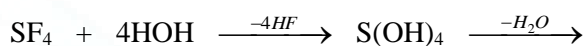
4. AX₇ type



Hydrolysis of TeF_6



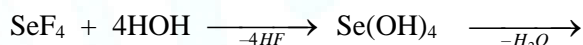
Hydrolysis of SF_4



H_2SO_3

Sp^3d

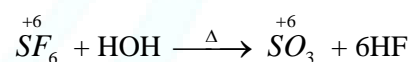
Hydrolysis of SeF_4



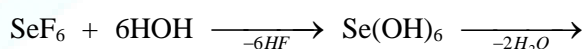
H_2SeO_3

Sp^3d

Hydrolysis of SF_6



Hydrolysis of SeF_6



H_2SeO_4

Sp^3d^2

ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1)

1. (B) 2. (D) 3. (A) 4. (C) 5. (A)

FUNDAMENTAL UNLOCKED- (FU#2)

1. (B) 2. (B) 3. (B) 4. (D) 5. (C)

FUNDAMENTAL UNLOCKED- (FU#3)

1. (B) 2. (C) 3. (B) 4. (B) 5. (D)




OBJECTIVE EXERCISE-I
Single Correct Type Question
Back Bonding

- Which of the following will not form adduct ?
(A) $(\text{CH}_3)_2\text{O} + \text{BF}_3$ (B) $(\text{SiH}_3)_2\text{O} + \text{BF}_3$
(C) $\text{NH}_3 + \text{BF}_3$ (D) $\text{CH}_3\text{NH}_2 + \text{BF}_3$
- Which of the following has highest bond energy?
(A) C-F in (CF_4) (B) C-Cl in (CCl_4)
(C) C-Br in (CBr_4) (D) B-F in (BF_3)

Multicentered Bond / Bridge Bonding

- In which of the following molecules/species all following characteristics are found ?
(a) sp^3 hybridisation of central atom
(b) Hybridisation can be considered to have taken place with the help of empty orbital(s).
(c) All bond lengths are identical i.e. all A-B bond lengths are identical.
(A) B_2H_6 (B) Al_2Cl_6
(C) $\text{BeCl}_2(\text{g})$ (D) BF_4^-
- Which of the following ligand is not responsible for symmetrical bond cleavage in B_2H_6 ?
(A) Me_2S (B) H^- (C) THF (D) NH_3
- Which of the following molecule have 3C-4e⁻ bond as well as planar geometry ?
(A) Al_2Br_6 (B) Al_2I_6
(C) I_2Cl_6 (D) $(\text{AlH}_3)_n$

Odd Electron Species

- Find the odd electron specie in which odd electron is present in pure 'p' atomic orbital.
(A) $\dot{\text{N}}\text{O}_2$ (B) $\dot{\text{Cl}}\text{O}_2$
(C) $\dot{\text{C}}\text{F}_3$ (D) $\dot{\text{C}}\text{H}_3$

Inert Pair Effect

- Which of the following statements is **incorrect**?
(A) Oxidizing power order: $\text{SiCl}_4 < \text{SnCl}_4 < \text{PbCl}_4$
(B) Ionic character order: $\text{CsBr} > \text{RbBr} > \text{KBr} > \text{NaBr} > \text{LiBr}$

- (C) The ionic character of lead(II) halides decreases with increase in atomic no. of halogen
(D) The oxidation state of Tl in TlI_3 is +3.

Ionic Compound

- Compound having lowest Melting point.
(A) BeCl_2 (B) MgCl_2 (C) CaCl_2 (D) SrCl_2
- Which of the following order is correct of the given property.
(A) $\text{LiCl} > \text{NaCl} > \text{KCl} > \text{RbCl} > \text{CsCl}$
: Thermal stability order
(B) $\text{BeF}_2 < \text{MgF}_2 < \text{CaF}_2 < \text{SrF}_2 < \text{BaF}_2$
: solubility order
(C) $\text{NO}^{2-} > \text{NO}^- > \text{NO} = \text{NO}^{2+} > \text{NO}^+$
: bond length order
(D) $\text{BaO} > \text{SrO} > \text{CaO} > \text{BeO} > \text{MgO}$
: basic character order
- Which of the following order is/are **INCORRECT**:
(A) $\text{NaF} < \text{MgF}_2 < \text{AlF}_3$ (Lattice energy)
(B) $\text{NaF} < \text{MgF}_2 < \text{AlF}_3$ (Melting point)
(C) $\text{NaF} < \text{MgF}_2 < \text{AlF}_3$ (Polarizing power of cation)
(D) $\text{NaF} < \text{MgF}_2 < \text{AlF}_3$ (% ionic character)
- The correct solubility order is/are:
(I) $\text{CaCO}_3 > \text{SrCO}_3 > \text{BaCO}_3$
(II) $\text{Li}_2\text{CO}_3 < \text{Na}_2\text{CO}_3 < \text{K}_2\text{CO}_3$
(III) $\text{K}_2\text{CO}_3 < \text{Rb}_2\text{CO}_3 < \text{Cs}_2\text{CO}_3$
(IV) $\text{Na}_2\text{CO}_3 > \text{K}_2\text{CO}_3 > \text{Rb}_2\text{CO}_3$
(A) II, IV (B) I, IV
(C) II, III, IV (D) I, II, III

Molecular Orbital Theory

- Which of the following species have more number of electrons in bonding MO's as compared to antibonding MO's
(A) O_2^- (B) N_2^+
(C) C_2 (D) All of these
- Which of the following statement is **INCORRECT**:
(A) KO_2 is paramagnetic in nature
(B) All halogens are coloured gases at room temperature
(C) O_2 is paramagnetic gas
(D) Bond order of OF is 1.5



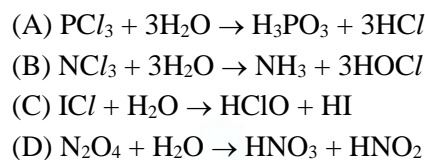


14. Assuming that if Hund's rule is violated, then the paramagnetic specie is:
(A) B_2 (B) O_2 (C) NO^\ominus (D) O_2^\oplus
15. The molecular orbital with highest energy in a nitrogen molecule is
(A) σ_{2p} (B) π_{2p} (C) σ_{2p}^* (D) π_{2p}^*
16. N_2 and O_2 are converted to monocations N_2^+ and O_2^+ respectively, which is wrong statement:
(A) In N_2^+ , the N—N bond weakens
(B) In O_2^+ , the O—O bond order increases
(C) In O_2^+ , the paramagnetism decrease
(D) N_2^+ becomes diamagnetic

Hydrolysis

17. Which of the following is an uncommon hydrolysis product of XeF_2 and XeF_4 ?
(A) Xe (B) XeO_3
(C) HF (D) O_2

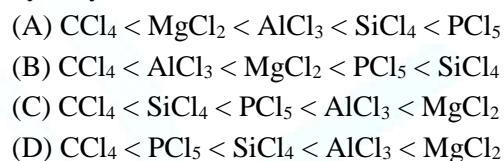
18. In which of the following reactions is **INCORRECT** according to its products formed.



19. Which of the following compound produce only oxyacid on hydrolysis:-



20. The correct increasing order of extent of hydrolysis is:





OBJECTIVE EXERCISE-II

Multiple Correct Type Question

- Which of the following statement(s) is/are **CORRECT**
 (A) $[B_3O_6]^{-3}$ ion is non-planar but $B_3N_3H_6$ is planar
 (B) $(SiH_3)_3N$ is planar but $(SiH_3)_3P$ is pyramidal
 (C) H_3C-NCS is bent but SiH_3-NCS is linear
 (D) $(CH_3)_3N$ is pyramidal but $(GeH_3)_3N$ is planar
- Which of the following statements is/are **INCORRECT**
 (A) CHF_3 is less acidic than $CHCl_3$
 (B) R_3C-O-H is more acidic than $R_3Si-O-H$
 (C) In BF_3 back bonding is possible but in CO back bonding is not possible
 (D) PH_3 is more basic than NH_3
- No $X-X$ bond exists in which of the following compounds having general form of X_2H_6 ?
 (A) B_2H_6 (B) C_2H_6 (C) Al_2H_6 (D) Si_2H_6
- Which of the following molecules have **CORRECT** indicated overlapping.
Molecule Overlapping in the bridge bond (if any)
 (A) Si_2Cl_6 sp^3-p-sp^3
 (B) Be_2H_4 sp^2-s-sp^2
 (C) Si_2H_6 sp^3-s-sp^3
 (D) B_2H_6 sp^3-s-sp^3
- Which of the following have $(18 + 2)$ electron configuration?
 (A) Pb^{2+} (B) Cd^{2+} (C) Bi^{3+} (D) S^{-2}
- Which of following stability order is/are correct due to inert pair effect.
 (A) $Hg > Hg^{2+}$ (B) $Bi^{3+} < Bi^{5+}$
 (C) $Pb^{2+} > Pb^{4+}$ (D) $Fe^{2+} < Fe^{3+}$
- Which of the following order is/are **CORRECT**:-
 (A) $MgCO_3 < BaCO_3$ (Thermal stability)
 (B) $LiF < CsF$ (Solubility)
 (C) $Li_3N > K_3N$ (Thermal stability)
 (D) $MgSO_3 > BaSO_3$ (Solubility)

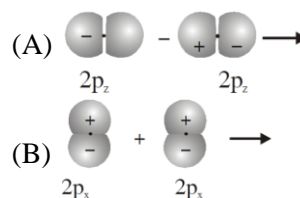
- Select the **CORRECT** order against the mentioned property:-
 (A) $NaNO_3 < KNO_3 < RbNO_3$ (Thermal stability)
 (B) $NaF > KF > RbF$ (Covalent nature)
 (C) $Si-O < P-O < S-O < Cl-O$ (π -bond strength)
 (D) $F_2 < Cl_2 < Br_2 < I_2$ (Bond length)
- Which of the following order is/are **CORRECT**:
 (A) $NaCl < LiCl$ (melting point)
 (B) $CaF_2 > CaO$ (lattice energy)
 (C) $LiNO_3 < NaNO_3$ (thermal stability)
 (D) $Be_3N_2 > Ba_3N_2$ (thermal stability)
- Which of the following have identical bond order?
 (A) O_2^{2+} (B) NO^+ (C) CN^- (D) CN^+
- Assuming that if Hund's rule is violated, then the diamagnetic specie(s) is/are:
 (A) B_2 (B) O_2 (C) N_2 (D) O_2^{\square}
- Which of the following compounds do not give free halogen acid (Hydra acid) on hydrolysis with excess water as a final product?
 (A) NCl_3 (B) PCl_3 (C) $SiCl_4$ (D) BF_3

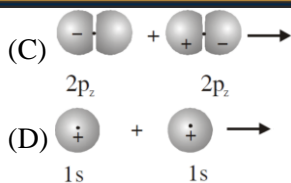
Comprehension Type Question

Paragraph for Question 13 to 14

Qualitatively, the formation of molecular orbitals can be understood in terms of the constructive or destructive interference of the electron waves of the combining atoms. In the formation of bonding molecular orbital, the two electron waves of the bonding atoms reinforce each other due to constructive interference while in the formation of antibonding molecular orbital, the electron waves cancel each other due to destructive interference.

- Which of the following combinations give(s) antibonding sigma molecular orbital if z-axis is the internuclear axis?





14. Which of the following species does not exist:
(A) HeH^+ (B) Be_2 (C) C_2^{2-} (D) NO^+

Paragraph for Question 15 to 16

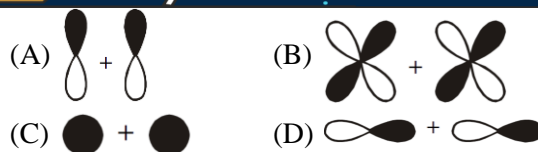
When a substance undergoes nucleophilic substitution reaction and the nucleophile is solvent itself, then the reaction is known as solvolysis, if the solvent used is water then the reaction is called as hydrolysis.

15. The product(s) of hydrolysis of NCl_3 is/are:-
(A) HNO_2 (B) HCl (C) NH_3 (D) HOCl
16. Which of the following compounds on hydrolysis produce oxyacid having basicity three in water:
(A) PCl_5 (B) AsCl_3 (C) PCl_3 (D) BCl_3

Paragraph for Question 17 to 18

Molecular orbital theory is based on linear combination of atomic orbitals (LCAO). According to LCAO when respective atomic orbitals of the atoms interact, they undergo constructive and destructive interference giving two types of molecular orbital i.e. bonding and antibonding molecular orbitals respectively.

17. Which of the following overlapping result ungerade molecular orbital.



18. Which of the following property does not change when O_2 is converted to O_2^{2-} :-
(A) Magnetic behaviour
(B) Magnetic moment
(C) Bond order
(D) Number of bonding electron

Matrix Match Type Question

19. Match column-I with column-II:-

Column-I Column-II

- (P) BF_3 (1) Exist in dimeric form
(Q) AlCl_3 (2) Effective back bond is present
(R) SiO_2 (3) Acts as Lewis acid
(S) CO (4) Exist in polymeric form

Select correct code of your answer:

Code:	(P)	(Q)	(R)	(S)
(A)	3	2	4	1
(B)	1	2	4	3
(C)	3	1	4	2
(D)	1	2	3	4

20. **Column-I Column-II**

- (A) BF_3 (P) Intra molecular Lewis acid-base interaction
(B) BCl_3 (Q) $2p_\pi - 3p_\pi$ back bond
(C) H_3BO_3 (R) Lewis acid
(D) $\text{B}(\text{OCH}_3)_3$ (S) Incomplete octet of central atom boron
(T) sp^2 hybridisation of boron


SUBJECTIVE EXERCISE-I

- | | |
|---|--|
| <p>1. Find the total number of $2C - 2e^-$ bond in Al_2Cl_6 -
 Fill your answer as sum of digits till you get the single digit answer.</p> <p>2. $6XeF_4 + 12H_2O \rightarrow 4X + 2Y + 24HF + 3O_2$
 In above reaction find the difference of oxidation state in central atom of X and Y.</p> <p>3. Total number of molecules in which bridge bond formed by sp^3-s-sp^3 type overlap
 B_2H_6, $Al_2(CH_3)_6$, I_2Cl_6, Al_2H_6, Si_2Cl_6</p> <p>4. For the given compounds, number of compounds which undergo complete hydrolysis in presence of excess amount of water in ordinary condition:
 SF_4, XeF_2, $BiCl_3$, NF_3, NCl_3, $POCl_3$, BF_3</p> <p>5. Find out the number (s) of molecule in which bond angle around under line atom is 120°:</p> | <p>$H_3\text{B}O_3$, $P(SiH_3)_3$, $N(SiH_3)_3$, $\overset{+}{C}H_3$, $\overset{-}{N}(CH_3)_3$, $\overset{-}{Sn}Cl_3$</p> <p>6. Find the number of chemical species which undergoes hydrolysis through redox reaction.
 XeF_2, XeF_4, XeF_6, SF_4, PCl_3, PCl_5</p> <p>7. Number of species having bond order 2 will be?
 O_2^{+2}, N_2^{+2}, N_2^{-2}, O_2^+, N_2^+, C_2, B_2^{-2}</p> <p>8. Find the total number of $2C - 2e^-$ bond in $Al_2(CH_3)_6$ (excluding π bond)
 Fill your answer as sum of digits till you get the single digit answer.</p> <p>9. Maximum number of atoms in one plane in $N(SiH_3)_3$ is:</p> <p>10. No. of boron atoms in anionic part of borax, which participate in back bonding.</p> |
|---|--|





SUBJECTIVE EXERCISE-II

- | | |
|--|--|
| <p>1. If AB molecule is isoelectronic with NO^+ then find the difference between total number of antibonding electron and bonding electrons in AB molecule.</p> <p>2. Maximum number of identical bond length in B_2H_6.</p> <p>3. Find the total number of properties which are changed due to backbonding in boric acid
(i) B-O bond length
(ii) $\text{O}-\hat{\text{B}}-\text{O}$ bond angle
(iii) $\text{B}-\hat{\text{O}}-\text{H}$ bond angle
(iv) Hybridisation of boron
(v) Hybridisation of oxygen</p> <p>4. Maximum number of atoms present in one plane in $\text{C}(\text{CH}_3)_4$</p> | <p>5. Find the sum of oxidation states of central atom in BiOCl and TlI_3</p> <p>6. Total number of moles of water molecules required for complete hydrolysis of 1 mole P_4O_{10}</p> <p>7. Sum of the most stable oxidation states of Al, Ga, Bi, Tl, Pb, Sn</p> <p>8. If XO_2 is an odd electron molecule then total number of valence electron of XO_2 molecule will be (X belongs to second period of periodic table)</p> <p>9. The difference between total number of electrons of pseudo inert gas configuration cation and non inert gas configuration cation of last element of carbon family is?</p> |
|--|--|



JEE-MAIN (PREVIOUS YEAR QUESTIONS)

1. Which of the alkaline earth metal halides given below is essentially covalent in nature:-

[JEE(Main)-2015]

- (1) SrCl_2 (2) CaCl_2
(3) BeCl_2 (4) MgCl_2

2. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

[JEE-MAIN-2015]

- (1) BaSO_4 (2) SrSO_4 (3) CaSO_4 (4) BeSO_4

3. Which intermolecular force is most responsible in allowing xenon gas to liquefy?

[JEE(Main)-2016]

- (1) Ionic
(2) Instantaneous dipole- induced dipole
(3) Dipole - dipole
(4) Ion - dipole

4. The increasing order of the boiling point for the following compounds is:

[JEE(Main)-2017]

