(Held On Wednesday 25th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let the function $f(x)=2x^3 + (2p–7)x^2+3(2p–9)x–6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(\frac{9}{2},\infty\right)$ (2) $\left(0,\frac{9}{2}\right)$

(3) $\left(-\infty,\frac{9}{2}\right)$ (4) $\left(-\frac{9}{2},\frac{9}{2}\right)$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $f(x) = 2x^3 + (2p – 7)\, x^2 + 3\,(2p – 9)x – 6$

$f'(x) = 6x^2 + 2(2p – 7)x + 3(2p – 9)$

$f'(0) < 0$

$\therefore\ 3(2p – 9) < 0$

$$p < \frac{9}{2}$$

$$p \in \left(-\infty,\frac{9}{2}\right)$$

2. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(1) (2, 0) (2) (0, 0)

(3) (0, 2) (4) (0, –2)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $(z – 2i)\ (\bar{z}+2i) = 4(z + i)\ (\bar{z}-i)$

$z\bar{z} + 4 + 2i\,(z – \bar{z}) = 4(z\bar{z} + 1 + i(\bar{z} – z))$

$3z\bar{z} – 6i(z – \bar{z}) = 0$

$x^2 + y^2 – 2i\,(2iy) = 0$

$x^2 + y^2 + 4y = 0$

3. If the function

$$f(x) = \begin{cases} (1+|\cos x|)\dfrac{\lambda}{|\cos x|}, & 0 < x < \dfrac{\pi}{2} \\ \mu, & x = \dfrac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \dfrac{\pi}{2} < x < \pi \end{cases}$$

is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(1) 11 (2) 8

(3) $2e^4+8$ (4) 10

Official Ans. by NTA (4)

QuestPix Ans. BONUS

Sol. $\Rightarrow \lim\limits_{x\to\frac{\pi}{2}^+} e^{\frac{\cot 6x}{\cot 4x}} = \lim\limits_{x\to\frac{\pi}{2}^+} e^{\frac{\sin 4x.\cos 6x}{\sin 6x.\cos 4x}} = e^{2/3}$

$\Rightarrow \lim\limits_{x\to\frac{\pi}{2}^-} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} = e^{\lambda}$

$\Rightarrow f(\pi/2) = \mu$

For continuous function $\Rightarrow e^{2/3} = e^{\lambda} = \mu$

$\lambda = \frac{2}{3}, \mu = e^{2/3}$

Now, $9\lambda + 6\log_e\mu + \mu^6 – e^{6\lambda} = 10$

4. Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and f(4)=133, f(5)=255. Then the sum of all the positive integer divisors of (f(3)–f(2)) is

(1) 61 (2) 60

(3) 58 (4) 59

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $f(x) = 2x^n + \lambda$

$f(4) = 133$

$f(5) = 255$

$133 = 2 \times 4^n + \lambda$ (1)

$255 = 2 \times 5^n + \lambda$ (2)

(2) – (1)

$122 = 2(5^n - 4^n)$

$\Rightarrow 5^n - 4^n = 61$

$\therefore \quad n = 3 \ \& \ \lambda = 5$

Now, $f(3) - f(2) = 2(3^3 - 2^3) = 38$

Number of Divisors is 1, 2, 19, 38 ; & their sum is 60

5. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}, \hat{i} + 2\hat{j} - \hat{k}, -2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{73}{17}$ (2) $-\frac{107}{17}$

(3) $-\frac{73}{17}$ (4) $\frac{107}{17}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Let A : (3, –4, 2) C : (–2, –1, 3)

B : (1, 2, –1) D : (5, -2α, 4)

A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$$\Rightarrow \alpha = \frac{73}{17}$$

6. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^T BA$, then the inverse of the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$

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$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$M = A^T BA$

$M^2 = M.M = A^T BA \ A^T BA = A^T B^2 A$

$M^3 = M^2.M = A^T B^2 A A^T BA = A^T B^3 A$

.

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$M^{2023} = \ldots\ldots\ldots\ldots A^T B^{2023} A$

$AM^{2023}A^T = \underline{AA^T} B^{2023} \underline{AA^T} = B^{2023}$

$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

7. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

(1) $\Delta = \wedge, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$

(3) $\Delta = \vee, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Given $(p \to q)\ \Delta\ (p\ \nabla\ q)$

Option I $\Delta = \wedge\ ,\ \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Option 2 $\Delta = \vee\ ,\ \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Option 3 $\Delta = \vee\ ,\ \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence, it is tautology.

Option 4 $\Delta = \wedge\ ,\ \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

8. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(1) 6 (2) 12

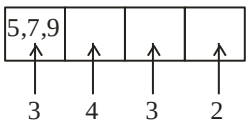
(3) 120 (4) 72

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Numbers between 5000 & 10000

Using digits 1, 3, 5, 7, 9



Total Numbers $= 3 \times 4 \times 3 \times 2 = 72$

9. The number of functions $f : \{1,2,3,4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(1) 3 (2) 4

(3) 1 (4) 2

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$

$f(n) + \frac{1}{n} f(n+1) = 1,\ \forall\ n \in \{1, 2, 3\}$

$f(n + 1)$ must be divisible by n

$f(4) \Rightarrow -6, -3, 0, 3, 6$

$f(3) \Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8$

$f(2) \Rightarrow -8, \ldots\ldots\ldots\ldots, 8$

$f(1) \Rightarrow -8, \ldots\ldots\ldots\ldots, 8$

$\frac{f(4)}{3}$ must be odd since $f(3)$ should be even therefore 2 solution possible.

$f(4)$	$f(3)$	$f(2)$	$f(1)$
–3	2	0	1
3	0	1	0

10. The equations of two sides of a variable triangle are x = 0 and y = 3, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is :

(1) $4y^2 - 18y - 3x - 18 = 0$ (2) $4y^2 + 18y + 3x + 18 = 0$

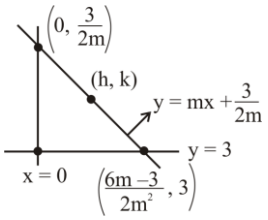
(3) $4y^2 - 18y + 3x + 18 = 0$ (4) $4y^2 - 18y - 3x + 18 = 0$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $y^2 = 6x$ & $y^2 = 4ax$

$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$



$y = mx + \frac{3}{2m}$; $(m \neq 0)$

$h = \frac{6m-3}{4m^2}$, $k = \frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

11. Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$, for some m, such that the range of f is [0, 2]. Then the value of m is _____

(1) 5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Since,

$-\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\therefore -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

(Assume $\sqrt{2}(\sin x - \cos x) = k$)

$-2 \le k \le 2$ ...(i)

$f(x) = \log_{\sqrt{m}}(k + m - 2)$

Given,

$0 \le f(x) \le 2$

$0 \le \log_{\sqrt{m}}(k + m - 2) \le 2$

$1 \le k + m - 2 \le m$

$-m + 3 \le k \le 2$ ...(ii)

From eq. (i) & (ii), we get $-m + 3 = -2$

12. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then,

(1) Only S2 is true

(2) Only S1 is true

(3) Both S1 and S2 are false

(4) Both S1 and S2 are true

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Given, $A^T = A$, $B^T = -B$, $C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

13. Let y=y(t) be a solution of the differential equation

$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

Where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim_{t\to\infty} y(t)$

(1) is 0 (2) does not exist

(3) is 1 (4) is –1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

I.F. $= e^{\int \alpha dt} = e^{\alpha t}$

Solution $\Rightarrow y.e^{\alpha t} = \int \gamma e^{-\beta T}.e^{\alpha t} dt$

$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)} + c$

$\Rightarrow y = \frac{\gamma}{e^{\beta t}(\alpha-\beta)} + \frac{c}{e^{\alpha t}}$

So, $\lim_{t\to\infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$

14. $\sum_{k=0}^{6} {}^{51-k}C_3$ is equal to

(1) ${}^{51}C_4 - {}^{45}C_4$ (2) ${}^{51}C_3 - {}^{45}C_3$

(3) ${}^{52}C_4 - {}^{45}C_4$ (4) ${}^{52}C_3 - {}^{45}C_3$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\sum_{k=0}^{6} {}^{51-k}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ... + {}^{45}C_3$

$= {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3$

$= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3 - {}^{45}C_4$

$({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$

$= {}^{52}C_4 - {}^{45}C_4$

15. The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

(1) 2 (2) 3

(3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$ and $\frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$\Rightarrow$ Shortest distance $= \frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$\text{S.D.} = \left(-\hat{i}+2\hat{j}-\hat{k}\right).\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$$\left\{\vec{p}\times\vec{q} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{-1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \text{ or } 2\hat{i} - 3\hat{j} + 6\hat{k}\right\}$$

$\text{S.D.} = \frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right).\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{2^2+3^2+6^2}} = \left|\frac{-14}{7}\right| = 2$

16. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(1) 2 (2) 4

(3) 16 (4) 8

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $n(s) = 36$

Given : $N-2$, $\sqrt{3N}$, $N+2$ are in G.P.

