(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6$, $z \in C$ is :

(A) 0 (B) 1

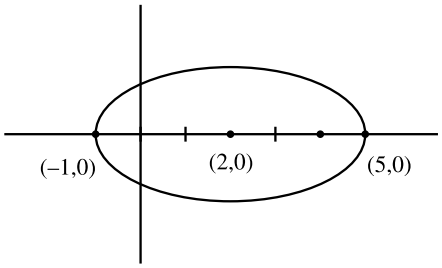
(C) 2 (D) 3

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $C : (x-4)^2 + (y-3)^2 = 4$

$E : \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$



Lower Extremity of vertical diameter of circle $\rightarrow (4,1)$

Put in ellipse $\Rightarrow \frac{(4-2)^2}{9} + \frac{1}{5} - 1$

$= \frac{4}{9} + \frac{1}{5} - 1$

$= \frac{29}{45} - 1 < 0$

Two Solutions

Answer (C)

2. Let $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of which the squares of all the values of a for $2f'(10)-f'(5)+100=0$ is :

(A) 117 (B) 106

(C) 125 (D) 136

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

$f(x) = a \begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$= a [1 (a^2 + ax) + 1 (ax + x^2)]$

$\Rightarrow f(x) = a (x + a)^2$

so, $f'(x) = 2a (x + a)$

as, $2f'(10) - f' (5) + 100 = 0$

$\Rightarrow 2 \times 2a (10 + a) - 2a (5 + a) + 100 = 0$

$\Rightarrow 40a + 4a^2 - 10a - 2a^2 + 100 = 0$

$2a^2 + 30a + 100 = 0$

$\Rightarrow a^2 + 15a + 50 = 0$

$(a + 10) (a + 5) = 0$

$a = -10$ or $a = -5$

Required $= (-10)^2 + (-5)^2 = 125$

3. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to :

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $a = \alpha - i\beta\ ;\ \alpha \in R\ ;\ \beta \in R$

$4ix + (1 + i)\, y = 0$ and

$$8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$$

$$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix} = 0$$

$$\Rightarrow 4i\,\bar{a} - (1+i)\,8e^{i2\pi/3} = 0$$

$$\Rightarrow 4i\,(\alpha + i\beta) - 8\,(1+i)\left(\frac{-1+i\sqrt{3}}{2}\right) = 0$$

$$\Rightarrow i\alpha - \beta + 1 + \sqrt{3} + i\,(1 - \sqrt{3}) = 0$$

$$\Rightarrow \beta = \sqrt{3} + 1$$

$$\alpha = \sqrt{3} - 1$$

So, $\dfrac{\alpha}{\beta} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

4. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$ then $|\text{adj}(B\,\text{adj}(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $AB = i$

$|\text{adj}\,(B\ \text{adj}\,(2A)| = |B\ \text{adj}\,(2A)|^2$

$= |B|^2\ |\text{adj}\,(2A)|^2$

$= |B|^2\,(|2A|^2)^2 = |B|^2\,(2^6\,|A|^2)^2$

$|A| = \frac{1}{8}$ and $|AB| = 1 \Rightarrow |A|\,|B| = 1$

$\Rightarrow \frac{1}{8}\,|B| = 1$

$\Rightarrow |B| = 8$

required value = 64

5. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots..$ then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$ (B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$ (D) $\frac{7^2}{3^3}$

Official Ans. by NTA (C)

QuestPix Ans. (C)

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots$$

Considering infinite sequence,

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots\ldots$$

$$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots\ldots\ldots$$

$$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \ldots\ldots$$

$$\Rightarrow \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + \ldots\ldots\ldots$$

$$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots\ldots\ldots$$

$$\Rightarrow \frac{6^2 S}{7^2} = \frac{2}{1 - \frac{1}{7}} = \frac{2}{6} \times 7$$

$$\Rightarrow S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$$

6. If $a_1, a_2, a_3 \ldots.$ and $b_1, b_2, b_3 \ldots.$ are A.P. and $a_1 = 2,\ a_{10} = 3,\ a_1 b_1 = 1 = a_{10} b_{10}$ then $a_4 b_4$ is equal to

(A) $\frac{35}{27}$ (B) 1

(C) $\frac{27}{28}$ (D) $\frac{28}{27}$

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. $a_1, a_2, a_3 \ldots.\text{A.P.}\ ;\ a_1 = 2\ ;\ a_{10} = 3; d_1 = \frac{1}{9}$

$b_1, b_2, b_3, \ldots.$ A.P. $;\ b_1 = \frac{1}{2}\ ;\ b_{10} = \frac{1}{3}\ ;\ d_2 = \frac{-1}{54}$

[Using $a_1 b_1 = 1 = a_{10} b_{10}$; d_1 & d_2 are common differences respectively]

$$a_4 . b_4 = (2 + 3d_1)\left(\frac{1}{2} + 3d_2\right)$$

$$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$$

$$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$$

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}\,dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Official Ans. by NTA (B)

QuestPix Ans. (B)

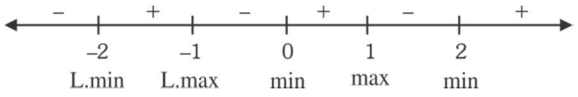
Sol. $m = L \cdot \max$

$N = L \cdot \min$

$$f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$$

$$f'(x) = \frac{(x^4 - 5x^2 + 4)2x}{2 + e^{x^2}} = \frac{2x(x^2 - 1)(x^2 - 4)}{2 + e^{x^2}}$$

$$= \frac{2x(x-1)(x+1)(x-2)(x+2)}{2 + e^{x^2}}$$



so, m = 2 and n = 3

8. Let f be a differentiable function in $\left(0, \frac{\pi}{2}\right)$.

If $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$ then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(A) $6 - 9\sqrt{2}$ (B) $6 - \frac{9}{\sqrt{2}}$

(C) $\frac{9}{2} - 6\sqrt{2}$ (D) $\frac{9}{\sqrt{2}} - 6$

Official Ans. by NTA (B)

QuestPix Ans. (Bonus)

Sol. At right hand vicinity of x = 0 given equation does not satisfy

$$\because \text{LHS} = \int_{1^-}^{1} t^2 f(t)\,dt = 0, \text{RHS} = \lim_{x \to 0^+} (\sin^3 x + \cos x) = 1$$

LHS ≠ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$$\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x - 1$$

Calculation for option

differentiating both sides

$-\cos^2 x\, f(\cos x).(-\sin x) = 3\sin^2 x.\cos x - \sin x$

$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$

$\Rightarrow f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x \tan x$

$$\Rightarrow f'(\cos x)\cos x = \frac{2}{\cos^2 x} - \frac{3}{\sin x . \cos x}$$

When $\cos x = \frac{1}{\sqrt{3}}$; $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$$\therefore f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}} = 6 - \frac{9}{\sqrt{2}}.$$

9. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$, where [.] denotes the greatest integer function is equal to

(A) $1 + 6\log_e\left(\frac{6}{7}\right)$ (B) $1 - 6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$ (D) $1 - 7\log_e\left(\frac{6}{7}\right)$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. $$\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx = -\int_1^0 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$$

$$= (-1)\left[\int_1^{1/2} \frac{1}{7} dx + \int_{1/2}^{1/3} \frac{1}{7^2} dx + \int_{1/3}^{1/4} \frac{1}{7^3} dx + ...\infty\right]$$

$$= \left(\frac{1}{7} + \frac{1}{2.7^2} + \frac{1}{3.7^3} + ...\infty\right) - \left(\frac{1}{7.2} + \frac{1}{7^2.3} + \frac{1}{7^2.4} ...\infty\right)$$

$$= -\ln\left(1 - \frac{1}{7}\right) - 7\left(\frac{1}{7^2.2} + \frac{1}{7^3.3} + \frac{1}{7^4.4} + ...\infty\right)$$

$$\left[\text{as } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} ....\infty\right]$$

$$\left[\text{as } \ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} ....\infty\right)\right]$$

$$= -\ln\frac{6}{7} - 7\left(-\ln\left(1 - \frac{1}{7}\right) - \frac{1}{7}\right)$$

$$= 6\ln\frac{6}{7} + 1$$

10. If the solution curve of the differential equation $\left((\tan^{-1} y) - x\right)dy = (1 + y^2)dx$ passes through the point (1, 0) then the abscissa of the point on the curve whose ordinate is tan (1) is :

(A) 2e (B) $\frac{2}{e}$

(C) 2 (D) $\frac{1}{e}$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$$

$$\text{I.f} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$x e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy$$

$x \cdot e^{\tan^{-1} y} = \left(\tan^{-1} y - 1\right) e^{\tan^{-1} y} + c$

$\because$ (1, 0) lies exit c = 2.

