

(HELD ON SUNDAY 05th APRIL 2026)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

SECTION-A

1. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots the equation $x^2 - 4x + q = 0$; $p, q \in \mathbb{Z}$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then $|p+q|$ equals:

- (1) 16 (2) 32
(3) 34 (4) 38

Ans. (3)

Sol. Let $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$$a + ar = 1$$

$$ar^2 + ar^3 = 4$$

$$\Rightarrow ar^2(1+r) = 4$$

$$\Rightarrow r = 2, a = \frac{1}{3} \text{ (reject as } p \in \mathbb{Z} \text{)}$$

$$\text{or } r = -2, a = -1$$

$$\text{Now } |P+q| = |a(ar) + ar^2 - ar^3|$$

$$= |a^2r(1+r^4)|$$

$$|1 \times -2 \times 17| = 34$$

2. Let $z_1, z_2 \in \mathbb{C}$ be the distinct solutions of the equation $z^2 + 4z - (1 + 12i) = 0$. Then $|z_1|^2 + |z_2|^2$ is equal to :

- (1) 18 (2) 22
(3) 29 (4) 34

Ans. (4)

Sol. $Z^2 + 4Z - (1 + 12i) = 0$

$$\Rightarrow Z = \frac{-4 \pm \sqrt{16 + 4(1 + 12i)}}{2}$$

$$\Rightarrow Z = -2 \pm \sqrt{5 + 12i}$$

$$\Rightarrow Z = -2 \pm (3 + 2i)$$

$$\Rightarrow Z = 1 + 2i, -5 - 2i$$

$$\therefore |Z_1|^2 + |Z_2|^2$$

$$= 5 + 29$$

$$= 34$$

TEST PAPER WITH SOLUTION

3. If $f : \mathbb{N} \rightarrow \mathbb{Z}$ is defined by

$$f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}, k \in \mathbb{N}, \text{ and}$$

$$\sum_{n=1}^k f(n) = 98, \text{ then } k \text{ is equal to:}$$

- (1) 3 (2) 4
(3) 5 (4) 6

Ans. (1)

Sol. $f(n) = \begin{vmatrix} n & -1 & -5 \\ -2n^2 & 3(2k+1) & 2k+1 \\ -3n^3 & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix}$

$$\sum_{n=1}^k f(n) = \begin{vmatrix} \frac{k(k+1)}{2} & -1 & -5 \\ -2 \frac{k(k+1)(2k+1)}{6} & 3(2k+1) & 2k+1 \\ -3 \frac{k^2(k+1)^2}{4} & 3k(2k+1) & 3k(k+2)+1 \end{vmatrix} = 98$$

$$\Rightarrow \frac{k(k+1)(2k+1)}{2} \cdot \frac{7}{3} = 98$$

$$\Rightarrow k(k+1)(2k+1) = 84$$

$$\Rightarrow k = 3$$

4. Let M be a 3×3 matrix such that

$$M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \text{ and}$$

$$M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}. \text{ If } M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}, \text{ then } x + y + z$$

equals :

- (1) 4 (2) 5
(3) 7 (4) 11

Ans. (3)

Sol. Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

then $M \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow a_1 = 1, b_1 = 2, c_1 = 3$

$M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \Rightarrow a_2 = 0, b_2 = 1, c_2 = 2$

& $M \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow a_3 = -1, b_3 = 1, c_3 = 1$

Now $M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 11 \end{pmatrix}$

$\Rightarrow x + 0y - z = 1$

$2x + y + z = 7$

$3x + 2y + z = 11$

on solving $x = 2, y = 2, z = 1$

$\therefore x + y + z = 5$

5. If the sum of the first 10 terms of the series

$\frac{1}{1+1^4 \times 4} + \frac{2}{1+2^4 \times 4} + \frac{3}{1+3^4 \times 4} + \frac{4}{1+4^4 \times 4} + \dots$ is

$\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

(1) 256 (2) 264

(3) 276 (4) 284

Ans. (3)

Sol. $T_r = \frac{r}{1+4r^4} = \frac{r}{4r^4+4r^2+1-4r^2} = \frac{r}{(2r^2+1)^2-(2r)^2}$

$= \frac{r}{(2r^2+2r+1)(2r^2-2r+1)}$

$S_{10} = \frac{1}{4} \left(\frac{1}{1} - \frac{1}{221} \right) = \frac{55}{221} = \frac{m}{n}$

$\therefore m + n = 276$

6. Let $A_1, A_2, A_3, \dots, A_{39}$ be 39 arithmetic means between the numbers 59 and 159. Then the mean of A_{25}, A_{28}, A_{31} and A_{36} is equal to:

- (1) 129 (2) 136
(3) 131.50 (4) 134

Ans. (4)

Sol. $59, A_1, A_2, A_3, \dots, A_{39}, 159$

$d = \frac{159-59}{39+1} = \frac{5}{2}$

Now, $A_{25} = 59 + 25d = 121.5$

$A_{28} = 59 + 28d = 129$

$A_{31} = 59 + 31d = 136.5$

$A_{36} = 59 + 36d = 149$

Mean of

$A_{25}, A_{28}, A_{31}, A_{36} = \frac{121.5+129+136.5+149}{4}$

$= \frac{536}{4} = 134$

7. The coefficient of x^2 in the expansion of

$\left(2x^2 + \frac{1}{x} \right)^{10}, x \neq 0$, is :

- (1) 3240 (2) 3360
(3) 3480 (4) 3600

Ans. (3)

Sol. $T_{r+1} = {}^{10}C_r (2x^2)^{10-r} (1/x)^r$

$= {}^{10}C_r 2^{10-r} x^{20-2r-r}$

$= {}^{10}C_r 2^{10-r} x^{20-3r}$

$\Rightarrow 20 - 3r = 2$

$r = 6$

So required coefficient is ${}^{10}C_6 2^4$

8. The probabilities that players A and B of a team are selected for the captaincy for a tournament are 0.6 and 0.4, respectively. If A is selected the captain, the probability that the team wins the tournament is 0.8 and if B is selected the captain, the probability that the team wins the tournament is 0.7. Then the probability, that the team wins the tournament, is :

- (1) 0.74 (2) 0.76
(3) 0.72 (4) 0.78

Ans. (2)

Sol. $P(\text{Win}) = P(A \cap \text{Win}) + P(B \cap \text{Win})$

$$= P(A) \cdot P\left(\frac{\text{Win}}{A}\right) + P(B) \cdot P\left(\frac{\text{Win}}{B}\right)$$

$$= (0.6)(0.8) + (0.4)(0.7)$$

$$= 0.48 + 0.28 = 0.76$$

9. A box contains 5 blue, 6 yellow and 4 red balls. The number of ways, of drawing 8 balls containing at least two balls of each colour, is:

(1) 4100 (2) 4140

(3) 4230 (4) 4290

Ans. (1)

Sol. (But in this questions not mentioned that balls are distinct/alike)?

Blue – 5 Yellow – 6 Red – 4

Reqd. No. of ways

$$= {}^5C_2 \times {}^6C_2 \times {}^4C_4 + {}^5C_2 \times {}^6C_3 \times {}^4C_3 + {}^5C_2 \times {}^6C_4 \times {}^4C_2$$

$$+ {}^5C_3 \times {}^6C_2 \times {}^4C_3 + {}^5C_3 \times {}^6C_3 \times {}^4C_2 + {}^5C_4 \times {}^6C_2 \times {}^4C_2$$

$$= 10 \times 15 \times 1 + 10 \times 20 \times 4 + 10 \times 15 \times 6 +$$

$$10 \times 15 \times 4 + 10 \times 20 \times 6 + 5 \times 15 \times 6$$

$$= 150 + 800 + 900 + 600 + 1200 + 450$$

$$= 4100$$

10. A variable X takes values 0, 0, 2, 6, 12, 20,..... $n(n-1)$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, {}^nC_4, {}^nC_5, \dots, {}^nC_n$, respectively. If the mean of this data is 60, then its median is:

(1) 56 (2) 42

(3) 72 (4) 90

Ans. (1)

Sol. $\bar{X} = \frac{0 \cdot {}^nC_0 + 0 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \dots + n(n-1) \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n}$

$$= \frac{\sum_{r=0}^n r(r-1) {}^nC_r}{2^n}$$

$$= \frac{\sum_{r=1}^n n(r-1) {}^{n-1}C_{r-1}}{2^n} = \frac{\sum_{r=2}^n n(n-1) {}^{n-2}C_{r-2}}{2^n}$$

$$= \frac{n(n-1)2^{n-2}}{2^n} = \frac{n(n-1)}{4}$$

$$\Rightarrow \frac{n(n-1)}{4} = 60 \Rightarrow n^2 - n - 240 = 0$$

$$\Rightarrow n = 16, -15 \text{ (Reject)}$$

$$N = \sum f_i = {}^{16}C_0 + {}^{16}C_1 + \dots + {}^{16}C_{16} = 2^{16}$$

$$N = 2^{16} \text{ (Even)}$$

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{\text{th}} + \left(\frac{N}{2} + 1\right)^{\text{th}}}{2} \text{ term}$$

$$\frac{N}{2} = 2^{15} = 32768$$

x_i	f_i	
0	${}^{16}C_0$	
0	${}^{16}C_1$	
2	${}^{16}C_2$	
6	${}^{16}C_3$	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
240	${}^{16}C_{16}$	
x_i	f_i	CF
0	${}^{16}C_0$	1
0	${}^{16}C_1$	16
2	${}^{16}C_2$	137
6	${}^{16}C_3$	697
12	${}^{16}C_4$	2517
20	${}^{16}C_5$	6885
30	${}^{16}C_6$	14893
42	${}^{16}C_7$	26333
56	${}^{16}C_8$	$39203 \rightarrow \frac{N}{2} \& \left(\frac{N}{2} + 1\right)$ lies
72	${}^{16}C_9$	$\frac{n(n-1)2^{n-2}}{2^n}$
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	

$$\Rightarrow \text{median} = \frac{56 + 56}{2} = 56$$



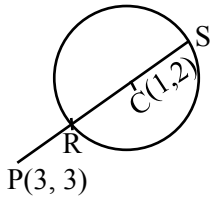
11. Let the point P be the vertex of the parabola $y = x^2 - 6x + 12$. If a line passing through the point P intersects the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the points R and S, then the maximum value of $(PR + PS)^2$ is:

- (1) 10 (2) 20
(3) 25 (4) 5

Ans. (2)

Sol. Parabola $y = x^2 - 6x + 12 = (x-3)^2 + 3$

$$y-3 = (x-3)^2 \Rightarrow \text{Vertex P (3, 3)}$$



$$x^2 + y^2 - 2x - 4y + 3 = 0$$

$$C(1, 2); r = \sqrt{2}$$

For max. of $(PR + PS)^2$

RS should be diameter

$$\begin{aligned} (PR + PS)_{\max}^2 &= (PC - r + PC + r)^2 \\ &= (2PC)^2 = 4(PC)^2 \\ &= 4 \times 5 = 20 \end{aligned}$$

12. Let the directrix of the parabola $P : y^2 = 8x$, cut x-axis at the point A. Let B (α, β) , $\alpha > 1$, be a point on P such that the slope of AB is $3/5$. If BC is a focal chord of P, then six times the area of $\triangle ABC$ is:

- (1) 80 (2) 160
(3) 174 (4) 192

Ans. (2)

Sol. Let $B(2t^2, 4t) \Rightarrow C\left(\frac{2}{t^2}, \frac{-4}{t}\right); A(-2, 0)$

$$\text{given } m_{AB} = \frac{4t}{2t^2 + 2} = \frac{3}{5}$$

$$20t = 6t^2 + 6$$

$$3t^2 - 10t + 3 = 0$$

$$t = 3, \frac{1}{3} \text{ but } t > 1 \text{ (given)}$$

$$\Rightarrow P(18, 12), Q\left(\frac{2}{9}, \frac{-4}{3}\right)$$

$$A = \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 18 & 12 & 1 \\ \frac{2}{9} & \frac{-4}{3} & 1 \end{vmatrix} = \frac{80}{3}$$

$$\Rightarrow 6A = 160$$

13. Let the eccentricity e of a hyperbola satisfy the equation $6e^2 - 11e + 3 = 0$. If the foci of the hyperbola are $(3, 5)$ $(3, -4)$, then the length of its latus rectum is:

- (1) $\frac{11}{3}$ (2) $\frac{17}{3}$
(3) $\frac{15}{2}$ (4) $\frac{17}{2}$

Ans. (3)

Sol. $S_1(3, 5)$ & $S_2(3, -4)$

$$\therefore S_1 S_2 = 2ae$$

$$\Rightarrow 9 = 2ae \quad \dots(1)$$

$$\text{and } \therefore 6e^2 - 11e + 3 = 0$$

$$\Rightarrow e = \frac{3}{2}, e = \frac{1}{3} < 1 \text{ (rejected)}$$

$$\text{From (1); } a = 3$$

$$\Rightarrow LR = 2 \cdot \frac{b^2}{a} = 2a(e^2 - 1) = 2 \cdot 3 \left(\frac{9}{4} - 1 \right) = \frac{15}{2}$$

14. Let a triangle PQR be such that P and Q lie on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ and are at a distance of 6 units from R (1, 2, 3). If (α, β, γ) is the centroid of ΔPQR , then $\alpha + \beta + \gamma$ is equal to:

- (1) 4 (2) 5
(3) 6 (4) 8

Ans. (3)

Sol. Let the point P is $(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

$$PR = 6 \Rightarrow PR^2 = 36$$

$$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$$

$$(4\lambda - 2)^2 + (\lambda + 1)^2 + (\lambda - 2)^2 = 9$$

$$\lambda^2 - \lambda = 0 \Rightarrow \lambda = 0, 1$$

$$\lambda = 0 \Rightarrow P(-3, 4, -1)$$

$$\lambda = 1 \Rightarrow P(5, 6, 1)$$

$$R(1, 2, 3)$$

$$G(1, 4, 1)$$

$$= \alpha + \beta + \gamma = 6$$

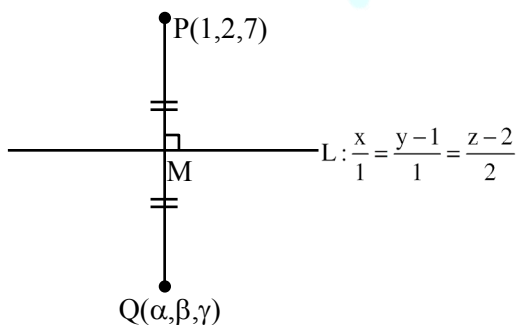
15. If the distance of the point (a, 2, 5) from the image of the point (1, 2, 7) in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ is 4, then the sum of all possible values of a is equal to :

- (1) 11 (2) 9
(3) 6 (4) 4

Ans. (3)

Sol. $PQ \perp L \Rightarrow (\alpha - 1) + (\beta - 2) + 2(\gamma - 7) = 0$

$$\Rightarrow \alpha + \beta + 2\gamma = 17 \quad \dots(1)$$



M is mid point of PQ which will satisfy L

$$\frac{\alpha + 1}{2} = \frac{\beta + 2}{2} - 1 = \frac{\gamma + 7}{2} - 2$$

$$\Rightarrow \frac{\alpha + 1}{2} = \frac{\beta}{2} = \frac{\gamma + 3}{4}$$

$$\Rightarrow \alpha + 1 = \beta \quad \dots(2)$$

$$\text{and } 2\beta = \gamma + 3 \quad \dots(3)$$

$$\Rightarrow \alpha = 3, \beta = 4, \gamma = 5$$

$$\text{Distance from } (a, 2, 5) \text{ is } = \sqrt{(a-3)^2 + 4 + 0} = 4$$

$$(a-3)^2 + 4 = 16 \Rightarrow a^2 - 6a - 3 = 0 \Rightarrow \text{sum of values of } a = 6$$

16. Let O be the origin, $\vec{OP} = \vec{a}$ and $\vec{OQ} = \vec{b}$. If R is the point on $\vec{OP}$ such that $\vec{OR} = 5\vec{OR}$, and M is the point such that $\vec{OQ} = 5\vec{RM}$, then $\vec{PM}$ is equal to :

(1) $\frac{1}{5}(\vec{a} - 4\vec{b})$ (2) $\frac{1}{5}(\vec{b} - 4\vec{a})$

(3) $\frac{1}{5}(-\vec{a} + 4\vec{b})$ (4) $\frac{1}{5}(-\vec{b} + 4\vec{a})$

Ans. (2)

Sol. $\vec{OR} = \frac{\vec{OP}}{5} = \frac{\vec{a}}{5}$

$$\vec{RM} = \frac{\vec{OQ}}{5} = \frac{\vec{b}}{5}$$

$$\vec{OM} - \vec{OR} = \frac{\vec{b}}{5}$$

$$\vec{OM} = \frac{\vec{a} + \vec{b}}{5}$$

$$\vec{PM} = \vec{OM} - \vec{OP}$$

$$\frac{\vec{a} + \vec{b}}{5} - \vec{a}$$

$$= \frac{\vec{b} - 4\vec{a}}{5}$$

17. Let $f(x) = \lim_{y \rightarrow 0} \frac{(1 - \cos(xy)) \tan(xy)}{y^3}$. Then the number of solutions of the equation $f(x) = \sin x$, $x \in \mathbb{R}$ is :

- (1) 0 (2) 2
(3) 3 (4) 1

Ans. (3)

Sol. $f(x) = \lim_{y \rightarrow 0} \frac{1 - \cos(xy)}{(xy)^2} \cdot \frac{\tan xy}{xy} \cdot \frac{(xy)^3}{y^3}$

$$f(x) = \frac{x^3}{2}$$

$$f(x) = \sin x$$

$$\frac{x^3}{2} = \sin x$$

3 points of intersection

18. Let $(2^{1-a} + 2^{1+a}), f(a), (3^a + 3^{-a})$ be a A. P. and α be the minimum value of $f(a)$. Then the value of the

integral $\int_{\log_e(\alpha-1)}^{\log_e(\alpha)} \frac{dx}{(e^{2x} - e^{-2x})}$ is :

- (1) $\frac{1}{2} \log_e \left(\frac{4}{3} \right)$ (2) $\frac{1}{4} \log_e \left(\frac{4}{3} \right)$
(3) $\frac{1}{2} \log_e \left(\frac{8}{5} \right)$ (4) $\frac{1}{4} \log_e \left(\frac{8}{5} \right)$

Ans. (2)

Sol. $f(a) = \frac{1}{2} (2(2^a + 2^{-a}) + (3^a + 3^{-a}))$

$$a = \frac{1}{2} (2 \times 2 + 2) = 3$$

$$I = \int_{\ln(3-1)}^{\ln 3} \frac{e^{2x}}{e^{4x} - 1} dx$$

put $e^{2x} = t$

$$I = \frac{1}{2} \int_4^9 \frac{dt}{t^2 - 1}$$

$$I = \frac{1}{4} \left(\ln \left(\frac{t-1}{t+1} \right) \right)_4^9$$

$$I = \frac{1}{4} \ln \left(\frac{4}{3} \right)$$

19. Let $f : [1, \infty) \rightarrow \mathbb{R}$ be a differentiable function defined as $f(x) = \int_1^x f(t) dt + (1-x)(\log_e x - 1) + e$.