$3N = (N-2)(N+2)$

$3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0$

$(N-4)(N+1) = 0 \Rightarrow \boxed{N=4}$ or $N = -1$ rejected

$(\text{Sum} = 4) \equiv \{(1,3), (3,1), (2,2)\}$

$n(A) = 3$

$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow \boxed{k=4}$

17. The integral $16\int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$ is equal to

(1) $\frac{11}{6} + \log_e 4$ (2) $\frac{11}{12} + \log_e 4$

(3) $\frac{11}{12} - \log_e 4$ (4) $\frac{11}{6} - \log_e 4$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $I = 16\int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$

$= 16\int_1^2 \frac{dx}{x^3x^4\left(1+\frac{2}{x^2}\right)^2}$

Let, $1+\frac{2}{x^2} = t \Rightarrow \frac{-4}{x^3}dx = dt$

$I = -4\int_3^{\frac{3}{2}} \frac{dt}{\left(\frac{2}{t-1}\right)^2 t^2}$

$I = -4\int_3^{\frac{3}{2}} \left(\frac{t-1}{2}\right)^2 \frac{dt}{t^2}$

$I = -\frac{4}{4}\int_3^{\frac{3}{2}} \left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$

$$I = -1\left[t - 2\ell n|t| - \frac{1}{t}\right]_3^{\frac{3}{2}}$$

$$I = -1\left[\left(\frac{3}{2} - 2\ell n\frac{3}{2} - \frac{2}{3}\right) - \left(3 - 2\ell n3 - \frac{1}{3}\right)\right]$$

$$I = -1\left[2\ell n2 - \frac{11}{6}\right]$$

$$I = \frac{11}{6} - \ell n4$$

18. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{44}{3}$ (2) $4\sqrt{6} + \frac{28}{3}$

(3) $4\sqrt{6} - \frac{44}{3}$ (4) $4\sqrt{6} - \frac{28}{3}$

Official Ans. by NTA (2)

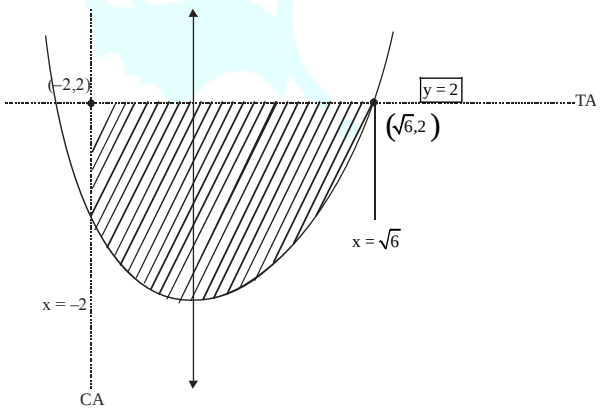
QuestPix Ans. (2)

Sol. $16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16(x + 2)^2 - 64 - (y - 2)^2 + 4 + 44 = 0$

$16(x + 2)^2 - (y - 2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$



$$A = \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right) dx$$

$$A = \int_{-2}^{\sqrt{6}} \left(6 - x^2\right) dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

$$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$$

$$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}$$

$$A = 4\sqrt{6} + \frac{28}{3}$$

19. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3\left(\hat{i} - \hat{j} - \hat{k}\right)$ (2) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

(3) $3\left(\hat{i} - \hat{j} + \hat{k}\right)$ (4) $3\left(\hat{i} + \hat{j} - \hat{k}\right)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\vec{a} \times \vec{b} = (\hat{i} - \hat{j})$

Taking cross product with $\vec{a}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$\Rightarrow \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

20. The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then. Which of the following is NOT correct?

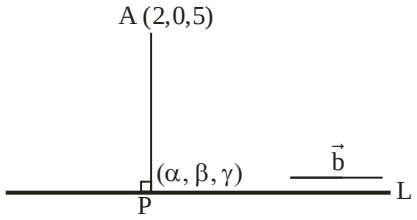
(1) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ (2) $\frac{\alpha}{\beta} = -8$

(3) $\frac{\beta}{\gamma} = -5$ (4) $\frac{\gamma}{\alpha} = \frac{5}{8}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $L: \frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{-1}=\lambda$ (let)



Let foot of perpendicular is

$P(2\lambda-1, 5\lambda+1, -\lambda-l)$

$\overrightarrow{PA}=(3-2\lambda)\hat{i}-(5\lambda+1)\hat{j}+(6+\lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b}=2\hat{i}+5\hat{j}-\hat{k}$

Now, $\Rightarrow \overrightarrow{PA}\cdot\vec{b}=0$

$\Rightarrow 2(3-2\lambda)-5(5\lambda+1)-(6+\lambda)=0$

$\Rightarrow \lambda=\frac{-1}{6}$

$P(2\lambda-1, 5\lambda+1, -\lambda-1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow \alpha=2\left(-\frac{1}{6}\right)-1=-\frac{4}{3} \Rightarrow \boxed{\alpha=-\frac{4}{3}}$

$\Rightarrow \beta=5\left(-\frac{1}{6}\right)+1=\frac{1}{6} \Rightarrow \boxed{\beta=\frac{1}{6}}$

$\Rightarrow \gamma=-\lambda-1=\frac{1}{6}-1 \Rightarrow \boxed{\gamma=-\frac{5}{6}}$

$\therefore$ Check options

SECTION-B

21. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then, 16a + 12b is equal to _____.

Official Ans. by NTA 3

QuestPix Ans. 3

Sol. $a, b, \frac{1}{18} \rightarrow GP$

$\frac{a}{18}=b^2$ ..... (i)

$\frac{1}{a}, 10, \frac{1}{b} \rightarrow AP$

$\frac{1}{a}+\frac{1}{b}=20$

$\Rightarrow a+b=20ab$, from eq. (i) ; we get

$\Rightarrow 18b^2+b=360b^3$

$\Rightarrow 360b^2-18b-1=0 \quad \{\because b\neq 0\}$

$\Rightarrow b=\frac{18\pm\sqrt{324+1440}}{720}$

$\Rightarrow b=\frac{18+\sqrt{1764}}{720} \quad \{\because b>0\}$

$\Rightarrow b=\frac{1}{12}$

$\Rightarrow a=18\times\frac{1}{144}=\frac{1}{8}$

Now, $16a+12b=16\times\frac{1}{8}+12\times\frac{1}{12}=3$

22. Points P(–3,2),Q(9,10) and R(α,4)lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then k is equal to _____.

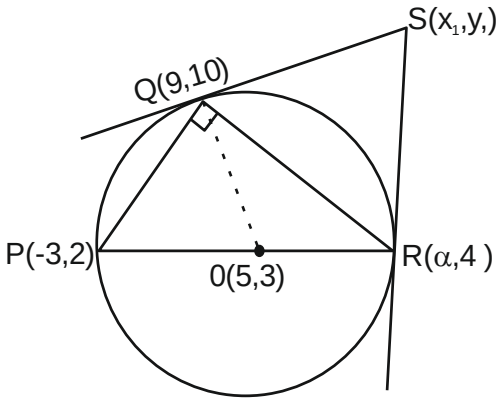
Official Ans. by NTA 3

QuestPix Ans. 3

Sol. $m_{PQ}\cdot m_{QR}=-1$

$\Rightarrow \frac{10-2}{9+3}\times\frac{10-4}{9-\alpha}=-1 \Rightarrow \boxed{\alpha=13}$

$m_{0P}\cdot m_{QS}=-1 \Rightarrow m_{QS}=-\frac{4}{7}$



Equation of QS

$y - 10 = -\frac{4}{7}(x-9)$

$\Rightarrow 4x + 7y = 106$ ….(1)

$m_{0R} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x-13)$

$\Rightarrow 8x + y = 108$ ….(2)

Solving eq. (1) & (2)

$x_1 = \frac{25}{2}$ $y_1=8$

$S(x_1,y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow \boxed{k = 3}$

23. Let $a \in R$ and let β, α be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is _____.

Official Ans. by NTA 45

QuestPix Ans. (45)

Sol. $x^2 + 60^{\frac{1}{4}}x + a = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\alpha + \beta = -60^{\frac{1}{4}}$ & $\alpha\beta = a$

Given $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$

$\Rightarrow \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2a^2 = -30$

$\Rightarrow \left\{60^{\frac{1}{2}} - 2a\right\}^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4a \times 60^{\frac{1}{2}} - 2a^2 = -30$

$\Rightarrow 2a^2 - 4.60^{\frac{1}{2}}a + 90 = 0$

Product $= \frac{90}{2} = 45$

24. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 orange, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is___

Official Ans. by NTA 6860

QuestPix Ans. 6860

Sol. 7 Red apple(RA),5 white apple(WA),8 oranges (O)

5 fruits to be selected (Note:- fruits taken different)

Possible selections :- (2O, 1RA, 2WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)

$\Rightarrow {}^8C_2\,{}^7C_1\,{}^5C_2 + {}^8C_2\,{}^7C_2\,{}^5C_1 + {}^8C_3\,{}^7C_1\,{}^5C_1$

$\Rightarrow 1960 + 2940 + 1960$

$\Rightarrow 6860$

25. If m and n respectively are the numbers of positive and negative value of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to _____.