- (I) $\text{C}_2\text{H}_5\text{OH}$ (II) $\text{C}_2\text{H}_5\text{Cl}$
(III) $\text{C}_2\text{H}_5\text{CH}_3$ (IV) $\text{C}_2\text{H}_5\text{OCH}_3$
(1) (III) < (II) < (I) < (IV)
(2) (II) < (III) < (IV) < (I)
(3) (IV) < (III) < (I) < (II)
(4) (III) < (IV) < (II) < (I)

5. Which of following is a Lewis acid?

[JEE(Main)-2018]

- (1) PH_3 (2) $\text{B}(\text{CH}_3)_3$
(3) NaH (4) NF_3

6. (I) (II)
 $\text{H} - \text{N} - - - \text{N} - - - \text{N}$

In hydrogen azide (above) the bond orders of bonds (I) and (II) are: [JEE(Main)-2018]

- (I) (II)
(1) <2 >2
(2) >2 <2
(3) >2 >2
(4) <2 <2

7. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X' and 'Y' and the oxidation state of Xe are respectively:

[JEE(Main)-2018]

- (1) $\text{XeO}_2(+4)$ and $\text{XeO}_3(+6)$
(2) $\text{XeOF}_4(+6)$ and $\text{XeO}_3(+6)$
(3) $\text{XeO}_2\text{F}_2(+6)$ and $\text{XeO}_2(+4)$
(4) $\text{XeOF}_4(+6)$ and $\text{XeO}_2\text{F}_2(+6)$

8. Among the oxides of nitrogen: N_2O_3 , N_2O_4 and N_2O_5 ; the molecule(s) having nitrogen-nitrogen bond is/are: [JEE(Main)-2018]

- (1) Only N_2O_5 (2) N_2O_3 and N_2O_5
(3) N_2O_5 and N_2O_4 (4) N_2O_3 and N_2O_4

9. A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3 is electron deficient and easily reacts with NH_3 to form $\text{Cl}_3\text{X} \leftarrow \text{NH}_3$ adduct; however, XCl_3 does not dimerize. X is: [JEE(Main)-2018]

- (1) B (2) Al (3) Ga (4) In

10. The number of P – O bonds in P_4O_6 is:

[JEE(Main)-2018]

- (1) 6 (2) 9 (3) 12 (4) 18

11. The chloride that cannot get hydrolysed is:

[JEE(Main)-2019]

- (1) SiCl_4 (2) CCl_4
(3) PbCl_4 (4) SnCl_4

12. The number of bonds between sulphur and oxygen atoms in $\text{S}_2\text{O}_8^{2-}$ and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are:

[JEE(Main)-2020]

- (1) 4 and 6 (2) 8 and 8 (3) 4 and 8 (4) 8 and 6

13. The number of sp^2 hybrid orbitals in a molecule of benzene is: [JEE(Main)-2020]

- (1) 6 (2) 24
(3) 18 (4) 12

14. Amongst BeF_2 , BF_3 , H_2O , NH_3 , CCl_4 and HCl , the number of molecules with non-zero net dipole moment is _____. [JEE(Main)-2022]

15. The total number of acidic oxides from the following list is: [JEE(Main)-2022]

- NO , N_2O , B_2O_3 , N_2O_5 , CO , SO_3 , P_4O_{10}
(1) 3 (2) 4 (3) 5 (4) 6



- | | |
|---|--|
| <p>16. Amongst the following the number of oxide(s) which are paramagnetic in nature is
 Na_2O, KO_2, NO_2, N_2O, ClO_2, NO, SO_2, Cl_2O
 [JEE(Main)-2022]</p> <p>17. The number of molecule(s) or ion(s) from the following having non-planar structure is
 NO_3^-, H_2O_2, BF_3, PCl_3, XeF_4, SF_4, XeO_3,
 PH_4^+, SO_3, $[\text{Al}(\text{OH})_4]^-$ [JEE(Main)-2022]</p> <p>18. Order of Covalent bond;
 [JEE(Main)-2023]
 A. $\text{KF} > \text{KI}$; $\text{LiF} > \text{KF}$
 B. $\text{KF} < \text{KI}$; $\text{LiF} > \text{KF}$
 C. $\text{SnCl}_4 > \text{SnCl}_2$; $\text{CuCl} > \text{NaCl}$</p> | <p>D. $\text{LiF} > \text{KF}$; $\text{CuCl} < \text{NaCl}$
 E. $\text{KF} < \text{KI}$; $\text{CuCl} > \text{NaCl}$
 (1) C, E only (2) B, C only
 (3) B, C, E only (4) A, B only</p> <p>19. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : N_2 : N_2^+; O_2; O_2^+ ?
 [JEE(Main)-2023]
 (1) 0, 1, 2, 1 (2) 2, 1, 2, 1
 (3) 0, 1, 0, 1 (4) 2, 1, 0, 1</p> <p>20. The ratio of sigma and π bonds present in pyrophosphoric acid is _____.
 [JEE(Main)-2023]</p> |
|---|--|





JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

1. Which of the following is soluble in water?
[IIT-98]

(A) CS_2 (B) $\text{C}_2\text{H}_5\text{OH}$
(C) CCl_4 (D) CHCl_3

2. Among the following, the paramagnetic compound is
[JEE-2007]

(A) Na_2O_2 (B) O_3
(C) N_2O (D) KO_2

3. **Statement-1:** In water, orthoboric acid behaves as a weak monobasic acid.

Statement-2: In water, orthoboric acid acts as a proton donor.
[JEE-2007]

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

4. Match each of the diatomic molecules/ions in Column I with its property/properties in Column II.
[JEE-2009]

Column I

Column II

- (A) B_2 (P) Paramagnetic
(B) N_2 (Q) undergoes oxidation
(C) O_2^- (R) Undergoes reduction
(D) O_2 (S) Bond order ≥ 2
(T) Mixing of 's' and 'p' orbitals

5. In the reaction
 $2\text{X} + \text{B}_2\text{H}_6 \longrightarrow [\text{BH}_2(\text{X})_2]^+ [\text{BH}_4]^-$
the amine(s) X is (are)
[JEE-2009]

(A) NH_3 (B) CH_3NH_2
(C) $(\text{CH}_3)_2\text{NH}$ (D) $(\text{CH}_3)_3\text{N}$

6. The value of n in the molecular formula $\text{Be}_n\text{Al}_2\text{Si}_6\text{O}_{18}$ is
[JEE-2010]

7. The total number of diprotic acids among the following is
[JEE-2010]

H_3PO_4 H_2SO_4 H_3PO_3 H_2CO_3 $\text{H}_2\text{S}_2\text{O}_7$
 H_3BO_3 H_3PO_2 H_2CrO_4 H_2SO_3

8. Among the following, the number of elements showing only one non-zero oxidation state is
O, Cl, F, N, P, Sn, Tl, Na, Ti [JEE-2010]

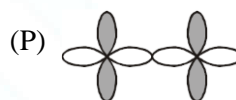
9. Assuming 2s-2p mixing is NOT operative, the paramagnetic species among the following is:
[JEE(Advanced)-2014]

(A) Be_2 (B) B_2
(C) C_2 (D) N_2

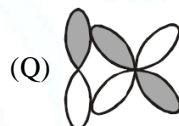
10. Match the orbital overlap figures shown in List-I with the description given in List-II and select the correct answer using the code given below the lists.
[JEE(Advanced)-2014]

List-I

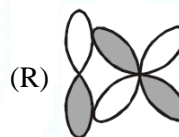
List-II



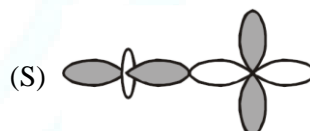
(1) p - d π antibonding



(2) d - d σ bonding



(3) p - d π bonding



(4) d - d σ antibonding

Code:

	P	Q	R	S
(A)	2	1	3	4
(B)	4	3	1	2
(C)	2	3	1	4
(D)	4	1	3	2

11. Three moles of B_2H_6 are completely reacted with methanol. The number of moles of boron containing product formed is -
[JEE(Advanced)-2015]

12. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. The is due to
[JEE(Advanced)-2017]

(A) the physical state of X_2 at room temperature





changes from gas to solid down the group
(B) Decrease in HOMO-LUMO gap down the group
(C) decrease in $\pi^* - \sigma^*$ gap down the group
(D) decrease in ionization energy down the group

13. Among the following, the correct statement(s) is (are) [JEE(Advanced)-2017]

(A) $\text{Al}(\text{CH}_3)_3$ has the three-centre two-electron bonds in its dimeric structure
(B) AlCl_3 has the three-centre two-electron bonds in its dimeric structure
(C) BH_3 has the three-centre two-electron bonds in its dimeric structure
(D) The Lewis acidity of BCl_3 is greater than that of AlCl_3

14. The option(s) with only amphoteric oxides is (are) [JEE(Advanced)-2017]

(A) Cr_2O_3 , BeO , SnO , SnO_2
(B) ZnO , Al_2O_3 , PbO , PbO_2
(C) NO , B_2O_3 , PbO , SnO_2
(D) Cr_2O_3 , CrO , SnO , PbO

15. The sum of the number of lone pairs of electrons on each central atom in the following species is

$[\text{TeBr}_6]^{2-}$, $[\text{BrF}_2]^+$, SNF_3 and $[\text{XeF}_3]^-$
(Atomic number: N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

[JEE(Advanced)-2017]

16. Each of the following options contains a set of four molecules. Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

[JEE(Advanced)-2019]

(A) NO_2 , NH_3 , POCl_3 , CH_3Cl
(B) BeCl_2 , CO_2 , BCl_3 , CHCl_3
(C) SO_2 , $\text{C}_6\text{H}_5\text{Cl}$, H_2Se , BrF_5
(D) BF_3 , O_3 , SF_6 , XeF_6

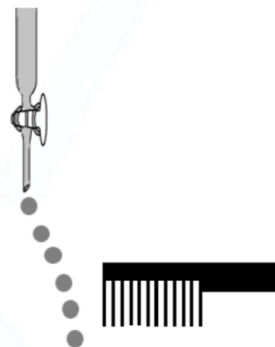
17. Among B_2H_6 , $\text{B}_3\text{N}_3\text{H}_6$, N_2O , N_2O_4 , $\text{H}_2\text{S}_2\text{O}_3$ and $\text{H}_2\text{S}_2\text{O}_8$, the total number of molecules containing covalent bond between two atoms of the same kind is _____

[JEE(Advanced)-2019]

18. Consider the following compounds in the liquid form: [JEE(Advanced)-2020]

O_2 , HF , H_2O , NH_3 , H_2O_2 , CCl_4 , CHCl_3 , C_6H_6 , $\text{C}_6\text{H}_5\text{Cl}$.

When a charged comb is brought near their flowing stream, how many of them show deflection as per the following figure?

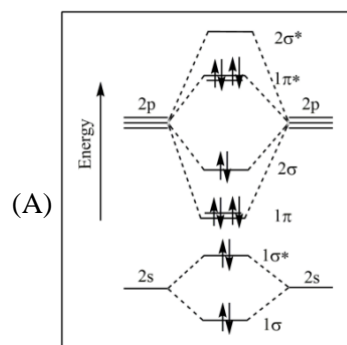


19. For diatomic molecules, the correct statement(s) about the molecular orbitals formed by the overlap of two $2p_z$ orbitals is(are) [JEE(Advanced)-2022]

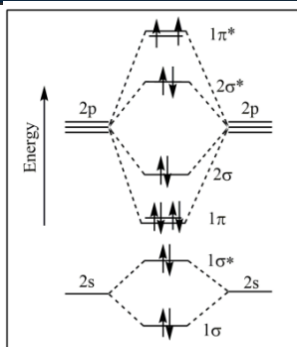
(A) σ -orbital has a total of two nodal planes.
(B) σ^* orbital has one node in the xz -plane containing the molecular axis.
(C) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.
(D) π^* orbital has one node in the xy -plane containing the molecular axis.

20. The correct molecular orbital diagram for F_2 molecule in the ground state is

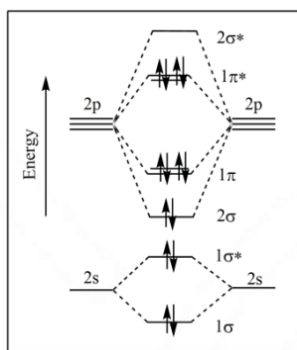
[JEE(Advanced)-2023]



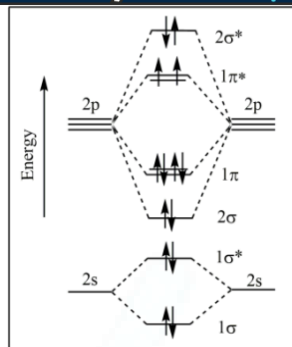
(B)



(C)



(D)



ANSWER KEY

OBJECTIVE EXERCISE - I

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (B) | 2. (D) | 3. (D) | 4. (D) | 5. (C) | 6. (D) | 7. (D) |
| 8. (A) | 9. (C) | 10. (D) | 11. (D) | 12. (D) | 13. (B) | 14. (D) |
| 15. (C) | 16. (D) | 17. (B) | 18. (C) | 19. (C) | 20. (A) | |

OBJECTIVE EXERCISE - II

- | | | | | | |
|-----------|---|----------|-----------|-----------|----------|
| 1. (BCD) | 2. (BCD) | 3. (AC) | 4. (BD) | 5. (AC) | 6. (AC) |
| 7. (ABCD) | 8. (ABCD) | 9. (CD) | 10. (ABC) | 11. (ABC) | 12. (AD) |
| 13. (A) | 14. (B) | 15. (CD) | 16. (AB) | 17. (AD) | 18. (AD) |
| 19. (C) | 20. (A) P,R,S,T; (B) P,Q,R,S,T; (C) P,R,S,T ; (D) P,R,S,T | | | | |

SUBJECTIVE EXERCISE - I

- | | | | | |
|------------|------------|------------|------------|-------------|
| 1. (04.00) | 2. (06.00) | 3. (02.00) | 4. (04.00) | 5. (02.00) |
| 6. (02.00) | 7. (04.00) | 8. (04.00) | 9. (07.00) | 10. (02.00) |

SUBJECTIVE EXERCISE - II

- | | | | | |
|------------|------------|------------|------------|------------|
| 1. (06.00) | 2. (04.00) | 3. (03.00) | 4. (05.00) | 5. (04.00) |
| 6. (06.00) | 7. (16.00) | 8. (17.00) | 9. (02.00) | |

JEE-MAIN (PREVIOUS YEAR QUESTIONS)

- | | | | | | | |
|---------|-------------|-------------|---------|---------|-------------|-------------|
| 1. (3) | 2. (4) | 3. (2) | 4. (4) | 5. (2) | 6. (1) | 7. (4) |
| 8. (4) | 9. (1) | 10. (3) | 11. (2) | 12. (2) | 13. (3) | 14. (03.00) |
| 15. (2) | 16. (04.00) | 17. (06.00) | 18. (3) | 19. (1) | 20. (06.00) | |

JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

- | | | | |
|-----------|------------|----------|---|
| 1. (B) | 2. (D) | 3. (C) | 4. A→P,Q,R,T; B→Q,R,S,T; C→P,Q,R; D→P,Q,R,S |
| 5. (BC) | 6. (3) | 7. (6) | 8. (2) |
| 9. (C) | 10. (C) | 11. (6) | |
| 12. (BC) | 13. (ACD) | 14. (AB) | 15. (6) |
| 16. (ABD) | 17. (4.00) | 18. (6) | |
| 19. (AD) | 20. (C) | | |

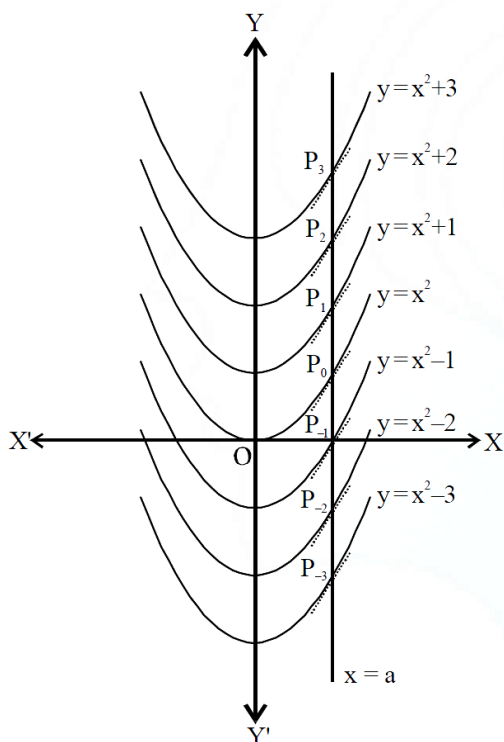

INDEFINITE INTEGRATION

If f and F are function of x such that $F'(x) = f(x)$ then the function F is called a **primitive or antiderivative or integral** of $f(x)$ w.r.t. x and is written symbolically as

$\int f(x) dx = F(x) + C \Leftrightarrow \frac{d}{dx} \{F(x) + C\} = f(x)$, where C is called the **constant of integration**.

