

(Held On Wednesday 01st February, 2023)
TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS	TEST PAPER WITH SOLUTION
SECTION-A 1. The sum $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$ is equal to : (1) $\frac{11e}{2} + \frac{7}{2e}$ (2) $\frac{13e}{4} + \frac{5}{4e} - 4$ (3) $\frac{11e}{2} + \frac{7}{2e} - 4$ (4) $\frac{13e}{4} + \frac{5}{4e}$ Official Ans. by NTA (2) QuestPix Ans. (2) Sol. $\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!}$ $\frac{1}{2} \sum_{n=1}^{\infty} \frac{2n(2n-1) + 8n + 8}{(2n)!}$ $\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{(2n-2)!} + 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} + 4 \sum_{n=1}^{\infty} \frac{1}{(2n)!}$ $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$ $e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots$ $\left(e + \frac{1}{e}\right) = 2 \left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots\right)$ $e - \frac{1}{e} = \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots\right)$ Now $\frac{1}{2} \left(\sum_{n=1}^{\infty} \frac{1}{(2n-2)!} \right) + 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} + 4 \sum_{n=1}^{\infty} \frac{1}{(2n)!}$ $= \frac{1}{2} \left[\frac{e + \frac{1}{e}}{2} \right] + 2 \left[\frac{e - \frac{1}{e}}{2} \right] + 4 \left[\frac{e + \frac{1}{e} - 2}{2} \right]$ $= \frac{\left(e + \frac{1}{e}\right)}{4} + e - \frac{1}{e} + 2e + \frac{2}{e} - 4$ $= \frac{13}{4}e + \frac{5}{4e} - 4$	2. Let $S = \left\{ x \in \mathbb{R} : 0 < x < 1 \text{ and } 2 \tan^{-1} \left(\frac{1-x}{1+x} \right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right\}$. If $n(S)$ denotes the number of elements in S then : (1) $n(S) = 2$ and only one element in S is less than $\frac{1}{2}$. (2) $n(S) = 1$ and the element in S is more than $\frac{1}{2}$. (3) $n(S) = 1$ and the element in S is less than $\frac{1}{2}$. (4) $n(S) = 0$ Official Ans. by NTA (3) QuestPix Ans. (3) Sol. $0 < x < 1$ $2 \tan^{-1} \left(\frac{1-x}{1+x} \right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$ $\tan^{-1} x = \theta \in \left(0, \frac{\pi}{4} \right) \therefore x = \tan \theta$ $2 \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right) = \cos^{-1} (\cos 2\theta)$ $2 \left(\frac{\pi}{4} - \theta \right) = 2\theta \therefore 4\theta = \frac{\pi}{2} \therefore \theta = \frac{\pi}{8}$ $x = \tan \frac{\pi}{8} \therefore x = \sqrt{2} - 1 \approx 0.414$ 3. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r}$ is equal to : (1) $\frac{11}{7}\sqrt{2}$ (2) $\frac{11}{7}$ (3) $\frac{11}{5}\sqrt{2}$ (4) $\frac{\sqrt{914}}{7}$ Official Ans. by NTA (1) QuestPix Ans. (1)

Sol. $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$
 $\vec{b} = \hat{i} + \hat{k}$
 $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$
 $\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$
 $\therefore \vec{r} = \vec{c} + \lambda \vec{a}$
 $\vec{r} \cdot \vec{b} = 0 \Rightarrow \vec{c} \cdot \vec{b} + \lambda \vec{b} \cdot \vec{a} = 0$
 $-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$
 $\therefore \vec{r} = \vec{c} + \frac{2\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$
 $|\vec{r}| = \frac{11\sqrt{2}}{7}$

4. If $A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

- (1) $A^{30} - A^{25} = 2I$
- (2) $A^{30} + A^{25} + A = I$
- (3) $A^{30} + A^{25} - A = I$
- (4) $A^{30} = A^{25}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $A = \frac{1}{2} \begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$
 $A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$
If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$
 $A^2 = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$
 $= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$
 $A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$
 $A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
 $A^{25} = \begin{bmatrix} \cos 25\alpha & \sin 25\alpha \\ -\sin 25\alpha & \cos 25\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
 $A^{25} = A$
 $A^{25} - A = 0$

5. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then
- (1) the number of favourable cases of the event $(A \cup B) \cap C$ is 6
 - (2) A and B are mutually exclusive
 - (3) The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively
 - (4) B and C are independent

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. A : no. on 1st die < no. on 2nd die
A : no. on 1st die = even & no. of 2nd die = odd
C : no. on 1st die = odd & no. on 2nd die = even
 $n(A) = 5 + 4 + 3 + 2 + 1 = 15$
 $n(B) = 9$
 $n(C) = 9$
 $n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$
 $= (3 + 2 + 1) + 0 = 6.$

6. Which of the following statements is a tautology ?
- (1) $p \rightarrow (p \wedge (p \rightarrow q))$
 - (2) $(p \wedge q) \rightarrow (\sim(p) \rightarrow q)$
 - (3) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
 - (4) $p \vee (p \wedge q)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (i) $p \rightarrow (p \wedge (p \rightarrow q))$
 $(\sim p) \vee (p \wedge (\sim p \vee q))$
 $(\sim p) \vee (p \wedge q)$
 $\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
 $= \sim p \vee q$
(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$
 $\sim(p \wedge q) \vee (p \vee q) = t$
 $\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
 $\sim(p \wedge (\sim p \vee q)) \vee \sim q = \sim(p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tautology
(iv) $p \vee (p \wedge q) = p$
Not tautology.



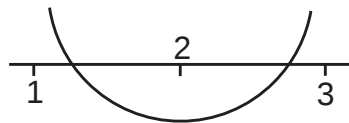
7. The number of integral values of k , for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval $(1, 2)$ and its other root lies in the interval $(2, 3)$, is :

- (1) 2
(2) 0
(3) 1
(4) 3

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $2x^2 - 8x + k = 0$



$$\begin{aligned} f(1) \cdot f(2) &< 0 & \& f(2) \cdot f(3) < 0 \\ (k-6)(k-8) &< 0 & \& (k-8)(k-6) < 0 \\ k &\in (6, 8) & k &\in (6, 8) \end{aligned}$$

integral value of $k = 7$

8. Let $f : \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}$ be a function such that

$$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x. \text{ Then } f(2) \text{ is equal to :}$$

- (1) $\frac{9}{2}$
(2) $\frac{9}{4}$
(3) $\frac{7}{4}$
(4) $\frac{7}{3}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

$$x = 2 \Rightarrow f(2) + f(-1) = 3 \quad (1)$$

$$x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0 \quad (2)$$

$$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2} \quad (3)$$

$$(1) + (3) - (2) \Rightarrow 2f(2) = \frac{9}{2}$$

$$\therefore f(2) = \frac{9}{4}$$

9. Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to :

- (1) $\frac{250}{83}$
(2) $\frac{15}{53}$
(3) $\frac{25}{83}$
(4) $\frac{250}{82}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $P \equiv P_1 + \lambda P_2 = 0$

$$(2 + \lambda)x + (3 + 2\lambda)y + (3\lambda - 1)z - 2 - 6\lambda = 0$$

$$\text{Plane } P \text{ is perpendicular to } P_3 \therefore \vec{n} \cdot \vec{n}_3 = 0$$

$$2(\lambda + 2) + (2\lambda + 3) - (3\lambda - 1) = 0$$

$$\lambda = -8$$

$$P \equiv -6x - 13y - 25z + 46 = 0$$

$$6x + 13y + 25z - 46 = 0$$

Dist from $(-7, 1, 1)$

$$d = \frac{|-42 + 13 + 25 - 46|}{\sqrt{36 + 169 + 625}} = \frac{50}{\sqrt{830}}$$

$$d^2 = \frac{50 \times 50}{830} = \frac{250}{83}$$

10. Let a, b be two real numbers such that $ab < 0$. If

the complex number $\frac{1+ai}{b+i}$ is of unit modulus and

$a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible

value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer

function, is :

- (1) $-\frac{1}{2}$
(2) -1
(3) 1
(4) $\frac{1}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (Bonus)

Sol. $ab < 0 \left| \frac{1+ai}{b+i} \right| = 1$

$$|1+ai| = |b+i|$$

$$a^2 + 1 = b^2 + 1 \Rightarrow a = \pm b \Rightarrow b = -a \quad \text{as } ab < 0$$

$$(a, b) \text{ lies on } |z-1| = |2z|$$

$$|a+ib-1| = 2|a+ib|$$

$$(a-1)^2 + b^2 = 4(a^2 + b^2)$$

$$(a-1)^2 = a^2 = 4(2a^2)$$

$$1-2a = 6a^2 \Rightarrow 6a^2 + 2a - 1 = 0$$

$$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-1 \pm \sqrt{7}}{6}$$

$$a = \frac{\sqrt{7}-1}{6} \text{ \& } b = \frac{1-\sqrt{7}}{6}$$

$$[a] = 0$$

$$\therefore \frac{1+[a]}{4b} = \frac{6}{4(1-\sqrt{7})} = -\left(\frac{1+\sqrt{7}}{4}\right)$$

$$\text{or } [a] = 0$$

$$\text{Similarly it is not matching with } a = \frac{-1-\sqrt{7}}{6}$$

No answer is matching.

