(Held On Wednesday 1st February, 2023) **TIME : 9 : 00 AM to 12 : 00**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. $\lim\limits_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right)$ is equal to :-

(1) 0 (2) $\log_e 2$

(3) $\log_e\left(\frac{3}{2}\right)$ (4) $\log_e\left(\frac{2}{3}\right)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\lim\limits_{n\to\infty}\left(\frac{1}{1+n}+....+\frac{1}{n+n}\right) = \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n+r}$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$

$$= \int_0^1 \frac{1}{1+x}dx = [\ell n(1+x]_0^1 = \ell n2$$

2. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $(\sim p) \wedge (\sim q)$ (2) $p \wedge (\sim q)$

(3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \vee q$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\sim(q \vee ((\sim q) \wedge p))$

$= \sim q \wedge \sim((\sim q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= (\sim q \wedge \sim p)$

3. In a binomial distribution B(n, p), the sum and product of the mean & variance are 5 and 6 respectively, then find 6(n + p – q) is equal to :-

(1) 51

(2) 52

(3) 53

(4) 50

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $np + npq = 5,\ np \cdot npq = 6$

$np(1 + q) = 5,\ n^2p^2q = 6$

$n^2p^2(1 + q)^2 = 25,\ n^2p^2q = 6$

$\frac{6}{q}(1 + q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 – 13q + 6 = 0$

$6q^2 – 9q – 4q + 6 = 0$

$(3q – 2)(2q – 3) = 0$

$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3}$ is accepted

$p = \frac{1}{3} \Rightarrow n.\frac{1}{3} + n.\frac{1}{3}.\frac{2}{3} = 5$

$\frac{3n + 2n}{9} = 5$

$n = 9$

So $6(n + p – q) = 6\left(9 + \frac{1}{3} – \frac{2}{3}\right) = 52$

4. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+....$ is:-

(1) $\frac{59}{111}$ (2) $\frac{55}{111}$

(3) $\frac{56}{111}$ (4) $\frac{58}{111}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $T_r = \frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$

$$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$$

$$T_3 = \frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right]$$

$$\vdots$$

$$T_{10} = \frac{1}{2}\left[\frac{1}{91}-\frac{1}{111}\right]$$

$$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1-\frac{1}{111}\right] = \frac{55}{111}$$

5. The value of

$$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+....+\frac{1}{49!2!}+\frac{1}{51!1!}$$ is

(1) $\frac{2^{50}}{50!}$ (2) $\frac{2^{50}}{51!}$

(3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{51}}{50!}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $$\sum_{r=1}^{26}\frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)}\frac{1}{51!}$$

$$= \frac{1}{51!}\{{}^{51}C_1 + {}^{51}C_3 + ... + {}^{51}C_{51}\} = \frac{1}{51!}(2^{50})$$

6. If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is

(1) $x^2 - 19x + 90 = 0$

(2) $x^2 - 18x + 80 = 0$

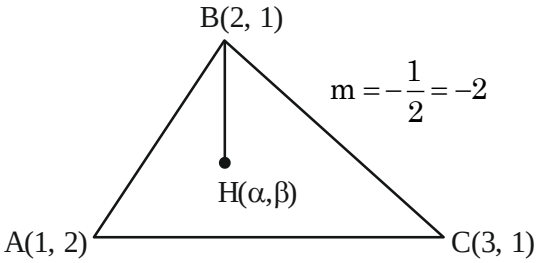
(3) $x^2 - 22x + 120 = 0$

(4) $x^2 - 20x + 99 = 0$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



$$m = -\frac{1}{2}$$

Here $m_{BH} \times m_{AC} = -1$

$$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$

$m_{AH} \times m_{BC} = -1$

$$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1$$

$\Rightarrow 2\beta - 4 = \alpha - 1$

$\Rightarrow 2(2\alpha - 1) = \alpha + 3$

$\Rightarrow 3\alpha = 5$

$$\alpha = \frac{5}{3}, \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$$

$$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$$

$$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$$

$x^2 - 20x + 99 = 0$

7. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) Perimeter of ΔABC is $18\sqrt{3}$

(2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

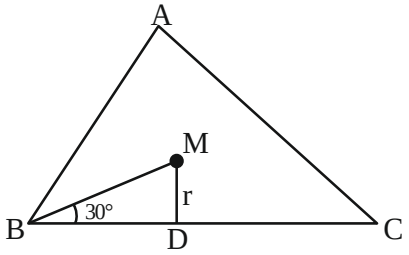
(3) $\overrightarrow{MA}.\overrightarrow{MB} = -18$

(4) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



If $\cos 2A + \cos 2B + \cos 2C$ is minimum then $A = B = C = 60^\circ$

So ΔABC is equilateral

Now in-radias $r = 3$

So in ΔMBD we have

$\text{Tan}30^\circ = \frac{MD}{BD} = \frac{r}{a/2} = \frac{6}{a}$

$1/\sqrt{3} = \frac{1}{a} = a = 6\sqrt{3}$

Perimeter of Δ ABC = $18\sqrt{3}$

Area of Δ ABC = $\frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

8. The combined equation of the two lines ax + by + c = 0 and a′x + b′y + c′ = 0 can be written as (ax + by + c) (a′x + b′y + c′) = 0

The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is

(1) $3x^2 + 5xy + 2y^2 = 0$

(2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + xy - 2y^2 = 0$

(4) $x^2 - y^2 - 10xy = 0$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

Equation of the pair of angle bisector for the homogenous equation $ax^2 + 2hxy + by^2 = 0$ is given as

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Here a = 2, h = ½ & b = – 3

Equation will become

$$\frac{x^2 - y^2}{2 - (-3)} = \frac{xy}{1/2}$$

$x^2 - y^2 = 10xy$

$x^2 - y^2 - 10xy = 0$

9. The shortest distance between the lines $\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and $\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is

(1) $7\sqrt{3}$

(2) $5\sqrt{3}$

(3) $6\sqrt{3}$

(4) $4\sqrt{3}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.

Shortest distance between two lines

$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3}$ &

$\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3}$ is given as

$$\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{(a_1b_3 - a_3b_2)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

$$\frac{\begin{vmatrix} 5-(3) & 2-(-5) & 4-1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(-10+12)^2 + (-5+3)^2 + (4-2)^2}}$$

$$\frac{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(2)^2 + (2)^2 + (2)^2}}$$

$$= \frac{|8(-10+12) - 7(-5+3) + 3(4-2)|}{\sqrt{4+4+4}}$$

$$= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

10. Let S denote the set of all real values of λ such that= the system of equations

λx + y + z = 1

x + λy + z = 1

x + y + λz = 1

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

(1) 2

(2) 12

(3) 4

(4) 6

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\begin{vmatrix}\lambda & 1 & 1\\ 1 & \lambda & 1\\ 1 & 1 & \lambda\end{vmatrix}=0$

$$(\lambda+2)\begin{vmatrix}1 & 1 & 1\\ 1 & \lambda & 1\\ 1 & 1 & \lambda\end{vmatrix}=0$$

$(\lambda+2)[1(\lambda^2-1)-1(\lambda-1)+(1-\lambda)]=0$

$(\lambda+2)[(\lambda^2-2\lambda+1)=0$

$(\lambda+2)(\lambda-1)^2=0 \Rightarrow \lambda=-2, \lambda=1$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

so $\sum(|-2|^2+|-2|)=6$

11. Let

$$S=\left\{x: x\in\mathbb{R} \text{ and } \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\right\}.$$

Then n (S) is equal to

(1) 2 (2) 4

(3) 6 (4) 0

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

$$t+\frac{1}{t}=10$$

$\Rightarrow$ $t=5+2\sqrt{6}, 5-2\sqrt{6}$

$\Rightarrow$ $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=5+2\sqrt{6},\ 5-2\sqrt{6}$

$\Rightarrow$ $x^2-4=2,-2$ or $x^2=6,2$

$\Rightarrow$ $x=\pm\sqrt{2}, \pm\sqrt{6}$

12. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi$, $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

Then $\sum_{x\in S}2\sin^{-1}(x^2-1)$ is equal to

(1) 0 (2) $\frac{-2\pi}{3}$

(3) $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\pi-2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\cos^{-1}(2x)-2\cos^{-1}\sqrt{1-x^2}=\pi$