Geometrical Interpretation of Indefinite Integral

$\int f(x) dx = F(x) + C = y$ (say), represents a family of curves. The different values of c will correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself. This is the geometrical interpretation of



Let $f(x) = 2x$. Then $\int f(x) dx = x^2 + C$. For different values of C , we get different integrals. But these integrals are very similar geometrically. Thus, $y = x^2 + C$, where C is arbitrary constant represents a family of integrals. By assigning different values of C , we get different members of the family. These together constitute the indefinite integral. In this case, each integral represents a parabola with its axis along y -axis. If the line $x = a$ intersects the parabolas $y = x^2$, y

$= x^2 + 1$, $y = x^2 + 2$, $y = x^2 - 1$, $y = x^2 - 2$ at P_0 , P_1 , P_2 , P_{-1} , P_{-2} etc., respectively, then $\frac{dy}{dx}$ at these points equals $2a$. This indicates that the tangents to the curves at these points are parallel. Thus, $\int 2x dx = x^2 + C = f(x) + C$ (say), implies that the tangents to all the curves $f(x) + C$, $C \in \mathbb{R}$, at the points of intersection of the curves by the line $x = a$, ($a \in \mathbb{R}$), are parallel.

Standard Formulae

$$(i) \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C; n \neq -1$$

$$(ii) \int \frac{dx}{ax + b} = \frac{1}{a} \ln |ax + b| + C$$

$$(iii) \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$(iv) \int a^{px+q} dx = \frac{1}{p} \frac{a^{px+q}}{\ln a} + C, (a > 0)$$

$$(v) \int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$(vi) \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

$$(vii) \int \tan(ax + b) dx = \frac{1}{a} \ln |\sec(ax + b)| + C$$

$$(viii) \int \cot(ax + b) dx = \frac{1}{a} \ln |\sin(ax + b)| + C$$

$$(ix) \int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + C$$

$$(x) \int \operatorname{cosec}^2(ax + b) dx = -\frac{1}{a} \cot(ax + b) + C$$

(xi)

$$\int \operatorname{cosec}(ax + b) \cdot \cot(ax + b) dx = -\frac{1}{a} \operatorname{cosec}(ax + b) + C$$

$$(xii) \int \sec(ax + b) \cdot \tan(ax + b) dx = \frac{1}{a} \sec(ax + b) + C$$

(xiii)

$$\int \sec x dx = \ln |\sec x + \tan x| + C = \ln \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + C$$

(xiv)

$$\int \operatorname{cosec} x dx = \ln |\operatorname{cosec} x - \cot x| + C = \ln \left| \tan \frac{x}{2} \right| + C$$





$$= -\ln |\operatorname{cosec} x + \cot x| + C$$

$$(xv) \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$(xvi) \quad \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$(xvii) \quad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$$

$$(xviii) \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln|x + \sqrt{x^2 + a^2}| + C$$

$$(xix) \quad \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$(xx) \quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$(xxi) \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$(xxii) \quad \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

(xxiii)

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}) + C$$

(xxiv)

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2}) + C$$

$$(xxv) \quad \int e^{ax} \cdot \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) +$$

$$C = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \frac{b}{a} \right) + C$$

$$(xxvi) \quad \int e^{ax} \cdot \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$+ C = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left(bx - \tan^{-1} \frac{b}{a} \right) + C$$

Techniques of Integration

(a) Substitution or Change of Independent Variable

If $\phi(x)$ is a continuous differentiable function, then to evaluate integrals of the form $\int f(\phi(x))\phi'(x)dx$, we substitute $\phi(x) = t$ and $\phi'(x)dx = dt$.

Hence $I = \int f(\phi(x))\phi'(x)dx$ reduces to $\int f(t)dt$.

(i) Fundamental Deductions of Method of Substitution

$\int [f(x)]^n f'(x)dx$ OR $\int \frac{f'(x)}{[f(x)]^n} dx$ put $f(x) = t$ and proceed.

Example :

Evaluate $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx$

Solution:

$$I = \int \frac{(1 - \sin^2 x) \cos x}{\sin x (1 + \sin x)} dx = \int \frac{1 - \sin x}{\sin x} \cos x dx$$

Put $\sin x = t \Rightarrow \cos x dx = dt$

$$\Rightarrow I = \int \frac{1-t}{t} dt = \ln|t| - t + C = \ln|\sin x| - \sin x + C$$

Example :

Evaluate $\int \frac{(x^2 - 1) dx}{(x^4 + 3x^2 + 1) \tan^{-1} \left(x + \frac{1}{x} \right)}$

Solution:

The given integral can be written as

$$I = \int \frac{\left(1 - \frac{1}{x^2} \right) dx}{\left[\left(x + \frac{1}{x} \right)^2 + 1 \right] \tan^{-1} \left(x + \frac{1}{x} \right)}$$

Let $\left(x + \frac{1}{x} \right) = t$. Differentiating we get

$$\left(1 - \frac{1}{x^2} \right) dx = dt$$

$$\text{Hence } I = \int \frac{dt}{(t^2 + 1) \tan^{-1} t}$$

Now make one more substitution $\tan^{-1} t = u$. Then

$$\frac{dt}{t^2 + 1} = du \text{ and } I = \int \frac{du}{u} = \ln|u| + C$$

Returning to t , and then to x , we have

$$I = \ln|\tan^{-1} t| + C = \ln \left| \tan^{-1} \left(x + \frac{1}{x} \right) \right| + C$$



FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 Evaluate: $\int \frac{x^2}{9+16x^6} dx$

Q.2 Evaluate: $\int \cos^3 x dx$

(i) Standard Substitutions

$$\int \frac{dx}{\sqrt{a^2 + x^2}} \text{ or } \int \frac{dx}{\sqrt{a^2 - x^2}}; \text{ put } x$$

$$= a \tan \theta \text{ or } x = a \cot \theta$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} \text{ or } \int \frac{dx}{\sqrt{x^2 - a^2}}; \text{ put } x$$

$$= a \sin \theta \text{ or } x = a \cos \theta$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} \text{ or } \int \frac{dx}{\sqrt{a^2 - x^2}}; \text{ put } x$$

$$= a \sec \theta \text{ or } x = a \operatorname{cosec} \theta$$

$$\int \frac{dx}{\sqrt{a-x}} \text{ or } \int \frac{dx}{\sqrt{a+x}}; \text{ put } x = a \cos 2\theta$$

$$\int \frac{dx}{\sqrt{\beta-x}} \text{ or } \int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}; \text{ put } x$$

$$= \alpha \cos^2 \theta + \beta \sin^2 \theta$$

$$\int \frac{dx}{\sqrt{x-\alpha}} \text{ or } \int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}; \text{ put } x$$

$$= \alpha \sec^2 \theta - \beta \tan^2 \theta$$

$$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}; \text{ put } x - \alpha = t^2 \text{ or } x - \beta = t^2.$$

Example :

Evaluate $\int \frac{dx}{\sqrt{(x-a)(b-x)}}$

Solution:

Put $x = a \cos^2 \theta + b \sin^2 \theta$, the given integral becomes

$$I = \int \frac{2(b-a) \sin \theta \cos \theta d\theta}{\{(a \cos^2 \theta + b \sin^2 \theta - a)(b - a \cos^2 \theta - b \sin^2 \theta)\}^{\frac{1}{2}}}$$

$$= \int \frac{2(b-a) \sin \theta \cos \theta d\theta}{(b-a) \sin \theta \cos \theta} = \left(\frac{b-a}{b-a} \right)$$

$$\int 2d\theta = 2\theta + C = 2 \sin^{-1} \sqrt{\frac{x-a}{b-a}} + C$$

Ans.

Example :

Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \cdot \frac{1}{x} dx$

Solution:

Put $x = \cos^2 \theta \Rightarrow dx = -2 \sin \theta \cos \theta d\theta$

$$\Rightarrow I = \int \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \cdot \frac{1}{\cos^2 \theta} (-2 \sin \theta \cos \theta) d\theta$$

$$= - \int 2 \tan \frac{\theta}{2} \tan \theta d\theta$$

$$= -4 \int \frac{\sin^2(\theta/2)}{\cos \theta} d\theta = -2 \int \frac{1-\cos \theta}{\cos \theta} d\theta$$

$$= -2 \ln |\sec \theta + \tan \theta| + 2\theta + C$$

$$= -2 \ln \left| \frac{1+\sqrt{1-x}}{\sqrt{x}} \right| + 2 \cos^{-1} \sqrt{x} + C$$

FUNDAMENTAL UNLOCKED- (FU#2) :

Q.1 Evaluate: $\int \sqrt{\frac{x-3}{2-x}} dx$

Q.2 Evaluate: $\int \frac{dx}{x\sqrt{x^2+4}}$

(b) Integration by Part

$$\int u \cdot v dx = u \int v dx - \int \left[\frac{du}{dx} \cdot \int v dx \right] dx \text{ where } u \text{ and } v$$

are differentiable functions and are commonly designated as first and second function respectively

Note: While using integration by parts, choose u and v such that

$$(i) \int v dx \quad \text{and} \quad (ii) \int \left[\frac{du}{dx} \cdot \int v dx \right] dx \text{ are}$$

simple to integrate.

This is generally obtained by choosing first function as the function which comes first in the word **ILATE**, where; I-Inverse function, L-Logarithmic function, A-Algebraic function, T-Trigonometric function and E-Exponential function.

Example :

Evaluate: $\int \cos \sqrt{x} dx$

Solution:

Consider $I = \int \cos \sqrt{x} dx$



Let $\sqrt{x} = t$ then $\frac{1}{2\sqrt{x}} dx = dt$

i.e. $dx = 2\sqrt{x} dt$ or $dx = 2t dt$

so $I = \int \cos t \cdot 2t dt$

Taking t as first function, integrate it by part

$$\Rightarrow I = 2 \left[t \int \cos t dt - \int \left\{ \frac{dt}{dt} \int \cos t dt \right\} dt \right]$$

$$I = 2 \left[t \sin t - \int 1 \cdot \sin t dt \right] = 2[t \sin t + \cos t] + C$$

$$I = 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + C$$

Example :

Evaluate: $\int \frac{x}{1 + \sin x} dx$

Solution:

Let $I = \int \frac{x}{1 + \sin x} dx = \int \frac{x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$

$$= \int \frac{x(1 - \sin x)}{1 - \sin^2 x} dx = \int \frac{x(1 - \sin x)}{\cos^2 x} dx$$

$$= \int x \sec^2 x dx - \int x \sec x \tan x dx$$

$$= \left[x \int \sec^2 x dx - \int \left\{ \frac{dx}{dx} \int \sec^2 x dx \right\} dx \right] -$$

$$\left[x \int \sec x \tan x dx - \int \left\{ \frac{dx}{dx} \int \sec x \tan x dx \right\} dx \right]$$

$$= [x \tan x - \int \tan x dx] - [x \sec x - \int \sec x dx]$$

$$= [x \tan x - \ln |\sec x|] - [x \sec x - \ln |\sec x + \tan x|] + C$$

$$= x(\tan x - \sec x) + \ln \left| \frac{(\sec x + \tan x)}{\sec x} \right| + C$$

$$= \frac{-x(1 - \sin x)}{\cos x} + \ln |1 + \sin x| + C \quad \text{Ans.}$$

FUNDAMENTAL UNLOCKED- (FU#3) :

Q.1 Evaluate: $\int x e^x dx$

Q.2 Evaluate: $\int x^3 \sin(x^2) dx$

Example :

Evaluate $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$

Solution:

$$\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx = \int e^x \frac{(1-2x+x^2)}{(1+x^2)^2} dx$$

$$= \int e^x \left(\frac{1}{(1+x^2)} - \frac{2x}{(1+x^2)^2} \right) dx = \frac{e^x}{1+x^2} + C$$

Example :

The value of $\int e^x \left(\frac{x^4 + 2}{(1+x^2)^{5/2}} \right) dx$ is equal to-

(A) $\frac{e^x(x+1)}{(1+x^2)^{3/2}} + C$ (B) $\frac{e^x(1-x+x^2)}{(1+x^2)^{3/2}} + C$

(C) $\frac{e^x(1-x)}{(1+x^2)^{3/2}} + C$ (D) None of these

Solution:

Let

$$I = \int e^x \left(\frac{x^4 + 2}{(1+x^2)^{5/2}} \right) dx = \int e^x \left(\frac{1}{(1+x^2)^{1/2}} + \frac{1-2x^2}{(1+x^2)^{5/2}} \right) dx$$

=

$$\int e^x \left(\frac{1}{(1+x^2)^{1/2}} - \frac{x}{(1+x^2)^{3/2}} + \frac{x}{(1+x^2)^{3/2}} + \frac{1-2x^2}{(1+x^2)^{5/2}} \right) dx$$

dx

$$= \frac{e^x}{(1+x^2)^{1/2}} + \frac{x e^x}{(1+x^2)^{3/2}} + C = \frac{e^x \{1+x^2+x\}}{(1+x^2)^{3/2}} + C$$

Ans.(D)

FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 Evaluate: $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$

Q.2 Evaluate: $\int x e^{x^2} (\sin x^2 + \cos x^2) dx$

Q.3 Prove that: $\int [f(x) + x f'(x)] dx = x f(x) + C$

Example :

Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$

Solution:

$$I = \int \frac{x + \sin x}{1 + \cos x} dx = \int \left(\frac{x + \sin x}{2 \cos^2 \frac{x}{2}} \right) dx =$$

$$\int \left(x \frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx = x \tan \frac{x}{2} + C$$




FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 Evaluate: $\int (\tan(e^x) + xe^x \sec^2(e^x)) dx$

Q.2 Evaluate: $\int (\ell nx + 1) dx$

(c) Integration of Trigonometric Functions

(i) $\int \sin^m x \cos^n x dx$

Case-I

When m and $n \in$ natural numbers.

If one of them is odd, then substitute for the term of even power.

If both are odd, substitute either of the term.

If both are even, use trigonometric identities to convert integrand into cosines of multiple angles.

Case-II

$m + n$ is a negative even integer.

In this case the best substitution is $\tan x = t$.

Example :

Evaluate $\int \sin^3 x \cos^5 x dx$

Solution:

Put $\cos x = t$; $-\sin x dx = dt$.

so that $I = - \int (1 - t^2) \cdot t^5 dt$

$$= \int (t^7 - t^5) dt = \frac{t^8}{8} - \frac{t^6}{6} + C = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$$

Alternate

Put $\sin x = t$; $\cos x dx = dt$

so that $I = \int t^3 (1 - t^2)^2 dt = \int (t^3 - 2t^5 + t^7) dt$

$$= \frac{\sin^4 x}{4} - \frac{2\sin^6 x}{6} + \frac{\sin^8 x}{8} + C$$

Note: This problem can also be handled by successive reduction or by trigonometric identities.

Example :

Evaluate $\int \sin^2 x \cos^4 x dx$

Solution:

$$\int \sin^2 x \cos^4 x dx = \int \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{\cos 2x + 1}{2} \right)^2 dx$$

$$\begin{aligned} &= \int \frac{1}{8} (1 - \cos 2x)(\cos^2 2x + 2\cos 2x + 1) dx \\ &= \frac{1}{8} \int (\cos^2 2x + 2\cos 2x + 1 - \cos^3 2x - 2\cos^2 2x - \cos 2x) dx \\ &= \frac{1}{8} \int (-\cos^3 2x - \cos^2 2x + \cos 2x + 1) dx \\ &= -\frac{1}{8} \int \left(\frac{\cos 6x + 3\cos 2x}{4} + \frac{1 + \cos 4x}{2} - \cos 2x - 1 \right) dx \\ &= -\frac{1}{32} \left[\frac{\sin 6x}{6} + \frac{3\sin 2x}{2} \right] - \frac{1}{16} x - \frac{\sin 4x}{64} + \frac{\sin 2x}{16} + \frac{x}{8} + C \\ &= -\frac{\sin 6x}{192} - \frac{\sin 4x}{64} + \frac{1}{64} \sin 2x + \frac{x}{16} + C \end{aligned}$$

Example :

Evaluate $\int \frac{\sqrt{\sin x}}{\cos^{9/2} x} dx$

Solution:

Let $I = \int \frac{\sin^{1/2} x}{\cos^{9/2} x} dx = \int \frac{dx}{\sin^{-1/2} x \cos^{9/2} x}$

Here $m + n = \frac{1}{2} - \frac{9}{2} = -4$ (negative even integer)

Divide Numerator and Denominator by $\cos^4 x$.

$$\begin{aligned} I &= \int \sqrt{\tan x} \sec^4 x dx = \int \sqrt{\tan x} (1 + \tan^2 x) \sec^2 x dx \\ &= \int \sqrt{t} (1 + t^2) dt \quad (\text{using } \tan x = t) \\ &= \frac{2}{3} t^{3/2} + \frac{2}{7} t^{7/2} + C = \frac{2}{3} t^{3/2} x + \frac{2}{7} \tan^{7/2} x + C \end{aligned}$$

FUNDAMENTAL UNLOCKED- (FU#6) :

Q.1 Evaluate: $\int \frac{\sin^2 x}{\cos^4 x} dx$

Q.2 Evaluate: $\int \frac{\sqrt{\sin x} dx}{\cos^{5/2} x}$

Q.3 Evaluate: $\int \sin^2 x \cos^5 x dx$

(ii) $\int \frac{dx}{a + b \sin^2 x}$ OR $\int \frac{dx}{a + b \cos^2 x}$ OR

$$\int \frac{dx}{a \sin^2 x + b \sin x \cos x + c \cos^2 x}$$

Divide N^r and D^r by $\cos^2 x$ and put $\tan x = t$.