Then the value of $f(f(1))$ is :

- (1) $(1 + e^e)$ (2) $(1 + e)$
(3) $(1 + e + e^e)$ (4) $1 + 2e$

Ans. (1)

Sol. $f(1) = \int_1^1 f(t) dt + (1-1)(\ln 1 - 1) + e$

$$f(1) = e$$

$$f(x) = \int_1^x f(t) dt + (1-x)(\ln x - 1) + e$$

diff. w.r.t. x

$$f'(x) = f(x) + \frac{1}{x} - \ln x$$

$$\text{I.F.} = e^{\int -1 dx} = e^{-x}$$

$$f(x) e^{-x} = \int e^{-x} \left(\frac{1}{x} - \ln x \right) dx$$

$$f(x) e^{-x} = e^{-x} \ln x + C$$

put $x = 1, y = e$

$$\Rightarrow C = 1$$

$$f(x) e^{-x} = e^{-x} \ln x + 1$$

$$f(x) = e^x + \ln x$$

$$f(e) = e^e + 1$$

20. Let $f(x)$ and $g(x)$ be twice differentiable functions satisfying $f''(x) = g''(x)$ for all $x \in \mathbb{R}$, $f'(1) = 2g'(1) = 4$ and $g(2) = 3f(2) = 9$. Then $f(25) - g(25)$ is equal to :

- (1) 20 (2) 40
(3) -20 (4) -40

Ans. (2)

Sol. $f'(x) = g''(x)$

Integrate

$$f(x) = g'(x) + C_1$$

Put $x = 1$

$$f(1) = g'(1) + C_1$$

$$4 = 2 + C_1$$

$$C_1 = 2$$

$$\Rightarrow f'(x) = g'(x) + 2$$

$$f(x) = g(x) + 2x + C_2$$

Put $x = 2$

$$f(2) = g(2) + 4 + C_2$$

$$3 = 9 + 4 + C_2$$

$$C_2 = -10$$

$$f(x) = g(x) + 2x - 10$$

Put $x = 25$

$$f(x) - g(25) = 40$$

SECTION-B

- 21.** Let $A = \{1, 4, 7\}$ and $B = \{2, 3, 8\}$. Then the number of elements, in the relation $R = \{(a_i, b_i), (a_2, b_2) \in ((A \times B) \times (A \times B)) : a_1 + b_2 \text{ divides } a_2 + b_1\}$ is

Ans. (18)

A / B	2	3	8
1	3	4	9
4	6	7	12
7	9	10	15

These are possible sum of $a_i + b_j$

Now $b_1 + a_2$ divides $a_1 + b_2$

So,

$a_1 + b_2$	$a_2 + b_1$	No. of pairs
15	3, 15	2
12	3, 4, 6, 12	4
10	10	1
9	3, 9	6
7	7	1
6	3, 6	2
4	4	1
3	3	1
		18

Number of relations = 18

- 22.** From the point $(-1, -1)$, two rays are sent making angles of 45° with the line $x + y = 0$. These rays get reflected from the mirror $x + 2y = 1$. If the equations of the reflected rays are $ax + by = 9$ and $cx + dy = 7$, $a, b, c, d \in \mathbb{Z}$, then the value of $ad + bc$ is

Ans. (7)

Sol. $\tan 45^\circ = \left| \frac{m+1}{1-m} \right|$

$$\Rightarrow \pm 1 = \frac{m+1}{1-m}$$

$\Rightarrow m = 0$ & one line is perpendicular to x axis.

$$L_1 : x = -1, L_2 : y = -1$$

Now taking mirror image of $x + 1 = 0$ in $x + 2y = 1$

$$A' : \frac{x+1}{1} = \frac{y+1}{2} = \frac{-2(-1-2-1)}{1^2+2^2}$$

$$x = \frac{3}{5}, y = \frac{11}{5} \quad A' \left(\frac{3}{5}, \frac{11}{5} \right)$$

$$\text{line} = 3x - 4y + 7 = 0$$

Now taking mirror image of $y + 1 = 0$ in $x + 2y = 1$

$$A'' : \frac{x+1}{1} = \frac{y+1}{2} = \frac{-2(-1-2-1)}{1^2+2^2}$$

$$x = \frac{3}{5}, y = \frac{11}{5}$$

$$A'' \left(\frac{3}{5}, \frac{11}{5} \right)$$

$$\text{line} : (y + 1) = \frac{\left(\frac{11}{5} + 1 \right)}{\left(\frac{3}{5} - 3 \right)} (x - 3)$$

$$4x + 3y = 9$$

comparing $4x + 3y = 9$ with $ax + by = 9$ &

$$a = 4, b = 3$$

by line

$$-3x + 4y = 7$$

comparing with

$$cx + dy = 7$$

$$\therefore c = -3, d = 4$$

So, $ad + bc$

$$= 4 \times 4 + 3 \times -3$$

$$= 16 - 9 = 7$$

23. If $S = \left\{ \theta \in [-\pi, \pi] : \cos \theta \cos \frac{5\theta}{2} = \cos 7\theta \cos \frac{7\theta}{2} \right\}$,
then $n(S)$ is equal to

Ans. (19)

Sol. $\cos \theta \cos \frac{5\theta}{2} = \cos 7\theta \cos \frac{7\theta}{2}$

$$\cos \frac{7\theta}{2} + \cos \frac{3\theta}{2} = \cos \frac{21\theta}{2} + \cos \frac{7\theta}{2}$$

$$\cos \frac{21\theta}{2} - \cos \frac{3\theta}{2} = 0$$

$$-2 \sin 6\theta \sin \frac{9\theta}{2} = 0$$

$$\sin 6\theta = 0$$

$$\theta = 0, \pm \frac{\pi}{6}, \pm \frac{2\pi}{6}, \dots, \pm \frac{5\pi}{6}, \pm \pi \quad (13 \text{ solutions})$$

$$\sin \frac{9\theta}{2} = 0$$

$$\theta = \pm \frac{2\pi}{9}, \pm \frac{4\pi}{9}, \pm \frac{8\pi}{9} \quad (6 \text{ more solutions})$$

Total = 19 solutions

24. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x$, $x \in \mathbb{R}$. Let maximum value of f on $\mathbb{R}$ be α . If the area of the region bounded by the curves $g(x) = x^2$ and $h(x) = \beta x^3$, $\beta > 0$, is α^2 , then $30\beta^3$ is equal to

Ans. (16)

Sol. $f(x) + 3f\left(\frac{\pi}{2} - x\right) = \sin x \quad \dots(1)$

Put $x \rightarrow \frac{\pi}{2} - x$

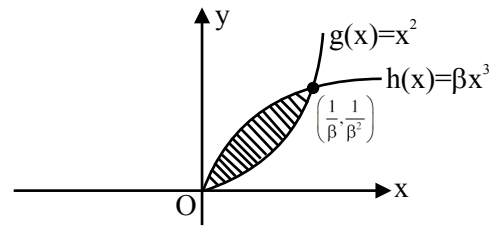
$$\Rightarrow f\left(\frac{\pi}{2} - x\right) + 3f(x) = \cos x \quad \dots(2)$$

From (1) & (2)

$$f(x) = \frac{1}{8}(3 \cos x - \sin x)$$

$$f_{\max} = \frac{\sqrt{10}}{8}$$

$y = g(x)$ & $y = h(x)$ intersect as shown in the figure



$$\therefore \text{Area bounded} = \Delta = \left| \int_0^{\frac{1}{\beta}} (\beta x^3 - x^2) dx \right|$$

$$= \frac{1}{12\beta^3} = \alpha^2 \text{ (given)}$$

$$\Rightarrow 30\beta^3 = 16$$

25. Let $y = y(x)$ be the solution of the differential equation $(\tan x)^{1/2} dy = (\sec^3 x - (\tan x)^{3/2} y) dx$, $0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$. If $y\left(\frac{\pi}{3}\right) = \frac{4}{5}\alpha$, then α^4 equals

Ans. (48)

Sol. $\frac{dy}{dx} + y \tan x = \frac{\sec^3 x}{\sqrt{\tan x}}$

$$\text{IF} = e^{\int \tan x dx} = \sec x$$

$$y \sec x = \int \frac{\sec^4 x}{\sqrt{\tan x}} dx$$

$$y \sec x = \int \frac{(1 + \tan^2 x)}{\sqrt{\tan x}} \sec^2 x dx$$

$$\tan x = t$$

$$y \sec x = 2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{6\sqrt{2}}{5}$$

$$\frac{6\sqrt{2}}{5} \times \sqrt{2} = 2 + \frac{2}{5} + c \Rightarrow \frac{12}{5} = \frac{12}{5} + c$$

$$\Rightarrow c = 0$$

$$y\left(\frac{\pi}{3}\right) = \frac{8}{5} \cdot 3^{1/4} \Rightarrow \alpha = 2 \times 3^{1/4}$$

$$\therefore \alpha^4 = 48$$

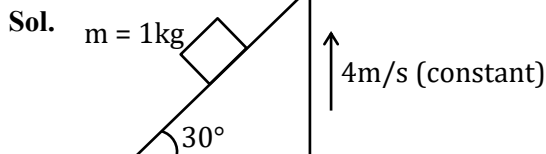


28. A mass of 1 kg kept in a inclined plane with 30° inclination with respect to horizontal plane and it is at rest initially. Then the whole assembly is moved up with constant velocity of 4 m/s. The work done by the frictional force in time 2s is ____ J.

