## (Held On Tuesday 20th July, 2021)

## TIME : 9 : 00 AM to 12 : 00 NOON

## MATHEMATICS

## TEST PAPER WITH SOLUTION

### SECTION-A

**1.** The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

**Official Ans. by NTA (2)**

**Sol.**

| p | q | ~p | ~q | $p \wedge \sim q$ | $q \vee \sim p$ | $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ | $p \Rightarrow q$ |
|---|---|---|---|---|---|---|---|
| T | F | F | T | T | F | F | F |
| F | T | T | F | F | T | T | T |
| T | T | F | F | F | T | T | T |
| F | F | T | T | F | T | T | T |

$\therefore \quad (p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$\equiv p \Rightarrow q$

So, option (2) is correct.

**2.** Let a be a positive real number such that $\int_0^a e^{x-[x]}\,dx = 10e - 9$ where [x] is the greatest integer less than or equal to x. Then a is equal to :

(1) $10 - \log_e(1 + e)$ (2) $10 + \log_e 2$

(3) $10 + \log_e 3$ (4) $10 + \log_e(1 + e)$

**Official Ans. by NTA (2)**

**Sol.** $a > 0$

Let $n \le a < n + 1, n \in W$

$\therefore a = [a] + \{a\}$

$\Downarrow \quad \Downarrow$

G.I.F Fractional part

Here $[a] = n$

Now, $\int_0^a e^{x-[x]}\,dx = 10e - 9$

$\Rightarrow \int_0^n e^{\{x\}}\,dx + \int_n^a e^{x-[x]}\,dx = 10e - 9$

$\therefore \quad n\int_0^1 e^x\,dx + \int_n^a e^{x-n}\,dx = 10e - 9$

$\Rightarrow n(e-1) + (e^{a-n} - 1) = 10e - 9$

$\therefore \quad \boxed{n = 0}$ and $\{a\} = \log_e 2$

So, $a = [a] + \{a\} = (10 + \log_e 2)$

$\Rightarrow$ Option (2) is correct.

**3.** The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 10, 11 (2) 3, 18

(3) 8, 13 (4) 1, 20

**Official Ans. by NTA (1)**

**Sol.** Let other two numbers be a, (21 – a)

Now,

$$10.25 = \frac{(4 + 16 + 25 + 49 + a^2 + (21 - a)^2)}{6} - (6.5)^2$$

(Using formula for variance)

$\Rightarrow 6(10.25) + 6(6.5)^2 = 94 + a^2 + (21 - a)^2$

$\Rightarrow a^2 + (21 - a)^2 = 221$

$\therefore a = 10$ and $(21 - a) = 21 - 10 = 11$

So, remaining two observations are 10, 11.

$\Rightarrow$ Option (1) is correct.

**4.** The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x} + \sqrt{1+x})\,dx$ is equal to :

(1) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$ (2) $2\log_e 2 + \frac{\pi}{4} - 1$

(3) $\log_e 2 + \frac{\pi}{2} - 1$ (4) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

**Official Ans. by NTA (2)**

**QuestPix Ans. (3)**

**Sol.** Let $I = 2\int_0^1 \underbrace{\ln\left(\sqrt{1-x}+\sqrt{1+x}\right)}_{(I)}\ \underbrace{1}_{(II)}\ dx$

(I.B.P.)

$$\therefore\ I = 2\Big[\left(x.\ln\left(\sqrt{1-x}+\sqrt{1-x}\right)\right)_0^1$$

$$-\int_0^1 x.\left(\frac{1}{\sqrt{1-x}+\sqrt{1+x}}\right)\cdot\left(\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}\right)dx\Big]$$

$$= 2\left(\ln\sqrt{2}-0\right)-\frac{2}{2}\int_0^1 \frac{x\sqrt{1-x}-\sqrt{1+x}\ dx}{\left(\sqrt{1-x}+\sqrt{1+x}\right)\sqrt{1-x^2}}$$

$$= \left(\log_e 2\right)-\int_0^1 \frac{x\cdot\left(2-2\sqrt{1-x^2}\right)}{-2x\sqrt{1-x^2}}\,dx$$

(After rationalisation)

$$= \left(\log_e 2\right)+\int_0^1\left(\frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)dx$$

$$= \left(\log_e 2\right)+\left(\sin^{-1}x\right)_0^1-1$$

$$= \log_e 2+\left(\frac{\pi}{2}-0\right)-1$$

$$\therefore\ I = \left(\log_e 2\right)+\frac{\pi}{2}-1$$

$\Rightarrow$ Option (3) is correct.

**5.** If $\alpha$ and $\beta$ are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12}-1)$ is equal to :

(1) $56 \times 3^{25}$ (2) $56 \times 3^{24}$

(3) $52 \times 3^{24}$ (4) $28 \times 3^{25}$

**Official Ans. by NTA (3)**

**Sol.** As, $\left(\alpha^2+\sqrt{3}\right) = -(3)^{1/4}\cdot\alpha$

$\Rightarrow \left(\alpha^4+2\sqrt{3}\alpha^2+3\right) = \sqrt{3}\alpha^2$ (On squaring)

$\therefore\ (\alpha^4+3) = (-)\sqrt{3}\alpha^2$

$\Rightarrow\ \alpha^8+6\alpha^4+9 = 3\alpha^4$ (Again squaring)

$\therefore\ \alpha^8+3\alpha^4+9 = 0$

$\Rightarrow\ \boxed{\alpha^8 = -9-3\alpha^4}$

(Multiply by $\alpha^4$)

So, $\alpha^{12} = -9\alpha^4-3\alpha^8$

$\therefore\ \alpha^{12} = -9\alpha^4-3(-9-3\alpha^4)$

$\Rightarrow\ \alpha^{12} = \cancel{-9\alpha^4}+27+\cancel{9\alpha^4}$

Hence, $\boxed{\alpha^{12} = (27)^2}$

$\Rightarrow\ (\alpha^{12})^8 = (27)^8$

$\Rightarrow\ \alpha^{96} = (3)^{24}$

Similarly $\beta^{96} = (3)^{24}$

$\therefore\ \alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1) = (3)^{24}\times 52$

$\Rightarrow$ Option (3) is correct.

**6.** Let $A = \begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}$, $a \in \mathbf{R}$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 36 (2) 24 (3) 45 (4) 18

**Official Ans. by NTA (1)**

**Sol.** $A = \begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}, a \in R$

and $P = \dfrac{A+A^T}{2} = \begin{bmatrix}2 & \dfrac{3+a}{2}\\ \dfrac{a+3}{2} & 0\end{bmatrix}$

and $Q = \dfrac{A-A^T}{2} = \begin{bmatrix}0 & \dfrac{3-a}{2}\\ \dfrac{a-3}{2} & 0\end{bmatrix}$

As, det (Q) = 9

$\Rightarrow\ (a-3)^2 = 36$

$\Rightarrow\ a = 3 \pm 6$

$\therefore\ \boxed{a = 9, -3}$

$$\therefore\quad \det.(P) = \begin{vmatrix}2 & \dfrac{3+a}{2}\\ \dfrac{a+3}{2} & 0\end{vmatrix}$$

$$= 0-\frac{(a-3)^2}{4} = 0, \text{ for } a = -3$$

$$= 0-\frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12), \text{ for } a = 9$$

$\therefore$ Modulus of the sum of all possible values of

det. (P) = $|-36|+|\ 0\ | = 36$ Ans.

$\Rightarrow$ Option (1) is correct

**7.** If z and $\omega$ are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then

$\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$ (2) $-\frac{3\pi}{4}$ (3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

**Official Ans. by NTA (3)**

**QuestPix Ans. (2)**

**Sol.** As $|z\omega| = 1$

$\Rightarrow$ If $|z| = r$, then $|\omega| = \frac{1}{r}$

Let $\arg(z) = \theta$

$\therefore \arg(\omega) = \left(\theta - \frac{3\pi}{2}\right)$

So, $z = re^{i\theta}$

$\Rightarrow \bar{z} = re^{i(-\theta)}$

$\omega = \frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}$

Now, consider

$$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i}\right)$$

$$= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i)$$

$\therefore$ prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$= \text{prin arg}\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

$= \left(-\frac{1}{2}(1+i)\right)$

$= -\left(\pi - \frac{\pi}{4}\right) = \frac{-3\pi}{4}$

So, option (2) is correct.

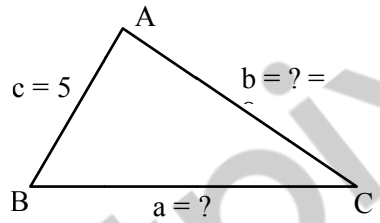
**8.** If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\Delta ABC$ is 5 units, then the area (in sq. units) of $\Delta ABC$ is :

(1) $10 + 6\sqrt{2}$ (2) $8 + 2\sqrt{2}$

(3) $6 + 8\sqrt{3}$ (4) $4 + 2\sqrt{3}$

**Official Ans. by NTA (3)**

**Sol.**




As, $\cos B = \frac{3}{5} \Rightarrow \boxed{B = 53°}$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow \boxed{C = 30°}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$

$\Rightarrow a^2 - 6a - 3g = 0$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\Rightarrow \boxed{3 + 4\sqrt{3}}$ (Reject $a = 3 - 4\sqrt{3}$)

Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct.

**9.** Let [x] denote the greatest integer $\le$ x, where $x \in \mathbf{R}$. If the domain of the real valued function

$$f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$$

is $(-\infty,a)\cup[b,c)\cup[4,\infty), a<b<c$, then the value of a + b + c is:

(1) 8 (2)1

(3) –2 (4) –3

**Official Ans. by NTA (3)**

**Sol.** For domain,

$$\frac{|[x]|-2}{|[x]|-3}\ge 0$$

**Case I :** When $|[x]|-2\ge 0$

and $|[x]|-3>0$

$\therefore\ x\in(-\infty,-3)\cup[4,\infty)$ .....(1)

**Case II :** When $|[x]|-2\le 0$

and $|[x]|-3<0$

$\therefore\ x\in[-2,3)$ .....(2)

So, from (1) and (2)

we get

Domain of function

$=(-\infty,-3)\cup[-2,3)\cup[4,\infty)$

$\therefore\ (a+b+c)=-3+(-2)+3=-2\ (a<b<c)$

$\Rightarrow$ Option (3) is correct.