Official Ans. by NTA 25

QuestPix Ans. 25

Sol. $\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$

$\Rightarrow 2\cos 2\theta \cdot \cos\frac{\theta}{2} = 2\cos\frac{9\theta}{2} \cdot \cos 3\theta$

$\Rightarrow \cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$

$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2}$

$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$

$5\theta = 2k\pi$ or $10\theta = 2k\pi$

$\theta = \frac{2k\pi}{5}$ $\quad$ $\theta = \frac{k\pi}{5}$

$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$

m = 5, n = 5

$\therefore$ m.n = 25

26. If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to _____.

Official Ans. by NTA 20

QuestPix Ans. (20)

Sol. $\int_{\frac{1}{3}}^{3} |\ell nx| dx = \int_{\frac{1}{3}}^{1} (-\ell nx) dx + \int_{1}^{3} (\ell nx) dx$

$= -\left[x\,\ell nx - x\right]_{1/3}^{1} + \left[x\,\ell nx - x\right]_{1}^{3}$

$= -\left[-1 - \left(\frac{1}{3}\ell n\frac{1}{3} - \frac{1}{3}\right)\right] + \left[3\ell n3 - 3 - (-1)\right]$

$= \left[-\frac{2}{3} - \frac{1}{3}\ell n\frac{1}{3}\right] + \left[3\ell n3 - 2\right]$

$= -\frac{4}{3} + \frac{8}{3}\ell n3$

$= \frac{4}{3}(2\ell n3 - 1)$

$= \frac{4}{3}\left(\ell n\frac{9}{e}\right)$

$\therefore$ m = 4, n = 3

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

27. The remainder when $(2023)^{2023}$ is divided by 35 is_____.

Official Ans. by NTA 7

QuestPix Ans. 7

Sol. $(2023)^{2023}$

$= (2030 - 7)^{2023}$

$= (35\,K - 7)^{2023}$

$= {}^{2023}C_0(35\,K)^{2023}(-7)^0 + {}^{2023}C_1(35\,K)^{2022}(-7) + \ldots\ldots + \ldots\ldots + {}^{2023}C_{2023}(-7)^{2023}$

$= 35\,N - 7^{2023}$.

Now, $-7^{2023} = -7 \times 7^{2022} = -7(7^2)^{1011}$

$= -7(50 - 1)^{1011}$

$= -7({}^{1011}C_0\,50^{1011} - {}^{1011}C_1(50)^{1010} + \ldots\ldots\ {}^{1011}C_{1011})$

$= -7(5\lambda - 1)$

$= -35\lambda + 7$

$\therefore$ when $(2023)^{2023}$ is divided by 35 remainder is 7

28. If the shortest distance between the line joining the points(1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to____.

Official Ans. by NTA 18

QuestPix Ans. 18

Sol. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ $\quad \vec{r} = \vec{a} + \lambda\vec{p}$

$\vec{r} = (+\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j})$ $\quad \vec{r} = \vec{b} + \mu\vec{q}$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$

$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

$d = \left|\frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}}\right|$

$= \left|\frac{-6 + 3}{\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$

$\alpha = \frac{3}{\sqrt{14}}$

Now, $28\alpha^2 = \cancel{28}^{2} \times \frac{9}{\cancel{14}} = 18$

29. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer then a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$ .Then the value of k is _____.

Official Ans. by NTA 9

QuestPix Ans. (9)

Sol. E_1 : Smokers

$P(E_1) = \frac{1}{4}$

E_2 : non-smokers

$P(E_2) = \frac{3}{4}$

E : diagnosed with lung cancer

$P(E/E_1) = \frac{27}{28}$

$P(E/E_2) = \frac{1}{28}$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)}$$

$$= \frac{\frac{1}{4}\times\frac{27}{28}}{\frac{1}{4}\times\frac{27}{28}+\frac{3}{4}\times\frac{1}{28}} = \frac{\cancel{27}^{9}}{\cancel{30}_{10}} = \frac{9}{10}$$

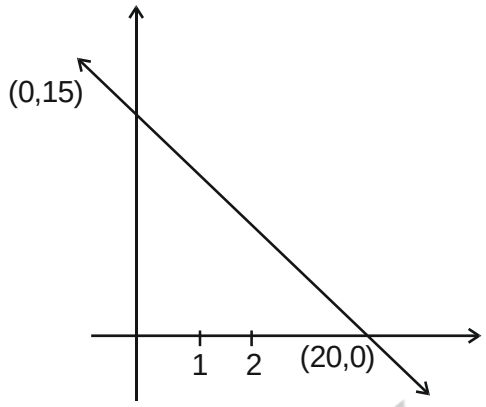
K = 9

30. A triangle is formed by X – axis, Y– axis and the line 3x + 4y = 60. Then the number of points P(a, b)which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is_____.

Official Ans. by NTA 31

QuestPix Ans. (31)

Sol. If x = 1, $y = \frac{57}{4} = 14.25$



(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 pts.

If x = 2, $y = \frac{27}{2} = 13.5$

(2, 2) (2, 4) …. (2, 12) $\Rightarrow$ 6 pts.

If x = 3, $y = \frac{51}{4} = 12.75$

(3, 3) (3, 6) – (3, 12) $\Rightarrow$ 4 pts.

If x = 4, y = 12

(4, 4) (4, 8) $\Rightarrow$ 2 pts.

If x = 5. $y = \frac{45}{4} = 11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If x = 6, $y = \frac{21}{2} = 10.5$

(6, 6) $\Rightarrow$ 1 pt.

If x = 7, $y = \frac{39}{4} = 9.75$

(7, 7) $\Rightarrow$ 1 pt.

If x = 8, y = 9

(8, 8) $\Rightarrow$ 1 pt.

If x = 9 $y = \frac{33}{4} = 8.25$ $\Rightarrow$ no pt.

Total = 31 pts.

(Held On Wednesday 25th Jan, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. Match List I with List II

List		List II	
A.	Young's Modulus (Y)	I.	$[M\ L^{-1}\ T^{-1}]$
B.	Co-efficient of Viscosity (η)	II.	$[M\ L^{2}\ T^{-1}]$
C.	Planck's Constant (h)	III.	$[M\ L^{-1}\ T^{-2}]$
D.	Work Function (ϕ)	IV.	$[M\ L^{2}\ T^{-2}]$

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-I, C-II, D-IV

(3) A-I, B-III, C-IV, D-II

(4) A-I, B-II, C-III, D-IV

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F/A}{\Delta\ell/\ell} = \dfrac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

$F = 6\pi\eta rv \Rightarrow \eta = \dfrac{F}{6\pi rv}$

$[\eta] = \dfrac{[MLT^{-2}]}{[L][LT^{-1}]} = [ML^{-1}T^{-1}]$

$E = h\nu \Rightarrow h = \dfrac{E}{\nu} = \dfrac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

Work function has same dimension as that of energy, so $[\phi] = [ML^2T^{-2}]$

32. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is :-

(1) $\frac{9}{2}R$ (2) $\frac{5}{2}R$ (3) $\frac{3}{2}R$ (4) $\frac{7}{2}R$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Diatomic gas molecules have three translational degree of freedom, two rotational degree of freedom & it is given that it has one vibrational mode so there are two additional degree of freedom corresponding to one vibrational mode, so total degree of freedom = 7

$C_V = \dfrac{fR}{2} = \dfrac{7R}{2}$

33. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are :-

A. Real

B. Erect

C. Smaller in size then object

D. Laterally inverted

Choose the most appropriate answer from the options given below :

(1) B and D only (2) B and C only

(3) A and D only (4) A, C and D only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Plane mirror forms erect, same sized, laterally inverted and virtual image of real object.

34. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of suspension wire is 4.0×10^{-5} Nm rad^{-1}, the magnetic field is 0.01 T and the number of turns in the coil is 200, the area of each turn (in cm^2) is :

(1) 2.0 (2) 1.0

(3) 1.5 (4) 0.5

Official Ans. by NTA (2)

QuestPix Ans. (2)

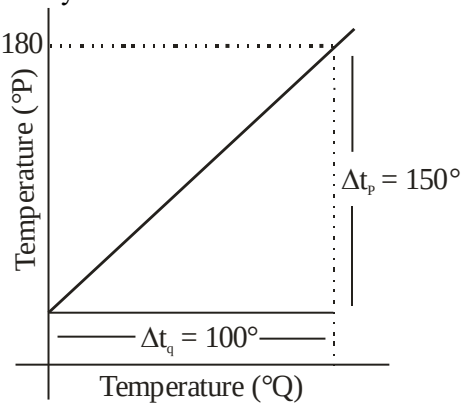
Sol. $\tau = K\theta$

$NiAB = K\theta$

$A = \dfrac{K\theta}{NiB} = \dfrac{4\times10^{-5}\times0.05}{200\times10\times10^{-3}\times0.01}$

On solving $A = 10^{-4}\ m^2 = 1\ cm^2$

35. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by :



(1) $\frac{t_Q}{150} = \frac{t_P - 180}{100}$ (2) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$

(3) $\frac{t_P}{180} = \frac{t_Q - 40}{100}$ (4) $\frac{t_P}{100} = \frac{t_Q - 180}{150}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\frac{\text{reading on scale} - \text{Lower fixed point}}{\text{upper fixed point} - \text{lower fixed point}} = \text{constant}$

$$\frac{t_P - 30}{180 - 30} = \frac{t_Q - 0}{100 - 0}$$

$$\frac{t_P - 30}{150} = \frac{t_Q}{100}$$

36. Match List I with List II :

A.	Gauss's Law in Electrostatics	I.	$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$
B.	Faraday's Law	II.	$\oint \vec{B}.d\vec{A} = 0$
C.	Gauss's Law in Magnetism	III.	$\oint \vec{B}.d\vec{l} = \mu_0 i_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$
D.	Ampere-Maxwell Law	IV.	$\oint \vec{E}.d\vec{s} = \frac{q}{\epsilon_0}$

Choose the correct answer from the options given below :

(1) A-IV, B-I, C-II, D-III

(2) A-I, B-II, C-III, D-IV

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Gauss's Law of electrostatic

$$\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\epsilon_0}$$

Faraday's law $\oint \vec{E}.d\vec{l} = \frac{-d\phi_B}{dt}$

Gauss's law of magnetism $\oint \vec{B}.d\vec{A} = 0$

Ampere's Maxwell law

$$\oint \vec{B}.d\vec{l} = \mu_0 i_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

Where i_C : Conduction current

$\epsilon_0 \frac{d\phi_E}{dt}$: Displacement current

37. **Statement I :** When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.

Statement II : When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.

In the light of above statements, choose the most appropriate answer from the options given below.

Options:

(1) Both Statement I and statement II are incorrect

(2) Statement I is incorrect but statement II is correct

(3) Both Statement I and statement II are correct

(4) Statement I is correct but statement II is incorrect

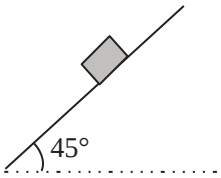
Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Statement – I is correct

When P-N junction is formed an electric field is generated form N-side to P-side due to which barrier potential arises & majority charge carrier can not flow through the junction due to barrier potential so current is zero unless we apply forward bias voltage.

38. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to :



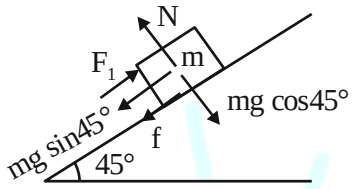
(1) 0.33 (2) 0.60

(3) 0.25 (4) 0.50

Official Ans. by NTA (1)

QuestPix Ans. (1)

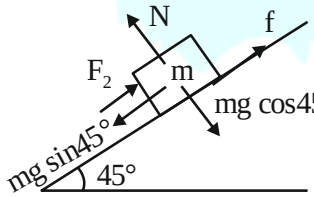
Sol.



$F_1 = mg\sin45° + f = mg\sin45° + \mu N$

$$F_1 = \frac{mg}{\sqrt{2}} + \mu mg\cos 45°$$

$$F_1 = \frac{mg}{\sqrt{2}}(1+\mu)$$



$F_2 = mg\sin 45° - f = mg\sin 45° - \mu N$

$$= \frac{mg}{\sqrt{2}}(1-\mu)$$

$F_1 = 2F_2$

$$\frac{mg}{\sqrt{2}}(1+\mu) = 2\frac{mg}{\sqrt{2}}(1-\mu)$$

$1 + \mu = 2 - 2\mu$

$\mu = 1/3 = 0.33$

39. A point charge of 10 μC is placed at the origin. At what location on the X-axis should a point charge of 40μC be placed so that the net electric field is zero at x = 2 cm on the X-axis ?

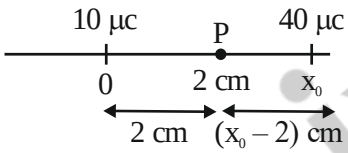
(1) x = 6 cm (2) x = 4 cm

(3) x = 8 cm (4) x = –4 cm

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



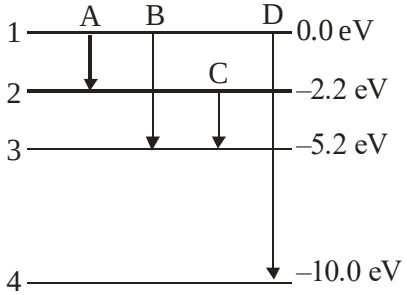
$$E_P = \frac{K\times 10}{2^2} - \frac{K\times 40}{(x_0-2)^2} = 0$$

$$\frac{1}{2} = \frac{2}{x_0 - 2}$$

$$x_0 - 2 = 4$$

$$x_0 = 6\,cm$$

40. The energy levels of an atom is shown is figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm ? Given ($h = 6.62 \times 10^{-34}$ Js)



(1) B (2) A

(3) C (4) D

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\lambda = \frac{hc}{\Delta E}$

$\Delta E_A = 2.2$ eV

$\Delta E_B = 5.2$ eV

$\Delta E_C = 3$ eV

$\Delta E_D = 10$ eV

$\lambda_A = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.2\times1.6\times10^{-19}}$

$= \frac{12.41\times10^{-7}}{2.2}$ m

$= \frac{1241}{2.2}$ nm $= 564$ nm

$\lambda_B = \frac{1241}{5.2}$ nm $= 238.65$ nm

$\lambda_C = \frac{1241}{3}$ nm $= 413.66$ nm

$\lambda_D = \frac{1241}{10} = 124.1$ nm

41. A particle executes simple harmonic motion between x = –A and x = +A. If time taken by particle to go from x = 0 to $\frac{A}{2}$ is 2s; then time taken by particle in going from $x = \frac{A}{2}$ to A is :

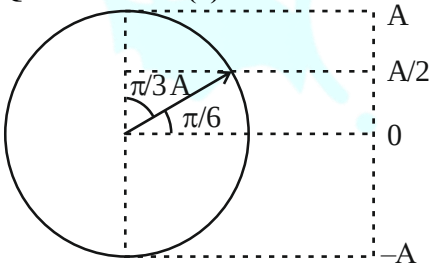
(1) 3 s (2) 2 s

(3) 1.5 s (4) 4 s

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



Let time from 0 to A/2 is t_1

& from A/2 to A is t_2

then $\omega t_1 = \pi/6$

$\omega t_2 = \pi/3$

$\frac{t_1}{t_2} = \frac{1}{2}$

$t_2 = 2t_1 = 2 \times 2 = 4$ sec

42. Match List I with List II :

	List I		List II
A.	Isothermal Process	I.	Work done by the gas decreases internal energy
B.	Adiabatic Process	II.	No change in internal energy
C.	Isochoric Process	III.	The heat absorbed goes partly to increase internal energy and partly to do work
D.	Isobaric Process	IV.	No work is done on or by the gas

Choose the correct answer from the options given below :

(1) A-II, B-I, C-III, D-IV

(2) A-II, B-I, C-IV, D-III

(3) A-I, B-II, C-IV, D-III

(4) A-I, B-II, C-III, D-IV

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\Delta U = nC_v\Delta T$

For isothermal process T is constant

So $\Delta U = 0$

A $\longrightarrow$ II

Adiabatic process

$\Delta Q = 0$

$\Delta Q = \Delta U + \Delta W$

$\Delta U = -\Delta W$

Work done by gas is positive

So ΔU is negative

B $\longrightarrow$ I

For Isochoric process $\Delta W = 0$

C $\longrightarrow$ IV

For Isobaric process

$\Delta W = P\Delta V \neq 0$

$\Delta U = nC_V\Delta T \neq 0$

Heat absorbed goes partly to increase internal energy and partly do work.

