(Held On Tuesday 24th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH ANSWER

SECTION-A

1. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to

(1) 220

(2) 210

(3) 200

(4) 105

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$\mathbf{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57}$

$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$

$\Rightarrow a_1 + a_6 = 19$

$\Rightarrow 2a_1 + 5d = 19$ and $a_1 + d = 5$

$\Rightarrow a_1 = 2, d = 3$

Numbers : 2, 5, 8, 11, 14, 17

Variance = σ^2 = mean of squares − square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$8\sigma^2 = 210$

2. Let f(x) be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 6

(2) 8

(3) 7

(4) 9

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $f(x + y) = f(x) \cdot f(y) \ \forall x, y \in N$, $f(1) = 3$

$f(2) = f^2(1) = 3^2$

$f(3) = f(1)\ f(2) = 3^3$

$f(4) = 3^4$

$f(k) = 3^k$

$\sum_{k=1}^{n} f(k) = 3279$

$f(1) + f(2) + f(3) + \ldots\ldots\ldots + f(k) = 3279$

$3 + 3^2 + 3^3 + \ldots\ldots\ldots 3^k = 3279$

$\frac{3(3^k - 1)}{3 - 1} = 3279$

$\frac{3^k - 1}{2} = 1093$

$3^k - 1 = 2186$

$3^k = 2187$

$\boxed{k = 7}$

3. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 4 (2) 0

(3) 3 (4) 2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Let $x + \frac{1}{x} = t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$3t(t - 1) + 1(t - 1) = 0$

$(t - 1)(3t + 1) = 0$

$t = 1, -\frac{1}{3}$

$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow$ No solution.

4. If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in R$,

then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+.....+f\left(\frac{2022}{2023}\right)$ is equal to

(1) 2011 (2) 1010

(3) 2010 (4) 1011

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $f(x)=\frac{4^x}{4^x+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2(4^x)}$

$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}$

$= 1$

$\Rightarrow f(x) + f(1 - x) = 1$

Now $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+.....+$

$.....+f\left(1-\frac{3}{2023}\right)+f\left(1-\frac{2}{2023}\right)+f\left(1-\frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum = 1 + 1 + 1 + …….+ 1 (1011 times)

= 1011

5. If $f(x) = x^3 - x^2f'(1)+xf''(2)-f'''(3)$, $x \in R$, then

(1) 3f(1) + f(2) = f(3)

(2) f(3) – f(2) = f(1)

(3) 2f(0) – f(1) + f(3) = f(2)

(4) f(1) + f(2) + f(3) = f(0)

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3),\ x\in R$

Let $f'(1)=a,\ f''(2)=b,\ f'''(3)=c$

$f(x) = x^3 - ax^2 + bx - c$

$f'(x)=3x^2-2ax+b$

$f''(x)=6x-2a$

$f'''(x)=6$

c = 6, a = 3, b = 6

$f(x) = x^3 - 3x^2 + 6x - 6$

f(1) = –2, f(2) = 2, f(3) = 12, f(0) = –6

2f(0) – f(1) + f(3) = 2 = f(2)

6. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(1) 120

(2) 168

(3) 220

(4) 48

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Four digit numbers greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

Five digit number = 5! = 120

Total number greater than 7000

= 120 + 48 = 168

7. If the system of equations

x + 2y + 3z = 3

4x + 3y – 4z = 4

8x + 4y – λz = 9 + μ

has infinitely many solutions, then the ordered pair (λ, μ) is equal to

(1) $\left(\frac{72}{5},\frac{21}{5}\right)$ (2) $\left(\frac{-72}{5},\frac{-21}{5}\right)$

(3) $\left(\frac{72}{5},\frac{-21}{5}\right)$ (4) $\left(\frac{-72}{5},\frac{21}{5}\right)$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. x + 2y + 3z = 3 …….(i)

4x + 3y – 4z = 4 …….(ii)

8x + 4y – λz = 9 + μ …….(iii)

(i) × 4 – (ii) $\Rightarrow$ 5y + 16z = 8 ……. (iv)

(ii) × 2 – (iii) $\Rightarrow$ 2y + (λ–8)z = –1–μ ……(v)

(iv) × 2 – (iii) × 5 $\Rightarrow$ (32–5(λ–8))z = 16–5(–1–μ)

For infinite solutions $\Rightarrow$ 72 – 5λ = 0 $\Rightarrow \lambda=\frac{72}{5}$

$21 + 5\mu = 0 \Rightarrow \mu=\frac{-21}{5}$

$\Rightarrow (\lambda,\mu)\equiv\left(\frac{72}{5},\frac{-21}{5}\right)$

8. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $\frac{-1}{2}(1-i\sqrt{3})$

(2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{-1}{2}(\sqrt{3}-i)$

(4) $\frac{1}{2}(\sqrt{3}+i)$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3=z^3$$

$$\Rightarrow\left(i\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)\right)^3$$

$$=-i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow\frac{-1}{2}(\sqrt{3}-i).$$

9. The equations of the sides AB and AC of a triangle ABC are

$(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$

respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is

(1) $\sqrt{6}$

(2) $2\sqrt{2}$

(3) 2

(4) 4

Official Ans. by NTA (2)

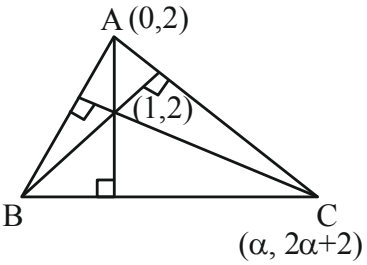
QuestPix Ans. (2)

Sol. $AB:(\lambda+1)x+\lambda y=4$

$AC:\lambda x+(1-\lambda)y+\lambda=0$

Vertex A is on y-axis

$\Rightarrow x=0$



So $y=\frac{4}{\lambda},y=\frac{\lambda}{\lambda-1}$

$\Rightarrow\frac{4}{\lambda}=\frac{\lambda}{\lambda-1}$

$\Rightarrow\lambda=2$

AB : $3x+2y=4$

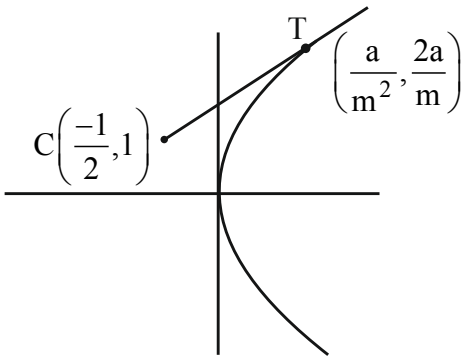
AC : $2x-y+2=0$

$\Rightarrow$ A(0,2) Let C $(\alpha, 2\alpha+2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right)=-1$

$$\left(\frac{2\alpha}{\alpha-1}\right)\left(-\frac{3}{2}\right)=-1\Rightarrow\alpha=-\frac{1}{2}$$

So $C\left(-\frac{1}{2},1\right)$



Let Equation of tangent be $y=mx+\frac{3}{2m}$

$m^2+2m-3=0$

$\Rightarrow m=1,-3$

So tangent which touches in first quadrant at T is

$T\equiv\left(\frac{a}{m^2},\frac{2a}{m}\right)$

$\equiv\left(\frac{3}{2},3\right)$

$\Rightarrow CT=\sqrt{4+4}=2\sqrt{2}$

10. The set of all values of a for which $\lim\limits_{x\to a}\left([x-5]-[2x+2]\right) = 0$, where $[\propto]$ denotes the greater integer less than or equal to $\propto$ is equal to

(1) (– 7.5, –6.5) (2) (– 7.5, –6.5]

(3) [– 7.5, –6.5] (4) [– 7.5, –6.5)

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\lim\limits_{x\to a}\left([x-5]-[2x+2]\right)=0$

$\lim\limits_{x\to a}\left([x]-5-[2x]-2\right)=0$

$\lim\limits_{x\to a}\left([x]-[2x]\right)=7$

$[a]-[2a]=7$

$a \in I,\ a=-7$

$a \notin I,\ a=I+f$

Now, $[a]-[2a]=7$

$-I-[2f]=7$

Case-I: $f \in \left(0,\frac{1}{2}\right)$

$2f \in (0, 1)$

$-I=7$

$I=-7 \Rightarrow a \in (-7, -6.5)$

Case-II: $f \in \left(\frac{1}{2},1\right)$

$2f \in (1, 2)$

$-I-1=7$

$I=-8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

11. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

(1) 30 (2) 60

(3) 15 (4) 10

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $S=0.({}^{30}C_0)^2+1.({}^{30}C_1)^2+2.({}^{30}C_2)^2+\ldots\ldots+30.({}^{30}C_{30})^2$