$\cos^{-1}(2x) - \cos^{-1}(2(1-x^2)-1) = \pi$

$\cos^{-1}(2x) - \cos^{-1}(1-2x^2) = \pi$

$-\cos^{-1}(1-2x^2) = \pi - \cos^{-1}(2x)$

Taking cos both sides we get

$\text{Cos}(-\cos^{-1}(1-2x^2)) = \cos(\pi-\cos^{-1}(2x))$

$1-2x^2 = -2x$

$2x^2-2x-1=0$

On solving, $x=\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}$

As $x=[-1/2, 1/2]$, $x=\frac{1+\sqrt{3}}{2}$ = rejected

So $x=\frac{1-\sqrt{3}}{2} \Rightarrow x^2-1=-\sqrt{3}/2$

$= 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{-2\pi}{3}$

13. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right|=2$ are respectively (α, β) and γ, then $3(\alpha+\beta+\gamma)$ is equal to

(1) 11

(2) 9

(3) 10

(4) 12

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

$$\sqrt{(x-2)^2+y^2}=2\sqrt{(x-3)^2+y^2}$$

$= x^2+y^2-4x+4 = 4x^2+4y^2-24x+36$

$= 3x^2+3y^2-20x+32=0$

$= x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0$

$= (\alpha,\beta)=\left(\frac{10}{3},0\right)$

$\gamma=\sqrt{\frac{100}{9}-\frac{32}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

$3(\alpha,\beta,\gamma)=3\left(\frac{10}{3}+\frac{2}{3}\right)$

$= 12$

14. If y = y(x) is the solution curve of the differential equation $\frac{dy}{dx} + y \tan x = x \sec x$, $0 \le x \le \frac{\pi}{3}$, $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Here I.F. = sec x

Then solution of D.E :

$y(\sec x) = x \tan x - \ln(\sec x) + c$

Given $y(0) = 1 \Rightarrow c = 1$

$\therefore\ y(\sec x) = x \tan x - \ln(\sec x) + 1$

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

15. Let R be a relation on $\mathbb{R}$, given by $R = \{(a, b) : 3a - 3b + \sqrt{7}$ is an irrational number$\}$. Then R is

(1) Reflexive but neither symmetric nor transitive

(2) Reflexive and transitive but not symmetric

(3) Reflexive and symmetric but not transitive

(4) An equivalence relation

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Check for reflexivity:

As 3(a – a) + $\sqrt{7}$ = $\sqrt{7}$ which belongs to relation so relation is reflexive

Check for symmetric:

Take $a = \frac{\sqrt{7}}{3}, b = 0$

Now (a, b) $\in$ R but (b, a) $\notin$ R

As 3(b – a) + $\sqrt{7}$ = 0 which is rational so relation is not symmetric.

Check for Transitivity:

Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$

& (b, c) as $\left(1, \frac{2\sqrt{7}}{3}\right)$

So now (a, b) $\in$ R & (b, c) $\in$ R but (a, c) $\notin$ R which means relation is not transitive

16. Let the image of the point P(2, –1, 3) in the plane x + 2y – z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is

(1) $\frac{22\sqrt{2}}{7}$

(2) $\frac{24\sqrt{2}}{7}$

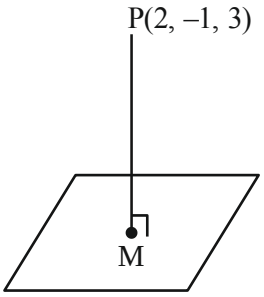
(3) $2\sqrt{14}$

(4) $3\sqrt{14}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



eq. of line PM $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \lambda$

any point on line = $(\lambda + 2, 2\lambda - 1, -\lambda + 3)$

for point 'm' $(\lambda + 2) + 2(2\lambda - 1) - (3 - \lambda) = 0$

$$\lambda = \frac{1}{2}$$

Point m $\left(\frac{1}{2} + 2, 2 \times \frac{1}{2} - 1, \frac{-1}{2} + 3\right)$

$= \left(\frac{5}{2}, 0, \frac{5}{2}\right)$

For Image Q (α, β, γ)

$\frac{\alpha+2}{2}=\frac{5}{2}, \frac{\beta-1}{2}=0,$

$\frac{\gamma+3}{2}=\frac{5}{2}$

Q : (3, 1, 2)

$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$

$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$

17. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}$,

$x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α a β respectively are the maximum and the minimum values of f, then

(1) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

(2) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

(3) $\alpha^2-\beta^2=4\sqrt{3}$

(4) $\alpha^2+\beta^2=\frac{9}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

$C_1 \to C_1+C_2+C_3$

$f(x)=\begin{vmatrix}2+\sin 2x & \cos^2 x & \sin 2x\\ 2+\sin 2x & 1+\cos^2 x & \sin 2x\\ 2+\sin 2x & \cos^2 x & 1+\sin 2x\end{vmatrix}$

$f(x)=(2+\sin 2x)\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 1 & 1+\cos^2 x & \sin 2x\\ 1 & \cos^2 x & 1+\sin 2x\end{vmatrix}$

$R_2 \to R_2-R_1$

$R_3 \to R_3-R_1$

$f(x)=2+\sin 2x)\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$

$=(2+\sin 2x)(1)=2+\sin 2x$

$=\sin 2x \in \left[\frac{\sqrt{3}}{2},1\right]$

Hence $2+\sin 2x\in\left[2+\frac{\sqrt{3}}{2},3\right]$

18. Let $f(x)=2x+\tan^{-1}x$ and $g(x)=\log_e(\sqrt{1+x^2}+x)$, $x\in[0,3]$. Then

(1) There exists $x\in[0,3]$ such that $f'(x)<g'(x)$

(2) max f(x) > max g(x)

(3) There exist $0<x_1<x_2<3$ such that $f(x)<g(x)$, $\forall x\in(x_1,x_2)$

(4) min $f'(x)$ = 1 + max $g'(x)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

$f(x)=2x+\tan^{-1}x$ and $g(x)=\ln\left(\sqrt{1+x^2}+x\right)$

and $x\in[0,3]$

$g'(x)=\frac{1}{\sqrt{1+x^2}}$

Now, $0\le x\le 3$

$0\le x^2\le 9$

$1\le 1+x^2\le 10$

So, $2+\frac{1}{10}\le f'(x)\le 3$

$\frac{21}{10}\le f'(x)\le 3$ and $\frac{1}{\sqrt{10}}\le g'(x)\le 1$

option (4) is incorrect

From above, $g'(x)<f'(x)\ \forall\ x\in[0,3]$

Option (1) is incorrect.

$f'(x)$ & $g'(x)$ both positive so f(x) & g(x) both are increasing

So, max (f(x) at x = 3 is $6+\tan^{-1}3$

Max (g(x) at x= 3 is $\ln(3+\sqrt{10})$

And $6+\tan^{-1}3>\ln(3+\sqrt{10})$

Option (2) is correct

19. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is

(1) 1072 (2) 1792

(3) 1216 (4) 1456

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{1+3+5+a+b}{5}=5$

$a+b=16$ .....(1)

$\sigma^2=\frac{\sum x_1^2}{5}-\left(\frac{\sum x}{5}\right)^2$

$8=\frac{1^2+3^2+5^2+a^2+b^2}{5}-25$

$a^2+b^2=130$ .....(2)

by (1), (2)

$a=7, b=9$

or $a=9, b=7$

20. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+a}{y-2}=0$, $y(1)=0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) $2\sqrt{3}$ (2) $\frac{2\sqrt{3}}{3}$

(3) 2 (4) $\frac{4\sqrt{3}}{3}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\frac{dy}{dx}+\frac{x+a}{y-2}=0$

$\frac{dy}{dx}=\frac{x+a}{2-y}$

$(2-y)\,dy=(x+a)\,dx$

$2y\frac{-y}{2}=\frac{x^2}{2}+ax+c$

$a+c=-\frac{1}{2}$ as $y(1)=0$

$X^2+y^2+2ax-4y-1-2a=0$

$\pi r^2=4\pi$

$r^2=4$

$4=\sqrt{a^2+4+1+2a}$

$(a+1)^2=0$

$P, Q=\left(0, 2\pm\sqrt{3}\right)$

Equation of normal at P, Q are $y-2=\sqrt{3}(x-1)$

$y-2=-\sqrt{3}(x-1)$

$R=\left(1-\frac{2}{\sqrt{3}}, 0\right)$

$S=\left(1+\frac{2}{\sqrt{3}}, 0\right)$

$RS=\frac{4}{\sqrt{3}}=4\frac{\sqrt{3}}{3}$

SECTION-B

21. Let $a_1=8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is _____.

Official Ans. by NTA (754)

QuestPix Ans. (754)

Sol. $a_1+a_2+a_3+a_4=50$

$\Rightarrow 32+6d=50$

$\Rightarrow d=3$

and, $a_{n-3}+a_{n-2}+a_{n-1}+a_n=170$

$\Rightarrow 32+(4n-10).3=170$

$\Rightarrow n=14$

$a_7=26, a_8=29$

$\Rightarrow a_7.a_8=754$

22. A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) and D(4, 5, 0), $|\lambda|\le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5-6\lambda$ is equal to _____.