43. Match List I with List II

	List I		List II
A.	Troposphere	I.	Approximate 65-75 km over Earth's surface
B.	E-Part of Stratosphere	II.	Approximate 300 km over Earth's surface
C.	F_2-Part of Thermosphere	III.	Approximate 10 km over Earth's surface
D.	D-Part of Stratosphere	IV.	Approximate 100 km over Earth's surface

Choose the correct answer from the options given below :

(1) A-III, B-IV, C-II, D-I

(2) A-I, B-II, C-IV, D-III

(3) A-I, B-IV, C-III, D-II

(4) A-III, B-II, C-I, D-IV

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. NCERT fact based

44. A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be : (g = acceleration due to gravity on the surface of Earth)

(1) $3mgR_e$ (2) $\frac{1}{3}mgR_e$

(3) $\frac{2}{3}mgR_e$ (4) $\frac{1}{2}mgR_e$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $U = \frac{-GM_em}{r}$

$U_i = \frac{-GM_em}{R_e}$

$U_f = \frac{-GM_em}{(R_e+h)} = \frac{-GM_em}{R_e+2R_e}$

$\frac{-GM_em}{3R_e}$

Increase in internal energy $\Delta U = U_f - U_i$

$= \frac{2}{3}\frac{GM_em}{R_e}$

$\frac{2}{3}\frac{GM_e}{R_e^2}mR_e$

$= \frac{2}{3}mgR_e$

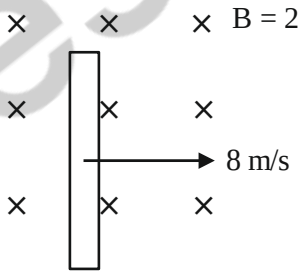
45. A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2T. The magnitude of induced emf, between the ends of wire will be _____ :

(1) 20 V (2) 8 V

(3) 12 V (4) 16 V

Official Ans. by NTA (4)

QuestPix Ans. (4)



Sol. Induced emf across the ends = $Bv\ell$

$= 2 \times 8 \times 1 = 16$ V

46. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at t = 5s is .

(1) 40 ms^{-1} (2) 25 ms^{-1}

(3) 20 ms^{-1} (4) 8 ms^{-1}

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $x = 4t^2$

$v = \frac{dx}{dt} = 8t$

At t = 5 sec

$v = 8 \times 5 = 40$ m/s.

47. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2nd object will be :

(1) 4 : 1 (2) 2 : 1

(3) 1 : 2 (4) 1 : 1

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\text{Range} = \dfrac{u^2 \sin 2\theta}{g}$

Range for projection angle "α"

$R_1 = \dfrac{u^2 \sin 2\alpha}{g}$

Range for projection angle "β"

$R_2 = \dfrac{u^2 \sin 2\beta}{g}$

$\alpha + \beta = 90°$ (Given)

$\Rightarrow \boxed{\beta = 90° - \alpha}$

$R_2 = \dfrac{u^2 \sin 2(90° - \alpha)}{g}$

$R_2 = \dfrac{u^2 \sin(180° - 2\alpha)}{g}$

$R_2 = \dfrac{u^2 \sin 2\alpha}{g}$

$\Rightarrow \dfrac{R_1}{R_2} = \dfrac{\left(\dfrac{u^2 \sin 2\alpha}{g}\right)}{\left(\dfrac{u^2 \sin 2\alpha}{g}\right)} = \dfrac{1}{1}$

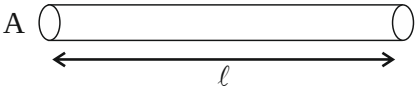
48. The resistance of a wire is 5 Ω. It's new resistance in ohm if stretched to 5 times of it's original length will be :

(1) 625 (2) 5 (3) 125 (4) 25

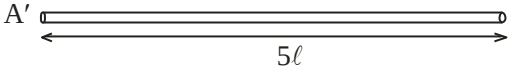
Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



$R_{initial} = \dfrac{\rho \ell}{A} = 5\Omega$



$\because$ Volume of wire is constant in stretching

$V_i = V_f$

$A_i \ell_i = A_f \ell_f$

$A\ell = A'(5\ell)$

$A' = \dfrac{A}{5}$

$R_f = \dfrac{\rho \ell_f}{A_f} = \dfrac{\rho(5\ell)}{\left(\dfrac{A}{5}\right)}$

$= 25\left(\dfrac{\rho \ell}{A}\right)$

$= 25 \times 5 = 125\ \Omega$

49. Given below are two statements :

Statement I : Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II : For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below.

Options :

(1) Statement I is incorrect but statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but statement II is incorrect

(4) Both statement I and statement II are correct

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Stopping potential $V_S = \dfrac{KE_{max}}{e}$

$V_S = \dfrac{\dfrac{hC}{\lambda} - \phi}{e}$

Stopping potential does not depend on intensity or power of light used, it only depends on frequency or wavelength of incident light.

So both statements I and II are correct

50. Every planet revolves around the sun in an elliptical orbit :

A. The force acting on a planet is inversely proportional to square of distance from sun.

B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun

C. The centripetal force acting on the planet is directed away from the sun.

D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

Choose the correct answer from the options given below :

Options :

(1) A and D only (2) C and D only

(3) B and C only (4) A and C only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $F = \frac{Gm_1m_2}{r^2}$

$\Rightarrow F \propto \frac{1}{r^2}$

$\Rightarrow F \propto m_1m_2$

$\Rightarrow$ This force provides centripetal force and acts towards sun

$\Rightarrow T^2 \propto a^3$ (Kepler's third law)

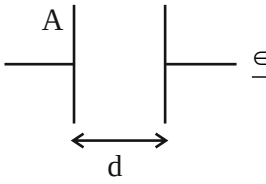
SECTION-B

51. A capacitor has capacitance 5μF when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be _____μF.

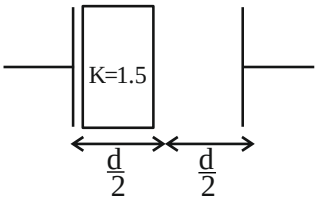
Official Ans. by NTA (6)

QuestPix Ans. (6)

Sol.



$\frac{\epsilon_0 A}{d} = 5\mu F$



$C_{new} = \frac{\epsilon_0 A}{\frac{\left(\frac{d}{2}\right)}{1.5} + \frac{\left(\frac{d}{2}\right)}{1}}$

$= \frac{\epsilon_0 A}{\left(\frac{d}{3} + \frac{d}{2}\right)} = \frac{6\epsilon_0 A}{5d}$

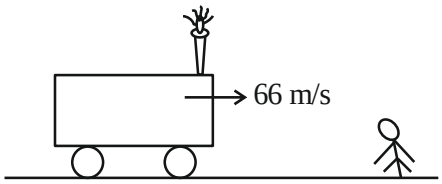
$= \frac{6}{5} \times 5\mu F = 6\mu F$

52. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms^{-1}) _____ Hz.

Official Ans. by NTA (400)

QuestPix Ans. (400)

Sol.

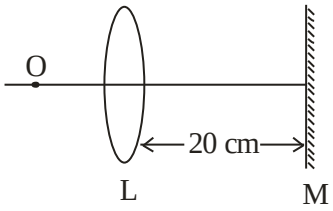


$f_{app} = f\left(\frac{v}{v - v_s}\right)$

$= 320\left(\frac{330}{330 - 66}\right)$

$= 400$ Hz

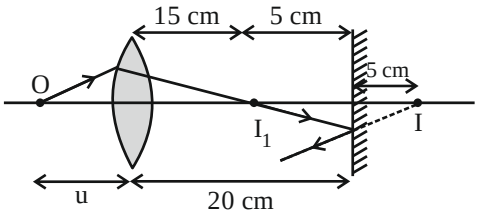
53. An object is placed on the principal axis of convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is ____ cm.



Official Ans. by NTA (30)

QuestPix Ans. (30)

Sol.



$f = 10$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{15}-\frac{1}{-u}=\frac{1}{10}$$

$$\Rightarrow \frac{1}{u}=\frac{1}{10}-\frac{1}{15}$$

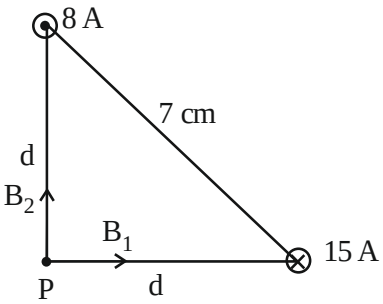
On solving we get value of u as 30 cm.

54. Two long parallel wires carrying currents 8A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is ____ $\times 10^{-6}$ T. (Given : $\sqrt{2} = 1.4$)

Official Ans. by NTA (68)

QuestPix Ans. (68)

Sol.