- 11.** The sum of the absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to :

(1) 10

(2) 12

(3) 13

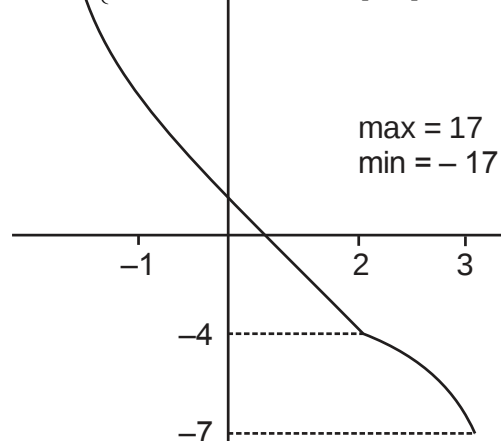
(4) 24

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $f(x) = |x^2 - 5x + 6| - 3x + 2$

$$f(x) = \begin{cases} x^2 - 8x + 8 & ; x \in [-1, 2] \\ -x^2 + 2x - 1 & ; x \in [2, 3] \end{cases}$$



- 12.** Let $P(S)$ denote the power set of $S = \{1, 2, 3, \dots, 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \emptyset$ and AR_2B if $A \cup B^c = B \cup A^c$, $\forall A, B \in P(S)$. Then :

(1) both R_1 and R_2 are equivalence relations

(2) only R_1 is an equivalence relation

(3) only R_2 is an equivalence relation

(4) both R_1 and R_2 are not equivalence relations

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $S = \{1, 2, 3, \dots, 10\}$

$P(S)$ = power set of S

$$AR, B \Rightarrow (A \cap \bar{B}) \cup (\bar{A} \cap B) = \emptyset$$

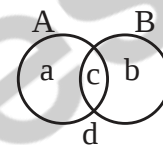
R_1 is reflexive, symmetric

For transitive

$$(A \cap \bar{B}) \cup (\bar{A} \cap B) = \emptyset; \{a\} = \emptyset = \{b\} \quad A = B$$

$$(B \cap \bar{C}) \cup (\bar{B} \cap C) = \emptyset \therefore B = C$$

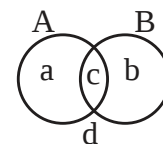
$$\therefore A = C \text{ equivalence.}$$



$$R_2 \equiv A \cup \bar{B} = \bar{A} \cup B$$

$R_2 \rightarrow$ Reflexive, symmetric

for transitive



$$A \cup \bar{B} = \bar{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$$

$$\{a\} = \{b\} \therefore A = B$$

$$B \cup \bar{C} = \bar{B} \cup C \Rightarrow B = C$$

$$\therefore A = C \quad \therefore A \cup \bar{C} = \bar{A} \cup C \therefore \text{Equivalence}$$

- 13.** The area of the region given by $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is :

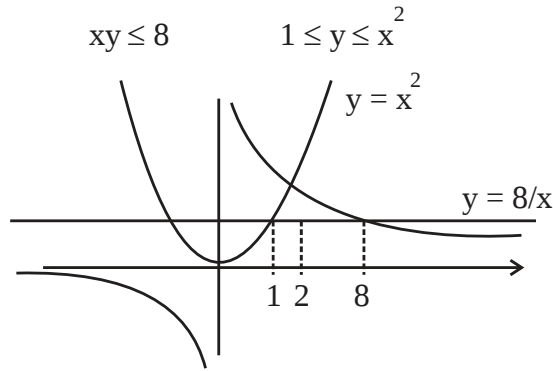
(1) $8 \log_e 2 - \frac{13}{3}$ (2) $16 \log_e 2 - \frac{14}{3}$

(3) $8 \log_e 2 + \frac{7}{6}$ (4) $16 \log_e 2 + \frac{7}{3}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$$\text{Area} = \int_1^2 (x^2 - 1) dx + \int_2^8 \left(\frac{8}{x} - 1 \right) dx$$

$$= \left(\frac{x^3}{3} \right)_1^2 + 8(\ln x)_2^8 - (x)_1^8$$

$$= \frac{7}{3} + 8(2\ln 2) - 7$$

$$= 16 \ln 2 - \frac{14}{3}$$

14. Let $\alpha x = \exp(x^\beta y^\gamma)$ be the solution of the differential equation $2x^2 y dy - (1 - xy^2) dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :

- (1) 1
(2) -1
(3) 0
(4) 3

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

$$\alpha x = e^{x^\beta \cdot y^\gamma}$$

$$2x^2 y \frac{dy}{dx} = 1 - x \cdot y^2 \quad y^2 = t$$

$$x^2 \frac{dt}{dx} = 1 - xt$$

$$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

$$t(x) = \int \frac{1}{x^2} \cdot x dx$$

$$y^2 \cdot x = \ln x + C$$

$$\therefore 2 \cdot \ln 2 = \ln 2 + C$$

$$\therefore C = \ln 2$$

$$\text{Hence, } xy^2 = \ln 2x$$

$$\therefore 2x = e^{x \cdot y^2}$$

$$\text{Hence } \alpha = 2, \beta = 1, \gamma = 2$$

15. The value of the integral $\int_{-\pi/4}^{\pi/4} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$ is :

(1) $\frac{\pi^2}{6}$

(2) $\frac{\pi^2}{12\sqrt{3}}$

(3) $\frac{\pi^2}{3\sqrt{3}}$

(4) $\frac{\pi^2}{6\sqrt{3}}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $I = \int_{-\pi/4}^{\pi/4} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx \quad (1)$

$$x \rightarrow -x$$

$$I = \int_{-\pi/4}^{\pi/4} \frac{-x + \frac{\pi}{4}}{2 - \cos 2x} dx \quad (2)$$

$$(1) + (2)$$

$$2I = \int_{-\pi/4}^{\pi/4} \frac{\frac{\pi}{2}}{2 - \cos 2x} dx$$

$$I = \frac{\pi}{4} \cdot 2 \int_0^{\pi/4} \frac{dx}{2 - \cos 2x}$$

$$I = \frac{\pi}{4} \cdot 2 \int_0^{\pi/4} \frac{(1 + \tan^2 x) dx}{2(1 + \tan^2 x) - (1 - \tan^2 x)}$$

$$I = \frac{\pi}{4} \int_0^1 \frac{dt}{3t^2 + 1}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{3}} \tan^{-1} \sqrt{3}$$

$$I = \frac{\pi^2}{6\sqrt{3}}$$

16. Let $9 = x_1 < x_2 < \dots < x_7$ be in an A.P. with common difference d . If the standard deviation of $x_1, x_2, \dots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to :

(1) $18\left(1 + \frac{1}{\sqrt{3}}\right)$

(2) 34

(3) $2\left(9 + \frac{8}{\sqrt{7}}\right)$

(4) 25

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $9 = x_1 < x_2 < \dots < x_7$

$$9, 9 + d, 9 + 2d, \dots, 9 + 6d$$

$$0, d, 2d, \dots, 6d$$

$$\bar{x}_{\text{new}} = \frac{21d}{7} = 3d$$

$$16 = \frac{1}{7}(0^2 + 1^2 + \dots + 6^2)d^2 - 9d^2$$

$$= \frac{1}{7} \left(\frac{6 \times 7 \times 13}{6} \right) d^2 - 9d^2$$

$$16 = 4d^2$$

$$d^2 = 4$$

$$d = 2$$

$$\bar{x} + x_6 = 6 + 9 + 10 + 9$$

17. For the system of linear equations $ax + y + z = 1$, $x + ay + z = 1$, $x + y + az = \beta$, which one of the following statements is NOT correct ?

(1) It has infinitely many solutions if $\alpha=2$ and $\beta=-1$

(2) It has no solution if $\alpha=-2$ and $\beta=1$

(3) $x + y + z = \frac{3}{4}$ if $\alpha=2$ and $\beta=1$

(4) It has infinitely many solutions if $\alpha=1$ and $\beta=1$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.
$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$$

$$\alpha^3 - 3\alpha + 2 = 0$$

$$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$$

$$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$$

$$\alpha = 1, \alpha = -2, 1$$

$$\text{For } \alpha = 1, \beta = 1$$

$$\begin{cases} x + y + z = 1 \\ x + y + z = \beta \end{cases} \text{ infinite solution}$$

$$\text{For } \alpha = 2, \beta = 1$$

$$\Delta = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & -1 \\ & 1 & \end{vmatrix} = 1 \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 1 \\ & 1 & 1 \end{vmatrix} = 1 \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 1 \\ & 1 & 1 \end{vmatrix} = 1 \Rightarrow z = \frac{1}{4}$$

For $\alpha = 2 \Rightarrow$ unique solution

18. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

Official Ans. by NTA (1)

QuestPix Ans. (Bonus)

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\vec{a} \cdot \vec{b} = \frac{5 - 3 - 15}{\sqrt{35}} = -\frac{13}{\sqrt{35}}$$

- 19.** Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$.