For y = tan1 $\Rightarrow x = \frac{2}{e}$

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$ then a + b + c +d + k is equal to

(A) 575 (B) –575

(C) 576 (D) –576

Official Ans. by NTA (D)

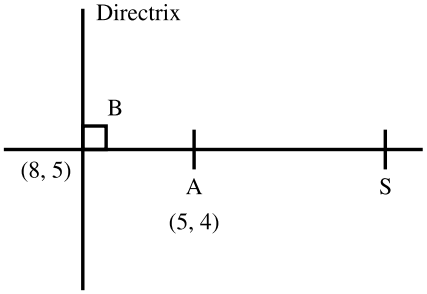
QuestPix Ans. (D)

Sol. Vertex (5,4)

Directrix : 3x + y – 29 = 0

Co-ordinates of B (foot of directrix)

$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$



x = 8, y = 5

S = (2, 3) (focus)

Equation of parabola

PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

a + b + c + d + k = 9 – 6 + 134 – 2 – 711 = – 576

12. The set of values of k for which the circle C : $4x^2+4y^2-12x+8y+k=0$ lies inside the fourth quadrant and the point $\left(1,-\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set (B) $\left(6, \frac{95}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$ (D) $\left(9, \frac{92}{9}\right]$

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. C : $4x^2 + 4y^2 - 12x + 8y + k = 0$

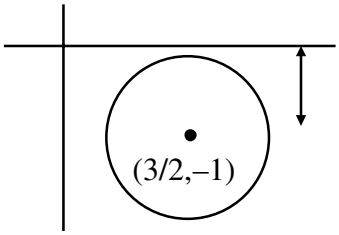
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}}$ $\Rightarrow k \le 13$ ... (1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ... (2)

(ii) C lies in 4th quadrant



r < 1

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k < 9$ .... (3)

Hence (1) $\cap$ (2) $\cap$ (3) $\Rightarrow k \in \left(9, \frac{92}{9}\right]$

13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$

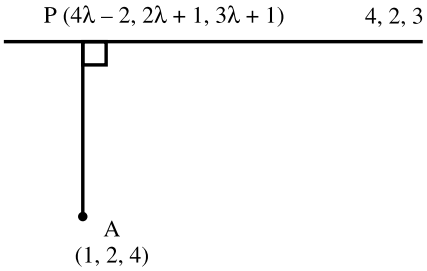
(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol.



$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\overrightarrow{AP} = (4\lambda - 3)\,\hat{i} + (2\lambda - 1)\,\hat{j} + (3\lambda - 5)\,\hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AP} \cdot \vec{b} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$P = (2, 3, 2)$

$3x + 4y + 12z + 23 = 0$

$d = \left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$

$d = \left|\frac{65}{13}\right| = 5$

14. The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

$A = (3, 2, 1)$ $B = (-3, 6, 5)$

$\overrightarrow{n_1} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\overrightarrow{n_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{BA} = 6\hat{i} - 4\hat{j} - 4\hat{k}$

$\text{SHORTEST DISTANCE} = \frac{\left[\overrightarrow{BA}\ \overrightarrow{n_1}\ \overrightarrow{n_2}\right]}{\left|\overrightarrow{n_1} \times \overrightarrow{n_2}\right|}$

$\overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$

$= 10\hat{i} - 8\hat{j} - 4\hat{k}$

$[\overrightarrow{BA}\ \overrightarrow{n_1}\ \overrightarrow{n_2}] = 60 + 32 + 16 = 108$

$|\overrightarrow{n_1} \times \overrightarrow{n_2}| = \sqrt{100+64+16} = \sqrt{180}$

$\text{S.D} = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$

15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

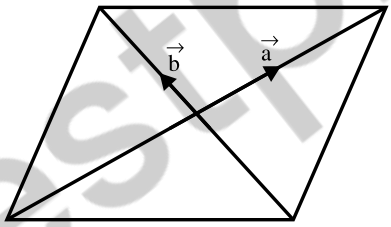
(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol.



$\text{Area} = \frac{1}{2}|\vec{a} \times \vec{b}| = 2\sqrt{2} \Rightarrow |\vec{a} \times \vec{b}| = 4\sqrt{2}$

$|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$\therefore |\vec{a} \times \vec{b}| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$

$\Rightarrow |\vec{b}| = 8$

Now, $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$

$|\vec{c}| = \sqrt{(2\sqrt{2})^2 |\vec{a} \times \vec{b}|^2 + \left(2|\vec{b}|\right)^2} = 16\sqrt{2}$

Now, $\vec{b}.\vec{c} = -2|\vec{b}|^2$

$\Rightarrow 8 \times 16\sqrt{2} \times \cos\alpha = -2.64$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$

16. The mean and variance of the data4, 5, 6, 6, 7, 8, x, y where $x < y$ are 6, and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

(A) 162 (B) 320
(C) 674 (D) 420

Official Ans. by NTA (B)
QuestPix Ans. (B)

Sol. mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

$= \frac{1}{8}(16+25+36+36+49+64+x^2+y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80$

$\therefore x = 4; y = 8$

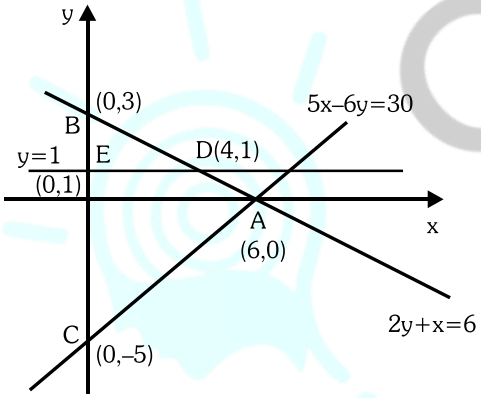
$x^4 + y^2 = 256 + 64 = 320$

17. If a point A(x, y) lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is :

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$
(C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Official Ans. by NTA (B)
QuestPix Ans. (B)

Sol. Required probability $= \frac{ar(ADEC)}{ar(ABC)}$



$= 1 - \frac{ar(BDE)}{ar(ABC)}$

$= 1 - \frac{\frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6} = 1 - \frac{1}{6} = \frac{5}{6}$

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$
(C) $\frac{50}{51}$ (D) $\frac{52}{51}$

Official Ans. by NTA (A)
QuestPix Ans. (A)

Sol. $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1} n$

so, $\sum_{n=1}^{50}(\tan^{-1}(n+1) - \tan^{-1} n)$

$= \tan^{-1} 51 - \tan^{-1} 1$

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right) = \cot(\tan^{-1} 51 + \tan^{-1} 1)$

$= \frac{1}{\tan(\tan^{-1} 51 - \tan^{-1} 1)} = \frac{1+51\times 1}{51-1} = \frac{52}{50} = \frac{26}{25}$

19. $\alpha = \sin 36°$ is a root of which of the following equation

(A) $10x^4 - 10x^2 - 5 = 0$ (B) $16x^4 + 20x^2 - 5 = 0$
(C) $16x^4 - 20x^2 + 5 = 0$ (D) $16x^4 - 10x^2 + 5 = 0$

Official Ans. by NTA (C)
QuestPix Ans. (C)

Sol. $\cos 72° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 1 - 2\sin^2 36° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow (5 - 8\alpha^2)^2 = 5$

$\Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$

20. Which of the following statement is a tautology?

(A) $((\sim q) \wedge p) \wedge q$
(B) $((\sim q) \wedge p) \wedge (p \wedge (\sim p))$
(C) $((\sim q) \wedge p) \vee (p \vee (\sim p))$
(D) $(p \wedge q) \wedge (\sim (p \wedge q))$

Official Ans. by NTA (C)
QuestPix Ans. (C)

Sol. (A) $(\sim q \wedge p) \wedge q = (\sim q \wedge q) \wedge p = f$

(B) $(\sim q \wedge p) \wedge (p \wedge \sim p) = \sim q \wedge (p \wedge \sim p) = f$

(C) $(\sim q \wedge p) \vee (p \vee \sim p) = (\sim q \wedge p) \vee (t) = t$

(D) $(p \wedge q) \wedge (\sim (p \wedge q)) = f$

SECTION-B

21. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define

$f: S \to S$ as $f(n)=\begin{cases} 2n, & \text{if } n=1,2,3,4,5 \\ 2n-11 & \text{if } n=6,7,8,9,10 \end{cases}$.