(Take $g = 10 \text{ m/s}^2$)

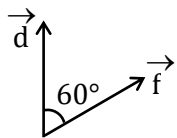
- (1) 20 (2) 25
(3) 30 (4) 10

Ans. (1)



$$f = mg \sin 30 = 5 \text{ N}$$

work done in 2 sec will be



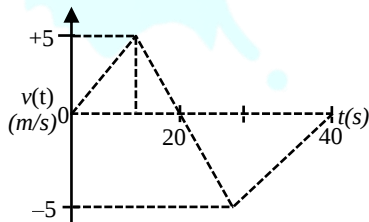
$$d = 4 \times 2 = 8 \text{ m}$$

$$W = fd \cos 60^\circ$$

$$W = (5)(8) \cos 60^\circ$$

$$W = 20 \text{ joule}$$

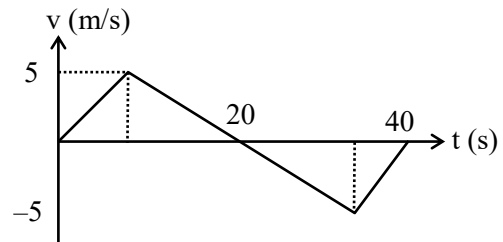
29. The velocity(v) versus time (t) plot of a particle is shown in the figure, for a time interval of 40s. The total distance travelled by the particle and the average velocity during this period are, respectively ____



- (1) 25 m and zero
(2) 50 m and zero
(3) 100 m and zero
(4) 100 m and 2.5 m/s

Ans. (3)

Sol.



Total distance travel

$$d = \frac{1}{2}(20)(5) + \frac{1}{2}(20) \times 5$$

$$d = 100 \text{ m}$$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$\langle v \rangle = \frac{0}{40} = 0$$

30. A wheel initially at rest is subjected to a uniform angular acceleration about its axis. In the first 2 s it rotates through an angle θ_1 and in the next 2 s it rotates through an angle θ_2 . The ratio $\frac{\theta_2}{\theta_1}$ is ____.

- (1) 6 (2) 3
(3) 4 (4) $\frac{1}{3}$

Ans. (2)

Sol. Let α be angular acceleration

In first 2 sec

$$\theta_1 = \frac{1}{2} \alpha (2)^2 = 2\alpha$$

Angular covered in 4 sec

$$\theta' = \frac{1}{2} (\alpha) (4)^2 = 8\alpha$$

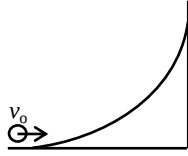
Angle in next 2 sec

$$\theta_2 = \theta' - \theta_1 = 8\alpha - 2\alpha$$

$$\theta_2 = 6\alpha$$

$$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$$

31. An object of uniform density rolls up the curved path with the initial velocity v_0 as shown in the figure. If the maximum height attained by an object is $\frac{7v_0^2}{10g}$ (g = acceleration due to gravity), the object is a _____.

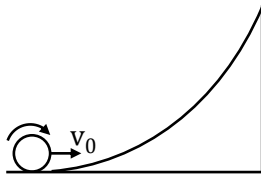


- (1) solid cylinder (2) ring
(3) disc (4) solid sphere

Ans. (4)

Sol. $h = \frac{7v_0^2}{10g}$

Applying work energy theorem



$$-mgh = 0 - \frac{1}{2}mv^2 - \frac{1}{2}I\omega^2$$

$$-mg\left(\frac{7v_0^2}{10g}\right) = -\frac{1}{2}mv_0^2 - \frac{1}{2}I\frac{v_0^2}{R^2}$$

$$\frac{7mv_0^2}{10} = \frac{mv_0^2}{2} + \frac{Iv_0^2}{2R^2}$$

$$\frac{mv_0^2}{5} = \frac{Iv_0^2}{2R^2}$$

$$I = \frac{2mR^2}{5}$$

32. A body of mass m is taken from the surface of earth to a height equal to twice the radius of earth (R_e). The increase in potential energy will be _____. (g is acceleration due to gravity at the surface of earth)

- (1) $\frac{1}{2}mgR_e$ (2) $\frac{3}{4}mgR_e$
(3) $\frac{1}{4}mgR_e$ (4) $\frac{2}{3}mgR_e$

Ans. (4)

Sol. $U_1 = \frac{-GMm}{R_e}$

$$U_2 = \frac{-GMm}{R_e + 2R_e} = -\frac{GMm}{3R_e}$$

$$\Delta U = U_2 - U_1 = \frac{-GMm}{3R_e} + \frac{GMm}{R_e}$$

$$\Delta U = \frac{2}{3} \frac{GMm}{R_e} = \frac{2}{3} \left[\frac{GM}{R_e^2} \right] (mR_e)$$

$$\Delta U = \frac{2}{3}mgR_e$$

33. Eight mercury drops, each of radius r , coalesce to form a bigger drop. The surface energy released in this process is _____. (S is the surface tension of mercury).

- (1) $8\pi r^2 S$ (2) $16\pi r^2 S$
(3) $64\pi r^2 S$ (4) $4\pi r^2 S$

Ans. (2)

Sol. Volume conservation

$$(8) \left[\frac{4}{3} \pi r^3 \right] = \frac{4}{3} \pi R^3$$

$$R = 2r$$

$$U_i = (8)(4\pi r^2)(s) ; s = \text{surface tension}$$

$$U_i = 32\pi r^2 s$$

$$U_f = (4\pi R^2)(s)$$

$$U_f = 4\pi(2r)^2(s) = 16\pi r^2 s$$

$$\text{Energy released} = U_i - U_f$$

$$\text{Energy released} = 16\pi r^2 s$$

34. An ideal gas at pressure P and temperature T is expanding such that $PT^3 = \text{constant}$. The coefficient of volume expansion of the gas is ____.

- (1) $\frac{2}{T}$ (2) $\frac{1}{T}$
(3) $\frac{4}{T}$ (4) $\frac{3}{T}$

Ans. (3)

Sol. $PT^3 = \text{constant}$ (C)

$$\frac{nRT}{V} \cdot T^3 = C$$

$$V = \frac{nR}{C} \cdot T^4$$

Volume expansion coeff.

$$\gamma = \frac{1}{V} \frac{dV}{dT}$$

$$\gamma = \frac{C}{nRT^4} \cdot \frac{nR}{C} \cdot 4T^3 = \frac{4}{T}$$

$$\text{Ans. } \gamma = \frac{4}{T}$$

35. Match List-I with List-II.

List-I		List-II	
A.	$\sin^2 \omega t$	I.	Periodic with time period $T = \frac{\pi}{\omega}$ but not simple harmonic motion (SHM)
B.	$\sin^3(2\omega t)$	II.	Periodic with time period $T = \frac{2\pi}{\omega}$ but Not SHM
C.	$\sin(\omega t) + \cos(\pi\omega t)$	III.	periodic with time period $T = \frac{\pi}{\omega}$ and SHM
D.	$\cos \omega t + \cos 2\omega t$	IV.	Non-periodic

Choose the correct answer from the options given below.

- (1) A-III, B-I, C-IV, D-II (2) A-II, B-I, C-III, D-IV
(3) A-III, B-II, C-IV, D-I (4) A-II, B-I, C-IV, D-III

Ans. (1)

Sol. (A) $\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$

it represents SHM motion having period = $\frac{\pi}{\omega}$

(B) $\sin^3 2\omega t = \frac{3}{4} \sin 2\omega t - \frac{1}{4} \sin 6\omega t$

common period $\frac{\pi}{\omega}$ but not SHM.

(C) $\sin \omega t + \cos \pi \omega t \rightarrow$ no common period hence it is a nonperiodic function

(D) $\cos \omega t + \cos 2\omega t$

common period $\frac{2\pi}{\omega}$ but it do not represent SHM.