Then $\sqrt{2} (y_0 - x_0)$ is equal to :

- (1) -3
(2) 9
(3) -9
(4) 3

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $3x^2 - 4y^2 = 36$ $3x + 2y = 1$

$$m = -\frac{3}{2}$$

$$m = +\frac{\sec \theta}{\sqrt{12} \cdot \tan \theta}$$

$$\Rightarrow \frac{3}{\sqrt{12}} \times \frac{1}{\sin \theta} = \frac{-3}{2}$$

$$\sin \theta = -\frac{1}{\sqrt{3}}$$

$$(\sqrt{12} \cdot \sec \theta, 3 \tan \theta)$$

$$\left(\sqrt{12} \cdot \frac{\sqrt{3}}{\sqrt{2}}, -3 \times \frac{1}{\sqrt{2}} \right) \Rightarrow \left(\frac{6}{\sqrt{2}}, \frac{-3}{\sqrt{2}} \right)$$

- 20.** If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to :

- (1) $8 \log_e 2 - 2$
(2) $4 \log_e 2 + 2$
(3) $4 (\log_e 2)^2 - 2$
(4) $4 (\log_e 2)^2 + 2$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $y' = x^x$

$$y' = x^x (1 + \frac{1}{x})$$

$$y'' = x^x (1 + \frac{1}{x})^2 + x^x \cdot \frac{1}{x^2}$$

$$y''(2) = 4(1 + \frac{1}{2})^2 + 2$$

$$y'(2) = 4(1 + \frac{1}{2})$$

$$y''(2) - 2y'(2) = 4(1 + \frac{1}{2})^2 + 2 - 8(1 + \frac{1}{2})$$

$$= 4(1 + \frac{1}{2}) [1 + \frac{1}{2} - 2] + 2$$

$$= 4(\frac{3}{2})^2 - 1 + 2$$

$$= 4(\frac{9}{4}) - 2$$

SECTION-B

- 21.** The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is _____.

Official Ans. by NTA (81)

QuestPix Ans. (81)

Sol. Taking single digit $\rightarrow 444444$ $\frac{6!}{6!} = 1$

Taking two digit $\rightarrow$

$$(4, 5) \quad 444555 \quad (4, 9) \quad 444999$$

$$\frac{5!}{3!2!} = 10$$

$$\frac{5!}{3!2!} = 10$$

Taking three digit

$$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!} = 20$$

$$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!} = 5$$

$$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!} = 30$$

Total = 81

- 22.** Number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1$, $y \geq 3$, $z \geq 4$, is equal to _____.

Official Ans. by NTA (105)

QuestPix Ans. (105)

Sol. ${}^{15}C_2 = \frac{15 \times 14}{2} = 105$

23. The line $x = 8$ is the directrix of the ellipse

$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus

$(2, 0)$. If the tangent to E at the point P in the first

quadrant passes through the point $(0, 4\sqrt{3})$ and

intersects the x -axis at Q , then $(3PQ)^2$ is equal to _____.

Official Ans. by NTA (39)

QuestPix Ans. (39)

Sol. $\frac{a}{e} = 8 \dots\dots\dots(1)$ $ae = 2 \dots\dots\dots(2)$

$8e = \frac{2}{e}$

$e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$

$a = 4$

$b^2 = a^2(1 - e^2)$

$= 16\left(\frac{3}{4}\right) = 12$

$\frac{x \cos \theta}{4} + \frac{y \sin \theta}{2\sqrt{3}} = 1$

$\sin \theta = \frac{1}{2}$

$\theta = 30^\circ$

$P(2\sqrt{3}, \sqrt{3})$

$Q\left(\frac{8}{\sqrt{3}}, 0\right)$

$(3PQ)^2 = 39$

24. If the x -intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to _____.

Official Ans. by NTA (16)

QuestPix Ans. (16)

Sol. $y^2 = 8x + 4y + 4$

$(y - 2)^2 = 8(x + 1)$

$y^2 = 4ax$

$a = 2, X = x + 1, Y = y - 2$

focus $(1, 2)$

$y - 2 = m(x - 1)$

Put $(3, 0)$ in the above line

$m = -1$

Length of focal chord = 16

25. If $\int_0^\pi \frac{5^{\cos x} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx}{1 + 5^{\cos x}} = \frac{k\pi}{16}$,

then k is equal to _____.

Official Ans. by NTA (26)

QuestPix Ans. (13)

Sol

$I = \int_0^\pi \frac{5^{\cos x} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx}{1 + 5^{\cos x}}$

$I = \int_0^\pi \frac{5^{-\cos x} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx}{1 + 5^{-\cos x}}$

$2I = \int_0^\pi (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx$

$\cancel{2}I = \cancel{2} \int_0^{\frac{\pi}{2}} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx$

$I = \int_0^{\frac{\pi}{2}} (1 + \sin x(-\sin 3x) + \sin^2 x - \sin^3 x \sin 3x) dx$

$2I = \int_0^{\frac{\pi}{2}} (3 + \cos 4x + \cos^3 x \cos 3x - \sin^3 x \sin 3x) dx$

$2I = \int_0^{\frac{\pi}{2}} 3 + \cos 4x + \left(\frac{\cos 3x + 3 \cos x}{4}\right) \cos 3x - \sin 3x \left(\frac{3 \sin x - \sin 3x}{4}\right) dx$

$2I = \int_0^{\frac{\pi}{2}} \left(3 + \cos 4x + \frac{1}{4} + \frac{3}{4} \cos 4x\right) dx$

$2I = \frac{13}{4} \times \frac{\pi}{2} + \frac{7}{4} \left(\frac{\sin 4x}{4}\right)_0^{\frac{\pi}{2}} \Rightarrow I = \frac{13\pi}{16}$

26. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is _____.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $T_6 = {}^m C_5 (10 - 3^x)^{\frac{m-5}{2}} \cdot (3^{x-2}) = 21 \dots (1)$

${}^m C_1, {}^m C_2, {}^m C_3$ are in A.P.

2. ${}^m C_2 = {}^m C_1 + {}^m C_3$

Solving for m , we get

$m = 2$ (rejected), 7

Put in equation (1)

$21 \cdot (10 - 3^x) \frac{3^x}{9} = 21$

$3^x = 3^0, 3^2$

$x = 0, 2$

Sum of the squares of all possible values of $x = 4$

27. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to _____.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $T_{r+1} = {}^{22} C_r \cdot \left(x^{\frac{2}{3}}\right)^{22-r} \cdot (\alpha)^r \cdot x^{-3r}$

$= {}^{22} C_r \cdot x^{\frac{44}{3} - \frac{2r}{3} - 3r} (\alpha)^r$

$\frac{44}{3} = \frac{11r}{3}$

$r = 4$

${}^{22} C_4 \cdot \alpha^4 = 7315$

$\frac{22 \times 21 \times 20 \times 19}{24} \cdot \alpha^4 = 7315$

$\alpha = 1$

28. The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15,, 399,

2, 5, 8, 11,, 359 and

2, 7, 12, 17,, 197, is equal to _____.

Official Ans. by NTA (321)

QuestPix Ans. (321)

Sol. 3, 7, 11, 15,, 399 $d_1 = 4$

2, 5, 8, 11,, 359 $d_2 = 3$

2, 7, 12, 17,, 197 $d_3 = 5$

LCM (d_1, d_2, d_3) = 60

Common terms are 47, 107, 167

Sum = 321

29. Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point (3, -2, 5) and perpendicular to the line joining the points (1, 2, 3) and (-2, 3, 5). Then the value of $\alpha\beta\gamma$ is equal to _____.

Official Ans. by NTA (6)

QuestPix Ans. (Bonus)

- Sol.** Given Equation is not equation of plane as γz is present. If we consider γ is γ then answer would be 6.

Normal vector of plane = $3\hat{i} - \hat{j} - 2\hat{k}$

Plane : $3x - y - 2z + \lambda = 0$

Point (3, -2, 5) satisfies the plane

$\lambda = -1$

$3x - y - 2z = 1$

$\alpha\beta\gamma = 6$



30. The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points $A(-3, -6, 1)$ and $B(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c ($|a|, |b|, |c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to _____.

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. Plane : $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

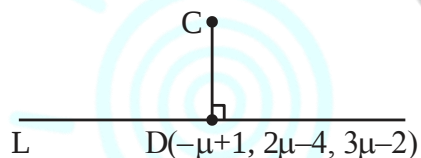
Point of intersection of line and plane

$$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$$

$$\lambda = -\frac{1}{3}$$

$$C\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$$

$$L : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$$



$$\overrightarrow{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$$

$$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$$

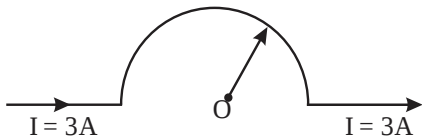
$$\mu = \frac{11}{14}$$

$$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$$

Direction ratios $\rightarrow (-1, -26, 17)$

$$|a + b + c| = 10$$

(Held On Wednesday 01st February, 2023)
TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS	TEST PAPER WITH SOLUTION
SECTION-A 31. A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of cold reservoir raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of hot reservoir is 99°C, will be: (1) 16.5 K (2) 33 K (3) 66 K (4) 62 K Official Ans. by NTA (4) QuestPix Ans. (4) Sol. $T_H = 99^\circ\text{C} = 99 + 273 = 372\text{K}$ $1 - \frac{T_C}{T_H} = \frac{1}{3}$ $\frac{T_C}{T_H} = \frac{2}{3} \quad (1) \Rightarrow T_C = \frac{2}{3} \times 372$ $= 2 \times 124 = 248\text{K}$ $1 - \frac{T_C + X}{T_H} = \frac{1}{6}$ $\frac{5}{6} = \frac{T_C + X}{T_H}$ $\frac{5}{6} = \frac{248 + X}{372}$ $248 + X = 5 \times 62$ $X = 310 - 248 = 62\text{K}$ 32. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R. Assertion A : Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one. Reason R : Capacitance of metallic spheres depend on the radii of spheres.	In the light of the above statements, choose the correct answer from the options given below. (1) A is false but R is true (2) Both A and R are true and R is the correct explanation of A (3) A is true but R is false (4) Both A and R are true but R is not the correct explanation of A Official Ans. by NTA (1) QuestPix Ans. (1) Sol. Potential of a conducting sphere is $V = \frac{KQ}{R} \quad (\text{Solid as well as hollow})$ $V_1 = V_2 \text{ and } R_1 = R_2$ $\therefore Q_1 = Q_2$ 33. As shown in the figure, a long straight conductor with semicircular arc of radius $\frac{\pi}{10}\text{m}$ is carrying current $I = 3\text{A}$. The magnitude of the magnetic field, at the center O of the arc is: (The permeability of the vacuum $= 4\pi \times 10^{-7}\text{NA}^{-2}$)  (1) $6\mu\text{T}$ (2) $1\mu\text{T}$ (3) $4\mu\text{T}$ (4) $3\mu\text{T}$ Official Ans. by NTA (4) QuestPix Ans. (4) Sol. $B_C = \frac{\mu_0 I}{4\pi R} (\pi)$ (B at centre of circular arc) $= \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 3}{4 \times \frac{\pi}{10}}$ $= 3 \times 10^{-6}\text{T} = 3\mu\text{T}$