Magnetic fields due to both wires will be perpendicular to each other.

$$B_1 = \frac{\mu_0 i_1}{2\pi d} \quad B_2 = \frac{\mu_0 i_2}{2\pi d}$$

$$B_{net} = \sqrt{B_1^2 + B_2^2} \Rightarrow \frac{\mu_0}{2\pi d}\sqrt{i_1^2 + i_2^2}$$

$$\Rightarrow \frac{4\pi\times10^{-7}}{2\pi\times(7/\sqrt{2})\times10^{-2}}\times\sqrt{8^2+15^2} \quad \left(d = \frac{7}{\sqrt{2}} \text{ cm}\right)$$

$\Rightarrow 68 \times 10^{-6}$ T

55. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation). $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is _____ :

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Surface Tension = T

R : Radius of bigger drop

r : Radius of smaller drop

Volume will remain same

$$\frac{4}{3}\pi R^3 = 1000\times\frac{4}{3}\pi r^3$$

$R = 10\ r$

$u_i = T.\ 4\pi R^2$

$u_f = T.\ 4\pi r^2 \times 1000$

$$\frac{u_f}{u_i} = \frac{1000r^2}{R^2}$$

$$\frac{u_f}{u_i} = \frac{10}{1}$$

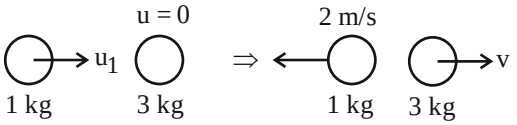
So, x = 1

56. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2m/s. The initial speed of the smaller body before collision is _____ ms^{-1}.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



$1 \times u_1 = -2 + 3v \Rightarrow u_1 = -2 + 3v$ ..... (1)

$1 = \frac{v+2}{u_1} \Rightarrow v + 2 = u_1$ ..... (2)

Solving (1) and (2)

$u_1 = 4$ m/s

57. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3 : 2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of 'x' is :

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

$\frac{v_1}{v_2} = \frac{3}{2}$

$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$

Since, Nuclear mass density is constant

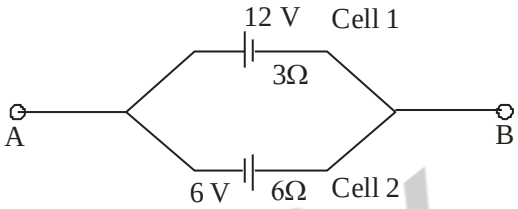
$\frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$

$\left(\frac{r_1}{r_2}\right)^3 = \frac{m_1}{m_2}$

$\frac{r_1}{r_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$

So, x = 2

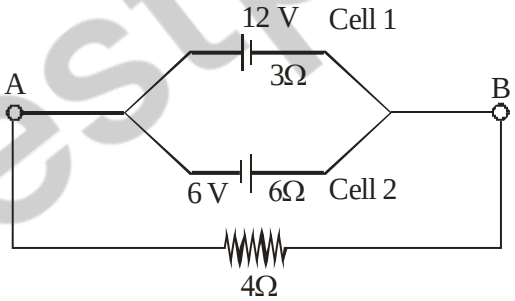
58. Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3Ω. Cell 2 has emf of 6V and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be _____ A.



Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

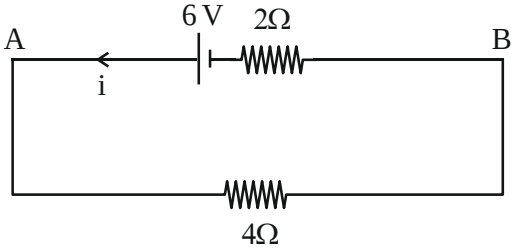


$E_{eq} = \frac{\frac{12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}}$

$E_{eq} = 6V$

$r_{eq} = 2\ \Omega$

$R = 4\ \Omega$



So, $i = \frac{6}{2+4} = 1$ A

59. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance R = 80Ω, an inductor of inductive reactance X_L = 70 Ω, and a capacitor of capacitive reactance X_C = 130 Ω. The power factor of circuit is $\frac{x}{10}$.

The value of x is :

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$

$\cos\phi = \frac{80}{\sqrt{(80)^2 + (60)^2}}$

$\cos\phi = \frac{80}{100} \Rightarrow \frac{8}{10}$

So, x = 8

60. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is _____.

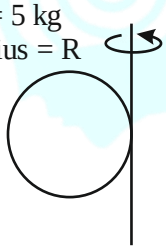
Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol.

m_1 = 5 kg
Radius = R

Solid sphere



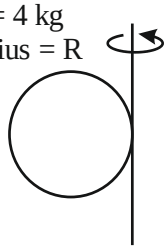
$I_1 = \frac{2}{5}m_1R^2 + m_1R^2$

$I_1 = m_1R^2\left(\frac{7}{5}\right)$

$I_1 = 7R^2$

m_2 = 4 kg
Radius = R

Disc



$I_2 = \frac{m_2R^2}{4} + m_2R^2$

$I_2 = \frac{5}{4}m_2R^2$

$I_2 = 5R^2$

$\frac{I_2}{I_1} = \frac{5}{7}$

x = 5

(Held On Wednesday 25th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

61. Match List I with List II

	List I		List II
A.	Cobalt catalyst	I.	(H_2 + Cl_2) production
B.	Syngas	II.	Water gas production
C.	Nickel catalyst	III.	Coal gasification
D.	Brine solution	IV.	Methanol production

Choose the correct answer from the options given below :-

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-IV, D-I

(4) A-IV, B-III, C-II, D-I

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Cobalt catalyst $\rightarrow$ Methanol production

Syn gas $\rightarrow$ Coal gasification

$\left(C_{(\text{Red hot coke})} + H_2O(g) \rightarrow CO + H_2\right)$

Nickel catalyst $\rightarrow$ Water gas production

Brine solution $\rightarrow$ Production

(aq. NaCl) $\begin{pmatrix} H_2 \rightarrow \text{Cathode} \\ Cl_2 \rightarrow \text{anode} \end{pmatrix}$

62. Given below are two statements :-

Statement I :- In froth floatation method a rotating paddle agitates the mixture to drive air out of it.

Statement II :- Iron pyrites are generally avoided for extraction of iron due to environmental reasons.
In the light of the above statements, choose the correct answer from the options given below :-

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. In froth floatation method a rotating paddle draws in air and stirs the pulp.

63. Which of the following represents the correct order of metallic character of the given elements ?

(1) Si < Be < Mg < K (2) Be < Si < Mg < K

(3) K < Mg < Be < Si (4) Be < Si < K < Mg

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Metallic character increases down the group and decreases along the period.

64. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R

Assertion A :- The alkali metals and their salts impart characteristic colour to reducing flame.

Reason R :- Alkali metals can be detected using flame tests.

In the light of the above statements, choose the most appropriate answer form the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A.

(2) A is correct but R is not correct.

(3) A is not correct but R is correct

(4) Both A and R are correct and R is the correct explanation of A.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. The alkali metals and their salts impart characteristic colour to **oxidizing** flame.

65. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g

of 0.25 molal aqueous solution and 250 mL of 0.25 molar aqueous solution ?

(1) 1 : 1 (2) 3 : 1

(3) 2 : 1 (4) 1 : 2

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Assume : Mass of solvent ≈ Mass of solution

Case I :-

$$0.25 = \frac{W_1}{62} \times \frac{1000}{500}$$

Case II :-

$$0.25 = \frac{W_2}{62} \times \frac{1000}{250}$$

$$\frac{W_1}{W_2} = \frac{2}{1}$$

66. **Statement I :-** Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.

Statement II :- The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Both Statement I and Statement II are correct.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is correct but Statement II is incorrect.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Statement II : The corssed arrow symbolises the direction of the shift of electron density in the molecule.

67. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A :- Butylated hydroxyl anisole when added to butter increases its shelf life.

Reason R :- Butylated hydroxyl anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Both A and R are correct and R is the correct explanation of A.

(2) A is correct but R is not correct.

(3) A is not correct but R is correct.

(4) Both A and R are correct but R is NOT the correct explanation of A.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Butylated hydroxyl anisole is an antioxidant.

68. A. Ammonium salts produce haze in atmosphere.

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.