Let g : S → S be a function such that $fog(n)=\begin{cases} n+1 & \text{,if n is odd} \\ n-1 & \text{,if n is even} \end{cases}$, then

g(10) ((g(1) + g(2) + g(3) + g(4) + g(5)) is equal to:

Official Ans. by NTA (190)

QuestPix Ans. (190)

Sol. $f^{-1}(n)=\begin{cases} \frac{n}{2} & ;\ n=2,4,6,8,10 \\ \frac{n+11}{2} & ;\ n=1,3,5,7,9 \end{cases}$

$f(g(n))=\begin{cases} n+1 & ;\ n\in \text{odd} \\ n-1 & ;\ n\in \text{even} \end{cases}$

$\Rightarrow g(n)=\begin{cases} f^{-1}(n+1) & ;\ n\in \text{odd} \\ f^{-1}(n-1) & ;\ n\in \text{even} \end{cases}$

$\therefore\ g(n)=\begin{cases} \frac{n+1}{2} & ;\ n\in \text{odd} \\ \frac{n+10}{2} & ;\ n\in \text{even} \end{cases}$

g(10) · [g(1) + g(2) + g(3) + g(4) + g(5)]

= 10 · [1 + 6 + 2 + 7 + 3] = 190

22. Let α, β be the roots of the equation $x^2-4\lambda x+5=0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$.

If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to :

Official Ans. by NTA (98)

QuestPix Ans. (98)

Sol. $x^2-4\lambda x+5=0\left\langle{}^{\alpha}_{\beta}\right.$

$x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+\left(7+3\lambda\sqrt{3}\right)=0\left\langle{}^{\alpha}_{\gamma}\right.$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$\beta+\lambda=3\sqrt{2}$ $\qquad$ $\alpha\gamma=7+3\lambda\sqrt{3}$

$\therefore\ \alpha=2\lambda+\sqrt{3}$ $\qquad$ $\alpha\beta=5$

$\beta=2\lambda-\sqrt{3}$ $\qquad$ $4\lambda^2=8 \Rightarrow\ \lambda=\sqrt{2}$

$\therefore\ \therefore\ (\alpha+2\beta+\lambda)^2=\left(4\alpha+3\sqrt{2}\right)^2=\left(7\sqrt{2}\right)^2=98$

23. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is :

Official Ans. by NTA (180)

QuestPix Ans. (180)

Sol. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

a + b + c + d = p, p ∈ {3, 5, 7}

Case-(i)

a + b + c + d = 3; a, b, c, d ∈ {0, 1, 2, 3}

No. of ways = ${}^{3+4-1}C_{4-1}={}^{6}C_3=56$ ..... (1)

Case-(ii)

a + b + c + d = 5; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

No. of ways = ${}^{5+4-1}C_{4-1}={}^{8}C_3=56$ ..... (2)

Case-(iii)

a + b + c + d = 7

No. of ways = total ways when a, b, c, d ∈ {0, 1, 2, 3, 4, 5, 6, 7}– total ways when a, b, c, d ∉ {6, 7}

No of ways $={}^{7+4-1}C_{4-1}=\left(\frac{\lfloor 4}{\lfloor 3}+\frac{\lfloor 4}{\lfloor 2}\right)$

$={}^{10}C_3-16=104$ ..... (3)

Hence total no. of ways = 180

24. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is :

Official Ans. by NTA (57)

QuestPix Ans. (57)

Sol. coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n} \to {}^7C_0$	1
$x^{6n-5} \to 2 \cdot {}^7C_1$	1+14
$x^{5n-10} \to 2^2 \cdot {}^7C_2$	1+14+84
$x^{4n-15} \to 2^3 \cdot {}^7C_3$	1+14+84+280
$x^{3n-20} \to 2^4 \cdot {}^7C_4$	1+4+84+280+560 = 939
$x^{2n-25} \to 2^5 \cdot {}^7C_5$	

$3n-20 \geq 0 \cap 2n-25<0 \cap n \in I$

$\therefore \quad 7 \leq n \leq 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

25. Let [t] denote the greatest integer ≤ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function

$f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at x = 0 is equal to

$\alpha-\dfrac{4}{3}$ is ____

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$\lim\limits_{x\to 0^-} f(x)=\alpha-\dfrac{4}{3} \Rightarrow 0+\dfrac{\alpha^{-1}-2}{-1}=\alpha-\dfrac{4}{3}$

$\Rightarrow 2-\dfrac{1}{\alpha}=\alpha-\dfrac{4}{3}$

$\Rightarrow \alpha+\dfrac{1}{\alpha}=\dfrac{10}{3}$

$\Rightarrow \alpha = 3; \alpha \in I$

26. If $y(x)=(x^{x^x}), x>0$ then $\dfrac{d^2x}{dy^2}+20$ at x = 1 is equal to:

Official Ans. by NTA (16)

QuestPix Ans. (16)

Sol. $y = (x) = (x^x)^x$

$ln\ y(x) = x^2 \cdot ln\ x$

$\dfrac{1}{y(x)} \cdot y'(x) = \dfrac{x^2}{x} + 2x \cdot ln\ x$

$y'(x) = y(x)[x + 2x\ ln\ x]$

$y(1) = 1; y'(1) = 1$

$y''(x) = y'(x)[x + 2x \cdot ln(x)]$

$+ y(x)[1 + 2(1 + ln\ x)]$

$y''(1) = 1[1 + 0] + 1(1 + 2) = 4$

$\dfrac{d^2y}{dx^2} = -\left(\dfrac{dy}{dx}\right)^3 \cdot \dfrac{d^2x}{dy^2}$

$\Rightarrow 4 = -\dfrac{d^2x}{dy^2}$

$\dfrac{d^2x}{dy^2} = -4$

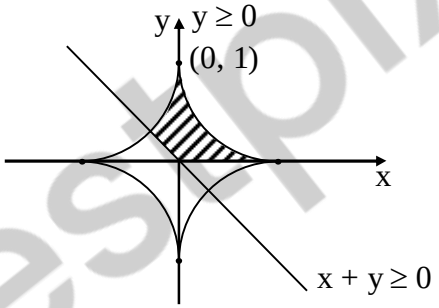
Ans. –4 + 20 = 16

27. If the area of the region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}} \leq 1 x+y \geq 0,\ y \geq 0\right\}$ is A, then $\dfrac{256A}{\pi}$ is

Official Ans. by NTA (36)

QuestPix Ans. (36)

Sol.



$A=\dfrac{3}{2}\int_0^1 (1-x^{2/3})^{3/2}dx$

Let $x = \sin^3\theta$

$A=\dfrac{3}{2}\int_0^{\pi/2}\left(1-\sin^2\theta\right)^{3/2}.3\sin^2\theta\cos\theta\,d\theta$

$=\dfrac{3}{2}\int_0^{\pi/2} 3\sin^2\theta\cos^4\theta\,d\theta$

$=\dfrac{9}{2}\int_0^{\pi/2} \sin^2\theta\cos^4\theta\,d\theta$

$A=\dfrac{9}{2}\times\dfrac{1.3.1}{(2+4)(4)(2)}.\dfrac{\pi}{2}$

$\Rightarrow A=\dfrac{9\pi}{64} \Rightarrow \dfrac{64A}{\pi}=9$

$\Rightarrow \dfrac{256A}{\pi}=36$ **Ans.**

28. Let v be the solution of the differential equation $(1-x^2)dy=\left(xy+(x^3+2)\sqrt{1-x^2}\right)dx, -1<x<1$

and y (0) = 0 if $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\,y(x)dx=k$ then k^{-1} is equal to :

Official Ans. by NTA (320)

QuestPix Ans. (320)

Sol. $(1-x^2)\dfrac{dy}{dx} = xy + (x^3+2)\sqrt{1-x^2}$

$\Rightarrow \dfrac{dy}{dx} + \left(\dfrac{-x}{1-x^2}\right)y = \dfrac{x^3+2}{\sqrt{1-x^2}}$

$IF = e^{\int \frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

$y(x)\cdot\sqrt{1-x^2} = \dfrac{x^4}{4} + 2x + c$

$y(0) = 0 \Rightarrow c = 0$

$\sqrt{1-x^2}\,y(x) = \dfrac{x^4}{4} + 2x$

required value $= \displaystyle\int_{-1/2}^{1/2}\left(\frac{x^4}{4}+2x\right)dx - \frac{1}{4}\cdot 2\int_0^{1/2} x^4 dx$

$= \dfrac{1}{10}\left(x^5\right)_0^{1/2} = \dfrac{1}{320}$

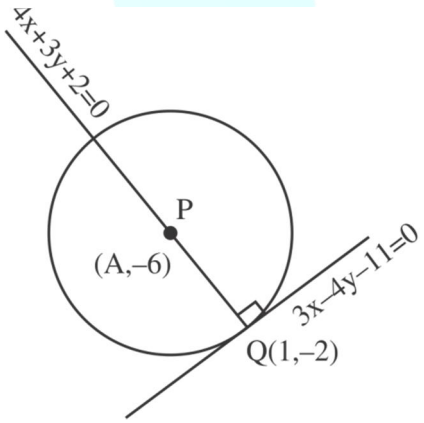
$k^{-1} = 320$

29. Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x+3y-2$ passes through the centre P of the circle C and intersects the line $L_2 : 3x-4y-11=0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x-12y+51=0$ is

Official Ans. by NTA (11)

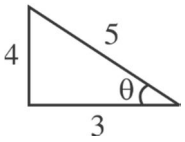
QuestPix Ans. (11)

Sol.