34. A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil can be changed:

A. By changing the magnitude of the magnetic field within the coil.
 B. By changing the area of coil within the magnetic field.
 C. By changing the angle between the direction of magnetic field and the plane of the coil.
 D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below:

- (1) A and B only (2) A, B and C only
 (3) A, B and D only (4) A and C only

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\phi = \vec{B} \cdot \vec{A}$

$$= BA \cos \theta$$

Most suitable ans is 2 [Otherwise ABCD]

35. In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude $2Y$ V. The ratio of modulation index in the two case respectively will be:

- (1) 1 : 2 (2) 1 : 1
 (3) 2 : 1 (4) 4 : 1

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Modulating Index

$$\mu = \frac{A_m}{A_c}$$

$$\mu_1 = \frac{X}{Y}$$

$$\mu_2 = \frac{X}{2Y}$$

$$\frac{\mu_1}{\mu_2} = \frac{2}{1}$$

36. For a body projected at an angle with the horizontal from the ground, choose the correct statement.

(1) Gravitational potential energy is maximum at the highest point.
 (2) The horizontal component of velocity is zero at highest point.
 (3) The vertical component of momentum is maximum at the highest point.
 (4) The kinetic energy (K.E.) is zero at the highest point of projectile motion.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. At highest point

$$V_y = 0$$

$$V_x = u_x = u \cos \theta$$

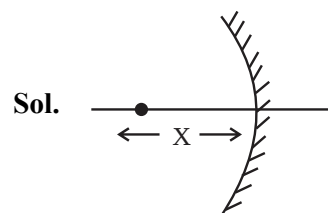
$$U_g = mgh, \text{ it is maximum at } H_{\max}.$$

37. Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is:

- (1) 40 cm (2) 60 cm
 (3) 160 cm (4) 100 cm

Official Ans. by NTA (3)

QuestPix Ans. (3)



Sol.

by mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_1} + \frac{1}{-15} = \frac{1}{(-20)}$$

$$\frac{1}{v_1} = -\frac{1}{20} + \frac{1}{15}$$

$$= \frac{-3 + 4}{60}$$

$$v_1 = 60 \text{ cm}$$

$$\frac{1}{v_2} + \frac{1}{(-25)} = \frac{1}{(-20)}$$

$$\frac{1}{v_2} = \frac{-1}{20} + \frac{1}{25}$$

$$= \frac{-5+4}{100} = \frac{-1}{100}$$

$$v_2 = -100 \text{ cm}$$

$$d = 60 + 100 = 160 \text{ cm}$$

- 38.** The Young's modulus of a steel wire of length 6 m and cross-sectional area 3 mm^2 , is $2 \times 10^{11} \text{ N/m}^2$. The wire is suspended from its support on a given planet. A block of mass 4 kg is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth = 10 m/s^2):

- (1) 1 cm (2) 1 mm
(3) 0.1 mm (4) 0.1 cm

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Tension (F) = mg

$$= 4 \times \frac{10}{4} = 10 \text{ N}$$

$$\Delta L = \frac{FL}{AY}$$

$$= \frac{10 \times 6}{3 \times 10^{-6} \times 2 \times 10^{11}}$$

$$= 10^{-4} \text{ m} = 0.1 \text{ mm}$$

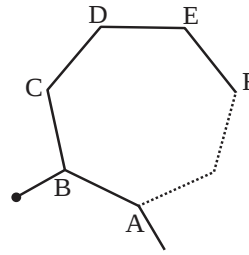
- 39.** Equivalent resistance between the adjacent corners of a regular n -sided polygon of uniform wire of resistance R would be:

- (1) $\frac{(n-1)R}{n^2}$ (2) $\frac{(n-1)R}{(2n-1)}$
(3) $\frac{n^2R}{n-1}$ (4) $\frac{(n-1)R}{n}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



Suppose resistance of each arm is r , then $r = R/n$

$$R_{eq(AB)} = \frac{R_1 R_2}{R_1 + R_2}$$

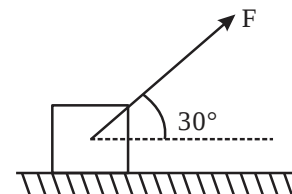
$$\frac{r(n-1)r}{r + (n-1)r}$$

$$= \frac{r(n-1)r}{nr}$$

$$= \frac{n-1}{n} r$$

$$= \frac{(n-1)R}{n^2}$$

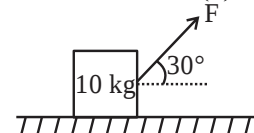
- 40.** As shown in the figure a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30° , with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F : [Given $g = 10 \text{ ms}^{-2}$]



- (1) 33.3 N (2) 25.2 N
(3) 20 N (4) 35.7 N

Official Ans. by NTA (2)

QuestPix Ans. (2)



Sol.

$$N = Mg - F \sin 30^\circ$$

$$= mg - \frac{F}{2} = 100 - \frac{F}{2} = \frac{200 - F}{2}$$

$$F \cos 30^\circ = \mu N$$

$$\sqrt{3} \frac{F}{2} = 0.25 \times \left(\frac{200 - F}{2} \right)$$

$$4\sqrt{3}F = 200 - F$$

$$F = \frac{200}{4\sqrt{3} + 1} = 25.22$$



41. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : For measuring the potential difference across a resistance of $600\ \Omega$, the voltmeter with resistance $1000\ \Omega$ will be preferred over voltmeter with resistance $4000\ \Omega$.

Reason R : Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

- (1) **A** is not correct but **R** is correct
- (2) Both **A** and **R** are correct and **R** is the correct explanation of **A**
- (3) Both **A** and **R** are correct but **R** is not the correct explanation of **A**
- (4) **A** is correct but **R** is not correct

Official Ans. by NTA (1)

QuestPix Ans. (1)

- Sol.** Error of voltmeter decreases with increase in its resistance.

42. Choose the correct statement about Zener diode:

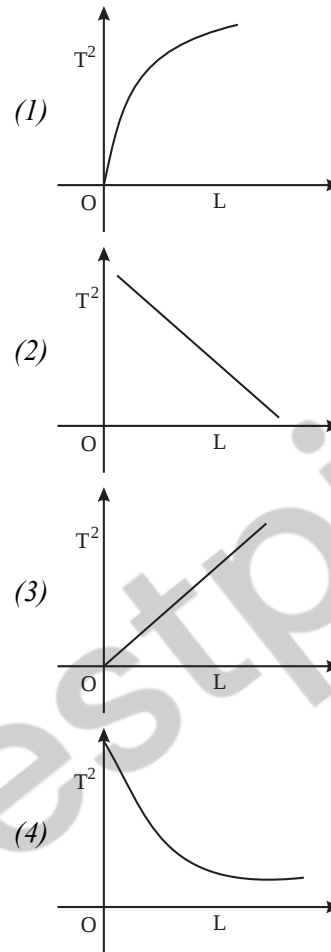
- (1) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.
- (2) It works as a voltage regulator in both forward and reverse bias.
- (3) It works a voltage regulator only in forward bias.
- (4) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

Official Ans. by NTA (1)

QuestPix Ans. (1)

- Sol.** Woks as voltage regulator in reverse bias and as simple P-n junction in forward bias.

43. Choose the correct length (L) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.



Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = \frac{4\pi^2}{g} \times \ell$$

$$T^2 \propto \ell$$

44. The escape velocities of two planets A and B are in the ratio $1 : 2$. If the ratio of their radii respectively is $1 : 3$, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet B will be:

- | | |
|-------------------|-------------------|
| (1) $\frac{4}{3}$ | (2) $\frac{3}{2}$ |
| (3) $\frac{2}{3}$ | (4) $\frac{3}{4}$ |

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho \frac{4}{3}\pi R^3}{R}} = C\sqrt{\rho \cdot R}$

$$\frac{V_{e_1}}{V_{e_2}} = \frac{R_1}{R_2} \sqrt{\frac{\rho_1}{\rho_2}} = \frac{1}{2}$$

$$\frac{R_1^2}{R_2^2} \times \frac{\rho_1}{\rho_2} = \frac{1}{4}$$

$$\frac{R_1}{R_2} = \frac{1}{3}$$

$$g = \frac{GM}{R^2} = \frac{G \frac{4}{3}\pi R^3 \times \rho}{R^2} = C \cdot \rho R$$

$$\frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{4} \frac{R_2^2}{R_1^2} \times \frac{R_1}{R_2}$$

$$= \frac{1}{4} \times \frac{R_2}{R_1} = \frac{3}{4}$$

45. An electron of a hydrogen like atom, having $Z = 4$, jumps from 4th energy state to 2nd energy state, The energy released in this process, will be:

(Given $R_{ch} = 13.6 \text{ eV}$)

