(HELD ON MONDAY 06th APRIL 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let [·] denote the greatest integer function. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x+[x]}{3}\right)$ is $[\alpha, \beta)$, then $\alpha^2 + \beta^2$ is equal to:

(1) 2 (2) 5
(3) 10 (4) 13

Ans. (2)

Sol. $-1 \le \frac{x+[x]}{3} \le 1$

$\Rightarrow -3 \le x - 1[x] \le 3$

$\Rightarrow -3 - [x] \le x \le 3 - [x]$

if $[x] = -1 \Rightarrow x \in [-1,0)$

if $[x] = 0 \Rightarrow x \in [0,1)$

if $[x] = 1 \Rightarrow x \in [1,2)$

Hence $x \in [-1,2)$

$\Rightarrow \alpha = -1$ and $\beta = 2$

$\Rightarrow \alpha^2 + \beta^2 = 5$

2. Let one root of the quadratic equation in x: $(k^2 - 15k + 27)x^2 + 9(k - 1)x + 18 = 0$ be twice the other. Then the length of the latus rectum of the parabola $y^2 = 6kx$ is equal to:

(1) 4 (2) 6
(3) 8 (4) 12

Ans. (4)

Sol. $\alpha + 2\alpha = \frac{-9(k-1)}{k^2-15k+27} = 3\alpha \quad \ldots(1)$

$\alpha \times 2\alpha = \frac{18}{k^2-15k+27} = 2\alpha^2 \quad \ldots(2)$

solving (1) & (2)

$k = 2$

$\Rightarrow$ parabola will be $y^2 = 6kx = 12x$

length of latus rectum = 12

3. Let e_1 and e_2 be two distinct roots of the equation $x^2 - ax + 2 = 0$. Let the sets $\{a \in \mathbb{R} : e_1$ and e_2 are the eccentricities of hyperbolas$\} = (\alpha, \beta)$, and $\{a \in \mathbb{R} : e_1$ and e_2 are the eccentricities of an ellipse and a hyperbola, respectively$\} = (\gamma, \infty)$.
The $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

(1) 18 (2) 22
(3) 26 (4) 34

Ans. (3)

$e_1 + e_2 = a$

$e_1e_2 = 2 \quad \Rightarrow e_2 = \frac{2}{e_1}$

Case 1 : Both are eccentricities of hyperbola

Now, $e_1 > 1$ and $e_2 > 1$

$\Rightarrow \frac{2}{e_1} > 1 \Rightarrow e_1 < 2 \quad \Rightarrow e_1 \in (1,2)$

Now, $a = e_1 + \frac{1}{e_1}$

Minimum occurs at $e_1 = \sqrt{2}$

$\Rightarrow a_{min} = 2\sqrt{2}$

if e = 1,2 (end points)

$a \to 2,3 \quad \Rightarrow a \in \left(2\sqrt{2}, 3\right)$

$\Rightarrow \alpha = 2\sqrt{2}$ and $\beta = 3$

Case 2 : One ellipse , one hyperbola
(e_1) (e_2)

$0 < e_1 < 1$ and $e_2 < 1$

$e_2 = \frac{2}{e_1} > 2$

Now $a = e_1 + \frac{2}{e_1}$

as $e_1 \to 1 \Rightarrow a \to 3$

$e_1 \to 0 \Rightarrow a \to \infty$

$\therefore (3,\infty) \Rightarrow r = 3$

Now $\alpha^2 + \beta^2 + \gamma^2 = 8 + 9 + 9 = 26$

4. Let the set of all values of $k \in R$ such that the equation $z(\bar{z}+2+i)+k(2+3i)=0$, $z \in C$, has at least one solution, be the interval $[\alpha, \beta]$. Then $9(\alpha+\beta)$ is equal to:

(1) -10 (2) -8

(3) $10\sqrt{13}$ (4) $8\sqrt{13}$

Ans. **(1)**

Sol. Put $z = x + iy$, $\bar{z} = x - iy$

$\Rightarrow \bar{z}+2+i = x-iy+2+i$

$\Rightarrow z(\bar{z}+2+i) = (x+iy)(x+2+i(1-y))$

$\Rightarrow$ Real part $= x(x+2) - y(1-y)$

Imaginary part $= x(1-y) + y(x+2)$

$\Rightarrow x^2 + y^2 + 2x - y + 2k = 0$ ..(1)

and $x + 2y + 3k = 0$ …(2)

eliminating x from (1) and (2)

$5y^2 + (12k-5)y + 9k^2 - 4k = 0$

since $y \in R$

use $D \geq 0$

$(12k-5)^2 - 4.5(9k^2 - 4k) \geq 0$

$\Rightarrow 36k^2 + 40k - 25 \leq 0$

$$\Rightarrow \alpha+\beta = \frac{-10}{9}$$

$$\Rightarrow 9(\alpha+\beta) = -10$$

5. The value of $1^3 - 2^3 + 3^3 - .... + 15^3$ is:

(1) 1706 (2) 1856

(3) 1982 (4) 2403

Ans. **(2)**

Sol. $(1^3 - 2^3) + (3^3 - 4^3) +…. 13^3 - 14^3 + 15^3$

use $n^3 - (n+1)^3 = -3n^2 - 3n - 1$

put $n = 1,3,5,….13$ (7 terms)

sum of pairs $= -1519$

add last term $= 15^3$

$\Rightarrow 3375 - 1519 = 1856$

6. The sum of the first ten terms of an A.P. is 160 and the sum of the first two terms of a G.P. is 8. If the first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to common difference of the A.P., then the sum of all possible values of the first term of the G.P. is:

(1) $\frac{34}{9}$ (2) $\frac{34}{13}$

(3) $\frac{32}{9}$ (4) $\frac{32}{13}$

Ans. **(1)**

Sol. Sum of 10 terms of A.P, $S_{10} = 160$

$5[2a + 9d] = 160$

$2a + 9d = 32$

G.P. $\Rightarrow d, da, da^2….$

$\Rightarrow d + da = 8$

$$\Rightarrow \frac{d+d(32-9d)}{2} = 8$$

$\Rightarrow 2d + 32d - 9d^2 = 8$

$\Rightarrow 9d^2 - 34d + 8 = 0$

$$d_1 + d_2 = \frac{34}{9}$$

7. The number of 4-letter words, with or without meaning, each consisting of two vowels and two consonants that can be formed from the letters of the word INCONSEQUENTIAL, without repeating any letter, is:

(1) 2670 (2) 2840

(3) 2920 (4) 3600

Ans. **(4)**

Sol. INCONSEQUENTIAL contains

(I, E,O, A,U) and (N,C,S,Q,T,L)

Total number of 4 letter words using two vowels and two consonants is

$= {}^5C_2 \times {}^6C_2 \times 4!$

$= 3600$

8. If the coefficients of the middle terms in the binomial expansions of $(1 + \alpha x)^{26}$ and $(1 - \alpha x)^{28}$, $\alpha \neq 0$, are equal, then the value of α is:

(1) 1 (2) $\frac{14}{13}$

(3) $\frac{27}{7}$ (4) $\frac{7}{27}$

Ans. (4)

Sol. $(1 + \alpha x)^{26}$

Middle term, $T_{14} = {}^{26}C_{13}\alpha^{13}$

For $(1 - \alpha x)^{28}$

Middle term $= {}^{28}C_{14}\alpha^{14}$

$\Rightarrow {}^{26}C_{13}\alpha^{13} = {}^{28}C_{14}\alpha^{14}$

$\Rightarrow {}^{26}C_{13} = {}^{28}C_{14}\alpha$

$\Rightarrow {}^{26}C_{13} = \frac{28}{14} \times \frac{27}{14} . {}^{26}C_{13}\alpha$

$\Rightarrow \alpha = \frac{7}{27}$

9. A data consists of 20 observations $x_1, x_2, \ldots\ldots, x_{20}$. If $\sum_{i=1}^{20}(x_i + 5)^2 = 2500$ and $\sum_{i=1}^{20}(x_i - 5)^2 = 100$, then the ratio of mean to standard deviation of this data is :

(1) 2 : 1 (2) 3 : 1

(3) 3 : 2 (4) 4 : 1

Ans. (2)

Sol. $\sum_{i=1}^{20}(x_i + 5)^2 = 2500 \quad \ldots(1)$

$\sum_{i=1}^{20}(x_i - 5)^2 = 100 \quad \ldots(2)$

on (1) - (2) $\Rightarrow 20\sum x_i = 2400 \Rightarrow \sum x_i = 120$

mean, $\bar{x} = 6$

(1) + (2) $\Rightarrow 2\sum x_i^2 + 40(5^2) = 2600$

$\sum x_i^2 = 800$

$\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 = \frac{800}{20} - 36 = 4$

$\bar{x} : \sigma = 6 : 2 = 3 : 1$

Ans. 3 : 1

10. A bag contains (N + 1) coins – N fair coins, and one coin with 'Head' on both sides. A coin is selected at random and tossed. If the probability of getting 'Head' is $\frac{9}{16}$, then N is equal to:

(1) 5 (2) 7

(3) 8 (4) 9

Ans. (2)

Sol. Total (N + 1) coins

N $\Rightarrow$ fair coins

1 $\Rightarrow$ Head on both side

$P(1^+) = \left(\frac{N}{N+1}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{N+1}\right)1$