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\dfrac{x}{-25} = \dfrac{y}{50} = \dfrac{1}{-25}$

$\dfrac{x-1}{\cos\theta} = \dfrac{y+2}{\sin\theta} = \pm 5$

$y = -2 + 5\left(-\dfrac{4}{5}\right) = -6$

$x = 1 + 5\left(\dfrac{3}{5}\right) = 4$

Req. distance

$\left|\dfrac{5(4)-12(-6)+51}{13}\right|$

$= \left|\dfrac{20+72+51}{13}\right|$

$= \dfrac{143}{13} = 11$

30. Let $S=\{E, E_2 ....E_8\}$ be a sample space of random experiment such that $P(E_n)=\dfrac{n}{36}$ for every n = 1, 2….8. Then the number of elements in the set $\left\{A \subset S : P(A) \geq \dfrac{4}{5}\right\}$ is _____

Official Ans. by NTA (19)

QuestPix Ans. (19)

Sol. $P(A') < \dfrac{1}{5} = \dfrac{36}{180}$

5 times the sum of missing number should be less than 36.

If 1 digit is missing = 7

If 2 digit is missing = 9

If 3 digit is missing = 2

If 0 digit is missing = 1

Alternate

A is subset of S hence

A can have elements:

type 1 : { }

type 2: $\{E_1\}, \{E_2\}, \ldots\ldots \{E_8\}$

type 3: $\{E_1, E_2\}, \{E_1, E_3\}\ldots\ldots \{E_1, E_8\}$

$\vdots$

$\vdots$

type 6: $\{E_1, E_2, \ldots\ldots E_5\}, \ldots\ldots\{E_4, E_5, E_6, E_7, E_8\}$

type 7: $\{E_1, E_2, \ldots\ldots E_6\}, \ldots\ldots\{E_3, E_4, \ldots\ldots\ldots E_8\}$

type 8: $\{E_1, E_2, \ldots\ldots E_7\}\{E_2, E_3, \ldots\ldots\ldots E_8\}$

type 9: $\{E_1, E_2, \ldots\ldots E_8\}$

As $P(A) \geq \frac{4}{5}$;

Note : Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5, E_6, E_7, E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for Type 5 acceptable elements let's call probability as P_5

$$P_5 = \frac{n_1+n_2+n_3+n_4+n_5}{36} \leq \frac{4}{5}$$

$\Rightarrow n_1 + n_2 + n_3 + n_4 + n_5 \geq 28.8$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8,$ **E_3 or E_4**$\}$

$P_6 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \geq 28.8$

$\Rightarrow$ 9 possible ways

$P_7 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots + n_7 \geq 288$

$\Rightarrow$ 7 possible ways

$P_8 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots + n_8 \geq 28.8$

$\Rightarrow$ 1 possible way

Total = 19

(Held On Monday 27th June, 2022) **TIME : 9:00 AM to 12:00 PM**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1} . After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be : ($g = 10\ ms^{-2}$)

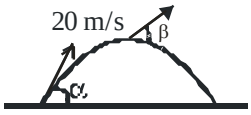
(A) $\tan\alpha + 5\sec\alpha$ (B) $\tan\alpha - 5\sec\alpha$

(C) $2\tan\alpha - 5\sec\alpha$ (D) $2\tan\alpha + 5\sec\alpha$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol.



$v_x = u_x = 20\cos\alpha$

$v_y = 20\sin\alpha - 10\times 10$

$$\tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$$

$= \tan\alpha - 5\sec\alpha$

32. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}\,kmh^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is :

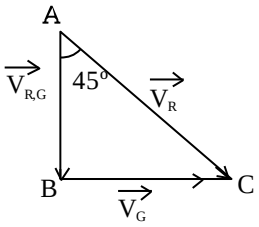
(A) $30\,kmh^{-1}$ (B) $\frac{25}{\sqrt{2}}kmh^{-1}$

(C) $\frac{30}{\sqrt{2}}kmh^{-1}$ (D) $25\,kmh^{-1}$

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol.



$$V = \tan\theta = \frac{V_G}{V_{RG}}$$

$$1 = \frac{V_G}{V_{RG}} \Rightarrow 15\sqrt{2} = V_{RG}$$

33. A sliver wire has mass (0.6 ± 0.006) g, radius (0.5 ± 0.005) mm and length (4 ± 0.04) cm. The maximum percentage error in the measurement of its density will be :

(A) 4% (B) 3%

(C) 6% (D) 7%

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. $M = (0.6 \pm 0.006)g$

$r = (0.5 \pm 0.005)mm$

$l = (4 \pm 0.04)cm$

$$\rho = \frac{m}{V}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

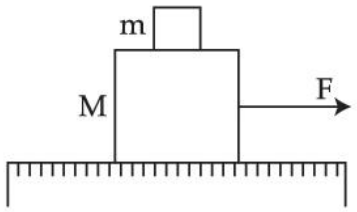
$(Volume\ of\ cylinder = \pi r^2 l)$

$$= \frac{0.006}{0.6} + \frac{2\times 0.005}{0.5} + \frac{0.04}{4}$$

$$100 \times \frac{\Delta\rho}{\rho} = 4\times 10^{-2}\times 100$$

$$\frac{\Delta\rho}{\rho}\times 100 = 4\%$$

34. A system of two blocks of masses m = 2 kg and M = 8 kg is placed on a smooth table as shown in figure. The coefficient of static friction between two blocks is 0.5. The maximum horizontal force F that can be applied to the block of mass M so that the blocks move together will be :



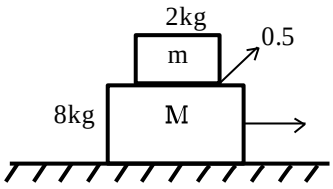
(A) 9.8 N (B) 39.2 N

(C) 49 N (D) 78.4 N

Official Ans. by NTA (C)

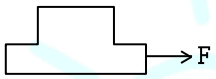
QuestPix Ans. (C)

Sol.



$(a_A)_{max} = 0.5g = 4.9 m/s^2$

For moving together



$F_{max} = m_T a_A$

$= 10 \times 4.9$

$= 49N$

35. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is :

(A) 4 cm towards the 10 kg block

(B) 2 cm away from the 10 kg block

(C) 2 cm towards the 10 kg block

(D) 4 cm away from the 10 kg block

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $\Delta x_G = \dfrac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$

$0 = \dfrac{10 \times 6 + 30(\Delta x_2)}{40}$

$\Delta x_2 = -2cm$

Block of mass 30 kg will to move towards 10 kg.

36. A 72 Ω galvanometer is shunted by a resistance of 8 Ω. The percentage of the total current which passes through the galvanometer is :

(A) 0.1% (B) 10 %

(C) 25% (D) 0.25%

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $S = \dfrac{R_G}{\dfrac{I}{I_g} - 1}$

$8 = \dfrac{72}{\dfrac{I}{I_g} - 1}$

$\dfrac{I}{I_g} - 1 = 9$

$\dfrac{I}{I_g} = 10 \Rightarrow \dfrac{I_g}{I} = \dfrac{1}{10} \qquad \%I = \dfrac{I_g}{I} \times 100 = 10\%$

37. Given below are two statements :

Statement I : The law of gravitation holds good for any pair of bodies in the universe.

Statement II : The weight of any person becomes zero when the person is at the centre of the earth.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both statement I and Statement II are true

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II are false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. Since it is universal law so it hold good for any pair of bodies.

The value of **g** at centre is zero.

So statement I and Statement II are true.

38. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision)

(A) 50.0% (B) 66.6%

(C) 55.5% (D) 33.3%

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. Velocity after collision

$$V_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$$

$$V_2 = \frac{(5m - m)0 + 2m.u_0}{m + 5m} = \frac{u_0}{3}$$

$$\%\Delta KE = \frac{\frac{1}{2}5m\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100$$

$$= \frac{5u_0^2}{9u_0^2} \times 100 = \frac{500}{9} = 55.6\%$$

39. The velocity of a small ball of mass 'm' and density d_1, when dropped in a container filled with glycerine, becomes constant after some time. If the density of glycerine is d_2, then the viscous force acting on the ball, will be :

(A) $mg\left(1 - \frac{d_1}{d_2}\right)$ (B) $mg\left(1 - \frac{d_2}{d_1}\right)$

(C) $mg\left(\frac{d_1}{d_2} - 1\right)$ (D) $mg\left(\frac{d_2}{d_1} - 1\right)$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol.
$$F_V = mg - F_B$$
$$= mg - \left(\frac{m}{d_1} \times d_2\right)g$$
$$= mg\left(1 - \frac{d_2}{d_1}\right)$$

40. The susceptibility of a paramagnetic material is 99. The permeability of the material in Wb/A-m is :

[Permeability of free space $\mu_0 = 4\pi \times 10^{-7} Wb / A - m$]

(A) $4\pi \times 10^{-7}$ (B) $4\pi \times 10^{-4}$

(C) $4\pi \times 10^{-5}$ (D) $4\pi \times 10^{-6}$

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. Susceptibility $\chi = 99$

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$$
$$\mu = \mu_0(1 + \chi)$$
$$= 4\pi \times 10^{-7}[1 + 99]$$
$$= 4\pi \times 10^{-5}$$

41. The current flowing through an ac circuit is given by

$$I = 5\sin(120\pi t)A$$

How long will the current take to reach the peak value starting from zero?