Where R = Rydberg constant

c = Speed of light in vacuum

h = Plank's constant

(1) 13.6 eV

(2) 10.5 eV

(3) 3.4 eV

(4) 40.8 eV

Official Ans. by NTA (4)

QuestPix Ans. (4)

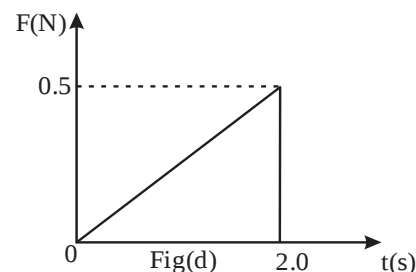
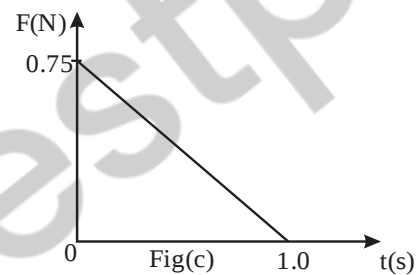
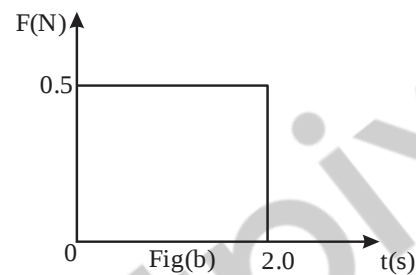
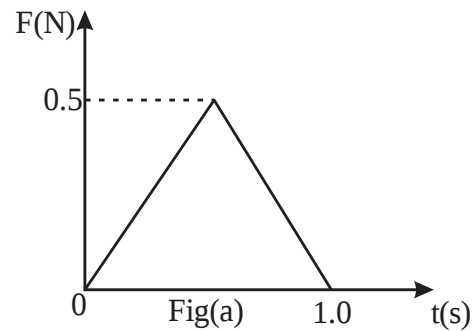
Sol. $\Delta E = 13.6 Z^2 \left[\frac{1}{2^2} - \frac{1}{4^2} \right] \text{eV}$

$$= 13.6 \times (4)^2 \left(\frac{1}{4} - \frac{1}{16} \right) \text{eV.}$$

$$= 13.6 [4 - 1] \text{eV}$$

$$= 13.6 \times 3 = 40.8 \text{ eV}$$

46. Figures (a), (b), (c) and (d) show variation of force with time.



The impulse is highest in figure.

- (1) Fig (c) (2) Fig (b)
(3) Fig (a) (4) Fig (d)

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Impulse = Area under $F = t$ curve

(a) $\frac{1}{2} \times 1 \times 0.5 = \frac{1}{4} \text{ N.s}$

(b) $0.5 \times 2 = 1 \text{ N.s (maximum)}$

(c) $\frac{1}{2} \times 1 \times 0.75 = \frac{3}{8} \text{ N.s}$

(d) $\frac{1}{2} \times 2 \times 0.5 = \frac{1}{2} \text{ N.s}$

47. If the velocity of light c , universal gravitational constant G and planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is:

(1) $\left[h^{\frac{1}{2}} c^{\frac{1}{2}} G^1 \right]$ (2) $\left[h^1 c^1 G^{-1} \right]$
 (3) $\left[h^{-\frac{1}{2}} c^{\frac{1}{2}} G^{\frac{1}{2}} \right]$ (4) $\left[h^{\frac{1}{2}} c^{\frac{1}{2}} G^{-\frac{1}{2}} \right]$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Say dimensional formale of mass is $H^x C^y G^z$

$$M^1 = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z$$

$$M^1 L^0 T^0 = M^{x-z} L^{2x+y+3z} T^{-x-y-2z}$$

on comparing both side

$$x - z = 1$$

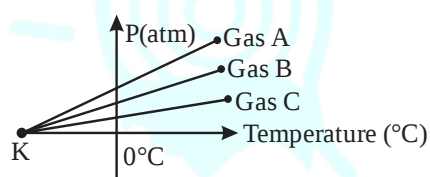
$$2x + y + 3z = 0$$

$$-x - y - 2z = 0$$

On solving above equations we get

$$x = \frac{1}{2} \quad y = \frac{1}{2} \quad z = -\frac{1}{2}$$

48. For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.



The temperature corresponding to the point 'K' is:

- (1) -273°C (2) -100°C
 (3) -373°C (4) -40°C

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. For isochoric process

$$\frac{P}{T} = n \frac{R}{V} = \text{constant}$$

$$P = \frac{nR}{V}(t + 273)$$

$$\text{If } P = 0 \Rightarrow t = -273^\circ\text{C}$$

49. The ratio of average electric energy density and total average energy density of electromagnetic wave is:

- (1) 2 (2) 1
 (3) 3 (4) $\frac{1}{2}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\langle u_E \rangle = \langle u_B \rangle = \frac{1}{2} \langle u_{\text{total}} \rangle$

So $\frac{\langle u_E \rangle}{\langle u_{\text{total}} \rangle} = \frac{1}{2}$

50. The threshold frequency of metal is f_0 . When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectron is v_1 . When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2 . The ratio of v_1 to v_2 is:

- (1) $\frac{v_1}{v_2} = \frac{1}{2}$ (2) $\frac{v_1}{v_2} = \frac{1}{8}$
 (3) $\frac{v_1}{v_2} = \frac{1}{16}$ (4) $\frac{v_1}{v_2} = \frac{1}{4}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $K_{\text{max}} = hf - hf_0$

For $f = 2f_0$

$$\frac{1}{2} m v_1^2 = 2 h f_0 - h f_0 = h f_0$$

For $f = 5f_0$

$$\frac{1}{2} m v_2^2 = 5 h f_0 - h f_0 = 4 h f_0$$

$$\frac{v_1}{v_2} = \frac{1}{2}$$

SECTION-B

51. For a train engine moving with speed of 20 ms^{-1} , the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x} \text{ ms}^{-1}$. The value of x is _____ (Assuming same retardation is produced by brakes)

Official Ans. by NTA (200)

QuestPix Ans. (200)

- Sol.** $u = 20 \text{ m/s}$, $S_1 = 500 \text{ m}$, $v = 0$

By third equation of motion

$$0 = (20)^2 - 2a \cdot 500 \Rightarrow a = \frac{4}{10} \text{ m/s}^2$$

$$u = 20 \text{ m/s}, S_2 = 250 \text{ m}, v = ?$$

$$v^2 = (20)^2 - 2a \cdot 250$$

$$= v = \sqrt{200} \text{ m/s}$$

$$x = 200$$

52. A force $F = (5 + 3y^2)$ acts on a particle in the y -direction, where F is newton and y is in meter. The work done by the force during a displacement from $y = 2 \text{ m}$ to $y = 5 \text{ m}$ is _____ J.

Official Ans. by NTA (132)

QuestPix Ans. (132)

- Sol.** $F = 5 + 3y^2$

$$W = \int_2^5 (5 + 3y^2) dy$$

$$= \left[5y + \frac{3y^3}{3} \right]_2^5$$

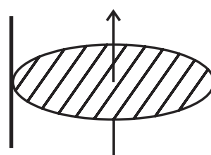
$$= 132 \text{ J}$$

53. Moment of inertia of a disc of mass M and radius ' R ' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2} MR^2$. The value of x is _____.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



$$I = I_{cm} + Md^2$$

$$= \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2} MR^2$$

$$x = 3$$

54. Nucleus a having $Z = 17$ and equal number of protons and neutrons has 1.2 MeV binding energy per nucleon.

Another nucleus B of $Z = 12$ has total 26 nucleons and 1.8 MeV binding energy per nucleons.

The difference of binding energy of B and A will be _____ MeV.

Official Ans. by NTA (6)

QuestPix Ans. (6)

- Sol.** For A mass number = 34

$$\text{Total binding energy} = 1.2 \times 34 = 40.8 \text{ MeV}$$

For B mass number = 26

$$\text{total binding energy} = 1.8 \times 26 \text{ MeV}$$

$$= 46.8 \text{ MeV}$$

$$\text{Difference of BE} = 6 \text{ MeV}$$

55. A square shaped coil of area 70 cm^2 having 600 turns rotates in a magnetic field of 0.4 wb m^{-2} , about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be _____ V.

$$(\text{Take } \pi = \frac{22}{7})$$

Official Ans. by NTA (44)

QuestPix Ans. (44)

- Sol.** $N = 600$, $A = 70 \times 10^{-4} \text{ m}^2$, $B = 0.4 \text{ T}$

$$\omega = \frac{500 \times 2\pi}{60} = \frac{100\pi}{6} \text{ rad/s}$$

$$E = NAB\omega \sin \omega t \quad \omega t \text{ is angle b/w } \vec{A} \text{ \& } \vec{B}$$

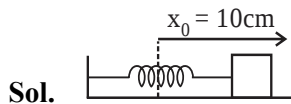
$$= 600 \times 70 \times 10^{-4} \times 0.4 \times \frac{100\pi}{6} \times \frac{1}{2}$$

$$= 44 \text{ V}$$

56. A block is fastened to a horizontal spring. The block is pulled to a distance $x = 10$ cm from its equilibrium position (at $x = 0$) on a frictionless surface from rest. The energy of the block at $x = 5$ cm is 0.25 J. The spring constant of the spring is $\text{---} \text{Nm}^{-1}$.