$= \frac{N+2}{2(N+1)} = \frac{9}{16}$

$8N + 16 = 9N + 9$

$\Rightarrow N = 7$

11. If the eccentricity e of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passing through $(6, 4\sqrt{3})$, satisfies $15(e^2 + 1) = 34e$, then the length of the latus rectum of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{2(a^2+1)} = 1$ is:

(1) 10 (2) 20

(3) 25 (4) 30

Ans. (1)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It passs through $(6, 4\sqrt{3})$

$\Rightarrow \frac{36}{a^2} - \frac{48}{b^2} = 1 \quad \ldots(1)$

also $15e^2 - 34e + 15 = 0$

$\Rightarrow 15e^2 - 25e - 9e + 15 = 0$

$e = \frac{5}{3}$ or $\frac{3}{5} \Rightarrow e = \frac{5}{3}$

$1 + \frac{b^2}{a^2} = \frac{25}{9} \Rightarrow \frac{b^2}{a^2} = \frac{16}{9} \quad \ldots(2)$

using (1) and (2)

$$\Rightarrow \frac{36}{a^2}-\frac{48}{16a^2}\times 9=1 \Rightarrow a=3, b=4$$

Length of L.R.

$$\frac{4\left(a^2+1\right)}{b}=\frac{4(10)}{4}=10$$

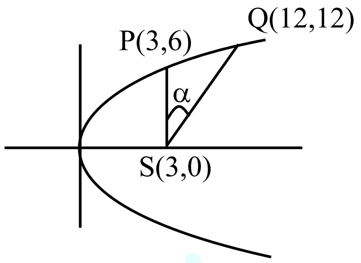
12. Let chord PQ of length $3\sqrt{13}$ of the parabola $y^2 = 12x$ be such that the ordinates of points P and Q are in the ratio 1 : 2. If the chord PQ subtends an angle α at the focus of the parabola, then sin α is equal to :

(1) $\frac{3}{5}$ (2) $\frac{4}{5}$

(3) $\frac{5}{13}$ (4) $\frac{12}{13}$

Ans. (1)

Sol.



$P(3t^2, 6t), Q(12t^2, 12t)$

$$PQ=\sqrt{81t^4+36t^2}$$

$81t^4 + 36t^2 = 117$

$\Rightarrow 9t^4 + 4t^2 - 13 = 0$

$(t^2 - 1)(9t^2 + 13) = 0 \Rightarrow t = 1$

P(3,6), Q(12,12)

slope of QS = $\frac{4}{3}$

$$\tan\alpha=\frac{3}{4}, \sin\alpha=\frac{3}{5}$$

13. Let $0<\alpha<1$, $\beta=\frac{1}{3\alpha}$ and $\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)$ $=\frac{\pi}{4}$. Then $6(\alpha+\beta)$ is equal to:

(1) 6 (2) 7

(3) 8 (4) 9

Ans. (2)

Sol. $\beta=\frac{1}{3\alpha}, \tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$

$$\Rightarrow \tan\left(\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)\right)=1$$

$$\Rightarrow \frac{1-\alpha+1-\beta}{1-(1-\alpha)(1-\beta)}=1$$

$$\Rightarrow 2-\alpha-\beta=1-(1-\alpha-\beta+\alpha\beta)$$

$$\Rightarrow 2-\alpha-\beta=\alpha+\beta-\alpha\beta$$

$$\Rightarrow 2(\alpha+\beta)=2+\alpha\beta$$

$$\Rightarrow 2(\alpha+\beta)=2+\frac{1}{3}=\frac{7}{3}$$

$$\alpha+\beta=\frac{7}{6}$$

$6(\alpha + \beta) = 7$

14. Let $S=\left\{\theta\in(-2\pi,2\pi):\cos\theta+1=\sqrt{3}\sin\theta\right\}$.

Then $\sum_{\theta\in S}\theta$ is equal to:

(1) $-\frac{2\pi}{3}$ (2) $-\frac{4\pi}{3}$

(3) $\frac{2\pi}{3}$ (4) $\frac{4\pi}{3}$

Ans. (2)

Sol. $\cos\theta + 1 = \sqrt{3}\sin\theta$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}+1=\sqrt{3}\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$$

$$2=2\sqrt{3}\tan\frac{\theta}{2}$$

$$\Rightarrow \tan\frac{\theta}{2}=\frac{1}{\sqrt{3}} \quad \theta\in(-2\pi, 2\pi), \ \frac{\theta}{2}\in(-\pi,\pi)$$

$$\frac{\theta}{2}=-\frac{5\pi}{6},\frac{\pi}{6}$$

$$\theta=\frac{-5\pi}{3},\frac{\pi}{3}$$

$$\text{Sum}=-\frac{-5\pi}{3}+\frac{\pi}{3}=\frac{-4\pi}{3}$$

15. Let the image of the point P(1, 6, a) in the line L: $\frac{x}{1}=\frac{y-1}{2}=\frac{z-a+1}{b}, b>0$, be $\left(\frac{a}{3},0,a+c\right)$. If S(α, β, γ), α > 0, is the point on L such that the distance of S from the foot of perpendicular from the point P on L is $2\sqrt{14}$, then α + β + γ is equal to :

(1) 19 (2) 20

(3) 21 (4) 22

Ans. (3)

Sol. Let Q be the point $\perp^r$ from P(1,6,a) to the line

$$L:\frac{x}{1}=\frac{y-1}{2}=\frac{z-a+1}{b}$$

The image of point P in line L is $P'\left(\frac{a}{3},0,a+c\right)$

∵ Q is the mid point PP' :

$$Q\equiv\left(\frac{1+\frac{a}{3}}{2},\frac{6+0}{2},\frac{a+a+c}{2}\right)\equiv\left(\frac{3+a}{6},3,\frac{2a+c}{2}\right)$$

comparing x and y, $\frac{3+a}{6}=1\Rightarrow a=3$

$$\therefore\ Q\equiv\left(1,3,\frac{6+c}{2}\right)$$

Direction vector of L is $\vec{v}=(1,2,b)$

vector PQ is $\perp^r$ to $\vec{v}$

$$\therefore\ \overrightarrow{PQ}=\left(1-1,3-6,\frac{6+c}{2}-3\right)$$

$$=\left(0,-3,\frac{c}{2}\right)$$

$$\overrightarrow{PQ}.\vec{v}=0$$

$$(0).(1)+(-3)(2)+\left(\frac{c}{2}\right)(b)=0$$

bc = 12

Now comparing z with $Q\left(1,3,\frac{6+c}{2}\right)$

$$\frac{3-1}{2}=\frac{\frac{6+c}{2}-3+1}{b}$$

2b – c = 2

solving bc = 12 and c = 2b – 2

b = 3, –2

b = 3 (b > 0)

c = 4

∴ Q(1,3,5)

Now, any point S on the line L can be written as (k, 2k + 1, 3k + 2)

$$SQ=\sqrt{(k-1)^2+(2k-2)^2+(3k-3)^2}=2\sqrt{14}$$

k = 3, –1

for k = 3, $S\equiv(3,7,11)$

$$\therefore\ \alpha+\beta+\gamma=21$$

16. Let a line L be perpendicular to both the lines

$L_1:\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and

$L_2:\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$.

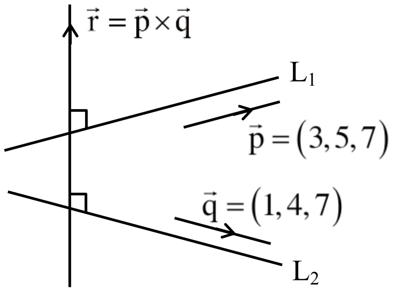
If θ is the acute angle between the lines L and $L_3:\frac{x-\frac{8}{7}}{2}=\frac{y-\frac{4}{7}}{1}=\frac{z}{2}$, then tan θ is equal to:

(1) $\frac{3}{2}\sqrt{2}$ (2) $\frac{5}{2}\sqrt{2}$

(3) $\frac{5}{3}\sqrt{2}$ (4) $\frac{4}{3}\sqrt{2}$

Ans. (2)

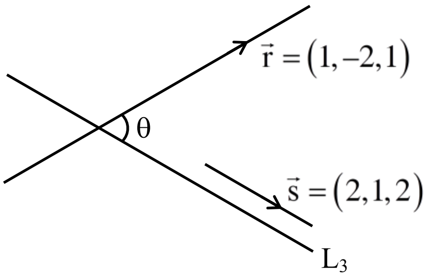
Sol.



$$\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$$

$$\equiv 7(\hat{i} - 2\hat{j} + \hat{k})$$

Now,



$$\because \cos\theta = \frac{2-2+2}{\sqrt{1+4+1}\sqrt{4+1+4}}$$

$$\Rightarrow \cos\theta = \frac{2}{3\sqrt{6}} \Rightarrow \tan\theta = \frac{5\sqrt{2}}{2}$$

17. The value of $\lim\limits_{x\to 0}\left(\dfrac{x^2 \sin^2 x}{x^2 - \sin^2 x}\right)$ is:

(1) 2 (2) 3

(3) 4 (4) 6

Ans. **(2)**

Sol. $\lim\limits_{x\to 0} \dfrac{x^2 \sin^2 x}{x^2 - \sin^2 x}$

Applying expansion

$$\lim_{x\to 0} \frac{x^2 \sin^2 x}{x^2 - \left(x - \dfrac{x^3}{3!} + \ldots\right)^2}$$

$$\lim_{x\to 0} \frac{x^2 \sin^2 x}{x^2 - x^2 + \dfrac{2x^4}{3!} \ldots} = \frac{3!}{2} = 3$$