Example :

Evaluate: $\int \frac{dx}{2 + \sin^2 x}$

Solution:

Divide numerator and denominator by $\cos^2 x$

$$I = \int \frac{\sec^2 x dx}{2\sec^2 x + \tan^2 x} = \int \frac{\sec^2 x dx}{2 + 3\tan^2 x}$$

Let $\sqrt{3} \tan x = t \quad \therefore \sqrt{3} \sec^2 x dx = dt$

So

$$\int \frac{dt}{2+t^2} = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + C$$

$$= \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

Example :

Evaluate: $\int \frac{dx}{(2\sin x + 3\cos x)^2}$

Solution:

Divide numerator and denominator by $\cos^2 x$

$$\therefore I = \int \frac{\sec^2 x dx}{(2\tan x + 3)^2}$$

Let $2 \tan x + 3 = t, \quad \therefore 2\sec^2 x dx = dt$

$$I = \frac{1}{2} \int \frac{dt}{t^2} = -\frac{1}{2t} + C = -\frac{1}{2(2\tan x + 3)} + C$$

FUNDAMENTAL UNLOCKED- (FU#7) :

Q.1 Evaluate: $\int x e^x dx$

Q.2 Evaluate: $\int \frac{dx}{3\sin^2 x + \sin x \cos x + 1}$

(iii)

$$\int \frac{dx}{a + b \sin x} \text{ OR } \int \frac{dx}{a + b \cos x} \text{ OR } \int \frac{dx}{a + b \sin x + c \cos x}$$

Convert sines and cosines into their respective tangents of half the angles and put $\tan \frac{x}{2} = t$ In this case sin

$$x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, x = 2 \tan^{-1} t; dx = \frac{2dt}{1+t^2}$$

Example :

Evaluate: $\int \frac{dx}{3\sin x + 4\cos x}$

Solution:

$$I = \int \frac{dx}{3\sin x + 4\cos x}$$

=

$$\int \frac{dx}{3 \left\{ \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right\} + 4 \left\{ \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right\}} = \int \frac{\sec^2 \frac{x}{2} dx}{4 + 6 \tan \frac{x}{2} - 4 \tan^2 \frac{x}{2}}$$

let $\tan \frac{x}{2} = t, \quad \therefore \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

so I

=

$$\int \frac{2dt}{4 + 6t - 4t^2} = \frac{1}{2} \int \frac{dt}{1 - \left(t^2 - \frac{3}{2}t \right)} = \frac{1}{2} \int \frac{dt}{\frac{25}{16} - \left(t - \frac{3}{4} \right)^2}$$

$$= \frac{1}{2} \cdot \frac{1}{2 \left(\frac{5}{4} \right)} \ln \left| \frac{\frac{5}{4} + \left(t - \frac{3}{4} \right)}{\frac{5}{4} - \left(t - \frac{3}{4} \right)} \right| + C = \frac{1}{5} \ln \left| \frac{1 + 2 \tan \frac{x}{2}}{4 - 2 \tan \frac{x}{2}} \right| + C$$

FUNDAMENTAL UNLOCKED- (FU#8) :

Q.1 Evaluate: $\int \frac{dx}{3 + \sin x}$

Q.2 Evaluate: $\int \frac{dx}{1 + 4\sin x + 3\cos x}$

(iv) $\int \frac{a \cos x + b \sin x + c}{p \cos x + q \sin x + r} dx$

Express Numerator (N^r) = $\ell(D^r) + m \frac{d}{dx}(D^r) + n$ and proceed.

Example :

Evaluate: $\int \frac{2 + 3\cos \theta}{\sin \theta + 2\cos \theta + 3} d\theta$





Solution:

Write the Numerator = $\ell(\text{denominator}) + m(\text{d.c. of denominator}) + n$

$$\Rightarrow 2 + 3 \cos \theta = \ell(\sin \theta + 2 \cos \theta + 3) + m(\cos \theta - 2 \sin \theta) + n.$$

Comparing the coefficients of $\sin \theta$, $\cos \theta$ and constant terms,

$$\text{We get } 3\ell + n = 2, 2\ell + m = 3, \ell - 2m = 0 \Rightarrow \ell = 6/5, m = 3/5 \text{ and } n = -8/5$$

$$\text{Hence } I = \int \frac{6}{5} d\theta + \frac{3}{5} \int \frac{\cos \theta - 2 \sin \theta}{\sin \theta + 2 \cos \theta + 3} d\theta - \frac{8}{5} \int \frac{d\theta}{\sin \theta + 2 \cos \theta + 3}$$

$$= \frac{6}{5} \theta + \frac{3}{5} \ell n |\sin \theta + 2 \cos \theta + 3| - \frac{8}{5} I_3$$

$$\text{where } I_3 = \int \frac{d\theta}{\sin \theta + 2 \cos \theta + 3}$$

$$\text{In } I_3, \text{ put } \tan \frac{\theta}{2} = t \Rightarrow \sec^2 \frac{\theta}{2} d\theta = 2dt$$

$$I_3 = 2 \int \frac{dt}{t^2 + 2t + 5} = 2 \int \frac{dt}{(t+1)^2 + 2^2}$$

$$= 2 \cdot \frac{1}{2} \tan^{-1} \left(\frac{t+1}{2} \right) = \tan^{-1} \left(\frac{\tan \theta / 2 + 1}{2} \right)$$

$$\text{Hence } I = \frac{6\theta}{5} + \frac{3}{5} \ell n |\sin \theta + 2 \cos \theta + 3| - \frac{8}{5} \tan^{-1} \left(\frac{\tan \theta / 2 + 1}{2} \right) + C \text{ Ans.}$$

FUNDAMENTAL UNLOCKED- (FU#9) :

Q.1 Evaluate: $\int \frac{\sin x}{\sin x + \cos x} dx$

Q.2 Evaluate: $\int \frac{3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx$

(d) Integration of Rational Function

(i) Rational function is defined as the ratio of two polynomials in the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and

$Q(x)$ are polynomials in x and $Q(x) \neq 0$. If the degree of $P(x)$ is less than the degree of $Q(x)$, then the rational function is called proper, otherwise,

it is called improper. The improper rational function can be reduced to the proper rational functions by long division process. Thus, if $\frac{P(x)}{Q(x)}$

is improper, then $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$, where $T(x)$

is a polynomial in x and $\frac{P_1(x)}{Q(x)}$ is proper rational

function. It is always possible to write the integrand as a sum of simpler rational functions by a method called partial fraction decomposition. After this, the integration can be carried out easily using the already known methods.

S.No	Form of the rational function	Form of the partial fraction
1	$\frac{px^2 + qx + r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
2	$\frac{px^2 + qx + r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
3	$\frac{px^2 + qx + r}{(x-a)(x^2 + bx + c)}$ where $x^2 + bx + c$ cannot be factorised further	$\frac{A}{x-a} + \frac{Bx + C}{x^2 + bx + c}$

Example :

Evaluate: $\int \frac{x}{(x-2)(x+5)} dx$

Solution:

$$\frac{x}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5}$$

$$\text{or } x = A(x+5) + B(x-2).$$

by comparing the coefficients, we get

$$A = 2/7 \text{ and } B = 5/7 \text{ so that}$$

$$\int \frac{x}{(x-2)(x+5)} dx = \frac{2}{7} \int \frac{dx}{x-2} + \frac{5}{7} \int \frac{dx}{x+5} = \frac{2}{7} \ell n |(x-2)| + \frac{5}{7} \ell n |(x+5)| + C$$





Example :

Evaluate $\int \frac{x^4}{(x+2)(x^2+1)} dx$

Solution:

$$\frac{x^4}{(x+2)(x^2+1)} = (x-2) + \frac{3x^2+4}{(x+2)(x^2+1)}$$

Now, $\frac{3x^2+4}{(x+2)(x^2+1)} = \frac{16}{5(x+2)} + \frac{-\frac{1}{5}x + \frac{2}{5}}{x^2+1}$

So, $\frac{x^4}{(x+2)(x^2+1)} = x-2 + \frac{16}{5(x+2)} + \frac{-\frac{1}{5}x + \frac{2}{5}}{x^2+1}$

Now, $\int \left((x-2) + \frac{16}{5(x+2)} + \frac{-\frac{1}{5}x + \frac{2}{5}}{x^2+1} \right) dx$

$$= \frac{x^2}{2} - 2x + \frac{2}{5} \tan^{-1} x + \frac{16}{5} \ln|x+2| - \frac{1}{10} \ln(x^2+1) + C$$

FUNDAMENTAL UNLOCKED- (FU#10) :

Q.1 Evaluate: $\int \frac{3x+2}{(x+1)(x+3)} dx$

Q.2 Evaluate: $\int \frac{x^2-1}{(x+1)(x+2)^2} dx$

(ii)

$$\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c} dx$$

Express ax^2+bx+c in the form of perfect square and then apply the standard results.

(iii) $\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

Express $px+q = \ell$ (differential coefficient of denominator) + m.

Example :

Evaluate $\int \frac{dx}{2x^2+x-1}$

Solution:

$$I = \int \frac{dx}{2x^2+x-1} = \frac{1}{2} \int \frac{dx}{x^2 + \frac{x}{2} - \frac{1}{2}}$$

$$= \frac{1}{2} \int \frac{dx}{x^2 + \frac{x}{2} + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}}$$

$$= \frac{1}{2} \int \frac{dx}{(x+1/4)^2 - 9/16} = \frac{1}{2} \int \frac{dx}{(x+1/4)^2 - (3/4)^2}$$

$$= \frac{1}{2} \cdot \frac{1}{2(3/4)} \log \left| \frac{x+1/4-3/4}{x+1/4+3/4} \right| + C$$

$$\left\{ \text{using } \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \right\}$$

$$= \frac{1}{3} \log \left| \frac{x-1/2}{x+1} \right| + C = \frac{1}{3} \log \left| \frac{2x-1}{2(x+1)} \right| + C$$

Example :

Evaluate: $\int \frac{3x+2}{4x^2+4x+5} dx$

Solution:

Express $3x+2 = \ell(\text{d.c. of } 4x^2+4x+5) + m$

or, $3x+2 = \ell(8x+4) + m$

Comparing the coefficients, we get

$$8\ell = 3 \text{ and } 4\ell + m = 2 \Rightarrow \ell = 3/8 \text{ and } m = 2 - 4\ell = 1/2$$

$$\Rightarrow I = \frac{3}{8} \int \frac{8x+4}{4x^2+4x+5} dx + \frac{1}{2} \int \frac{dx}{4x^2+4x+5} =$$

$$\frac{3}{8} \log|4x^2+4x+5| + \frac{1}{8} \int \frac{dx}{x^2+x+\frac{5}{4}}$$

$$= \frac{3}{8} \log|4x^2+4x+5| + \frac{1}{8} \tan^{-1} \left(x + \frac{1}{2} \right) + C \text{ Ans.}$$

FUNDAMENTAL UNLOCKED- (FU#11) :

Q.1 Evaluate: $\int \frac{dx}{x^2+x+1}$

Q.2 Evaluate: $\int \frac{5x+4}{\sqrt{x^2+4x+1}} dx$

(iv) Integrals of form $\int \frac{x^2+1}{x^4+Kx^2+1} dx$ OR

$\int \frac{x^2-1}{x^4+Kx^2+1} dx$, where K is any constant. Divide N^r

and D^r by x^2 and proceed.



Note: Sometimes it is useful to write the integral as a sum of two related integrals, which can be evaluated by making suitable substitutions e.g.

$$\int \frac{2x^2}{x^4+1} dx = \int \frac{x^2+1}{x^4+1} dx + \int \frac{x^2-1}{x^4+1} dx$$

$$\int \frac{2}{x^4+1} dx = \int \frac{x^2+1}{x^4+1} dx - \int \frac{x^2-1}{x^4+1} dx$$

These integrals can be called as **Algebraic Twins**.

Example :

Evaluate: $\int \frac{4}{\sin^4 x + \cos^4 x} dx$

Solution:

$$I = 4 \int \frac{1}{\sin^4 x + \cos^4 x} dx = 4 \int \frac{\sec^4 x}{1 + \tan^4 x} dx$$

$$= 4 \int \frac{(\tan^2 x + 1) \sec^2 x}{(\tan^4 x + 1)} dx$$

Now, put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\Rightarrow I = 4 \int \frac{1+t^2}{1+t^4} dt = 4 \int \frac{1/t^2+1}{t^2+1/t^2} dt$$

Now, put $t - 1/t = z \Rightarrow \left(1 + \frac{1}{t^2}\right) dt = dz$

$$\Rightarrow$$

$$I = 4 \int \frac{dz}{z^2+2} = \frac{4}{\sqrt{2}} \tan^{-1} \frac{z}{\sqrt{2}} + C = 2\sqrt{2} \tan^{-1} \frac{t-1/t}{\sqrt{2}} + C$$

$$= 2\sqrt{2} \tan^{-1} \left(\frac{\tan x - 1/\tan x}{\sqrt{2}} \right) + C$$

Example :

Evaluate: $\int \frac{1}{x^4+5x^2+1} dx$

Solution:

$$I = \frac{1}{2} \int \frac{2}{x^4+5x^2+1} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1+x^2}{x^4+5x^2+1} dx + \frac{1}{2} \int \frac{1-x^2}{x^4+5x^2+1} dx$$

$$= \frac{1}{2} \int \frac{1+1/x^2}{x^2+5+1/x^2} dx - \frac{1}{2} \int \frac{1-1/x^2}{x^2+5+1/x^2} dx$$

{dividing N^r and D^r by x^2 }

$$= \frac{1}{2} \int \frac{(1+1/x^2)}{(x-1/x)^2+7} dx - \frac{1}{2} \int \frac{(1-1/x^2)}{(x+1/x)^2+3} dx$$

$$= \frac{1}{2} \int \frac{dt}{t^2+(\sqrt{7})^2} - \frac{1}{2} \int \frac{du}{u^2+(\sqrt{3})^2}$$

where $t = x - \frac{1}{x}$ and $u = x + \frac{1}{x}$

$$I = \frac{1}{2} \cdot \frac{1}{\sqrt{7}} \left(\tan^{-1} \frac{t}{\sqrt{7}} \right) - \frac{1}{2} \cdot \frac{1}{\sqrt{3}} \left(\tan^{-1} \frac{u}{\sqrt{3}} \right) + C$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{7}} \tan^{-1} \left(\frac{x-1/x}{\sqrt{7}} \right) - \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x+1/x}{\sqrt{3}} \right) \right] + C$$

FUNDAMENTAL UNLOCKED- (FU#12) :

Q.1 Evaluate: $\int \frac{x^2+1}{x^4-x^2+1} dx$

Q.2 Evaluate: $\int \frac{1}{1+x^4} dx$

(e) Integration of Irrational Functions

(i) $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ & $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$; put $px+q = t^2$

(ii) $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put $ax+b = \frac{1}{t}$;

$$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}, \text{ put } x = \frac{1}{t}$$

Example :

Evaluate $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$

Solution:

Let, $I = \int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$ Put $x+1 = t^2 \Rightarrow dx = 2t dt$

$$\therefore I =$$

$$\int \frac{(t^2-1)+2}{\{(t^2-1)^2+3(t^2-1)+3\}\sqrt{t^2}} \cdot (2t) dt = 2 \int \frac{t^2+1}{t^4+t^2+1} dt$$

$$= 2 \int \frac{1+1/t^2}{t^2+1+1/t^2} dt$$

$$=$$

$$\left\{ \text{where } u = t - \frac{1}{t} \right\}$$



$$= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{u}{\sqrt{3}} \right) + C = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{t^2 - 1}{\sqrt{3}t} \right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right) + C \quad \text{Ans.}$$

Example :

Evaluate $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$

Solution:

Let, $I = \int \frac{dx}{(x-1)\sqrt{x^2+x+1}}$ put $x - 1 = \frac{1}{t} \Rightarrow dx = -1/t^2 dt$

$$I = \int \frac{-1/t^2 dt}{1/t \sqrt{\left(\frac{1}{t} + 1\right)^2 + \left(\frac{1}{t} + 1\right) + 1}} = - \int \frac{dt}{\sqrt{3t^2 + 3t + 1}}$$

$$= - \frac{1}{\sqrt{3}} \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 + 1/12}}$$

$$= - \frac{1}{\sqrt{3}} \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 + 1/12} \right| + C$$

$$= - \frac{1}{\sqrt{3}} \log \left| \left(\frac{1}{x-1} + \frac{1}{2}\right) + \sqrt{\frac{12\left(\frac{1}{x-1} + \frac{1}{2}\right)^2 + 1}{12}} \right| + C$$

Example :

Evaluate: $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Solution:

Let, $I = \int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ Put $x = \frac{1}{t}$, So that

$$dx = \frac{-1}{t^2} dt$$

$$\therefore I = \int \frac{-1/t^2 dt}{(1+1/t^2)\sqrt{1-1/t^2}} = - \int \frac{t dt}{(t^2+1)\sqrt{t^2-1}}$$

Again let, $t^2 = u$. So that $2t dt = du$.

$$= - \frac{1}{2} \int \frac{du}{(u+1)\sqrt{u-1}} \text{ which reduces to the form}$$

$$\int \frac{dx}{P\sqrt{Q}} \text{ where both P and Q are linear so that we put}$$

$$u - 1 = z^2 \text{ so that } du = 2z dz$$

$$\therefore I = - \frac{1}{2} \int \frac{2z dz}{(z^2+1+1)\sqrt{z^2}} = - \int \frac{dz}{(z^2+2)}$$

$$I = - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{z}{\sqrt{2}} \right) + C$$

$$I = - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{u-1}}{\sqrt{2}} \right) + C = - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{t^2-1}}{\sqrt{2}} \right) + C$$

$$= - \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}x} \right) + C \quad \text{Ans.}$$

FUNDAMENTAL UNLOCKED- (FU#13) :

Q.1 Evaluate: $\int \frac{x}{(x-3)\sqrt{x+1}} dx$

Q.2 Evaluate: $\int \frac{dx}{x^2 \sqrt{1+x^2}}$

(f) Manipulating Integrands

(i) $\int \frac{dx}{x(x^n+1)}$, $n \in \mathbb{N}$, take x^n common and put $1+x^{-n} = t$.