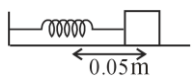
Official Ans. by NTA (50)

QuestPix Ans. (67)



$$U_i = \frac{1}{2} kx_0^2$$

$$K_i = 0$$



$$U_f = \frac{1}{2} k \left(\frac{x_0}{2} \right)^2$$

$$K_f = 0.25 \text{ J}$$

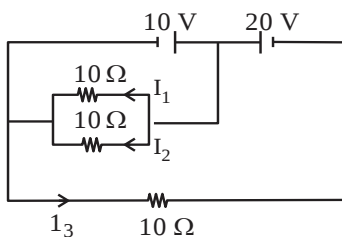
$$\frac{1}{2} kx_0^2 + 0 = \frac{1}{2} k \frac{x_0^2}{4} + 0.25$$

$$\frac{1}{2} kx_0^2 \frac{3}{4} = \frac{1}{4}$$

$$\frac{1}{2} k \frac{3}{100} = 1 \Rightarrow k = \frac{200}{3} \text{ N/m}$$

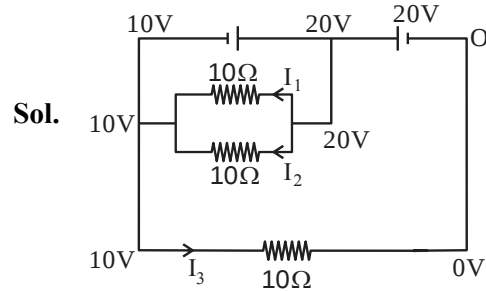
$$= 67 \text{ N/m}$$

57. In the given circuit the value of $\left| \frac{I_1 + I_3}{I_2} \right|$ is:



Official Ans. by NTA (2)

QuestPix Ans. (2)

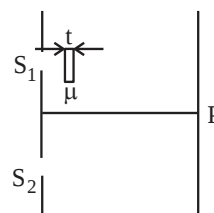


$$I_1 = I_2 = \frac{20 - 10}{10} = 1 \text{ A}$$

$$I_3 = 1 \text{ A}$$

$$\left| \frac{I_1 + I_3}{I_2} \right| = 2$$

58. As shown in the figure, in Young's double slit experiment, a thin plate of thickness $t = 10 \mu\text{m}$ and refractive index $\mu = 1.2$ is inserted in front of slit S_1 . The experiment is conducted in air ($\mu = 1$) and uses a monochromatic light of wavelength $\lambda = 500 \text{ nm}$. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0$. β_0 is the fringe-width before the insertion of the plate. The value of the x is --- .



Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Fringe shift $= \frac{t(\mu - 1)}{\lambda} B$

$$= \frac{10 \times 10^{-6} (1.2 - 1)}{5 \times 10^{-7}} B$$

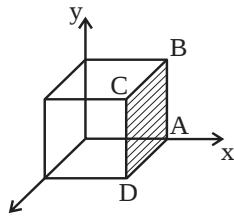
$$= \frac{10^{-5} \times 0.2}{5 \times 10^{-7}} = 4$$

59. A cubical volume is bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0, z = a$. The electric field in the region is given by $\vec{E} = E_0 x \hat{i}$. Where $E_0 = 4 \times 10^4 \text{ NC}^{-1} \text{ m}^{-1}$. If $a = 2 \text{ cm}$, the charge contained in the cubical volume is $Q \times 10^{-14} \text{ C}$. The value of Q is ____.

Take $\epsilon_0 = 9 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

Official Ans. by NTA (288)

QuestPix Ans. (288)



Sol.

$$\vec{E} = E_0 x \hat{i}$$

$$\phi_{\text{net}} = \phi_{\text{ABCD}} = E_0 a \cdot a^2$$

$$\frac{q_{\text{en}}}{\epsilon_0} = E_0 a^3$$

$$\begin{aligned} q_{\text{en}} &= E_0 \epsilon_0 a^3 \\ &= 4 \times 10^4 \times 9 \times 10^{-12} \times 8 \times 10^{-6} \\ &= 288 \times 10^{-14} \text{ C} \end{aligned}$$

$$Q = 288$$

Ans. 288

60. The surface of water in a water tank of cross section area 750 cm^2 on the top of a house is $h \text{ m}$. above the tap level. The speed of water coming out through the tap of cross section area 500 mm^2 is 30 cm/s . At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3} \text{ m/s}$. The value of x will be ____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $A_1 V_1 = A_2 V_2$

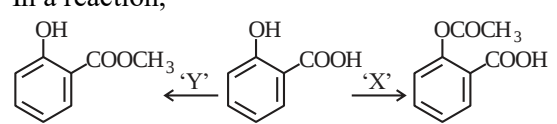
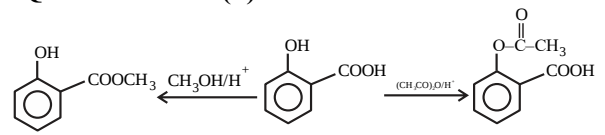
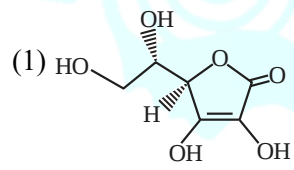
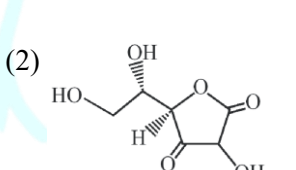
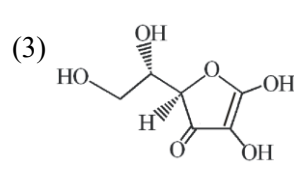
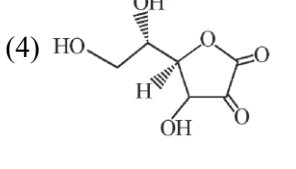
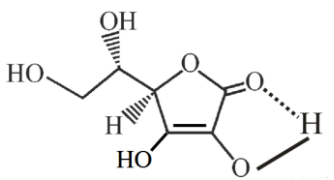
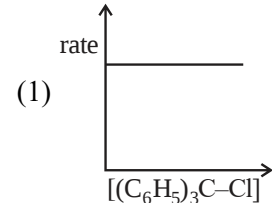
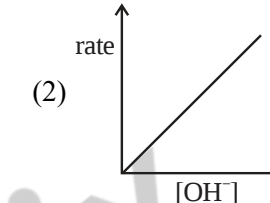
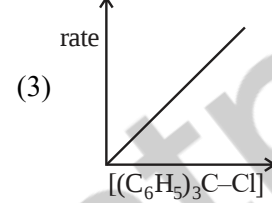
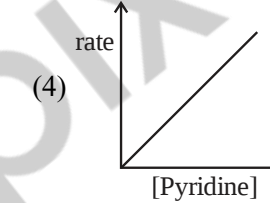
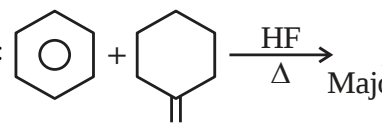
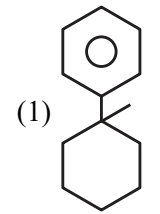
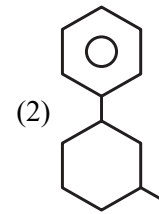
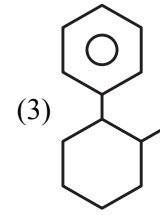
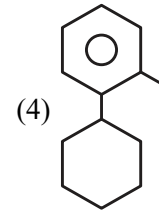
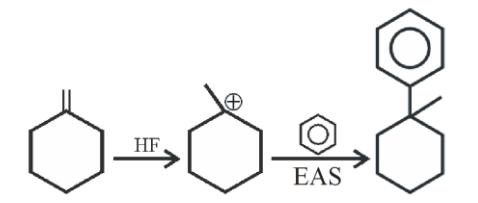
$$750 \times 10^{-4} V_1 = 500 \times 10^{-6} \times 0.3$$

$$V_1 = \frac{500 \times 3 \times 10^{-3}}{750} \text{ m/s}$$

$$= 2 \times 10^{-3} \text{ m/s}$$

$$\frac{dh}{dt} = -2 \times 10^{-3} \text{ m/s}$$

(Held On Wednesday 01st February, 2023)
TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY	TEST PAPER WITH SOLUTION
SECTION-A 61. In a reaction,  reagents 'X' and 'Y' respectively are : (1) $(\text{CH}_3\text{CO})_2\text{O}/\text{H}^+$ and $\text{CH}_3\text{OH}/\text{H}^+$, Δ (2) $(\text{CH}_3\text{CO})_2\text{O}/\text{H}^+$ and $(\text{CH}_3\text{CO})_2\text{O}/\text{H}^+$ (3) $\text{CH}_3\text{OH}/\text{H}^+$, Δ and $\text{CH}_3\text{OH}/\text{H}^+$, Δ (4) $\text{CH}_3\text{OH}/\text{H}^+$, Δ and $(\text{CH}_3\text{CO})_2\text{O}/\text{H}^+$ Official Ans. by NTA (1) QuestPix Ans. (1)  Sol. 62. The correct order of bond enthalpy (kJ mol^{-1}) is : (1) $\text{Si} - \text{Si} > \text{C} - \text{C} > \text{Sn} - \text{Sn} > \text{Ge} - \text{Ge}$ (2) $\text{Si} - \text{Si} > \text{C} - \text{C} > \text{Ge} - \text{Ge} > \text{Sn} - \text{Sn}$ (3) $\text{C} - \text{C} > \text{Si} - \text{Si} > \text{Sn} - \text{Sn} > \text{Ge} - \text{Ge}$ (4) $\text{C} - \text{C} > \text{Si} - \text{Si} > \text{Ge} - \text{Ge} > \text{Sn} - \text{Sn}$ Official Ans. by NTA (4) QuestPix Ans. (4) Sol. (Bond enthalpy order $\text{C} - \text{C} > \text{Si} - \text{Si} > \text{Ge} - \text{Ge} > \text{Sn} - \text{Sn}$) 63. All structures given below are of vitamin C. Most stable of them is : (1)  (2)  (3)  (4)  Official Ans. by NTA (1) QuestPix Ans. (1) Sol. H-bonding stabilised vitamin C 	64. The graph which represents the following reaction is : $(\text{C}_6\text{H}_5)_3\text{C} - \text{Cl} \xrightarrow[\text{Pyridine}]{\text{OH}^-} (\text{C}_6\text{H}_5)_3\text{C} - \text{OH}$ (1)  (2)  (3)  (4)  Official Ans. by NTA (3) QuestPix Ans. (3) Sol. (It is $\text{S}_{\text{N}}1$ reaction so rate of reaction depends on the concentration of alkyl halide only.) 65. 'X' is :  Major product (1)  (2)  (3)  (4)  Official Ans. by NTA (1) QuestPix Ans. (1) Sol. 