18. The value of the integral $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{32\cos^4 x}{1+e^{\sin x}}\right)dx$ is :

(1) $4\pi + 2$ (2) $3\pi + 8$

(3) $3\pi + 4$ (4) $4\pi + 3$

Ans. **(2)**

Sol. Apply (P-5)

$$I = \int_{-\pi/4}^{\pi/4} \frac{32\cos^4\theta}{1+e^{-\sin\theta}} d\theta$$

$$\text{Add } 2I = \int_{-\pi/4}^{\pi/4} 32\cos^4\theta\, d\theta = 2\int_0^{\pi/4} 32\cos^4\theta\, d\theta$$

$$I = 32\int_0^{\pi/4} \cos^4\theta\, d\theta$$

$$= 8\int_0^{\pi/4} (2\cos^2\theta)^2 d\theta = 8\int_0^{\pi/4} (1+\cos 2\theta)^2 d\theta$$

$$= 8\int_0^{\pi/4} 1 + 2\cos 2\theta + \cos^2 2\theta\, d\theta$$

$$= 8\int_0^{\pi/4} 1 + 2\cos 2\theta + \frac{1+\cos 4\theta}{2} d\theta$$

$$= 8\left[\frac{3\theta}{2} + \frac{2\sin 2\theta}{2} + \frac{\sin 4\theta}{8}\right]_0^{\pi/4}$$

$$= 8\left[\left(\frac{3\pi}{8} + 1 + 0\right)\right] = 3\pi + 8$$

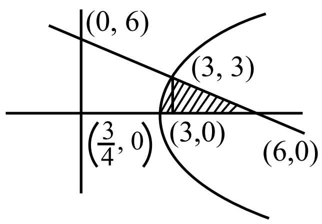
19. The area of the region $\{(x, y) : 0 \le y \le 6 - x, y^2 \ge 4x - 3, x \ge 0\}$ is :

(1) 8 (2) 9

(3) 12 (4) 15

Ans. **(2)**

Sol. $A = \displaystyle\int_0^3 \left(6 - y - \frac{y^2+3}{4}\right)dy = 9$



20. Let e be the base of natural logarithm and let $f: \{1, 2, 3, 4\} \to \{1, e, e^2, e^3\}$ and $g: \{1, e, e^2, e^3\} \to \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$ be two bijective functions such that f is strictly decreasing and g is strictly increasing. If $\phi(x) = \left[f^{-1}\left\{g^{-1}\left(\frac{1}{2}\right)\right\}\right]^x$, then the area of the region $R = \{(x, y) : x^2 \le y \le \phi(x), 0 \le x \le 1\}$ is :

(1) $\frac{3-\log_e(2)}{3\log_e(2)}$ (2) $\frac{1}{3\log_e(2)}$

(3) $3 + \log_e(2)$ (4) $\frac{3+\log_e(2)}{2+\log_e(3)}$

Ans. (1)

Sol. $f(x) = e^{x-1}$

$$g(x) = \frac{1}{\log_e x + 1}$$

$f^{-1}(g^{-1}(1/2)) = f^{-1}(e) = 2$

$\phi(x) = 2^x$

$$\therefore \int_0^1 (2^x - x^2)dx = \left(\frac{2^x}{\ln 2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{\ln 2} - \frac{1}{3}$$

$$= \frac{3-\ell n2}{3\ell n2}$$

SECTION-B

21. Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ satisfy

$A^2 + \alpha(\text{adj (adj (A))}) + \beta(\text{adj (A) (adj (adj (A)))})$ $= \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ for

Some $\alpha, \beta \in \mathbb{R}$.

Then $(\alpha - \beta)^2$ is equal to ___________

Ans. (4)

Sol. $|A| = -1$

$\text{adj(adj(A))} = |A|^{n-2}A = |A|^{3-2}.A = |A|.A$

$\text{adj(A) (adj (adj(A)))} = \text{adj(A) } (|A|.A)$

$= |A| \text{ (adj(A).A)}$

since $\text{adj(A)A} = |A|I$,

then $\text{adj(A) (adj(adj(A)))} = |A|^2 I$

or $\text{adj(A)(adj (adj (A)))} = I$

solving the matrix equation

$$A^2 + \alpha(-A) + \beta(I) = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$A^2 - \alpha A + \beta I = M$

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \alpha \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\alpha = 1$ and $\beta = -1$

$\therefore (\alpha - \beta)^2 = 4$

22. Let the centre of the circle $x^2 + y^2 + 2gx + 2fy + 25 = 0$ be in the first quadrant and lie on the line $2x - y = 4$. Let the area of an equilateral triangle inscribed in the circle be $27\sqrt{3}$. Then the square of the length of the chord of the circle on the line $x = 1$ is ________.

Ans. (80)

Sol. $x^2 + y^2 + 2gx + 2fy + 25 = 0$

$C \equiv (-g, -f)$ lies on line

$\therefore -2g + f = 4$ ...(i)

Also $g^2 + f^2 - 25 = r^2$ ...(ii)

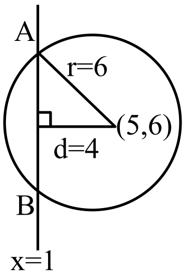
Area of equation triangle inside the circle

$$= \frac{3\sqrt{3}}{4} r^2$$

$$27\sqrt{3} = \frac{3\sqrt{3}}{4} \times r^2$$

$r^2 = 36$

Solving (i) and (ii) $g = -5$, $f = -6$



$$\ell_{AB} = \sqrt{36-16} = 2\sqrt{20}$$

$$(\ell_{AB})^2 = 80$$

23. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{j}-\hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$, then $\vec{c}\cdot(\vec{a}-2\vec{b})$ is equal to ________.

Ans. **(3)**

Sol. Using triple product identity with $\vec{a}\times\vec{c} = \vec{b}$

$$\vec{a}\times(\vec{a}\times\vec{c}) = \vec{a}\times\vec{b}$$

$$(\vec{a}.\vec{c})\vec{a} - (\vec{a}.\vec{a})\vec{c} = \vec{a}\times\vec{b}$$

$$\text{As } \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{i}+\hat{j}+\hat{k}$$

Substituting given values

$$3\vec{a} - 3\vec{c} = -2\hat{i}+\hat{j}+k$$

$$3(\hat{i}+\hat{j}+\hat{k}) + 2\hat{i}+\hat{j}+\hat{k} = 3\vec{c}$$

$$\vec{c} = \frac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})$$

$$\text{Now } \vec{c}.\left(\vec{a}-2\vec{b}\right) = \frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right).\left(\hat{i}-\hat{j}+3\hat{k}\right)$$

$$= \frac{1}{3}[5-2+6] = 3$$

24. For the functions $f(\theta) = \alpha\tan^2\theta + \beta\cot^2\theta$, and $g(\theta) = \alpha\sin^2\theta + \beta\cos^2\theta$, $\alpha > \beta > 0$, let $\min\limits_{0<\theta<\frac{\pi}{2}} f(\theta) = \max\limits_{0<\theta<\pi} g(\theta)$. If the first term of a G.P. is $\left(\frac{\alpha}{2\beta}\right)$, its common ratio is $\left(\frac{2\beta}{\alpha}\right)$ and the sum of its first 10 terms is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ________.

Ans. **(1279)**

Sol. $f(\theta)\big|_{min} = 2\sqrt{\alpha\beta}$ (AM ≥ GM)

$$g(\theta) = \alpha\sin^2\theta + \beta(1-\sin^2\theta) = (\alpha-\beta)\sin^2\theta + \beta$$

$$\therefore\ g(\theta)\big|_{max} = \alpha$$

Equation $2\sqrt{\alpha\beta} = \alpha$

$$4\alpha\beta = \alpha^2$$

$$\alpha = 4\beta \ (\alpha \neq 0)$$

Now first term $\left(\frac{\alpha}{2\beta}\right) = 2$ and common ratio $\left(\frac{2\beta}{\alpha}\right) = \frac{1}{2}$

$$S_{10} = \frac{a\left(1-r^{10}\right)}{1-r} = \frac{1023}{256} = \frac{m}{n}$$

m + n = 1279

25. Let y = y (x) be the solution of the differential equation $\left(x^2 - x\sqrt{x^2-1}\right)dy + \left(y\left(x-\sqrt{x^2-1}\right)-x\right)dx = 0$, $x \geq 1$. If y(1) = 1, then the greatest integer less than $y\left(\sqrt{5}\right)$ is ________.

Ans. **(3)**

Sol. $$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x-\sqrt{x^2-1}}$$

$$\frac{dy}{dx} + \frac{y}{x} = x + \sqrt{x^2-1}$$

$$I.f = e^{\ln x} = x$$

$$y.x = \int\left(x+\sqrt{x^2-1}\right)x\,dx$$

$$= \frac{x^2}{2} + \frac{(x^2-1)^{3/2}}{3} + C$$

Given y(1) = 1

$$1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$$

$$\left[y\left(\sqrt{5}\right)\right] = \left[\frac{\sqrt{5}}{2} + \frac{8}{3} + \frac{1}{2\sqrt{5}}\right] = 3$$

(HELD ON MONDAY 06th APRIL 2026) **TIME : 9:00 AM TO 12:00 NOON**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

26. The density ρ of a uniform cylinder is determined by measuring its mass m, length *l* and diameter d. The measured values of m, *l* and d are 97.42 ± 0.02 g, 8.35 ± 0.05 mm and 20.20 ± 0.02 mm, respectively. Calculated percentage fractional error in ρ is ________ .

