(HELD ON MONDAY 06th APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : R \to R$ be defined as $f(x) = \dfrac{2x^2 - 3x + 2}{3x^2 + x + 3}$.

Then f is :

(1) both one-one and onto

(2) one-one but not onto

(3) onto but not one-one

(4) neither one-one nor onto

Ans. (4)

Sol. $\dfrac{2x^2 - 3x + 2}{3x^2 + x + 3} = y$

$(3y - 2)\,x^2 + (y + 3)x + (3y - 2) = 0$

$D \geq 0$

$(7y - 1)\,(5y - 7) \leq 0$

$y \in \left[\dfrac{1}{7}, \dfrac{7}{5}\right]$

Hence f(x) is into and f(x) is many-one because f(x) is non-monotonic

2. Consider the quadratic equation $(n^2 - 2n + 2)x^2 - 3x + (n^2 - 2n + 2)^2 = 0$, $n \in R$. Let α be the minimum value of the product of its roots and β be the maximum value of the sum of its roots. Then the sum of the first six terms of the G.P., whose first term is α and the common ratio is $\dfrac{\alpha}{\beta}$, is :

(1) $\dfrac{61}{37}$ (2) $\dfrac{121}{81}$

(3) $\dfrac{364}{243}$ (4) $\dfrac{1093}{729}$

Ans. (3)

Sol. $\alpha = n^2 - 2n + 2$

$\alpha = (n - 1)^2 + 1$

$\therefore$ minimum value of α is 1

Similarly $\beta = \dfrac{3}{(n-1)^2 + 1}$

$\therefore$ maximum value of β is 3

required G.P. is $1, \dfrac{1}{3}, \dfrac{1}{3^2}, \ldots$

$$S_6 = \frac{1\left(1 - \left(\frac{1}{3}\right)^6\right)}{1 - \frac{1}{3}} = \frac{3}{2}\left[1 - \left(\frac{1}{3}\right)^6\right] = \frac{364}{243}$$

3. Let $S = \left\{z \in \mathbb{C} : z^2 + \sqrt{6}\,iz - 3 = 0\right\}$. Then $\sum_{z \in S} z^8$ is equal to :

(1) 162 (2) 184

(3) 262 (4) 324

Ans. (1)

Sol. $z = \dfrac{-\sqrt{6}i \pm \sqrt{-6+12}}{2}$

$= \dfrac{-\sqrt{6}i \pm \sqrt{6}}{2}$

$= \dfrac{\sqrt{6}}{2}(\pm 1 - i)$

$\alpha = \dfrac{-\sqrt{6}}{2}(1+i), \beta = \dfrac{\sqrt{6}}{2}(1-i)$

$\alpha = \dfrac{-\sqrt{6}}{2} \times \sqrt{2}e^{i\pi/4}, \beta = +\dfrac{\sqrt{6}}{2} \times \sqrt{2}^{-i\pi/4}$

$\alpha = -\sqrt{3}e^{i\pi/4}, \beta = +\sqrt{3}e^{-i\pi/4}$

$\alpha^8 + \beta^8 = 81.\,e^{i2\pi} + 81.\,e^{i(-2\pi)}$

$= 81\,(1 + 1) = 162$

4. The sum of all possible values of $\theta \in [0, 2\pi]$, for which the system of equations :

$x\cos3\theta - 8y - 12z = 0$

$x\cos2\theta + 3y + 3z = 0$

$x + y + 3z = 0$

has a non-trivial solution, is equal to :

(1) π (2) 2π

(3) 3π (4) 4π

Ans. (4)

Sol. $\begin{vmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 3 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$

$\begin{vmatrix} \cos 3\theta & -8 & -4 \\ \cos 2\theta & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$(C_1 \to C_1 - C_2)$ & $C_2 \to C_2 - C_3$

$\begin{vmatrix} \cos 3\theta + 8 & -4 & -3 \\ \cos 2\theta - 3 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$2\cos 3\theta + 16 + 4\cos 2\theta - 12 = 0$

$(4\cos^3\theta - 3\cos\theta) + 2(2\cos^2\theta - 1) + 2 = 0$

$4\cos^3\theta + 4\cos^2\theta - 3\cos\theta = 0$

$\cos\theta\,(4\cos^2\theta + 4\cos\theta - 3) = 0$

$\cos\theta\,(2\cos\theta + 3)(2\cos\theta - 1) = 0$

$\cos\theta = 0, \frac{1}{2}, -\frac{3}{2}$ (rejected)

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

Sum $= 4\pi$

5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $B = [b_{ij}]$, $1 \le i, j \le 3$.

If $B = A^{99} - I$, then the value of $\frac{b_{31} - b_{21}}{b_{32}}$ is :

(1) 99 (2) 199

(3) 149 (4) 159

Ans. (3)

Sol. $A = P + I$

$A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = P + I$ where $P^3 = 0$

$A^{99} = I + 99P + {}^{99}C_2 P^2$

$A^{99} - I = 99\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + {}^{99}C_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 99\times 3 & 0 & 0 \\ 44550 & 99\times 3 & 0 \end{bmatrix}$

$b_{31} = 44550$, $b_{21} = 99 \times 3$, $b_{32} = 99 \times 3$

$\frac{44550 - 297}{297} = 149$

6. The sum $1 + \frac{1}{2}(1^2 + 2^2) + \frac{1}{3}(1^2 + 2^2 + 3^2) + \ldots\ldots..$ upto 10 terms is equal to :

(1) 130 (2) 155

(3) $\frac{315}{2}$ (4) $\frac{325}{2}$

Ans. (3)

Sol. $T_n = \frac{1}{n}(1^2 + 2^2 + \ldots + n^2)$

$T_n = \frac{n(n+1)(2n+1)}{6n}$

$T_n = \frac{1}{6}[2n^2 + 3n + 1]$

$S_n = \Sigma T_n = \frac{1}{6}[2\Sigma n^2 + 3\Sigma n + \Sigma 1]$

$\Sigma T_n = \frac{1}{6}\left[2.\frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n\right]$

$S_{10} = \frac{1}{6}\left[\frac{2\times 10\times 11\times 21}{6} + \frac{3}{2}\times 10\times 11 + 10\right]$

$= \frac{10}{6}\left[77 + \frac{33}{2} + 1\right] = \frac{315}{2}$

7. A building has ground floor and 10 more floors. Nine persons enter in a lift at the ground floor. The lift goes up to the 10^{th} floor. The number of ways, in which any 4 persons exit at a floor and the remaining 5 persons exit at a different floor, if the lift does not stop at the first and the second floors, is equal to :

(1) 2184 (2) 3064

(3) 7056 (4) 11340

Ans. (3)

Sol. Form group of 4 and 5 person $\frac{9!}{4!5!} = 126$

choose two floor out of 8 floor ${}^8C_2 = 28$

Total ways in which 4 & 5 persons can leave at different floor $126 \times 28 \times 2 = 7056$

8. Let the mean and the variance of seven observations 2, 4, α, 8, β, 12, 14, $\alpha < \beta$, be 8 and 16 respectively. Then the quadratic equation whose roots are $3\alpha + 2$ and $2\beta + 1$ is :

(1) $x^2 - 35x + 306 = 0$ (2) $x^2 - 41x + 420 = 0$

(3) $x^2 - 45x + 506 = 0$ (4) $x^2 - 37x + 342 = 0$

Ans. **(2)**

Sol. $\frac{2+4+\alpha+\beta+8+12+14}{7} = 8$

$\alpha + \beta = 56 - 40 = 16 \Rightarrow \alpha + \beta = 16$ .....(i)

$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{2^2+4^2+\alpha^2+\beta^2+8^2+12^2+14^2}{7} - 64$

$560 = \alpha^2 + \beta^2 + 424$

$\Rightarrow \alpha^2 + \beta^2 = 136$ ....(ii)

From (i) & (ii) $\alpha = 6$ & $\beta = 10$

$x_1 = 3\alpha + 2 = 20$; $x_2 = 2\beta + 1 = 21$

Q.E. : $x^2 - 41x + 420 = 0$

9. A bag contains 6 blue and 6 green balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each drawn pair consists of one blue and one green ball is :

(1) $\frac{63}{925}$ (2) $\frac{17}{231}$

(3) $\frac{16}{231}$ (4) $\frac{64}{925}$

Ans. **(3)**

Sol. $\text{Probability} = \frac{\lfloor 6 \times \lfloor 6}{\left(\frac{\lfloor 12}{(\lfloor 2)^6 \lfloor 6}\right) \times \lfloor 6} = \frac{16}{231}$

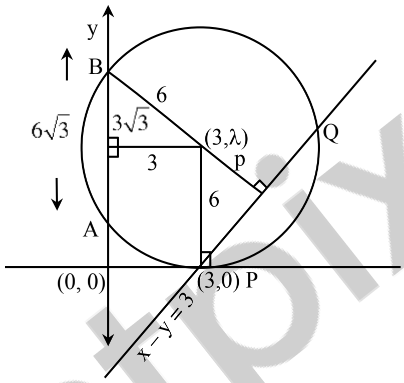
10. Let C be a circle having centre in the first quadrant and touching the x-axis at a distance 3 units from the origin. If the circle C has an intercept of length $6\sqrt{3}$ on y-axis, then the length of the chord of the circle C on the line $x - y = 3$ is :

(1) 8 (2) 6

(3) $6\sqrt{2}$ (4) $8\sqrt{2}$

Ans. **(3)**

Sol. Centre of circle is (3, 6) and radius = 6 units.