(A) $\frac{1}{60}s$ (B) 60s

(C) $\frac{1}{120}s$ (D) $\frac{1}{240}s$

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. $\omega = 120\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{60}$ sec

time taken to reach peak value $= \frac{T}{4} = \frac{1}{240}s$

42. Mach List-I with List – II :

List – I List – II

	List-I		List-Ii
(a)	Ultraviolet rays	(i)	Study crystal structure
(b)	Microwaves	(ii)	Greenhouse effect
(c)	Infrared waves	(iii)	Sterilizing surgical instrument
(d)	X-rays	(iv)	Radar system

Choose the correct answer from the options given below :

(A) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)

(B) (a) – (iii), (b) – (i), (c) – (ii), (d) – (iv)

(C) (a) – (iv), (b) – (iii), (c) – (ii), (d) – (i)

(D) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. (Fact)

43. An α particle and a carbon 12 atom has same kinetic energy K. The ratio of their de-Broglie wavelength $(\lambda_\alpha : \lambda_{C12})$ is :

(A) $1:\sqrt{3}$ (B) $\sqrt{3}:1$

(C) 3 : 1 (D) $2:\sqrt{3}$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $k = \frac{P^2}{2m} \Rightarrow P\alpha\sqrt{m}$

Now $\lambda = \frac{h}{p}$

So, $\lambda\alpha\frac{1}{p} \Rightarrow \lambda\ \alpha\ \frac{1}{\sqrt{m}}$

$\frac{\lambda_\alpha}{\lambda_{C12}} = \frac{\sqrt{3}}{1}$

44. A force of 10N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be :

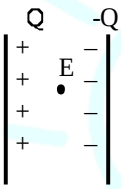
(A) 5 N (B) 10 N

(C) 20 N (D) Zero

Official Ans. by NTA (A)

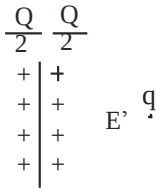
QuestPix Ans. (A)

Sol.



$F = qE = q\left(\frac{Q}{A\in_0}\right) = \frac{qQ}{A\in_0} = 10N$

Now, when one plate is removed.



$E' = \frac{Q}{2A\in_0}$

$F = qE' = \frac{Qq}{2A\in_0} = 5N$

45. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :

(A) 6 s (B) 8 s

(C) 12s (D) 36 s

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. $X = A\sin\omega t \left(t = 3,\ X = \frac{A}{2}\right)$

$\Rightarrow \frac{A}{2} = A\sin 3\omega$

$\Rightarrow \sin 3\omega = \frac{1}{2}$

$\Rightarrow 3\omega = \frac{\pi}{6}$

$\Rightarrow \omega = \frac{\pi}{18} = \frac{2\pi}{T}$

$\Rightarrow T = 36s$

46. An observer moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be :

(A) 20% (B) 10%

(C) 5% (D) 0%

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. $f_0 = \left(\frac{v + v_0}{v}\right) f_s$

$f_0 = \left(\frac{v + \frac{v}{5}}{v}\right) f_s$

$f_0 = \frac{6}{5} f_s$

$\%\ \text{change} = \frac{f_0 - f_s}{f_s} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

47. Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that of refracting angle. Then, the angle of incidence will be :

(A) $\sin^{-1}\left(\sqrt{n}\right)$ (B) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(C) $\sin^{-1}\left(\sqrt{2n}\right)$ (D) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol.

$i = 2r$

$\sin i \times n_1 = \sin r \times n_2$

$\sin i \times 1 = \sin\frac{i}{2} \times \sqrt{2n}$

$\frac{\sin i}{\sin\frac{i}{2}} = \sqrt{2n}$

$\frac{2\sin\frac{i}{2}\cos\frac{i}{2}}{\sin\frac{i}{2}} = \sqrt{2n}$

$\cos\frac{i}{2} = \sqrt{\frac{n}{2}}$

$\frac{i}{2} = \cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

$i = 2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

48. A hydrogen atom in is ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of : (Given, Plank's constant = 6.6 × 10^{-34} Js)

(A) $2.10\times10^{-34}\,Js$ (B) $1.05\times10^{-34}\,Js$

(C) $3.15\times10^{-34}\,Js$ (D) $4.2\times10^{-34}\,Js$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $13.6\left(\frac{1}{1^2}-\frac{1}{n^2}\right)=10.2$

$n = 2$

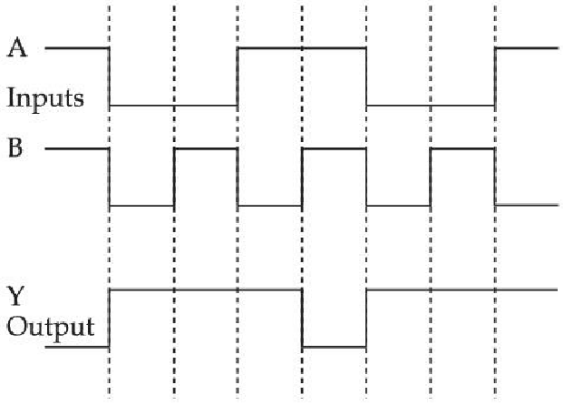
$L_i = \frac{h}{2\pi}\times1$

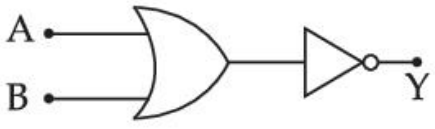
$L_F = \frac{2h}{2\pi}$

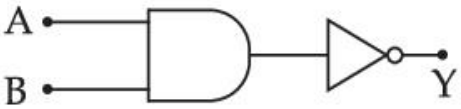
$\Delta L = L_F - L_i = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\times\frac{22}{7}}$

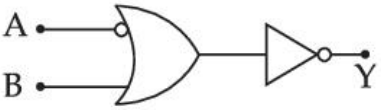
$= 1.05\times10^{-34}\,J-s$

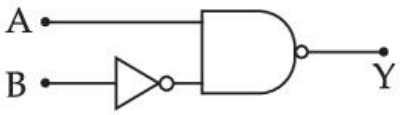
49. Identify the correct Logic Gate for the following output (Y) of two inputs A and B.



(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol.

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1
1	0	1
NAND Gate		

$Y = \overline{A.B}$

50. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 kPa and mass 0.76 g The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be :

(A) $\frac{1}{3}$ (B) $\frac{3}{1}$

(C) $\frac{1}{16}$ (D) $\frac{16}{1}$

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. PV = nRT

$$n = \frac{100\times10^3\times2000\times10^{-6}}{\frac{25}{3}\times300}$$

$n = 80\times10^{-3}$

$n_1 + n_2 = 0.08$

$n_1\times2 + n_2\times32 = 0.76$

$(0.08 - n_2)2 + n_2(32) = 0.76$

$n_2 = 0.02$

$n_1 = 0.06$

$\frac{n_1}{n_2} = \frac{3}{1}$

SECTION-B

51. In a carnot engine, the temperature of reservoir is 527ºC and that of sink is 200 K. If the workdone by the engine when it transfers heat from reservoir to sink is 12000 kJ, the quantity of heat absorbed by the engine from reservoir is _____ $\times 10^6$ J.

Official Ans. by NTA (16)

QuestPix Ans. (16)

Sol. $(T_2)T_{sink} = 200K$

$(T_1)T_{Reservoir} = 527 + 273 = 800K$

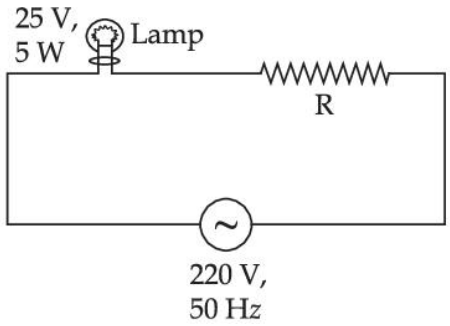
$W = 12000KJ = 12\times10^6 J$

$Q_1 = ?$

$$\eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = 1 - \frac{200}{800} = \frac{12\times10^6}{Q_1}$$

$$\frac{3}{4} = \frac{12\times10^6}{Q_1} = Q_1 = 16\times10^6 J$$

52. A 220 V, 50 Hz AC source is connected to a 25 V, 5 W lamp and an additional resistance R in series (as shown in figure) to run the lamp at its peak brightness, then the value of R (in ohm) will be _____ .



Official Ans. by NTA (975)

QuestPix Ans. (975)

Sol. P = Vi

5 = 25i

$i = \frac{1}{5}$

$V_R = iR$

$(220 - 25) = \frac{1}{5}R$

$R = 195\times5 = 975\Omega$

53. In Young's double slit experiment the two slits are 0.6 mm distance apart. Interference pattern is observed on a screen at a distance 80 cm from the slits. The first dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light will be _____ nm.