(1) 0.63%

(2) 0.82%

(3) 0.72%

(4) 0.25%

Ans. (2)

Sol. $m = 97.42 \pm 0.02$ g

$l = 8.35 \pm 0.05$ mm

$d = 20.20 \pm 0.02$ mm

$$\rho = \frac{m}{\frac{\pi}{4}d^2\ell}$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta d}{d} + \frac{\Delta\ell}{\ell}$$

$$= \frac{0.02}{97.42} + 2\times\frac{0.02}{20.20} + \frac{0.05}{8.35} = 0.0082$$

% error = 0.82%

27. The potential energy of a particle changes with distance x from a fixed origin as $V = \frac{A\sqrt{x}}{x+B}$, where A and B are constant with appropriate dimensions. The dimension of AB are _______ .

(1) $[M^1 L^{5/2} T^{-2}]$

(2) $[M^{3/2} L^{5/2} T^{-2}]$

(3) $[M^1 L^2 T^{-2}]$

(4) $[M^1 L^{7/2} T^{-2}]$

Ans. (4)

Sol. $[B] = [x] = L$

$$[U] = \left[\frac{A\sqrt{x}}{B+x}\right] \Rightarrow M L^2 T^{-2} = \frac{[A]L^{1/2}}{L}$$

$[A] = M L^{5/2} T^{-2}$

28. The rain drop of mass 1 g, starts with zero velocity from a height of 1 km. It hits the ground with a speed of 5 m/s. The work done by the unknown resistive force is __________ J. (take $g = 10\ m/s^2$)

(1) – 8.75

(2) – 8.35

(3) – 9.55

(4) – 9.98

Ans. (4)

Sol. $w_g + w_{res} = \Delta k$

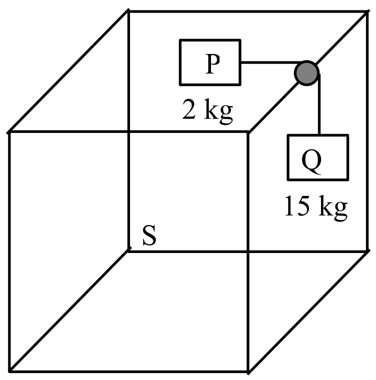
$$mgh + w_{res} = \frac{1}{2}mv^2$$

$$\therefore w_{res} = \frac{1}{2}mv^2 - mgh$$

$$= 10^{-3}\left(\frac{25}{2} - 10^4\right)$$

$= -9.98$ J

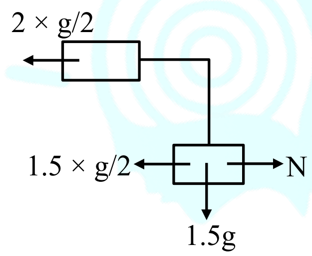
29. Two blocks (P and Q) with respectively masses 2 kg and 1.5 kg are joined by a massless thread. These blocks are mounted on a frictionless pully which is fixed on the edge of a cube (S), as shown in the figure below. Block P is positioned on the top surface which has no friction and block Q is in contact with side-surface, having coefficient friction μ. The cube (S) moves towards the right with acceleration of $\frac{g}{2}$, where g is gravitational acceleration. During this movement the block P and Q remain stationary. The value of μ is _____ .
(take $g = 10\ m/s^2$)



(1) 0.33 (2) 0.67
(3) 1 (4) 0.5

Ans. **(2)**

Sol. w.r.t bigger block



$$f = \mu N = \mu \times \frac{1.5g}{2}$$

$$1.5g = g + \mu + \frac{1.5g}{2} \qquad \Rightarrow 0.5 = \mu \times \frac{1.5}{2}$$

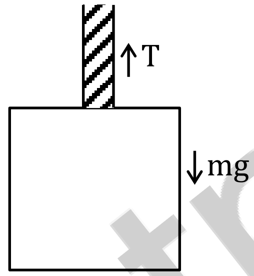
$$\left(\mu = \frac{2}{3}\right)$$

30. A lift of mass 1600 kg is supported by thick iron wire. If the maximum stress which the wire can withstand is $4 \times 10^8\ N/m^2$ and its radius is 4 mm, then maximum acceleration the lift can take is _____ m/s^2. (take $g = 10\ m/s^2$ and $\pi = 3.14$)

(1) 2.56 (2) 3.89
(3) 4.32 (4) 5.16

Ans. **(1)**

Sol.



$T - mg = ma$

$(\sigma\pi r^2) - mg = ma$

$\Rightarrow 4 \times 10^8\ \pi\ (4 \times 10^{-3})^2 - 16000 = 1600\ a$

$\Rightarrow\ a = 2.56\ m/s^2$

31. A solid sphere of radius 4 cm and mass 5 kg is rotating (rotation axis is passing through the centre of the sphere) with an angular velocity of 1200 rpm. It is brought to rest in 10 s by applying a constant torque. The torque applied and the number of rotations it made before it comes to rest are _____ and ______ respectively.

(1) $0.128\ \pi$ Nm, 100 (2) $0.0128\ \pi$ Nm, 50
(3) $0.128\ \pi$ Nm, 50 (4) $0.0128\ \pi$ Nm, 100

Ans. **(4)**

Sol. $\omega_i = 1200\ rpm = 1200 \times \frac{2\pi}{60} = 40\ \pi\ rad/sec$

$\rightarrow \omega_f = \omega_i + \alpha t \Rightarrow 0 = 40p + \alpha(10)$

$\alpha = -4\pi\ rad/sec^2$

$\rightarrow I = \frac{2}{5}MR^2 = \frac{2}{5}(5)\,(0.04)^2 = 2 \times 0.0016$

$I = 0.0032$ Kg-m^2

$\rightarrow \tau = I\alpha = 0.0032 \times 4\pi$

$\tau = 0.0128\pi$ N-m

$\rightarrow \omega_f^2 = \omega_i^2 + 2\alpha\theta$

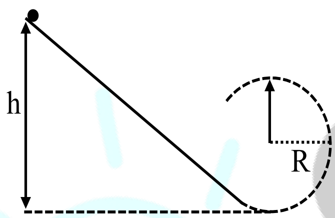
$0 = (40\pi)^2 + 2(4\pi)\theta$

$\theta = \frac{1600\pi^2}{8\pi} = 200\pi$

No. of revolution $= \frac{\theta}{2\pi} = \frac{200\pi}{2\pi}$

No. of revolution = 100

32. A smooth inclined plane ends in a vertical circular loop, as shown in the figure. A small body is released from height h as shown. If the body exerts a force of three times its weight on the plane at the highest point of circle then the height h = αR. The value of α is



(1) 2 (2) 4

(3) 3 (4) 6

Ans. **(2)**

Sol. at highest point

$mg + N = \frac{mv^2}{R}$

$4\,mg = \frac{mv^2}{R}$

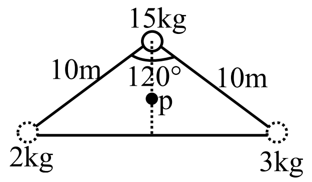
$v = \sqrt{4gR}$

Now energy conservation

$mg\,(h - 2R) = \frac{1}{2}m \times 4gR$

$(h = 4R)$

33. The position of center of mass of three masses 2 kg, 3 kg and 15 kg placed with respect to mid point (p) of normal bisector, as shown in the figure is _____ .

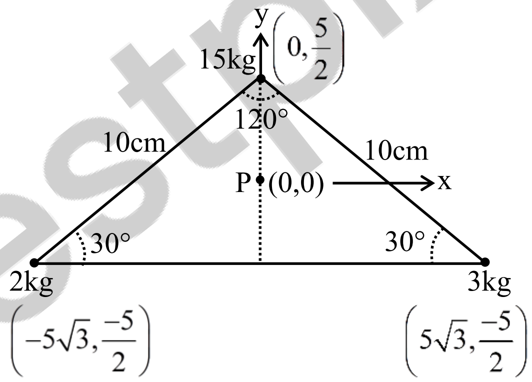


(1) $\left(\frac{\sqrt{3}}{4}, 1.25\right)$ (2) $\left(\frac{\sqrt{3}}{4}, 1.0\right)$

(3) (0, 0) (4) (1.25, 0)

Ans. **(1)**

Sol.



$$X_{cm} = \frac{2\times(-5\sqrt{3}) + 3(5\sqrt{3}) + 15\times 0}{20} = \frac{\sqrt{3}}{4}\text{ cm}$$

$$Y_{cm} = \frac{2\times(-5/2) + 3(5/2) + 15\times(5/2)}{20}$$

$$Y_{cm} = 1.25\text{ cm}$$

34. The two wires A and B of equal cross-section but of different materials are joined together. The ratio of Young's modulus of wire A and wire B is 20/11. When the joined wire is kept under certain tension the elongations in the wires A and B are equal. If the length of wire A is 2.2 m, then the length of wire B is____m.

(1) 1.1 (2) 2.22

(3) 1.21 (4) 4.44

Ans. (3)

Sol. $\Delta\ell = \dfrac{T\ell}{YA}$

$\ell \propto Y$

$\dfrac{\ell_1}{\ell_2} = \dfrac{Y_1}{Y_2} = \dfrac{20}{11}$

$\because \ell_1 = 2.2\,\text{m}$

$\therefore\ \ell_1 = 1.21\,\text{m}$

35. Two closed vessels of same volume are joined through a narrow tube and both vessels are filled with air of pressure 90 kPa and temperature 400 K. Keeping the temperature of one vessel constant at 400 K the second vessel temperature is raised to 500 K. The final pressure in the vessels is _____ kPa.