$S=30.({}^{30}C_0)^2+29.({}^{30}C_1)^2+28.({}^{30}C_2)^2+\ldots\ldots+0.({}^{30}C_0)^2$

$2S = 30.({}^{30}C_0{}^2+{}^{30}C_1{}^2+\ldots\ldots+{}^{30}C_{30}{}^2)$

$S = 15.{}^{60}C_{30} = 15\cdot\frac{60!}{(30!)^2}$

$\frac{15\cdot 10!}{(30!)^2}=\frac{\alpha\cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

12. Let the plane containing the line of intersection of the planes

P1: $x + (\lambda + 4)y + z = 1$ and

P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is

(1) $5\sqrt{6}$

(2) $4\sqrt{6}$

(3) $2\sqrt{6}$

(4) $3\sqrt{6}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Equation of plane passing through point of intersection of P1 and P2

$P = P1 + kP2$

$\left(x+(\lambda+4)y+z-1\right)+k\left(2x+y+z-2\right)=0$

Passing through (0, 1, 0) and (1, 0, 1)

$(\lambda + 4 - 1) + k(1 - 2) = 0$

$(\lambda + 3) - k = 0$ ....(1)

Also passing (1, 0, 1)

$(1 + 1 - 1) + k(2 + 1 - 2) = 0$

$1 + k = 0$

$k = -1$

put in (1)

$\lambda + 3 + 1 = 0$

$\lambda = -4$

Then point $(2\lambda, \lambda, -\lambda)$

$(-8, -4, 4)$

$d=\left|\frac{-16-4+4-2}{\sqrt{6}}\right|$

$d=\frac{18}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=3\sqrt{6}$

13. Let $\vec{\alpha}=4\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{\beta}=\hat{i}+2\hat{j}-4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, then the value of $5\vec{\beta}_2\cdot(\hat{i}+\hat{j}+\hat{k})$ is

(1) 6

(2) 11

(3) 7

(4) 9

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Let $\vec{\beta}_1 = \lambda\vec{\alpha}$

Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$

$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$

$\vec{\beta}_2 . \vec{\alpha} = 0$

$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$

$\Rightarrow 4 - 16\alpha + 6 - 9\lambda - 25\lambda - 20 = 0$

$\Rightarrow 50\lambda = -10$

$\Rightarrow \boxed{\lambda = \frac{-1}{5}}$

$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$

$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$

$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

$5\vec{\beta}_2 . (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

14. The locus of the mid points of the chords of the circle C_1: $(x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

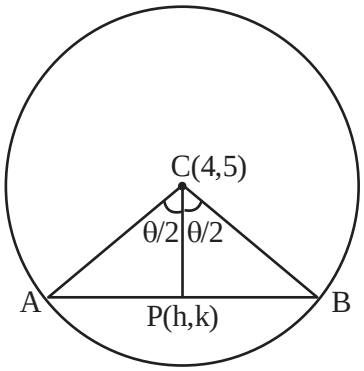
(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. In ΔCPB



$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h - 4)^2 + (k - 5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x - 4)^2 + (y - 5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2}}$

15. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) –1

(2) 3

(3) 1

(4) 5

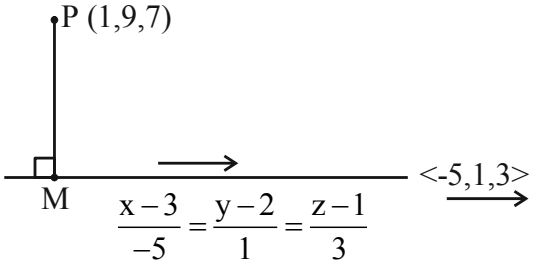
Official Ans. by NTA (4)

QuesetPix Ans. (4)

Sol. Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$



$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point M = (–2, 3, 4) = (α, β, γ)

$\alpha + \beta + \gamma = 5$

16. Let y = y(x) be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy\,dy = 0$, y(1) = 1. Then $6y^2(e)$ is equal to

(1) $3e^2$

(2) e^2

(3) $2e^2$

(4) $\frac{3e^2}{2}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$

$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y}$ (1)

Put y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$

$\Rightarrow vdv = \frac{-1}{3x}$

Integrating both side

$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$

y (1) = 1

$\Rightarrow \boxed{\frac{1}{2} = c}$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$

$\Rightarrow y^2 = -\frac{2}{3}x^2 \ln x + x^2$

$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow \boxed{6y^2(e) = 2e^2}$

17. Let p and q be two statements.

Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge (\sim q))$

(2) $p \vee ((\sim p) \wedge q)$

(3) $(\sim p) \vee q$

(4) $p \vee (p \wedge q)$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\sim (p \wedge (p \rightarrow \sim q))$

$\equiv \sim p \vee \sim(\sim p \vee \sim q)$

$\equiv \sim p \vee (p \wedge q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$

$\equiv t \wedge (\sim p \vee q)$

$\equiv \sim p \vee q$

18. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

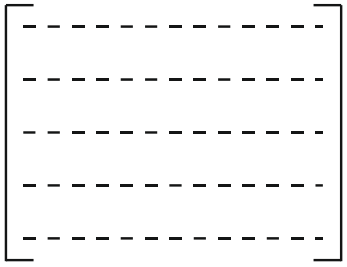
(3) 150

(4) 125

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



In each row and each column exactly one is to be placed –

$\therefore$ No. of such materials = $5 \times 4 \times 3 \times 2 \times 1 = 120$

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow 5 \text{ ways} \\ \rightarrow 4 \text{ ways} \\ \rightarrow 3 \text{ ways} \\ \rightarrow 2 \text{ ways} \\ \rightarrow 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways = $5 \times 4 \times 3 \times 2 \times 1 = 120$

19. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$ is equal to

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$

(4) 2π

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$

We have $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

Hence $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx = \frac{48}{2} \times \left[\sin^{-1}\frac{2x}{3}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$= 24 \times \left[\sin^{-1}\left(\frac{2}{3} \times \frac{3\sqrt{3}}{4}\right) - \sin^{-1}\left(\frac{2}{3} \times \frac{3\sqrt{2}}{4}\right)\right]$

$= 24 \times \left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]$

$= 24 \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= 24 \times \frac{\pi}{12} = 2\pi$

20. Let A be a 3×3 matrix such that $\left|\text{adj}\left(\text{adj}(\text{adj}A)\right)\right| = 12^4$. Then $|A^{-1}\text{adj } A|$ is equal to

(1) $2\sqrt{3}$

(2) $\sqrt{6}$

(3) 12

(4) 1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. **Given $|\text{adj}(\text{adj}(\text{adj}.A))| = 12^4$**

$\Rightarrow |A|^{(n-1)^3} = 12^4$

Given n = 3

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A|^2 = 12$

$|A| = 2\sqrt{3}$

We are asked

$|A^{-1}.\text{adj}\,A|$

$= |A^{-1}| . |\text{adj } A|$

$= \frac{1}{|A|} . |A|^{3-1}$

$= |A| = 2\sqrt{3}$

21. The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is

Official Ans. by NTA (432)

QuestPix Ans. (432)

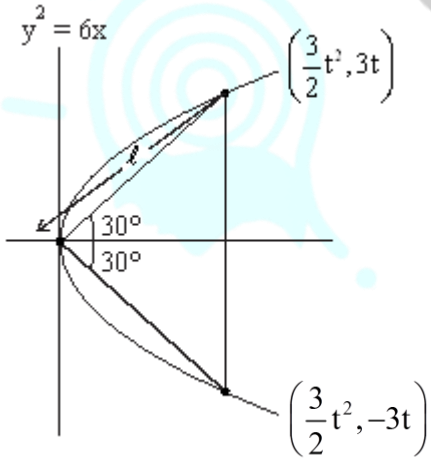
Sol.