**10.** Let y = y(x) be the solution of the differential equation $x\tan\left(\frac{y}{x}\right)dy=\left(y\tan\left(\frac{y}{x}\right)-x\right)dx$,

$-1\le x\le 1, y\left(\frac{1}{2}\right)=\frac{\pi}{6}$. Then the area of the region bounded by the curves x = 0, $x=\frac{1}{\sqrt{2}}$ and y = y (x) in the upper half plane is:

(1) $\frac{1}{8}(\pi-1)$ (2) $\frac{1}{12}(\pi-3)$

(3) $\frac{1}{4}(\pi-2)$ (4) $\frac{1}{6}(\pi-1)$

**Official Ans. by NTA (1)**

**Sol.** We have

$$\frac{dy}{dx}=\frac{x\left(\frac{y}{x}\cdot\tan\frac{y}{x}-1\right)}{x\tan\frac{y}{x}}$$

$$\therefore\frac{dy}{dx}=\frac{y}{x}-\cot\left(\frac{y}{x}\right)$$

$$\text{Put}\ \frac{y}{x}=v$$

$$\Rightarrow y=vn$$

$$\therefore\frac{dy}{dx}=v+\frac{ndv}{dx}$$

**Now, we get**

$$v+n\frac{dv}{dx}=v-\cot(v)$$

$$\Rightarrow\int(\tan)dv=-\int\frac{dx}{x}$$

$$\therefore\ \ell n\left|\sec\left(\frac{y}{x}\right)\right|=-\ell n|x|+c$$

$$\text{As}\left(\frac{1}{2}\right)=\left(\frac{y}{x}\right)\Rightarrow\boxed{C=0}$$

$$\therefore\sec\left(\frac{y}{x}\right)=\frac{1}{x}$$

$$\Rightarrow\cos\left(\frac{y}{x}\right)=x$$

$$\therefore\boxed{y=x\cos^{-1}(x)}$$

So, required bounded area

$$=\int_0^{1/\sqrt{2}}\underset{(II)}{x}\left(\underset{(I)}{\cos^{-1}x}\right)dx=\left(\frac{\pi-1}{8}\right)$$

(I.B.P.)

$\therefore$ option (1) is correct.

**11.** The coefficient of $x^{256}$ in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is:

(1) $^{100}C_{16}$ (2) $^{100}C_{15}$

(3) $-^{100}C_{16}$ (4) $-^{100}C_{15}$

**Official Ans. by NTA (2)**

**Sol.** $(1-x)^{100}\cdot(x^2+x+1)^{100}\cdot(1-x)$

$$=\left((1-x)(x^2+x+1)\right)^{100}(1-x)$$

$$=(1^3-x^3)^{100}(1-x)$$

$$=(1-x^3)^{100}(1-x)$$

$$=\underbrace{(1-x^3)^{100}}_{\text{No term of } x^{256}}-\underbrace{x(1-x^3)^{100}}_{\text{We find cofficient of } x^{255}}$$

Required coefficient $(-1)\times(-^{100}C_{85})$

$=^{100}C_{85}=^{100}C_{15}$

**12.** Let A = $[a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i-j| = 1 \\ 2x+1 & , \text{ otherwise.} \end{cases}$$

Let a function f : **R** → **R** be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on **R** is equal to:

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$

(3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

**Official Ans. by NTA (4)**

**Sol.** $A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$

$|A| = 4x^3 - 4x^2 - 4x = f(x)$

$f'(x) = 4(3x^2 - 2x - 1) = 0$

$\Rightarrow x = 1 \; ; x = \frac{-1}{3}$

$\therefore \underbrace{f(1) = -4}_{\text{min}} \; ; \underbrace{f\left(-\frac{1}{3}\right) = \frac{20}{27}}_{\text{max.}}$

Sum = $-4 + \frac{20}{27} = -\frac{88}{27}$

**13.** Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right|$ is :

(1) $\frac{2}{3}$ (2) 4

(3) 3 (4) $\frac{3}{2}$

**Official Ans. by NTA (4)**

**Sol.** $|\vec{a}| = 3 = a \; ; \; \vec{a}.\vec{c} = c$

Now $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow c^2 + 9 - 2(c) = 8$

$\Rightarrow c^2 - 2c + 1 = 0 \Rightarrow c = 1 = |\vec{c}|$

Also, $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

Given $(\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}||\vec{c}| \sin\frac{\pi}{6}$

= (3) (1) (1/2)

= 3/2

**14.** The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is :

(1) 1 (2) 2

(3) 4 (4) 0

**Official Ans. by NTA (4)**

**Sol.** $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$

For equation to be defined,

$x^2 + x \geq 0$

$\Rightarrow \quad x^2 + x + 1 \geq 1$

$\therefore$ only possibility that the equation is defined

$x^2 + x = 0 \quad \Rightarrow x = 0; x = -1$

None of these values satisfy

$\therefore$ No of roots = 0

**15.** Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0, y(1) = -1.$

Then the value of $(y(3))^2$ is equal to:

(1) $1 - 4e^3$ (2) $1 - 4e^6$

(3) $1 + 4e^3$ (4) $1 + 4e^6$

**Official Ans. by NTA (2)**

**Sol.** $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

$\Rightarrow e^x\sqrt{1-y^2}\,dx + \frac{-y}{x}dy$

$\Rightarrow \int \frac{-y}{\sqrt{1-y^2}}\,dy = \int_{\text{II}}^{e^x} \frac{x}{1}\,dx$

$\Rightarrow \sqrt{1-y^2} = e^x(x-1) + c$

Given : At x = 1, y = – 1

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\therefore \sqrt{1-y^2} = e^x(x-1)$

At x = 3 $\quad 1 - y^2 = (e^3\,2)^2 \Rightarrow y^2 = 1 - 4e^6$

**16.** Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15,\ x \in \mathbf{R}$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15,\ x \in \mathbf{R}$ has a:

(1) local maximum at $x = -\frac{3}{4}$

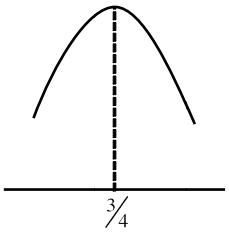
(2) local minimum at $x = -\frac{3}{4}$

(3) local maximum at $x = \frac{3}{4}$

(4) local minimum at $x = \frac{3}{4}$

**Official Ans. by NTA (1)**

**Sol.**




$\frac{-B}{2A} = \frac{3}{4}$

$\Rightarrow \frac{-(6)}{2a} = \frac{3}{4}$

$\Rightarrow a = \frac{-6 \times 4}{6} \Rightarrow a = -4$

$\therefore\ g(x) = 4x^2 - 6x + 15$

Local max. at $x = \frac{-B}{2A} = -\frac{(-6)}{2(-4)}$

$= \frac{-3}{4}$

**17.** Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$$

Where [x] is the greatest integer less than or equal to x. If $f$ is continuous on $\mathbf{R}$, then (a + b) is equal to:

(1) 4 (2) 3

(3) 2 (4) 5

**Official Ans. by NTA (2)**

**Sol.** Continuous at x = 0

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at x = 1

$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore\ a + b = 3$

**18.** Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is:

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$ (3) $\frac{1}{9}$ (4) $\frac{2}{11}$

**Official Ans. by NTA (2)**

**Sol.** AAEIIMNNOTX

--------M----------

Total words with M at fourth Place = $\frac{10!}{2!2!2!}$

Total words = $\frac{11!}{2!2!2!}$

Required probability = $\frac{10!}{11!} = \frac{1}{11}$

**19.** The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a + 4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$, is:

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

**Official Ans. by NTA (2)**

**Sol.** $D < 0$

$\Rightarrow 4(a + 4)^2 - 4(-5a + 64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a + 16)(a - 3) < 0$

$\Rightarrow a \in (-16, 3)$

$\therefore$ Possible a : {–5, –4, ....., 3}

$\therefore$ Required probability = $\frac{8}{36}$

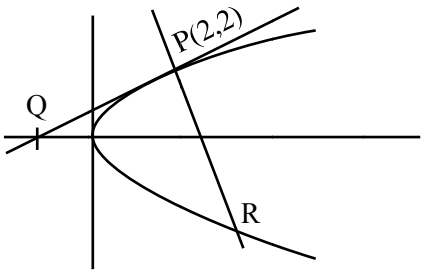
$= \frac{2}{9}$

**20.** Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to:

(1) $\frac{25}{2}$ (2) $\frac{35}{2}$ (3) $\frac{15}{2}$ (4) 25

**Official Ans. by NTA (1)**

**Sol.**




$y^2=2x$
$a=1/2$

Tangent at P : $y(2) = 2\,(1/2)\,(x + 2)$

$\Rightarrow 2y = x + 2$

$\therefore \quad Q = (-2, 0)$

Normal at P : $y - 2 = -\frac{(2)}{2.\frac{1}{2}}(x - 2)$

$\Rightarrow y - 2 = -2(x - 2)$

$\Rightarrow y = 6 - 2x$

$\therefore$ Solving with $y^2 = 2x \Rightarrow R\left(\frac{9}{2} - 3\right)$

$$\therefore \text{ Ar } (\Delta PQR) = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & 3- & 1 \end{vmatrix}$$

$= \frac{25}{2}$ sq.units

## SECTION-B

**21.** Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle $\theta$, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to _____.

**Official Ans. by NTA (4)**

**Sol.** $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{c})$

$= 3$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}| + |\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$\Rightarrow 1 = \sqrt{3}\cos\theta$

$\Rightarrow \cos 2\theta = -\frac{1}{3}$

$\Rightarrow 36\cos^2 2\theta = \boxed{4}$

**22.** Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order $3 \times 3$. If $B = [b_{ij}]$, then $b_{13}$ is equal to _____.

**Official Ans. by NTA (910)**

**Sol.** Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = I + C$

where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$C^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$C^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = C^4 = C^5 = .....$

$B = 7\,A^{20} - 20\,A^7 + 2I$

$= 7\,(I + C)^{20} - 20\,(I + C)^7 + 2I$

$= 7(I + 20C + {}^{20}C_2\,C^2) - 20\,(I + 7C + {}^7C_2\,C^2) + 2I$

So

$b_{13} = 7 \times {}^{20}C_2 - 20 \times {}^7C_2 = \boxed{910}$

**23.** Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{a}\cdot\left(\hat{i} + \hat{j} + 2\hat{k}\right) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____.