(1) 100 (2) 120

(3) 90 (4) 105

Ans. (1)

Sol. Using mole conservation

$$\frac{2\times 9\times 10^4\times V}{R\times 400} = \frac{P\times V}{R\times 400} + \frac{P\times V}{R\times 500}$$

P = 100 kPa

36. In interference experiment the path difference between two interfering waves at a point A on the screen is $\lambda/3$, where λ is the wavelength of these waves, and at another point B the path difference is $\lambda/6$. The ratio of intensities at points A and B is _____ .

(1) 3 (2) 4

(3) 1/3 (4) 1/4

Ans. (3)

Sol. $\Delta\phi_A = \dfrac{2\pi}{\lambda}\times\dfrac{\lambda}{3} = \dfrac{2\pi}{3}$

$\therefore\ I_A = I + I + 2\sqrt{I}.\sqrt{I}.\cos\left(\dfrac{2\pi}{3}\right) = I$

$\Delta\phi_B = \dfrac{2\pi}{\lambda}\times\dfrac{\lambda}{6} = \dfrac{\pi}{3}$

$\therefore\ I_B = I + I + 2\sqrt{I}.\sqrt{I}.\cos\left(\dfrac{\pi}{3}\right) = 3I$

$\dfrac{I_A}{I_B} = \dfrac{I}{3I} = \dfrac{1}{3}$

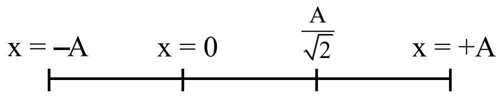
37. A particle is executing simple harmonic motion. Its amplitude is A and time period is 5 sec. the time required by it to move from x = A to $x = \dfrac{A}{\sqrt{2}}$ is_____sec.

(1) 1/4 (2) 5/4

(3) 5/8 (4) 3/8

Ans. (3)

Sol.



$x = 0 \rightarrow \theta = 0^\circ$

$x = \dfrac{A}{\sqrt{2}} \rightarrow \theta = \dfrac{\pi}{4}$

$\therefore$ phase covered $= \dfrac{\pi}{4}$

$\text{time} = \dfrac{\text{phase}}{w} = \dfrac{\pi}{4w} = \dfrac{\pi}{4}\times\dfrac{T}{2\pi} = \dfrac{T}{8}$

$\therefore\ t = \dfrac{5}{8}$ secs.

38. A thin half ring of radius 35 cm is uniformly charged with a total charge of Q coulomb. If the magnitude of the electric field at centre of the half ring is 100 V/m, then the value of Q is ______ nC.

($\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 and $\pi = 3.14$)

(1) 2.14 (2) 2.44

(3) 3.25 (4) 0.7

Ans. **(1)**

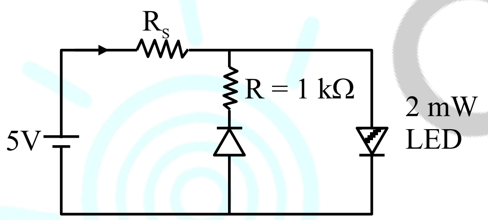
Sol. $E = \dfrac{2k\lambda}{R}\sin\left(\dfrac{\theta}{2}\right)$

$$100 = 2 \times \frac{9\times 10^9 \times Q}{\pi R^2}$$

$Q = 2.14 \times 10^{-9}$ C

39. The maximum rated power of the LED is 2 mW and it is used in the circuit with input voltage of 5 V as shown in the figure below. The current through resistance R_s is 0.5 mA.

The minimum value of the resistance of R_S, to ensure that the LED is not damaged is ______ kΩ.



(1) 6 (2) 2

(3) 4 (4) 5

Ans. **(2)**

Sol. Diode in reverse bias so no current in it.

$P_{bulb} = 2 \times 10^{-3} = V_{bulb}$

$V_{bulb} = 4V$

So $5V = V_{RS} + V_{bulb}$

$V_{rs} = 5 - 4 = 1$

$1 = 0.5 \times 10^{-3}\ R_s \Rightarrow R_s = 2k\Omega$

40. A point light source emits E.M. waves in free space. A detector, placed at a distance of L m, measures the intensity as I_0. The detector is now shifted to another location on the same spherical surface ensuring the angle between original location and new location as 45°. The measured intensity at new location will be ______.

(1) $\dfrac{I_0}{4}$ (2) I_0

(3) $\dfrac{I_0}{\sqrt{2}}$ (4) $\dfrac{I_0}{2}$

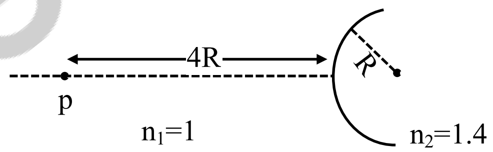
Ans. **(2)**

Sol. Distance is same

So intensity is also same

$$I = \frac{P}{4\pi R^2}$$

41. A spherical interface lens of radius R separates two media of refractive indices 1 and 1.4 respectively as shown in the figure below. A point source is placed at a distance of 4R in front of spherical interface. The magnitude of the magnification of point source image is _________.



(1) 1.66 (2) 2.33

(3) 2.66 (4) 1.33

Ans. **(1)**

Sol. $m = \dfrac{n_1}{n_2}\dfrac{v}{u}$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.4}{v} + \frac{1}{4R} = \frac{1.4-1}{R}$$

$$\frac{1.4}{v} = \frac{4}{10R} - \frac{1}{4R}$$

$$v = \frac{56R}{6} = +\frac{28}{3}R$$

$$m = \frac{1}{1.4}\cdot\frac{28R/3}{(-4R)} = -1.67$$

42. A small cube of side 1 mm is placed at the centre of a circular loop of radius 10 cm carrying a current of 2 A. The magnetic energy stored inside the cube is $\alpha \times 10^{-14}$ J. The value of α is ________.

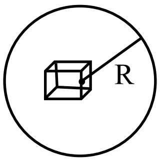
$(\mu_0 = 4\pi \times 10^{-7}$ Tm/A, $\pi = 3.14)$

(1) 6.28 (2) 6.28×10^{-6}

(3) 628 (4) 6.28×10^{-4}

Ans. (1)

Sol. Magnetic field at centre $B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi i}{R}$



$$= \frac{10^{-7} \times 2 \times \pi \times 2}{10 \times 10^{-2}}$$

So energy stored

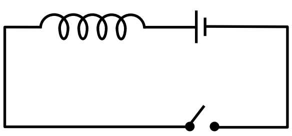
$$U = \frac{B^2}{2\mu_0} \times V$$

$$= \frac{10^{-14} \times 4 \times \pi^2 \times 4 \times 100}{2 \times 4\pi \times 10^{-7}} \times (1 \times 10^{-9})$$

$$= 2\pi \times 10^{-14}\,\text{J}$$

$= 6.28 \times 10^{-14}$ J

43. An inductor of inductance 10 mH having resistance of 100 Ω is connected to battery of E.M.F. 1.0 V through a switch as shown in the figure below. After switch is closed, the ratio of instantaneous voltages across the inductor when the current passing through it is 2 mA and 4 mA is _______.



(1) 4/3 (2) 3/4

(3) 5/3 (4) 3/5

Ans. (1)

Sol. Apply KVL

$E - iR - V_L = 0$

$V_L = E - iR = 1 - 100\,i$

at $i_1 = 2 \times 10^{-3}$ A

$V_{L_1} = 0.8V$

at $i_2 = 4 \times 10^{-3}$ A

$V_{L_2} = 0.6V$

$$\therefore \frac{V_{L_1}}{V_{L_2}} = \frac{4}{3}$$

44. The ratio of momentum of the photons of the 1st and 2nd line of Balmer series of Hydrogen atoms is α/β. The possible values of α and β are :-

(1) 27 and 20 (2) 3 and 16

(3) 5 and 36 (4) 20 and 27

Ans. (4)

Sol. $p = \frac{h}{\lambda}$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_1} = R\frac{5}{36}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\frac{1}{\lambda_2} = \frac{3R}{16}$$

$$\frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1} = \frac{\frac{16}{3R}}{\frac{36}{5R}} = \frac{20}{27}$$

$\alpha = 20$ & $\beta = 27$

45. A LCR series circuit driven with $E_{rms} = 90$ V at frequency $f_d = 30$ Hz has resistance R = 80 Ω, an inductance with inductive reactance $X_L = 20.0$ Ω and capacitance with capacitive reactance $X_C = 80.0$ Ω. The power factor of the circuit is ______.