(ii) $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}$, $n \in \mathbb{N}$, take x^n common and put $1+x^{-n} = t^n$

(iii) $\int \frac{dx}{x^n(1+x^n)^{1/n}}$, take x^n common and put $1+x^{-n} = t^n$.

Example :

Evaluate: $\int \frac{dx}{x^n(1+x^n)^{1/n}}$

Solution:

$$\text{Let } I = \int \frac{dx}{x^n(1+x^n)^{1/n}} = \int \frac{dx}{x^{n+1} \left(1 + \frac{1}{x^n}\right)^{1/n}}$$





Put $1 + \frac{1}{x^n} = t^n$, then $\frac{1}{x^{n+1}} dx = -t^{n-1} dt$

$$I = - \int \frac{t^{n-1} dt}{t} = - \int t^{n-2} dt = - \frac{t^{n-1}}{n-1} + C$$

$$= \frac{-1}{n-1} \left(1 + \frac{1}{x^n} \right)^{\frac{n-1}{n}} + C$$

Ans.

FUNDAMENTAL UNLOCKED- (FU#14) :

(i) Evaluate: $\int \frac{dx}{x(x^2+1)}$

(ii) Evaluate: $\int \frac{dx}{x^2(x^3+1)^{2/3}}$

(iii) Evaluate: $\int \frac{dx}{x^3(x^3+1)^{1/3}}$

Miscellaneous Examples
Example 27:

Evaluate: $\int \frac{\cos^4 x dx}{\sin^3 x \{ \sin^5 x + \cos^5 x \}^{3/5}}$

Solution:

$$I = \int \frac{\cos^4 x}{\sin^3 x \{ \sin^5 x + \cos^5 x \}^{3/5}} dx$$

$$= \int \frac{\cos^4 x}{\sin^6 x \{ 1 + \cot^5 x \}^{3/5}} dx = \int \frac{\cot^4 x \operatorname{cosec}^2 x dx}{(1 + \cot^5 x)^{3/5}}$$

Put $1 + \cot^5 x = t$

$$5 \cot^4 x \operatorname{cosec}^2 x dx = - dt$$

$$= - \frac{1}{5} \int \frac{dt}{t^{3/5}} = - \frac{1}{2} t^{2/5} + C = - \frac{1}{2} (1 + \cot^5 x)^{2/5} + C$$

Example :

$\int \frac{dx}{\cos^6 x + \sin^6 x}$ is equal to-

(A) $\ln |\tan x - \cot x| + C$ (B) $\ln |\cot x - \tan x| + C$

(C) $\tan^{-1}(\tan x - \cot x) + C$ (D) $\tan^{-1}(-2 \cot 2x) + C$

Solution:

$$\text{Let } I = \int \frac{dx}{\cos^6 x + \sin^6 x} = \int \frac{\sec^6 x}{1 + \tan^6 x} dx$$

$$= \int \frac{(1 + \tan^2 x)^2 \sec^2 x dx}{1 + \tan^6 x}$$

If $\tan x = p$, then $\sec^2 x dx = dp$

$$\Rightarrow I = \int \frac{(1+p^2)^2 dp}{1+p^6}$$

$$= \int \frac{(1+p^2)}{p^4 - p^2 + 1} dp = \int \frac{p^2 \left(1 + \frac{1}{p^2} \right)}{p^2 \left(p^2 + \frac{1}{p^2} - 1 \right)} dp$$

$$= \int \frac{dk}{k^2 + 1} = \tan^{-1}(k) + C$$

$$\left(\text{where } p - \frac{1}{p} = k, \left(1 + \frac{1}{p^2} \right) dp = dk \right)$$

$$= \tan^{-1} \left(p - \frac{1}{p} \right) + C = \tan^{-1}(\tan x - \cot x) + C = \tan^{-1}(-$$

$$2 \cot 2x) + C$$

Ans. (C, D)

Example :

Evaluate: $\int \frac{2 \sin 2x - \cos x}{6 - \cos^2 x - 4 \sin x} dx$

Solution:

$$I = \int \frac{2 \sin 2x - \cos x}{6 - \cos^2 x - 4 \sin x} dx = \int \frac{(4 \sin x - 1) \cos x}{6 - (1 - \sin^2 x) - 4 \sin x}$$

$$dx = \int \frac{(4 \sin x - 1) \cos x}{\sin^2 x - 4 \sin x + 5} dx$$

Put $\sin x = t$, so that $\cos x dx = dt$.

$$\therefore I = \int \frac{(4t-1)dt}{(t^2-4t+5)} \quad \dots(i)$$

Now, let $(4t-1) = \lambda(2t-4) + \mu$

Comparing coefficients of like powers of t , we get

$$2\lambda = 4, -4\lambda + \mu = -1 \quad \dots(ii)$$

$$\lambda = 2, \mu = 7 \quad \therefore I = \int \frac{2(2t-4) + 7}{t^2 - 4t + 5} dt$$

{using (i) and (ii)}

=

$$2 \int \frac{2t-4}{t^2-4t+5} dt + 7 \int \frac{dt}{t^2-4t+5} = 2 \log |t^2-4t+5| + 7$$

$$\int \frac{dt}{t^2-4t+4-4+5}$$

$$= 2 \log |t^2-4t+5| + 7 \int \frac{dt}{(t-2)^2 + (1)^2}$$

$$= 2 \log |t^2-4t+5| + 7 \tan^{-1}(t-2) + C$$

$$= 2 \log |\sin^2 x - 4 \sin x + 5| + 7 \tan^{-1}(\sin x - 2) + C.$$





ANSWER KEY

FUNDAMENTAL UNLOCKED- (FU#1) :

Q.1 $\frac{1}{36} \tan^{-1} \left(\frac{4x^3}{3} \right) + C$

Q.2 $\sin x - \frac{1}{3} \sin^3 x + C$

FUNDAMENTAL UNLOCKED- (FU#2) :

Q.1 $\sqrt{(x-2)(3-x)} - \sin^{-1} \sqrt{3-x} + C$

Q.2 $\frac{1}{2} \ln \left[\frac{\sqrt{x^2+4}-2}{x} \right] + C$

FUNDAMENTAL UNLOCKED- (FU#3) :

Q.1 (i) $xe^x - e^x + C$

Q.2 $\frac{1}{2} [-x^2 \cos x^2 + \sin x^2] + C$

FUNDAMENTAL UNLOCKED- (FU#4) :

Q.1 $e^x \tan^{-1} x + C$

Q.2 $\frac{1}{2} e^{x^2} \sin(x^2) + C$

FUNDAMENTAL UNLOCKED- (FU#5) :

Q.1 $x \tan(e^x) + C$

Q.2 $x \ln x + C$

FUNDAMENTAL UNLOCKED- (FU#6) :

Q.1 $\frac{1}{3} \tan^3 x + C$

Q.2 $\frac{2}{3} \tan^{3/2} x + C$

Q.3 $\frac{1}{3} \sin^3 x - \frac{2}{5} \sin^5 x + \frac{1}{7} \sin^7 x + C$

FUNDAMENTAL UNLOCKED- (FU#7) :

Q.1 $\frac{1}{\sqrt{5}} \tan^{-1}(\sqrt{5} \tan x) + C$

Q.2 $\frac{2}{\sqrt{15}} \tan^{-1} \left(\frac{8 \tan x + 1}{\sqrt{15}} \right) + C$

FUNDAMENTAL UNLOCKED- (FU#8) :

Q.1 $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{3 \tan x / 2 + 1}{2\sqrt{2}} \right) + C$

Q.2 $\frac{1}{2\sqrt{6}} \ln \left| \frac{\sqrt{6} + \tan x / 2 - 2}{\sqrt{6} - \tan x / 2 + 2} \right| + C$




FUNDAMENTAL UNLOCKED- (FU#9) :

Q.1 $\frac{1}{2}x - \frac{1}{2}\ell n |\sin x + \cos x| + C$

Q.2 $\frac{12}{13}x - \frac{5}{13}\ell n |3\cos x + 2\sin x| + C$

FUNDAMENTAL UNLOCKED- (FU#10) :

Q.1 $-\frac{1}{2}\ell n |x+1| + \frac{7}{2}\ell n |x+3| + C$

Q.2 $\ell n |x+2| + \frac{3}{x+2} + C$

FUNDAMENTAL UNLOCKED- (FU#11) :

Q.1 $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

Q.2 $5\sqrt{x^2+4x+1} - 6\ell n \left[(x+2) + \sqrt{x^2+4x+1} \right] + C$

FUNDAMENTAL UNLOCKED- (FU#12) :

Q.1 $\tan^{-1}\left(\frac{x^2-1}{x}\right) + C$

Q.2 $\frac{1}{2\sqrt{2}}\left[\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{2}\ell n \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right]$

FUNDAMENTAL UNLOCKED- (FU#13) :

Q.1 $2\sqrt{x+1} + \frac{3}{2}\ell n \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + C$

Q.2 $-\frac{1}{x}\sqrt{1+x^2} + C$

FUNDAMENTAL UNLOCKED- (FU#14) :

Q.1 $-\frac{1}{2}\ell n \left(\frac{x^2+1}{x^2} \right) + C$

Q.2 $-\left(1 + \frac{1}{x^3}\right)^{1/3} + C$

Q.3 $-\frac{1}{2}\left(1 + \frac{1}{x^3}\right)^{2/3} + C$





OBJECTIVE EXERCISE - I

Single Correct Type Questions

- Integrate with respect to x : $\sqrt{x+1}$
 (A) $\frac{(x+1)^{3/2}}{2} + C$ (B) $\frac{3(x+1)^{3/2}}{2} + C$
 (C) $\frac{(x+1)^{3/2}}{3} + C$ (D) $\frac{2(x+1)^{3/2}}{3} + C$
- Integrate with respect to x : $\frac{1}{\sqrt{2x+1}}$
 (A) $\sqrt{2x+1} + C$ (B) $(2x+1)^{3/2} + C$
 (C) $-\sqrt{2x+1} + C$ (D) $\frac{1}{(2x+1)^{3/2}} + C$
- If $\int \frac{1}{1+\sin x} dx = \tan\left(\frac{x}{2} + a\right) + C$, then
 (A) $a = -\frac{\pi}{4}$, $C \in \mathbb{R}$ (B) $a = \frac{\pi}{4}$, $C \in \mathbb{R}$
 (C) $a = \frac{5\pi}{4}$, $C \in \mathbb{R}$ (D) $a = \frac{\pi}{3}$, $C \in \mathbb{R}$
- If $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + C$, then
 (A) $a = \frac{5\pi}{4}$, $C \in \mathbb{R}$ (B) $a = -\frac{5\pi}{4}$, $C \in \mathbb{R}$
 (C) $a = \frac{\pi}{4}$, $C \in \mathbb{R}$ (D) $a = \frac{\pi}{2}$, $C \in \mathbb{R}$
- The value of $\int \frac{\cos 2x}{\cos x} dx$ is equal to
 (A) $2 \sin x - \ln |\sec x + \tan x| + C$
 (B) $2 \sin x - \ln |\sec x - \tan x| + C$
 (C) $2 \sin x + \ln |\sec x + \tan x| + C$
 (D) $\sin x - \ln |\sec x - \tan x| + C$
- If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos 4x + B$; where A & B are constants, then
 (A) $A = -1/4$ & B may have any value
 (B) $A = -1/8$ & B may have any value
 (C) $A = -1/2$ & $B = -1/4$
 (D) $A = B = 1/2$
- Let $g(x)$ be an antiderivative for $f(x)$. Then $\ln(1 + (g(x))^2)$ is an antiderivative for
 (A) $\frac{2f(x)g(x)}{1 + (f(x))^2}$ (B) $\frac{2f(x)g(x)}{1 + (g(x))^2}$
 (C) $\frac{2f(x)}{1 + (f(x))^2}$ (D) none
- The value of $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to
 (A) $\frac{a^{\sqrt{x}}}{\sqrt{x}} + C$ (B) $\frac{2a^{\sqrt{x}}}{\ln a} + C$
 (C) $2a^{\sqrt{x}} \cdot \ln a + C$ (D) $2a^{\sqrt{x}} + C$
- The value of $\int 5^{5^x} \cdot 5^{5^x} \cdot 5^x dx$ is equal to
 (A) $\frac{5^{5^x}}{(\ln 5)^3} + C$ (B) $5^{5^x} (\ln 5)^3 + C$ (C) $\frac{5^{5^x}}{(\ln 5)^3} + C$
 (D) $\frac{5^{5^x}}{(\ln 5)^2} + C$
- The value of $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to
 (A) $2\sqrt{\tan x} + C$ (B) $2\sqrt{\cot x} + C$
 (C) $\frac{\sqrt{\tan x}}{2} + C$ (D) $\sqrt{\tan x} + C$
- If $\int \frac{2^x}{\sqrt{1-4^x}} dx = K \sin^{-1}(2^x) + C$, then the value of K is equal to
 (A) $\ln 2$ (B) $\frac{1}{2} \ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$
- If $y = \int \frac{dx}{(1+x^2)^{3/2}}$ and $y = 0$ when $x = 0$, then value of y when $x = 1$, is:
 (A) $\sqrt{\frac{2}{3}}$ (B) $\sqrt{2}$
 (C) $3\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$



13. The value of $\int \tan^3 2x \sec 2x \, dx$ is equal to:

- (A) $\frac{1}{3} \sec^3 2x - \frac{1}{2} \sec 2x + C$
 (B) $-\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + C$
 (C) $\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + C$
 (D) $\frac{1}{3} \sec^3 2x + \frac{1}{2} \sec 2x + C$

14. If $\int x^{13/2} \cdot (1+x^{5/2})^{1/2} \, dx = P(1+x^{5/2})^{7/2} + Q(1+x^{5/2})^{5/2} + R(1+x^{5/2})^{3/2} + C$, then P, Q and R are

- (A) $P = \frac{4}{35}$, $Q = -\frac{8}{25}$, $R = \frac{4}{15}$
 (B) $P = \frac{4}{35}$, $Q = \frac{8}{25}$, $R = \frac{4}{15}$
 (C) $P = -\frac{4}{35}$, $Q = -\frac{8}{25}$, $R = \frac{4}{15}$
 (D) $P = \frac{4}{35}$, $Q = -\frac{8}{25}$, $R = -\frac{4}{15}$

15. $\int \frac{1-x^7}{x(1+x^7)} \, dx$ equals -

- (A) $\ln x + \frac{2}{7} \ln(1+x^7) + C$
 (B) $\ln x - \frac{2}{7} \ln(1-x^7) + C$
 (C) $\ln x - \frac{2}{7} \ln(1+x^7) + C$
 (D) $\ln x + \frac{2}{7} \ln(1-x^7) + C$

16. The value of $\int \frac{1}{x^2(x^4+1)^{3/4}} \, dx$ is equal to

- (A) $\left(1 + \frac{1}{x^4}\right)^{1/4} + C$ (B) $(x^4+1)^{1/4} + C$
 (C) $\left(1 - \frac{1}{x^4}\right)^{1/4} + C$ (D) $-\left(1 + \frac{1}{x^4}\right)^{1/4} + C$

17. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$

- (A) $-\left(\frac{x+1}{x}\right)^{1/6} + C$ (B) $6\left(\frac{x+1}{x}\right)^{-1/6} + C$
 (C) $\left(\frac{x}{x+1}\right)^{5/6} + C$ (D) $-\left(\frac{x}{x+1}\right)^{5/6} + C$

18. $\int \frac{x^3}{(2x^2+1)^3} \, dx$ is equal to -

- (A) $\frac{1}{4}\left(2 + \frac{1}{x^2}\right)^{-2} + C$ (B) $-\frac{1}{4}\left(2 + \frac{1}{x^2}\right)^{-2} + C$
 (C) $\frac{1}{2}\left(2 + \frac{1}{x^2}\right)^{-2} + C$ (D) $\frac{1}{4}\left(2 + \frac{1}{x^2}\right)^2 + C$
 (where 'C' is integration constant)

19. Consider $f(x) = \frac{x^2}{1+x^3}$; $g(t) = \int f(t) \, dt$.

If $g(1) = 0$ then $g(x)$ equals

- (A) $\frac{1}{3} \ln(1+x^3)$ (B) $\frac{1}{3} \ln\left(\frac{1+x^3}{2}\right)$
 (C) $\frac{1}{2} \ln\left(\frac{1+x^3}{3}\right)$ (D) $\frac{1}{3} \ln\left(\frac{1+x^3}{3}\right)$

20. The value of $\int (x-1) e^{-x} \, dx$ is equal to

- (A) $-xe^x + C$ (B) $xe^x + C$
 (C) $-xe^{-x} + C$ (D) $xe^{-x} + C$

21. The value of $\int e^{\tan^{-1}x} \left(\frac{1+x+x^2}{1+x^2}\right) \, dx$ is equal to

- (A) $x e^{\tan^{-1}x} + C$ (B) $x^2 e^{\tan^{-1}x} + C$
 (C) $\frac{1}{x} e^{\tan^{-1}x} + C$ (D) $x e^{\cot^{-1}x} + C$

22. The value of $\int [f(x)g''(x) - f''(x)g(x)] \, dx$ is equal to

- (A) $\frac{f(x)}{g'(x)} + C$
 (B) $f'(x)g(x) - f(x)g'(x) + C$
 (C) $f(x)g'(x) - f'(x)g(x) + C$
 (D) $f(x)g'(x) + f'(x)g(x) + C$