**Official Ans. by NTA (81)**

**Sol.** Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$

$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ || to $3x - z - 2 = 0$

$\Rightarrow \boxed{3\alpha - 8 = 0}$ .....(1)

$\vec{a} \perp i + 2\hat{j} + 3\hat{k}$

$\Rightarrow \alpha + 2\beta + 38 = 0$ ..... (2)

$\vec{a}.(\hat{i} + \hat{j} + 2\hat{k}) = 0$

$\Rightarrow \alpha + \beta + 28 = 2$ ..... (3)

on solving 1, 2 & 3

$\alpha = 1, \quad \beta = -5, \quad 8 = 3$

So $(\alpha - \beta + 8) = \boxed{81}$

**24.** The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is _____.

**Official Ans. by NTA (21)**

**Sol.** $\left(4^{1/4} + 5^{1/6}\right)^{120}$

$T_{r+1} = {}^{120}C_r (2^{1/2})^{120-r} (5)^{r/6}$

for rational terms $r = 6\lambda \quad 0 \le r \le 120$

so total no of forms are 21.

**25.** If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in \mathbf{R}$, $\alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in \mathbf{R}$ is 9, then $\alpha$ is equal to _____.

**Official Ans. by NTA (6)**

**Sol.** If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$

then shortest distance between two lines is

$$L = \frac{(\vec{a} - \vec{c}).(\vec{b} \times \vec{d})}{|b \times d|}$$

$\therefore \vec{a} - \vec{c} = ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k})$

$$\frac{\vec{b} \times \vec{d}}{|b \times d|} = \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3}$$

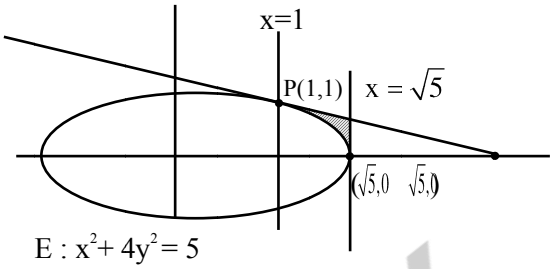
$$\therefore ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}).\frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = 9$$

or $\alpha = 6$

**26.** Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to _____.

**Official Ans. by NTA (1)**

**Sol.**




E : $x^2 + 4y^2 = 5$

Tangent at P : $x + 4y = 5$

Required Area

$$= \int_1^{\sqrt{5}} \left(\frac{5 - x}{4} - \frac{\sqrt{5 - x^2}}{2}\right) dx$$

$$= \left[\frac{5x}{4} - \frac{x^2}{8} - \frac{x}{4}\sqrt{5 - x^2} - \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$$

$$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

It we assume $\alpha, \beta, \gamma, \in Q$ (Not given in question)

then $\alpha = \frac{5}{4}, \beta = -\frac{5}{4}$ & $\gamma = -\frac{5}{4}$

$|\alpha + \beta + \gamma| = 1.25$

**27.** Let a, b, c, d be in arithmetic progression with common difference $\lambda$. If

$$\begin{vmatrix} x + a - c & x + b & x + a \\ x - 1 & x + c & x + b \\ x - b + d & x + d & x + c \end{vmatrix} = 2,$$

then value of $\lambda^2$ is equal to _____.

**Official Ans. by NTA (1)**

**Sol.** $$\begin{vmatrix} x + a - c & x + b & x + a \\ x - 1 & x + c & x + b \\ x - b + d & x + d & x + c \end{vmatrix} = 2$$

$C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} x - 2\lambda & \lambda & x + a \\ x - 1 & \lambda & x + b \\ x + 2\lambda & \lambda & x + c \end{vmatrix} = 2$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$$\Rightarrow \lambda\begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$$

$$\Rightarrow 1\,(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$$

$$\Rightarrow \boxed{\lambda^2 = 1}$$

**28.** There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is_____.

**Official Ans. by NTA (777)**

**Sol.** 15 : Players

6 : Bowlers

7 : Batsman

2 : Wicket keepers

Total number of ways for :

at least 4 bowlers, 5 batsman & 1 wicket keeper

$= {}^6C_4({}^7C_6 \times {}^2C_1 + {}^7C_5 \times {}^2C_2) + {}^6C_5 \times {}^7C_5 \times {}^2C_1$

$= \boxed{777}$

**29.** Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}\,(m + c)$ is equal to_____.

**Official Ans. by NTA (34)**

**Sol.** $y^2 = -64x$

focus : $(-16, 0)$

$y = mx + c$ is focal chord

$\Rightarrow c = 16m$ .....(1)

$y = mx + c$ is tangent to $(x + 10)^2 + y^2 = 4$

$$\Rightarrow y = m(x+10) \pm 2\sqrt{1+m^2}$$

$$\Rightarrow c = 10m \pm 2\sqrt{1+m^2}$$

$$\Rightarrow 16m = 10m \pm 2\sqrt{1+m^2}$$

$$\Rightarrow 6m = 2\sqrt{1+m^2} \qquad (m > 0)$$

$\Rightarrow 9m^2 = 1 + m^2$

$$\Rightarrow m = \frac{1}{2\sqrt{2}} \;\&\; c = \frac{8}{\sqrt{2}}$$

$$4\sqrt{2}(m+c) = 4\sqrt{2}\left(\frac{17}{2\sqrt{2}}\right) = \boxed{34}$$

**30.** If the value of $\lim\limits_{x\to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to $e^a$, then a is equal to _____.

**Official Ans. by NTA (3)**

**Sol.** $\lim\limits_{x\to 0}\left(2 - \cos x\sqrt{\cos x}\right)^{\frac{x+2}{x^2}}$

form: $1^\infty$

$$= e^{\lim\limits_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$$

Now $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$=$

$$\lim_{x\to 0}\frac{\sin x\sqrt{\cos 2x} - \cos x \times \frac{1}{2\sqrt{\cos 2x}} \times (-2\sin 2x)}{2x}$$

(by L' Hospital Rule)

$$\lim_{x\to 0}\frac{\sin x\cos 2x + \sin 2x.\cos x}{2x}$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

So, $e^{\lim\limits_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$$= e^{\frac{3}{2}\times 2} = e^3$$

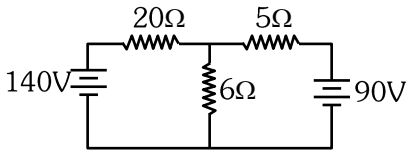
$$\Rightarrow \boxed{a = 3}$$

**(Held On Tuesday 20th July, 2021)** **TIME : 9 : 00 AM to 12 : 00 NOON**

**PHYSICS** | **TEST PAPER WITH SOLUTION**

**SECTION-A**

**31.**

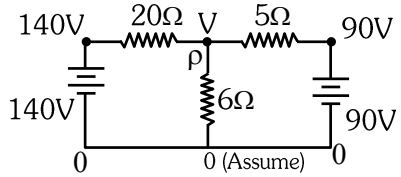



The value of current in the 6Ω resistance is :

(1) 4A (2) 8A (3) 10A (4) 6A

**Official Ans. by NTA (3)**

**Sol.**




Applying KCL at point P,

$$\frac{V-0}{6}+\frac{V-90}{5}+\frac{V-140}{20}=0$$

$\Rightarrow 10V + 12V - 1080 + 3V - 420 = 0$

$\Rightarrow V = 60$

$$\therefore \text{ current in } 6\Omega = \frac{V-0}{6} = 10A$$

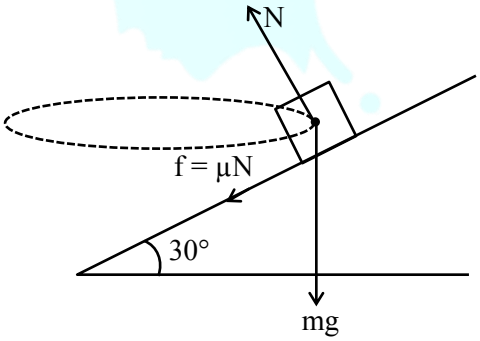
Hence option 3.

**32.** The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3$ kg m/s$^2$.

(1) 10.2 (2) 7.2 (3) 12.4 (4) 6.96

**Official Ans. by NTA (1)**

**Sol.**




At $v_{max}$, f will be limiting in nature.

$\therefore$ Balancing force in vertical direction,

$N\cos30° - mg - \mu N\cos60° = 0$

$\Rightarrow N[\cos30° - \mu\cos60°] = mg$

$$\therefore N = \frac{800\times10}{(0.87-0.1)} \approx 10.2\times10^3 \text{ kg m/s}$$

Hence option 1.

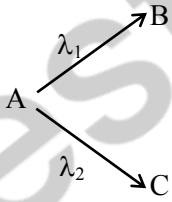
**33.** A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after the which one third of the material remains ? (Take ln 3 = 1.1)

(1) 1110 years (2) 700 years

(3) 340 years (4) 740 years

**Official Ans. by NTA (4)**

**Sol.**




Given $\lambda_1 = \frac{\ell n2}{700}$ / year , $\lambda_2 = \frac{\ell n2}{1400}$ / year

$$\therefore \lambda_{net} = \lambda_1 + \lambda_2 = \ell n2\left[\frac{1}{700}+\frac{1}{1400}\right]$$

$$= \frac{3\ell n2}{1400} \text{ / year}$$

Now, Let initial no. of radioactive nuclei be No.

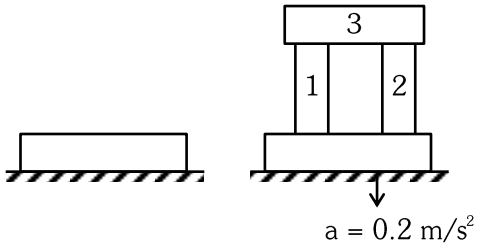
$$\therefore \frac{N_0}{3} = N_0 e^{-\lambda_{net} t}$$

$$\Rightarrow \ell n\frac{1}{3} = -\lambda_{net} t$$

$$\Rightarrow 1.1 = \frac{3\times0.693}{1400} t \Rightarrow t \approx 740 \text{ years}$$

Hence option 4

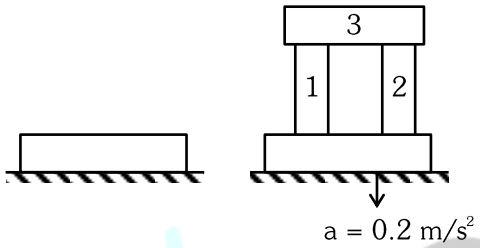
**34.** A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s$^2$. The normal reaction R' by the floor if mass of the iron cylinders are equal and of 20 kg each, is _____ N. [Take g = 10 m/s$^2$ and $\mu_s = 0.2$]




(1) 716 (2) 686 (3) 714 (4) 684

**Official Ans. by NTA (2)**

**Sol.**




Writing force equation in vertical direction

Mg – N = Ma

$\Rightarrow$ 70g – N = 70 × 0.2

$\Rightarrow$ N = 70 [g – 0.2] = 70 × 9.8

$\therefore$ N = 686 Newton

Note : Since there is no compressive normal from the sides, hence friction will not act.