Urn A		Urn B		Urn C	
Red	Black	Red	Black	Red	Black
4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right)=\frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}$$

$$0.4=\frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$\Rightarrow \lambda = 6$



$\tan 30° = 3t = \frac{3}{2}t^2$

$\frac{1}{\sqrt{3}} = \frac{2}{t}$

$t = 2\sqrt{3}$

$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$

$\ell^2 = 18^2 + (6\sqrt{3})^2$

$= 324 + 108$

$= 432$

22. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8A is equal to

Official Ans. by NTA (36)

QuestPix Ans. (36)

Sol. $y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

$(y - 1)^2 = -(x - 1)$

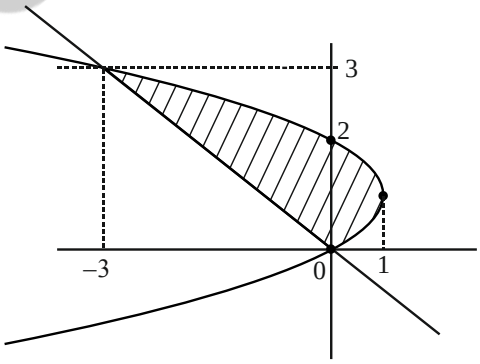
$y = -x$

Points of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$

$x = 0, -3$



$A = \int_0^3 (-y^2 + 2y + y)dy$

$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$

$8A = 36$

23. If $\frac{1^3 + 2^3 + 3^3 + \ldots\ldots\text{upto n terms}}{1\cdot 3 + 2\cdot 5 + 3\cdot 7 + \ldots\ldots\text{upto n terms}} = \frac{9}{5}$, then the value of n is

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $1^3+2^3+3^3.....+n^3=\left(\frac{n(n+1)}{2}\right)^2$

$1\cdot 3+2\cdot 5+3\cdot 7+\ldots\ldots+n$ terms $=$

$\sum_{r=1}^{n} r(2r+1)=\sum_{r=1}^{n}\left(2r^2+r\right)$

$=\frac{2\cdot n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$=\frac{n(n+1)}{6}\left(2(2n+1)+3\right)$

$=\frac{n(n+1)}{2}\times\frac{(4n+5)}{3}$

$=\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}\times\frac{(4n+5)}{3}}=\frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2}=\frac{9(4n+5)}{3}$

$\Rightarrow 15n(n+1)=18(4n+5)$

$\Rightarrow 15n^2+15n=72n+90$

$\Rightarrow 15n^2-57n-90=0\Rightarrow 5n^2-19n-30=0$

$\Rightarrow (n-5)(5n+6)=0$

$\Rightarrow n=\frac{-6}{5}$ or 5

$\Rightarrow n=5.$

24. Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right)$ such that $f(x)>0$ and

$f(x)+\int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\,dt=e,\ \forall x\in\left[0,\frac{\pi}{2}\right).$

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _____.

Official Ans. by NTA (27)

QuestPix Ans. (27)

Sol. $f(x)+\int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\,dt=e$

$\Rightarrow f(0)=e$

$f'(x)+f(x)\sqrt{1-\left(\ln f(x)\right)^2}=0$

$f(x)=y$

$\frac{dy}{dx}=-y\sqrt{1-(\ln y)^2}$

$\int\frac{dy}{y\sqrt{1-(\ln y)^2}}=-\int dx$

Put $\ln y=t$

$\int\frac{dt}{\sqrt{1-t^2}}=-x+C$

$\sin^{-1}t=-x+C\Rightarrow\sin^{-1}(\ln y)=-x+C$

$\sin^{-1}\left(\ln f(x)\right)=-x+C$

$f(0)=e$

$\Rightarrow\frac{\pi}{2}=C$

$\Rightarrow\sin^{-1}\left(\ln f(x)\right)=-x+\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{-\pi}{6}+\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{\pi}{3}$

$\Rightarrow\ln f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, we need $\left(6\times\frac{\sqrt{3}}{2}\right)^2=27.$

25. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _____.

Official Ans. by NTA (13)

QuestPix Ans. (13)

Sol. Given R = {(a, b), (b, c), (b, d)}

In order to make it equivalence relation as per given set, R must be

{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}

There already given so 13 more to be added.

26. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$, $\vec{a}\cdot\vec{c}=7$, $2\vec{b}\cdot\vec{c}+43=0$, $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$, $\vec{a}\cdot\vec{c}=7$

$\vec{a}\times\vec{c}-\vec{b}\times\vec{c}=\vec{0}$,

$(\vec{a}-\vec{b})\times\vec{c}=0\Rightarrow(\vec{a}-\vec{b})$ is paralleled to $\vec{c}$

$\vec{a}-\vec{b}=\mu\vec{c}$, where μ is a scalar

$-2\hat{i}+7\hat{j}+2\lambda\hat{k}=\mu\cdot\vec{c}$

Now $\vec{a}\cdot\vec{c}=7$ gives $2\lambda^2+12=7\mu$

And $\vec{b}\cdot\vec{c}=-\frac{43}{2}$ gives $4\lambda^2+82=43\mu$

$\mu=2$ and $\lambda^2=1$

$|\vec{a}\cdot\vec{b}|=8$

27. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$, $x\neq0$, $n\in N$, be 376. Then the coefficient of x^4 is _____.

Official Ans. by NTA (405)

QuestPix Ans. (405)

Sol. Given Binomial $\left(x-\frac{3}{x^2}\right)^n$, $x\neq0, n\in N$,

Sum of coefficients of first three terms

${}^nC_0-{}^nC_1\cdot3+{}^nC_2 3^2=376$

$\Rightarrow 3n^2-5n-250=0$

$\Rightarrow (n-10)(3n+25)=0$

$\Rightarrow n=10$

Now general term ${}^{10}C_r x^{10-r}\left(\frac{-3}{x^2}\right)^r$

$={}^{10}C_r x^{10-r}(-3)^r\cdot x^{-2r}$

$={}^{10}C_r(-3)^r\cdot x^{10-3r}$

Coefficient of x^4 $\Rightarrow 10-3r=4$

$\Rightarrow r=2$

${}^{10}C_2(-3)^2=405$

28. If the shortest between the lines

$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and

$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of λ is

Official Ans. by NTA (384)

QuestPix Ans. (384)

Sol. Shortest distance between the lines

$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$

$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{2+2\sqrt{6}}{5}$ is 6

Vector along line of shortest distance

$=\begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$, $\Rightarrow -\hat{i}+2\hat{j}-k$ (its magnitude is $\sqrt{6}$)

Now $\frac{1}{\sqrt{6}}\begin{vmatrix} \sqrt{6}+\lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=\pm6$

$\Rightarrow\lambda=-2\sqrt{6},10\sqrt{6}$

So, square of sum of these values is 384.

29. Let $S=\{[\in\theta0,2\pi):\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0\}$.

Then $\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0$

$\tan(\pi\cos\theta)=-\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta)=\tan(-\pi\sin\theta)$

$\pi\cos\theta=n\pi-\pi\sin\theta$

$\sin\theta+\cos\theta=n$ where $n\in I$

possible values are n = 0, 1 and –1 because

$-\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$

Now it gives $\theta\in\left\{0,\frac{\pi}{2},\frac{3\pi}{4},\frac{7\pi}{4},\frac{3\pi}{2},\pi\right\}$

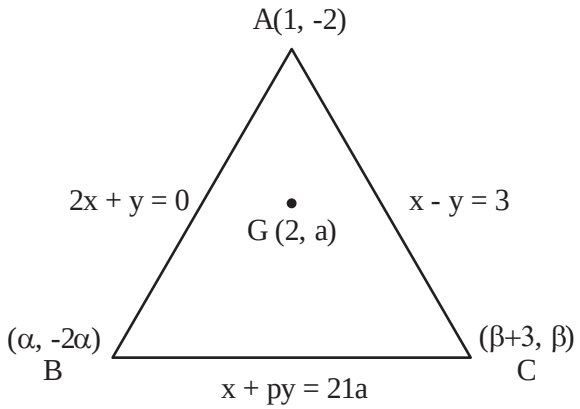
So $\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)=2(0)+4\left(\frac{1}{2}\right)=2$

30. The equations of the sides AB, BC and CA of a triangle ABC are: $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P (2, a) be the centroid of ΔABC. Then $(BC)^2$ is equal to

Official Ans. by NTA (122)

QuestPix Ans. (122)

Sol.