$PQ = 2\sqrt{6^2 - p^2}$ $\therefore p = \frac{6}{\sqrt{2}}$

$p = \frac{|3-6-3|}{\sqrt{2}}$

$PQ = 2\sqrt{36 - \frac{36}{2}}$

$= 2 \times \frac{6}{\sqrt{2}} = 6\sqrt{2}$

11. The eccentricity of an ellipse E with centre at the origin O is $\frac{\sqrt{3}}{2}$ and its directrices are $x = \pm\frac{4\sqrt{6}}{3}$. Let H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola whose eccentricity is equal to the length of semi-major axis of E, and whose length of latus rectum is equal to the length of minor axis of E. Then the distance between the foci of H is :

(1) $\frac{4\sqrt{2}}{\sqrt{7}}$ (2) $\frac{4\sqrt{2}}{7}$

(3) $\frac{4}{\sqrt{7}}$ (4) $\frac{8}{7}$

Ans. **(4)**

Sol. $e = \frac{\sqrt{3}}{2}$

$\frac{a}{e} = \frac{4\sqrt{6}}{3}$

$a = 2\sqrt{2}$

$(ae)^2 = a^2 - b^2$

$b^2 = 2$

so E : $\frac{x^2}{8} + \frac{y^2}{2} = 1$

Now $e_H = 2\sqrt{2} \Rightarrow b^2 = 7a^2$

$\frac{2b^2}{a} = 2\sqrt{2} \Rightarrow a = \frac{\sqrt{2}}{7}$

$(ae_H)^2 = a^2 + b^2 = 8a^2 = \frac{16}{49}$

$ae_H = \frac{4}{7}$

Distance between focii = $2ae_H = \frac{8}{7}$

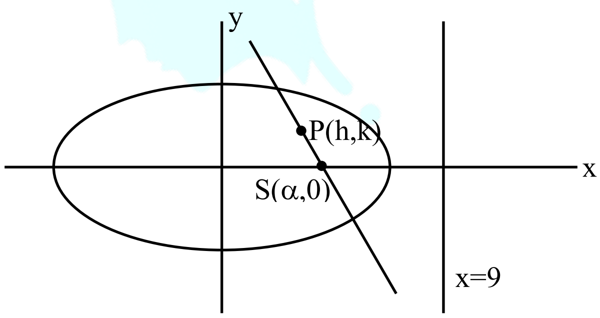
12. Let x = 9 be a directrix of an ellipse E, whose centre is at the origin and eccentricity is $\frac{1}{3}$. Let P (α, 0), α > 0, be a focus of E and AB be a chord passing through P. Then the locus of the mid point of AB is :

(1) $9y^2 = 8x(1 - x)$ (2) $3y^2 = 4x(1 - x)$

(3) $9y^2 = 8x(x - 1)$ (4) $3y^2 = 4x(x - 1)$

Ans. **(1)**

Sol.



$\frac{a}{e} = 9 \Rightarrow a = 3$

$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 8$

$\frac{x^2}{9} + \frac{y^2}{8} = 1$

$S(\alpha, 0) \equiv S(1, 0)$

$T = S_1 \Rightarrow \frac{hx}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}$

$(1, 0) \Rightarrow \frac{h}{9} + 0 = \frac{h^2}{9} + \frac{k^2}{8}$

$h = h^2 + \frac{9}{8}k^2$

$9y^2 = 8x(1 - x)$

13. If $\sin\left(\tan^{-1}(x\sqrt{2})\right) = \cot\left(\sin^{-1}\sqrt{1-x^2}\right)$, $x \in (0, 1)$, then the value of x is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{2}{3}$ (4) $\frac{5}{8}$

Ans. **(1)**

Sol. $\frac{x\sqrt{2}}{\sqrt{2x^2+1}} = \frac{1}{\sqrt{1-x^2}}$

$2(1 - x^2) = (2x^2 + 1)$

$2 - 2x^2 = 2x^2 + 1$

$x^2 = \frac{1}{4}$

$x = \frac{1}{2}$

14. The shortest distance between the lines $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-2}{-3}$ and $\frac{x+2}{2} = \frac{y-6}{4} = \frac{z-5}{-5}$ is:

(1) $\frac{5\sqrt{6}}{6}$ (2) $2\sqrt{5}$

(3) $3\sqrt{5}$ (4) $4\sqrt{5}$

Ans. **(3)**

Sol. $SD = \dfrac{\begin{vmatrix} 4-(-2) & 3-6 & 2-5 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}$

$SD = \dfrac{|6(-10+12)-(-3)(-5+6)-3(4-4)|}{|2\hat{i}-\hat{j}|}$

$SD = \left|\dfrac{12+3}{\sqrt{5}}\right|$

$= \dfrac{15}{\sqrt{5}}$

$SD = 3\sqrt{5}$

15. Let $\vec{a} = 2\hat{i}+3\hat{j}+3\hat{k}$ and $\vec{b} = 6\hat{i}+3\hat{j}+3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is :

(1) 450

(2) 900

(3) 1800

(4) 2400

Ans. **(3)**

Sol. $2\vec{a}+3\vec{b} = 2(2\hat{i}+3\hat{j}+3\hat{k})+3(6\hat{i}+3\hat{j}+3\hat{k})$

$\Rightarrow 22\hat{i}+15\hat{j}+15\hat{k}$

$\vec{a}-\vec{b} = -4\hat{i}$

$\text{Area} = \dfrac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right|$

$= \dfrac{1}{2}\left|-60\hat{j}+60\hat{k}\right| = \dfrac{1}{2}\sqrt{(60)^2\times 2}$

$A = \dfrac{60}{\sqrt{2}}$

Square of area is $A^2 = 1800$

16. Let $\lim\limits_{x\to 2}\dfrac{(\tan(x-2))\left(rx^2+(p-2)x-2p\right)}{(x-2)^2} = 5$ for some r, p $\in$ R. If the set of all possible values of q, such that the roots of the equation $rx^2 - px + q = 0$ lie in (0, 2), be the interval $(\alpha, \beta]$, then $4(\alpha + \beta)$ equals.

(1) 11

(2) 13

(3) 17

(4) 21

Ans. **(3)**

Sol. $\lim\limits_{x\to 2}\dfrac{\tan(x-2)}{x-2}\cdot\dfrac{[rx^2+(p-2)x-2p]}{x-2} = 5$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{rx^2-2x+p(x-2)}{x-2} = 5$

$\because D^r \to 0 \therefore N^r \to 0 \Rightarrow r = 1$

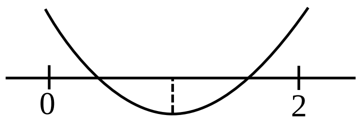
$\Rightarrow \lim\limits_{x\to 2}\dfrac{x(x-2)+p(x-2)}{x-2} = 5$

$\Rightarrow 2 + p = 5$

$\Rightarrow p = 3$

Now, quadratic equation is $x^2 - 3x + q = 0$

its both roots lie in (0, 2)



$D \geq 0, \quad 0 < \dfrac{-b}{2a} < 2, \quad f(0) > 0, \quad f(2) > 0$

$\Rightarrow 9 - 4q \geq 0$ and $q > 2$

$\Rightarrow \; q \in \left(2, \dfrac{9}{4}\right]$

$\alpha = 2, \beta = \dfrac{9}{4} \Rightarrow \left(2+\dfrac{9}{4}\right) = 17$

17. Let $A = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 1 & \alpha \\ 0 & 1 & -1 \end{vmatrix}$ be a singular matrix. Let $f(x) = \int_0^x (t^2 + 2t + 3)dt$, $x \in [1, \alpha]$. If M and m are respectively the maximum and the minimum values of f in $[1, \alpha]$ then 3(M – m)is equal to :

(1) 64 (2) 68

(3) 72 (4) 76

Ans. **(2)**

Sol. $|A| = 0$

$\Rightarrow 1.(-1-\alpha)-3(-2-0)-1(2-0) = 0$

$\Rightarrow \alpha = 3$

$$f(x) = \int_0^x (t^2 + 2t + 3)dt \ ;\ x \in [1, 3]$$

$$f(x) = \frac{x^3}{3} + x^2 + 3x$$

$f'(x) = x^2 + 2x + 3$ where $D < 0$

$\Rightarrow$ f(x) is strictly increasing

$M = f(3) = 9 + 9 + 9 = 27$

$$m = f(1) = \frac{1}{3} + 1 + 3 = \frac{13}{3}$$

$$3(M - m) = 3\left(27 - \frac{13}{3}\right) = 68$$

18. Let $f : R \to R$ be such that $f(xy) = f(x)f(y)$, for all $x, y \in R$ and $f(0) \neq 0$. Let $g: [1, \infty) \to R$ be a differentiable function such that

$$x^2 g(x) = \int_1^x (t^2 f(t) - tg(t))dt$$

Then g(2) is equal to :

(1) $\frac{13}{8}$ (2) $\frac{11}{16}$

(3) $\frac{15}{32}$ (4) $\frac{17}{64}$

Ans. **(3)**

Sol. $\because f(xy) = f(x)f(y)$

$\therefore f(x) = x^n$ or $f(x) = k$

But $f(0) \neq 0 \Rightarrow f(x) \neq x^n$

For $f(x) = k \Rightarrow k = k.k \Rightarrow k = 1$ ($\because f(0) \neq 0$)