Hence option 2.

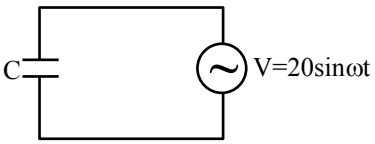
**35.** AC voltage V(t) = 20 sin$\omega$t of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m$^2$. The amplitude of the oscillating displacement current for the applied AC voltage is _____. [Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m]

(1) 21.14 μA (2) 83.37 μA

(3) 27.79 μA (4) 55.58 μA

**Official Ans. by NTA (3)**

**Sol.**




From the given information,

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \times 1}{2\times10^{-3}}$$

$$\therefore X_C = \frac{1}{\omega C} = \frac{2\times10^{-3}}{2\times50\pi\times\epsilon_0} = \frac{2}{25\times4\pi\epsilon_0}\ \Omega$$

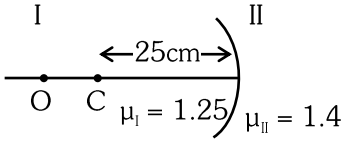
$$\therefore X_C = \frac{2\times10^{-3}}{25}\times9\times10^9 = \frac{18}{25}\times10\ \Omega$$

$$\therefore i_0 = \frac{V_0}{X_C} = \frac{20\times25}{18}\times10^{-6}\ A = 27.47\ \mu A.$$

The value of amplitude of displacement current will be same as value of amplitude of conventional current.

Hence option 3.

**36.** Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is :




(1) 55.44 cm (2) 9.52 cm

(3) 18.23 cm (4) 37.58 cm

**Official Ans. by NTA (4)**

**Sol.** $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

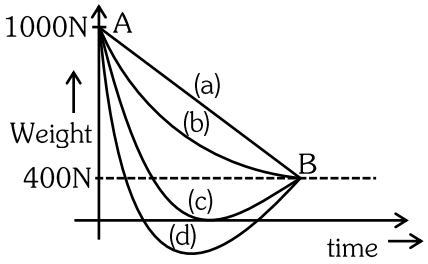
$$\frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4-1.25}{-25}$$

$$\frac{1.4}{v} = -\frac{0.15}{25} - \frac{1.25}{40}$$

v = – 37.58 cm

Hence option (4)

**37.** A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s$^2$ and 4 m/s$^2$ respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.




(1) (c) (2) (a) (3) (d) (4) (b)

**Official Ans. by NTA (1)**

**Sol.** At neutral point g = 0 so graph (C) is correct

Hence option (1).

**38.** The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is _____. (R is the universal gas constant)

(1) 250 R (2) 750 R (3) 175 R (4) 500 R

**Official Ans. by NTA (4)**

**Sol.** $\Delta Q = \Delta U + \Delta W$

Here $\Delta W = 0$

$\Delta Q = \Delta U = nC_V\Delta T$

$$\Delta Q = 4\times\frac{5R}{2}(50) = 500R$$

Hence option (4).

**39.** If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be :

(1) $\sqrt{A^2+B^2}$

(2) $\sqrt{A^2+B^2+\sqrt{2}AB}$

(3) $\sqrt{A^2+B^2+2AB}$

(4) $\sqrt{A^2+B^2-\sqrt{2}AB}$

**Official Ans. by NTA (4)**

**Sol.** $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$

$AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos 45°}$$

$$= \sqrt{A^2+B^2-\sqrt{2}AB}$$

Hence option (4).

**40.** A deuteron and an alpha particle having equal kinetic energy enter perpendicular into a magnetic field. Let $r_d$ and $r_\alpha$ be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) 1 (4) 2

**Official Ans. by NTA (2)**

**Sol.** $$r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$$

$$\frac{r_d}{r_\alpha} = \sqrt{\frac{m_d}{m_\alpha}\frac{q_\alpha}{q_d}} = \sqrt{\frac{2}{4}}\left(\frac{2}{1}\right) = \sqrt{2}$$

Hence option (2).

**41.** A nucleus of mass M emits γ-ray photon of frequency 'ν'. The loss of internal energy by the nucleus is :

[Take 'c' as the speed of electromagnetic wave]

(1) hν (2) 0

(3) $h\nu\left[1-\frac{h\nu}{2Mc^2}\right]$ (4) $h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$

**Official Ans. by NTA (4)**

**Sol.** Energy of γ ray $[E_\gamma] = h\nu$

Momentum of γ ray $\left[P_\gamma\right] = \frac{h}{\lambda} = \frac{h\nu}{C}$

Total momentum is conserved.

$\vec{P}_\gamma + \vec{P}_{Nu} = 0$

Where $\vec{P}_{Nu}$ = Momentum of decayed nuclei

$\Rightarrow P_\gamma = P_{Nu}$

$\Rightarrow \frac{h\nu}{C} = P_{Nu}$

$\Rightarrow$ K.E. of nuclei

$$= \frac{1}{2}Mv^2 = \frac{(P_{Nu})^2}{2M} = \frac{1}{2M}\left[\frac{h\nu}{C}\right]^2$$

Loss in internal energy = $E_\gamma + K.E_{Nu}$

$$= h\nu + \frac{1}{2M}\left[\frac{h\nu}{C}\right]^2$$

$$= h\nu\left[1+\frac{h\nu}{2MC^2}\right]$$

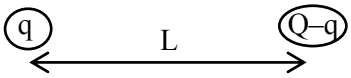
**42.** A certain charge Q is divided into two parts q and (Q–q). How should the charges Q and q be divided so that q and (Q–q) placed at a certain distance apart experience maximum electrostatic repulsion ?

(1) $Q = \frac{q}{2}$ (2) $Q = 2q$

(3) $Q = 4q$ (4) $Q = 3q$

**Official Ans. by NTA (2)**

**Sol.**




$$F_q = \frac{kq(Q-q)}{L^2} = \frac{k}{L^2}\left(qQ - q^2\right)$$

$\frac{dF}{dq} = 0$ when force is maximum

$$\frac{dF}{dq} = \frac{k}{L^2}[Q - 2q] = 0$$

$\Rightarrow Q - 2q = 0 \Rightarrow Q = 2q$

**43.** A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 $m^2$. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :

(Resistivity of magnesium $\rho = 44 \times 10^{-8}\ \Omega m$)

(1) $11 \times 10^{-2}$ V/m (2) $11 \times 10^{-7}$ V/m

(3) $11 \times 10^{-5}$ V/m (4) $11 \times 10^{-3}$ V/m

**Official Ans. by NTA (3)**

**Sol.** $I = \vec{J} \cdot \vec{A} = JA\cos(\theta)$

$$5 = J\left(\frac{4}{100}\right) \times \cos(60)$$

$J = 5 \times 50 = 250\ A/m^2$

Now, $\vec{E} = \rho \cdot \vec{J}$

$= 44 \times 10^{-8} \times 250 = 11 \times 10^{-5}$ V/m

**44.** Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

(1) $v_A = v_B = v_C = 0$

(2) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$

(3) $v_A = v_B \neq v_C$

(4) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

**Official Ans. by NTA (4)**

**Sol.** $V_{RMS} = \sqrt{\frac{3RT}{M}}$

$m_A < m_B < m_C$

$\Rightarrow V_A > V_B > V_C$

$\Rightarrow \frac{1}{V_A} < \frac{1}{V_B} < \frac{1}{V_C}$

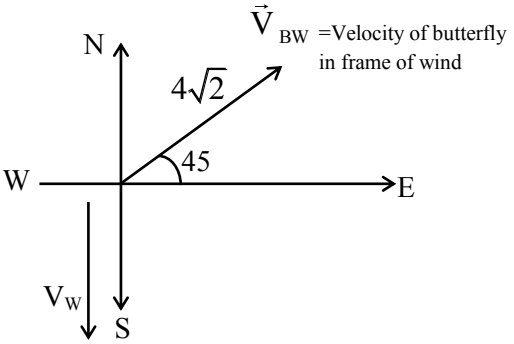
**45.** A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

(1) 3 m (2) 20 m

(3) $12\sqrt{2}$ m (4) 15 m

**Official Ans. by NTA (4)**

**Sol.**




$\vec{V}_{BW} = 4\sqrt{2}\cos 45\hat{i} + 4\sqrt{2}\sin 45\hat{j}$

$= 4\hat{i} + 4\hat{j}$

$\vec{V}_W = -\hat{j}$

$\vec{V}_B = \vec{V}_{BW} + \vec{V}_W = 4\hat{i} + 3\hat{j}$

$\vec{S}_B = \vec{V}_B \times t = \left(4\hat{i} + 3\hat{j}\right) \times 3 = 12\hat{i} + 9\hat{j}$

$\left|\vec{S}_B\right| = \sqrt{(12)^2 + (9)^2} = 15m$

**46.** The value of tension in a long thin metal wire has been changed from $T_1$ to $T_2$. The lengths of the metal wire at two different values of tension $T_1$ and $T_2$ are $\ell_1$ and $\ell_2$ respectively. The actual length of the metal wire is :

(1) $\frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$ (2) $\frac{T_1\ell_1 - T_2\ell_2}{T_1 - T_2}$

(3) $\frac{\ell_1 + \ell_2}{2}$ (4) $\sqrt{T_1 T_2 \ell_1 \ell_2}$

**Official Ans. by NTA (1)**

**Sol.** $Y = \frac{FL}{A\Delta L}$

$\Rightarrow Y = \frac{T_1\ell_0}{A(\ell_1 - \ell_0)} = \frac{T_2\ell_0}{A(\ell_2 - \ell_0)}$

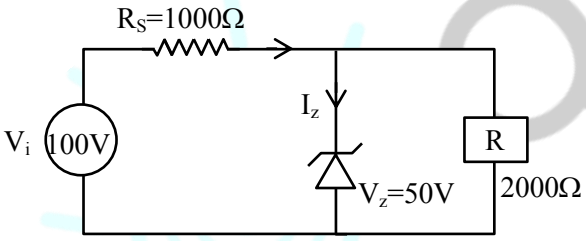
$1 = \frac{T_1(\ell_2 - \ell_0)}{T_2(\ell_1 - \ell_0)}$