Assume $B(\alpha, -2\alpha)$ and $C(\beta + 3, \beta)$

$\frac{\alpha+\beta+3+1}{3} = 2$ also $\frac{-2\alpha-2+\beta}{3} = a$

$\Rightarrow \alpha+\beta = 2$ $\quad -2\alpha-2+\beta = 3a$

$\Rightarrow \beta = 2-\alpha$ $\quad -2\alpha-\cancel{2}+\cancel{2}-\alpha = 3a \Rightarrow \alpha = -a$

Now both B and C lies as given line

$\alpha - p\cdot 2\alpha = 21a$

$\alpha(1-2p) = 21\,a$ ….. (1)

$-\alpha(1-2p) = 21\,a \Rightarrow p = 11$

$\beta + 3 + p\beta = 21\,a$

$\beta + 3 + 11\beta = 21\,a$

$21\alpha + 12\beta + 3 = 0$

Also $\beta = 2-\alpha$

$21\alpha + 12(2-\alpha) + 3 = 0$

$21\alpha + 24 - 12\alpha + 3 = 0$

$9\alpha + 27 = 0$

$\alpha = -3, \beta = 5$

So $BC = \sqrt{122}$ and $(BC)^2 = 122$

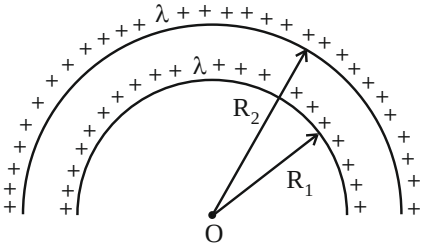
(Held On Tuesday 24th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is



(1) $\frac{2\lambda}{\epsilon_0}$ (2) $\frac{\lambda}{2\epsilon_0}$

(3) $\frac{\lambda}{4\epsilon_0}$ (4) $\frac{\lambda}{\epsilon_0}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Potential at centre

$$V = \frac{(\lambda \cdot \pi R_2)}{4\pi\varepsilon_0 R_2} + \frac{(\lambda \cdot \pi R_1)}{4\pi\varepsilon_0 R_1}$$

$$= \frac{\lambda}{2\varepsilon_0}$$

32. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is

(1) $\frac{27}{35}$ (2) $\frac{35}{27}$

(3) $\frac{25}{21}$ (4) $\frac{21}{25}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. For monoatomic gas $\gamma_1 = \frac{5}{3}$

For diatomic gas at low temperatures

$$\gamma_2 = \frac{7}{5}$$

$$\therefore \frac{\gamma_1}{\gamma_2} = \frac{\frac{5}{3}}{\frac{7}{5}} = \frac{25}{21}$$

33. An α-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their De-Broglie wavelength:

(1) $\lambda_\alpha > \lambda_p > \lambda_e$

(2) $\lambda_\alpha < \lambda_p < \lambda_e$

(3) $\lambda_\alpha = \lambda_p = \lambda_e$

(4) $\lambda_\alpha > \lambda_p < \lambda_e$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\lambda_D = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

$$\therefore \lambda \propto \frac{1}{\sqrt{m}}$$

$\because m_\alpha > m_p > m_e$

$\lambda_e > \lambda_p > \lambda_\alpha$

34. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is:

(1) 6×10^7 km

(2) 6×10^6 km

(3) 3×10^6 km

(4) 3×10^7 km

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$$\Rightarrow \left(\frac{1}{2.83}\right)^2 = \left(\frac{1.5\times10^6}{R_2}\right)^3$$

$$\Rightarrow R_2 = \left[(2.83)^2 \times (1.5\times10^6)^3\right]^{1/3}$$

$$= 8^{1/3} \times 1.5 \times 10^6 = 3 \times 10^6 \text{ km}$$

35. Match List I with List II

	LIST I		LIST II
A.	AM Broadcast	I.	88-108 MHz
B.	FM Broadcast	II.	540-1600 kHz
C,	Television	III.	3.7-4.2 GHz
D.	Satellite Communication	IV.	54MH_z - 590MHz

Choose the correct answer from the options given below:

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-I, B-III, C-II, D-IV

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. AM Broadast $\rightarrow$ 540 – 1600 KHz

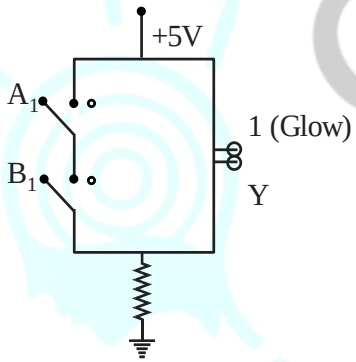
FM Broadcast $\rightarrow$ 88 – 108 MHz

Television $\rightarrow$ 54 – 890 MHz

Salellite communication $\rightarrow$ 3.7 – 4.2 GHz

$\therefore$ A-II, B-I, C-IV, D-III

36. The logic gate equivalent to the given circuit diagram is :



(1) OR

(2) NAND

(3) NOR

(4) AND

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

A_1	B_1	Y
0	0	1
0	1	1
1	0	1
1	1	0

$Y = \overline{A_1 \odot B_1}$ NAND

37. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is _____ ($\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})

(1) 1232×10^{-4} T

(2) 176×10^{-4} T

(3) 352×10^{-4} T

(4) 88×10^{-4} T

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $B = \mu_0 nI$

$= 4\pi \times 10^{-7} \times 70 \times 10^2 \times 2$

$= 56\pi \times 10^{-4}$ T

$= 176 \times 10^{-4}$ T

38. Given below are two statements:

Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II: Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.

In the light of above statements, choose the ***most appropriate*** answer form the options given below

(1) Statement I is incorrect but statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but statement II is incorrect

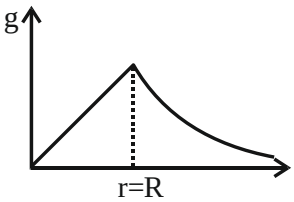
(4) Both Statement I and II are correct

Official Ans. by NTA (3)

QuestPix Ans. (3)

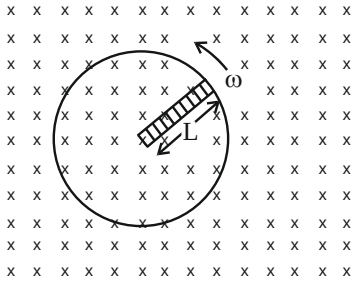
Sol. $g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$g' = g\left\{1-\dfrac{d}{R}\right\}$



Statement I is correct & Statement II is incorrect

39. A metallic rod of length 'L' is rotated with an angular speed of 'ω' normal to a uniform magnetic field 'B' about an axis passing through one end of rod as shown in figure. The induced emf will be :



(1) $\frac{1}{4}B^2L\omega$

(2) $\frac{1}{4}BL^2\omega$

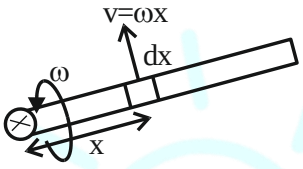
(3) $\frac{1}{2}BL^2\omega$

(4) $\frac{1}{2}B^2L^2\omega$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



$$\int d\varepsilon = \int B(\omega x)dx$$

$$\varepsilon = B\omega\int_0^L x dx = \frac{B\omega L^2}{2}$$

40. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called :

(1) Scattering

(2) Chromatic aberration

(3) Spherical aberration

(4) Polarisation

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Based on fact.

41. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $\nu = r^a \rho^b s^c$. The values of a, b and c respectively are

(1) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\left[T^{-1}\right] = \left[L^1\right]^a \left[M^1L^{-3}\right]^b \left[\frac{MLT^{-2}}{L}\right]^c$

$$\Rightarrow T^{-1} = M^{b+c} \cdot L^{a-3b} \cdot T^{-2c}$$

$$c = \frac{1}{2},\ b = -\frac{1}{2},\quad a - 3b = 0$$

$$a + \frac{3}{2} = 0 \Rightarrow a = -\frac{3}{2}$$

42. A body of mass 200g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s, then the ratio of extension in the spring to its natural length will be :

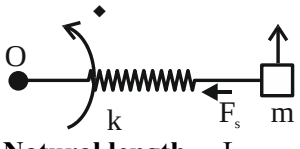
(1) 1:2 (2) 1:1

(3) 2:3 (4) 2:5

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



Natural length = L_0

Extension = x

$$kx = m(L_0 + x)\omega^2$$

$$\Rightarrow 12.5x = \frac{1}{5}(L_0 + x)25 \quad \Rightarrow 1.5x = L_0$$

$$\Rightarrow \frac{x}{L_0} = \frac{2}{3}$$

43. A cell of emf 90 V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400 Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be :

(1) 40 V

(2) 45 V

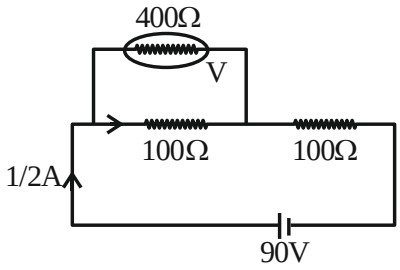
(3) 80 V

(4) 90 V

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



$$R_{eq} = \frac{400 \times 100}{500} + 100$$

$$= 180\Omega$$

$$i = \frac{90}{180} = \frac{1}{2}A$$

$$\text{Reading} = \frac{1}{2} \times \frac{400}{500} \times 100$$

$$= 40\text{volt}$$

44. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$E_x = E_0 \sin(kz - \omega t)$