Official Ans. by NTA (450)

QuestPix Ans. (450)

Sol. $d = 0.6 \times 10^{-3}$

$D = 80 \times 10^{-2}$

1st Dark fringe $= \frac{D\lambda}{2d} = \frac{d}{2}, \qquad \lambda = \frac{d^2}{D}$

$= 450 \times 10^{-9} m$

54. A beam of monochromatic light is used to excite the electron in Li^{++} from the first orbit to the third orbit. The wavelength of monochromatic light is found to be $x \times 10^{-10} m$. The value of x is _____ . [Given hc = 1242 eV nm]

Official Ans. by NTA (114)

QuestPix Ans. (114)

Sol. $Z = 3$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$n_1 = 1, \quad n_2 = 3,$

$$\frac{1}{\lambda} = R(9)\left(\frac{1}{1} - \frac{1}{9}\right) = 8R$$

$$\lambda = \frac{1}{8R} = 114 \times 10^{-10} m$$

55. A cell, shunted by a 8 Ω resistance, is balanced across a potentiometer wire of length 3m. The balancing length is 2 m when the cell is shunted by 4 Ω resistance. The value of internal resistance of the cell will be _____ Ω.

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. $\frac{V_1}{V_2} = \frac{3}{2} = \frac{E - i_1 r}{E - i_2 r}$

$$= \frac{E - \frac{E}{8+r} \times r}{E - \frac{E}{4+r} \times r}$$

$$\frac{3}{2} = \frac{8(4+r)}{4(8+r)}$$

$24 + 3r = 16 + 4r$

$r = 8\Omega$

56. The current density in a cylindrical wire of radius 4 mm is $4 \times 10^6 Am^{-2}$. The current through the outer portion of the wire between radial distance $\frac{R}{2}$ and R is ____ π A.

Official Ans. by NTA (48)

QuestPix Ans. (48)

Sol. $J = \frac{I}{A}$

$I = JA$

$$= 4 \times 10^6 \times \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]$$

$$= 4 \times 10^6 \times \pi R^2 \times \frac{3}{4}$$

$$= 4 \times 10^6 \times \pi \times \left(4 \times 10^{-3}\right)^2 \times \frac{3}{4} = 48\pi A.$$

57. A capacitor of capacitance 50 pF is charged by 100 V source. It is then connected to another uncharged identical capacitor. Electrostatic energy loss in the process is ____ nJ.

Official Ans. by NTA (125)

QuestPix Ans. (125)

Sol. Energy loss $= \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{1}{2}\frac{50 \times 50 \times 10^{-12} \times 10^{-12}}{(50+50)10^{-12}}(100-0)^2 = 125\, nJ$$

58. The height of a transmitting antenna at the top of a tower is 25 m and that of receiving antenna is, 49 m. The maximum distance between them, for satisfactory communication in LOS (Line-Of-Sight) is $K\sqrt{5}\times10^2 m$. The value of K is _____ .

[Assume radius of Earth is $64\times10^{+5} m$] (Calculate upto nearest integer value)

Official Ans. by NTA (192)

QuestPix Ans. (192)

Sol. $LOS=\sqrt{2Rh_T}+\sqrt{2Rh_R}$

$=\sqrt{2R}\left(\sqrt{h_T}+\sqrt{h_R}\right)$

$=\sqrt{2\times64\times10^5}\left(\sqrt{25}+\sqrt{49}\right)$

$=192\sqrt{5}\times10^2\ m.$

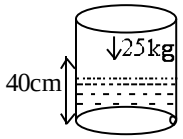
$K=192$

59. The area of cross-section of a large tank is 0.5 m^2. It has a narrow opening near the bottom having area of cross-section 1 cm^2. A load of 25 kg is applied on the water at the top in the tank. Neglecting the speed of water in the tank, the velocity of the water, coming out of the opening at the time when the height of water level in the tank is 40 cm above the bottom, will be _____ cms^{-1}. [Take g = 10 ms^{-2}]

Official Ans. by NTA (300)

QuestPix Ans. (300)

Sol.



$$P_0+\frac{250}{0.5}+\rho g\left(40\times10^{-2}\right)=P_0+\frac{1}{2}\rho v^2$$

$$500+\frac{1000\times10\times40}{100}=\frac{1}{2}\times1000\times v^2$$

V = 3 m/s

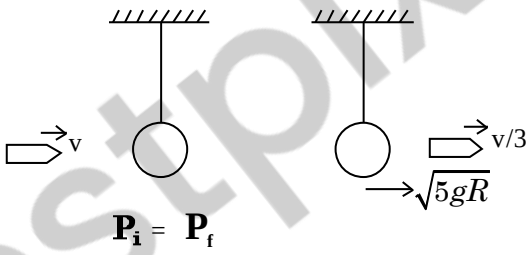
V = 300 cm/s

60. A pendulum of length 2 m consists of a wooden bob of mass 50 g. A bullet of mass 75 g is fired towards the stationary bob with a speed v. The bullet emerges out of the bob with a speed $\frac{v}{3}$ and the bob just completes the vertical circle. The value of v is _____ ms^{-1}. (if g = 10 m/s^2)

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. Considering Only Horizontal direction



$\mathbf{P_i} = \mathbf{P_f}$

$$(75v)+0=50\left(\sqrt{5gR}\right)+75\frac{v}{3}$$

$$75\left(v-\frac{v}{3}\right)=50\sqrt{100}$$

$v=10\,m/s$

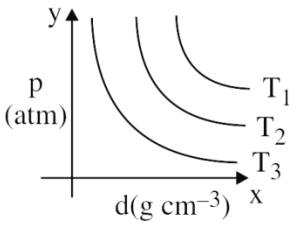
(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTIONS

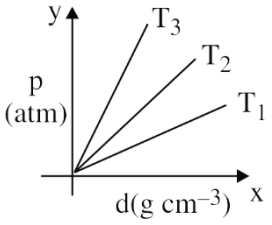
SECTION-A

61. Which amongst the given plots is the correct plot for pressure (p) vs density (d) for an ideal gas ?

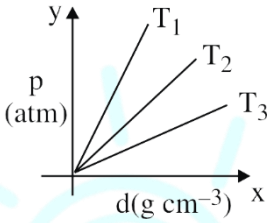
(A) $T_3 > T_2 > T_1$



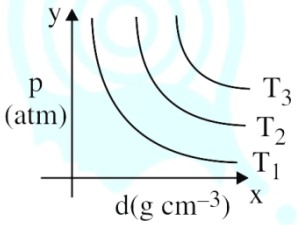
(B) $T_3 > T_2 > T_1$



(C) $T_3 > T_2 > T_1$



(D) $T_3 > T_2 > T_1$

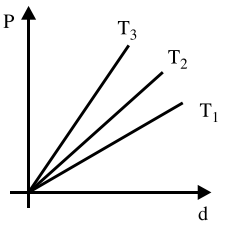


Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. P vs d :

$P = \left(\frac{RT}{M}\right) d$



$T_3 > T_2 > T_1$

62. Identify the **incorrect** statement for PCl_5 from the following.

(A) In this molecule, orbitals of phosphorous are assumed to undergo sp^3d hybridization.

(B) The geometry of PCl_5 is trigonal bipyramidal.

(C) PCl_5 has two axial bonds stronger than three equatorial bonds.

(D) The three equatorial bonds of PCl_5 lie in a plane.

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. In PCl_5, axial bonds are weaker than equatorial.

63. **Statement I : Leaching of gold** with cyanide ion in absence of air / O_2 leads to cyano complex of Au(III).

Statement II : Zinc is oxidized during the displacement reaction carried out for gold extraction.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. Statement-1 : wrong, Au^+ is correct, not Au^{+3}

Statement-2 : correct

64. The correct order of increasing intermolecular hydrogen bond strength is

(A) $HCN < H_2O < NH_3$

(B) $HCN < CH_4 < NH_3$

(C) $CH_4 < HCN < NH_3$

(D) $CH_4 < NH_3 < HCN$

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. Order of H-Bonding

$CH_4 < HCN < NH_3$

NCH . . . NCH

H_2NH . . . NH_3

65. The correct order of increasing ionic radii is

(A) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

(B) $N^{3-} < O^{2-} < F^{-} < Na^{+} < Mg^{2+}$

(C) $F^{-} < Na^{+} < O^{2-} < Mg^{2+} < N^{3-}$

(D) $Na^{+} < F^{-} < Mg^{2+} < O^{2-} < N^{3-}$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. $N^{-3} > O^{-2} > F^{-} > Na^{+} > Mg^{+2}$ (Radii)

(Isoelectronic species)

66. The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is

(A) NH_3 (B) N_2

(C) N_2O (D) Cl_2

Official Ans. by NTA (B)

QuestPix Ans. (B)

Sol. $NH_4Cl + NaNO_2 \rightarrow NH_4NO_2 + NaCl$

$\downarrow$

$N_2 + 2H_2O$

67. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Flourine forms one oxoacid.

Reason R : Flourine has smallest size amongst all halogens and is highly electronegative

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol. Both A and R are correct and R is the correct explanation of A.