$T_2\ell_1 - T_2\ell_0 = T_1\ell_2 - T_1\ell_0$

$(T_1 - T_2)\ell_0 = T_1\ell_2 - T_2\ell_1$

$\ell_0 = \left(\frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}\right)$

**47.** For the circuit shown below, calculate the value of $I_z$ :

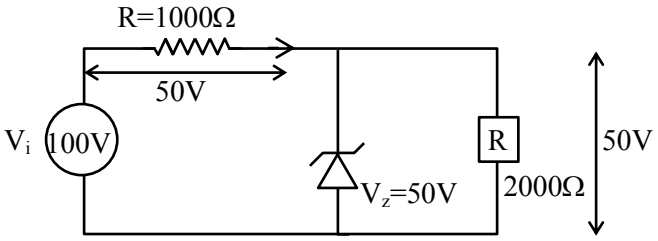



(1) 25 mA (2) 0.15 A

(3) 0.1 A (4) 0.05 A

**Official Ans. by NTA (1)**

**Sol.** $I = \frac{50}{1000} = 50mA$

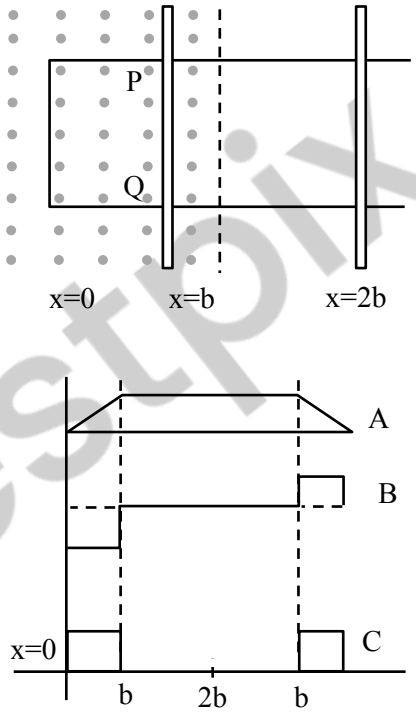



$I = \frac{50}{2000} = 25mA$

$I_Z = I_{1000} - I_{2000}$

$= 50 - 25 = 25$ mA

**48.** The arm PQ of a rectangular conductor is moving from x = 0 to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph showing the variation of different quantities with distance :




(1) A-Flux, B-Power dissipated, C-EMF

(2) A-Power dissipated, B-Flux, C-EMF

(3) A-Flux, B-EMF, C-Power dissipated

(4) A-EMF, B-Power dissipated, C-Flux

**Official Ans. by NTA (3)**

**Sol.** As rod moves in field area increases upto x = b then field is absent and again flux is generated on return journey from x = b to x = 0. Thus plot A for flux.

$\Rightarrow e = -\frac{d\phi}{dt} \Rightarrow$ curve B for emf

$\Rightarrow$ Power dissipated = vi $\Rightarrow$ curve C for power dissipated

**49.** The entropy of any system is given by

$$S = \alpha^2\beta \ln\left[\frac{\mu kR}{J\beta^2} + 3\right]$$

where $\alpha$ and $\beta$ are the constants. $\mu$, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

$$\left[\text{Take } S = \frac{dQ}{T}\right]$$

Choose the incorrect option from the following :

(1) $\alpha$ and J have the same dimensions.

(2) S, $\beta$, k and $\mu$R have the same dimensions.

(3) S and $\alpha$ have different dimensions.

(4) $\alpha$ and k have the same dimensions.

**Official Ans. by NTA (4)**

**Sol.** $S = \alpha^2\beta\ell n\left(\frac{\mu KR}{J\beta^2} + 3\right)$

$S = \frac{Q}{T} = \text{joulek} / k$

$[\alpha^2\ \beta] = \text{Joule/k}$

$PV = nRT \qquad \left[\frac{\mu KR}{J\beta^2}\right] = 1$

$R = \frac{\text{Joule}}{K}$

$\Rightarrow R = \frac{\text{Joule}}{K}, K = \frac{\text{Joule}}{R}$

$\Rightarrow \beta = \left(\frac{\text{Joule}}{K}\right)$

$\alpha^2\beta = \left(\frac{\text{Joule}}{K}\right)$

$\Rightarrow \alpha =$ dimensionless

**50.** The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of $5 \times 10^{-4}$ T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is :

(Mass of electron = $9.1 \times 10^{-31}$ kg)

(1) 1.36 eV (2) 1.88 eV

(3) 0.16 eV (4) 0.82 eV

**Official Ans. by NTA (4)**

**Sol.** 1.51 ——————— n = 3

3.4 ——————— n =

13.6 ——————— n = 1

$3 \rightarrow 2 \Rightarrow 1.89$ eV

$5 \times 10^{-4}$ T r = 7mm

$r = \frac{mv}{qB} \Rightarrow mv = qrB$

$\Rightarrow E = \frac{P^2}{2m} = \frac{(qRB)^2}{2m}$

$= \frac{\left(1.6\times10^{-19}\times7\times10^{-3}\times5\times10^{-4}\right)^2}{2\times9.1\times10^{-31}\text{ Joule}}$

$= \frac{3136\times10^{-52}}{18.2\times10^{-31}\times1.6\times10^{-19}}\text{ eV}$

= 1.077 eV

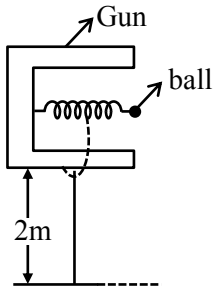
We know work function = energy incident – $(KE)_{electron}$

$\phi = 1.89 - 1.077 = 0.813$ eV

## SECTION-B

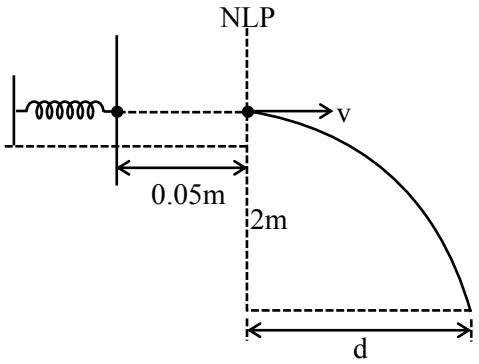
**51.** In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is ____ m.

($g = 10\ m/s^2$)




**Official Ans. by NTA (1)**

**Sol.**




$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$Kx^2 = mv^2$$

$$v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{100}{0.1}} = 0.05\times 10\sqrt{10}$$

$$v = 0.5\sqrt{10}$$

From $h = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{10}} = \frac{2}{\sqrt{10}}$$

$$\therefore d = vt = 0.5\sqrt{10}\times\frac{2}{\sqrt{10}} = 1m$$

**52.** In an LCR series circuit, an inductor 30 mH and a resistor 1 Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x}\times 10^{-3}$ F. Then the value of x is ____.

**Official Ans. by NTA (3)**

**Sol.** $\tan\phi = \frac{x_C - x_L}{R}$

$$\tan 45 = \frac{x_C - x_L}{R}$$

$$x_C - x_L = R$$

$$\frac{1}{\omega C} - \omega L = R$$

$$\frac{1}{\omega C} - 300\times 0.03 = 1$$

$$\frac{1}{\omega C} = 10$$

$$C = \frac{1}{10\omega} = \frac{1}{10\times 300}$$

$$C = \frac{1}{3}\times 10^{-3}$$

$$X = 3$$

**53.** The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time t = 0 and $y = \frac{1}{1+(x-2)^2}$ at t = 1s, where x and y are in meres. The shape of wave does not change during the propagation. The velocity of the wave will be ____m/s.

**Official Ans. by NTA (2)**

**Sol.** At $t = 0,\ y = \frac{1}{1+x^2}$

At time $t = t,\ y = \frac{1}{1+(x-vt)^2}$

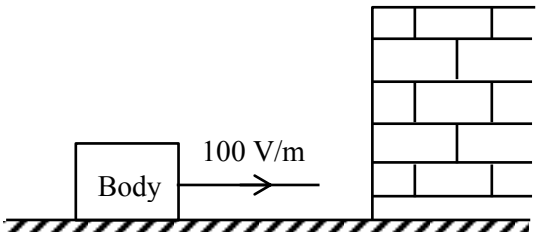
At $t = 1,\ y = \frac{1}{1+(x-v)^2}$ … (i)

At $t = 1,\ y = \frac{1}{1+(x-2)^2}$ … (ii)

Comparing (i) & (ii)

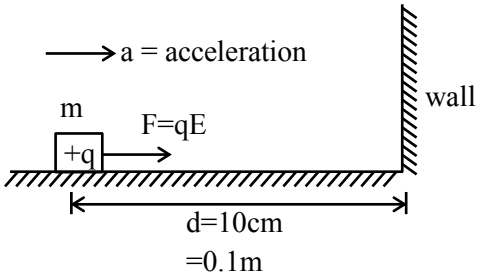
v = 2m/s

**54.** A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be _____ s.




**Official Ans. by NTA (1)**

**Sol.**




$F = ma$

$qE = ma$

$a = \frac{qE}{m}$

Now $d = \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}}$

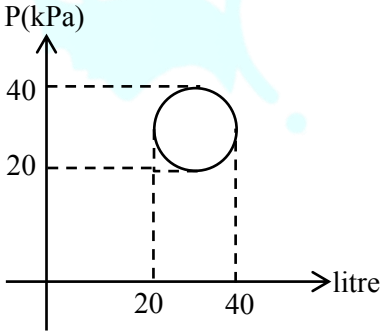
$t = \sqrt{\frac{2d}{\left(\frac{qE}{m}\right)}}$

$t = \sqrt{\frac{2\times 0.1}{\left(\frac{8\times 10^{-6}}{10^{-3}}\right)\times 100}} = \frac{1}{2}$

$\therefore$ Time period = 2t = 1 sec

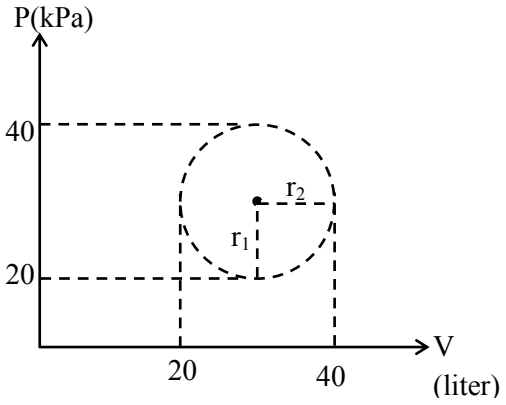
Ans. = 1.00

**55.** In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ πJ.




**Official Ans. by NTA (100)**

**Sol.**




For complete cyclic process

$\Delta U = 0$

$\therefore$ from $\Delta Q = \Delta U + W$

$= 0 + W$

$\Delta Q = W$

= Area

$= \pi r_1 \cdot r_2$

$= \pi \times (10 \times 10^3) \times (10 \times 10^{-3})$

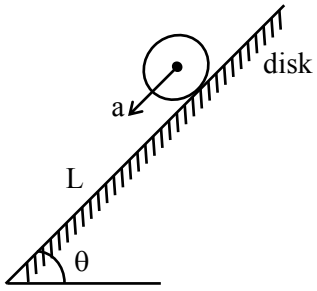
$\Delta Q = 100\ \pi$

$\therefore$ Ans. = 100

**56.** A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time '$t_1$'. When it rolls down the plane, it takes time $t_2$. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be _____.