$\Rightarrow f(x) = 1$

$$x^2 g(x) = \int_1^x \left(t^2(1) - tg(t)\right).dt$$

diff w/r to x

$\Rightarrow x^2 g'(x) + g(x) . 2x = x^2 - xg(x)$

$\Rightarrow xg'(x) + 3g(x) = x$

$$\Rightarrow g'(x) + \frac{3}{x} g(x) = 1$$

$$IF = e^{\int \frac{3}{x} dx} = e^{3\ln(x)} = x^3$$

Solution of DE :

$$\Rightarrow g(x) . x^3 = \int x^3 dx$$

$$\Rightarrow x^3.g(x) = \frac{x^4}{4} + C$$

$$\Rightarrow C = -\frac{1}{4} \ \{\because g(1) = 0\}$$

$$g(x) . x^3 = \frac{x^4}{4} - \frac{1}{4}$$

Put x = 2

$$(8)\ g(2) = 4 - \frac{1}{4}$$

$$g(2) = \frac{15}{32}.$$

19. The area of the region $\{(x, y) : x^2 - 8x \leq y \leq -x\}$ is:

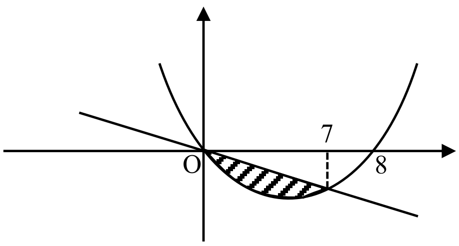
(1) $\frac{343}{6}$ (2) $\frac{637}{6}$

(3) $\frac{437}{6}$ (4) $\frac{523}{6}$

Ans. **(1)**

Sol. Required area $= \int_0^7 (7x - x^2)\,dx$

$$= \left[\frac{7x^2}{2} - \frac{x^3}{3}\right]_0^7 = \frac{343}{6}$$



20. The value of the integral $\int_{-1}^{1}\left(\frac{x^3+|x|+1}{x^2+2|x|+1}\right)dx$ is equal to

(1) $3\log_e 2$ (2) $2\log_e 2$

(3) $5\log_e 3$ (4) $3\log_e 3$

Ans. (2)

Sol. $I = \int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}.dx$

$$I = \int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}.dx + \int_{-1}^{1} \frac{|x|+1}{x^2+2|x|+1}.dx$$

$$I = 0 + \int_{-1}^{1} \frac{|x|+1}{|x|^2+2|x|+1}.dx$$

$$2\int_0^1 \frac{1}{x+1}.dx$$

$$I = 2\left[\ell n|x+1|\right]_0^1$$

$$I = 2\ell n(2)$$

SECTION-B

21. Let R = {(x,y) $\in$ N $\times$ N : $\log_e (x + y) \le 2$}. Then the minimum number of elements, required to be added in R to make it a transitive relation, is ______.

Ans. (15)

Sol. $x + y \le e^2$ ($e^2 \simeq 7.29$)

R = {(1, 1), (1,2), (1, 4), (1,5), (1,6)}

(2, 1), (2,2), (2,3), (2,4), (2,5)

(3,1), (3,2), (3,3), (3,4)

(4,1), (4,2), (4,3)

(5,1), (5,2)

(6,1)

For R to be transitive, we have to add

(6,2), (6,3), (6,4), (6,5), (6,6)

(5,3), (5,4), (5,5), (5,6)

(4,4), (4,5), (4,6)

(3,5), (3,6)

(2,3)

= 15 elements

22. If $(1-x^3)^{10} = \sum_{r=0}^{10} a_r x^r (1-x)^{30-2r}$, then $\frac{9a_9}{a_{10}}$ is equal to ______

Ans. (30)

Sol. $(1 - x)^3 = 1 - x^3 - 3x(1 - x)$

$(1 - x)^3 = 1 - x^3 - 3x + 3x^2$

$(1 - x)^3 + 3x - 3x^2 = 1 - x^3$

$(1 - x^3)^{10} = [(1 - x)^3 + 3x(1 - x)]^{10}$

$\Rightarrow {}^{10}C_r(3x(1-x))^r.((1-x)^3)^{10-r}$

$(1 - x^3)^{10} = {}^{10}C_r.3^r.x^r.(1 - x)^{30-2r}$

$\therefore a_r = {}^{10}C_r.3^r$

$$\frac{a_9}{a_{10}} = \frac{{}^{10}C_9 3^9}{{}^{10}C_{10}.3^{10}}$$

$$\frac{a_9}{a_{10}} = \frac{10}{3}$$

$$\therefore \frac{9a_9}{a_{10}} = 30$$

23. Let the line $x - y = 4$ intersect the circle C : $(x - 4)^2 + (y + 3)^2 = 9$ at the points Q and R. If P(α, β) is a point on C such that PQ = PR, then $(6\alpha + 8\beta)^2$ is equal to ________.

Ans. (18)

Sol. $x - y = 4$ ... (1)

$(x-4)^2 + (y+3)^2 = 9$ ... (2)

(α, β) lies on circle

$\therefore\ (\alpha-4)^2 + (\beta+3)^2 = 9$ ... (3)

eq. (1) & (2) $\Rightarrow y^2 + (y+3)^2 = 9 \Rightarrow 2y^2 + 6y = 0$

$\Rightarrow\ y = 0, -3$

$\therefore\ x = 4, 1$

Q(4, 0), R(1, –3)

now

$(PQ)^2 = (PR)^2$

$\Rightarrow\ (\alpha-4)^2 + \beta^2 = (\alpha-1)^2 + (\beta+3)^2$

$\Rightarrow \alpha + \beta = 1$ ... (4)

eq. (3) & (4)

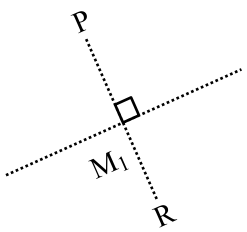
$$\Rightarrow \alpha = 4 + \frac{3}{\sqrt{2}},\ \beta = -3 - \frac{3}{\sqrt{2}}$$

$\therefore (6\alpha + 8\beta)^2 = 18$

24. Let the image of the point P(0, –5, 0) in the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2}$ be the point R and the image of the point $Q\left(0, \frac{-1}{2}, 0\right)$ in the line $\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1}$ be the point S. Then the square of the area of the parallelogram PQRS is ______.

Ans. **(162)**

Sol. P(0, –5, 0)



$$\frac{x-1}{2} = \frac{y}{1} = \frac{z+1}{-2} = k$$

Let $R(x_1, y_1, z_1)$

$2k + 1, k, -2k - 1$

$(2k+1)(2) + (k+5)(1) + (-2k-1)(-2) = 0$

$\Rightarrow 9k + 9 = 0 \Rightarrow\ k = -1$

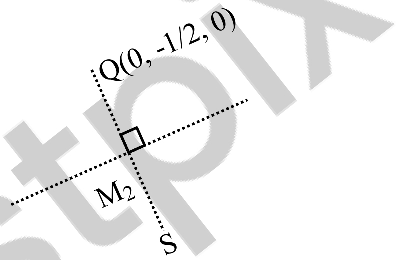
$\therefore\ M_1 \Rightarrow (-1, -1, 1)$

$R \Rightarrow (-2, 3, 2)$

Q(0, –1/2, 0)

$$\frac{x-1}{-1} = \frac{y+9}{4} = \frac{z+1}{1} = \lambda$$

$(-\lambda + 1, 4\lambda - 9, \lambda - 1)$



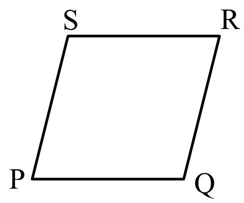
$S(x_2, y_2, z_2)$

$$(-\lambda+1)(-1) + \left(4\lambda - \frac{17}{2}\right)(4) + (\lambda-1)(1) = 0$$

$\Rightarrow 18\lambda - 36 = 0 \Rightarrow \lambda = 2$

$M_2 \Rightarrow (-1, -1, 1)$

$S \Rightarrow (-2, -3/2, 2)$



$$PR = \sqrt{4+64+4} = \sqrt{72} = 6\sqrt{2}$$

$$QS = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$Ar. = \frac{1}{2} \cdot 6\sqrt{2} \cdot 3 = 9\sqrt{2}$$

$$(Ar.)^2 = \left(9\sqrt{2}\right)^2 = 162 \text{ Sq.unit}$$

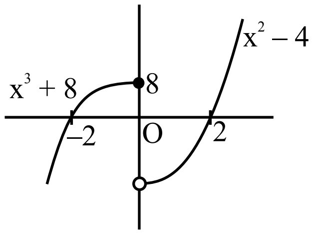
25. Let $f(x)=\begin{cases} x^3+8 & x<0 \\ x^2-4 & x\ge 0 \end{cases}$ and

$$g(x)=\begin{cases} (x-8)^{1/3}; & x<0 \\ (x+4)^{1/2}; & x\ge 0 \end{cases}$$

Then the number of points, where the function gof is discontinuous, is _______.

Ans. **(3)**

Sol. $$g(f(x))=\begin{cases} (f(x)-8)^{1/3} & f(x)<0 \\ (f(x)+4)^{1/2} & f(x)\ge 0 \end{cases}$$



$$g(f(x))=\begin{cases} (x^3)^{1/3} & , \quad x<-2 \\ (x^2-12)^{1/3}, & 0<x<2 \\ (x^3+12)^{1/2}, & -2\le x\le 0 \\ (x^2)^{1/2} & , \quad x\ge 2 \end{cases}$$

$$g(f(x))=\begin{cases} x & , \quad x<-2 \\ (x^3+12)^{1/2} & , \quad -2\le x\le 0 \\ (x^2-12)^{1/3} & , \quad 0<x<2 \\ x & , \quad x\ge 2 \end{cases}$$

Number of points of discontinuity is equal to 3.