66. The complex cation which has two isomers is :

- (1) $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$ (2) $[\text{Co}(\text{NH}_3)_5\text{Cl}]^{2+}$
 (3) $[\text{Co}(\text{NH}_3)_5\text{NO}_2]^{2+}$ (4) $[\text{Co}(\text{NH}_3)_5\text{Cl}]^+$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $[\text{Co}(\text{NH}_3)_5\text{NO}_2]^{2+}$

Two linkage isomers possible

$\text{NO}_2 \rightarrow$ Ambidentate ligand

67. Given below are two statements :

Statement I : Sulphanilic acid gives esterification test for carboxyl group.

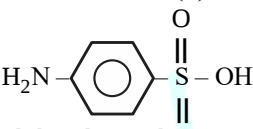
Statement II : Sulphanilic acid gives red colour in Lassaigne's test for extra element detection.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

- (1) **Statement I** is correct but **Statement II** is incorrect.
 (2) Both **Statement I** and **Statement II** are incorrect.
 (3) Both **Statement I** and **Statement II** are correct.
 (4) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. 

Sulphanilic acid

Does not show esterification test.

Presence of both sulphur and nitrogen give red colour in Lassaigne's test.

68. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Gypsum is used for making fireproof wall boards.

Reason (R) : Gypsum is unstable at high temperatures.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**.
 (2) **(A)** is correct but **(R)** is not correct.
 (3) **(A)** is not correct but **(R)** is correct.
 (4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. (Gypsum is used for making fireproof wall boards.

69. Which element is not present in Nessler's reagent ?

- (1) Mercury
 (2) Potassium
 (3) Iodine
 (4) Oxygen

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. (Nessler's Reagent $\rightarrow \text{K}_2[\text{HgI}_4]$)

70. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : α -halocarboxylic acid on reaction with dil. NH_3 gives good yield of α -amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.

Reason (R) : Amino acids exist in zwitter ion form in aqueous medium.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.
 (2) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**.
 (3) **(A)** is correct but **(R)** is not correct.
 (4) **(A)** is not correct but **(R)** is correct.

Official Ans. by NTA (1)

QuestPix Ans. (2)

71. The industrial activity held least responsible for global warming is :

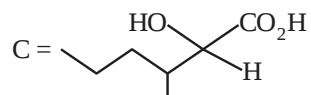
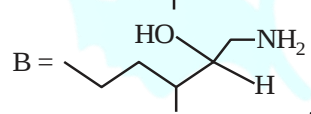
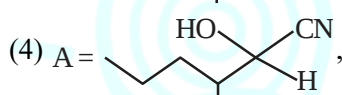
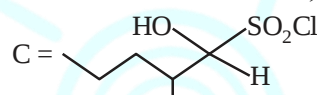
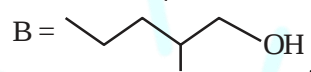
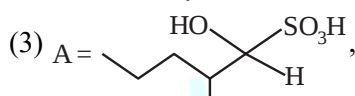
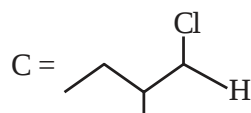
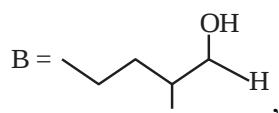
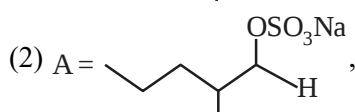
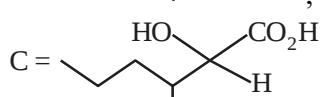
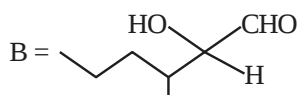
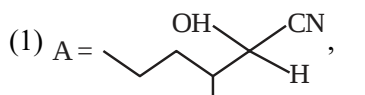
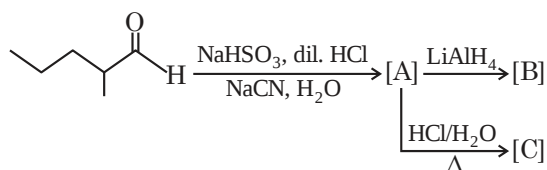
- (1) manufacturing of cement
 (2) steel manufacturing
 (3) Electricity generation in thermal power plants.
 (4) Industrial production of urea

Official Ans. by NTA (4)

QuestPix Ans. (4)

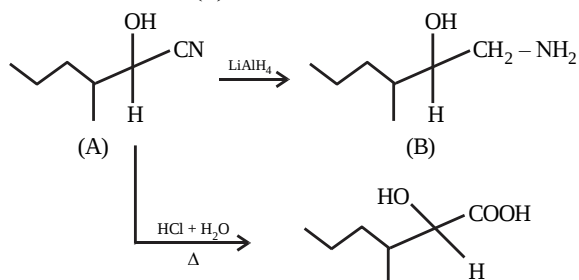
Sol. In urea production NH_3 and CO_2 consumed so least responsible for global warming.

72. The structures of major products A, B and C in the following reaction are sequence.



Official Ans. by NTA (4)

QuestPix Ans. (4)



73. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Cu^{2+} in water is more stable than Cu^+ .

Reason (R) : Enthalpy of hydration for Cu^{2+} is much less than that of Cu^+ .

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both (A) and (R) are correct and (R) is the correct explanation of (A).
 (2) (A) is correct but (R) is not correct.
 (3) (1) is not correct but (R) is correct.
 (4) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A).

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $2\text{Cu}^+ \rightarrow \text{Cu}^{2+} + \text{Cu}$

The stability of $\text{Cu}^{2+}(\text{aq})$ rather than $\text{Cu}^+(\text{aq})$, is due to the much more negative $\Delta_{\text{hyd}}H$ of $\text{Cu}^{2+}(\text{aq})$ than $\text{Cu}^+(\text{aq})$, which more than compensates for the second ionisation enthalpy of Cu.

74. The starting material for convenient preparation of deuterated hydrogen peroxide (D_2O_2) in laboratory is:

- (1) $\text{K}_2\text{S}_2\text{O}_8$ (2) 2-ethylanthraquinol
 (3) BaO_2 (4) BaO

Official Ans. by NTA (1)

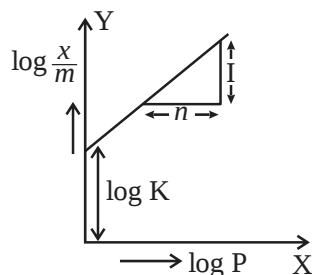
QuestPix Ans. (1)

Sol. $(\text{K}_2\text{S}_2\text{O}_8(\text{s}) + 2\text{D}_2\text{O}(\text{l}) \rightarrow 2\text{KDSO}_4(\text{aq.}) + \text{D}_2\text{O}_2$

75. In figure, a straight line is given for Freundlich

Adsorption ($y = 3x + 2.505$). The value of $\frac{1}{n}$ and

$\log K$ are respectively.



- (1) 0.3 and $\log 2.505$ (2) 0.3 and 0.7033
 (3) 3 and 2.505 (4) 3 and 0.7033

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\frac{x}{m} = Kp^{1/n}$

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$$

$$Y = 3x + 2.505, \frac{1}{n} = 3, \log K = 2.505$$

- 76.** Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : An aqueous solution of KOH when for volumetric analysis, its concentration should be checked before the use.

Reason (R) : On aging, KOH solution absorbs atmospheric CO₂.

In the light of the above statements, choose the correct answer from the options given below.

- (1) (A) is not correct but (R) is correct
- (2) Both (A) and (R) are correct but (R) is **not** the correct explanation of (A)
- (3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (4) (A) is correct but (R) is not correct

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. KOH absorb CO₂

So its concentration should be checked.

- 77.** Which one of the following sets of ions represents a collection of isoelectronic species?

(Given : Atomic Number : F : 9, Cl : 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)

- (1) (Li⁺, Na⁺, Mg²⁺, Ca²⁺)
- (2) (Ba²⁺, Sr²⁺, K⁺, Ca²⁺)
- (3) (N³⁻, O²⁻, F⁻, S²⁻)
- (4) (K⁺, Cl⁻, Ca²⁺, Sc³⁺)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. K⁺ Cl⁻ Ca²⁺ Sc³⁺
18 18 18 18

- 78.** The effect of addition of helium gas to the following reaction in equilibrium state, is :



- (1) the equilibrium will shift in the forward direction and more of Cl₂ and PCl₃ gases will be produced.
- (2) the equilibrium will go backward due to suppression of dissociation of PCl₅.
- (3) helium will deactivate PCl₅ and reaction will stop.
- (4) addition of helium will not affect the equilibrium.