Official Ans. by NTA (11)

QuestPix Ans. (11)

Sol. A(2, 6, 2) B(–4, 0, λ), C(2, 3, –1) D(4, 5, 0)

Area = $\frac{1}{2}\left|\overrightarrow{BD}\times\overrightarrow{AC}\right| = 18$

$$\overrightarrow{AC}\times\overrightarrow{BD} = \begin{vmatrix} \hat{i} & j & k \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

$= (3\lambda+15)\hat{i} - j(-24) + k(-24)$

$\overrightarrow{AC}\times\overrightarrow{BD} = (3\lambda+15)\hat{i} + 24j - 24k$

$= \sqrt{(3\lambda+15)^2 + (24)^2 + (24)^2} = 36$

$= \lambda^2 + 10\lambda + 9 = 0$

$= \lambda = -1, -9$

$|\lambda| \le 5 \Rightarrow \lambda = -1$

$5 - 6\lambda = 5 - 6(-1) = 11$

23. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is ____.

Official Ans. by NTA (514)

QuestPix Ans. (514)

Sol. Divisible by 2 → 450

Divisible by 3 → 300

Divisible by 7 → 128

Divisible by 2 & 7 → 64

Divisible by 3 & 7 → 43

Divisible by 2 & 3 → 150

Divisible by 2, 3 & 7 → 21

∴ Total numbers =

450 + 300 – 150 – 64 – 43 + 21 = 514

24. The remainder when $19^{200} + 23^{200}$ is divided by 49, is ____ .

Official Ans. by NTA (29)

QuestPix Ans. (29)

Sol. $(21 + 2)^{200} + (21 - 2)^{200}$

$\Rightarrow 2[{}^{100}C_0 21^{200} + {}^{200}C_2\, 21^{198}.\, 2^2 + \ldots\ldots + {}^{200}C_{198}\, 21^2.\, 2^{198} + 2^{200}]$

$\Rightarrow 2[49\, I_1 + 2^{200}] = 49I_1 + 2^{201}$

Now, $2^{201} = (8)^{67} = (1 + 7)^{67} = 49I_2 + {}^{67}C_0\, {}^{67}C_1.\, 7 =$

$49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

∴ Remainder is 29

25. If

$$\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7}\, dx = \frac{1}{l}(11)^{m/n}$$

where l, m, n ∈ ℕ, m and n are coprime then l + m + n is equal to ____.

Official Ans. by NTA (63)

QuestPix Ans. (63)

Sol. $\int \left(x^{20} + x^{13} + x^6\right)\left(2x^{21} + 3x^{14} + 6x^7\right)^{1/7} dx$

$2x^{21} + 3x^{14} + 6x^7 = t$

$42(x^{20} + x^{13} + x^6)\, dx = dt$

$$\frac{1}{42}\int_0^{11} t^{\frac{1}{7}} dt = \left(\frac{t^{\frac{8}{7}}}{\frac{8}{7}} \times \frac{1}{42}\right)_0^{11}$$

$$= \frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11} = \frac{1}{48}(11)^{8/7}$$

$l = 48$, m = 8, n = 7

l + m + n = 63

26. If $f(x) = x^2 + g'(1)x + g''(2)$ and

$g(x) = f(1)x^2 + xf'(x) + f''(x)$,

then the value of f(4) – g(4) is equal to _____.

Official Ans. by NTA (14)

QuestPix Ans. (14)

Sol. $f(x) = x^2 + g'(1)x + g''(2)$

$f'(x) = 2x + g'(1)$

$f''(x) = 2$

$g(x) = f(1)\, x^2 + x\,[2x + g'(1)] + 2$

$g'(x) = 2f(1)\, x + 4x + g'(1)$

$g''(x) = 2f(1) + 4$

$g''(x) = 0$

$2f(1) + 4 = 0$

$f(1) = -2$

$-2 = 1 + g'(1) = g'(1) = -3$

So, $f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

$c = 0$

$f(x) = x^2 - 3x$

$g(x) = -3x + 2$

$f(4) - g(4) = 14$

27. Let $\vec{v}=\alpha\hat{i}+2j-3k$, $\vec{w}=2\alpha\hat{i}+j-k$, and $\vec{u}$ be a vector such that $|\vec{u}|=\alpha>0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}.\hat{i}|^2=\frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to _____.

Official Ans. by NTA (3501)

QuestPix Ans. (3501)

Sol. $[\vec{u}\vec{v}\vec{w}]=\vec{u}.(\vec{v}\times\vec{w})$

min. $\left(|u||\vec{v}\times\vec{w}|\cos\theta\right)=-\alpha\sqrt{3401}$

$\Rightarrow\cos\theta = -1$

$|u|=\alpha$ (Given)

$|\vec{v}\times\vec{w}| = \sqrt{3401}$

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & j & k\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$\vec{v}\times\vec{w}=\hat{i}-5\alpha j-3\alpha k$

$|\vec{v}\times\vec{w}|=\sqrt{1+25\alpha^2+9\alpha^2}=\sqrt{3401}$

$34\alpha^2=3400$

$\alpha^2=100$

$\alpha=10$ (as $\alpha>0$)

So $\vec{u}=\lambda\left(\hat{i}-5\alpha j-3\alpha k\right)$

$\vec{u}=\sqrt{\lambda^2+25\alpha^2\lambda^2+9\alpha^2\lambda}$

$\alpha^2=\lambda^2\left(1+25\alpha^2+9\alpha^2\right)$

$100=\lambda^2(1+34\times100)$

$\lambda^2=\frac{100}{3401}=\frac{m}{n}$

28. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____.

Official Ans. by NTA (50400)

QuestPix Ans. (50400)

Sol. Vowels : A,A,A,I,I,O

Consonants : S,S,S,S,N,N,T

☐ Total number of ways in which vowels come together

$=\frac{\lfloor 8}{\lfloor 4\lfloor 2}\times\frac{\lfloor 6}{\lfloor 3\lfloor 2}=50400$

29. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to _____.

Official Ans. by NTA (62)

QuestPix Ans. (62)

Sol. $A=\int_{-1}^{0}(x^2-3x)dx+\int_{0}^{2}(3x-x^2)\,dx$

$\Rightarrow A=\frac{x^3}{3}-\frac{3x^2}{2}\Big|_{-1}^{0}+\frac{3x^2}{2}-\frac{x^3}{3}\Big|_{0}^{2}$

$\Rightarrow A=\frac{11}{6}+\frac{10}{3}=\frac{31}{6}$

$\therefore$ $12A = 62$

30. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f'(x)+f(x)=\int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to _____.

Official Ans. by NTA (1)

QuestPix Ans. (1)

$\frac{dy}{dx}+y=k$

$y\cdot e^x = k\cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow$ $c = e^{-2} - k$

$\therefore$ $y = k + (e^{-2}-k)e^{-x}$

now $k=\int_0^2\left(k+\left(e^{-2}-k\right)e^{-x}\right)dx$

$\Rightarrow$ $k = e^{-2} - 1$

$\therefore$ $y = (e^{-2}-1)+e^{-x}$

$f(2) = 2e^{-2} - 1$, $f(0) = e^{-2}$

$2f(0) - f(2) = 1$

(Held On Wednesday 1st February, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. Match the List I with List II

	List I		List II
A.	Intrinsic Semiconductor	I.	Fermi-level near conduction band
B.	n-type semiconductor	II.	Fermi-level at middle
C.	p-type semiconductor	III.	Fermi-level near valence band
D.	Metals	IV.	Fermi-level inside conduction band

Choose the **correct** answer from the options given below:

(1) (A) → I, (B) → II, (C) → III, (D) → IV

(2) (A) → II, (B) → I, (C) → III, (D) →IV

(3) (A) → II, (B) → III, (C) → I, (D) → IV

(4) (A) → III, (B) → I, (C) → II, (D) → IV

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Based on theory.