A $\rightarrow$ III, B $\rightarrow$ I, C $\rightarrow$ IV, D $\rightarrow$ II

36. A metal rod of length L rotates about on end at origin with a uniform angular velocity ω . The magnetic field radially falls off as $B(r) = B_0 e^{-\lambda r}$; λ being a positive constant. The emf induced (neglecting the centripetal force on electrons in the rod) is:

(1) $B_0 \omega \left[\frac{1}{\lambda^2} - e^{-\lambda L} \left(\frac{1}{\lambda^2} + \frac{L}{\lambda} \right) \right]$

(2) $B_0 \omega \left[\frac{1}{\lambda^2} + e^{-\lambda L} \left(\frac{1}{\lambda^2} + \frac{L}{\lambda} \right) \right]$

(3) $B_0 \omega \left[\frac{4}{\lambda^2} - e^{-2\lambda L} \left(\frac{1}{\lambda^2} + \frac{2L}{\lambda} \right) \right]$

(4) $B_0 \omega \left[\frac{3}{\lambda^2} - e^{-3\lambda L} \left(\frac{3}{\lambda^2} + \frac{L}{\lambda} \right) \right]$

Ans. (1)

Sol.
$$e = \int_0^L B(\omega r) dr = B_0 \omega \int_0^L e^{-\lambda r} \cdot r dr$$

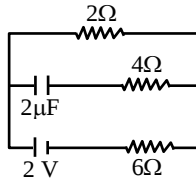
$$= B_0 \omega \left[r \int e^{-\lambda r} dr - \int 1 \cdot \frac{e^{-\lambda r}}{-\lambda} dr \right]_0^L$$

$$= B_0 \omega \left[\frac{re^{-\lambda r}}{-\lambda} - \frac{e^{-\lambda r}}{\lambda^2} \right]_0^L$$

$$= B_0 \omega \left[\left(0 + \frac{1}{\lambda^2} \right) - \left(\frac{Le^{-\lambda L}}{\lambda} + \frac{e^{-\lambda L}}{\lambda^2} \right) \right]$$

$$e = B_0 \omega \left[\frac{1}{\lambda^2} - e^{-\lambda L} \left(\frac{1}{\lambda^2} + \frac{L}{\lambda} \right) \right]$$

37. Under steady state condition the potential difference across the capacitor in the circuit is ____ V.



- (1) 0.5 (2) 1.5
(3) 0 (4) 2

Ans. (1)

Sol.
$$i = \frac{2}{6+2} = \frac{1}{4} \text{ A}$$

$$V_c = V_{2\Omega} = i \times 2 = \frac{1}{2} \text{ V} = 0.5 \text{ V}$$

38. A particle of charge q and mass m is projected from origin with an initial velocity $\vec{v} = \left(\frac{v_0}{\sqrt{2}} \hat{x} + \frac{v_0}{\sqrt{2}} \hat{y} \right)$. There exists a uniform magnetic field $\vec{B} = B_0 \hat{z}$ and a space varying electric field $\vec{E} = E_0 e^{-\lambda x} \hat{x}$ within the region $0 \leq x \leq L$. After travelling a distance such that x -coordinate has changed from $x = 0$ to $x = L$, the change in the kinetic energy is ____.

- (1) $\frac{qE_0}{\lambda} [1 - e^{-\lambda L}]$
(2) $\left(\frac{v_0 q B_0}{2\lambda} \right) [2 - e^{-2\lambda L}]$
(3) $\frac{qE_0}{\lambda} [1 + e^{-\lambda L}]$
(4) $q \left[\frac{E_0 + v_0 B_0}{\lambda} \right] [1 - e^{-\lambda L/2}]$

Ans. (1)

Sol.
$$\Delta K = W_e + W_m = W_e + 0$$

$$= \int_0^L q E dx = q E_0 \int_0^L e^{-\lambda x} dx$$

$$= \frac{q E_0}{\lambda} (1 - e^{-\lambda L})$$

39. Given below are two statements : one is labelled as **Assertion (A)** : and the other is labelled as

Reason (R).

Assertion (A) : The electromagnetic wave exerts pressure on the surface on which they are allowed to fall.

Reason (R) : There is no mass associated with the electromagnetic waves.

In the light of the above statements, choose the **correct answer** from the options given below :

- (1) Both (A) and (R) are true and (R) is the correct explanation of (A)
(2) Both (A) and (R) are true but (R) is **not** the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

Ans. (2)

Sol. Assertion and reason both are correct but not related.

40. A thin convex lens and a thin concave lens are kept in contact and are co-axial. Which of the following statements is correct for this combination of two lenses?

- (1) behaves as concave lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$
- (2) behaves as concave lens if $|f_{\text{convex}}| < |f_{\text{concave}}|$
- (3) behaves as convex lens if $|f_{\text{convex}}| > |f_{\text{concave}}|$
- (4) Focal length of the lens system will change if the position of two lenses are interchanged

Ans. (1)

Sol. 1 → convex lens

2 → concave lens

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{-f_2} \Rightarrow \frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2}$$

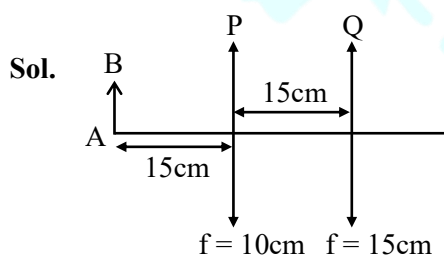
for $f_1 > f_2 \therefore f < 0$

$\therefore$ concave lens.

41. An object AB is placed 15 cm on the left of a convex lens P of focal length 10 cm. Another convex lens Q is now placed 15 cm right of lens P. If the focal length of lens Q is 15 cm, the final image is _____

- (1) virtual, formed at 7.5 cm right of lens Q, with a size bigger than that of AB
- (2) real, formed at 7.5 cm right of lens Q, with a size same than that of AB
- (3) formed at infinity.
- (4) real, formed at 7 cm right of lens Q, with a size smaller than that of AB

Ans. (2)



for P $\frac{1}{V} - \frac{1}{-15} = \frac{1}{10}$

$$\therefore v = +30 \text{ cm} = +15 \text{ cm right of Q.}$$

for Q $\frac{1}{V'} - \frac{1}{15} = \frac{1}{15}$

$$V' = 7.5 \text{ cm}$$

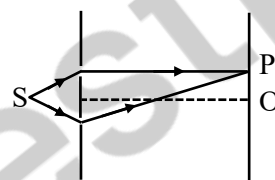
$$m = m_1 m_2 = \frac{30}{-15} \times \frac{7.5}{15} = -1$$

42. The maximum intensity in a Young's double slit experiment is I_0 . Distance between the slits (d) is 5λ , where λ is the wavelength of light used. The intensity of the fringe, exactly opposite to one of the slits on the screen, placed at $D = 10d$ is _____

- (1) $\frac{I_0}{4}$
- (2) $\frac{I_0}{2}$
- (3) I_0
- (4) $\frac{3I_0}{4}$

Ans. (2)

Sol.



$$\text{At P } y = \frac{d}{2}$$

$$\Delta Q = \frac{2\pi y d}{\lambda D} = \frac{2\pi d}{\lambda} \frac{d}{2D} = \frac{\pi d^2}{\lambda D}$$

$$\Delta Q = \frac{\pi \times 25\lambda^2}{\lambda \times 10d} = \frac{5\pi\lambda}{2 \times 5\lambda} = \frac{\pi}{2}$$

$$I = I_0 \cos^2\left(\frac{\Delta Q}{2}\right) = I_0 \cos^2\left(\frac{\pi}{2 \times 2}\right) = \frac{I_0}{2}$$

43. An electron is travelling with a velocity v in free space and when it enters a medium, its velocity is reduced by 20%. The de Broglie wavelength of electron in the medium is $\alpha\lambda_0$, where λ_0 is its de Broglie wavelength in free space. The value of α is _____

- (1) 1.20
- (2) 1.0
- (3) 1.25
- (4) 0.75

Ans. (3)

Sol. $\lambda_0 = \frac{h}{mv_0}$

$$\lambda = \frac{h}{mv} = \frac{h}{m(0.8v_0)} = \frac{1}{0.8} \frac{h}{mv_0} = 1.25 \lambda_0$$

$\therefore \alpha = 1.25$

- 44.** Assuming the experimental mass of ${}^{12}_6\text{C}$ as 12u, the mass defect of ${}^{12}_6\text{C}$ atom is ____ MeV/c².
(Mass of proton = 1.00727u, mass of neutron = 1.00866 u, 1 u = 931.5 MeV/c² and c is the speed of the light in vacuum.)

- (1) 127.5 (2) 89.03
(3) 272.0 (4) 92.0

Ans. (2)

Sol. $\Delta m = (6m_p + 6m_n) - m_c$
 $= 6(1.00727 + 1.00866) - 12$
 $= 0.09558 \text{ amu} = 89.03 \text{ MeV/C}^2$

- 45.** In a semiconductor p-n diode, the doping concentrations on p-side and n-side are 10^{15} atoms/cm³ and 10^{18} atoms/cm³, respectively. Which one of the following statement is true?

- (1) Widths of depletion region on either side of the interface are equal
 (2) The depletion region width is more on p-side compared to that in n-side
 (3) The depletion region width is more on n-side compared to that in p-side
 (4) No depletion region forms because of unequal doping concentrations on p and n-sides

Ans. (2)

Sol. $N_p x_p = N_n x_n$

$N_p < N_n \therefore x_p > x_n$ in depletion region

SECTION-B

- 46.** A copper wire of length 3 m is stretched by 3 mm by applying an external force. The volume of the wire is $600 \times 10^{-6} \text{ m}^3$. The elastic potential energy stored in the wire in stretched condition would be ____ J.

(Given Young modulus of copper = $1.1 \times 10^{11} \text{ N/m}^2$)

Ans. (33)

Sol. $U = \frac{1}{2} y (\text{strain})^2 \times \text{volume}$

$$= \frac{1}{2} \times 1.1 \times 10^{11} \times \left(\frac{3 \times 10^{-3}}{3} \right)^2 \times 600 \times 10^{-6}$$

$U = 33 \text{ J}$

- 47.** The heat extracted out of x gram of water initially at 50°C to cool it down to 0°C is sufficient to evaporate (1000 - x) gram of water also initially at 50°C. The value of x (closest integer) is ____.

(Take latent heat of water 2256 kJ/kg. K, specific heat capacity of water 4200 J/kg. K)

Ans. (922)

Sol. $x \times 4200 \times 50 = (1000 - x) \times 4200 \times 50 + (1000 - x) \times 2256 \times 10^3$
 $x = 1000 - x + (1000 - x) \times 10.74$
 $x = 921.52$

- 48.** A series LCR circuit with $R = 20\Omega$, $L = 1.6 \text{ H}$ and $C = 40 \mu\text{F}$ is connected to a variable frequency a.c. source. The inductive reactance at resonant frequency is ____ Ω .