Official Ans. by NTA (1)

QuestPix Ans. (1 & 4)

Sol. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(Case 1 : At constant P – volume will increase so reaction will shift in forward direction then answer will be A

Case 2 : At constant volume no change in active mass so reaction will not shift in any direction then answer will be D.

- 79.** For electron gain enthalpies of the elements denoted as Δ_{eg}H, the incorrect option is :

- (1) Δ_{eg}H (Cl) < Δ_{eg}H (F)
- (2) Δ_{eg}H (Se) < Δ_{eg}H (S)
- (3) Δ_{eg}H (I) < Δ_{eg}H (At)
- (4) Δ_{eg}H (Te) < Δ_{eg}H (Po)

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (1) Δ_{eg}H (Cl) < Δ_{eg}H (F)

(-345) (-328) Correct

(2) Δ_{eg}H (Se) < Δ_{eg}H (S)

(-195) (-200) Incorrect

(3) Δ_{eg}H (I) < Δ_{eg}H (At)

(-295) (-270) Correct

(4) Δ_{eg}H (Te) < Δ_{eg}H (Po)

(-190) (-183) Correct

80. O–O bond length in H_2O_2 is X than the O–O bond length in F_2O_2 . The O – H bond length in H_2O_2 is Y than that of the O–F bond in F_2O_2 . Choose the correct option for X and Y from the given below.

- (1) X – shorter, Y – shorter
 (2) X – shorter, Y – longer
 (3) X – longer, Y – longer
 (4) X – longer, Y – shorter

Official Ans. by NTA (4)

QuestPix Ans. (4)

- Sol.** According to bent rule more electronegative atom occupy less s-characters so bond length increases. O – H bond will be short than O – F bond due to small size of H than F.

SECTION-B

81. 0.3 g of ethane undergoes combustion at 27°C in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by 0.5°C . The heat evolved during combustion of ethane at constant pressure is _____ kJ mol^{-1} .

(Nearest integer)

[Given : The heat capacity of the calorimeter system is 20 kJ K^{-1} , $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$.

Assume ideal gas behaviour.

Atomic mass of C and H are 12 and 1 g mol^{-1} respectively]

Official Ans. by NTA (1006)

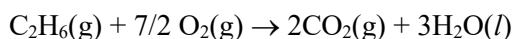
QuestPix Ans. (1006)

- Sol.** (Bomb calorimeter $\rightarrow$ const volume

Heat released

By combustion of 1 mole

$$\text{C}_2\text{H}_6 (\Delta U) = - \frac{20 \times 0.5}{0.3} \times 30 = -1000 \text{ kJ}$$



$$\Delta n_g = 2 - (2 + 7/2) = - (7/2)$$

$$\Delta H = \Delta U + \Delta n RT$$

$$= -1000 - 7/2 \times 8.3 \times 300 \text{ kJ}$$

$$= -1000 - 6.225$$

$$= -1006 \text{ kJ}$$

So heat released = 1006 kJ mol^{-1}

82. Among following compounds, the number of those present in copper matte is _____.

- A. CuCO_3
 B. Cu_2S
 C. Cu_2O
 D. FeO

Official Ans. by NTA (3)

QuestPix Ans. (1)

- Sol.** FeS and Cu_2S , present in copper matte.

83. Among the following, the number of tranquilizer/s is/are _____.

- A. Chloroliazepoxide
 B. Veronal
 C. Valium
 D. Salvarsan

Official Ans. by NTA (3)

QuestPix Ans. (3)

- Sol.** (chlorodiazepoxide, Veronal, Valium is tranquilizer where as salvarsan is antibiotic.

84. $A \rightarrow B$

The above reaction is of zero order. Half life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is _____ min.

(Nearest integer)

Official Ans. by NTA (75)

QuestPix Ans. (75)

- Sol.** Assume reaction starts with 1 mole A

$$(t_{1/2} = \frac{a}{2k}, K = \frac{1}{2 \times 50})$$

For 75% completion

$$a - \frac{a}{4} = kt$$

$$t = \frac{3}{4} \frac{a}{k} = \frac{3}{4} \times \frac{100}{a} = 75$$

85. 20% of acetic acid is dissociated when its 5 g is added to 500 mL of water. The depression in freezing point of such water is _____ $\times 10^{-3}^{\circ}\text{C}$. Atomic mass of C, H and O are 12, 1 and 16 a.m.u. respectively.

[Given : Molal depression constant and density of water are $1.86 \text{ K kg mol}^{-1}$ and 1 g cm^{-3} respectively.]

Official Ans. by NTA (372)

QuestPix Ans. (372)

Sol. $i = 1 + (n - 1) \alpha$

$$(i = 1 + 0.2(2 - 1) = 1.2)$$

$$\Delta T_f = i K_f m$$

$$\Delta T_f = 1.2 \times 1.86 \times \frac{5 \times 1000}{60 \times 500}$$

$$\Delta T_f = 3.72$$

$$\Delta T_f = 372 \times 10^{-2}$$

86. The molality of a 10% (v/v) solution of di-bromine solution in CCl_4 (carbon tetrachloride) is 'x'. $x =$ _____ $\times 10^{-2} \text{ M}$. (Nearest integer)

[Given : molar mass of $\text{Br}_2 = 160 \text{ g mol}^{-1}$

atomic mass of C = 12 g mol^{-1}

atomic mass of Cl = 35.5 g mol^{-1}

density of dibromine = 3.2 g cm^{-3}

density of $\text{CCl}_4 = 1.6 \text{ g cm}^{-3}$]

Official Ans. by NTA (139)

QuestPix Ans. (139)

Sol. (10 ml solute in 90 ml solvent
mass of solute = $10 \times 3.2 = 32 \text{ g}$
mass of solvent = 90×1.6

$$m = \frac{32 \times 1000}{160 \times 90 \times 1.6} = 1.388$$

$$m = 138.8 \times 10^{-2} = 139$$

87. $1 \times 10^{-5} \text{ M AgNO}_3$ is added to 1 L of saturated solution of AgBr. The conductivity of this solution at 298 K is _____ $\times 10^{-8} \text{ S m}^{-1}$.

[Given : $K_{sp}(\text{AgBr}) = 4.9 \times 10^{-13}$ at 298K

$$\lambda_{\text{Ag}^+}^0 = 6 \times 10^{-3} \text{ Sm}^2 \text{ mol}^{-1}$$

$$\lambda_{\text{Br}^-}^0 = 8 \times 10^{-3} \text{ Sm}^2 \text{ mol}^{-1}$$

$$\lambda_{\text{NO}_3^-}^0 = 7 \times 10^{-3} \text{ Sm}^2 \text{ mol}^{-1}]$$

Official Ans. by NTA (14)

QuestPix Ans. (Bonus)

Sol. $[\text{Ag}^+] = 10^{-5}$

$$[\text{NO}_3^-] = 10^{-5}$$

$$[\text{Br}^-] = \frac{K_{sp}}{[\text{Ag}^+]} = 4.9 \times 10^{-8}$$

$$\Lambda_m = \frac{k}{1000 \times M}$$

For Ag^+

$$6 \times 10^{-3} = \frac{K_{\text{Ag}^+}}{1000 \times 10^{-5}}$$

$$K_{\text{Ag}^+} = 6 \times 10^{-5}$$

$$\Rightarrow 6000 \times 10^{-8}$$

for Br^-

$$8 \times 10^{-3} = \frac{K_{\text{Br}^-}}{1000 \times 4.9 \times 10^{-8}}$$

$$K_{\text{Br}^-} = 39.2 \times 10^{-8}$$

for NO_3^-

$$7 \times 10^{-3} = \frac{K_{\text{NO}_3^-}}{1000 \times 10^{-5}}$$

$$K_{\text{NO}_3^-} = 7 \times 10^{-5}$$

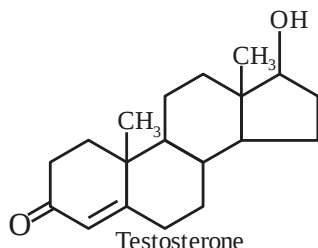
$$= 7000 \times 10^{-8}$$

Conductivity of solution

$$\Rightarrow (6000 + 7000 + 39.2) \times 10^{-8}$$

$$\Rightarrow 13039.2 \times 10^{-8} \text{ S m}^{-1}$$

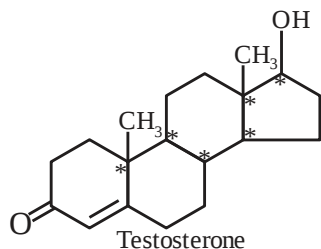
88. Testosterone, which is a steroidal hormone, has the following structure.



The total number of asymmetric carbon atom/s in testosterone is _____

Official Ans. by NTA (6)

QuestPix Ans. (6)



Sol.

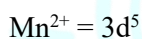
89. The spin only magnetic moment of $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$ complexes is _____ B.M. (Nearest integer)

(Given : Atomic no. of Mn is 25)

Official Ans. by NTA (6)

QuestPix Ans. (6)

Sol. $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$



$$\mu = \sqrt{5(5+2)} = 5.91 \text{ BM}$$

90. A metal M crystallizes into two lattices :- face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5 Å respectively. The ratio of densities of lattices fcc to bcc for the metal M is _____.

(Nearest integer)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $d = \frac{Z \times M}{N_A a^3}$

$$\frac{d_{\text{FCC}}}{d_{\text{BCC}}} = \frac{\frac{4 \times M_w}{N_A \times (2)^3}}{\frac{2 \times M_w}{N_A \times (2.5)^3}} = 3.90$$