32. An object moves with speed v_1, v_2, and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3 AB, then average speed of the object will be :

A B C D

(1) $\frac{(v_1+v_2+v_3)}{3}$

(2) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$

(3) $\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

(4) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. AB = x

BC = x

$2x$ + CD = $3x$

CD = x

$$< v >= \frac{3x}{\frac{x}{v_1}+\frac{x}{v_2}+\frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3+v_1v_3+v_1v_2}$$

33. Given below are two statements :

Statement-I: Acceleration due to gravity is different at different places on the surface of earth.

Statement-II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $g_{eff} = g - \omega^2 R_e \sin^2\theta$, $\theta \rightarrow$ co-latitude angle

$g_{eff} = g\left(1-\frac{d}{R_e}\right)$, d here depth

34. Match the List-I with List-II.

	List I		List II
A.	AC generator	I.	Presence of both L and C
B.	Transformer	II.	Electromagnetic Induction
C.	Resonance phenomenon to occur	III.	Quality factor
D.	Sharpness of resonance	IV.	Mutual Inductance

Choose the **correct** answer from the options given below:

(1) A-IV, B-II, C-I, D-III

(2) A-II, B-I, C-III, D-IV

(3) A-II, B-IV, C-I, D-III

(4) A-IV, B-III, C-I, D-II

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Based on theory.

35. Match the List-I with List-II:

	List I		List II
A.	Microwaves	I.	Radio active decay of the nucleus
B.	Gamma rays	II.	Rapid acceleration and deceleration of electron in aerials
C.	Radio waves	III.	Inner shell electrons
D.	X-rays	IV.	Klystron valve

Choose the **correct** answer from the options given below:

(1) A-I, B-II, C-III, D-IV

(2) A-IV, B-I, C-II, D-III

(3) A-I, B-III, C-IV, D-II

(4) A-IV, B-III, C-II, D-I

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Based on theory.

36. If earth has a mass nine times and radius twice to the of a planet P. Then $\frac{v_e}{3}\sqrt{x}$ ms^{-1} will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is

(1) 2 (2) 3

(3) 18 (4) 1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $v_{(\text{escape})\text{plant}} = \sqrt{\frac{2GM_P}{R_P}}$

$$= \sqrt{\frac{2G\left(\frac{M_e}{9}\right)}{\left(\frac{R_e}{2}\right)}} = \frac{v_e\sqrt{2}}{3} \therefore x = 2$$

37. 'n' polarizing sheets are arranged such that each makes an angle 45° with the proceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $\frac{I}{64}$. The value of n will be:

(1) 3 (2) 6

(3) 5 (4) 4

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. After passing through first sheet

$$I_1 = \frac{I}{2}$$

After passing through second sheet

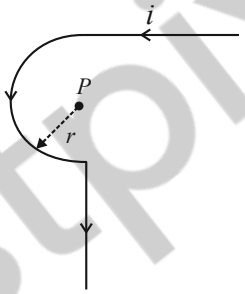
$$I_2 = I_1\cos^2(45^\circ) = \frac{I}{4}$$

After passing through n^{th} sheet

$$I_n = \frac{I}{2^n} = \frac{I}{64}$$

$$n = 6$$

38. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 i}{2r}\left(1+\frac{2}{\pi}\right)$ (2) $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$

(3) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$ (4) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $B_P = \left(\frac{\mu_0 i}{4r} + \frac{\mu_0 i}{4\pi r}\right) = \frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$

39. Which of the following frequencies does not belong to FM broadcast.

(1) 106 MHz (2) 64 MHz

(3) 99 MHz (4) 89 MHz

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. FM broadcast range is 88MHz to 108MHz

40. A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be:

(1) 200 π m/s (2) 100 m/s

(3) 10 m/s (4) 50 m/s

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{70\times10^{-3}}} = 100$ m/s

41. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be _____ ms^{-1} (given, g = 10 ms^{-2}).

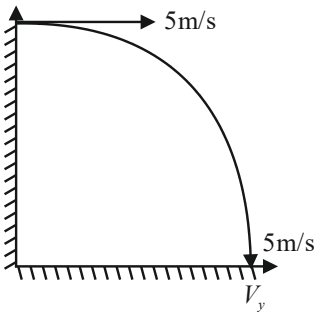
(1) 20 (2) 15

(3) 30 (4) 25

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$v_y = \sqrt{2gh} = \sqrt{200}$

$v_{net} = \sqrt{25 + 200} = 15$ m/s

42. A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is:

(1) 2 : 1 (2) 4 : 1

(3) 1 : 2 (4) 1 : 4

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

$\frac{KE_p}{KE_\alpha} = \frac{m_\alpha}{m_p} = 4:1$

43. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\text{given}, \gamma = \frac{3}{2}\right)$:

(1) $W = TR\,[\sqrt{2} - 2]$ (2) $W = \frac{T}{R}[\sqrt{2} - 2]$

(3) $W = \frac{R}{T}[2 - \sqrt{2}]$ (4) $W = RT\,[2 - \sqrt{2}]$

Official Ans. by NTA (4)

QuestPix Ans. (4)

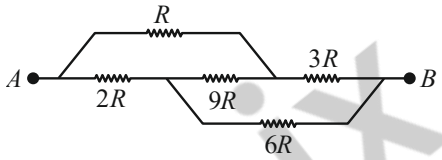
Sol. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$TV^{1/2} = T_2(2V)^{1/2}$

$T_2 = \frac{T}{\sqrt{2}}$

$W = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{1}{2}} = RT\left(2 - \sqrt{2}\right)$

44. The equivalent resistance between A and B of the network shown in figure:



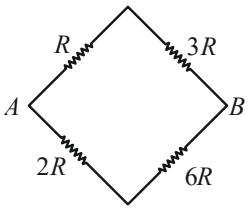
(1) $11\frac{2R}{3}$ (2) $14R$

(3) $21\,R$ (4) $\frac{8}{3}R$

Official Ans. by NTA (4)

QuestPix Ans. (4)

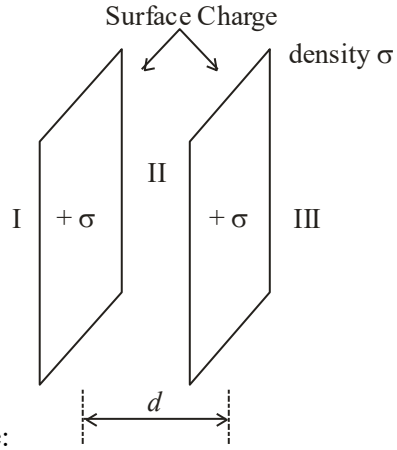
Sol. Wheat stone bridge is in balanced condition.



$\frac{1}{R_{eq}} = \frac{1}{4R} + \frac{1}{8R}$

$R_{eq} = \frac{8R}{3}$

45. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_{II} and E_{III} are:



(1) $\vec{E}_{\mathrm{I}}=\frac{2\sigma}{\epsilon_0}\hat{n}, \vec{E}_{\mathrm{II}}=0, \vec{E}_{\mathrm{III}}=\frac{2\sigma}{\epsilon_0}\hat{n}$

(2) $\vec{E}_{\mathrm{I}}=0, \vec{E}_{\mathrm{II}}=\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{\mathrm{III}}=0$

(3) $\vec{E}_{\mathrm{I}}=\frac{\sigma}{2\epsilon_0}\hat{n}, \vec{E}_{\mathrm{II}}=0, \vec{E}_{\mathrm{III}}=\frac{\sigma}{2\epsilon_0}\hat{n}$

(4) $\vec{E}_{\mathrm{I}}=-\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{\mathrm{II}}=0, \vec{E}_{\mathrm{III}}=\frac{\sigma}{\epsilon_0}\hat{n}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Assuming RHS to be $\hat{n}$

$$\vec{E}_{\mathrm{I}}=\frac{\sigma}{2\epsilon_0}(-\hat{n})+\frac{\sigma}{2\epsilon_0}(-\hat{n})=-\frac{\sigma}{\epsilon_0}\hat{n}$$

$$\vec{E}_{II}=0,$$

$$\vec{E}_{III}=\frac{\sigma}{2\epsilon_0}(\hat{n})+\frac{\sigma}{2\epsilon_0}(\hat{n})=\frac{\sigma}{\epsilon_0}(\hat{n})$$

46. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45 Nm^{-1}. The gain in surface energy is:

(1) 2.26×10^{-5} J (2) 28×10^{-5} J

(3) 17.5×10^{-5} J (4) 5×10^{-5} J

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Initial surface energy = $0.45 \times 4\pi (10^{-3})^2$

$$\frac{4}{3}\pi(10^{-3})^3=125\times\frac{4\pi}{3}R_{\text{new}}^3$$

$\therefore$ $10^{-3}=5R_{\text{new}}$

$\therefore$ $R_{\text{new}}=\frac{10^{-3}}{5}\text{m}$

So, final surface energy = $0.45 \times 125 \times 4\pi\left(\frac{10^{-3}}{5}\right)^2$

Increase in energy = $0.45 \times 4\pi \times (10^{-3})^2\left[\frac{125}{25}-1\right]$

$= 4 \times 0.45 \times 4\pi \times 10^{-6}$

$= 2.26 \times 10^{-5}$ J

47. The mass of proton, neutron and helium nucleus are respectively 1.0073 u, 1.0087 u and 4.0015u. The binding energy of helium nucleus is:

(1) 14.2 MeV (2) 28.4 MeV

(3) 56.8 MeV (4) 7.1 MeV

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $B.E$ of Helium = $(2m_P + 2m_N - m_{He})\,c^2$

= 28.4 MeV

48. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume, T is the temperature and a, b, R are the constants. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$, will be :

(1) Bulk modulus (2) Modulus of rigidity

(3) Compressibility (4) Energy density

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $[b] = [V]$

$$\left[\frac{a}{b^2}\right]=[P] \qquad \therefore \left[\frac{b^2}{a}\right]=\frac{1}{[P]}=\frac{1}{[B]}=[K]$$

49. The average kinetic energy of a molecule of the gas is

(1) proportional to absolute temperature

(2) proportional to volume

(3) proportional to pressure

(4) dependent on the nature of the gas

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Basic theory

Translational $K.E$ on average of a molecule is $\frac{3}{2}$ KT which is independent of nature, pressure and volume.

50. A block of mass 5 kg is placed at rest on a table of rough surface. Now, if a force of 30N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10s. Coefficient of kinetic friction is (given, g = 10 ms^{-2}):

(1) 0.60 (2) 0.75

(3) 0.50 (4) 0.25

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $S = ut + \frac{1}{2}at^2$

$50 = 0 + \frac{1}{2} \times a \times 100$

$a = 1$ m/s^2

$F - \mu mg = ma$

$30 - \mu \times 50 = 5 \times 1$

$50\mu = 25$

$\mu = \frac{1}{2}$

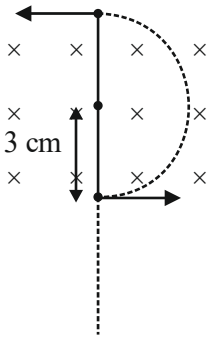
SECTION-B

51. A charge particle of 2 μC accelerated by a potential difference of 100V enters a region of uniform magnetic field of magnitude 4 mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is_____ $\times 10^{-18}$ kg.

Official Ans. by NTA (144)

QuestPix Ans. (144)

Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2km}}{qB}, m = \frac{r^2q^2B^2}{2k}$



$$m = \frac{\frac{1}{100} \times \frac{3}{100} \times 2 \times 2 \times 4 \times 10^{-3} \times 4 \times 10^{-3} \times 10^{-12}}{2 \times (100) \times 10^{-6}}$$

$= 144 \times 10^{-18}$ kg

52. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E. the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}$ V. The value of x is _____.

Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol. $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

$\frac{1.5}{E_2} = \frac{60}{60+40} = \frac{6}{10} = \frac{3}{5}$

$E_2 = \frac{5}{2} = \frac{x}{10}$

$x = 25$

53. A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N. The value of work done will be _____ J.

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. $W = \vec{F} \cdot \left(\vec{r}_f - \vec{r}_i\right)$

$= \left(5\hat{i} + 2\hat{j} + 7\hat{k}\right) \cdot \left(\left(5\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)\right)$

$W = 40$ J

54. A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi} \times 10^{-17}$ eVs. The value of x is _____ (use $h = 4.14 \times 10^{-15}$eVs, $c = 3 \times 10^8$ ms^{-1}).

Official Ans. by NTA (828)

QuestPix Ans. (828)

Sol. In the ground state energy = –13.6 eV

So energy

$\frac{-13.6\text{eV}}{n^2} = -13.6 + 12.75$

$\frac{-13.6\text{eV}}{n^2} = -0.85$

$n = \sqrt{16}$

$\boxed{n = 4}$

Angular momentum = $\frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$

Angular momentum = $\frac{2}{\pi} \times 4.14 \times 10^{-15}$

$= \frac{828 \times 10^{-17}}{\pi}$ eVs

55. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$. The value of x is _____.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $B_{water} = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{-\Delta P}{\frac{0.01}{100}}$

$B_{liquid} = \frac{-\Delta P}{\frac{0.03}{100}}$

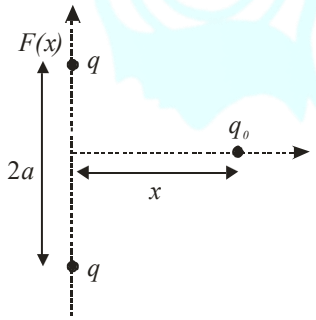
$\frac{B_{water}}{B_{liquid}} = 3$

$x = 1$

56. Two equal positive point charges are separated by a distance $2a$. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)



Sol.

$F = \frac{2Kqq_0x}{(x^2 + a^2)^{3/2}}$

For F to be maximum

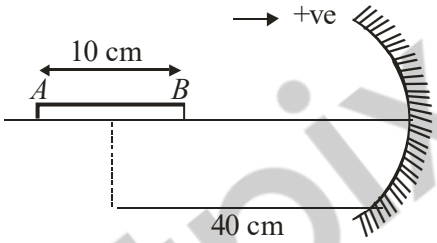
$\frac{dF}{dx} = 0$

$x = \frac{a}{\sqrt{2}}$

57. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is _____.

Official Ans. by NTA (32)

QuestPix Ans. (32)



Sol.

$U_A = -45$ cm, $f = -20$ cm

$V_A = \frac{-45 \times (-20)}{-45 - (-20)} = \frac{-900}{25} = -36$ cm

And $U_B = -35$cm

$\therefore V_B = \frac{-35 \times (-20)}{-35 - (-20)} = \frac{700}{-15}$

$\therefore V_A - V_B =$ length of image

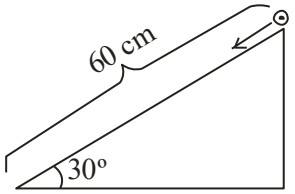
$= \left(-36 + \frac{140}{3}\right)$ cm

$= \frac{-108 + 140}{3}$ cm

$= \frac{32}{3}$ cm

$\therefore x = 32$

58. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is _____ ms^{-1}. (Given g = 10 ms^{-2})



Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$

Where $h = 60 \sin 30^\circ = 30$ cm

$$k^2 = \frac{R^2}{2}$$

$v = 2\ ms^{-1}$

59. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: _____ cm.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $KE = PE + \dfrac{PE}{4}$

$KE = \dfrac{5}{4}PE$

$\dfrac{1}{2}m\omega^2(A^2 - x^2) = \dfrac{5}{4} \times \dfrac{1}{2}m\omega^2x^2$

$[v = \omega\sqrt{A^2 - x^2}]$

$A^2 - x^2 = \dfrac{5}{4}x^2$

$\dfrac{9x^2}{4} = A^2$

$\boxed{x = \dfrac{2}{3}A}$

$\therefore x = \dfrac{2}{3} \times 3\text{cm}$

$x = 2$ cm

60. A series LCR circuit is connected to an ac source of 220V, 50Hz. The circuit contain a resistance $R = 100\Omega$ and an inductor of inductive reactance $X_L = 79.6\ \Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _____ μF.

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. To maximize the average rate at which energy supplied i.e. power will be maximum.