68. In 3d series, the metal having the highest M^{2+}/M standard electrode potential is

(A) Cr (B) Fe

(C) Cu (D) Zn

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $Cr^{+2}/Cr \rightarrow -0.90$ V

$Fe^{+2}/Fe \rightarrow -0.44$ V

$Cu^{+2}/Cu \rightarrow +0.34$ V

$Zn^{+2}/Zn \rightarrow -0.76$ V

So Ans. Cu^{+2}/Cu

69. The 'f' orbitals are half and completely filled, respectively in lanthanide ions

(Given: Atomic no. Eu, 63; Sm, 62; Tm, 69; Tb, 65; Yb, 70; Dy, 66]

(A) Eu^{2+} and Tm^{2+} (B) Sm^{2+} and Tm^{3+}

(C) Tb^{4+} and Yb^{2+} (D) Dy^{3+} and Yb^{3+}

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $Tb \rightarrow 4f^9 6s^2$

$Tb^{+4} \rightarrow 4f^7$

$Yb \rightarrow 4f^{14} 6s^2$

$Yb^{+2} \rightarrow 4f^{14}$

70. Arrange the following coordination compounds in the increasing order of magnetic moments. (Atomic numbers: Mn = 25; Fe = 26)

(A) $[FeF_6]^{3-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[MnCl_6]^{3-}$ (high spin)

(D) $[Mn(CN)_6]^{3-}$

(A) A < B < D < C (B) B < D < C < A

(C) A < C < D < B (D) B < D < A < C

Official Ans. by NTA (B)

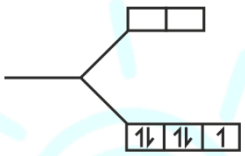
QuestPix Ans. (B)

Sol. (A) $[FeF_6]^{3-}$

$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 5

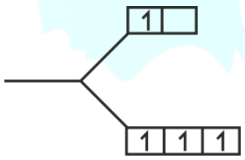
(B) $[Fe(CN)_6]^{3-}$



$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 1

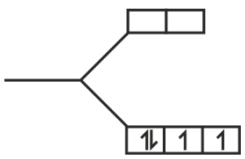
(C) $[MnCl_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 4

(D) $[Mn(CN)_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 2

$\mu \Rightarrow A > C > D > B$

71. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrate gives A and B while its reaction with HCl produces B and C. A, B and C are, respectively

(A) $HOCl, HNO_3, Cl_2$

(B) $Cl_2, HNO_3, HOCl$

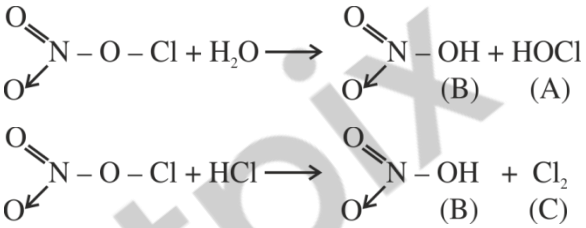
(C) $HClO_2, HNO_2, HOCl$

(D) $HOCl, HNO_2, Cl_2O$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol.

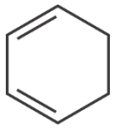


72. Which of the following is most stable?

(A) 

(B) 

(C) 

(D) 

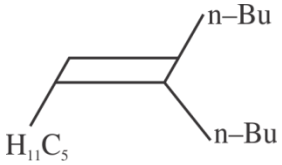
Official Ans. by NTA (A)

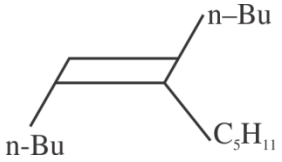
QuestPix Ans. (A)

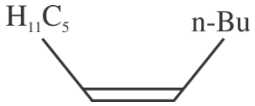
Sol.  is most stable as it is aromatic.

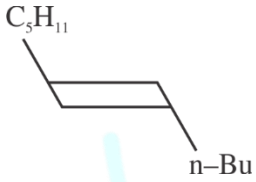
73. What will be the major product of following sequence of reactions?

$$n-Bu-\equiv \xrightarrow[(ii)\ \text{Lindlar cat, } H_2]{(i)\ n-BuLi,\ n-C_5H_{11}Cl}$$

(A) 

(B) 

(C) 

(D) 

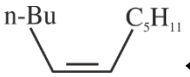
Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. $n-Bu-C\equiv CH \xrightarrow[\text{(acid base reaction)}]{nBuLi}$

$n-Bu-C\equiv C^{-}Li^{+}$

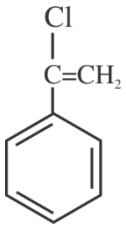
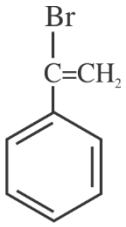
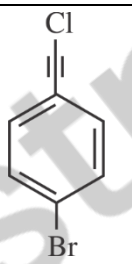
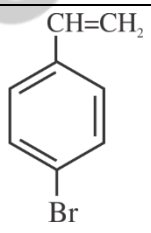
$n-C_5H_{11}Cl \downarrow$ (SN reaction)



$\xleftarrow[\text{Lindlar's Catalyst}]{H_2} n-Bu-C\equiv C-C_5H_{11}$

74. Product 'A' of following sequence of reactions is

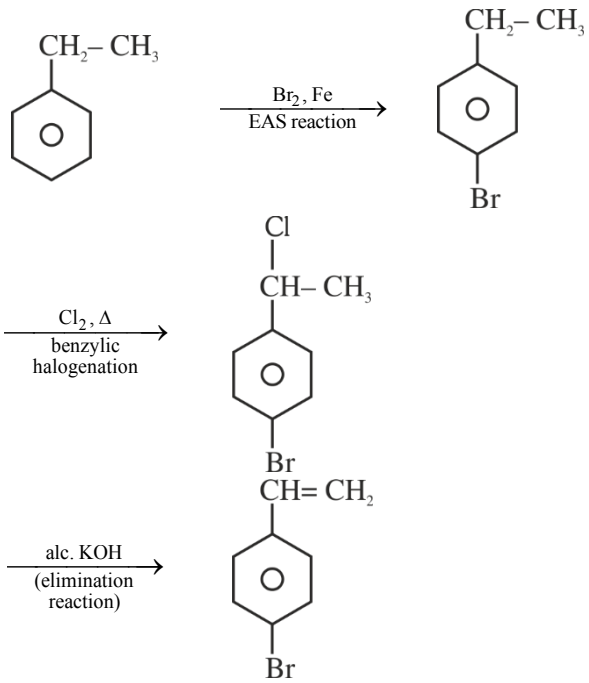
$$\text{Ethylbenzene} \xrightarrow[(c)\ \text{alc. KOH}]{(a)\ Br_2.Fe,\ (b)\ Cl_2, \Delta} \text{'A'(Major product)}$$

(A)	
(B)	
(C)	
(D)	

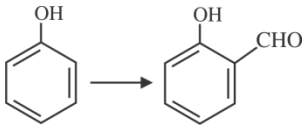
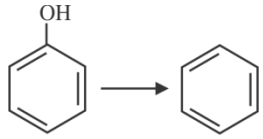
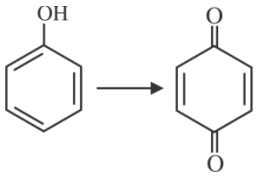
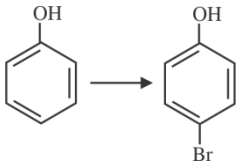
Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol.



75. Match List I with List II

List I	List II
A. 	I. Br_2 in CS_2
B. 	II. $Na_2Cr_2O_7/H_2SO_4$
C. 	III. Zn
D. 	IV. $CHCl_3$/NaOH

Choose the correct answer from the options given below:

(A) A-IV, B-III, C-II, D-I

(B) A-IV, B-III, C-I, D-II

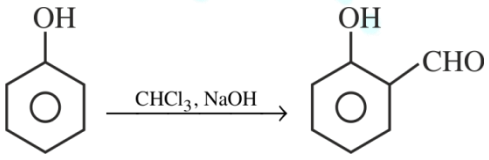
(C) A-II, B-III, C-I, D-IV

(D) A-IV, B-II, C-III, D-I

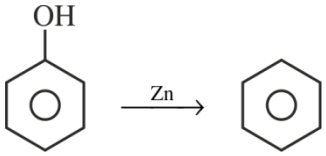
Official Ans. by NTA (A)

QuestPix Ans. (A)

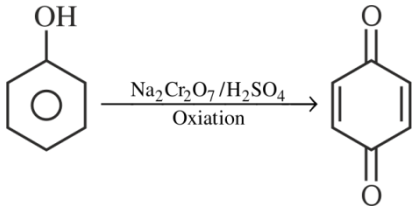
Sol. (A)



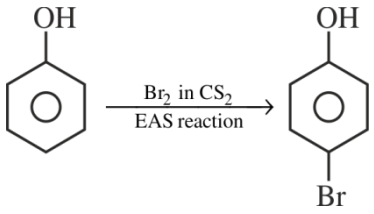
(B)



(C)



(D)



76. Decarboxylation of all six possible forms of diaminobenzoic acids $C_6H_3(NH_2)_2COOH$ yields three products A, B and C. Three acids give a product 'A', two acids gives a product 'B' and one acid give a product 'C'. The melting point of product 'C' is

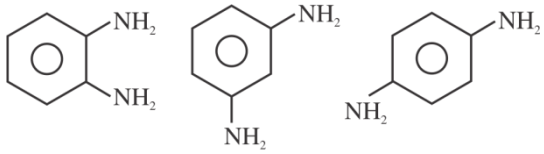
(A) 63ºC (B) 90ºC

(C) 104ºC (D) 142ºC

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol.