Ans. (200)

Sol. For resonance

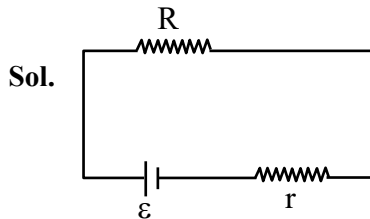
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.6 \times 40 \times 10^{-6}}} = \frac{10^3}{8} = 125$$

$X_L = \omega L = 125 \times 1.6 = 200 \Omega$



49. When an external resistance of 5Ω is connected across terminals of a cell, a current of 0.25 A flows through it. When the 5Ω resistor is replaced by a 2Ω resistor, a current of 0.5 A flows through it. The internal resistance of the cell is ____ Ω .

Ans. (1)



$$i = \frac{\varepsilon}{R + r}$$

$$0.25 = \frac{\varepsilon}{5 + r}$$

$$0.5 = \frac{\varepsilon}{2 + r}$$

on solving, $r = 1\Omega$

50. A circular loop of radius 20 cm and resistance 2Ω is placed in a time varying magnetic field $\vec{B} = (2t^2 + 2t + 3)\text{ T}$. At $t = 0$, for the plane of the loop being perpendicular to the magnetic field and, the induced current in the loop at $t = 3\text{ s}$ is $\frac{\alpha}{50}\text{ A}$.

The value of α is ____ (Take $\pi = 22/7$)

Ans. (44)

Sol. $\phi = B \cdot \pi r^2$

$$\varepsilon = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt}$$

$$i = \frac{\varepsilon}{R} = \frac{\pi r^2}{R} \cdot \left(\frac{dB}{dt} \right) = \frac{\pi}{2} \left(\frac{1}{5} \right)^2 (4t + 2)$$

for $t = 3$

$$i = \frac{11}{7} \times \frac{1}{25} \times 14$$

$$i = \frac{44}{50}$$

(HELD ON SUNDAY 05th APRIL 2026)
TIME : 3:00 PM TO 6:00 PM

CHEMISTRY	TEST PAPER WITH SOLUTION																						
SECTION-A 51. What volume of hydrogen gas at STP would be liberated by action of 50 mL of H₂SO₄ of 50% purity (density = 1.3 g mL⁻¹) on 20 g of zinc ? Given : Molar mass of H, O, S, Zn are 1, 16, 32, 65 g mol⁻¹ respectively. (1) 5.824 L (2) 7.428 L (3) 6.892 L (4) 8.375 L Ans. (3) Sol. Mass of H₂SO₄ = $\frac{50 \times 1.3}{98} \times \frac{50}{100} = \frac{32.5}{98}$ Zn(s) + H₂SO₄(aq) → ZnSO₄(aq.) + H₂(g) $\frac{20}{65} \qquad \frac{32.5}{98}$ Moles of H₂ formed = $\frac{20}{65}$ Volume of H₂ = $\frac{20}{65} \times 22.7 = 6.9$ L. 52. Which of the following statement(s) is/are true ? (A) If two orbitals have the same value of (n + l), the orbital with lower value of n will have lower energy. (B) Energies of the orbitals in the same subshell increase with increase in atomic number. (C) The size of 2p_x orbital is less than the size of 3p_x orbital. (D) Among 5f, 6s, 4d, 5p and 5d orbitals, none of the orbitals have 2 radial nodes. Choose the correct answer from the options given below : (1) A, B and C only (2) A and C only (3) C and D only (4) A only Ans. (2)	Sol. (a) If two orbitals are having same value of “n+l” then the orbital having lower value of ‘n’ will have lower energy. (b) Energies of the orbitals in the same subshell decreases with increase in atomic number. (c) Size of 2p_x < 3p_x (d) Orbital Radial node (n-ℓ-1) <table><tr><td>5f</td><td>1</td></tr><tr><td>6s</td><td>5</td></tr><tr><td>4d</td><td>1</td></tr><tr><td>5p</td><td>3</td></tr><tr><td>5d</td><td>2</td></tr></table> 53. The covalent radii of atoms A and B are r_A and r_B respectively. The covalent bond length and total length of AB molecule are respectively : (1) (r_A + r_B), 2(r_A + r_B) (2) $\frac{1}{2}$ (r_A + r_B), (r_A + r_B) (3) (r_A + r_B), (r_A + r_B) (4) 2(r_A + r_B), $\frac{1}{2}$ (r_A + r_B) Ans. (1) Sol. According to NCERT Bond length = r_A + r_B Total length of molecule = 2(r_A + r_B) 54. Consider the following data for the reaction X₂(g) + Y₂(g) ⇌ 2XY (g) at 600 K. The Δ_rG° (in kJ mol⁻¹) for the reaction is: <table><tr><th>Compound</th><th>Δ_fH°_{600K} (kJ mol⁻¹)</th><th>S°_{600K} (J mol⁻¹K⁻¹)</th></tr><tr><td>XY(g)</td><td>42</td><td>200</td></tr><tr><td>X₂(g)</td><td>8</td><td>140</td></tr><tr><td>Y₂(g)</td><td>80</td><td>250</td></tr></table> (1) -21000 (2) -10 (3) -1000 (4) -9.012 Ans. (2)	5f	1	6s	5	4d	1	5p	3	5d	2	Compound	Δ _f H° _{600K} (kJ mol ⁻¹)	S° _{600K} (J mol ⁻¹ K ⁻¹)	XY(g)	42	200	X ₂ (g)	8	140	Y ₂ (g)	80	250
5f	1																						
6s	5																						
4d	1																						
5p	3																						
5d	2																						
Compound	Δ _f H° _{600K} (kJ mol ⁻¹)	S° _{600K} (J mol ⁻¹ K ⁻¹)																					
XY(g)	42	200																					
X ₂ (g)	8	140																					
Y ₂ (g)	80	250																					

Sol. $\Delta_r G = \Delta_r H - T \Delta_r S$

$$\Delta_r H = (2 \times 42 - 80 - 8) = -4 \text{ kJ/mol}$$

$$\Delta_r S = (400 - 250 - 140) = +10 \text{ J/K.mol}$$

$$\Delta_r G = -4000 - 600 \times (10)$$

$$= -10,000 \text{ J/mol} = -10 \text{ kJ/mol}$$

55. The **correct** order of molar heat capacities measured at 298 K and 1 bar is :

(1) Copper (s) > Bromine (l) > Helium (g)

(2) Bromine (l) > Copper (s) > Helium (g)

(3) Helium (g) > Bromine (l) > Copper (s)

(4) Helium (g) > Bromine (l) = Copper (s)

Ans. (2)

Sol. Generally molar heat capacity of liquids is greater than solids because liquids have greater degrees of freedom & thus can store more energy with lesser temperature rise.

Also for monoatomic gases molar heat capacity is low as only translational degrees of freedom are present.

$\therefore$ order is :

$$\text{Br}_2(\text{l}) > \text{Cu}(\text{s}) > \text{He}(\text{g})$$

56. The reaction $\text{A}(\text{g}) \rightleftharpoons \text{B}(\text{g}) + \text{C}(\text{g})$ was initiated with the amount 'a' of A(g). At equilibrium it is found that the amount of A(g) remaining is (a - x) at a total pressure of p.

The equilibrium constant K_p of the reaction can be calculated from the expression :

$$(1) \frac{x^2}{a^2 + x^2} \times p \quad (2) \frac{x^2}{a^2 - x^2} \times p$$

$$(3) \frac{a + x^2}{x^2} \times p \quad (4) \frac{a - x^2}{x^2} \times p$$

Ans. (2)

Sol. $\text{A}(\text{g}) \rightleftharpoons \text{B}(\text{g}) + \text{C}(\text{g})$

$$t = 0 \quad a \quad 0 \quad 0$$

$$t_{\text{eq}} \quad a - x \quad x \quad x$$

$$K_p = \frac{(x)(x)}{(a - x)} \times \left(\frac{P}{a + x} \right)$$

$$K_p = \frac{x^2 P}{a^2 - x^2}$$

57. One half cell in a voltaic cell is constructed by dipping silver rod in AgNO_3 solution of unknown concentration, other half cell is Zn rod dipped in 1 molar solution of ZnSO_4 .

A voltage of 1.60 V is measured at 298 K for this cell. What is the concentration of Ag^+ ions used in terms of log x ($x = [\text{Ag}^+]$) ?

$$E_{\text{Zn}^{2+}/\text{Zn}}^\ominus = -0.76 \text{ V}, E_{\text{Ag}^+/\text{Ag}}^\ominus = +0.80 \text{ V},$$

$$\frac{2.303RT}{F} = 0.059 \text{ V}$$

$$(1) \frac{2}{3.9}$$

$$(2) \frac{4}{5.9}$$

$$(3) \frac{2.9}{2}$$

$$(4) \frac{5.9}{4}$$

Ans. (2)

Sol. Cell Reaction :



$$E_{\text{cell}} = E_{\text{cell}}^\ominus - \frac{0.059}{n} \log(Q)$$

$$1.6 = 1.56 - \frac{0.059}{2} \log \frac{1}{(\text{Ag}^+)^2}$$

$$0.04 = + 0.059 \log[\text{Ag}^+]$$

$$\text{Log}[\text{Ag}^+] = \frac{4}{5.9}$$

58. Given below are two statements :

Statement-I : The number of pairs among $[\text{Al}_2\text{O}_3, \text{Cr}_2\text{O}_3]$, $[\text{Cl}_2\text{O}_7, \text{Mn}_2\text{O}_7]$, $[\text{Na}_2\text{O}, \text{V}_2\text{O}_3]$ and $[\text{CO}, \text{N}_2\text{O}]$ that contain oxides of same nature (acidic, basic, neutral or amphoteric) is 4.