(HELD ON MONDAY 06th APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

26. The percentage error in the calculated volume of a sphere, if there is 2% error in its diameter measurement, is ____.

(1) 1 (2) 2

(3) 6 (4) 8

Ans. (3)

Sol. $v = \frac{4}{3}\pi R^3$

$\frac{\Delta v}{v} = \frac{3\Delta R}{R}$

$= 6\ \%$

27. Match List-I with List-II.

	List-I		List-II
A.	Boltzmann constant	I.	$[M^{-1}L^{3}T^{-2}]$
B.	Stefan's constant	II.	$[M\ L^{2}T^{-1}]$
C.	Planck's constant	III.	$[ML^{2}T^{-2}K^{-1}]$
D.	Gravitational constant	IV.	$[ML^{0}T^{-3}K^{-4}]$

(1) A-I, B-II,C-III, D-IV

(2) A-IV, B-III,C-II, D-I

(3) A-III, B-IV,C-II, D-I

(4) A-II, B-I,C-IV, D-III

Ans. (3)

Sol. $(A) \rightarrow ML^{2}T^{-2}K^{-1}$ (III)

$(B) \rightarrow MT^{-3}K^{-4}$ (IV)

$(C) \rightarrow ML^{2}T^{-1}$ (II)

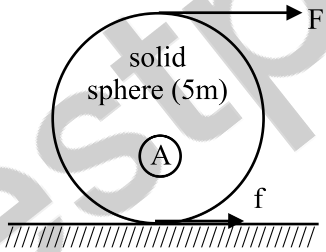
$(D) \rightarrow M^{-1}L^{3}T^{-2}$ (IV)

28. A solid sphere (A) of mass, 5m and spherical shell (B) of mass m, both having same radius, are placed on a rough surface. When a force of same magnitude is applied tangentially at the highest points of A and B, they start rolling without slipping with an acceleration of a_A and a_B, respectively. The ratio of a_A and a_B is _____.

(1) 5 : 21 (2) 6 : 10

(3) 21 : 25 (4) 1 : 5

Ans. (1)



Sol.

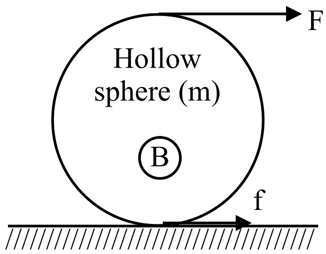
$F + f = 5m\ a_A$

$F(R) - f(R) = \frac{2}{5}(5m)R^2.\alpha$

$\alpha = R\alpha$

$2F = 7m\ a_A$

$a_A = \frac{2F}{7m}$



$F + f = ma_B$

$F(R) - f(R) = \frac{2}{3}mR^2 \times \alpha$

$a = R\alpha$

$$2F = \frac{5ma_B}{3}$$

$$\frac{a_A}{a_B} = \frac{2F}{7m} \times \frac{5m}{6F} = \frac{5}{21}$$

29. A body of mass 1kg moves along a straight line with a velocity $v = 2x^2$. The work done by the body during displacement from x = 0 to 5m is ____J.

(1) 0 (2) 250

(3) 1250 (4) 1000

Ans. (3)

Sol. $w.\Delta = \Delta K.E$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$= \frac{1}{2}\times 1\left[\left(2(5)^2\right)^2 - \left(2(0)\right)^2\right]^0$$

$$= \frac{1}{2}\times 50\times 50 = 1250J$$

30. A cylinder with adiabatic walls is closed at both ends and is divided into two compartments by a frictionless adiabatic piston. Ideal gas is filled in both (left and right) the compartments at same P,V, T. Heating is started from left side until pressure changes to $\frac{27P}{8}$. If initial volume of each compartment was 9 litres then the final volume in right-hand side compartment is ____ litres. (for this ideal gas $C_P/C_V = 1.5$)

(1) 3 (2) 4

(3) 14 (4) 9

Ans. (2)

Sol. $P_{Left} = P_{Right}$ (final)

for right chamber

$$P_iV_i^\gamma = P_fV_f^\gamma$$

$$P(9)^{3/2} = \frac{275}{8}\times V_f^{3/2}$$

$$27 = \frac{27}{8}V_f^{3/2}$$

$\therefore V_f = 4$ litres

31. For an electromagnetic wave propagating through vacuum, $\vec{k}$, $\vec{E}$ and ω represent propagation vector, electric field and angular frequency, respectively. The magnetic field associated with this wave is represented by :

(1) $\frac{\vec{E}\times\vec{k}}{\omega}$ (2) $\frac{\vec{k}\times\vec{E}}{\omega}$

(3) $\omega\left(\vec{E}\times\vec{k}\right)$ (4) $\omega\left(\vec{k}\times\vec{E}\right)$

Ans. (2)

Sol. $\vec{B} = \frac{1}{\omega}\left(\vec{k}\times\vec{E}\right)$

32. Two identical bodies A and B of equal masses have initial velocities $\vec{v}_1 = 4\hat{i}\,m/s$ and $\vec{v}_2 = 4\hat{j}\,m/s$ respectively. The body A has acceleration $\vec{a}_1 = 6\hat{i} + 6\hat{j}\,m/s^2$ while the acceleration of the other body B is zero. The centre of mass of the two bodies moves in _____path.

(1) circular (2) parabolic

(3) straight line (4) elliptical

Ans. (3)

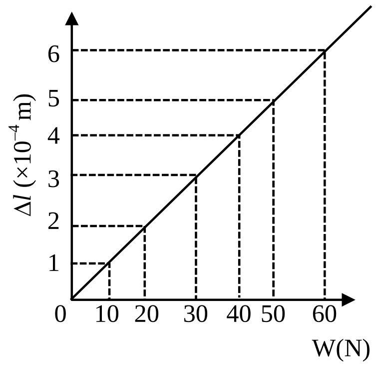
Sol. $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{m(4\hat{i}) + m(4\hat{j})}{2m} = 2\hat{i} + 2\hat{j}$

$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{m(6\hat{i}+6\hat{j}) + m(0)}{2m} = 3\hat{i} + 3\hat{j}$ -

$\vec{v}_{cm} \parallel \vec{a}_{cm}$

so, path is straight line

33. Figure represents the extension (Δl) of a wire of length 1 meter, suspended from the ceiling of the room at one end with load W connected to the other end. If the cross- sectional area of the wire is 10^{-5} m^2 then the Young's modulus of the wire is ___N/m^2.



(1) 1.0×10^{11} (2) 2.0×10^{10}

(3) 1.0×10^{10} (4) 2.0×10^{11}

Ans. (3)

Sol. $\frac{\Delta\ell}{w} = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{1\times10^{-4}}{10} = 10^{-5}$

$\frac{w}{\Delta\ell} = 10^5$

$\Rightarrow y = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell} = \left(\frac{w}{\Delta\ell}\right)\frac{\ell}{A}$

$\Rightarrow y = \frac{10^5\times1}{10^{-5}} = 10^{10} N/m^2$

34. A cylindrical vessel of 40 cm radius is completely filled with water and its capacity is 528 dm^3 (dm : decimeter). The vessel is placed on a solid block of exactly same height as vessel. If a small hole is made at 70 cm below the top of water level, then horizontal range of water falling on the ground in the beginning is ____cm.

(1) $120\sqrt{2}$ (2) $140\sqrt{2}$

(3) $140\sqrt{3}$ (4) $120\sqrt{3}$

Ans. **(2)**

Sol. $v = \pi r^2 h \Rightarrow 528\times10^{-3} = \pi(0.4)^2 h$

$h = \frac{528\times10^{-3}\times7}{(0.4)^2\times22} = 1.05m = 105cm$

$\rightarrow t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times(105+35)\times10^{-2}}{10}}$

$= \sqrt{\frac{2(140)\times10^{-2}}{10}} \quad \sqrt{28}\times10^{-1}sec$

$R = ut = \sqrt{2\times10\times70\times10^{-2}}\times\sqrt{28}\times10^{-1}$

$= \left(\sqrt{14}\right)\left(\sqrt{28}\right)\times10^{-1} = 14\sqrt{2}\times10^{-1}m$

$R = 140\sqrt{2}$ cm

35. If 2 mole of an ideal monoatomic gas at temperature T, is maxed with 6 mole of another ideal monoatomic gas at temperature 2T then the temperature of mixture is :

(1) $\frac{5}{2}T$ (2) $\frac{5}{4}T$

(3) $\frac{7}{2}T$ (4) $\frac{7}{4}T$

Ans. **(4)**

Sol. $T_{mix} = \frac{n_1C_1T_1 + n_2C_2T_2}{n_1C_1 + n_2C_2}$

$= \frac{\left(2\times\frac{3}{2}R\times T\right)+\left(6\times\frac{3}{2}R\times2T\right)}{\left(2\times\frac{3}{2}R\right)+\left(6\times\frac{3}{2}R\right)}$

$T_{mix} = \frac{2T+12T}{8} = \frac{7}{4}T$

36. A spring stretches by 2 mm when it is loaded with a mass of 200 g. From equilibrium position the mass is further pulled down by 2 mm and released. The frequency associated with the system and maximum energy in the spring are ______ Hz and ______ J, respectively. (Take g = 10 m/s^2)

(1) $\frac{5\sqrt{50}}{\pi}$ and 8×10^{-3}

(2) $\frac{5\sqrt{50}}{\pi}$ and 8

(2) $10\sqrt{50}$ and 2×10^{-3}

(4) $\frac{5\sqrt{50}}{\pi}$ and 16×10^{-3}

Ans. **(1)**

Sol. $kx = 0.2g;$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$k\times2\times10^{-3} = 2$ $\quad f = \frac{1}{2\pi}\sqrt{\frac{1000}{0.2}}$

$k = 1000$ N/m

$= \frac{1}{2\pi}\sqrt{5000} = \frac{5}{\pi}\sqrt{50}$

$E_{max} = \frac{1}{2}k\ x_{max}^2$

$\frac{1}{2}\times1000\times\left(4\times10^{-3}\right)^2 = 8\times10^{-3}$ Joule

37. The electric potential as a function of x, y is given by $V = 5(x^2 - y^2)V$. The electric field at a point (2, 3) m is ______ V/m.