23. $\int \frac{x \ln x}{(x^2 - 1)^{3/2}} dx$ equals

(A) $\operatorname{arcsec} x - \frac{\ln x}{\sqrt{x^2 - 1}} + C$

(B) $\sec^{-1} x + \frac{\ln x}{\sqrt{x^2 - 1}} + C$

(C) $\cos^{-1} x - \frac{\ln x}{\sqrt{x^2 - 1}} + C$

(D) $\sec x - \frac{\ln x}{\sqrt{x^2 - 1}} + C$

24. The value of $\int (x e^{n \sin x} - \cos x) dx$ is equal to:

(A) $x \cos x + C$ (B) $\sin x - x \cos x + C$

(C) $-e^{n x} \cos x + C$ (D) $\sin x + x \cos x + C$

25. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} (x + \sqrt{x}) dx$

(A) $2e^{\sqrt{x}} [x - \sqrt{x} + 1] + C$

(B) $e^{\sqrt{x}} [x - 2\sqrt{x} + 1] + C$

(C) $e^{\sqrt{x}} (x + \sqrt{x}) + C$

(D) $e^{\sqrt{x}} (x + \sqrt{x} + 1) + C$

26. Let $f(x) = \frac{2 \sin^2 x - 1}{\cos x} + \frac{\cos x (2 \sin x + 1)}{1 + \sin x}$

Then $\int e^x (f(x) + f'(x)) dx$

(where C is the constant of integration)

(A) $e^x \tan x + C$ (B) $e^x \cot x + C$

(C) $e^x \operatorname{cosec}^2 x + C$ (D) $e^x \sec^2 x + C$

27. $\int e^x \left(\frac{x^2 - 3}{(x - 1)^2} \right) dx$ is equal to—

(A) $e^x \left(\frac{x + 3}{x - 1} \right) + C$ (B) $e^x \left(\frac{x - 3}{x - 1} \right) + C$

(C) $e^x \left(\frac{x + 1}{x - 1} \right) + C$ (D) $e^x \left(\frac{1}{x - 1} \right)^2 + C$

(where 'C' is integration constant)

28. The value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to

(A) $\frac{-1}{\sin x + \cos x} + C$

(B) $\ln (\sin x + \cos x) + C$

(C) $\ln (\sin x - \cos x) + C$

(D) $\ln (\sin x + \cos x)^2 + C$

29. The value of $\int [1 + \tan x \cdot \tan(x + \alpha)] dx$ is equal to

(A) $\cos \alpha \cdot \ln \left| \frac{\sin x}{\sin(x + \alpha)} \right| + C$

(B) $\tan \alpha \cdot \ln \left| \frac{\sin x}{\sin(x + \alpha)} \right| + C$

(C) $\cot \alpha \cdot \ln \left| \frac{\sec(x + \alpha)}{\sec x} \right| + C$

(D) $\cot \alpha \cdot \ln \left| \frac{\cos(x + \alpha)}{\cos x} \right| + C$

30. The value of $\int \sqrt{\sec x - 1} dx$ is equal to

(A) $2 \ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$

(B) $\ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$

(C) $-2 \ln \left(\cos \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$

(D) $-2 \ln \left(\sin \frac{x}{2} + \sqrt{\cos^2 \frac{x}{2} - \frac{1}{2}} \right) + C$

31. The value of $\int \frac{dx}{\cos^3 x \sqrt{\sin 2x}}$ is equal to

(A) $\sqrt{2} \left(\sqrt{\cos x} + \frac{1}{5} \tan^{5/2} x \right) + C$

(B) $\sqrt{2} \left(\sqrt{\tan x} + \frac{1}{5} \tan^{5/2} x \right) + C$

(C) $\sqrt{2} \left(\sqrt{\tan x} - \frac{1}{5} \tan^{5/2} x \right) + C$

(D) $\sqrt{2} \left(\sqrt{\cos x} - \frac{1}{5} \tan^{5/2} x \right) + C$





32. Antiderivative of $\frac{\sin^2 x}{1 + \sin^2 x}$ w.r.t. x is :

(A) $x - \frac{\sqrt{2}}{2} \arctan(\sqrt{2} \tan x) + C$

(B) $x - \frac{1}{\sqrt{2}} \arctan\left(\frac{\tan x}{\sqrt{2}}\right) + C$

(C) $x - \sqrt{2} \arctan(\sqrt{2} \tan x) + C$

(D) $x - \sqrt{2} \arctan\left(\frac{\tan x}{\sqrt{2}}\right) + C$

33. Integrate $\frac{1}{1 - \cot x}$

(A) $\frac{1}{2} \log |\sin x - \cos x| + \frac{1}{2} x + C$

(B) $\frac{1}{2} \log |\sin x + \cos x| + \frac{1}{2} x + C$

(C) $\frac{1}{2} \log |\sin x + \cos x| - \frac{1}{2} x + C$

(D) $\frac{1}{2} \log |\sin x - \cos x| - \frac{1}{2} x + C$

34. The value of $\int \frac{dx}{x^2 + x + 1}$ is equal to

(A) $\frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

(B) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

(C) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

(D) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

35. Primitive of $\frac{3x^4 - 1}{(x^4 + x + 1)^2}$ w.r.t. x is -

(A) $\frac{x}{x^4 + x + 1} + C$ (B) $-\frac{x}{x^4 + x + 1} + C$

(C) $\frac{x+1}{x^4 + x + 1} + C$ (D) $-\frac{x+1}{x^4 + x + 1} + C$

36. Integral $\int \sqrt{1 + 2 \cot x (\cot x + \operatorname{cosec} x)} dx$ w.r.t. x is

(A) $2 \ln \cos \frac{x}{2} + C$

(B) $2 \ln \sin \frac{x}{2} + C$

(C) $\frac{1}{2} \ln \cos \frac{x}{2} + C$

(D) $\ln \sin x - \ln(\operatorname{cosec} x - \cot x) + C$

37. $\int \frac{(2x+3)}{x(x+1)(x+2)(x+3)+1} dx = C - \frac{1}{f(x)}$, where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals

(A) 4 (B) 5 (C) 6 (D) none

38. The value of $\int \frac{dx}{x\sqrt{1-x^3}}$ is equal to

(A) $\frac{1}{3} \ln \left| \frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1} \right| + C$

(B) $\frac{1}{3} \ln \left| \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1} \right| + C$

(C) $\frac{1}{3} \ln \left| \frac{1}{\sqrt{1-x^3}} \right| + C$

(D) $\frac{1}{3} \ln |1 - x^3| + C$

39. The value of $\int \sqrt{\frac{e^x - 1}{e^x + 1}} dx$ is equal to

(A) $\ln(e^x + \sqrt{e^{2x} - 1}) - \sec^{-1}(e^x) + C$

(B) $\ln(e^x + \sqrt{e^{2x} - 1}) + \sec^{-1}(e^x) + C$

(C) $\ln(e^x - \sqrt{e^{2x} - 1}) - \sec^{-1}(e^x) + C$

(D) $\ln(e^x + \sqrt{e^{2x} - 1}) - \sin^{-1}(e^x) + C$

40. If $\int \frac{dx}{x^4 + x^3} = \frac{A}{x^2} + \frac{B}{x} + \ln \left| \frac{x}{x+1} \right| + C$, then

(A) $A = \frac{1}{2}, B = 1$ (B) $A = 1, B = -\frac{1}{2}$

(C) $A = -\frac{1}{2}, B = 1$ (D) $A = -\frac{1}{2}, B = \frac{1}{2}$





OBJECTIVE EXERCISE – II

Single Correct Type Questions

1. If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x} dx = \ln(f(x)) + g(x) + C$ where C is the constant of integration and $f(x)$ is positive, then $f(x) + g(x)$ has the value equal to
 (A) $e^x + \sin x + 2x$ (B) $e^x + \sin x$
 (C) $e^x - \sin x$ (D) $e^x + \sin x + x$

2. $\int x \cdot \frac{\ln(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ equals –
 (A) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - x + C$
 (B) $\frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) - \frac{x}{\sqrt{1+x^2}} + C$
 (C) $\frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}} + C$
 (D) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) + x + C$

3. A function $y = f(x)$ satisfies
 $f''(x) = -\frac{1}{x^2} - \pi^2 \sin(\pi x); f'(2) = \pi + \frac{1}{2}$ and
 $f(1) = 0$. The value of $f\left(\frac{1}{2}\right)$ is
 (A) $\ln 2$ (B) 1
 (C) $\frac{\pi}{2} - \ln 2$ (D) $1 - \ln 2$

4. $\int \frac{x^2(1 - \ln x)}{\ln^4 x - x^4} dx$ equals
 (A) $\frac{1}{2} \ln\left(\frac{x}{\ln x}\right) - \frac{1}{4} \ln(\ln^2 x - x^2) + C$
 (B) $\frac{1}{4} \ln\left(\frac{\ln x - x}{\ln x + x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{\ln x}{x}\right) + C$
 (C) $\frac{1}{4} \ln\left(\frac{\ln x + x}{\ln x - x}\right) + \frac{1}{2} \tan^{-1}\left(\frac{\ln x}{x}\right) + C$
 (D) $\frac{1}{4} \left(\ln\left(\frac{\ln x - x}{\ln x + x}\right) + \tan^{-1}\left(\frac{\ln x}{x}\right) \right) + C$

5. $\int (\sin(101x) \cdot \sin^{99} x) dx$ equals
 (A) $\frac{\sin(100x)(\sin x)^{100}}{100} + C$
 (B) $\frac{\cos(100x)(\sin x)^{100}}{100} + C$
 (C) $\frac{\cos(100x)(\cos x)^{100}}{100} + C$
 (D) $\frac{\sin(100x)(\sin x)^{101}}{101} + C$

6. The evaluation of $\int \frac{p x^{p+2q-1} - q x^{q-1}}{x^{2p+2q} + 2x^{p+q} + 1} dx$ is
 (A) $-\frac{x^p}{x^{p+q} + 1} + C$ (B) $\frac{x^q}{x^{p+q} + 1} + C$
 (C) $-\frac{x^q}{x^{p+q} + 1} + C$ (D) $\frac{x^p}{x^{p+q} + 1} + C$
7. The integral $\int \sqrt{\cot x} e^{\sqrt{\sin x}} \sqrt{\cos x} dx$ equals
 (A) $\frac{\sqrt{\tan x} e^{\sqrt{\sin x}}}{\sqrt{\cos x}} + C$ (B) $2e^{\sqrt{\sin x}} + C$
 (C) $-\frac{1}{2} e^{\sqrt{\sin x}} + C$ (D) $\frac{\sqrt{\cot x} e^{\sqrt{\sin x}}}{2\sqrt{\cos x}} + C$

One or More than One Correct Type Questions

8. Which one of the following is FALSE?
 (A) $x \cdot \int \frac{dx}{x} = x \ln |x| + C$
 (B) $x \cdot \int \frac{dx}{x} = x \ln |x| + Cx$
 (C) $\frac{1}{\cos x} \cdot \int \cos x dx = \tan x + C$
 (D) $\frac{1}{\cos x} \cdot \int \cos x dx = x + C$
9. If $I_n = \int (\sin x)^n dx$ $n \in \mathbb{N}$, then $5I_4 - 6I_6$ is equal to –
 (A) $\sin x \cdot (\cos x)^5 + C$
 (B) $\cos x \cdot (\sin x)^5 + C$
 (C) $\frac{\sin 2x}{8} [\cos^2 2x + 1 - 2\cos 2x] + C$
 (D) $\frac{\sin 2x}{8} [\cos^2 2x + 1 + 2\cos 2x] + C$



10. Let $f(x) = \sin^3 x + \sin^3 \left(x + \frac{2\pi}{3}\right) + \sin^3 \left(x + \frac{4\pi}{3}\right)$ then the primitive of $f(x)$ w.r.t. x is

(A) $-\frac{3\sin 3x}{4} + C$ (B) $\frac{1}{2}\cos^2\left(\frac{3x}{2}\right) + C$
 (C) $\frac{\sin 3x}{4} + C$ (D) $\frac{\cos 3x}{4} + C$

Where C is an arbitrary constant.

11. Suppose $J = \int \frac{\sin^2 x + \sin x}{1 + \sin x + \cos x} dx$ and $K = \int \frac{\cos^2 x + \cos x}{1 + \sin x + \cos x} dx$. If C is an arbitrary constant of integration then which of the following is/are correct?

(A) $J = \frac{1}{2}(x - \sin x + \cos x) + C$
 (B) $J = K - (\sin x + \cos x) + C$
 (C) $J = x - K + C$
 (D) $K = \frac{1}{2}(x - \sin x + \cos x) + C$

12. $\int \frac{\cot^{-1}(e^x)}{e^x} dx$ equals to

(A) $\frac{1}{2}\ln(e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} + x + c$
 (B) $\frac{1}{2}\ln(e^{2x} + 1) + \frac{\cot^{-1}(e^x)}{e^x} + x + c$
 (C) $\frac{1}{2}\ln(e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} - x + c$
 (D) $\frac{1}{2}\ln(e^{2x} + 1) + \frac{\cot^{-1}(e^{-x})}{e^x} - \frac{\pi}{2}e^{-x} - x + c$

13. $\int \sec^2 \theta (\sec \theta + \tan \theta)^2 d\theta$

(A) $\frac{(\sec \theta + \tan \theta)}{2} [2 + \tan \theta (\sec \theta + \tan \theta)] + C$
 (B) $\frac{(\sec \theta + \tan \theta)}{3} [2 + 4 \tan \theta (\sec \theta + \tan \theta)] + C$
 (C) $\frac{(\sec \theta + \tan \theta)}{3} [2 + \tan \theta (\sec \theta + \tan \theta)] + C$
 (D) $\frac{(\sec \theta + \tan \theta)}{3} [1 + \sec \theta (\sec \theta + \tan \theta)] + C$

14. If $f'(x^2) = \frac{\ln x}{x^2}$ and $f(1) = -\frac{1}{4}$, then -

(A) $f(e) = 0$ (B) $f'(e) = \frac{1}{2e}$
 (C) $f''(e) = f(e)$ (D) $f''(e) = f'(e)$

15. $\int \frac{\sin 2x}{(\sin x + \cos x)^2} dx$ is equal to -

(A) $x + \frac{1}{2}\cot\left(x + \frac{\pi}{4}\right) + C$
 (B) $x + \frac{1}{2}\tan\left(\frac{\pi}{4} - x\right) + C$
 (C) $x - \frac{\tan 2x}{2} + \frac{\sec 2x}{2} + C$
 (D) $x + \frac{1}{1 + \tan x} + C$

(where C is constant of integration)

16. $I = \int \frac{2x - 1 - x^2}{(1 + x^2)^2} dx$ is equal to -

(A) $\alpha - \frac{1}{1 + x^2} - \tan^{-1} x$
 (B) $\cot^{-1} x - \frac{1}{1 + x^2} + \beta$
 (C) $\frac{x^2}{1 + x^2} - \tan^{-1} x + \gamma$
 (D) $\frac{2x^2 + 1}{1 + x^2} - \tan^{-1} x + \delta$

(where $\alpha, \beta, \gamma, \delta$ are arbitrary constants)

17. $\int \frac{x+1}{2x^{3/2}} dx$ equal to -

(A) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} + C$
 (B) $\frac{x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{x} + C$
 (C) $\frac{x^{\frac{3}{2}} + \sqrt{x}(\sqrt{x} - 1)}{x} + C$
 (D) $\frac{x^2 - 1}{x^{\frac{3}{2}} + x^{\frac{1}{2}}} + C$

(where C is constant of integration)



18. $\int \frac{\sin 2x + 2}{\sin 2x + \cos 2x + 1} dx = f(x) + C$, Where C is integration constant and $f(0) = 0$, then -

- (A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{8} + \frac{3}{2} \ln 2$
 (B) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{8} + \frac{3}{4} \ln 2$
 (C) $\lim_{x \rightarrow \frac{3\pi}{4}} f(x)$ does not exist
 (D) $\lim_{x \rightarrow \frac{3\pi}{4}} f(x) = \frac{3\pi}{8} + \ln 3$

19. If $f(x) = \frac{x}{x^2 - 1}$, then which of the following is/are correct -

- (A) $\int f(e^x) dx = \frac{1}{2} \ln \left| \frac{e^x - 1}{e^x + 1} \right| + C$
 (B) $\int f(\sin x) dx = \sec x + C$
 (C) $\int f(\sec \theta) d\theta = -\operatorname{cosec} \theta + C$
 (D) $\int f(x^2) dx = \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C$

Match the List Type Questions

20. $I_1 = \int \tan x \tan(ax + b) dx$ and

$I_2 = \int \cot x \cot(ax + b) dx$

List - I

- (P) Value of I_1 for $a = 1$ is
 (Q) Value of I_2 for $a = 1$ is
 (R) Value of I_1 for $a = -1$ is
 (S) Value of I_2 for $a = -1$ is

List-II

- (1) $x - \cot b \ln \frac{\cos(x-b)}{\cos x} + C$
 (2) $\cot b \ln \frac{\sin x}{\sin(x+b)} - x + C$
 (3) $\cot b \ln \left(\frac{\cos x}{\cos(x+b)} \right) - x + C$
 (4) $x + \cot b \ln \left(\frac{\sin x}{\sin(b-x)} \right) + C$

- (A) (P)→(3); (Q)→(2); (R)→(1); (S)→(4)
 (B) (P)→(2); (Q)→(3); (R)→(1); (S)→(4)
 (C) (P)→(3); (Q)→(2); (R)→(4); (S)→(1)
 (D) (P)→(3); (Q)→(1); (R)→(2); (S)→(4)




SUBJECTIVE EXERCISE – I

1. Integrate with respect to x:

- (i) $(2x + 3)^5$ (ii) $\sin 2x$
 (iii) $\sec^2(4x + 5)$ (iv) $\sec(3x + 2)$
 (v) $\tan(2x + 1)$ (vi) 2^{3x+4}
 (vii) $\frac{1}{2x+1}$ (viii) e^{4x+5}

2. Integrate with respect to x:

- (i) $\sin^2 x$ (ii) $\cos^3 x$
 (iii) $\sin 2x \cos 3x$ (iv) $4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$
 (v) $\frac{1}{\sqrt{x+3} - \sqrt{x+2}}$

3. A function g defined for all positive real numbers, satisfies $g'(x^2) = x^3$ for all $x > 0$ and $g(1) = 1$. Compute $g(4)$.