(1) 0.8 (2) 0.64

(3) 0.9 (4) 0.5

Ans. (1)

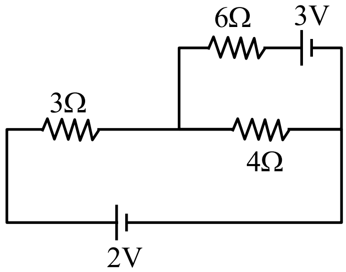
Sol. Power factor is $\cos\phi = \frac{R}{Z}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 100\Omega$$

$$\cos\phi = \frac{80}{100} = 0.8$$

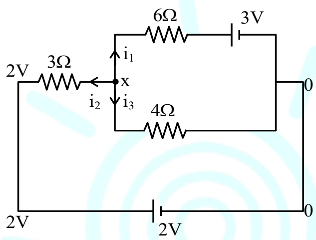
SECTION-B

46. Refer to the circuit diagram given below. The heat generated across the 6 Ω resistance in 100 second is $\frac{\alpha}{100}$ J. The value of α is _______. (Nearest integer)



Ans. **(3477)**

Sol.



$$i_1 + i_2 + i_3 = 0$$

$$\frac{x - 0 - 3}{6} + \frac{x - 2}{3} + \frac{x - 0}{4} = 0$$

$$x = \frac{14}{9}\text{V}$$

Current across 6Ω $= \frac{3 - (14/9)}{6} = \frac{13}{54}$A

$$H = \left(\frac{13}{54}\right)^2 \times 6 \times 100 = 34.77 = \frac{\alpha}{100}$$

$$\alpha = 3477$$

47. An unpolarized light of intensity I_0 passes through polarizer and then through a certain optically active solution and finally it goes to analyser. If the angle between analyser and polariser is 0° and intensity of light emerged from analyser is $\frac{3}{8}I_0$, the angle of rotation of the light by the solution with respect to analyser is ________ degrees.

Ans. **(30)**

Sol. $\underset{\text{(Unpolarised)}}{I_0} \rightarrow \underset{\text{(Polarised)}}{\frac{I_0}{2}} \xrightarrow{\text{Analyser}}$

$$I = I' \cos^2\theta$$

$$\frac{3I_0}{8} = \frac{I_0}{2}\cos^2\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30°$$

48. The energy released when $\frac{7}{17.13}$ kg of ${}_3^7\text{Li}$ is converted into ${}_2^4\text{He}$ by proton bombardment is $\alpha \times 10^{32}$ eV. The value of α is ______. (Nearest integer)

(Mass of ${}_3^7\text{Li}$ = 7.0183 u, mass of ${}_2^4\text{He}$ = 4.004 u, mass of proton = 1.008 u and 1 u = 931 MeV/c^2 and Avogadro number = 6.0×10^{23})

Ans. **(6)**

Sol. ${}_3^7\text{Li} + {}_1^1\text{P} \rightarrow 2\,{}_2^4\text{He}$

$\Delta m = (7.018 + 1.008) - 2(4.004) = 0.0183$ u

Energy released per reaction $E = \Delta mc^2$

Number of moles of Li $= \frac{7}{17.13} \times \frac{10^3}{7} = \frac{10^3}{17.13}$

Number of molecules $= \frac{10^3}{17.13} \times 6 \times 10^{23}$

Total energy released

$$E_{Total} = 0.0183 \times 931 \times \frac{6}{17.13} \times 10^{32}$$

$$\simeq 6 \times 10^{32}$$

$$\alpha = 6$$

49. A three coulomb charge moves from the point (0, –2, –5) to the point (5, 1, 2) in an electric field expressed as $\vec{E} = 2x\hat{i} + 3y^2\hat{j} + 4\hat{k}$ N/C. The work done in moving the charge is ________ J.

Ans. **(186)**

Sol. $\Delta U = -w_{\text{electrostatic}}$

$w_{\text{electrostatic}} = -q\Delta V$

$\Delta V = -\int 2x dx - \int 3y^2 dy - \int 4dz$

$= \left(-x^2 - y^3 - 4z\right)_{(0,-2,-5)}^{(5,1,2)}$

$= -62$

$w_{\text{electrostatic}} = 3(62) = 186$ J

50. A certain gas is isothermally compressed to $\left(\frac{1}{3}\right)^{\text{rd}}$ of its initial volume (V_0 = 3 litre) by applying required pressure. If the bulk modulus of the gas is 3×10^5 N/m^2, the magnitude of work done on the gas is _______ J.

Ans. **(989)**

Sol. For isothermal process initially $B = P_0$.

$W = nRT\ell n\left(\frac{V_2}{V_1}\right)$

$= P_0 V_0 \ell n\left(\frac{1}{3}\right)$

$W = BV_0 \ell n\left(\frac{1}{3}\right)$

$W = -988.75 \text{ J} \approx 989 \text{ J}$

Note : Bulk moldulus of gas for isothermal process is not constant. In the given solution we have considered it as initial bulk modulus.

(HELD ON MONDAY 06th APRIL 2026) **TIME : 9:00 AM TO 12:00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

51. An oxide of iron contains 69.9% iron, its empirical formula, is :

(Given : Molar mass of Fe and O are 56 and 16 g mol^{-1} respectively.)

(1) FeO (2) Fe_2O_3

(3) Fe_3O_4 (4) FeO_3

Ans. (2)

Sol.

	Fe	Oxygen
Moles :	$\frac{69.9}{56}$	$\frac{30.1}{16}$
Molar Ratio :	1 :	1.5
Molar Ratio :	2 :	3

Answer : $\mathbf{Fe_2O_3}$

52. If shortest wavelength of hydrogen atom in Lyman series is x, then longest wavelength in Balmer series of He^+ is :

(1) $\frac{9x}{5}$ (2) $\frac{36x}{5}$

(3) $\frac{x}{4}$ (4) $\frac{5x}{9}$

Ans. (1)

Sol. $$\frac{1}{x} = R \times 1^2 \times \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \text{ ....(1)}$$

$$\frac{1}{\lambda} = R \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2}\right) \text{ ....(2)}$$

$$\frac{x}{\lambda} = R \times 4 \times \frac{5}{4 \times 9}$$

$$\frac{1}{\lambda} = \frac{1}{x} \times \frac{5}{9}$$

$$\lambda = \frac{9x}{5}$$

53. Match the **List-I** with **List-II** :

List-I Orbital		List-II Radial nodes and nodal plane	
A.	2s	I.	1 Radial node + two nodal planes
B.	3s	II.	1 Radial node + one nodal plane
C.	3p	III.	2 Radial nodes + No nodal plane
D.	4d	IV.	1 Radial node + No nodal plane.

Choose the **correct** answer from the options given below :

(1) A-IV, B-I, C-III, D-II

(2) A-IV, B-II, C-III, D-I

(3) A-III, B-I, C-IV, D-II

(4) A-IV, B-III, C-II, D-I

Ans. (4)

Sol. No. of Radial nodes = $n - \ell - 1$

No. of Angular nodes or nodal planes = ℓ

$2s \Rightarrow n = 2, \ell = 0$

1 radial node + zero nodal plane

$3s \Rightarrow n = 3, \ell = 0$

2 radial node + zero nodal plane

$3p \Rightarrow n = 3, \ell = 1$

1 radial node + 1 nodal plane

$4d \Rightarrow n = 4, \ell = 2$

1 radial node + 2 nodal plane

54. The pairs among

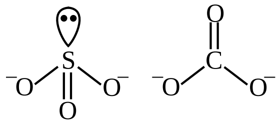
A = $[SO_3^{2-}, CO_3^{2-}]$, B = $[O_2^{2-}, F_2]$, C = $[CN^-, CO]$, D = $[NH_3, H_3O^+]$ and E = $[MnO_4^{2-}, CrO_4^{2-}]$ that do not have similar Lewis dot structure are :

(1) A, B and E (2) A and E

(3) B, C and D (4) C and D

Ans. (2)

Sol. A and E do not have similar Lewis structure



MnO_4^{2-} have are unpaired e but CrO_4^{2-} does not have unpaired e.

In B, C, D pair of species haved same no of e.

They have similar Lewis dot structure.

55. Arrange the following isothermal processes in order of the magnitude of the work (p – V) involved between states 1 and 2.

A. Expansion in single stage w_A

B. Expansion in multi stages w_B

C. Compression in single stage w_C

D. Compression in multi stages w_D

(1) $|w_B| > |w_A| > |w_C| > |w_D|$

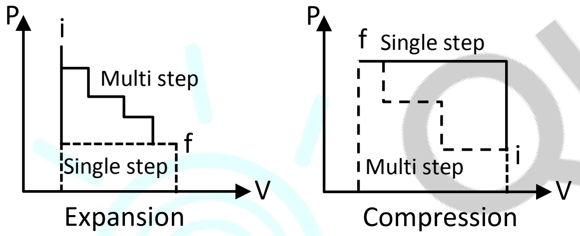
(2) $|w_C| > |w_D| > |w_A| > |w_B|$

(3) $|w_C| > |w_D| > |w_B| > |w_A|$

(4) $|w_B| > |w_A| > |w_D| > |w_C|$

Ans. (3)

Sol.



$\therefore |w_A| < |w_B| < |w_D| < |w_C|$

56. When 0.25 moles of a non-volatile, non-ionizable solute was dissolved in 1 mole of a solvent the vapor pressure of solution was x% of vapor pressure of pure solvent. What is x% ?

(1) 50% (2) 60%

(3) 70% (4) 80%

Ans. (4)

Sol. $P_S = P^0.X_A$

$$\frac{P_S}{P^0} = X_A = \frac{1}{1+0.25} = 0.80$$

% = 0.8 × 100 = 80

57. One mole each of He and A(g) are taken in a 10 L closed flask and heated to 400 K to establish the following equilibrium.

$A(g) \rightleftharpoons B(g)$

K_C for this reaction at 400 K is 4.0. The partial pressures (in atm) of He and B(g) are respectively (at equilibrium).