(1) $(-20\hat{i}+30\hat{j})$ (2) $(20\hat{i}-30\hat{j})$

(2) $(20\hat{i}+45\hat{j})$ (4) $(-4\hat{i}+6\hat{j})$

Ans. **(1)**

Sol. $V = 5(x^2 - y^2)V$

$$\varepsilon_x = \frac{-dv}{dx} = -10x\hat{i}$$

$$\varepsilon_y = \frac{-dv}{dx} = +10y\hat{i} \Rightarrow \varepsilon(2,3) = -20\hat{i} + 30\hat{j}$$

38. A current of 30 A each flows in opposite directions in two conducting wires, placed parallel to each other at a distance of 8 cm. The magnetic field at the mid point between the two wires is _____ μT.

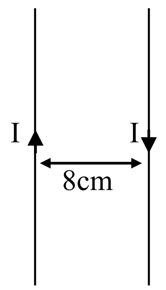
$(\frac{\mu_0}{4\pi} = 10^{-7}\ N/A^2)$

(1) 30 (2) 300

(2) 150 (4) 0.1

Ans. (2)

Sol.



$$B = \frac{\mu_0 I}{2\pi(4)\times 10^{-2}} \times 2$$

$= 10^{-7} \times 30 \times 10^2$

$= 30 \times 10^{-5}$ T

$= 300$ μT

39. A square loop of side 2cm is placed in a time varying magnetic field with magnitude as B = 0.4 sin(300t) Tesla. The normal to the plane of loop makes an angle of 60° with the field. The maximum induced emf produced in the loop is ____ mV.

(1) 12 (2) 18

(3) 21 (4) 24

Ans. (4)

Sol. $B = 0.4\sin 300t$ $\theta = 60°$

$A = (0.02)^2 = 4 \times 10^{-4}\ m^2$

$\phi = BA\cos\theta$

$\phi = 0.4 \sin(300t) A\cos\theta$

$$\text{emf} = \frac{d\phi}{dt} = 0.4\ A\cos\theta\cos(300t) \times 300$$

$$\text{max emf} = 0.4 \times 4 \times 10^{-4} \times 300 \times \frac{1}{2}$$

$= 240 \times 10^{-4}$ V

$= 24$ mV

40. A sphere of capacitance 100 pF is charged to a potential of 100V. Another identical undercharged metal sphere is brought in contact with the charged sphere, then the change in the total energy stored on these spheres, when they touch is $\alpha \times 10^{-7}$ J. The value of α is ______. (combined capacitance of spheres is 200 pF).

(1) 5 (2) $\frac{5}{2}$

(3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Ans. (2)

Sol. $Q_1 = CV = 100 \times 100\ pC = 10^4\ pC$

$$V_1 = V_2 = \frac{V}{2} = 50$$

$$\varepsilon_i = \frac{1}{2} \times 100 \times (100)^2\ pJ = \varepsilon$$

$$\varepsilon_f = 2 \times \frac{1}{2} \times 100 \times (50)^2 = \frac{\varepsilon}{2}$$

$$\Delta\varepsilon = \frac{\varepsilon}{2} = 100 \times (50)^2\ pJ$$

$= 25 \times 10^4 \times 10^{-12} = 2.5 \times 10^{-7}$

41. The energy released if hydrogen atoms are combined to form ^4_2He is_______.

(Take binding energies per nucleon of ^2_1H and ^4_2He as 1.1 meV and 7.2 MeV, respectively).

(1) 6.1 (2) 24.4

(3) 26.6 (4) 5

Ans. (2)

Sol. ${}_1^2H + {}_1^2H \rightarrow {}_2^4He$

$Q = BE\ (RHS) - BE\ (LHS)$

$= 4 \times 7.2 - 4 \times 1.1$

$= 4 \times 6.1$

$Q = 24.4$ MeV

42. Angle of minimum deviation is equal to the half of the angle of prism is an equilateral prism. The refreactive index of the prism is ______.

(1) 1.5 (2) $\sqrt{3}$

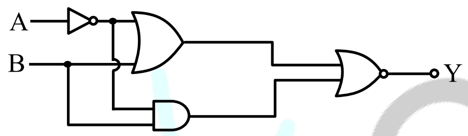
(3) $\sqrt{2}$ (4) 1.65

Ans. (3)

Sol. $$\mu_p = \mu_s \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\frac{3A}{4}}{\sin\frac{A}{2}}$$

$$\mu_p = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2} = \sqrt{2}$$

43. Refer to the logic circuit given below. For two inputs (A = 1, B = 1) and (A = 0, B = 1), output (Y) will be ______.



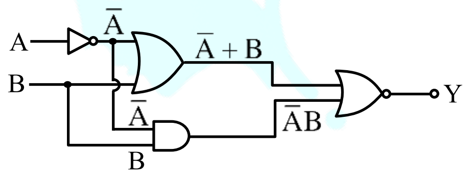
(1) 1, 0 respectively (2) 0,1 rsepectively

(3) 0,0 respectively (4) 1,1 respectively

(5) 0.0

Ans. (3)

Sol.



$y = \overline{\bar{A}+B+\bar{A}B}$

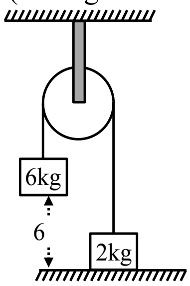
$= \overline{\bar{A}+B} = \bar{\bar{A}} \cdot \bar{B}$

$= A\bar{B}$

for A = 1, B = 1, y = 0

for A = 0, B = 1, y = 0

44. The velocity at which 6 kg mass (shown in figure) strikes the ground when it is released from a height of 6m above the ground is ________ m/s. Assume pulley is massless and string is light and inextensible (Take g = 10 m/s^2).

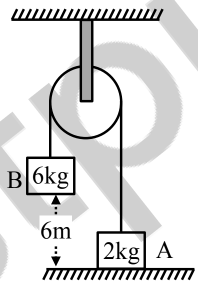


(1) 7.74 (2) 7.20

(3) 6.55 (4) 4.50

Ans. (1)

Sol.



$W_G = \Delta K$

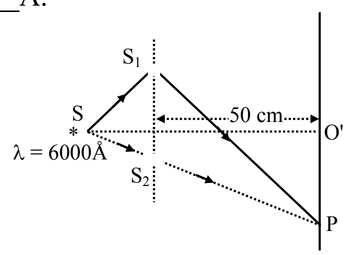
$$m_B gh - m_A gh = \frac{1}{2}(m_A + m_B)V^2$$

$$(6-2) \times 10 \times 6 = \frac{1}{2}(8)v^2$$

$v = \sqrt{60}$ m / s

$v = 7.746$ m/s

45. In a Young double slit experiment, the wavelength of incident of incident light is 6000 Å, the separation between slits S_1 and 5 cm the distance between slits plane and screen is 50 cm, as shown in the figure below. If the resultant intensity at P is equal to the intensity due to individual slits, the path different between interfering waves is _______Å.



(1) 4000 (2) 3000

(3) 2000 (4) 1000

Ans. (3)

Sol. $I_P = I_0 = 4I_0 \cos^2\frac{\Delta\phi}{2}$

$$\cos\frac{\Delta\phi}{2} = \frac{1}{2}$$

$$\frac{\Delta\phi}{2} = \frac{\pi}{3}$$

$$\frac{2\pi}{\lambda} = \Delta x = \Delta\phi = \frac{2\pi}{3}$$

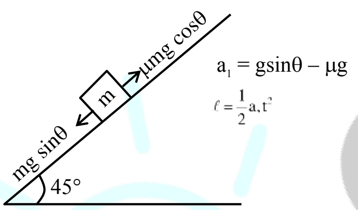
$$\Delta x = \frac{\lambda}{3} = 2000A^\circ$$

SECTION-B

46. A block takes t time to slide down a plane inclined at 45° to the horizontal. If the surface is made smooth (frictionless), the block takes times $\frac{t}{2}$ to slide down the plane. The coeffcient of friction between the block and the inclined plane is $\left(\frac{\alpha}{100}\right)$. The value of α is ____.

Ans. (75)

Sol.



for frictionless surface $a_2 = g \sin\theta$

$$\ell = \frac{1}{2}a_2\frac{t^2}{4}$$

$$a_1 t^2 = a_2\frac{t^2}{4}$$

$$g(\sin 45 - \mu\cos 45^\circ) = \frac{g\sin 45}{4}$$

$$1 - \mu = \frac{1}{4}$$

$$\mu = \frac{3}{4} = \frac{\alpha}{100}$$

$$\alpha = 75$$

47. The de Broglie weave length for an electron accelerated through the potential difference V_1 volt is λ_1. When the potential difference is changed to V_2 volt, the associated de Broglie wavelength is increased by 50%. If $(V_1/V_2) = (9/\alpha)$, then the value of α is _____.