C. Polychlorinated biphenyls act as cleansing solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.

Choose the correct answer from the options given below :-

(1) A, B and C only

(2) B and C only

(3) A and D only

(4) A and C only **Official**

Ans. by NTA (4)

QuestPix Ans. (4)

Sol. **B.** $\dot{Cl} + O_3 \longrightarrow O_2 + Cl\dot{O}$

D. 'Blue baby' syndrome occurs due to the presence of excess of nitrate ions in water.

69. Match List I with List II

	List I (Amines)		List II (pK_b)
A.	Aniline	I.	3.25
B.	Ethanamine	II.	3.00
C.	N-Ethylethanamine	III.	9.38
D.	N, N-Diethylethanamine	IV.	3.29

Choose the correct answer from the options given below :-

(1) A-I, B-IV, C-II, D-III

(2) A-III, B-II, C-I, D-IV

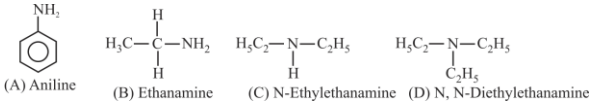
(3) A-III, B-II, C-IV, D-I

(4) A-III, B-IV, C-II, D-I

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



$$\text{Basic Strength } \alpha \frac{1}{pK_b}$$

Order for pK_b : A > B > D > C

70. Which one among the following metals is the weakest reducing agent ?

(1) K (2) Rb

(3) Na (4) Li

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Sodium have lowest oxidation potential in alkali metals. Hence it is weakest reducing agent among alkali metals.

71. Match List I with List II.

	List I Isomeric pairs		List II Type of isomers
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

Choose the correct answer from the options given below :-

(1) A-III, B-IV, C-I, D-II

(2) A-IV, B-III, C-I, D-II

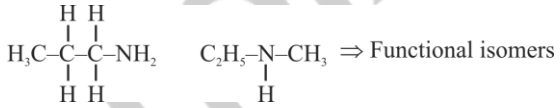
(3) A-II, B-III, C-I, D-IV

(4) A-III, B-I, C-IV, D-II

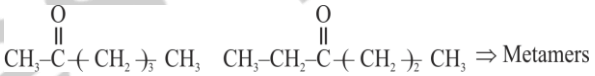
Official Ans. by NTA (4)

QuestPix Ans. (4)

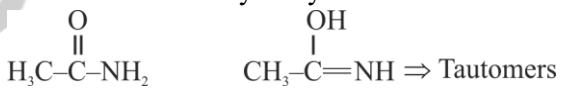
Sol. A. Propanamine N–Methylethanamine



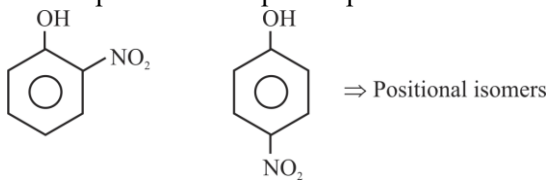
B. Hexan–2–one Hexan–3–one



C. Ethanamide Hydroxyethanimine



D. o–Nitrophenol p–nitrophenol



72. Match List I with List II

	List I (Name of polymer)		List II (Uses)
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III.	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below :-

(1) A-III, B-II, C-IV, D-I

(2) A-III, B-IV, C-II, D-I

(3) A-III, B-IV, C-I, D-II

(4) A-III, B-I, C-IV, D-II

Official Ans. by NTA (2)

QuestPix Ans. (2)

73. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A :- Carbon forms two important oxides – CO and CO_2. CO is neutral whereas CO_2 is acidic in nature.

Reason R :- CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water.

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Both A and R are correct but R is NOT the correct explanation of A.

(2) Both A and R are correct and R is the correct explanation of A.

(3) A is not correct but R is correct.

(4) A is correct but R is not correct.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. The oxide which form acid on dissolving in water is acidic oxide.

74. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

(1) + 3 to + 1 (2) + 6 to + 3

(3) + 2 to + 1 (4) + 6 to + 2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $14H^+ + 6e^- + \underset{+6}{Cr_2O_7^{-2}} \longrightarrow 2\underset{+3}{Cr^{+3}} + 7H_2O$

75. When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000, the value of pH of the solution _____.

(1) increases by 1000 units

(2) decreases by 3 units

(3) decreases by 2 units

(4) increases by 2 units

Official Ans. by NTA (4)

QuestPix Ans. (2)

Sol. $\Delta[H^+] = 1000$

$$\Delta pH = -\log \Delta[H^+] = -\log 10^3$$

$$= -3$$

76. Match List I with List II

	List I Coordination entity		List II Wavelength of light absorbed in nm
A.	$[CoCl(NH_3)_5]^{2+}$	I.	310
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	535
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

Choose the correct answer from the options given below :-

(1) A-IV, B-I, C-III, D-II

(2) A-III, B-II, C-I, D-IV

(3) A-III, B-I, C-II, D-IV

(4) A- II, B-III, C-IV, D-I

Official Ans. by NTA (2)

QuestPix Ans. (2)

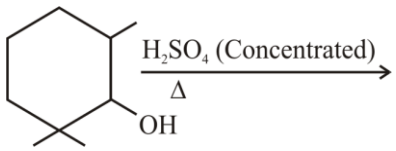
Sol.

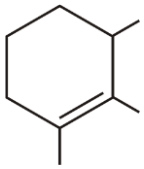
	List I Coordination entity		List II Wavelength of light absorbed in nm
A.	$[CoCl(NH_3)_5]^{2+}$	I.	535
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	310
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

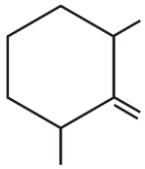
$$E = \frac{hc}{\lambda} \Rightarrow E \propto \frac{1}{\lambda}$$

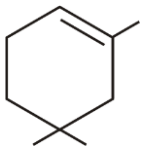
$$\Rightarrow \Delta(CFSE) \propto \frac{1}{\lambda_{absorb}} \propto \text{strength of ligand.}$$

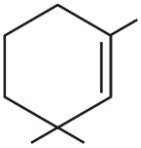
77. Find out the major product from the following reaction.



(1) 

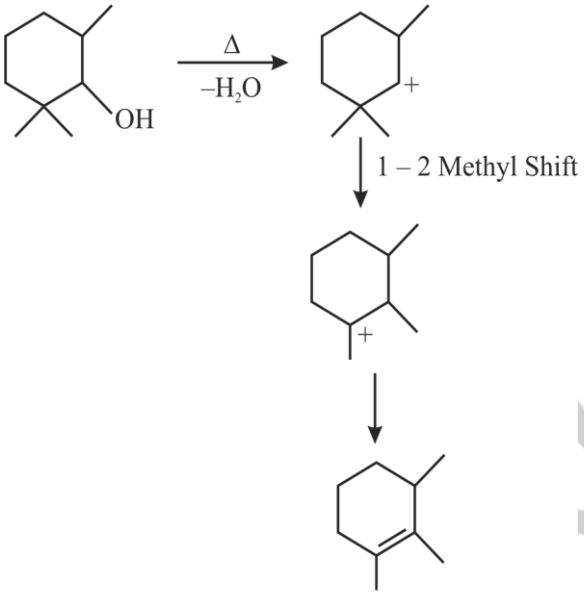
(2) 

(3) 

(4) 

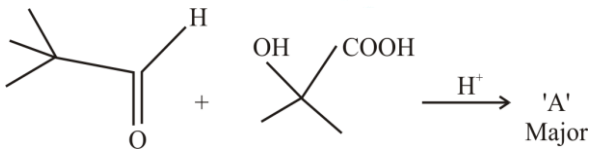
Official Ans. by NTA (1)

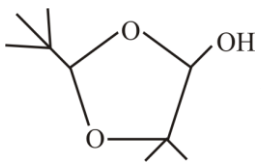
QuestPix Ans. (1)

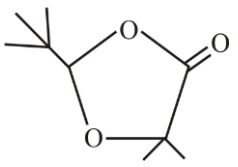
Sol. 

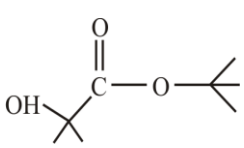
Thermodynamically controlled product

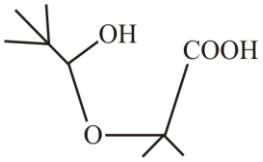
78. 'A' in the given reaction is



(1) 

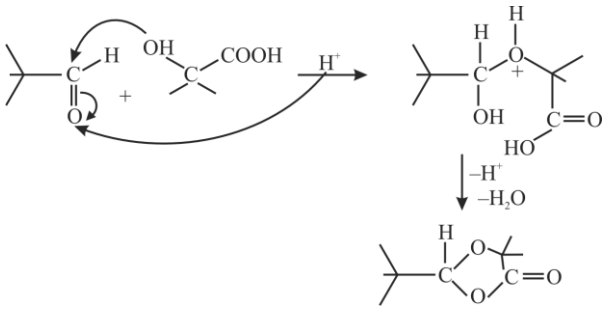
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. 

79. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is

(1) 2–Bromo–1–deuterobutane

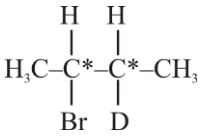
(2) 2–Bromo–2–deuterobutane

(3) 2–Bromo–3–deuterobutane

(4) 2–Bromo–1–deutero–2–methylpropane

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. 