$B_y = B_0 \sin(kz - \omega t)$

Then the correct relation between E_0 and B_0 is given by

(1) $kE_0 = \omega B_0$

(2) $E_0 B_0 = \omega k$

(3) $\omega E_0 = kB_0$

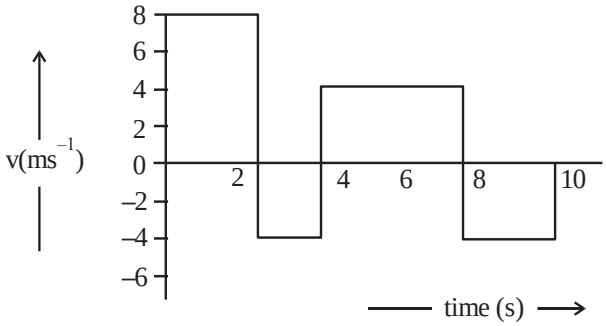
(4) $E_0 = kB_0$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $C = \frac{\omega}{k} = \frac{E_0}{B_0}$

45. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10s is

(1) 1:1

(2) 1:4

(3) 1:2

(4) 1:3

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\text{Displacement} = \Sigma\text{area} = 16 - 8 + 16 - 8 = 16\text{ m}$

$\text{Distance} = \Sigma|\text{area}| = 48\text{ m}$

$$\frac{\text{displacement}}{\text{Distance}} = \frac{1}{3}$$

46. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Steel is used in the construction of buildings and bridges.

Reason R: Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) **A** is not correct but **R** is correct

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(4) **A** is correct but **R** is not correct

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Concept based

47. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A pendulum clock when taken to Mount Everest becomes fast.

Reason R: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(3) **A** is not correct but **R** is correct

(4) **A** is correct but **R** is not correct

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $T \propto \frac{1}{\sqrt{g}}$

48. A photon is emitted in transition from n = 4 to n=1 level in hydrogen atom. The corresponding wavelength for this transition is

(given, $h = 4 \times 10^{-15}$ eVs) :

(1) 94.1 nm

(2) 941 nm

(3) 97.4 nm

(4) 99.3 nm

Official Ans. by NTA (1)

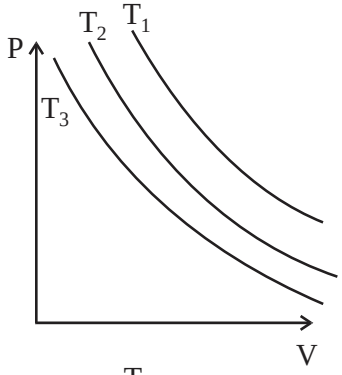
QuestPix Ans. (1)

Sol. $\frac{hc}{\lambda} = \left[1 - \frac{1}{16}\right](13.6\ eV)$

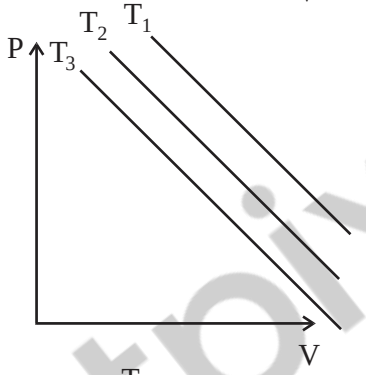
So, $\lambda = 94.1 nm$

49. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as :

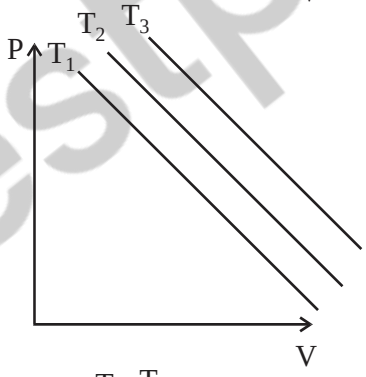
(1)



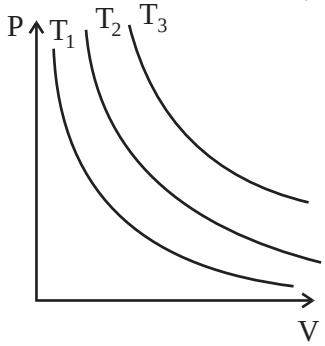
(2)



(3)



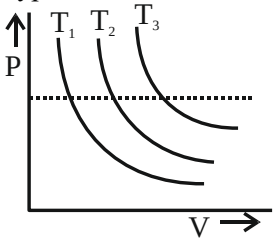
(4)



Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. For isothermal process P-V graph is rectangular hyperbola



As dotted line is isobaric line which implies $T_3 > T_2 > T_1$ as volume is increasing.

50. If two vectors $\vec{P} = \hat{i} + 2m\hat{j} + m\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be :

(1) 1 (2) –1

(3) – 3 (4) 2

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\vec{P} \cdot \vec{Q} = 0$

$\left(\hat{i} + 2m\hat{j} + m\hat{k}\right) \cdot \left(4\hat{i} - 2\hat{j} + m\hat{k}\right) = 0$

$\Rightarrow 4 - 4m + m^2 = 0$

$\Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$

Section-B

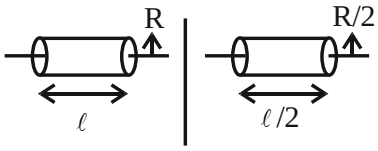
51. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is caned out of the original cylinder. If I_2 is the moment of inertia of the carved out portion ot the cylinder then $\frac{I_1}{I_2} =$ _____

(Both I_1 and I_2 are about the axis of the cylinder)

Official Ans. by NTA (32)

QuestPix Ans. (32)

Sol.



$I_1 = \frac{m_1 R^2}{2}$ $I_2 = \frac{m_2 (R/2)^2}{2}$

$\frac{I_1}{I_2} = \frac{4m_1}{m_2} = \frac{4 \cdot \rho\pi R^2 \ell}{\rho \cdot \frac{\pi R^2}{4} \times \frac{\ell}{2}} \Rightarrow \frac{I_1}{I_2} = 32$

52. A mass m attached to free end of a spring executes SHM with a period of 1s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass m is _____ kg.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $T = 2\pi\sqrt{\frac{m}{k}} = 1$

$T' = 2\pi\sqrt{\frac{m+3}{k}} = 2$

$\frac{T}{T'} = \sqrt{\frac{m}{m+3}} = \frac{1}{2}$

$\Rightarrow \frac{m}{m+3} = \frac{1}{4}$

$m = 1$

53. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120g of pure ^{240}X undergo fission is _____ $\times 10^{25}$ MeV.

(Given $N_A = 6 \times 10^{23}$)

Official Ans. by NTA (6)

QuestPix Ans. (6)

Sol. No. of mole $= \frac{120}{240} = \frac{1}{2}$

No. of molecules $= \frac{1}{2} \times N_A$

Energy released $= \frac{1}{2} \times 6 \times 10^{23} \times 200$

$= 6 \times 10^{25}$ MeV

54. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$ pF. The value of x is _____.

Official Ans. by NTA (105)

QuestPix Ans. (105)

Sol. $C_0 = \frac{\epsilon_0 A}{d} = 15pF$

$C = \frac{K\epsilon_0 A}{2d} = \frac{3.5}{2} \times 15pF = \frac{105}{4} pF$

55. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = \left(t\hat{i} + 3t^2\hat{j}\right)N$. where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time t = 2s. will be _____W.

Official Ans. by NTA (100)

QuestPix Ans. (100)

Sol. $\vec{F} = t\hat{i} + 3t^2\hat{j}$

$\frac{md\vec{v}}{dt} = t\hat{i} + 3t^2\hat{j}$

$m = 1kg, \int_0^{\vec{v}} dv = \int_0^t tdt\hat{i} + \int_0^t 3t^2 dt\hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t^3\hat{j}$

Power $= \vec{F} \cdot \vec{V} = \frac{t^3}{2} + 3t^5$

At t = 2, power $= \frac{8}{2} + 3 \times 32$

$=100$

56. If a copper wire is stretched to increase its length by 20°%. The percentage increase in resistance of the wire is _____ %.