Statement-II : Among Na_2O , Al_2O_3 , CO and Cl_2O_7 , the most basic and acidic oxides are Na_2O and Cl_2O_7 , respectively.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. (1)

Sol. Statement I :

$\text{Al}_2\text{O}_3, \text{Cr}_2\text{O}_3$: Amphoteric

$\text{CO}, \text{N}_2\text{O}$: Neutral

$\text{Na}_2\text{O}, \text{V}_2\text{O}_3$: Basic

$\text{Cl}_2\text{O}_7, \text{Mn}_2\text{O}_7$: Acidic

Statement II :

Most basic oxide : Na_2O

Most acidic oxide : Cl_2O_7

59. Given below are two statements :

Statement-I : Aluminium upon reaction with NaOH forms $[\text{Al}(\text{OH})_6]^{3-}$ ion.

Statement-II : The geometry of ICl_4^- , ClO_3^- and IBr_2^- is square planar, pyramidal and linear respectively.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **Statement-I** and **Statement-II** are true.
- (2) Both **Statement-I** and **Statement-II** are false.
- (3) **Statement-I** is true but **Statement-II** is false.
- (4) **Statement-I** is false but **Statement-II** is true.

Ans. (4)

Sol. Statement I :

$\text{Al} + \text{NaOH} (\text{excess}) \rightarrow \text{Na}[\text{Al}(\text{OH})_4] + \text{H}_2(\text{g})$

Statement II :

ClO_3^- is sp^3 and pyramidal

ICl_4^- is sp^3d^2 and square planar

IBr_2^- is sp^3d and linear.

60. Given below are two statements :

Statement-I : Presence of large number of unpaired electrons in transition metal atoms results in higher enthalpies of their atomisation.

Statement-II : $d_{xy} = d_{xz} = d_{yz} < d_{x^2-y^2} = d_{z^2}$ and

$d_{x^2-y^2} = d_{z^2} < d_{xy} = d_{xz} = d_{yz}$ are the d-orbital splittings in $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ and $[\text{Ni}(\text{Cl})_4]^{2-}$ complex ions respectively.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **Statement-I** and **Statement-II** are correct.
- (2) Both **Statement-I** and **Statement-II** are incorrect.
- (3) **Statement-I** is correct but **Statement-II** is incorrect.
- (4) **Statement-I** is incorrect but **Statement-II** is correct.

Ans. (1)

Sol. Statement I :

More are the number of unpaired electrons, stronger is metal-metal bonding thus higher enthalpy of atomisation.

Statement II :

$[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ is octahedral complex.

Thus, $[t_{2g}(d_{xy} = d_{yz} = d_{zx}) < e_g(d_{x^2-y^2} = d_{z^2})]$

$[\text{NiCl}_4]^{2-}$ is tetrahedral complex.

Thus, $[e(d_{x^2-y^2} = d_{z^2}) < t_2(d_{xy} = d_{yz} = d_{zx})]$

61. Identify the **correct** statements from the following:

- (A) $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ is the most stable complex among $[\text{Fe}(\text{OH})_6]^{3+}$, $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ and $[\text{Fe}(\text{SCN})_6]^{3-}$
- (B) The stability of $[\text{Cu}(\text{NH}_3)_4]^{2+}$ is greater than that of $[\text{Cu}(\text{en})_2]^{2+}$
- (C) The hybridization of Fe in $\text{K}_4[\text{Fe}(\text{CN})_6]$ is $d^2\text{sp}^3$
- (D) $[\text{Fe}(\text{NO})_3\text{Cl}_3]^{3-}$ exhibits linkage isomerism
- (E) NO_2^- and SCN^- ligands are NOT ambidentate ligands

Choose the **correct** answer from the options given below :

- (1) A, B, C, D and E
- (2) B, C and D only
- (3) A, C and D only
- (4) A, C and E only

Ans. (3)

Sol. (A) $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$ has oxalate ion as ligand which leads to chelation thus maximum stability among given complexes.

(B) Stability : $[\text{Cu}(\text{NH}_3)_4]^{2+} < [\text{Cu}(\text{en})_2]^{2+}$; due to higher CFSE and chelation.

(C) $\text{K}_4[\text{Fe}(\text{CN})_6]$

$\Rightarrow \text{Fe}^{2+} = [\text{Ar}]3d^6$

In presence of SFL the electronic configuration of Fe^{+2} $[\text{t}_{2g}^6, \text{e}_g^0]$, thus d^2sp^3 hybridized.

(D) $[\text{Fe}(\text{NO}_2)_3\text{Cl}_3]^{3-}$ has NO_2^- as ambidentate ligand thus can show linkage isomerism.

(E) NO_2^- and SCN^- both are ambidentate ligands.

62. Match List-I with List-II.

	List-I (Purification technique)		List-II (Used to separate)
A.	Simple distillation	I.	Steam volatile compound
B.	Fractional distillation	II.	Two liquids with large difference in boiling points
C.	Steam distillation	III.	Liquid decomposing at its boiling point
D.	Distillation under reduced pressure	IV.	Two liquids with close boiling points

Choose the **correct** answer from the options given below :

- (1) A-II, B-III, C-I, D-IV (2) A-II, B-IV, C-I, D-III
(3) A-II, B-IV, C-III, D-I (4) A-IV, B-III, C-II, D-I

Ans. (3)

Sol. It is theory based question.

63. IUPAC name of the some alkenes are given below :
Find out the correct stability order.

A. 2-Methylbut-2-ene

B. *cis*-But-2-ene

C. 2,3-Dimethylbut-2-ene

D. Prop-1-ene

Choose the **correct** answer from the options given below :

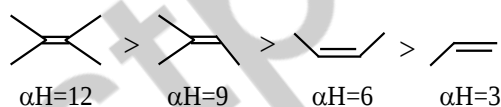
(1) $C > A > B > D$

(2) $C > A > D > B$

(3) $B > D > A > C$

(4) $A > B > C > D$

Ans. (1)

Sol. 

Stability of alkene $\propto$ Number of α hydrogen.

64. Identify the correct IUPAC name of hydrocarbon (x) containing three primary carbon atoms and with molar mass 72 g mol^{-1} .

(1) 1,1-Dimethylcyclopropane

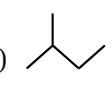
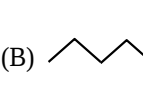
(2) 2,2-Dimethylpropane

(3) 2-Methylbutane

(4) n-pentane

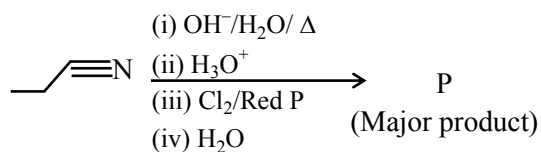
Ans. (3)

Sol. (A)  (1°C = 2) (B)  (1°C = 4)

(C)  (1°C = 3) (D)  (1°C = 2)

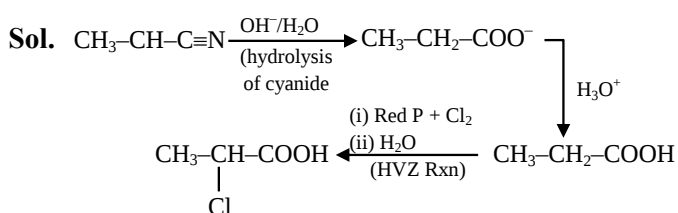
Mol. mass = 72

65. Complete the following reaction sequence and give the name of major product 'P'.



- (1) 2-Chloropropanoic acid
(2) 3-Chloropropanoic acid
(3) 1-Chloropropane
(4) 2-Chloropropane

Ans. (1)



66. Given below are two statements :

Statement-I : The condensation reaction between $\text{CH}_3\text{CH}=\text{O}$ and $\text{H}_2\text{N}-\text{N}(\text{H})-\text{C}(=\text{O})-\text{NH}_2$ under optimum

pH will produce $\text{CH}_3\text{CH}=\text{N}-\text{C}(=\text{O})-\text{NH}_2$

Statement-II : The molecule,

$\text{Ph}-\text{CH}(\text{OCH}_3)(\text{OH})$ will generate $\text{Ph}-\text{CH}=\text{O}$ in the

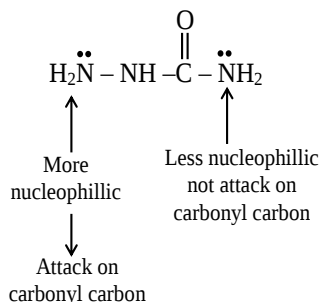
presence of dilute acid.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **Statement-I** and **Statement-II** are true.
(2) Both **Statement-I** and **Statement-II** are false.
(3) **Statement-I** is true but **Statement-II** is false.
(4) **Statement-I** is false but **Statement-II** is true.

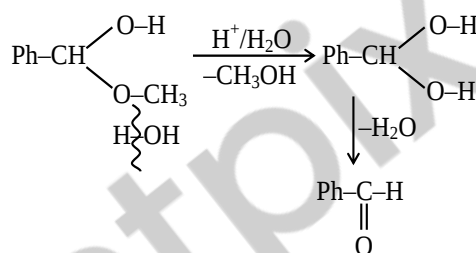
Ans. (4)

Sol. Statement-I



Hence statement-I is incorrect.

Statement-II



Statement-II is correct.