So in LCR circuit power will be maximum at the condition of resonance and in resonance condition

$X_L = X_C$

$79.6 = \dfrac{1}{\omega C}$

$\therefore C = \dfrac{1}{2\pi \times 50 \times 79.6}$

$\therefore \boxed{C = 40\mu F}$

(Held On Wednesday 1st February, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

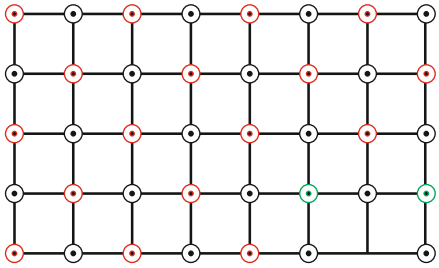
CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

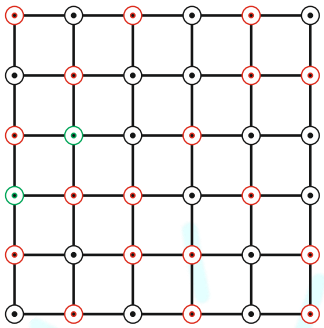
61. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions?

$\odot$ A^{2+} $\odot$ A^{3+} $\odot$ O^{2-}

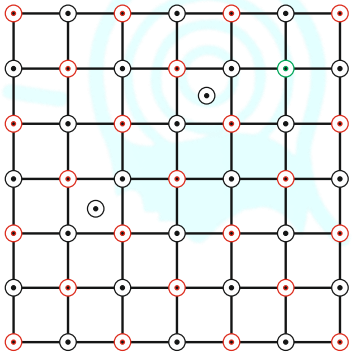
A.



B.



C.



(1) B and C only

(2) B only

(3) A and B only

(4) A only

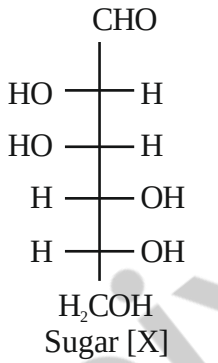
Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Applying electrical neutrality principle in metal defficiency defect.

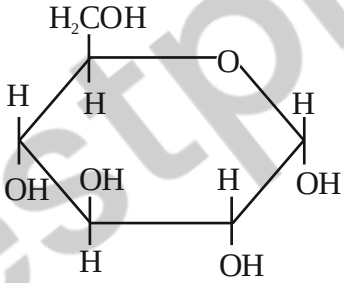
3 A^{2+} are replaced by $2A^{3+}$, thus one vacant site per pair of A^{3+} is created

62. The correct representation in six membered pyranose form for the following sugar [X] is

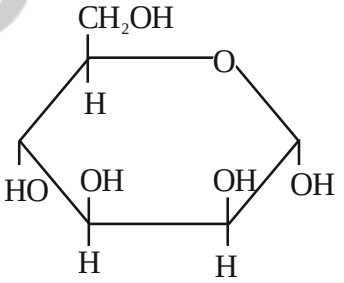


Sugar [X]

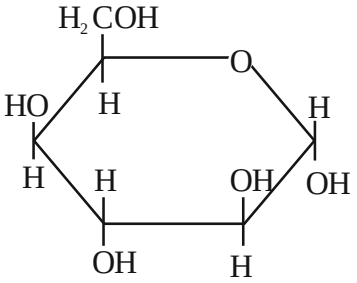
(1)



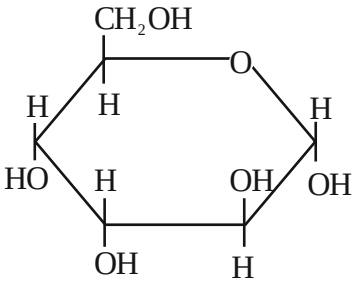
(2)



(3)



(4)



Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. By Haworth structure of mannose.

63. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms

(B) Mn is octahedrally surrounded by oxygen atoms

(C) Contains Mn-O-Mn bridge

(D) Contains Mn-Mn bond.

Choose the correct answer from the options given below

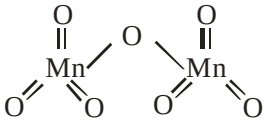
(1) A and C only (2) A and D only

(3) B and D only (4) B and C only

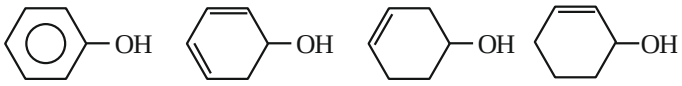
Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



64. Decreasing order of dehydration of the following alcohols is



(1) a > d > b > c (2) b > d > c > a

(3) b > a > d > c (4) d > b > c > a

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Dehydration of alcohol is directly proportional to the stability of carbocation.

65. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Amongst He, Ne, Ar and Kr;

1 g of activated charcoal adsorbs more of Kr.

Reason R : The critical volume V_c ($cm^3\ mol^{-1}$) and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation A

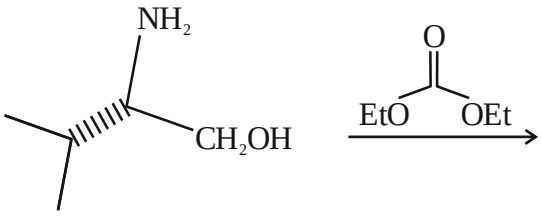
Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Adsorption $\propto$ vanderwaal attraction forces

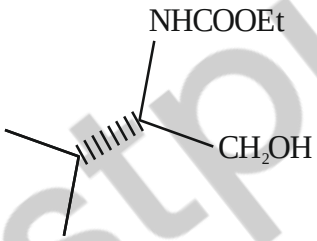
$Z_c = \frac{3}{8}$ for all real gases

66. In the following reaction, 'A' is

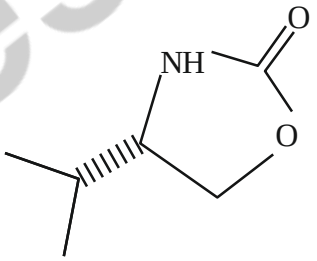


'A' Major product.

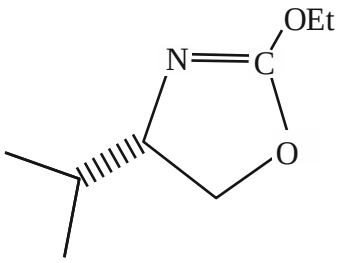
(1)



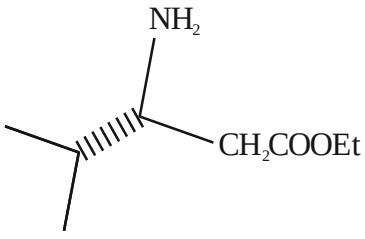
(2)



(3)



(4)

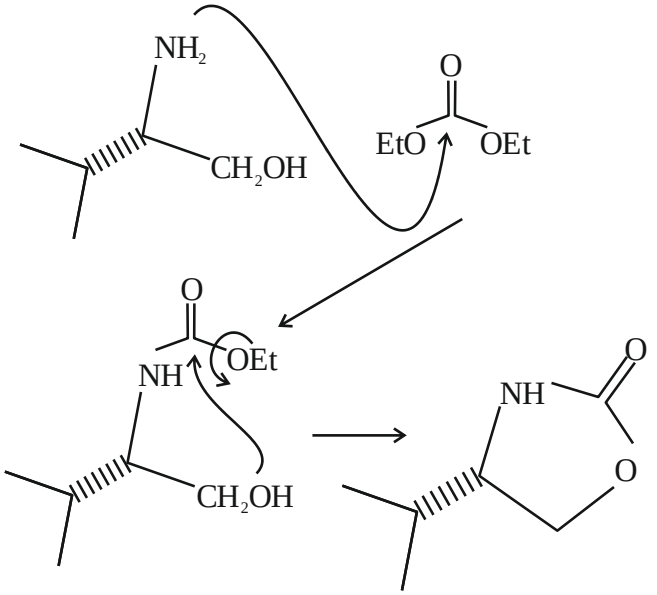


Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Initially lone pair electron of $-NH_2$ attack on electrophilic carbon, after then lone pair electron of

oxygen attacks leading to formation of cyclic compound.



67. Match List I with List II

List-I	List-II
(A) Tranquilizers	(I) Anti blood clotting
(B) Aspirin	(II) Salvarsan
(C) Antibiotic	(III) Antidepressant drugs
(D) Antiseptic	(IV) Soframicine

Choose the correct answer from the options given below:

(1) (A) – IV, (B) – II, (C) – I, (D) – III

(2) (A) – II, (B) – I, (C) – III, (D) – IV

(3) (A) – III, (B) – I, (C) – II, (D) – IV

(4) (A) – II, (B) – IV, (C) – I, (D) – III

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. NCERT (Chemistry in every day life)

68. Given below are two statements:

Statement I: Chlorine can easily combine with oxygen to from oxides: and the product has a tendency to explode.

Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both the statements I and II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

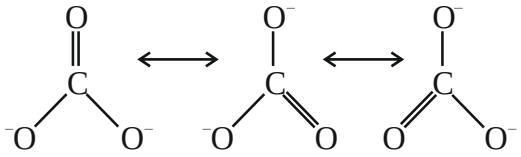
(4) Both the Statements I and II are false

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are highly reactive oxidising agents and tend to explode.

69. Resonance in carbonate ion (CO_3^{2-}) is



Which of the following is true?

(1) It is possible to identify each structure individually by some physical or chemical method.

(2) All these structures are in dynamic equilibrium with each other.

(3) Each structure exists for equal amount of time.

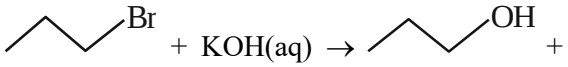
(4) CO_3^{2-} has a single structure i.e., resonance hybrid of the above three structures.

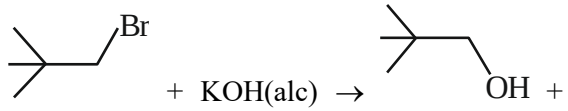
Official Ans. by NTA (4)

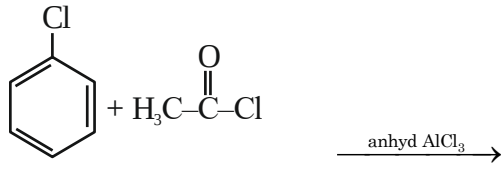
QuestPix Ans. (4)

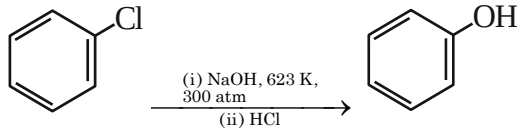
Sol. Resonating structure are hypothetical and resonance hybrid is real structure which is weighted average of all the resonating structures.

70. Identify the incorrect option from the following:

(1)  + KBr

(2)  + KBr

(3) 

(4) 

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. In alcoholic KOH, elimination reaction takes place.

71. A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussiun blue precipitate due to the formation of

(1) $K[Fe_2(CN)_6]$

(2) $Fe[Fe(CN)_6]$

(3) $Fe_3[Fe(CN)_6]_2$

(4) $Fe_4[Fe(CN)_6]_3$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Formation of Prussian blue complex takes place.

72. Which of the following are the example of double salt?

(A) $FeSO_4.(NH_4)_2SO_4.6H_2O$

(B) $CuSO_4.4NH_3.H_2O$

(C) $K_2SO_4.Al_2(SO_4)_3.24H_2O$

(D) $Fe(CN)_2.4KCN$

Choose the correct answer.

(1) A and C only

(2) A and B only

(3) A, B and D only

(4) B and D only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Double salt contain's two or more types of salts. $CuSO_4.4NH_3.H_2O$ and $Fe(CN)_2.4KCN$ are complex compounds.

73. Which of the following complex will show largest splitting of d-orbitals?

(1) $[Fe(C_2O_4)_3]^{3-}$

(2) $[FeF_6]^{3-}$

(3) $[Fe(CN)_6]^{3-}$

(4) $[Fe(NH_3)_6]^{3+}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\bar{C}N$ is a strong field ligand so maximum splitting in d orbitals take place.

74. How can photochemical smog be controlled?

(1) By using tall chimneys

(2) By complete combustion of fuel

(3) By using catalytic converters in the automobiles/industry

(4) By using catalyst

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. NCERT (Environmental chemistry)

75. Match List I with List II

(A)	Slaked lime	(I)	NaOH
(B)	Dead burnt plaster	(II)	$Ca(OH)_2$
(C)	Caustic soda	(III)	$Na_2CO_3 \cdot 10H_2O$
(D)	Washing soda	(IV)	$CaSO_4$

Choose the correct answer form the options given below:

(1) (A) – I, (B) – IV, (C) – II, (D) – III

(2) (A) – III, (B) – IV, (C) – II, (D) – I

(3) (A) – II, (B) – IV, (C) – I, (D) – III

(4) (A) – III, (B) – II, (C) – IV, (D) – I

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. From S-block NCERT

76. Choose the correct statement(s):

A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2.

C. Beryllium sulphate is readily soluble in water.

D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

(1) A, B and C only

(2) B, C and D only

(3) A and B only

(4) A only

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. A. Beryllium oxide is amphoteric in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2 because it is thermally less stable.

C. Beryllium sulphate is readily soluble in water due to high degree of hydration.

D. Beryllium shows anomalous behaviour due to small size, high ionization energy and high value of ϕ (polarising power).

77. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) Both A and R are correct but R is NOT the correct explanation of A

(4) A is correct but R is not correct

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $2C(s) + O_2(g) \rightarrow 2CO(g)$

Δ_rS^o is +ve, $\Delta_rG^o = \Delta_rH^o - T\Delta_rS^o$; thus slope is negative

As temperature increases Δ_rG^o becomes more negative thus it has lower tendency to get decomposed.

78. But-2-yne is reacted separately with one mole of Hydrogen as shown below:

$$\underline{B} \xleftarrow[\text{liq } NH_3]{Na} CH_3 - C \underset{+H_2}{\equiv} C - CH_3 \xrightarrow[\Delta]{Pd/C} \underline{A}$$

Identify the incorrect statements from the options given below:

A. A is more soluble than B.

B. The boiling point & melting point of A are higher and lower than B respectively.

C. A is more polar than B because dipole moment of A is zero.

D. Br_2 adds easily to B than A.

(1) B and C only

(2) B, C and D only

(3) A, C and D only

(4) A and B only

Official Ans. by NTA (2)

QuestPix Ans. (Bonus)

Sol. Incorrect statements are C and D only, correct choice is not available.

79. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Hydrogen is an environment friendly fuel.

Reason R: Atomic number of hydrogen is 1 and it is a very light element.

In the light of the above statements, choose the correct answer from the options given below

(1) A is true but R is false

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is false but R is true

(4) Both A and R are true and R is the correct explanation of A

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. No pollution occurs by combustion of hydrogen and very low density of hydrogen.

80. Match List I and List II

List I	List II
Test	**Functional group / Class of Compound**
(A) Molisch's Test	(I) Peptide
(B) Biuret Test	(II) Carbohydrate
(C) Carbylamine Test	(III) Primary amine
(D) Schiff s Test	(IV) Aldehyde

Choose the correct answer from the options given below:

(1) (A) – I, (B) – II, (C) – III, (D) – IV

(2) (A) – III, (B) – IV, (C) –I, (D) – II

(3) (A) – II, (B) – I, (C) – III, (D) – IV

(4) (A) – III, (B) – IV, (C) –II, (D) – I

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.

List I	List II
Test	**Functional group / Class of Compound**
(A) Molisch's Test	(II) Carbohydrate
(B) Biuret Test	(I) Peptide
(C) Carbylamine Test	(III) Primary amine
(D) Schiff s Test	(IV) Aldehyde

SECTION-B

81. The density of 3 M solution of NaCl is 1.0 g mL^{-1}. Molality of the solution is _____ $\times 10^{-2}$ m. (Nearest integer).

Given: Molar mass of Na and Cl is 23 and 35.5 g mol^{-1} respectively.

Official Ans. by NTA (364)

QuestPix Ans. (364)

Sol. $m = \dfrac{1000 \times M}{1000 \times d - M \times M.W \text{ of solute}}$

$= \dfrac{1000 \times 3}{1000 \times 1 - (3 \times 58.5)} = 3.64$

$= 364 \times 10^{-2}$

82. Electrons in a cathode ray tube have been emitted with a velocity of 1000 ms^{-1}. The number of following statements which is/are <u>true</u> about the emitted radiation is _____ .

Given : $h = 6 \times 10^{-34}$ Js, $m_e = 9 \times 10^{-31}$ kg.

(A) The deBroglie wavelength of the electron emitted is 666.67nm.

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (A) $V_e = 1000$ m/s ; $h = 6 \times 10^{-34}$ Js ;

$m_e = 9 \times 10^{-31}$ kg

$\lambda = \dfrac{h}{mv} = \dfrac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000} = 666.67 \times 10^{-9}$m

$= 666.67$ nm

(B) The characteristic of electrons emitted is independent of the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons is independent on the nature of the gas present in cathode ray tube.