(Assume He, A(g) and B(g) behave as ideal gases)

(Given : R = 0.082 L atm K^{-1} mol^{-1})

(1) 3.28, 2.624 (2) 2.624, 3.28

(3) 3.28, 0.656 (4) 0.656, 6.56

Ans. (1)

Sol.

$$\begin{array}{lcc} & A \rightleftharpoons & B \\ t=0 & 1 \text{ mol} & - \\ t_{eq^m} & 1-x & x \end{array}$$

$$K_C = \frac{x/10}{\frac{1-x}{10}} = 4$$

x = 0.8

$$P_{He} = \frac{1\times 0.082\times 400}{10} = 3.28 \text{ atm}$$

$$P_B = \frac{0.8\times 0.082\times 400}{10} = 2.624 \text{ atm}$$

58. Consider the following data :

Electrolyte	Λ_m^o $(S\,cm^2\,mol^{-1})$
$BaCl_2$	x_1
H_2SO_4	x_2
HCl	x_3

$BaSO_4$ is sparingly soluble in water. If the conductivity of the saturated $BaSO_4$ solution is x S cm^{-1} then the solubility product of $BaSO_4$ can be given as.

(Here $\wedge_m = \wedge_m^o$)

(1) $\dfrac{10^6x^2}{\alpha^2(x_1+x_2-2x_3)^2}$ (2) $\dfrac{x^2}{(x_1+x_2-2x_3)^2}$

(3) $\dfrac{\alpha^2(x_1+x_2-2x_3)^2}{10^6x^2}$ (4) $\dfrac{x^2}{(x_1+x_2+2x_3)^2}$

Ans. (1)

Sol. $BaSO_4(s) \rightleftharpoons Ba^{+2}_{(aq)} + SO^{2-}_{4(aq)}$

s s

$K_{sp} = S^2$

k of $BaSO_4 = x\ S\ cm^{-1}$

Λ^o_m of $BaSO_4 = (x_1 + x_2 - 2x_3)$

$$\Lambda_m = \alpha \Lambda^o_M = \frac{k \times 1000}{S}$$

$$S = \frac{x \times 1000}{\alpha(x_1 + x_2 - 2x_3)}$$

$$k_{sp} = S^2 = \frac{1}{\alpha^2}\left[\frac{x \times 1000}{x_1 + x_2 - 2x_3}\right]^2$$

59. Given below are two statements :

Statement-I : Aluminium is more electropositive than thallium as the standard electrode potential value of $E^o_{Al^{3+}/Al}$ is negative and $E^o_{Tl^{3+}/Tl}$ is positive.

Statement-II : The sum of first three ionization enthalpies of boron is very high when compared to that of aluminium. Due to this reason boron forms covalent compounds only and aluminium forms Al^{3+} ion.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement-I and Statement-II are true.

(2) Both Statement-I and Statement-II are false.

(3) Statement-I is true but Statement-II is false.

(4) Statement-I is false but Statement-II is true.

Ans. **(1)**

Sol. $E^o_{Al^{3+}/Al} = -1.66\ V$

$E^o_{Tl^{3+}/Tl} = +1.26\ V$

For B : $IE_1 + IE_2 + IE_3 = 801 + 2427 + 3659 = 6887\ kJ$

Both statements are true

60. The **correct** statements among the following are :

A. Basic vanadium oxide is used in the manufacture of H_2SO_4.

B. The spin-only magnetic moment value of the transition metal halide employed in Ziegler-Natta polymerization is 2.84 BM.

C. The p-block metal compound employed in Ziegler-Natta polymerization has the metal in +3 oxidation state.

D. The number of electrons present in the outer most 'd' orbital of metal halide employed in Wacker process is 8.

Choose the correct answer from the options given below :

(1) A and B only (2) A, C and D only

(3) C and D only (4) B, C and D only

Ans. **(3)**

Sol. A. V_2O_5 is amphoteric. It is used in manufacture of H_2SO_4 .

B. Ziegler natta catalyst is $\underset{+4}{\underline{Ti}}Cl_4 + \underset{+3}{\underline{Al}}Me_3$

$Ti^{4+} : 3d^0\ 4s^0\ n = 0\ \mu = 0$

C. O.S. of Al is +3

D. $\underset{+2}{\underline{Pd}}Cl_2$ is used in wacker process

$Pd^{2+} : 4d^8 5s^0$

C and D are correct.

61. Match the **List-I** with **List-II**

List-I Electronic configuration of tetrahedral metal ion		**List-II Crystal Field Stabilization Energy (Δ_t)**	
A.	d^2	I.	–0.6
B.	d^4	II.	–0.8
C.	d^6	III.	–1.2
D.	d^8	IV.	–0.4

Choose the **correct** answer from the options given below :

(1) A-III, B-IV, C-II, D-I

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-IV, D-III

Ans. **(3)**

Sol. In tetrahedral complex

$$CFSE=\left[-\frac{3}{5}n_1+\frac{2}{5}n_2\right]\Delta_t$$

$d^2 : \underbrace{e^{1,1}}_{n_1=2}\ \underbrace{t_2^{0,0,0}}_{n_2=0}$; $C.F.S.E=\left[\frac{-3}{5}(2)\right]\Delta t=-1.2\Delta t$

$d^4 : \underbrace{e^{1,1}}_{n_1=2}\ \underbrace{t_2^{1,1,0}}_{n_2=2}$; $C.F.S.E=\left[\frac{-3}{5}(2)+\frac{2}{5}(2)\right]\Delta t=-0.4$

Δt

$d^6 : \underbrace{e^{2,1}}_{n_1=3}\ \underbrace{t_2^{1,1,1}}_{n_2=3}$; $C.F.S.E=\left[\frac{-3}{5}(3)+\frac{2}{5}(3)\right]\Delta t=-0.6\ \Delta t$

$d^8 : \underbrace{e^{2,2}}_{n_1=4}\ \underbrace{t_2^{2,1,1}}_{n_2=4}$; $C.F.S.E=\left[\frac{-3}{5}(4)+\frac{2}{5}(4)\right]\Delta t=-0.8\ \Delta t$

A - III,B - IV, C - I, D – II

62. Which of the following are true about the energy of the given d-orbitals of a tetrahedral complex ?

A. $d_{xy}=d_{xz}>d_{x^2-y^2}$

B. $d_{xy}=d_{yz}>d_{z^2}$

C. $d_{x^2-y^2}>d_{z^2}>d_{xz}$

D. $d_{x^2-y^2}=d_{z^2}<d_{xz}$

Choose the **correct** answer from the options given below :

(1) A, B and D only (2) A and B only

(3) B and D only (4) B, C and D only

Ans. **(1)**

Sol. Order of energy in tetrahedral splitting

$(dxy = dyz = dzx) < d_{x^2-y^2}=d_{z^2}$

A, B and D are correct.

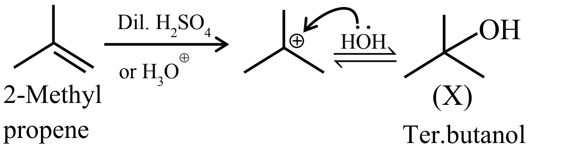
63. R_f value for 2-methylpropene in a solvent system (Ethyl acetate + ether) is 0.42. 2-methylpropene is treated with dilute H_2SO_4 to give major organic product (X). R_f value for (X) in the same solvent system under identical condition will be :

(1) 0.42 (2) 0.82

(3) 0.62 (4) 0.12

Ans. **(4)**

Sol.



Ter butanol (X) is more polar than 2-Methyl propene and $R_f\propto\frac{1}{\text{polarity}}$ so R_f of ter.butanol is less than 2-Methylpropen

So. Ans is 0.12

64. Given below are two statements :

Statement-I : 2, 6-diethylcyclohexanone and 6-methyl-2-n-propylcyclohexanone are metamers.

Statement-II : 2, 2, 6, 6-tetramethylcyclohexanone exhibits keto-enol tautomerism.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement-I and Statement-II are true.

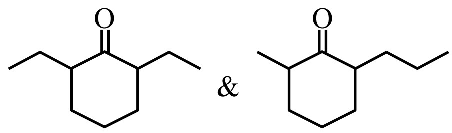
(2) Both Statement-I and Statement-II are false.

(3) Statement-I is true but Statement-II is false.

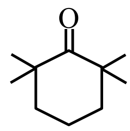
(4) Statement-I is false but Statement-II is true.

Ans. **(3)**

Sol. Statement-I is correct and Statement-II is incorrect



are metamers



No α-H

No Keto enol tautomerism

65. Given below are two statements :

Statement-I : Methane can be prepared by decarboxylation of sodium ethanoate, Kolbe's electrolysis of sodium acetate and reaction of CH_3MgBr with water.

Statement-II : Methane cannot be prepared from unsaturated hydrocarbons and by Wurtz reaction.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement-I and Statement-II are true.

(2) Both Statement-I and Statement-II are false.

(3) Statement-I is true but Statement-II is false.

(4) Statement-I is false but Statement-II is true.

Ans. (4)

Sol. Methane can not be prepared by Kolbe's electrolysis process. So, statement-1 is false

$$2CH_3\text{–}\overset{\overset{\Large O}{\|}}{C}\text{–ONa} \xrightarrow{\text{Kolbe electrolysis}} \underset{\text{Ethane}}{CH_3\text{–}CH_3} + 2CO_2 + 2NaOH + H_2$$

Methane also can not be prepared by Wurtz reaction so statement-2 is true

$$2R-X \xrightarrow[\text{Wurtz reaction}]{\text{Na.oxyether}} R\text{–}R + 2NaX$$

If $R = \text{–}CH_3$ then ethane is formed not methane.