**Official Ans. by NTA (2)**

**Sol.**




If disk slips on inclined plane, then it's acceleration

$a_1 = g\sin\theta$

$$L = \frac{1}{2}a_1 t_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2L}{a_1}} \quad \text{.... (i)}$$

If disk rolls on inclined plane, its acceleration,

$$a_2 = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

$$a_2 = \frac{g\sin\theta}{1+\frac{mR^2}{2mR^2}}$$

$$a_2 = \frac{2}{3}g\sin\theta$$

Now $L = \frac{1}{2}a_2 \cdot t_2^2$

$$\Rightarrow t_2 = \sqrt{\frac{2L}{a_2}} \quad \text{... (ii)}$$

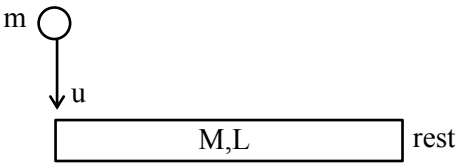
Now $\frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{3}{2}}$

$\Rightarrow x = 2$

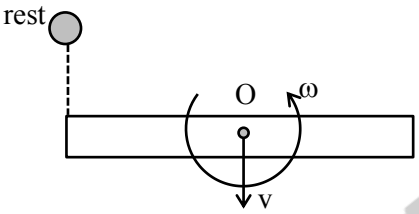
**57.** A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be _____.

**Official Ans. by NTA (4)**

**Sol.**




Just before collision




Just after collision

From momentum conservation, $P_i^0 = P_f$

$mu = Mv$ .... (i)

From angular momentum conservation about O,

$$mu \cdot \frac{L}{2} = \frac{ML^2}{12}\omega$$

$$\Rightarrow \omega = \frac{6mu}{ML} \quad \text{... (ii)}$$

From $e = \frac{R.V.S}{R.V.A}$

$$1 = \frac{V + \frac{\omega L}{2}}{u}$$

$$v + \frac{\omega L}{2} = u$$

$$v + \frac{3mu}{M} = u$$

$$\frac{mu}{M} + \frac{3mu}{M} = u$$

$$\frac{4mu}{M} = u$$

$$\frac{m}{M} = \frac{1}{4}$$

$X = 4$

**58.** An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to _____ cm.

**Official Ans. by NTA (25)**

**Sol.** For simple microscope,

$$m = 1 + \frac{D}{f_0}$$

$$6 = 1 + \frac{D}{f_0}$$

$$5 = \frac{25}{f_0}$$

$f_0 = 5$ cm

For compound microscope,

$$m = \frac{\ell \cdot D}{f_0 \cdot f_e}$$

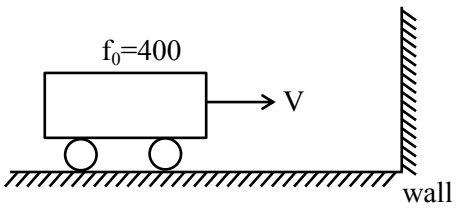
$$12 = \frac{60 \times 25}{5 \cdot f_e}$$

$f_e = 25$ cm

**59.** The frequency of a car horn encountered a change from 400 Hz to 500 Hz. When the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is _____ km/h.

**Official Ans. by NTA (132)**

**Sol.**




Wall as an observer

Frequency received by wall

$$f_1 = f_0\left(\frac{C}{C-V}\right)$$

Again wall as a source

Frequency received by observer on car

$$f_2 = f_1\left(\frac{C+V}{C}\right)$$

$$f_2 = f_0\left(\frac{C+V}{C-V}\right)$$

$$500 = 400\left(\frac{C+V}{C-V}\right)$$

$$\frac{5}{4} = \frac{C+V}{C-V}$$

$C = 9V$

$$V = \frac{C}{9} = \frac{330}{9} m/s$$

$$V = \frac{330}{9} \times \frac{18}{5} = 132 km/hr$$

**60.** A carrier wave $V_C(t) = 160 \sin(2\pi \times 10^6 t)$ volts is made to vary between $V_{max} = 200$ V and $V_{min} = 120$ V by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ volts. The peak voltage $A_m$ of the modulating signal is _____.

**Official Ans. by NTA (40)**

**Sol.** Maximum amplitude

$A_{max} = A_m + A_C$

$\Rightarrow V_{max} = V_m + V_C$

$200 = V_m + 160$

$V_m = 40$

$\therefore$ Peak voltage $A_m = 40$

Ans. 40

**(Held On Tuesday 20th July, 2021)** **TIME : 9 : 00 AM to 12 : 00 NOON**

## CHEMISTRY | TEST PAPER WITH SOLUTION

### SECTION-A

**61.** According to the valence bond theory the hybridization of central metal atom is $dsp^2$ for which one of the following compounds?

(1) $NiCl_2.6H_2O$ (2) $K_2[Ni(CN)_4]$

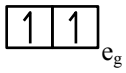
(3) $[Ni(CO)_4]$ (4) $Na_2[NiCl_4]$

**Official Ans. by NTA (2)**

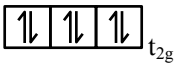
**Sol.** 1) $NiCl_2.6H_2O$

$Ni^{+2} \rightarrow [Ar]_{18}3d^8 4s^0$

C.N. = 6 octahedral

splitting  $e_g$

Hybridisation $sp^3d^2$

 $t_{2g}$

2) $K_2[Ni(CN)_4]$

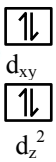
C.N. 4 $Ni^{+2} \rightarrow [Ar]_{18}3d^8 4s^0$

$CN^- \rightarrow$ Strong field ligand

 $d_{x^2-y^2}$

Hybridisation $dsp^2$

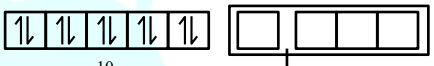
Square planar splitting



$d_{xy}$, $d_{z^2}$, $d_{yz}$ $d_{xz}$

3) $Ni(CO)_4$

CO - Strong field ligand

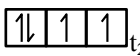
$Ni \rightarrow [Ar]_{18}$ 

$3d^{10}$ 4s $sp^3$ 4p

Hybridisation

4) $Na_2[NiCl_4]$ $Ni \rightarrow [Ar]_{18}3d^8 4s^0$

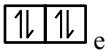
$Cl^{\ominus} \rightarrow$ weak field ligand

 $t_2$

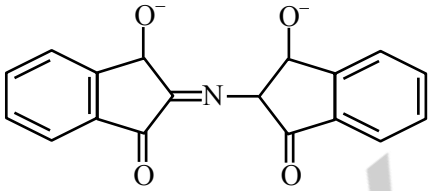
Hybridisation $sp^3$

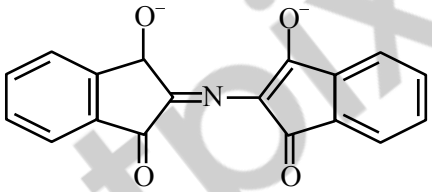
C.N. 4

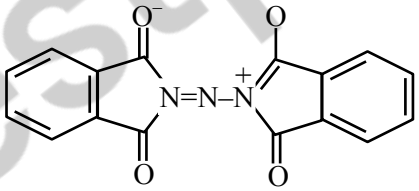
tetrahedral splitting

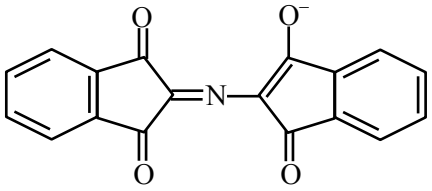
 e

**62.** The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is :

(1) 

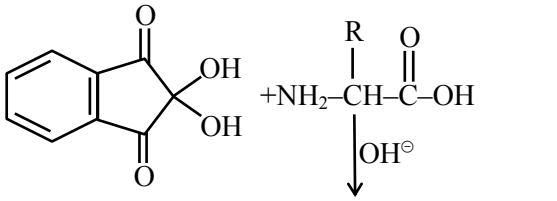
(2) 

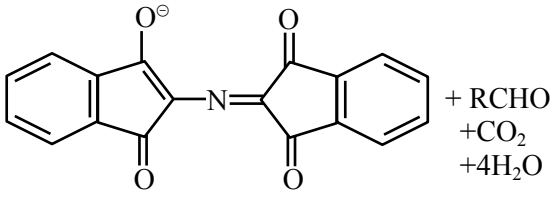
(3) 

(4) 

**Official Ans. by NTA (4)**

**Sol.**





Ninhydrin Test

**63.** Green chemistry in day–to–day life is in the use of:

(1) Chlorine for bleaching of paper

(2) Large amount of water alone for washing clothes

(3) Tetrachloroethene for laundry

(4) Liquified $CO_2$ for dry cleaning of clothes

**Official Ans. by NTA (4)**

**Sol.** Chlorine gas was used earlier for bleaching paper. These days, hydrogen peroxide ($H_2O_2$) with suitable catalyst.

Tetra chlroroethene ($Cl_2C{=}CCl_2$) was earlier used as solvent for dry cleaning. The compound contaminates the ground water and is also a suspected carcinogen. Replacement of halogenated solvent by liquid $CO_2$ will result in less harm to groundwater.
Hence given statement (4) is correct.