83. Sum of oxidation states of bromine in bromic acid and perbromic acid is _____ .

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $HBrO_3$ (Bromic acid)

Ox. State of Br = +5

$HBrO_4$ (per bromic acid)

OX. State of Br = +7

Sum of Ox. State = 12

84. At what pH, given half cell MnO_4^- (0.1 M) | Mn^{2+} (0.001 M) will have electrode potential of 1.282 V? _____ (Nearest Integer)

Given $E^o_{MnO_4^-/Mn^{2+}} = 1.54$ V, $\frac{2.303RT}{F} = 0.059$V

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $MnO_4^- + 8H^+ + 5e^- \rightleftharpoons Mn^{2+} + 4H_2O$

$$E = E° - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5}\log\frac{10^{-3}}{10^{-1}\times[H^+]^8}$$

$$\frac{0.258\times 5}{0.059} = \log\frac{10^{-2}}{[H^+]^8}$$

$\Rightarrow$ $21.86 = -2 + 8pH$

$\therefore$ $pH = 2.98$

$\simeq 3$

85. Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2, 4 – DNP (iv) on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is _____ .

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. As per the language of given question, the best possible isomeric structure is Ph – CH = CH – O – CH_3 (cis and trans). So, the answer is 2.

86. (i) $X(g) \rightleftharpoons Y(g) + Z(g)$ $K_{p1} = 3$

(ii) $A(g) \rightleftharpoons 2B(g)$ $K_{p2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x : 1. The value of x is _____ (Nearest integer)

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $x(g) \rightleftharpoons y(g) + z(g)$ $k_{p_1} = 3$

Initial moles	n	–	–
at equilibrium	$n - \alpha n$	αn	αn

$$k_{p_1} = \frac{\left(\frac{\alpha}{1+\alpha}\times p_1\right)^2}{\frac{1-\alpha}{1+\alpha}p_1}$$

$$3 = \frac{\alpha^2\times p_1}{1-\alpha^2}$$

$A(g) \rightleftharpoons 2B(g)$ $k_{p_2} = 1$

Initial mole	n	–	
at equilibrium	$x - \alpha n$	$2\alpha n$	$p_{total} = p_2$

$$k_{p_2} = \frac{\left(\frac{2\alpha}{1+\alpha}\times p_2\right)^2}{\frac{1-\alpha}{1+\alpha}\times p_2}$$

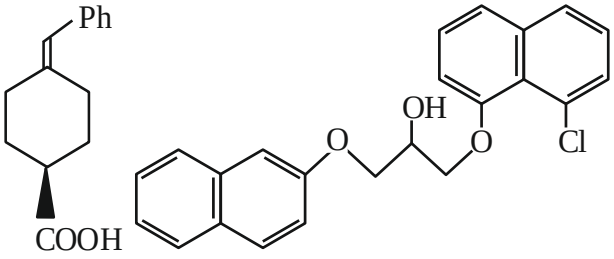
$$1 = \frac{4\alpha^2\times p_2}{1-\alpha^2}$$

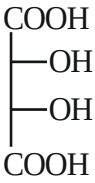
$$\frac{k_{p_1}}{k_{p_2}} = \frac{p_1}{4p_2}$$

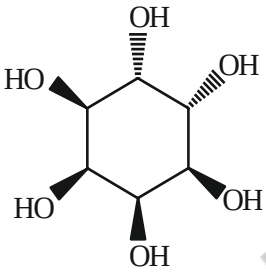
$$\frac{3}{1} = \frac{p_1}{4p_2} \qquad \therefore \; p_1 : p_2 = 12 : 1$$

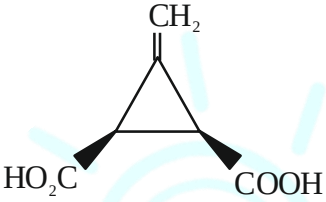
$x = 12$

87. The total number of chiral compound/s from the following is _____.





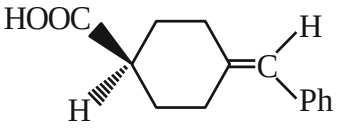




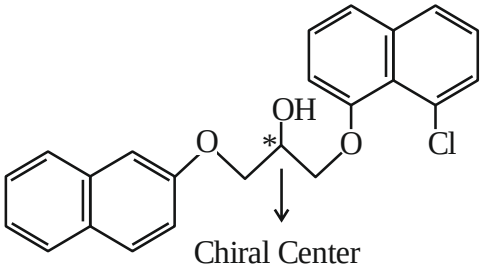
Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

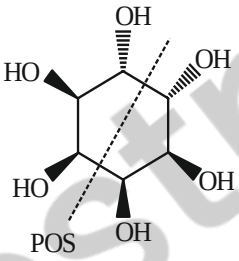


No POS, COS
(Chiral)

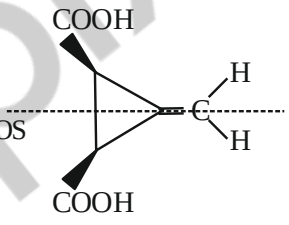


Chiral Center

No POS, COS
(Chiral)



(Achiral)



(Achiral)

88. A and B are two substances undergoing radioactive decay in a container. The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same? _____ min.

Official Ans. by NTA (15)

QuestPix Ans. (15)

Sol. $[A]_t = [A]_0 e^{-kt}$

For A : Let $[A]_t$ be y and $[A]_0$ be x ; $k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{15 \text{ min}}$

$y = xe^{-kt}$

$= xe^{-\left(\frac{\ln 2}{15}\right)t}$

For B : $[B]_t = [B]_0 e^{-kt}$

Let $[B]_t = y$; $[B]_0 = 4x$; $k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5 \text{ min}}$

$y = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$

$\Rightarrow \quad xe^{-\left(\frac{\ln 2}{15}\right)t} = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$

$e^{t\left(\frac{\ln 2}{5} - \frac{\ln 2}{15}\right)} = 4$

$t \times \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] = \ln 4$

$t \times \ln 2 \left[\frac{1}{5} - \frac{1}{15}\right] = 2 \ln 2$

$t = 15$ min

89. At 25°C, the enthalpy of the following processes are given:

$H_2(g) + O_2(g) \rightarrow 2OH(g) \quad \Delta H^o = 78 kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(g) \quad \Delta H^o = -242\ kJ\ mol^{-1}$

$H_2(g) \rightarrow 2H(g) \quad \Delta H^o = 436\ kJ\ mol^{-1}$

$\frac{1}{2} O_2(g) \rightarrow O(g) \quad \Delta H^o = 249\ kJ\ mol^{-1}$

What would be the value of X for the following reaction? _____ (Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g) \quad \Delta H^o = X\ kJ\ mol^{-1}$

Official Ans. by NTA (499)

QuestPix Ans. (499)

Sol. $2H_2O(g) \rightarrow 2H_2(g) + O_2(g) \quad +(242 \times 2)\ kJ\ mol^{-1}$

$H_2(g) + O_2(g) \rightarrow 2OH \quad +78\ kJ\ mol^{-1}$

$H_2(g) \rightarrow 2H \quad +436\ kJ\ mol^{-1}$

$2H_2O \rightarrow 2H + 2OH \quad +998\ kJ\ mol^{-1}$

$H_2O \rightarrow H + OH \quad 998 \times \frac{1}{2} = +499\ kJ\ mol^{-1}$

90. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration? _____ (Nearest integer).

(Given : $K_f = 2.0\ K\ kg\ mol^{-1}$)

Assume

1) 100% ionization and

2) density of the aqueous solution as $1\ g\ mL^{-1}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.

$$\underset{\downarrow}{KCl} + \underset{\downarrow}{AgNO_3} \rightarrow AgCl + KNO_3$$

V=25ml V=20ml
M = 1M

At equivalence point,

mmole of KCl = mmole of $AgNO_3$

= 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent

= 25 – mass of solute

$= 25 - [20 \times 10^{-3} \times 74.5]$

= 23.51 gm

Molality of KCl = $\frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$

$= \frac{20\times10^{-3}}{23.51\times10^{-3}} = 0.85$

i of KCl = 2 (100% ionisation)

$\Delta T_f = i \times K_f \times m$

$= 2 \times 2 \times 0.85$

$= 3.4$

$\simeq 3$