4. Integrate with respect to x:

- (i) $x \sin x^2$ (ii) $\frac{x}{x^2+1}$
 (iii) $\sec^2 x \tan x$ (iv) $\frac{e^x + 1}{e^x + x}$
 (v) $\frac{1 - \sin x}{x + \cos x}$ (vi) $\frac{e^{2x}}{e^{2x} - 2}$
 (vii) $\frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x}$ (viii) $\frac{\sec x}{\ln(\sec x + \tan x)}$
 (ix) $\frac{x}{\sqrt{x+2}}$ (x) $\left(e^x + \frac{1}{e^x}\right)^2$
 (xi) $(e^x + 1)^2 e^x$ (xii) $\frac{1}{x(x^5 + 1)}$
 (xiii) $\frac{1}{x^5(1 + x^5)^{\frac{1}{5}}}$ (xiv) $\frac{\sqrt{x^2 - 8}}{x^4}$

5. Find the value of $\int \frac{d(x^2 + 1)}{\sqrt{(x^2 + 2)}}$.

6. Evaluate the following:

- (i) $\int \left(\frac{x \cos x - \sin x}{x \sin x} \right) dx$

(ii) $\int \left(\frac{\frac{x}{x+1} - \ln(x+1)}{x(\ln(x+1))} \right) dx$

7. $\int \left[\sin \alpha \sin(x - \alpha) + \sin^2 \left(\frac{x}{2} - \alpha \right) \right] dx$

8. Integrate with respect to x:

- (i) $x \ell n x$ (ii) $x \sin^2 x$
 (iii) $x \tan^{-1} x$ (iv) $\ell n x$
 (v) $\sec^3 x$ (vi) $2x^3 e^{x^2}$
 (vii) $\sin^{-1} \sqrt{x}$ (viii) $\frac{x^2 \tan^{-1} x}{1 + x^2}$
 (ix) $e^x \sin x$ (x) $e^x (\sec^2 x + \tan x)$

9. Find the antiderivative of $f(x) = \ln(\ln x) + (\ln x)^{-2}$ whose graph passes through (e, e) .

10. $\int \frac{\ln \left(\ln \left(\frac{1+x}{1-x} \right) \right)}{1-x^2} dx$

11. $\int \left[\left(\frac{x}{e} \right)^x + \left(\frac{e}{x} \right)^x \right] \ell n x dx$

12. Integrate with respect to x :

- (i) $\frac{1}{x^2+4}$ (ii) $\frac{1}{x^2+5}$
 (iii) $\frac{1}{x^2+2x+5}$ (iv) $\frac{2x+1}{x^2+3x+4}$
 (v) $\frac{x^3-1}{x^3+x}$ (vi) $\frac{1}{\sqrt{x^2-4}}$
 (vii) $\sqrt{x^2+4}$ (viii) $\sqrt{x^2+2x+5}$
 (ix) $(x-1) \sqrt{1-x-x^2}$ (x) $x^5 \sqrt{a^3+x^3}$

13. Integrate with respect to x:

- (i) $\frac{1}{(x+1)(x+2)}$ (ii) $\frac{1}{(x^2+1)(x+3)}$
 (iii) $\frac{3x+2}{(x+1)^2(x+2)}$ (iv) $\frac{1}{(x+1)(x+2)(x+3)}$





14. Integrate with respect to x:

(i) $\frac{1}{x^4 + x^2 + 1}$ (ii) $\frac{1+x^2}{1+x^4}$

(iii) $\frac{1-x^2}{1-x^2+x^4}$

15. Integrate with respect to x:

(i) $\frac{1}{(x+1)\sqrt{x+2}}$ (ii) $\frac{1}{(x^2-4)\sqrt{x+1}}$

(iii) $\frac{1}{(x+1)\sqrt{x^2+2}}$ (iv) $\frac{1}{(x^2+1)\sqrt{x^2+2}}$

16. $\int \frac{x^2+3}{x^6(x^2+1)} dx$

17. $\int \frac{x^5+3x^4-x^3+8x^2-x+8}{x^2+1} dx$

18. Evaluate the following:

(i) $\int \sqrt{\frac{1+x}{x}} dx$ (ii) $\int \sqrt{\frac{x-1}{x+1}} dx$

(iii) $\int \left(\frac{x\sqrt{1+x}}{\sqrt{1-x}} \right) dx$

19. $\int \frac{5x^4+4x^5}{(x^5+x+1)^2} dx$

20. Integrate with respect to x:

(i) $\frac{1}{2+\cos x}$

(ii) $\frac{1}{2-\cos x}$

(iii) $\frac{2\sin x + 2\cos x}{3\cos x + 2\sin x}$

(iv) $\frac{1}{1+\sin x + \cos x}$

(v) $\frac{1}{2+\sin^2 x}$

(vi) $\frac{\operatorname{cosec}^2 x \cdot \sin x}{(\sin x - \cos x)}$

(vii) $\frac{\sin^4 x}{\cos^2 x}$

21. Evaluate the following

(i) $\int \left(\frac{\sin x + \cos x}{9 + 16\sin 2x} \right) dx$

(ii) $\int \left(\frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} \right) dx$

22. If $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}} dx = -2 \left(A \tan^{\frac{-9}{2}} x + B \tan^{\frac{-5}{2}} x \right) + C$, then find A and B.

23. $\int (\sin x)^{-11/3} (\cos x)^{-1/3} dx$




SUBJECTIVE EXERCISE – II

1. $\int \frac{3x^2 + 1}{(x^2 - 1)^3} dx = C - \frac{x}{(x^2 - \lambda)^2}$, then λ is

2. $\int \frac{(2x^2 - 1) dx}{x \sqrt{x^2 - (2x^2 + 1)^2}} = \sin^{-1} \left(\frac{ax^2 + b}{cx} \right) + k$,
then $a + b + c$

3. $\int \frac{e^{\cos x} (x \sin^3 x + \cos x)}{\sin^2 x} dx = C - e^{\cos x} f(x)$ if
 $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 1$, then $f\left(\frac{\pi}{6}\right) - \frac{\pi}{6}$

4. $\int \frac{4x^5 - 7x^4 + 8x^3 - 2x^2 + 4x - 7}{x^2(x^2 + 1)^2} dx = a \ln x + \frac{b}{x}$
 $+ 6 \tan^{-1}(x) + \frac{cx}{1 + x^2} + K$, then $a + b + c$ is equal
to

5. Let $\int \frac{f'(x)g(x) - g'(x)f(x)}{(f(x) + g(x))\sqrt{f(x)g(x) - g^2(x)}} dx$.
 $= \sqrt{m} \tan^{-1} \left(\sqrt{\frac{f(x) - g(x)}{ng(x)}} \right) + C$

Where $m, n \in \mathbb{N}$ and 'C' is constant of integration ($g(x) > 0$). Find the value of $(m^2 + n^2)$.

6. If the value

$$\int \frac{1 - (\cot x)^{2008}}{\tan x + (\cot x)^{2009}} dx = \frac{1}{k} \ln |\sin^k x + \cos^k x| + C,$$

Then find k .

7. Suppose $\int \frac{1 - 7 \cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{g(x)}{\sin^7 x} + C$, where
C is arbitrary constant of integration. Then find
the value of $g'(0) + g''\left(\frac{\pi}{4}\right)$

8. If $\int \frac{3z^3 - 8z + 5}{\sqrt{z^2 - 4z - 7}} dz = (z^2 + az + 36)$
 $\sqrt{z^2 - 4z - 7} + b \ln |z - 2 + \sqrt{z^2 - 4z - 7}| + C$,

where $a, b \in \mathbb{I}$ and C is integration constant,
then $a + b$ is equal to

9. If $\int \frac{1 + x \cos x}{x(1 - x^2 e^{2 \sin x})} dx = k \ln$
 $\sqrt{\frac{x^2 e^{2 \sin x}}{1 - x^2 e^{2 \sin x}}} + C$ then k is equal to :

10. If $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx = p \sin x - \frac{q}{\sin x} - r$
 $\tan^{-1}(\sin x) + C$ then $p + 2q + r$ is equal to :

Comprehension # 1 (Q.No. 11 to 13)

Let $I_{n,m} = \int \sin^n x \cos^m x \cdot dx$. Then we can relate $I_{n,m}$
with each of the following

- | | |
|-------------------|--------------------|
| (i) $I_{n-2,m}$ | (ii) $I_{n+2,m}$ |
| (iii) $I_{n,m-2}$ | (iv) $I_{n,m+2}$ |
| (v) $I_{n-2,m+2}$ | (vi) $I_{n+2,m-2}$ |

Suppose we want to establish a relation between $I_{n,m}$
and $I_{n,m-2}$, then we set

$$P(x) = \sin^{n+1} x \cos^{m-1} x \quad \dots\dots\dots(1)$$

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of $\cos x$ is m and $m-2$
respectively, the minimum of the two is $m-2$, adding
1 to the minimum we get $m-2+1 = m-1$. Now
choose the exponent $m-1$ of $\cos x$ in $P(x)$. Similarly
choose the exponent of $\sin x$ for $P(x)$

Now differentiating both sides of (1), we get

$$\begin{aligned} P'(x) &= (n+1) \sin^n x \cos^m x - (m-1) \sin^{n+2} x \cos^{m-2} x \\ &= (n+1) \sin^n x \cos^m x - (m-1) \sin^n x (1 - \cos^2 x) \cos^{m-2} x \\ &= (n+1) \sin^n x \cos^m x - (m-1) \sin^n x \cos^{m-2} x + (m-1) \sin^n x \cos^m x \\ &= (n+m) \sin^n x \cos^m x - (m-1) \sin^n x \cos^{m-2} x \end{aligned}$$

Now integrating both sides, we get

$$\sin^{n+1} x \cos^{m-1} x = (n+m) I_{n,m} - (m-1) I_{n,m-2}.$$

Similarly we can establish the other relations.

11. The relation between $I_{4,2}$ and $I_{2,2}$ is

(A) $I_{4,2} = \frac{1}{6} (-\sin^3 x \cos^3 x + 3I_{2,2})$

(B) $I_{4,2} = \frac{1}{6} (\sin^3 x \cos^3 x + 3I_{2,2})$





(C) $I_{4,2} = \frac{1}{6} (\sin^3 x \cos^3 x - 3I_{2,2})$

(D) $I_{4,2} = \frac{1}{6} (-\sin^3 x \cos^3 x + 2I_{2,2})$

12. The relation between $I_{4,2}$ and $I_{6,2}$ is

(A) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x + 8I_{6,2})$

(B) $I_{4,2} = \frac{1}{5} (-\sin^5 x \cos^3 x + 8I_{6,2})$

(C) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x - 8I_{6,2})$

(D) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x + 8I_{6,2})$

13. The relation between $I_{4,2}$ and $I_{4,4}$ is

(A) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x + 8I_{4,4})$

(B) $I_{4,2} = \frac{1}{3} (-\sin^5 x \cos^3 x + 8I_{4,4})$

(C) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x - 8I_{4,4})$

(D) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x + 6I_{4,4})$

Comprehension # 2 (Q. No. 14 to 16)

It is known that

$$\sqrt{\tan x} + \sqrt{\cot x} = \begin{cases} \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } 0 < x < \frac{\pi}{2} \\ \frac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \frac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \pi < x < \frac{3\pi}{2} \end{cases}$$

$$\frac{d}{dx} (\sqrt{\tan x} - \sqrt{\cot x}) = \frac{1}{2}$$

$$(\sqrt{\tan x} + \sqrt{\cot x})(\tan x + \cot x),$$

$$\forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$$

$$\text{and } \frac{d}{dx} (\sqrt{\tan x} + \sqrt{\cot x}) = \frac{1}{2}$$

$$(\sqrt{\tan x} - \sqrt{\cot x})(\tan x + \cot x),$$

$$\forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right).$$

14. Value of integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$,

where $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$ is

(A) $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C$

(B) $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}} \right) + C$

(C) $-\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C$

(D) $-\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}} \right) + C$

15. Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x})$

dx , where $x \in \left(0, \frac{\pi}{2}\right)$, is

(A) $\sqrt{2} \sin^{-1} (\cos x - \sin x) + C$

(B) $\sqrt{2} \sin^{-1} (\sin x - \cos x) + C$

(C) $\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$

(D) $-\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$

16. Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x})$,

where $x \in \left(\pi, \frac{3\pi}{2}\right)$, is

(A) $\sqrt{2} \sin^{-1} (\cos x - \sin x) + C$

(B) $\sqrt{2} \sin^{-1} (\sin x - \cos x) + C$

(C) $\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$

(D) $-\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$



JEE-MAIN (PREVIOUS YEAR QUESTIONS)

1. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$ is equal to

[JEE(Main)-2014]

- (1) $(x-1)e^{x+\frac{1}{x}} + c$ (2) $xe^{x+\frac{1}{x}} + c$
 (3) $(x+1)e^{x+\frac{1}{x}} + c$ (4) $-xe^{x+\frac{1}{x}} + c$

2. The integral $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ equals:

[JEE(Main)-2015]

- (1) $-(x^4+1)^{\frac{1}{4}} + c$ (2) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$
 (3) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (4) $(x^4+1)^{\frac{1}{4}} + c$

3. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ equals to

[JEE(Main)-2016]

- (1) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$
 (2) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$
 (3) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$
 (4) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$

(Where C is an arbitrary constant)

4. Let

$$I_n = \int \tan^n x dx, (n > 1). I_4 + I_6 = a \tan^5 x + bx^5 + C,$$

where C is a constant of integration, then the ordered pair (a, b) is equal to:

[JEE(Main)-2017]

- (1) $\left(-\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, 1\right)$

- (3) $\left(\frac{1}{5}, 0\right)$ (4) $\left(\frac{1}{5}, -1\right)$

5. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to:

- (1) $\frac{-1}{1 + \cot^3 x} + C$ (2) $\frac{1}{3(1 + \tan^3 x)} + C$
 (3) $\frac{-1}{3(1 + \tan^3 x)} + C$ (4) $\frac{1}{1 + \cot^3 x} + C$

(Where C is a constant of integration)

[JEE(Main)-2018]

6. For $x^2 \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of natural numbers), the integral [JEE(Main)-2019]

$$\int x \sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}} dx$$
 is equal to:

(where c is a constant of integration)

- (1) $\log_e \left| \sec \left(\frac{x^2-1}{2} \right) \right| + c$
 (2) $\log_e \left| \frac{1}{2} \sec^2(x^2-1) \right| + c$
 (3) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$
 (4) $\frac{1}{2} \log_e |\sec(x^2-1)| + c$

7. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$), and $f(0) = 0$, then the value of $f(1)$ is:

[JEE(Main)-2019]

- (1) $\frac{1}{4}$ (2) $-\frac{1}{4}$ (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

8. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$.

$$\text{Then } \int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$$
 is equal to:

(where C is a constant of integration)

[JEE(Main)-2019]



$$(1) \frac{n}{n^2-1} \left(1 + \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$$

$$(2) \frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n+1} \theta} \right)^{\frac{n+1}{n}} + C$$

$$(3) \frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$$

$$(4) \frac{n}{n^2+1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$$

9. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to:

[JEE(Main)-2019]

$$(1) -4x^3 - 1$$

$$(2) -2x^3 - 1$$

$$(3) -2x^3 + 1$$

$$(4) 4x^3 + 1$$

10. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x) \sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to:

[JEE(Main)-2019]

$$(1) \frac{1}{3}(x+4)$$

$$(2) \frac{2}{3}(x+2)$$

$$(3) \frac{1}{3}(x+1)$$

$$(4) \frac{2}{3}(x-4)$$

11. The integral $\int \cos(\log_e x) dx$ is equal to:
(where C is a constant of integration)

[JEE(Main)-2019]

$$(1) \frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)] + C$$

$$(2) x [\cos(\log_e x) + \sin(\log_e x)] + C$$

$$(3) \frac{x}{2} [\sin(\log_e x) - \cos(\log_e x)] + C$$

$$(4) x [\cos(\log_e x) - \sin(\log_e x)] + C$$

12. The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to:
(where C is a constant of integration)

[JEE(Main)-2019]

$$(1) \frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$$

$$(2) \frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$$

$$(3) \frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$$

$$(4) \frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$$

13. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = x f(x) (1+x^6)^{\frac{1}{3}} + C$ where C

is a constant of integration, then the function f(x) is equal to: [JEE(Main)-2019]

$$(1) -\frac{1}{2x^2} \quad (2) -\frac{1}{2x^3} \quad (3) \frac{3}{x^2} \quad (4) -\frac{1}{6x^3}$$

14. If $\int e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)) dx = e^{\sec x} f(x) + C$, then a possible choice of f(x) is