B A C

M.P. 142ºC

77. Which is true about Buna-N?

(A) It is a linear polymer of 1, 3-butadiene.

(B) It is obtained by copolymerization of 1, 3-butadiene and styrene.

(C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile.

(D) The suffix N in Buna-N stands for its natural occurrence

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol. It is copolymerization of 1, 3-butadiene and acrylonitrile.

78. Given below are two statements.

Statments I: Maltose has two α-D-glucose units linked at C_1 and C_4 and is a reducing sugar.

Statement II: Maltose has two monosaccharides: α-D-glucose and β-D-glucose linked at C_1 and C_6 and it is a non-reducing sugar.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

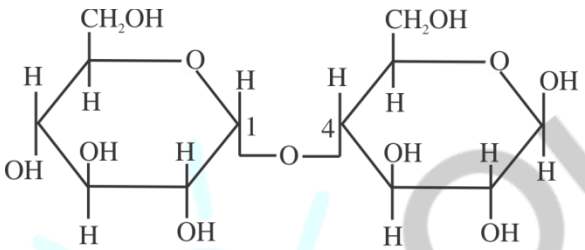
(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol.



Maltose

79. Match List I with List Ii

List I	List II
A. Antipyretic	I. Reduces pain
B. Analgesic	II. Reduces stress
C. Tranquilizer	III. Reduces fever
D. Antacid	IV. Reduces acidity (Stomach)

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-III, C-II, D-IV

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol.

A. Antipyretic	Reduces fever
B. Analgesic	Reduces pain
C. Tranquilizer	Reduces stress
D. Antacid	Reduces acidity (Stomach)

80. Match List I with List II

List I (Anion)	List II (Gas evolved on reaction with dil. H_2SO_4)
A. CO_3^{2-}	I. Colourless gas which turns lead acetate paper black
B. S^{2-}	II. Colourless gas which turns acidified potassium dichromate solution green.
C. SO_3^{2-}	III. Brown fumes which turns acidified KI solution containing starch blue.
D. NO_2^-	IV. Colourless gas evolved with brisk effervescence, which turns lime water milky.

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-I, C-IV, D-III

(C) A-IV, B-I, C-III, D-II

(D) A-IV, B-I, C-II, D-III

Official Ans. by NTA (D)

QuestPix Ans. (D)

Sol. CO_3^{2-} will give $CO_2(g)$ which will turns lime water milky.

S^{2-} will give H_2S (g), will turns lead acetate paper black

SO_3^{2-} will give SO_2 (g), which will turns acidified potassium dichromate solution green.

NO_2^- will give brown $NO_2(g)$ will turn KI solution blue.

SECTION-B

81. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12 respectively. The data agrees with how many formulae of the following?

(A) CH_3COOH (B) HCHO

(C) CH_3OOCH_3 (D) CH_3CHO

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\%H = \frac{7.5}{116} \times 100 = 6.5$

$\%O = \frac{60}{116} \times 100 = 51.7$

$\%C = \frac{48.5}{116} \times 100 = 41.8$

Relative atomicities = $H \Rightarrow 6.5$

$O \Rightarrow \frac{51.7}{16} = 3.25$

$C \Rightarrow \frac{41.8}{12} = 3.5$

Emperically formula is approx.. CH_2O

(A) $C_2H_4O_2$ (B) CH_2O relate to this formula.

82. Consider the following set of quantum numbers

	n	l	m_l
A.	3	3	-3
B.	3	2	-2
C.	2	1	+1
D.	2	2	+2

The number of correct sets of quantum numbers is _____

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Quantum no. of set (B) and (C) can be correct.

(A) and (D) are wrong as $n = \ell$ is not possible.

83. BeO reacts with HF in presence of ammonia to give [A] which on thermal decomposition produces [B] and ammonium fluoride. Oxidation state of Be in [A] is_____

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

$$BeO + HF + NH_3 \rightarrow \underset{[A]}{(NH_4)_2[BeF_4]} \xrightarrow{\Delta} \underset{[B]}{NH_4F} + BeF_2$$

Oxidation state of Be in A is (+2)

84. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is ____ J. [nearest integer] (Given: R = 8.3 J $K^{-1}mol^{-1}$ and log 2 = 0.3010)

Official Ans. by NTA (8630)

QuestPix Ans. (8630)

Sol. n = 5 mol

T = 300 K

V_1 = 10 L

V_2 = 20 L

$w = -nRT\,\ell n\frac{V_2}{V_1}$

$= -5 \times 8.3 \times 300 \times \ell n\frac{20}{10}$

$= -8630.38$ J

85. A solution containing 2.5×10^{-3} kg of a solute dissolved in 75×10^{-3} kg of water boils at 373.535 K. The molar mass of the solute is _____ g mol^{-1}. [nearest integer] (Given: K_b (H_2O) = 0.52 K Kg mol^{-1} , boiling point of water = 373.15K)

Official Ans. by NTA (45)

QuestPix Ans. (45)

Sol. $w = 2.5\ g$ $K_b = 0.52$

$w_{solvent} = 75\ g$ M = Mol. Wt. of solute

$T'_B = 373.535\ K$

$T_B^o = 373.15\ K$

$\Delta T_B = 0.385 = K_b$ molality

$$0.385 = 0.52 \times \left(\frac{2.5}{M} \times \frac{1000}{75}\right)$$

$M = 45\ g\ mol^{-1}$

86. pH value of 0.001 M NaOH solution is_____.

Official Ans. by NTA (11)

QuestPix Ans. (11)

Sol. 0.001 M NaOH

$[OH^-] = 10^{-3}$

pOH = 3

pH = 11

87. For the reaction taking place in the cell:

$Pt(s) \mid H_2(g) \mid H^+(aq) \parallel Ag^+(aq) \mid Ag(s)$

$E^o_{Cell} = +0.5332$ V.

The value of $\Delta_f G^0$ is _____ kJ mol^{-1}. (in nearest integer)

Official Ans. by NTA (51)

QuestPix Ans. (51 or 103)

Sol. $\frac{1}{2}H_2 + Ag^+ \rightarrow H^+ + Ag$

$\Delta G° = -nE° F$

$= -1 \times 0.5332 \times 96500\ J$

$= -51.35\ kJ$

$(n = 2 \text{ for } H_2 + 2Ag^+ \rightarrow 2H^+ + 2Ag)$

88. It has been found that for a chemical reaction with rise in temperature by 9K the rate constant gets doubled. Assuming a reaction to be occurring at 300 K, the value of activation energy is found to be _____ kJ mol^{-1}. [nearest integer]

(Given ln 10 = 2.3, R = 8.3 $JK^{-1}mol^{-1}$, log2 = 0.30)

Official Ans. by NTA (59)

QuestPix Ans. (59)

Sol. $$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{300} - \frac{1}{309}\right)$$

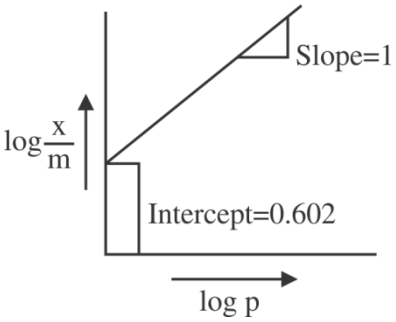
$$0.3 = \frac{E_a}{2.303 \times 8.3}\left(\frac{9}{300 \times 309}\right)$$

$$E_a = \frac{0.3 \times 2.303 \times 8.3 \times 300 \times 309}{9}$$

$= 59065.04\ J$

$E_a = 59.06\ kJ$

89.



If the initial pressure of a gas is 0.03 atm, the mass of the gas adsorbed per gram of the adsorbent is _____ $\times 10^{-2}$g.

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

From graph

$$\text{Slope} = \frac{1}{n} = 1 \Rightarrow n = 1$$

Intercept = log k = 0.602

k = 4

$$\frac{x}{m} = 4 \times (0.03)^{\frac{1}{1}}$$

$$\frac{x}{m} = 12 \times 10^{-2}$$

90. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is _____. [in nearest integer]

(Given: Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1})

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. wt. of organic compound = 0.25 g

$$\text{mass of Cl} = \frac{35.5}{143.5} \times 0.4g$$

mass % of Cl in the organic compound

$$= \frac{35.5 \times 0.4}{143.5 \times 0.25} \times 100$$

$= 39.58\%$