67. Given below are two statements :

Statement-I : Heating benzamide with bromine in an ethanolic solution of sodium hydroxide will give benzylamine.

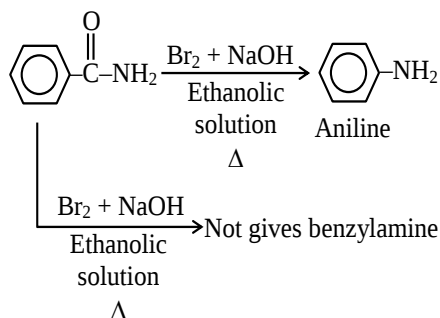
Statement-II : Nitration of aniline with $\text{HNO}_3/\text{H}_2\text{SO}_4$ at 288 K produces *m*-nitroaniline in higher amount than *o*-nitroaniline (pH adjusted).

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **Statement-I** and **Statement-II** are true.
(2) Both **Statement-I** and **Statement-II** are false.
(3) **Statement-I** is true but **Statement-II** is false.
(4) **Statement-I** is false but **Statement-II** is true.

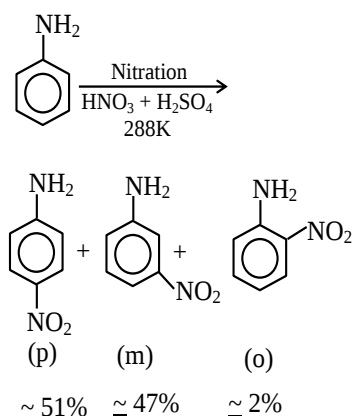
Ans. (4)

Sol. Statement-I



Hence Statement-I is incorrect

Statement-II



Statement-II is correct

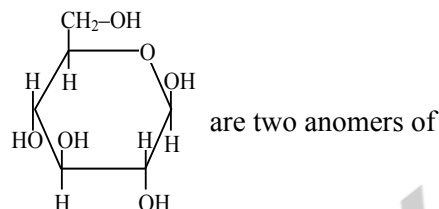
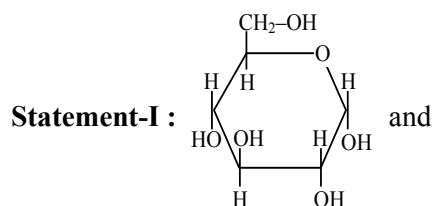
68. Identify the **incorrect** statement about tertiary structure of proteins.

- (1) They can be fibrous or globular in structure.
- (2) The main forces that stabilize the structure are hydrogen bonding, disulphide links, van der Waals and electrostatic forces of attraction.
- (3) The structure remains intact when exposed to pH changes.
- (4) A linear polypeptide chain will convert to a secondary structure and then further folding of the secondary structure will convert to tertiary structure.

Ans. (3)

Sol. On changing pH tertiary structure of protein disrupted and amino acids intact only by peptide linkage.

69. Given below are two statements :



D-(+)-glucose.

Statement-II : The open chain forms of D-glucose and D-fructose contain three similar chiral carbons at C₃, C₄ and C₅.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both **Statement-I** and **Statement-II** are true.
- (2) Both **Statement-I** and **Statement-II** are false.
- (3) **Statement-I** is true but **Statement-II** is false.
- (4) **Statement-I** is false but **Statement-II** is true.

Ans. (1)

Sol. α-D-Glucose and β-D-Glucose are anomers.

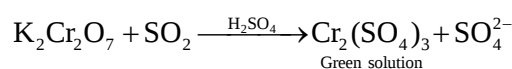
At C-3, C-4 and C-5 are identical configuration in glucose and fructose.

70. A paper dipped in a dil. H₂SO₄ solution of 'X' upon treatment with SO₂ gas turns into green. The compound 'X' is :

- | | |
|--|---|
| (1) KI-starch | (2) KMnO ₄ |
| (3) Pb(CH ₃ COO) ₂ | (4) K ₂ Cr ₂ O ₇ |

Ans. (4)

Sol. SO₂ gas acts as reducing agent.



SECTION-B

71. The total number of unpaired electrons present in the d^3 , d^4 (low spin) d^5 (high spin), d^6 (high spin) and d^7 (low spin) octahedral complex systems is ____.

Ans. (15)

Sol. $d^3 \rightarrow [t_{2g}^3, e_g^0]$, 3 unpaired electrons

d^4 (low spin) $\rightarrow [t_{2g}^4, e_g^0]$, 2 unpaired electrons

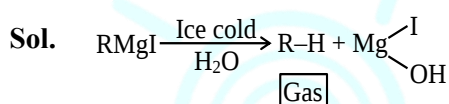
d^5 (high spin) $\rightarrow [t_{2g}^3, e_g^2]$, 5 unpaired electrons

d^6 (high spin) $\rightarrow [t_{2g}^4, e_g^2]$, 4 unpaired electrons

d^7 (low spin) $\rightarrow [t_{2g}^6, e_g^1]$, 1 unpaired electrons

72. RMgI when treated with ice cold water liberated a gas which occupied $1.4 \text{ dm}^3/\text{g}$ at STP. The gas produced is further reacted with iodine in presence of HIO_3 to give compound (X). Compound (X) in presence of Na and dry ether produced compound (Y). Molar mass of compound (Y) is ____ g mol^{-1} . (Nearest integer)

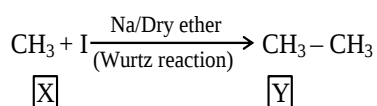
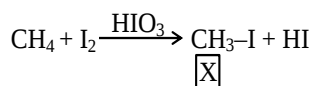
Ans. (30)



Vol. = $1.4 \text{ dm}^3/\text{gm}$ at STP

Hence according to this mol. Mass of gas is = 16

Hence liberated gas is = CH_4 (Methane)



Molar mass = 30

73. 20 g hemoglobin in a 1 L aqueous solution (A) at 300 K is separated from pure water by semi permeable membrane. At equilibrium the height of solution in a tube dipped in a solution (A) is found to be 80.0 mm higher than the tube dipped in water.

The molar mass of hemoglobin is ____ kg mol^{-1} . (Nearest integer)

(Given : $g = 10 \text{ ms}^{-2}$, $R = 8.3 \text{ kPa dm}^3\text{K}^{-1}\text{mol}^{-1}$, density of solution = 1000 kg m^{-3})

Ans. (62)

Sol. Osmotic pressure, $\pi = \rho gh$

$$= 1000 \times 10 \times 80 \times 10^{-3}$$

$$= 800 \text{ Pa} \quad \dots\dots(1)$$

Let molar mass of haemoglobin = $M \text{ g/mol}$

$$\text{conc. of haemoglobin} = \frac{20/M}{10^{-3}} \left(\frac{\text{mol}}{\text{m}^3} \right)$$

$$\pi = CRT \quad (\text{S.I. units})$$

$$\Rightarrow \pi = \left[\frac{20}{10^{-3}M} \right] \times 8.3 \times 300$$

$$= 800 \quad [\text{From (1)}]$$

Solving we get :

$$M = \frac{20 \times 8.3 \times 300}{0.8} \text{ g/mol}$$

$$= 62.25 \text{ kg mol}^{-1}$$

Ans = 62

74. At 298 K, the molar conductivity of $x\%$ (w/w) MX solution (aqueous) is $123.5 \text{ S cm}^2 \text{ mol}^{-1}$. The conductance of same solution is $1.9 \times 10^{-3} \text{ S}$. The value of x is ____ $\times 10^{-2}$.

(Given : cell constant = 1.3 cm^{-1} ; molar mass of MX is 75 g mol^{-1} , density of aqueous solution of MX at 298 K is 1.0 g mL^{-1})

Ans. (15)

Sol. Let molarity of MX solution = y M

$$\Rightarrow \Lambda_m = c(\ell/A) \times \frac{1000}{y}$$

$$\Rightarrow 123.5 = 1.9 \times 10^{-3} \times 1.3 \times \frac{1000}{y}$$

$$\Rightarrow y = 0.02 \text{ M}$$

Let molar mass of MX = M g mol⁻¹

0.2 mol MX is dissolve in 1L solution

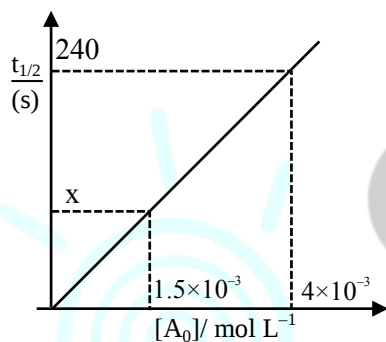
(0.02 × 75) gm MX is dissolved in 1000 g solution

$$\% \text{ w/w} = x\% = \frac{0.02 \times 75}{1000} \times 100$$

$$= 0.15\%$$

Ans = 15

- 75.** For a reaction $A \rightarrow P$ at T K, the half life ($t_{1/2}$) is plotted as a function of initial concentration $[A]_0$ of A as given below :



The value of x in the given figure is _____ s
(Nearest integer)

Ans. (90)

Sol. From the plot, $t_{1/2} \propto [A]_0$

$$\propto [A_0]^{1-N}$$

$$1 - N = 1$$

$$N = 0 \text{ (order)}$$

$$t_{1/2} = \frac{A_0}{2K}$$

$$240 = \frac{4 \times 10^{-3}}{2K} \quad \dots(1)$$

$$x = \frac{1.5 \times 10^{-3}}{2K} \quad \dots(2)$$

Divide (1) / (2)

$$\frac{240}{x} = \frac{4}{1.5}$$

$$x = 90$$