**64.** The correct order of intensity of colors of the compounds is :

(1) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(2) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(3) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(4) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

**Official Ans. by NTA (3)**

**Sol.** $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

Splitting energy order: $\Delta_t < \Delta_0 < \Delta_{sq}$

absorbed energy order: $[NiCl_4]^{2-} < [Ni(H_2O)_6]^{2+} < [Ni(CN)_4]^{2-}$

intensity of colour of compound: $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

**65.** The set in which compounds have different nature is :

(1) $B(OH)_3$ and $H_3PO_3$

(2) $B(OH)_3$ and $Al(OH)_3$

(3) NaOH and $Ca(OH)_2$

(4) $Be(OH)_2$ and $Al(OH)_3$

**Official Ans. by NTA (2)**

**Sol.** 1) $B(OH)_3$ acidic and $H_3PO_3$ acidic

2) $B(OH)_3$ acidic and $Al(OH)_3$ amphoteric

3) NaOH basic and $Ca(OH)_2$ basic

4) $Be(OH)_2$ amphoteric and $Al(OH)_3$ amphoteric

**66.** The species given below that does NOT show disproportionation reaction is :

(1) $BrO_4^-$ (2) $BrO^-$

(3) $BrO_2^-$ (4) $BrO_3^-$

**Official Ans. by NTA (1)**

**Sol.** In $BrO_4^{\ominus}$, Br is in highest oxidation state (+7), So it cannot oxidise further hence it cannot show disproportionation reaction.

**67.** Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A :** Sharp glass edge becomes smooth on heating it upto its melting point.

**Reason R :** The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below.

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(3) **A** is false but **R** is true.

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**.

**Official Ans. by NTA (2)**

**Sol.** Hence given statement (A) is not correct

But statement (B) is correct

**68.** Orlon fibres are made up of :

(1) Polyacrylonitrile

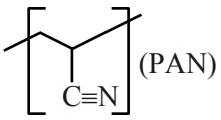
(2) Polyesters

(3) Polyamide

(4) Cellulose

**Official Ans. by NTA (1)**

**Sol.** → orlon fibers are made up of Polyacrylonitrile

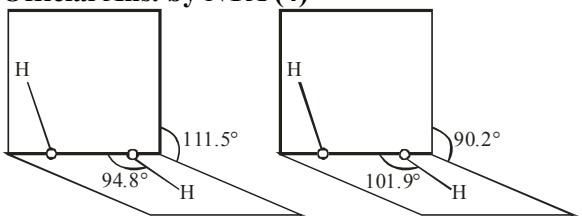


(PAN)

**69.** Given below are two statements : One is labelled as **Assertion A** and other is labelled as **Reason R**.
**Assertion A** : The dihedral angles in $H_2O_2$ in gaseous phase is 90.2° and in solid phase is 111.5°.
**Reason R** : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.
Choose the most appropriate answer from the options given below for **A** and **R**.
(1) **A** is correct but **R** is not correct.
(2) Both **A** and **R** are correct but R is not the correct explanation of A.
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
(4) **A** is not correct but **R** is correct.

**Official Ans. by NTA (4)**

**Sol.**




(a) Gas phase (a) Solid phase

(a) $H_2O_2$ structure in gas phase, dihedral angle is 111.5°. (b) $H_2O_2$ structure in solid phase at 110K, dihedral angle is 90.2°.
Hence given statement (A) is not correct But statement (B) is correct.

**70.** Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and $P_4O_{10}$ (in 4 : 1 ratio) is :
(1) acidic (2) basic
(3) amphoteric (4) neutral

**Official Ans. by NTA (1)**

**Sol.** $4HNO_3 + P_4O_{10}$
$\downarrow$
$2N_2O_5 + (HPO_3)_4$
Ans. $N_2O_5$ is acidic in nature.

**71.** An inorganic Compound 'X' on treatment with concentrated $H_2SO_4$ produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated $H_2SO_4$. Also Compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with $H_2S$ gas. The precipitate 'Y' on treatment with concentrated $HNO_3$ followed by excess of $NH_4OH$ further gives deep blue coloured solution, Compound 'X' is:
(1) $Co(NO_3)_2$ (2) $Pb(NO_2)_2$
(3) $Cu(NO_3)_2$ (4) $Pb(NO_3)_2$

**Official Ans. by NTA (3)**

**Sol.** $\underset{\text{(Anion)}}{\underset{X}{NO_3^-}} + \underset{\text{(Conc.)}}{H_2SO_4} \rightarrow \underset{\text{Brown fumes}}{NO_2\uparrow} + H_2O$

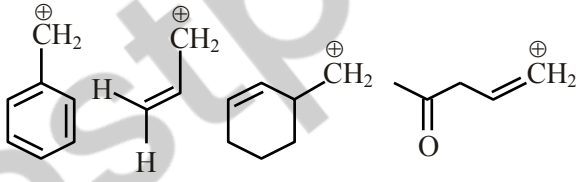
$FeSO_4 + H_2SO_4 + NO_3^-$
$Sol^n$ conc. X
$\downarrow$
$[Fe(H_2O)_5(NO)]SO_4$
(Dark brown ring)

$Cu^{2+} + (\text{dil HCl} + H_2S)$
$\underset{\text{(cation)}}{X}$ (Group-II reagent)
$\downarrow$
$CuS\downarrow$
(Black ppt)
(Y)

$CuS \underset{(Y)}{} \xrightarrow[HNO_3]{Conc^n} \overset{\text{Soluble}}{Cu(NO_3)_2} + NO_2 + S + H_2O$
$\downarrow$ Excess $NH_4OH$ $Sol^n$.
$[Cu(NH_3)_4]^{2+}$
Deep blue colour solution.

$\therefore X \rightarrow Cu(NO_3)_2$

**72.**

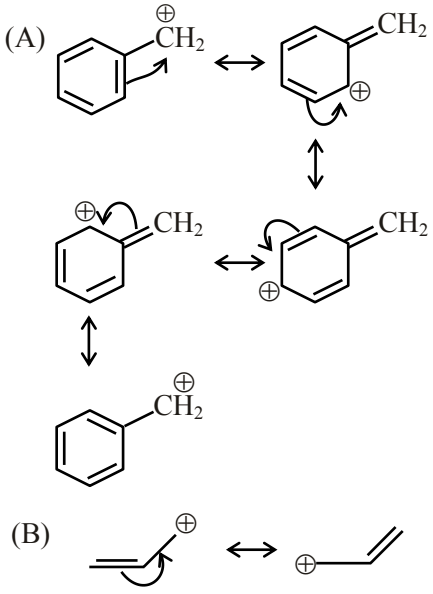



(A) (B) (C) (D)

Among the given species the Resonance stabilised carbocations are:
(1) (C) and (D) only
(2) (A), (B) and (D) only
(3) (A) and (B) only
(4) (A), (B) and (C) only

**Official Ans. by NTA (3)**

**Sol.** (A) and (B) only in Resonance



**73.** A s-block element (M) reacts with oxygen to form an oxide of the formula $MO_2$. The oxide is pale yellow in colour and paramagnetic. The element (M) is:

(1) Mg (2) Na

(3) Ca (4) K

**Official Ans. by NTA (4)**

**Sol.** (A) $2Mg + O_2 \rightarrow 2MgO$ (Diamagnetic)

(B) $2Na + O_2 \rightarrow Na_2O$ (Diamagnetic)

$2Na + \underset{\text{(excess)}}{O_2} \rightarrow Na_2O_2$ (Diamagnetic)

(C) $2Ca + O_2 \rightarrow 2CaO$ (Diamagnetic)

$Ca + O_2 \rightarrow CaO_2$(Diamagnetic)

(D) $K + \underset{\text{(excess)}}{O_2} \rightarrow KO_2$ (Paramagnetic)

**74.** In the given reaction 3-Bromo-2, 2-dimethyl butane $\xrightarrow{C_2H_5OH}$ $\underset{\text{(Major Product)}}{\text{'A'}}$ Product A is:

(1) 2-Ethoxy-3, 3-dimethyl butane

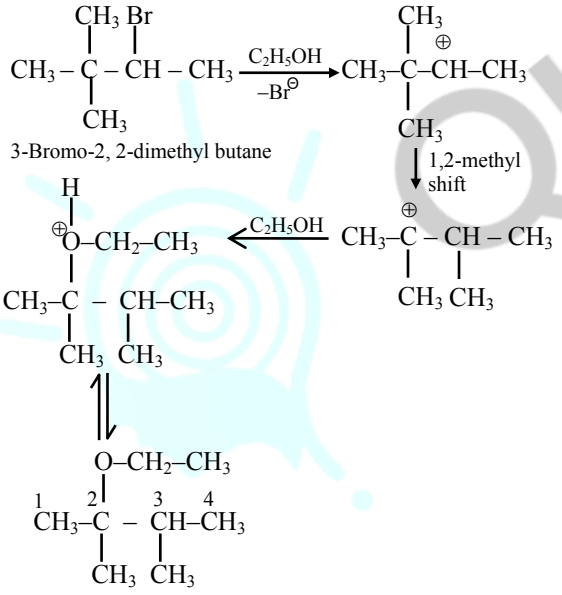
(2) 1-Ethoxy-3, 3-dimethyl butane

(3) 2-Ethoxy-2, 3-dimethyl butane

(4) 2-Hydroxy-3, 3-dimethyl butane

**Official Ans. by NTA (3)**

**Sol.**




2 Ethoxy –2,3–dimethyl butane

**75.** The metal that can be purified economically by fractional distillation method is:

(1) Fe (2) Zn (3) Cu (4) Ni

**Official Ans. by NTA (2)**

**Sol.** Zinc can be purified economically by fractional distillation.

**76.** Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are :

(1) primary amine, nitrile compound, conc. HCl

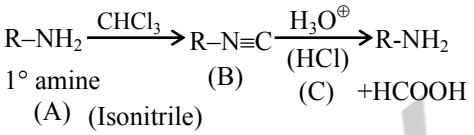
(2) secondary amine, isonitrile compound, conc. NaOH

(3) primary amine, isonitrile compound, conc. HCl

(4) secondary amine, nitrile compound, conc. NaOH

**Official Ans. by NTA (3)**

**Sol.**




**77.** The conditions given below are in the context of observing Tyndall effect in colloidal solutions:

(A) The diameter of the colloidal particles is comparable to the wavelength of light used.

(B) The diameter of the colloidal particles is much smaller than the wavelength of light used.

(C) The diameter of the colloidal particles is much larger than the wavelength of light used.