Ans. (4)

Sol. $\lambda_1 = \frac{h}{mv} = \frac{h}{\sqrt{2meV_1}} = \lambda$

$$\lambda_2 = \frac{h}{\sqrt{2meV_2}} = \frac{3\lambda}{2} = \frac{3}{2}\frac{h}{\sqrt{2meV_1}}$$

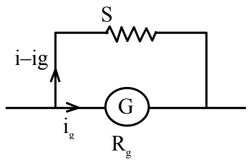
$$V_2 = \frac{4}{9}.V_1$$

$$\frac{V_1}{V_2} = \frac{9}{4} \Rightarrow \alpha = 4$$

48. A moving coil of a golvanometer when shunted with 2Ω resistance gives a full scale deflection for a current of 500 mA. When a resistance of 470Ω is connected in series it gives a full scale deflection for 10V potential applied on it. The value of resistance of galvanometer coil is _____Ω.

Ans. (50)

Sol. $i_g R_g = 2\left(\frac{1}{2} - ig\right)$



$$i_g = \frac{1}{R_g + 2}$$

$$10 = (R_g + 470)i_g$$

$$10 = \frac{R_g + 470}{R_g + 2}$$

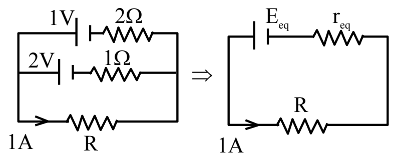
$$10R_g + 20 = R_g + 470$$

$$R_g = 50\Omega$$

49. Two cells of emfs 1V and 2V and internal reseistance 2Ω and 1Ω, respectively connected in parallel, gave current of 1A through an external resistance. If the polarity of one cell is reversed, then value of current through the external resistance will be $\frac{\alpha}{5}$ A. The value of α is _____.

Ans. **(3)**

Sol.



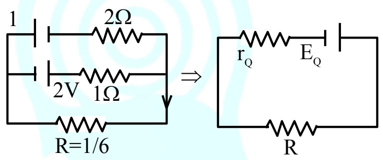
$$E_{eq} = \frac{\frac{1}{2}+\frac{2}{1}}{\frac{1}{2}+\frac{1}{1}} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}$$

$$r_Q = \frac{3}{2}$$

$$i = \frac{E_{eq}}{r_{eq}+R} = \frac{\frac{5}{3}}{\frac{3}{2}+R} = 1$$

$$\frac{5}{3} = \frac{3}{2}+R$$

$$R = \frac{1}{6}\Omega$$



$$r_{eq} = \frac{3}{2}$$

$$E_{eq} = \frac{\frac{2}{1}-\frac{1}{2}}{\frac{1}{1}+\frac{1}{2}} = 1$$

$$i = \frac{E_{eq}}{r_{eq}+R} = \frac{1}{\frac{3}{2}+\frac{1}{6}} = \frac{3}{5} \quad \Rightarrow \alpha = 3$$

50. A concave mirror of focal length 10 cm froms an image which is double the size of object when the object is placed at two different positions. The distance between the two positions of the object is ____ cm.

Ans. **(10)**

Sol. **Case I** $v = 2u$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{-2u}+\frac{1}{-u}=\frac{1}{-f}$$

$$\frac{3}{2u}=\frac{1}{f}$$

$$u_1 = \frac{3f}{2}$$

Case 2 $v = -2u$.

$$\frac{1}{+2u}+\frac{1}{-u}=\frac{1}{-f}$$

$$\frac{-1}{2u}=\frac{1}{-f}$$

$$u_2 = \frac{f}{2}$$

$$\Delta u = f = 10 \text{ cm}$$

(HELD ON MONDAY 06th APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

51. Which of the following contain the same number of atoms ?

(Given : Molar mass in g mol^{-1} of H, He, O and S are 1,4,16 and 32 respectively)

A. 2g of O_2 gas

B. 4g fo SO_2 gas

C. 1400 mL of O_2 at STP

D. 0.05 L of He at STP

E, 0.0625 mol of H_2 gas

Choose the **correct** answer from the options given below :

(1) A and B only (2) B and C only

(3) C and D only (4) A, C and E only

Ans. (4)

Sol.

$$N_{atoms}\text{ in }O_2 = \frac{m_{O_2}}{M_{O_2}}N_A = \frac{2}{32}\times 2N_A = \frac{N_A}{8} = 0.125N_A$$

$$N_{atoms}\text{ in }SO_2 = \frac{m_{SO_2}}{M_{SO_2}} = 3N_A = \frac{4}{64}\times 3N_A = \frac{3N_A}{16} = 0.1875\,N_A$$

$$N_{atoms}\text{ in }O_2\text{ at STP} = \frac{V_{O_2}}{22.7\,\text{lit}}N_A = \frac{1400\,\text{ml}}{22400\,\text{ml}}\times 2N_A = 0.125\,N_A$$

$$N_{atoms}\text{ in He(g) at STP} = \frac{V_{He}}{22.4\text{ lit.}}N_A = \frac{0.05}{22.4}N_A = 0.002N_A$$

$$N_{atoms}\text{ in }H_2(g) = \text{moles}\times\text{atomicity}\times N_A = 0.0625\times 2N_A$$
$$= 0.125\,N_A$$

52. The Bohr radius of a hydrogen like species is 70.53 pm. The species and the stationary state(n) are respectively

(Given: Hydrogen atom Bohr radius is 52.9 pm)

(1) Li^{2+}, 3 (2) He^{+}, 3

(3) He^{+}, 2 (4) Li^{2+}, 2

Ans. (4)

Sol. $r_{species} = a_0\dfrac{n^2}{Z}$

$$70.53 = 52.9\left(\frac{n^2}{Z}\right)$$

$$\frac{n^2}{Z} = \frac{70.53}{52.9} = 1.33 = \frac{4}{3}$$

Data will satisfy if n = 2 and z = 3

$$Li^{+2}, 2 \Rightarrow \frac{4}{3} = 1.33$$

53. Given below are two statements:

Statement I : The number of compounds among SO_2, SO_3, SF_4, SF_6 and H_2S in which sulphur does not obey the Octet rule is 3.

Statement II : Among [H_2O, ClF_3, SF_4], [NH_3, BrF_5, SF_4], [BrF_5, ClF_3, XeF_4] and [XeF_4, ClF_3,H_2O], the number of sets in which all the molecules have one lone pair of electrons on the central atom is 1.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true **Statement II** is false

(4) **Statement I** is false **Statement II** is true

Ans. (4)

Sol. SO_2 , SO_3 , SF_6 & SF_4 do not follow octet rule

NH_3 , BrF_5 , SF_4 have one lone pair on central atom

ClF_3 , XeF_4 & H_2O have two lone pair on central atom.

54. Match **List-I** with **List-II**

Given V_1 and V_2 are initial and final volumes respectively:

List-I (Isothermal process)		**List-II (Expression)**	
A.	Reversible expansion	I.	$q=0$
B.	Free expansion	II.	$q = nRT \ln\frac{V_2}{V_1}$
C.	Irreversible Compression	III.	$w = -p_{ext}(V_1 - V_2)$
D.	Cyclic reversible	IV.	$\frac{q_{rev}}{T} = 0$

Choose the correct answer from the options given below :

(1) A-II, B-III, C-I, D-IV (2) A-II, B-I, C-IV, D-III

(3) A-II, B-I, C-III, D-IV (4) A-I, B-II, C-III, D-IV

Ans. (3)

Sol. (A) For isothermal process, $\Delta U = 0$

$$W = -nRT\ell n\left(\frac{V_2}{V_1}\right) \text{ so } q = nRT\ell n\left(\frac{V_2}{V_1}\right)$$

(B) Free expansion which is isothermal must be adiabatic, so $q = 0$

(C) Irreversible process $W = -P_{ext}[V_2 - V_1]$

(D) Cyclic process, change in state function (S) so change in entropy must be zero.

$$dS = \frac{dq_{rev}}{T}$$

$$\oint dS = 0$$

55. Given below are two statements :

Chamber 1	**Semi-permeable membrane**	**Chamber 2**
18 g glucose in 100 mL aqueous solution		30 g glucose in 250 mL aqueous solution

Statement I : H_2O molecules move from the chamber 1 to chamber 2.

Statement II : The osmotic pressure of a solution prepared by dissolving 50 mg of potassium sulphate (molar mass = 174 g/mol) in 2 L of water (at 27°C) is 0.0107 bar. (Given : R = 0.083dm^3 bar K^{-1} mol^{-1} and assume complete dissociation of electrolyte)

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (4)

Sol. On side (I) $C_1 = \frac{18/180}{0.1} = 1M$

On side (II) $C_2 = \frac{30/180}{250/1000} = \frac{2}{3}M$; $\{\pi_1 > \pi_2\}$

Hence water will flow from chamber-II to Chamber-I.

$\pi = iCRT$; $i = 3$ for K_2SO_4

And $\left(\pi = 3 \times \frac{50\times10^{-3}/174}{2}\right) \times 0.083 \times 300$

$= 0.0107$ bar.