80. A chloride salt solution acidified with dil. HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution, B.

(1) $H[AgCl_3]$ & $[Ag(NH_3)_2]Cl$

(2) $H[AgCl_3]$ & $(NH_4)[Ag(OH)_2]$

(3) AgCl & $[Ag(NH_3)_2]Cl$

(4) AgCl & $(NH_4)[Ag(OH)_2]$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $Cl^- + AgNO_3 \longrightarrow \underset{\text{Curdy white precipitate}}{\overset{[A]}{AgCl}}$

$$AgCl + NH_4OH \longrightarrow \underset{\text{(Soluble Complex)}}{\overset{[B]}{\left[Ag(NH_3)_2\right]Cl}}$$

SECTION-B

81. The number of given orbitals which have electron density along the axis is _____

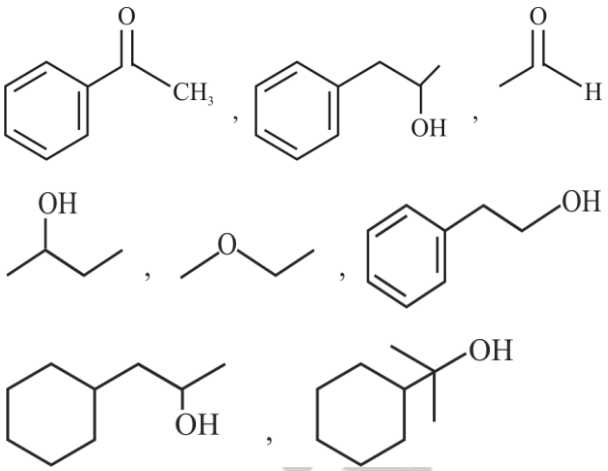
$p_x, p_y, p_z, d_{xy}, d_{yz}, d_{xz}, d_{z^2}, d_{x^2-y^2}$

Official Ans. by NTA (5.00)

QuestPix Ans. (5.00)

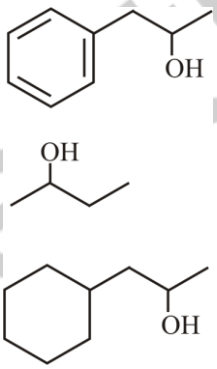
Sol. p_x, p_y, p_z, d_{z^2} & $d_{x^2-y^2}$ are axial orbitals.**52.**

82. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is _____



Official Ans. by NTA (3.00)

QuestPix Ans. (3.00)

Sol. 

83. The number of pairs of the solution having the same value of the osmotic pressure from the following is _____.

(Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M NaCl (aq)

D. 0.15 M NaCl (aq) and 0.1 M $BaCl_2$ (aq)

E. 0.02 M KCl. $MgCl_2$. $6H_2O$ (aq) and 0.05 M KCl (aq)

Official Ans. by NTA (4.00)

QuestPix Ans. (4.00)

Sol. $\pi = iCRT$

$\pi \propto iC$

A, B, D and E have same value of osmatic pressure.

84. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is _____ kJ.

Given : $\Delta H_C\ (CH_4) = -900\ kJ\ mol^{-1}$

$\Delta H_C\ (C_2H_4) = -1400\ kJ\ mol^{-1}$

Official Ans. by NTA (925.00)

QuestPix Ans. (847.00)

Sol. Let, Volume of C_2H_4 is x litre

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$$

Initial x

Final – 2x

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Initial (16.8 – x)

Final – (16.8 – x)

Total volume of $CO_2 = 2x + 16.8 - x$

$\Rightarrow$ $28 = 16.8 + x$

$x = 11.2$ L

$$n_{CH_4} = \frac{PV}{RT} = \frac{1\times5.6}{0.082\times298} = 0.229 \text{ mole}$$

$$n_{C_2H_2} = \frac{11.2}{0.082\times298} = 0.458 \text{ mole}$$

$\therefore$ Heat evolved = 0.229 × 900 + 0.458 × 1400

= 206.1 + 641.2

= 847.3 kJ

85. Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is

Official Ans. by NTA (5.00)

QuestPix Ans. (5.00)

Sol. $[Co(NH_3)_4Cl_2]Cl \Rightarrow$ Gives 1 mole AgCl

$[Ni(H_2O)_6]Cl_2 \Rightarrow$ Gives 2 moles AgCl

$[Pt(NH_3)_2Cl_2] \Rightarrow$ Gives No AgCl

$[Pd(NH_3)_4]Cl_2 \Rightarrow$ Gives 2 moles AgCl

Total number of moles of AgCl = 5 mole.

86. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is _____

(Given : Molar mass of A = 84 g mol^{-1})

Official Ans. by NTA (12.00)

QuestPix Ans. (12.00)

Sol.

Element	Percentage	Mole	Mole ratio
C	85.8	$\frac{85.8}{12} = 7.15$	1
H	14.2	$\frac{14.2}{1} = 14.2$	2

Empirical formula (CH_2)

$14 \times n = 84$

$n = 6$

$\therefore$ Molecular formula C_6H_{12}

87. $Pt(s)H_2(g)(1bar)|H^+(aq)(1M)||M^{3+}(aq),M^+(aq)|Pt(s)$

The E_{cell} for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$

The value of a is _____

Given : $E^\theta_{M^{3+}/M^+} = 0.2$ V

$\frac{2.303\ RT}{F} = 0.059$ V

Official Ans. by NTA (3.00)

QuestPix Ans. (3.00)

Sol. Overall reaction :–

$$H_{2(g)} + M^{3+}_{(aq)} \longrightarrow M^{+}_{(aq)} + 2H^{+}_{(aq)}$$

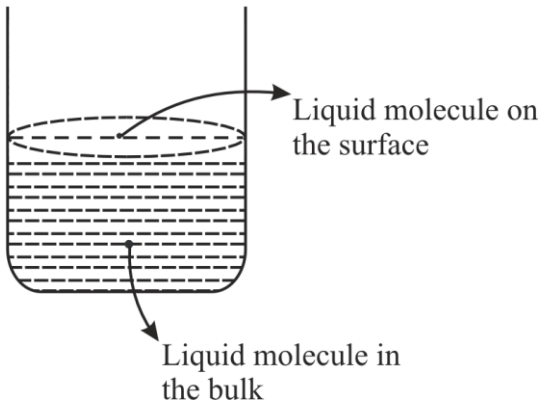
$$E_{Cell} = E^\circ_{Cathode} - E^\circ_{anode} - \frac{0.059}{2}\log\frac{[M^+]\times1^2}{[M^{+3}]\,1}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log\frac{[M^{+}]}{[M^{+3}]}$$

$$3 = \log\frac{[M^{+}]}{[M^{+3}]}$$

$\therefore$ a = 3

88. Based on the given figure, the number of **correct** statement/s is/are _____



A. Surface tension is the outcome of equal attractive and repulsion forces acting on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

C. The molecule in the bulk can never come to the liquid surface.

D. The molecules on the surface are responsible for vapour pressure if the system is a closed system.

Official Ans. by NTA (2.00)

QuestPix Ans. (2.00)

Sol. B and D options are correct.

89. A first order reaction has the rate constant, $k = 4.6 \times 10^{-3}\ s^{-1}$. The number of correct statement/s from the following is/are _____.

Given : log 3 = 0.48

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to $(1 - e^{-kt})$.

E. The rate and the rate constant have the same unit.

Official Ans. by NTA (2.00)

QuestPix Ans. (2.00)

Sol. $t_{10\%} = \frac{1}{K}\ln\left(\frac{a}{a-x}\right) = \frac{1}{K}\ln\left(\frac{100}{90}\right)$

$$t_{10\%} = \frac{2.303}{K}(\log 10 - \log 9)$$

$$t_{10\%} = \frac{2.093}{K} \times (0.04)$$

Similarly

$$t_{90\%} = \frac{1}{K}\ln\left(\frac{100}{10}\right)$$

$$t_{90\%} = \frac{2.303}{K}$$

$$\frac{t_{90\%}}{t_{10\%}} = \frac{1}{0.04} = 25$$

$$e^{kt} = \frac{a}{a-x}$$

$$\frac{a-x}{a} = e^{-kt}$$

$$1 - \frac{x}{a} = e^{-kt}$$

$$x = a(1 - e^{-kt})$$

$$\alpha = \frac{x}{a} = \left(1 - e^{-kt}\right)$$

90. The number of **incorrect** statement/s from the following is/are _____

A. Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C. As the adsorption proceeds, ΔH becomes more and more negative.

D. Adsorption is accompanied by decrease in entropy of the system.

Official Ans. by NTA (2.00)

QuestPix Ans. (2.00)

Sol. 'A' water vapours are absorbed by calcium chloride.

C. As the adsorption proceeds, ΔH becomes less and less negative.