[JEE(Main)-2019]

$$(1) \sec x - \tan x - \frac{1}{2} \quad (2) x \sec x + \tan x + \frac{1}{2}$$

$$(3) \sec x + x \tan x - \frac{1}{2} \quad (4) \sec x + \tan x + \frac{1}{2}$$

15. If $\int \frac{dx}{(x^2 - 2x + 10)^2} =$

$$A \left(\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{f(x)}{x^2 - 2x + 10} \right) + C, \text{ where C}$$

is constant of integration, then:

[JEE(Main)-2019]

$$(1) A = \frac{1}{54} \text{ and } f(x) = 3(x-1)$$

$$(2) A = \frac{1}{81} \text{ and } f(x) = 3(x-1)$$

$$(3) A = \frac{1}{54} \text{ and } f(x) = 9(x-1)^2$$

$$(4) A = \frac{1}{27} \text{ and } f(x) = 9(x-1)$$

16. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to: (Here C is constant of integration) [JEE(Main)-2019]

$$(1) \log_e \left| \frac{x^3 + 1}{x} \right| + C \quad (2) \frac{1}{2} \log_e \frac{(x^3 + 1)^2}{|x^3|} + C$$



$$(3) \log_e \frac{|x^3+1|}{x^2} + C \quad (4) \frac{1}{2} \log_e \frac{|x^3+1|}{x^2} + C$$

17. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$

is equal to: **[JEE(Main)-2020]**

(1) $\frac{9}{8}$ (2) $-\frac{9}{8}$ (3) 2 (4) -2

18. The integral $\int \frac{dx}{(x+4)^{8/7} \cdot (x-3)^{6/7}}$ is equal to:

[JEE(Main)-2020]

(where C is a constant of integration)

(1) $-\frac{1}{13} \left(\frac{x-3}{x+4} \right)^{-13/7} + C$ (2) $-\left(\frac{x-3}{x+4} \right)^{-1/7} + C$

(3) $\frac{1}{2} \left(\frac{x-3}{x+4} \right)^{3/7} + C$ (4) $\left(\frac{x-3}{x+4} \right)^{1/7} + C$

19. If $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} = \lambda \tan \theta +$

$2 \log_e |f(\theta)| + C$ where C is constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to:

[JEE(Main)-2020]

(1) $(-1, 1 + \tan \theta)$ (2) $(1, 1 + \tan \theta)$

(3) $(-1, 1 - \tan \theta)$ (4) $(1, 1 - \tan \theta)$

20. If $\int \sin^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x)$

+ C , where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be:

[JEE(Main)-2020]

(1) $(x+1, -\sqrt{x})$ (2) $(x+1, \sqrt{x})$

(3) $(x-1, \sqrt{x})$ (4) $(x-1, -\sqrt{x})$

21. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to

[JEE(Main)-2020]

(where C is a constant of integration):

(1) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

(2) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

(3) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(4) $\tan x - \frac{x \tan x}{x \sin x + \cos x} + C$

22. If $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$

can be: **[JEE(Main)-2020]**

(1) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$ (2) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$

(3) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$ (4) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$

23. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx =$

$a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + C$, where c is a

constant of integration, then the ordered pair (a, b) is equal to: **[JEE(Main)-2021]**

(1) $(1, -3)$ (2) $(1, 3)$ (3) $(-1, 3)$ (4) $(3, 1)$

24. The integral $\int \frac{(2x-1) \cos \sqrt{(2x-1)^2 + 5}}{\sqrt{4x^2 - 4x + 6}} dx$ is

equal to (where c is a constant of integration):

[JEE(Main)-2021]

(1) $\frac{1}{2} \sin \sqrt{(2x-1)^2 + 5} + c$

(2) $\frac{1}{2} \cos \sqrt{(2x+1)^2 + 5} + c$

(3) $\frac{1}{2} \cos \sqrt{(2x-1)^2 + 5} + c$

(4) $\frac{1}{2} \sin \sqrt{(2x+1)^2 + 5} + c$

25. If $\int \frac{1}{x} \sqrt{\frac{1-x}{1+x}} dx = g(x) + c$, $g(1) = 0$, then g

$\left(\frac{1}{2} \right)$ is equal to: **[JEE(Main)-2022]**

(1) $\log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) + \frac{\pi}{3}$



$$(2) \log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-11} \right) + \frac{\pi}{3}$$

$$(3) \log_e \left(\frac{\sqrt{3}+1}{\sqrt{3}-11} \right) - \frac{\pi}{3}$$

$$(4) \frac{1}{2} \log_e \left(\frac{\sqrt{3}-1}{\sqrt{3}+11} \right) - \frac{\pi}{6}$$

26. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then : **[JEE(Main)-2022]**

$$(1) 3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$$

$$(2) 3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$$

$$(3) 3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$$

$$(4) 3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$$

27. If $\int \sqrt{\sec 2x - 1} dx = \alpha \log_e$

$$\left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x \right)} \right| +$$

constant, then $\beta - \alpha$ is equal to _____.

[JEE(Main)-2023]

28. If $I(x) = \int e^{\sin^2 x} (\cos x \sin 2x - \sin x) dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to **[JEE(Main)-2023]**

$$(1) -\frac{1}{2} e^{\frac{3}{4}} \quad (2) e^{\frac{3}{4}}$$

$$(3) \frac{1}{2} e^{\frac{3}{4}} \quad (4) -e^{\frac{3}{4}}$$

29. For, $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if

$$\int \left(\left(\frac{x}{e} \right)^{2x} + \left(\frac{e}{x} \right)^{2x} \right) \log_e x dx = \frac{1}{\alpha} \left(\frac{x}{e} \right)^{\beta x} - \frac{1}{\gamma} \left(\frac{e}{x} \right)^{\delta x} + C,$$

where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is constant of

integration then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

[JEE(Main)-2023]

$$(1) 1 \quad (2) -4 \quad (3) -8 \quad (4) 4$$

30. Let $I(x) = \int \sqrt{\frac{x+7}{x}} dx$ and $I(9) = 12 + 7 \log_e 7$.

If $I(1) = \alpha + 7 \log_e (1 + 2\sqrt{2})$, then α^4 is equal to _____. **[JEE(Main)-2023]**




JEE-ADVANCED (PREVIOUS YEAR QUESTIONS)

1. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equals to –

[JEE-2006]

(A) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$

(B) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$

(C) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$

(D) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

2. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x)$

$= \underbrace{(f \text{ of } (f \text{ of } \dots \text{ of } f))}_{f \text{ occurs } n \text{ times}}(x).$

Then $\int x^{n-2} g(x) dx$ equals.

[JEE-2007]

(A) $\frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + K$

(B) $\frac{1}{n-1} (1+nx^n)^{1-\frac{1}{n}} + K$

(C) $\frac{1}{n(n+1)} (1+nx^n)^{1+\frac{1}{n}} + K$

(D) $\frac{1}{n+1} (1+nx^n)^{1+\frac{1}{n}} + K$

3. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

[JEE-2007]

Statement-1: The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x .

because

Statement-2: $\sin^2(x + \pi) = \sin^2 x$ for all real x .

(A) Statement – 1 is True, Statement-2 is True;
Statement-2 is a correct explanation for Statement-1.

(B) Statement – 1 is True, Statement-2 is True;
Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement – 1 is True, Statement-2 is False.

(D) Statement – 1 is False, Statement-2 is True.

4. Let

$$I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx, \quad J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx.$$

Then, for an arbitrary constant c , the value of $J - I$ equals:

[JEE-2008]

(A) $\frac{1}{2} \ln \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + c$

(B) $\frac{1}{2} \ln \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + c$

(C) $\frac{1}{2} \ln \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c$

(D) $\frac{1}{2} \ln \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + c$

5. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K):

[JEE(Advanced)-2012]

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$





ANSWER KEY

OBJECTIVE EXERCISE - I

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	A	A	B	A	B	B	B	C	A	D	D	C	A	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	B	A	B	C	A	C	A	C	A	A	C	B	C	C
Que.	31	32	33	34	35	36	37	38	39	40					
Ans.	B	A	A	B	B	B	B	A	A	C					

OBJECTIVE EXERCISE - II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	D	B	A	C	B	ACD	BC	BD	BC	CD	CD	ABC	ABCD
Que.	16	17	18	19	20										
Ans.	ABCD	ABCD	BC	AC	A										

SUBJECTIVE EXERCISE - I

1. (i) $\frac{(2x+3)^6}{12} + C$ (ii) $-\frac{\cos 2x}{2} + C$
 (iii) $\frac{\tan(4x+5)}{4} + C$ (iv) $\frac{1}{3} \ln |\sec(3x+2) + \tan(3x+2)| + C$
 (v) $\frac{1}{2} \ln |\sec(2x+1)| + C$ (vi) $\frac{2^{3x+4}}{3 \ln 2} + C$
 (vii) $\frac{1}{2} \ln |2x+1| + C$ (viii) $\frac{e^{4x+5}}{4} + C$
2. (i) $\frac{x}{2} - \frac{1}{4} \sin 2x + C$ (ii) $\frac{\sin 3x}{12} + \frac{3}{4} \sin x + C$
 (iii) $-\frac{1}{10} \cos 5x + \frac{1}{2} \cos x + C$ (iv) $\cos x - \frac{1}{2} \cos 2x - \frac{1}{3} \cos 3x + C$
 (v) $\frac{2}{3} ((x+3)^{3/2} + (x+2)^{3/2}) + C$
3. $\frac{67}{5}$
4. (i) $-\frac{1}{2} \cos x^2 + C$ (ii) $\frac{1}{2} \ln |x^2 + 1| + C$
 (iii) $\frac{1}{2} (\tan x)^2 + C$ or $\frac{\sec^2 x}{2} + C$ (iv) $\ln |e^x + x| + C$
 (v) $\ln |x + \cos x| + C$ (vi) $\frac{1}{2} \ln |e^{2x} - 2| + C$
 (vii) $\frac{1}{2} \ln |x^2 + \sin 2x + 2x| + C$ (viii) $\ln |\ln(\sec x + \tan x)| + C$
 (ix) $\frac{2}{3} (x+2)^{3/2} - 4(x+2)^{1/2} + C$ (x) $\frac{1}{2} (e^{2x} - e^{-2x}) + 2x + C$
 (xi) $\frac{1}{3} e^{3x} + e^{2x} + e^x + C$ (xii) $-\frac{1}{5} \ln \left| 1 + \frac{1}{x^5} \right| + C$
 (xiii) $-\frac{1}{4} \left(1 + \frac{1}{x^5} \right)^{4/5} + C$ (xiv) $\frac{(x^2 - 8)^{3/2}}{24 x^3} + C$
5. $2\sqrt{x^2 + 2} + C$



6. (i) $\ln \left(\frac{\sin x}{x} \right) + C$ (ii) $\ln \left(\frac{\ln(x+1)}{x} \right) + C$
7. $\frac{1}{2}(x - \sin x) + C$
8. (i) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$ (ii) $\frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + C$
- (iii) $\frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C$ (iv) $x(\ln x - 1) + C$
- (v) $\frac{\sec x \tan x}{2} + \frac{1}{2} \ln |\sec x + \tan x| + C$ (vi) $(x^2 - 1)e^{x^2} + C$
- (vii) $x \sin^{-1} \sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} - \frac{1}{2} \sin^{-1} \sqrt{x} + C$ (viii) $x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) - \frac{(\tan^{-1} x)^2}{2} + C$
- (ix) $\frac{e^x}{2} (\sin x - \cos x) + C$ (x) $e^x \tan x + C$
9. $y = x \left[\ln(\ln x) - \frac{1}{\ln x} \right] + 2e$ 10. $\frac{1}{2} \left[\ln \left(\frac{1+x}{1-x} \right) \cdot \ln \left(\ln \frac{1+x}{1-x} \right) - \ln \left(\frac{1+x}{1-x} \right) \right] + C$
11. $\left(\frac{x}{e} \right)^x - \left(\frac{e}{x} \right)^x + C$
12. (i) $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$ (ii) $\frac{1}{\sqrt{5}} \tan^{-1} \frac{x}{\sqrt{5}} + C$
- (iii) $\frac{1}{2} \tan^{-1} \left(\frac{(x+1)}{2} \right) + C$ (iv) $\ln |x^2 + 3x + 4| - \frac{4}{\sqrt{7}} \tan^{-1} \frac{2x+3}{\sqrt{7}} + C$
- (v) $x - \arctan x + \ln \frac{\sqrt{1+x^2}}{x} + C$ (vi) $\ln |x + \sqrt{x^2 - 4}| + C$
- (vii) $\frac{x}{2} \sqrt{x^2 + 4} + 2 \ln |x + \sqrt{x^2 + 4}| + C$ (viii) $\frac{x+1}{2} \sqrt{x^2 + 2x + 5} + 2 \ln |x+1 + \sqrt{x^2 + 2x + 5}| + C$
- (ix) $-\frac{(1-x-x^2)^{3/2}}{3} - \frac{3}{8} (2x+1) \sqrt{1-x-x^2} - \frac{15}{16} \sin^{-1} \left(\frac{2x+1}{\sqrt{5}} \right) + C$
- (x) $\frac{2}{15} (a^3 + x^3)^{5/2} - \frac{2a^3}{9} (a^3 + x^3)^{3/2} + C$
13. (i) $\ln \left| \frac{x+1}{x+2} \right| + C$ (ii) $\frac{1}{10} \ln |x+3| - \frac{1}{20} \ln |x^2+1| + \frac{3}{10} \tan^{-1} x + C$
- (iii) $4 \ln |x+1| + \frac{1}{(x+1)} - 4 \ln |x+2| + C$ (iv) $\frac{1}{2} \ln |x+1| - \ln |x+2| + \frac{1}{2} \ln |x+3| + C$
14. (i) $\frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{3}x} \right) - \frac{1}{4} \ln \left| \frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1} \right| + C$
- (ii) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) + C$ (iii) $-\frac{1}{2\sqrt{3}} \ln \left| \frac{x + \frac{1}{x} - \sqrt{3}}{x + \frac{1}{x} + \sqrt{3}} \right| + C$
15. (i) $\ln \left| \frac{\sqrt{x+2}-1}{\sqrt{x+2}+1} \right| + C$



$$(ii) \quad \frac{1}{4\sqrt{3}} \ln \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| - \frac{1}{2} \tan^{-1}(t) + C, \text{ where } t = \sqrt{x+1}$$

$$(iii) \quad -\frac{1}{\sqrt{3}} \ln \left| \left(t - \frac{1}{3} \right) + \sqrt{\left(t - \frac{1}{3} \right)^2 + \frac{2}{9}} \right| + C, \text{ where } t = \frac{1}{x+1}$$

$$(iv) \quad -\tan^{-1} \sqrt{\frac{x^2+2}{x^2}} + C$$

$$16. \quad C - \frac{2}{x} + \frac{2}{3x^3} - \frac{3}{5x^5} - 2 \tan^{-1} x$$

$$17. \quad \frac{x^4}{4} + x^3 - x^2 + 5x + \frac{1}{2} \ln(x^2 + 1) + 3 \tan^{-1} x + C$$

$$18. \quad (i) \quad \frac{1}{2} \ln \left| \left(x + \frac{1}{2} \right) + \sqrt{x^2 + x} \right| + \sqrt{x^2 + x} + C \quad (ii) \quad \sqrt{x^2 - 1} - \ln \left| x + \sqrt{x^2 - 1} \right| + C$$

$$(iii) \quad \frac{1}{2} \sin^{-1} x - \frac{x}{2} \sqrt{1-x^2} - \sqrt{1-x^2} + C$$

$$19. \quad C - \frac{x+1}{x^5+x+1} \text{ or } C + \frac{x^5}{x^5+x+1}$$

$$20. \quad (i) \quad \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x/2}{\sqrt{3}} \right) + C$$

$$(ii) \quad \frac{2}{\sqrt{3}} \tan^{-1} \left(\sqrt{3} \tan \frac{x}{2} \right) + C$$

$$(iii) \quad \frac{10}{13} x - \frac{2}{13} \ln |3 \cos x + 2 \sin x| + C$$

$$(iv) \quad \ln \left| 1 + \tan \frac{x}{2} \right| + C$$

$$(v) \quad \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

$$(vi) \quad \ln |1 - \cot x| + C$$

$$(vii) \quad \tan x + \frac{1}{4} \sin 2x - \frac{3x}{2} + C$$

$$21. \quad (i) \quad \frac{1}{40} \ln \left(\frac{4(\sin x - \cos x) + 5}{4(\sin x + \cos x) - 5} \right) + C$$

$$(ii) \quad \sin^{-1} \left(\frac{\sin x + \cos x}{3} \right) + C$$

$$22. \quad A = \frac{1}{9}, B = \frac{1}{5}$$

$$23. \quad -\frac{3(1+4 \tan^2 x)}{8(\tan x)^{8/3}} + C$$

SUBJECTIVE EXERCISE - II

1.	1	2.	4	3.	2	4.	17	5.	8	6.	2010	7.	5
8.	117	9.	1	10.	11	11.	(A)	12.	(A)	13.	(B)	14.	(A)
15.	(B)	16.	(A)										

JEE MAIN (PREVIOUS YEAR QUESTIONS)

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	3	3	3	1	1	3	1	1	1	4	2	4	1	1	4	4	3	1
Que.	21	22	23	24	25	26	27	28	29	30										
Ans.	2	2	2	1	1	1	1	3	4	64										

JEE ADVANCED (PREVIOUS YEAR QUESTIONS)

1.	(D)	2.	(A)	3.	(D)	4.	(C)	5.	(C)
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