56. Given is a concentrated solution of a weak electrolyte A_xB_y of concentration 'c' and dissociation constant 'K'. the degree of dissociation is given by:

(1) $[K \times c^{x+y-1}x^xy^y]^{x+y}$

(2) $\left(\frac{K}{c^{x+y-1}x^xy^y}\right)^{\frac{1}{x+y}}$

(3) $\left(\frac{c^{x+y-1}x^xy^y}{K}\right)^{x+y}$

(4) $\left(\frac{c^{x+y-1}x^xy^y}{K}\right)^{\frac{1}{x+y}}$

Ans. (2)

Sol. $A_xB_y \rightleftharpoons xA^{y+} + yB^{x-}$

$C(1-\alpha) \quad xC\alpha \quad yC\alpha$

$$K = \frac{(xC\alpha)^x(xC\alpha)^y}{C(1-\alpha)}$$

As $\alpha << 1 \Rightarrow (1-\alpha) \simeq 1$

$K = x^xy^yC^{x+y-1}\alpha^{x+y}$

$$\alpha = \left(\frac{K}{C^{x+y-1}x^xy^y}\right)^{\frac{1}{x+y}}$$

57. For a general redox reaction

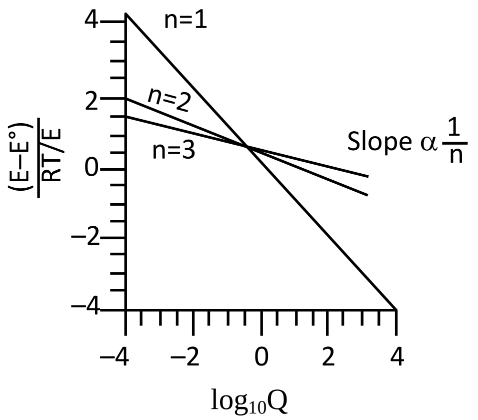
Anode $Red_1 \rightarrow Ox_1^{n_1^+} + n_1e^-$

Cathode $Ox_2 + n_2e^- \rightarrow Red_2^{n_2^-}$

Which of the following statement is **incorrect** ?

(1) The overall reaction can be written as
$n_2 Red_1 + n_1Ox_2 \rightleftharpoons n_2Ox_1^{n_1^+} + n_1 Red_2^{n_2^-}$

(2) The electrons do not appear in the overall reaction because electrons produced at the anode are consumed at the cathode.



(3) Here n is the number of electrons transferred in redox reaction.

(4) If the reaction is carried out reversibly, the electrical work done is equal to the ratio of charge and potential difference through which charge is moved.

Ans. (4)

Sol. **Anode :** $\left[Red_1 \rightarrow Ox_1^{n_1^+} + n_1e^-\right] \times n_2$

Cathode : $\left[Ox_2 + n_2e^- \rightarrow Red_2^{n_2^-}\right] \times n_1$

Overall reaction :

$n_2Red_1 + n_1Ox_2 \rightarrow n_2Ox_1^{n_1^+} + n_1 Red_2^{n_2^-}$

The electrons does not appear in overall reaction

$$E = E^o - 2.303 \frac{RT}{nF} \log Q$$

$$\frac{(E - E^o)}{RT/F} = -\frac{2.303}{n} \log Q$$

$$\text{Slope} = \frac{-2.303}{n}$$

Straight line passing through origin.

Electrical work = Charge × Potential difference

58. In a period, the first ionisation enthalpy of the element at extreme left and the negative electron gain enthalpy of the extreme right element, except noble gases, are respectively.

(1) lowest and lowest (2) highest and lowest

(3) lowest and highest (4) highest and highest

Ans. **(3)**

Sol. Left most element in a period will be of group 1 which will have lowest IE, while right most element will be of group 17 which will have most negative ΔH_{eg}.

59. Given below are two statements :

Statement I : $F_2O < H_2O < Cl_2O$ is the correct trend in terms of bond angle.

Statement II : SiF_4, SnF_4 and PbF_4 are ionic in nature.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. **(3)**

Sol.

Specie	Bond angle
OCl_2	111°
H_2O	104.5°
OF_2	103°

SiF_4 is covalent.

60. The correct order of first (Δ_iH_1) and second (Δ_iH_2) ionisation enthalpy values of cr and Mn are :

A. $\Delta_iH_1 : Cr > Mn$

B. $\Delta_iH_2 : Cr > Mn$

C. $\Delta_iH_1 : Mn > Cr$

D. $\Delta_iH_2 : Mn > Cr$

Choose the **correct** answer from the options given below :

(1) A and B only (2) B and C only

(3) A and D only (4) C and D only

Ans. **(2)**

Sol. $Cr - [Ar]3d^54s^1$

$Mn - [Ar]3d^54s^2$

$IE_1 - Mn > Cr$

$IE_2 - Mn < Cr$

61. Which of the following sequence of hybridisation, geometry and magnetic nature are correct for the given coordination compounds ?

A. $[NiCl_4]^{2-}$ – sp^3, tetrahedral, paramagnetic

B. $[Ni(NH_3)_6]^{2+}$ – sp^3d^2, octahedral, paramagnetic

C. $[Ni(CO)_4]$ – sp^3, tetrahedral, paramagnetic

D. $[Ni(CN)_4]^{2-}$ – dsp^2, square planar diamagnetic

Choose the **correct** answer from the options given below :

(1) A, B, C and D (2) B, C and D only

(3) A, C and D only (4) A, B and D only

Ans. **(4)**

Sol. $[Ni(CO)_4]$ is sp^3 , tetrahedral and diamagnetic

62. Given below are two statements :

Statement I : A mixture of $C_{12}H_{22}O_{11}$ (sugar) and NaCl can be separated by dissolving sugar in alcohol, due to differential solubility

Statement II : Rose essence from rose petals is separated by steam distillation due to its high volatility and insolubility in H_2O.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true **Statement II** is false

(4) **Statement I** is false **Statement II** is true

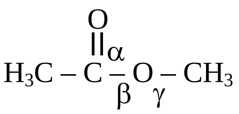
Ans. **(1)**

Sol. Sugar is soluble in alcohol not NaCl

So they are separated by solvent extraction (or differential extraction)

Rose essence oil is steam volatile so it is separated by steam distillation.

63. Shown below is the structure of methyl acetate with three different α, β and γ carbon-oxygen bonds.



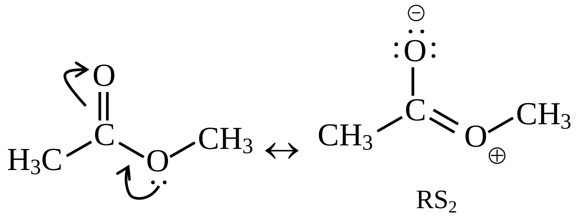
The correct order of bond length of these bonds is :

(1) $\alpha > \beta > \gamma$ (2) $\alpha < \beta < \gamma$

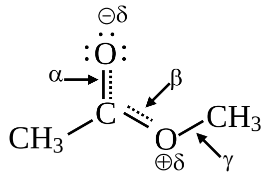
(3) $\alpha = \beta = \gamma$ (4) $\alpha < \beta = \gamma$

Ans. **(2)**

Sol.



RS$_1$ RS$_2$



R.Hybrid

RS1 is more stable than RS2

∴ contribution of RS1 is more than RS2 in it's R.H.

∴ Order of bond length $\alpha < \beta < \gamma$

64. 'x' is the product which is obtained by the hydrolysis of prop-1-yne in the presence of mercuric sulphate under dilute acidic medium at 333 K. 'y' is the product which is obtained by the reaction of ethane nitrile with methyl magnesium bromide in dry ether followed by hydrolysis. IUPAC name of product obtained from 'x' and 'y' in the presence of barium hydroxide followed by heating is:

(1) 2-Methylpent-4-en-3-one

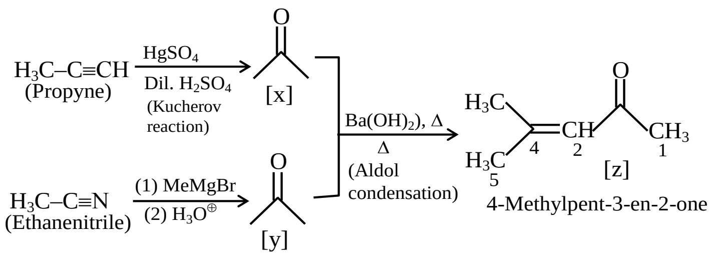
(2) 4-Methylpent-3-en-2-one

(3) 4-Methylpent-1-ene

(4) 2-Methylpent-3-one

Ans. **(2)**

Sol.

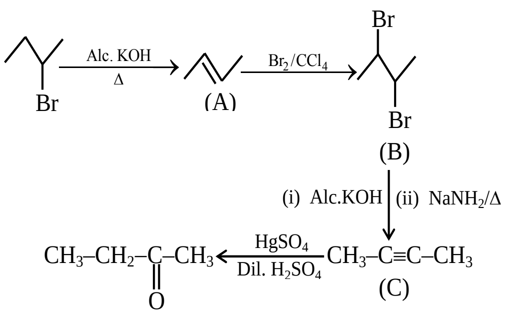


65. An optically active alkyl bromide C_4H_9Br, reacts with ethanolic KOH to form major compound [A] which reacts with bromine to give compound [B]. Compound [B] reacts with ethanolic KOH and sodamide to give compound [C]. One molecule of water adds to compound [C] on warming with mercuric sulphate and dilute sulphuric acid at 333 K to form compound [D]. The functional group in compound D will be confirmed by:

(1) Haloform test
(2) Lucas test
(3) Silver mirror test
(4) Benedict test

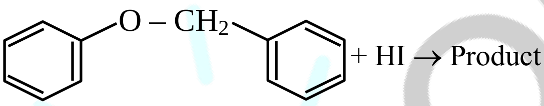
Ans. **(1)**

Sol.



Final product gives +ve haloform test

66. Consider the following reaction.



Statement I : In the above reaction, product formed will be a mixture of benzyl alcohol and iodobenzene.

Statement II : In the above reaction, the $– O – CH_2 –$ bond is cleaved to give the product.