(D) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(E) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below:

(1) (A) and (E) only (2) (C) and (D) only

(3) (A) and (D) only (4) (B) and (E) only

**Official Ans. by NTA (1)**

**Sol.** The phenomenon of scattering of light by colloidal particles as a result of which the path of the beam becomes visible is called a tyndall effect.

smaller the diameter and similar the magnitude of refractive indices, lesser is the scattering and hence the tyndall effect and viced-versa.

The diameter of the dispersed phase particle should not be smaller than the wavelength of light used because they won't be able to scatter the light so, therefore, the diameter of the dispersed particles should be equal or not much smaller than the wavelength of the light used.

2. The refractive indies (i.e. the ratio of the velocity of light in vacuum to the velocity of light in any medium) of the dispersed phase and the dispersion medium should differ greatly in magnitude than only the particles will be able to

scatter the light and tyndall effect will be obersved. On the other hand, if the refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude, then there will be no scattering of light and hence, therefore, no tyndall effect effect is observed.
Hence answer A and E are correct.

**78.** Identify the incorrect statement from the following

(1) Amylose is a branched chain polymer of glucose

(2) Starch is a polymer of α-D glucose

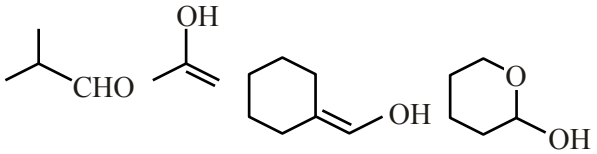
(3) β-Glycosidic linkage makes cellulose polymer

(4) Glycogen is called as animal starch

**Official Ans. by NTA (1)**

**Sol.** Amylose is a linear chain polymer of α-D-glucose while amylopectine is branched chain polymer of α-D-glucose.

**79.**




(I) (II) (III) (IV)

Which among the above compound/s does/do not form Silver mirror when treated with Tollen's reagent?

(1) (I), (III) and (IV) only

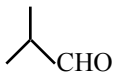
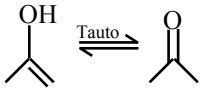
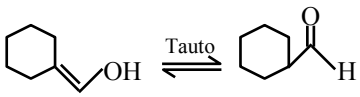
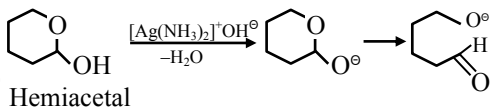
(2) Only (IV)

(3) Only (II)

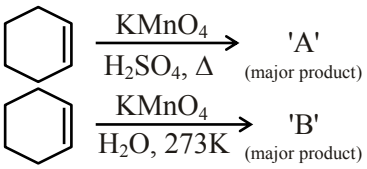
(4) (III) and (IV) only

**Official Ans. by NTA (3)**

Aldehydes give ⊕ve Tollen's Test (Silver mirror test)

| | | Tollen's test |
|---|---|---|
| (I) |  | Positive |
| (II) |  Ketone | Negative |
| (III) |  | Positive |
| (IV) |  Hemiacetal | Positive |

**80.**




For above chemical reactions, identify the correct statement from the following:

(1) Both compound 'A' and compound 'B' are dicarboxylic acids

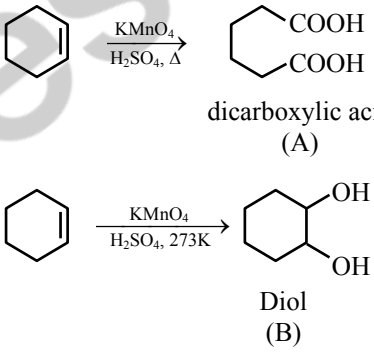
(2) Both compound 'A' and compound 'B' are diols

(3) Compound 'A' is diol and compound 'B' is dicarboxylic acid

(4) Compound 'A' is dicarboxylic acid and compound 'B' is diol

**Official Ans. by NTA (4)**

**Sol.**

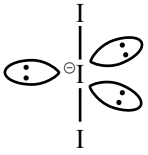



## SECTION-B

**81.** The number of lone pairs of electrons on the central I atom in $I_3^-$ is _____.

**Official Ans. by NTA (3)**

**Sol.** $I_3^-$ :




The number of lone pairs of electron on the central atom is 3.

**82.** 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is _____ $\times 10^{21}$. (Nearest integer)

($N_A = 6.022 \times 10^{23}$)

**Official Ans. by NTA (226)**

**Sol.** We known that no. of moles = $V_{litre} \times$ Molarity

& No. of millimoles = $V_{ml} \times$ Molarity

so millimoles of NaOH = $250 \times 0.5$

$= 125$

Millimoles of HCl = $500 \times 1 = 500$

Now reaction is

$$NaOH + HCl \rightarrow NaCl + H_2O$$

| | | | | |
|---|---|---|---|---|
| t = 0 | 125 | 500 | 0 | 0 |
| t = t | 0 | 375 | 125 | 125 |

so millimoles of HCl left = 375

Moles of HCl = $375 \times 10^{-3}$

No. of HCl molecules = $6.022 \times 10^{23} \times 375 \times 10^{-3}$

$= 225.8 \times 10^{21}$

$\simeq 226 \times 10^{21} = 226$

**83.** The Azimuthal quantum number for the valence electrons of $Ga^+$ ion is _____.

(Atomic number of Ga = 31)

**Official Ans. by NTA (0)**

**Sol.** $Ga^+ : Is^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2$

The azimuthal quantum number for the valence electrons (4s-subshell) of Ga+ ion is zero(0).

**84.** The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is _____ BM.

[At. no. of Co = 27]

**Official Ans. by NTA (2)**

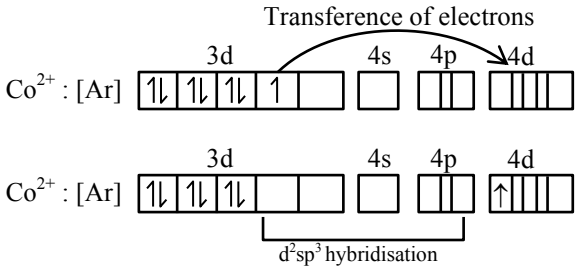
**Sol.** $[Co(CN)_6]^{4-}$

$x + 6 \times (-1) = -4$

$\boxed{x = +2}$

$Co^{2+} : [Ar]\ 3d^7$

and $CN^-$ is a strong field ligand which can pair electron of central atom.




It has one unpaired electron (n) in 4d-subshell.

So spin only magnetic moment ($\mu$) = $\sqrt{n(n+2)}$ B.M

where n = number of unpaired electrons.

$\mu = \sqrt{3}$ B.M

$\boxed{\mu = 1.73\,BM}$

**85.** $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In an equilibrium mixture, the partial pressures are $P_{SO_3} = 43$ kPa ; $P_{O_2} = 530$ Pa and $P_{SO_2} = 45$ kPa. The equilibrium constant $K_P =$ _____ $\times 10^{-2}$. (Nearest integer)

**Official Ans. by NTA (172)**

**QuestPix Ans. (17228)**

**Sol.** $2SO_{2(g)} + O_{2(g)} = 2SO_{3(g)}$

$$K_P = \frac{(pSO_{3(g)})^2}{pSO2(g)} \times pO_{2(g)}$$

$$= \frac{43 \times 43}{45 \times 45} \times 530\ Pa^{-1}$$

$= 172.28 \times 10^{-5}\ Pa^{-1}$

$= 172.28$ atm

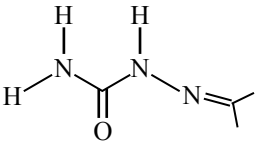
$= 17228 \times 10^{-2}$ atm

Ans is 17228

**86.** The number of nitrogen atoms in a semicarbazone molecule of acetone is _____.

**Official Ans. by NTA (3)**

**Sol.**

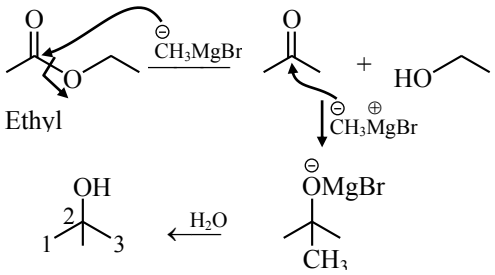


Semicarbazone molecule of acetone

**87.** To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate _____ equivalents of $CH_3MgBr$ reagent will be required. (Integer value)

**Official Ans. by NTA (2)**

**Sol.**




2-Methylpropan-2-ol

**88.** The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is _____ $\times 10^{-3}$ $min^{-1}$. (Nearest integer)

[Use : ln 10 = 2.303 ; $\log_{10} 3 = 0.477$;

property of logarithm : $\log x^y = y \log x$]

**Official Ans. by NTA (106)**

**Sol.** As the unit of rate constant is $min^{-1}$ so it must be a first order reaction

$K \times t = 2.303 \log A_0/A_t$

in 1 min 10% is in activated so tabing

$A_0 = 100$ $A_t = 90$ in 1 min

$$\text{So } K \times 1 = 2.303 \times \log \frac{100}{90}$$

$$= 2.303 \times (\log 10 - 2\log 3)$$

$$= 2.303 \times (1 - 2 \times 0.477)$$

$$= 0.10593$$

$$= 105.93 \times 10^{-3}$$

$$\approx 106$$

**89.** An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g $mol^{-1}$) needed to meet this energy requirement is _____ g.

(Use : $\Delta_C H$(glucose) = –2700 kJ $mol^{-1}$)

**Official Ans. by NTA (667)**

**Sol.** 1 mole glucose give 2700 kJ energy

so mole of glucose needed for $10^5$ kJ energy

$$= \frac{10000}{2700} = 370 \text{ moles}$$

wt. of glucose = 3.10 × 180

= 666.666

≈ 667 gm

$$\frac{Y_{Benzene}}{Y_{M.B}} = \frac{P_B^0 X_B}{P_{MB}^0 X_{MB}} = \frac{70 \times 1}{20 \times 1} = \frac{7}{2}$$

$$Y_{Benzene} = \frac{7}{9} = 77.77 \times 10^{-2}$$

$$= 78 \times 10^{-12}$$

**90.** At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20°C above an equimolar mixture of benzene and methyl benzene is _____ $\times 10^{-2}$. (Nearest integer)

**Official Ans. by NTA (78)**

**Sol.** $P_B^o = 40$ $P_T^o = 20$ $K_B = 0.5 = K_T$

Now $y_B = \dfrac{K_B P_B^o}{K_B P_B^o + K_T P_T^o}$

$$= \frac{70 \times 0.5}{70 \times 0.5 + 20 \times 0.5}$$