66. Given below are two statements :

Statement-I : 3-phenylpropene reacts with HBr and gives secondary alkyl bromide having a chiral carbon atom as the major product.

Statement-II : Aryl chlorides and aryl cyanides can be prepared by Sandmeyer reaction as well as Gattermann reaction.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement-I and Statement-II are true.

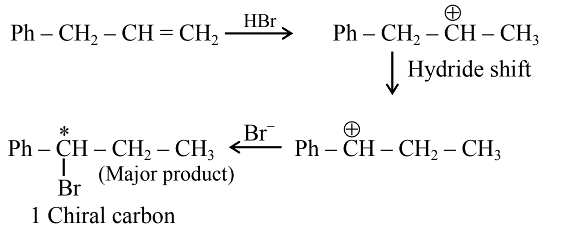
(2) Both Statement-I and Statement-II are false.

(3) Statement-I is true but Statement-II is false.

(4) Statement-I is false but Statement-II is true.

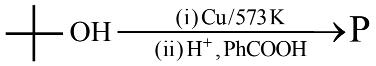
Ans. (3)

Sol.

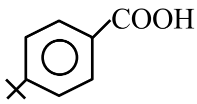


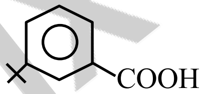
Aryl cyanide is not formed by Gattermann reaction.

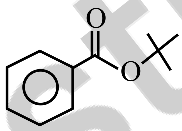
67. Consider the following sequence of reactions

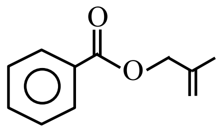


The major product P is :

(1) 

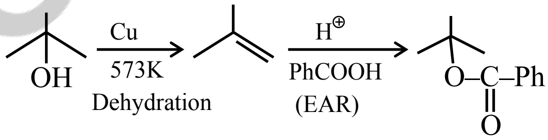
(2) 

(3) 

(4) 

Ans. (3)

Sol.



68. Arrange the following compounds according to increasing order of boiling points.

n-C_4H_9OH (A), n-$C_4H_9NH_2$ (B), n-C_4H_{10} (C) and $C_2H_5NHC_2H_5$(D).

(1) C < B < A < D

(2) D < C < B < A

(3) C < D < B < A

(4) D < B < A < C

Ans. (3)

Sol. Boiling point $\propto$ Intermolecular attraction

$\propto$ Number of H-bonding

n-butanol has strong H-bonding.

69. Match the **List-I** with **List-II** :

List-I Deficiency Disease		List-II Vitamin	
A.	Scurvy	I.	Pyridoxine
B.	Convulsions	II.	Vitamin A
C.	Cheilosis	III.	Ascorbic Acid
D.	Xerophthalmia	IV.	Riboflavin

Choose the **correct** answer from the options given below :

(1) A-I, B-III, C-II, D-IV

(2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-III, B-I, C-II, D-IV

Ans. (3)

Sol. Vitamin–A → Xerophthalmia

Vitamin-C (Ascorbic acid) → Scurvy

Vitamin-B_2 (Riboflavin) → Cheilosis

Vitamin-B_6 (Pyridoxine) → Convulsions

70. Match the **List-I** with **List-II**

List-I Amino acid		List-II Positive reaction/Test for functional group present in side chain of amino acid	
A.	Glutamine	I.	Hinsberg's test
B.	Lysine	II.	Neutral $FeCl_3$ test
C.	Tyrosine	III.	Ceric ammonium nitrate test
D.	Serine	IV.	Hoffman bromamide degradation

Choose the **correct** answer from the options given below :

(1) A-IV, B-II, C-I, D-III

(2) A-IV, B-I, C-II, D-III

(3) A-III, B-II, C-I, D-IV

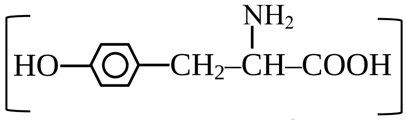
(4) A-IV, B-I, C-III, D-II

Ans. (2)

Sol. Glutamine $\left[H_2N-\overset{\overset{\displaystyle O}{\|}}{C}-CH_2-CH_2-\overset{\overset{\displaystyle NH_2}{|}}{CH}-COOH\right]$

has amide functional group inside chain. So it gives Hoffmann bromamide reaction.

Lysine $H_2N-\left(CH_2\right)_4-\overset{\overset{\displaystyle NH_2}{|}}{CH}-COOH$ has 1° amine group. So it gives +ve Hinsberg test.

Tyrosine  has phenolic OH. So it gives +ve test with neutral $FeCl_3$.

Serine $HO-CH_2-\overset{\overset{\displaystyle NH_2}{|}}{CH}-COOH$ has alcohlic –OH group. So it gives +ve test with cerric ammonium nitrate.

SECTION-B

71. First and second ionization enthalpies of lithium are 520 kJ mol^{-1} and 7297 kJ mol^{-1} respectively. Energy required to convert 3.5 mg lithium (g) into Li^{2+}(g) [Li(g) → Li^{2+}(g)] is _____ kJ mol^{-1}. (nearest integer)

[Molar mass of Li = 7 g mol^{-1}]

Ans. (4)

Sol. Number of moles of Li = $\frac{3.5\times10^{-3}}{7}=5\times10^{-4}$

Total energy needed to convert

Li(g) → Li^{2+}(g) is

5×10^{-4} (520 + 7297) = 3.9085 kJ = 4 kJ

72. Consider the following sequence of reactions.

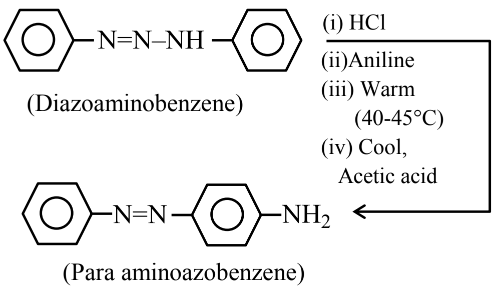
The percentage of nitrogen in the yellow product (X) formed is _______ %.

(Nearest Integer)

(Given Molar mass in g mol^{-1} H : 1, C : 12, N : 14)

Ans. (21)

Sol.



- HCl catalysed hydrolysis.
- Aniline acts as a solvent and reactant.
- The mixture is gently heated to facilitate rearrangement.
- After the reaction is complete, the mixture is cooled and treated with acetic acid to neutralise excess acid and precipitate the final product.

Mass percentage of nitrogen in final product

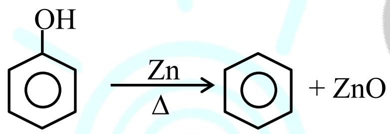
$$= \frac{42}{197} \times 100 = 21.30\ \% \approx 21$$

73. 4.7 g of phenol is heated with Zn to give product X. If this reaction goes to 60% completion then the number of moles of compound X formed will be _____ $\times 10^{-2}$ (Nearest Integer)

(Given molar mass in g mol^{-1} : H : 1, C : 12, O : 16)

Ans. **(3)**

Sol.



Moles of phenol $= \frac{4.7}{94.0} = 0.05\,\text{mol}$

Reaction is 1 : 1 so 0.05 mole of phenol will give 0.05 mole of benzene on 100% yield

For 60% yield = 0.05 × 0.60

= 0.03 mol.

mol of 'X' = 3×10^{-2} mol

So answer is (3)

74. Sucrose hydrolyses in acidic medium into glucose and fructose by first order rate law with $t_{1/2} = 3$ hour. The percentage of sucrose remaining after 6 hours is _____ . (Nearest integer)

(Given : log 2 = 0.3010 and log 3 = 0.4771)

Ans. **(25)**

Sol. Sucrose + $H_2O \rightarrow$ glucose + fructose

$t_{1/2} = 3$ hr

For 1^{st} order $\qquad t_{75\%} = 2 \times t_{1/2}$

So t = 6 hr, $\qquad$ 75% of sucrose decomposed

So 25% of sucrose remains after 6 hr.

75. Consider the reaction $X \rightleftharpoons Y$ at 300 K. If ΔH^{θ} and K are 28.40 kJ mol^{-1} and 1.8×10^{-7} at the same temperature, then the magnitude of ΔS^{θ} for the reaction in J K^{-1} mol^{-1} is ______.(Nearest Integer). (Given : R = 8.3 JK^{-1} mol^{-1}, ln10 = 2.3, log 3 = 0.48, log 2 = 0.30

Ans. **(34)**

Sol. $X \rightleftharpoons Y \quad K_{eq} = 1.8 \times 10^{-7}$

$$\ln(K_{eq}) = \frac{\Delta_r S^o}{R} - \frac{\Delta_r H^o}{R}\left(\frac{1}{T}\right)$$

$$2.3 \log (1.8 \times 10^{-7}) = \frac{\Delta_r S^o}{R} - \frac{28400}{R} \times \frac{1}{300}$$

$$\Delta_r S^o = 2.3 \times R \log (1.8 \times 10^{-7}) + \frac{284}{3}$$

$$= 19.09\left[-8 + 2 \times 0.48 + 0.3\right] + \frac{284}{3}$$

$$= -128.66 + \frac{284}{3}$$

$$\Delta_r S^o = -33.99 \simeq -34\ J/K.mol$$