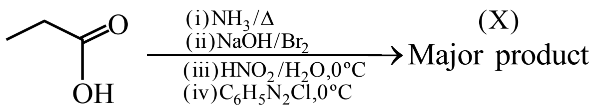
In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true
(2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is true but **Statement II** is false
(4) **Statement I** is false but **Statement II** is true

Ans. **(4)**

Sol. $Ph–O–CH_2–Ph \xrightarrow{HI} \underset{\text{Phenol}}{Ph–OH} + \underset{\text{Benzyl Iodide}}{I–CH_2–Ph}$

67. Consider the following organic reaction sequence. Choose the final product (X) from the following (consider the major product in all intermediate reactions)

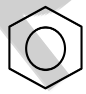


(1) Benzene
(2) Phenol
(3) Propanol
(4) Chlorobenzene

Ans. **(1)**

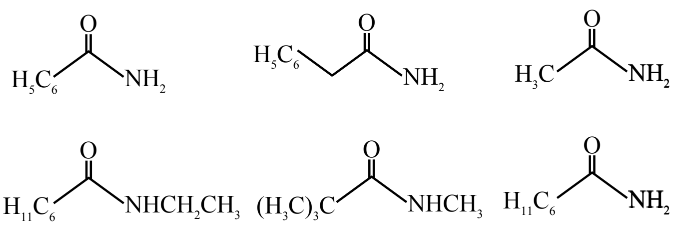
Sol. $CH_3–CH–COOH \xrightarrow[\Delta]{NH_3} CH_3-CH_2-\overset{O}{\overset{\|}{C}}-NH_2$

$\xrightarrow{NaOH+Br_2} CH_3–CH_2–NH_2 \xrightarrow[H_2O/0°C]{HNO_2}$

$CH_3–CH_2–OH \xrightarrow[0°C]{Ph–N_2^+Cl^-}$ 

$+ \; CH_3-\overset{O}{\overset{\|}{C}}-H + N_2 + HCl$

68. The number of compounds from the following which can undergo reaction with Br_2/KOH/(alcoholic) to give respective products and these respective products can also be obtained separately by Gabriel phthalimide reaction is:



(1) 5
(2) 4
(3) 3
(4) 6

Ans. **(3)**

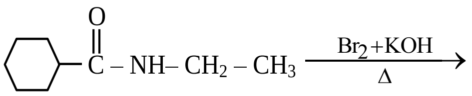
Sol. (ii), (iii), (vi)

(i) $Ph-\overset{\overset{\Large O}{\|}}{C}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} Ph-NH_2$

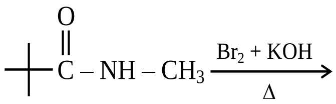
Aniline is not formed by Gabriel phtalimide synthesis.

(ii) $Ph-CH_2-\overset{\overset{\Large O}{\|}}{C}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} Ph-CH_2-NH_2$

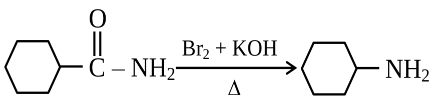
(iii) $CH_3-\overset{\overset{\Large O}{\|}}{C}-NH_2 \xrightarrow[\Delta]{Br_2 + KOH} CH_3-NH_2$

(iv) 

No 1° amine is form

(v) 

No 1° amine is form

(vi) 

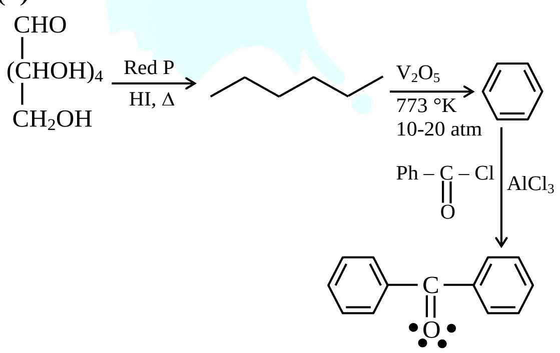
69. Consider the following reactions. Total number of electrons in the π bonds and lone pair of electrons in the product (X) is:

$$\begin{array}{l} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \xrightarrow[\text{(iii) Benzoyl chloride/Anhyd. } AlCl_3]{\text{(i) HI, }\Delta \quad \text{(ii) } V_2O_5\text{/10-20 atm/773 K}} \underset{\text{Major product}}{(X)}$$

(1) 12
(2) 16
(3) 14
(4) 18

Ans. **(4)**

Sol.



Total number of π and lone pair electrons in final product = 18

70. Treatment of a gas 'X' with a freshly prepared ferrous sulphate solution gives a compound 'Y' as a brown ring. The compounds X and Y are.

(1) NO and $[Fe(NO)]SO_4$
(2) NO_2 and $[Fe(NO_2)]SO_4$
(3) N_2O and $[Fe(N_2O)]SO_4$
(4) N_2O_4 and $[Fe(N_2O_4)]SO_4$

Ans. **(1)**

Sol. $NO + FeSO_4 \xrightarrow{\text{dil. } H_2SO_4} [\overset{I}{Fe}(NO)]SO_4$

SECTION-B

71. An excess of $AgNO_3$ is added to 100 mL of a 0.05 M solution of tetraaquadichloridochromium (III) chloride. The number of moles of AgCl precipitated will be________ × 10^{-3}. (Nearest integer)

Ans. **(5)**

Sol. $[Cr(H_2O)_4Cl_2]Cl + AgNO_3 \rightarrow AgCl\downarrow + [Cr(H_2O)_4Cl_2]NO_3$

⇒ 1 mole of complex will precipitate 1 mole of AgCl.

⇒ Given moles of complex =

$M \times V = (0.05 \times 100 \times 10^{-3})$ moles

⇒ Moles of AgCl = 5×10^{-3} moles

⇒ x = 5

72. An alkane (Y) requires 8 moles of oxygen for complete combustion and on chlorination with Cl_2/hv, (Y) gives only one monochlorinated product (Z). The total number of primary carbon atoms in (Y) is______.

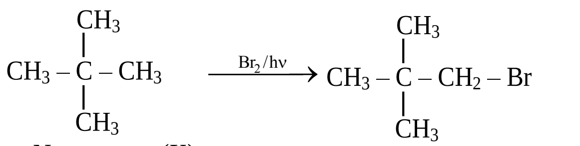
Ans. **(4)**

Sol. $C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow nCO_2 + (n+1)H_2O$

$\frac{3n+1}{2} = 8$

$3n + 1 = 8 \times 2$

$n = 5 \quad (\therefore C_5H_{12})$



Neopentane (X)

Only one mono-bromo product

Number of primary carons = 4

73. 500 mL of 0.2 M MnO_4^- solution in basic medium when mixed with 500 mL of 1.5 M KI solution, oxidises iodide ions to liberate molecular iodine. This liberated iodine is then titrated with a standard x M thiosulphate solution in presence of starch till the end point. If 300 mL of thiosulphate was consumed, then the value of x is______.

Ans. (1)

Sol. $3e^- + MnO_4^- + 2H_2O \rightarrow MnO_2 + 4OH^{\ominus}$

$2I^{\ominus} \rightarrow I_2 + 2e^-$

$I_2 + 2S_2O_3^{-2} \rightarrow 2I^{\ominus} + S_4O_6^{-2}$

gm equivalent of MnO_4^-

$= 0.2 \times \frac{500}{1000} \times 3 = 0.3$ (Limiting reagent)

gm equivalent of KI

$= 1.5 \times \frac{500}{1000} \times 1 = 0.75$

gm equivalent of MnO_4^-

= gm equivalent of hypo

$0.3 = x \times \frac{300}{1000} \times 1$

$x = 1$

74. In a closed flask at 600 K, one mole of $X_2Y_4(g)$ attains equilibrium as given below:

$X_2Y_4(g) \rightleftharpoons 2XY_2(g)$

At equilibrium, 75% $X_2Y_4(g)$ was dissociated and the total pressure is 1 atm. The magnitude of $\Delta_rG^{\ominus}$ (in kJ mol^{-1}) at this temperature is_____. (Nearest Integer)

(Given: R = 8.3 J mol^{-1} K^{-1}; ln 10 = 2.3, $\log 2 = 0.3, \log 3 = 0.48, \log 5 = 0.69, \log 7 = 0.84$)

Ans. (8)

Sol. $X_2Y_4(g) \rightleftharpoons 2XY_2(g)$

t=0 $\quad P^o$

t=t $\quad P^o(1-\alpha) \quad 2P^o\alpha$

$P^o(1+\alpha) = 1$

$\alpha = \frac{3}{4}$; $P^o = \frac{4}{7}$

$k_P = \frac{4P_o\alpha^2}{1-\alpha^2} = \frac{36}{7}$

$\Delta G^\circ = -RT\ell nk_p$

$|\Delta G^\circ| = 8.314 \times 600 \ \ell n\left(\frac{36}{7}\right)$

$= 8169.5$ J mol^{-1}

$= 8.169$ kJ/mol

75. Decomposition of a hydrocarbon follows the equation $k = (5.5 \times 10^{11}\ s^{-1})\ e^{\frac{-28000K}{T}}$. The activation energy of reaction is______ kJ mol^{-1}. (Nearest Integer)

Given : R = 8.3 JK^{-1} mol^{-1}

Ans. (232)

Sol. $K = (5.5 \times 10^{11}\ S^{-1})\ e^{-\frac{28000K}{T}}$

$K = Ae^{-E_a/RT}$

$\frac{E_a}{R} = 28000$

$E_a = \frac{28000 \times 8.3}{1000}$ kJ / mol = 232.4 kJ / mol