Official Ans. by NTA (44)

QuestPix Ans. (44)

Sol. As volume is constant,

So resistance $\propto$ (length)^2

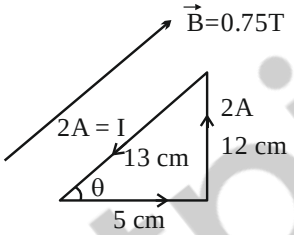
$\Rightarrow$ % change in resistance $= 20 + 20 + \frac{400}{100} = 44\%$

57. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of x is _____.

Official Ans. by NTA (9)

QuestPix Ans. (9)

Sol.



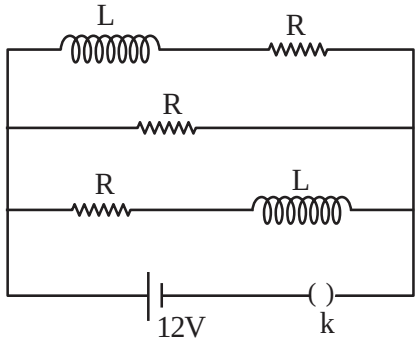
Force on 5 cm side is

$\left|\vec{F}\right| = ILB\sin\theta$

$= (2)(5 \times 10^{-2}) \times \frac{3}{4} \times \frac{12}{13} = \frac{9}{130} N$

So, x = 9

58. Three identical resistors with resistance R = 12Ω and two identical inductors with sell inductance L=5 mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be_____A.



Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. After long time an inductor behaves as a resistance-less path.

So current through cell

$I = \frac{12}{R/3} = 3A \ \{\because R = 12\Omega\}$

59. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be_____cm.

(Given refractive index of water = $\frac{4}{3}$)

Official Ans. by NTA (54)

QuestPix Ans. (54)

Sol. $$\frac{I}{f_{H_2O}} = \left(\frac{\mu_g}{\mu_{H_2O}} - 1\right)\left(\frac{2}{R}\right)$$

$$= \frac{1}{8}\left(\frac{2}{R}\right)$$

$$= \frac{1}{(4f_{air})}$$

So, $f_{H_2O} = 4f_{air} = 72$ cm

So change in focal length = 72 – 18= 54 cm

60. A Spherical ball of radius 1mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696 x 10^{-x} N. The value of x is

(Given, g = 9.8 m/s^2 and $\pi = \frac{22}{7}$)

Official Ans. by NTA (7)

QuestPix Ans. (7)

Sol. When the ball attain terminal velocity

$$F_v = (mg - F_B) \ (\because a = 0)$$

$$= V\sigma_b g - V\rho_\ell g$$

$$= Vg(\sigma_b - \rho_\ell)$$

$$= \frac{4}{3}\pi(10^{-3})^3 \times 9.8\ (10.5 - 1.5) \times 10^3$$

$$= 3696 \times 10^{-7}\ N$$

So, x = 7

(Held On Tuesday 24th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH ANSWER

SECTION-A

61. Which one amongst the following are good oxidizing agents?

A. Sm^{2+} B. Ce^{2+}

C. Ce^{4+} D. Tb^{4+}

Choose the most appropriate answer from the options given below :

(1) C only

(2) D only

(3) A and B only

(4) C and D only

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Ce^{+4} and Tb^{+4} act as oxidising agent.

62. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : $N_2 : N_2^+; O_2; O_2^+$?

(1) 0, 1, 2, 1 (2) 2, 1, 2, 1

(3) 0, 1, 0, 1 (4) 2, 1, 0, 1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. N_2 -

$$\sigma ls^2\ \sigma^* ls^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \underbrace{\sigma 2p_z^2}_{\text{HOMO}}$$

$$N_2^+ - \sigma ls^2\ \sigma^* ls^2\ \sigma^* 2s^2 \sigma^* 2s^2\ \pi 2p_x^2 = \pi 2p_y^2\ \underbrace{\sigma 2p_z^1}_{\text{HOMO}}$$

$$O_2 - \sigma ls^2\ \sigma^* ls^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2$$

$$\pi 2p_x^2 = \pi 2p_y^2$$

$$\pi^* 2p_x^1 = \pi^* 2p_y^1 (\text{HOMO})$$

$$O_2^+ - \sigma ls^2\ \sigma^* ls^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2$$

$$\pi^* 2p_x^1 = \pi^* 2p_y^0 (\text{HOMO})$$

$N_2 \Rightarrow$ 0 unpaired e^- in HOMO

$N_2^+ \Rightarrow$ 1 unpaired e^- in HOMO

$O_2 \Rightarrow$ 2 unpaired e^- in HOMO

$O_2^+ \Rightarrow$ 1 unpaired e^- in HOMO

63. Which of the following cannot be explained by crystal field theory?

(1) The order of spectrochemical series

(2)Magnetic properties of transition metal complexes

(3) Colour of metal complexes

(4) Stability of metal complexes

Official Ans. by NTA (4)

QuestPix Ans. (1)

Sol. Crystal field theory introduce spectrochemical series based upon the experimental values of Δ but can't explain it's order. While other three points are explained by CFT. Specially when the CFSE increases thermodynamic stability of the complex increases.

64. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p (mm Hg)	50	100	200	400
Relative $t_{1/2}$ (s)	4	2	1	0.5

The order of the reaction is

(1) 0.5 (2) 2

(3) 1 (4) 0 (zero)

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $t_{\frac{1}{2}} \propto (P_O)^{1-n}$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{(P_0)_1^{1-n}}{(P_{0_2})_2^{1-n}}$$

$$\Rightarrow \left(\frac{4}{2}\right) = \left(\frac{50}{100}\right)^{1-n}$$

$$\Rightarrow 2 = \left(\frac{1}{2}\right)^{1-n}$$

$$\Rightarrow 2 = (2)^{n-1}$$

$$\Rightarrow n - 1 = 1 \quad \Rightarrow n = 2$$

65. The number of s-electrons present in an ion with 55 protons in its unipositive state is

(1) 8 (2) 9

(3) 12 (4) 10

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $Z = 55$ [Cs] $\Rightarrow$ [Xe] $6s^1$

$[Cs^+] \Rightarrow$ [Xe] i.e. upto 5s count e^- of s-subshell

i.e. 1s, 2s, 3s, 4s, 5s 10 electrons

66. In which of the following reactions the hydrogen peroxide acts as a reducing agent?

(1) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(2) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

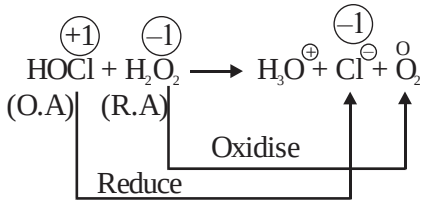
(3) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(4) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



67. The metal which is extracted by oxidation and subsequent reduction from its ore is :

(1) Al (2) Ag

(3) Cu (4) Fe

Official Ans. by NTA (2)

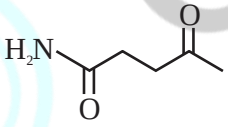
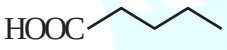
QuestPix Ans. (2)

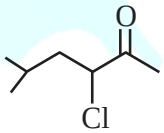
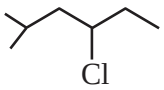
Sol. Ag.

$4Ag + 8CN^- + O_2 + 2H_2O \rightarrow 4[Ag(CN)_2]^- + 4OH^-$

$2[Ag(CN)_2]^{-1} + Zn \rightarrow 2Ag\downarrow + [Zn(CN)_4]^{-2}$

68. Given below are two statements :

Statement I :  under Clemmensen reduction conditions will give  .

Statement II :  under Wolff-Kishner reduction condition will give 

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

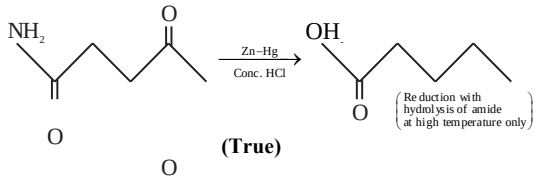
(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Official Ans. by NTA (3)

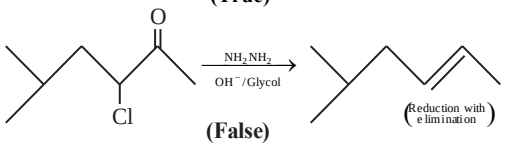
QuestPix Ans. (3)

Sol. S – I



(True)

S – II



(False)

69. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason R : Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) A is correct but R is not correct

(2) Both A and R are correct and R is the correct explanation of A.

(3) A is not correct but R is correct

(4) Both A and R are correct and R is NOT the correct explanation of A.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Be has less negative value compared to other AEM. However it's reducing nature is due to large hydration energy associated with the small size of Be^{2+} ion and relatively large value of the atomization enthalpy of metal.

70. Match List I with List II

	LIST I Type		**LIST II Name**
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III.	Seldane
D.	Antibiotic	IV.	Ampicillin

Choose the correct answer from the options given below:

(1) A-II, B-I, C-III, D-IV

(2) A-IV, B-III, C-II, D-I

(3) A-I, B-III, C-II, D-IV

(4) A-I, B-II, C-III, D-IV

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Theoretical, NCERT based.

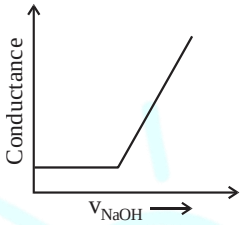
71. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : Benzene is more stable than hypothetical cyclohexatriene.
Reason R : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.
In the light of the above statements, choose the correct answer from the options given below:
(1) A is true but R is false.
(2) A is false but R is true.
(3) Both A and R are correct and R is the correct explanation of A.
(4) Both A and R are correct but R is NOT the correct explanation of A.
Official Ans. by NTA (3)
QuestPix Ans. (3)

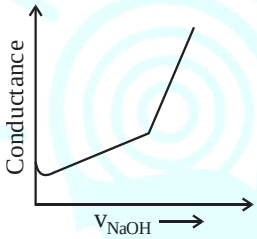
Sol. **Assertion – A :** Benzene is more stable than cyclohexatriene **(True)**

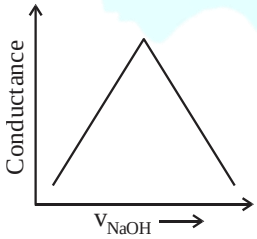
Reason – R : Delocalised $\pi-e$ cloud lies B.M.O so more attracted by nuclei of carbon atom.

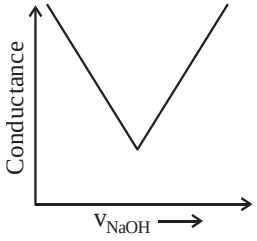
(True & Correct Explanation)

72. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

(1) 

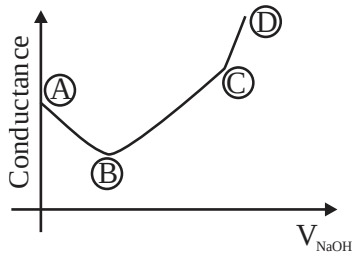
(2) 

(3) 

(4) 

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. $\underset{(WA)}{C_6H_5COOH} + \underset{(SB)}{NaOH} \longrightarrow \underset{(Salt)}{C_6H_5COONa} + H_2O$

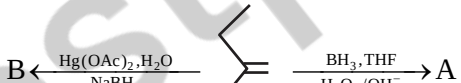


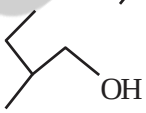
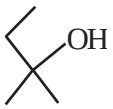
(A) $\rightarrow$ (B) Free H^+ ions are replaced by $Na^{\oplus}$ which decreases conductance.

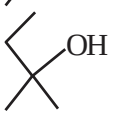
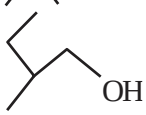
(B) $\rightarrow$ (C) Un-dissociated benzoic acid reacts with NaOH and forms salt which increases ions & conductance increases.

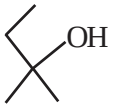
(C) $\rightarrow$ (D) After equivalence point at (3), NaOH added further increases $Na^{\oplus}$ & $OH^{\odot}$ ions which further increases the conductance.

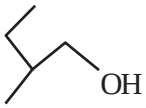
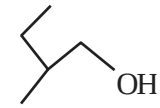
73. Find out the major products from the following reactions.



(1) A =  , B = 

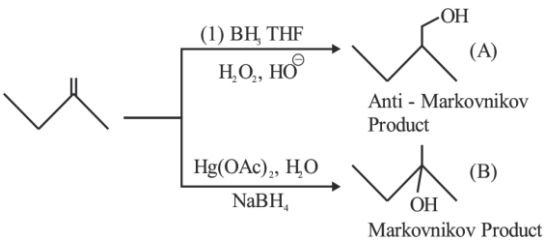
(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol.



74. Correct statement is :
(1) An average human being consumes more food than air
(2) An average human being consumes nearly 15 times more air than food
(3) An average human being consumes equal amount of food and air
(4) An average human being consumes 100 times more air than food

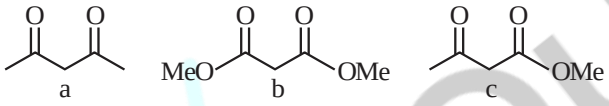
Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. Theoretical.

75. Given below are two statements :
Statement I : Pure Aniline and other arylamines are usually colourless.
Statement II : Arylamines get coloured on storage due to atmospheric reduction.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Both Statement I and Statement II are incorrect
(2) Both Statement I and Statement II are correct
(3) Statement I is correct but Statement II is incorrect
(4) Statement I is incorrect but Statement II is correct
Official Ans. by NTA (3)
QuestPix Ans. (3)

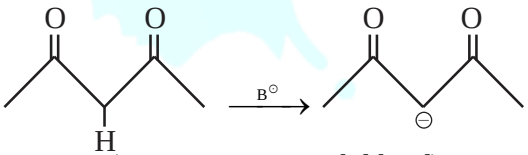
Sol. **Statement – 1 is (True)**
Pure aniline is colourless liquid
Statement – 2 is (False)
Aniline becomes dark brown due to action of air and light [oxidation]

76. Which will undergo deprotonation most readily in basic medium?



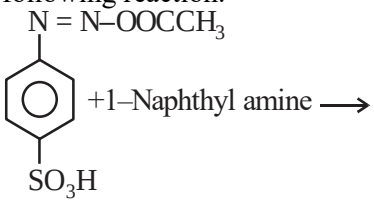
(1) a only
(2) c only
(3) Both a and c
(4) b only
Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol. Most easily deprotonation



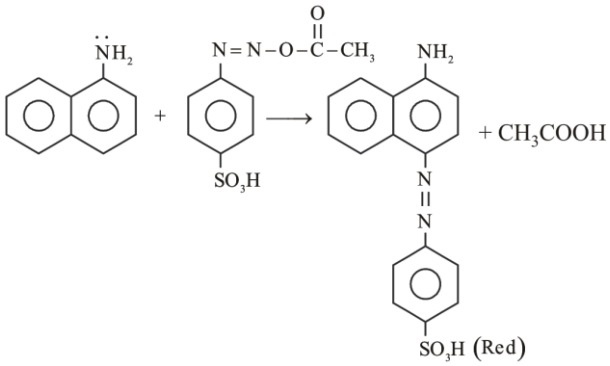
(More resonance stabilibsed)

77. Choose the correct colour of the product for the following reaction.



(1) Yellow (2) White
(3) Red (4) Blue
Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol.



78. Identify the correct statements about alkali metals.
A. The order of standard reduction potential $(M^+ | M)$ for alkali metal ions is Na > Rb > Li.
B. CsI is highly soluble in water.
C. Lithium carbonate is highly stable to heat.
D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.
E. All the alkali metal hydrides are ionic solids.
Choose the correct answer from the options given below
(1) A, B, D only (2) C and E only
(3) A and E only (4) A, B and E only
Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol. (1) Na > Cs > Li – true {If considered with sign}
The low solubility of CsI is due to smaller hydration enthalpy of it's two ions .
Li_2CO_3 is highly stable to heat - false
In Conc. NH_3, K formed blue solution – true
All the alkali metal hydrides are ionic solid (True).

79. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is
(1) sp^3d^2 and diamagnetic
(2) d^2sp^3 and paramagnetic
(3) d^2sp^3 and diamagnetic
(4) sp^3d^2 and paramagnetic
Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol. $[Co(NH_3)_6]^{+3}$ – d^2sp^3, diamagnetic

80. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to
(1) Carbon dioxide
(2) Sulphur trioxide
(3) Hydrogen sulphide
(4) Sulphur dioxide
Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. $3SO_2 + Cr_2O_7^{2-} + 2H^+ \rightarrow 3SO_4^{2-} + 2Cr^{+3} + H_2O$
green

SECTION-B

81. The number of statement/s which are the characteristics of physisorption is _____.

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increase in temperature

D. It results into unimolecular layer

E. No activation energy is needed

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. **For physisorptions**

(a)Decreases with increase in temperature

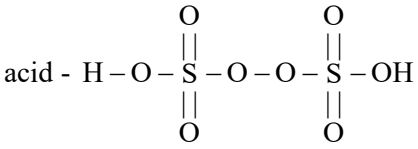
(b)No appreciable activation energy is required

82. Sum of π-bonds present in peroxodisulphuric acid and pyrosulphuric acid is

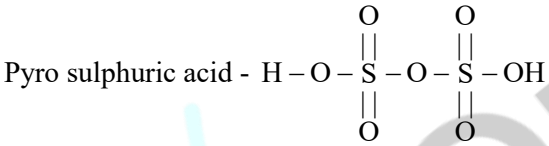
Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. Peroxodisulphuric



No. of π – bonds = 4



No. of π – bonds = 4

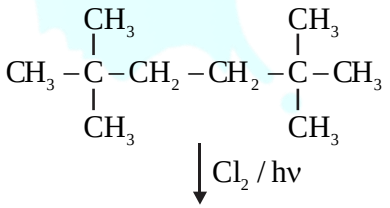
Total π – bonds = 8

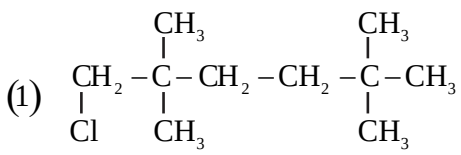
83. Maximum number of isomeric monochloro derivatives which can be obtained from 2,2,5,5-tetramethylhexane by chlorination is _____

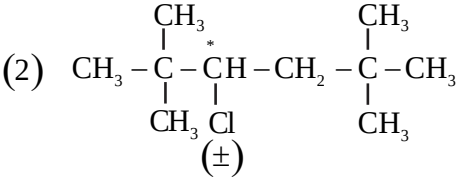
Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.







Total numbers of isomer = 03

84. Total number of tripeptides possible by mixing of valine and proline is _____

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. No. of possible tripeptide :

Val & Pro is 2^3

(1) val – val – val (2)pro – pro – pro

(3)val – pro – pro (4)pro – val – pro

(5)val – val – pro (6)val – pro – val

(7)pro – pro – val (8)pro – val – val

85. The number of units, which are used to express concentration of solutions from the following is _____.

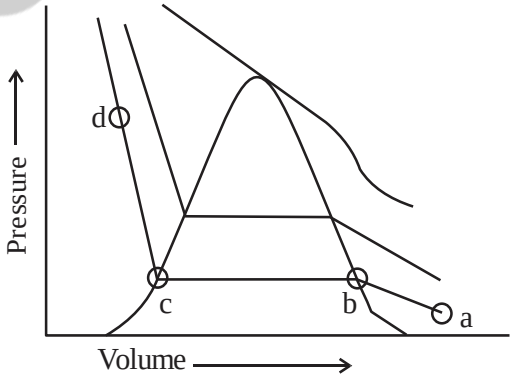
Mass percent, Mole, Mole fraction, Molarity, ppm, Molality.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. Mass percent, mole fraction, molarity, ppm, molality are used for measuring concentration terms.

86. The number of statement's, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is _____.



A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

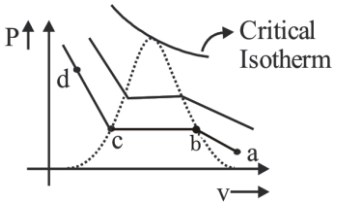
C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) to (c), the amount of liquid decreases

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



At

(a) $\rightarrow$ CO_2 exist as gas

(b) $\rightarrow$ liquefaction of CO_2 starts

(c) $\rightarrow$ liquefaction ends

(d) $\rightarrow$ CO_2 exist as liquid.

Between (b) & (c) $\rightarrow$ liquid and gaseous CO_2 co-exist.

As volume changes from (b) to (c) gas decreases and liquid increases.

(A), (C) $\rightarrow$ correct

87. The Total pressure observed by mixing two liquid A and B is 350 mm Hg when their mole fractions are 0.7 and 0.3 respectively.

The Total pressure becomes 410 mm Hg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is _____ mm Hg. (Nearest integer)

Consider the liquids and solutions behave ideally.

Official Ans. by NTA (314)

QuestPix Ans. (314)

Sol. Let V.P. of pure A be P_A^0

Let V.P of pure B be P_B^0

When $X_A = 0.7$ & $X_B = 0.3$

$P_s = 350$

$\Rightarrow P_A^0 \times 0.7 + P_B^0 \times 0.3 = 350$ ...(i)

When $X_A = 0.2$ & $X_B = 0.8$

$P_s = 410$

$\Rightarrow P_A^0 \times 0.2 + P_B^0 \times 0.8 = 410$ ...(ii)

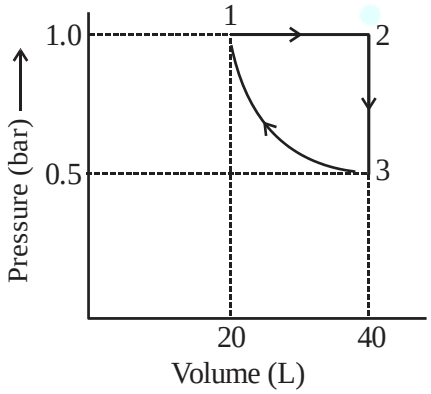
Solving (i) and (ii)

$P_A^0 = 314$ mm Hg

$P_B^0 = 434$ mm Hg

= (314)

88. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is _____ J (nearest integer).



Given : log 2 = 0.3, ln 10 = 2.3

Official Ans. by NTA (620)

QuestPix Ans. (620)

Sol. $1 \rightarrow 2 \Rightarrow$ Isobaric process

$2 \rightarrow 3 \Rightarrow$ Isochoric process

$3 \rightarrow 1 \Rightarrow$ Isothermal process

$$W = W_{1\rightarrow 2} + W_{2\rightarrow 3} + W_{3\rightarrow 1}$$

$$= \left(-P(V_2 - V_1) + 0 \left| -P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \right| \right)$$

$$= \left| -1 \times (40 - 20) + 0 + \left| -1 \times 20 \ln \left(\frac{20}{40} \right) \right| \right|$$

$$= -20 + 20 \ln 2$$

$$= -20 + 20 \times 2.3 \times 0.3$$

$$= -6.2 \text{ bar L}$$

$$|W| = 6.2 \text{ bar l} = 620 \text{ J}$$

89. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate solution at 25° C is _____ $\times 10^{-1}$ (Nearest integer)

Lactic acid $CH_3 - \underset{\underset{OH}{|}}{\overset{\overset{H}{|}}{C}} - COOH$

Official Ans. by NTA (85)

QuestPix Ans. (85)

Sol. Concentration of calcium lactate = 0.005 M,:

concentration of lactate ion = (2 × 0.005) M.

Calcium lactate is a salt of weak acid + strong base

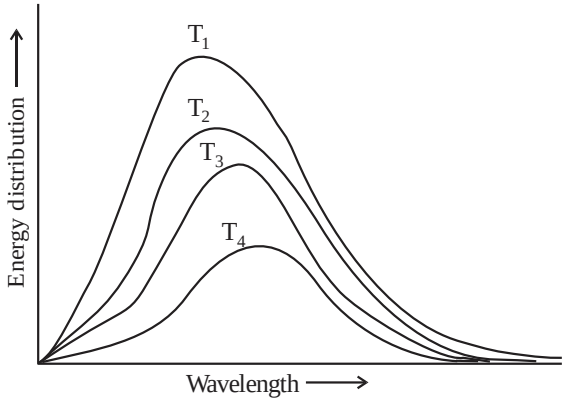
$\therefore$ Salt hydrolysis will take place.

$$pH = 7 + \frac{1}{2}(pKa + \log C)$$

$$= 7 + \frac{1}{2}(5 + \log(2 \times 0.005))$$

$$= 7 + \frac{1}{2}[5 - 2\log 10] = 7 + \frac{1}{2} \times 3 = 8.5 = 85 \times 10^{-1}$$

90. Following figure shows spectrum of an ideal black body at four different temperatures. The number of **correct** statement/s from the following is _____.



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{v_1} = \frac{T_2}{v_2} = \frac{T_3}{v_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. The spectrum of Black body radiation is explained using quantization of energy. With increase in temperature, peak of spectrum shifts to shorter wavelength or higher frequency. For above graph $\rightarrow T_1 > T_2 > T_3 > T_4$.