(Held On Thursday 22nd July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

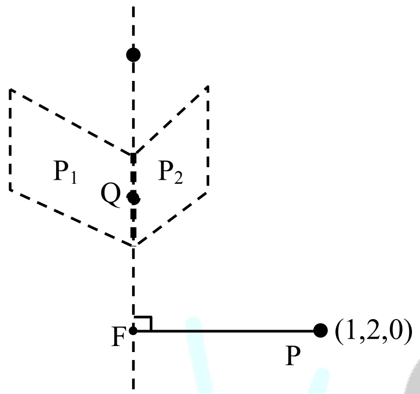
1. Let L be the line of intersection of planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})=2$. If $P(\alpha,\beta,\gamma)$ is the foot of perpendicular on L from the point (1,2,0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :

(1) 101 (2) 119 (3) 143 (4) 134

Official Ans. by NTA (2)

Sol. $P_1 : x - y + 2z = 2$

$P_2 = 2x + y - 3 = 2$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q.

$\Rightarrow$ z–coordinate of point Q is zero

$$\Rightarrow \left.\begin{matrix} x-y=2 \\ \text{and } 2x+y=2 \end{matrix}\right\} \Rightarrow x=\frac{4}{3}, y=\frac{-2}{3}$$

$$\Rightarrow Q\left(\frac{4}{3},\frac{-2}{3},0\right)$$

Vector parallel to the line of intersection

$$\vec{a}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}=-\hat{i}+5\hat{j}+3\hat{k}$$

Equation of Line of intersection

$$\frac{x-\frac{4}{3}}{-1}=\frac{y+\frac{2}{3}}{5}=\frac{z-0}{3}=\lambda(\text{say})$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda+\frac{4}{3},5\lambda-\frac{2}{3},3\lambda\right)$$

$$\overrightarrow{PF}=\left(-\lambda+\frac{1}{3}\right)\hat{i}+\left(5\lambda-\frac{8}{3}\right)\hat{j}+(3\lambda)\hat{k}$$

$$\overrightarrow{PF}\cdot\vec{a}=0$$

$$\Rightarrow \lambda-\frac{1}{3}+25\lambda\frac{-40}{3}+9\lambda=0$$

$$\Rightarrow 35\lambda=\frac{41}{3} \Rightarrow \boxed{\lambda=\frac{41}{105}}$$

$$\text{Now, } \alpha=-\lambda+\frac{4}{3}, \beta=5\lambda-\frac{2}{3}, \gamma=3\lambda$$

$$\Rightarrow \alpha+\beta+\gamma = 7\lambda+\frac{2}{3}$$

$$=7\left(\frac{41}{105}\right)+\frac{2}{3}$$

$$=\frac{51}{15}$$

$$\Rightarrow 35(\alpha+\beta+\gamma)=\frac{51}{15}\times35=119$$

2. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to :

(1) 1862 (2) 1842 (3) 1852 (4) 1872

Official Ans. by NTA (1)

Sol. $S_{10}=530 \Rightarrow \frac{10}{2}\{2a+9d\}=530$

$\Rightarrow 2a + 9d = 106$ ….(1)

and $S_5 = 140 \Rightarrow \frac{5}{2}\{2a+4d\}=140$

$\Rightarrow 2a + 4d = 56$ …..(2)

$\Rightarrow 5d = 50 \Rightarrow \boxed{d=10} \Rightarrow \boxed{a=8}$

Now, $S_{20}-S_6=\frac{20}{2}\{2a+19d\}-\frac{6}{2}\{2a+5d\}$

$= 14a + 175d$

$= (14 \times 8) + (175 \times 10)$

$= 1862$

3. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

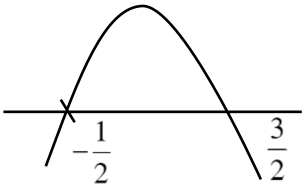
$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x &, \ x > 0 \\ 3xe^x &, \ x \le 0 \end{cases}.$$ Then f is increasing function in the interval

(1) $\left(-\frac{1}{2}, 2\right)$ (2) $(0,2)$

(3) $\left(-1, \frac{3}{2}\right)$ (4) $(-3, -1)$

Official Ans. by NTA (3)

Sol. $f'(x)\begin{cases} -4x^2 + 4x + 3 & x > 0 \\ 3e^x(1+x) & x \le 0 \end{cases}$



For $x > 0$, $f'(x) = -4x^2 + 4x + 3$

f(x) is increasing in $\left(-\frac{1}{2}, \frac{3}{2}\right)$

For $x \le 0$, $f'(x) = 3e^x(1 + x)$

$f'(x) > 0 \ \forall \ x \in (-1, 0)$

$\Rightarrow$ f(x) is increasing in $(-1, 0)$

So, in complete domain, f(x) is increasing in $\left(-1, \frac{3}{2}\right)$

4. Let $y = y(x)$ be the solution of the differential equation $\text{cosec}^2 x dy + 2dx = (1 + y \cos 2x) \text{cosec}^2 x dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to :

(1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$

$\Rightarrow \frac{dy}{dx} + (-\cos 2x)y = \cos 2x$

$\text{I.F.} = e^{\int -\cos 2x \, dx} = e^{-\frac{\sin 2x}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int (\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx + c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$

Given

$y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 0 = -e^{-1/2} + c \Rightarrow \boxed{c = e^{-1/2}}$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$

at x = 0

$y = -1 + e^{-1/2}$

$\Rightarrow y(0) = -1 + e^{-1/2} \Rightarrow (y(0) + 1)^2 = e^{-1}$

5. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(1) $\frac{45}{162}$ (2) $\frac{23}{81}$ (3) $\frac{22}{81}$ (4) $\frac{43}{162}$

Official Ans. by NTA (4)

Sol. $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $|A| = ad - bc$

Total case $= 6^4$

For non-singular matrix $|A| \ne 0 \Rightarrow ad - bc \ne 0$

$\Rightarrow ad \ne bc$

And a, b, c, d are all different numbers in the set $\{1, 2, 3, 4, 5, 6\}$

Now for $ad = bc$

(i) $6 \times 1 = 2 \times 3$

$\left.\begin{array}{l} \Rightarrow a = 6, b = 2, c = 3, d = 1 \\ \text{or } a = 1, b = 2, c = 3, d = 6 \\ \quad \vdots \end{array}\right\}$ 8 such cases

(ii) $6 \times 2 = 3 \times 4$

$\left.\begin{array}{l} \Rightarrow a = 6, b = 3, c = 4, d = 2 \\ \text{or } a = 2, b = 3, c = 4, d = 6 \\ \quad \vdots \end{array}\right\}$ 8 such cases

favourable cases

$= {}^6C_4 \lfloor 4 - 16$

required probability

$= \frac{{}^6C_4 \lfloor 4 - 16}{6^4} = \frac{43}{162}$

6. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]+\left[\vec{a}\ \vec{b}\ \vec{d}\right]+\left[\vec{a}\ \vec{c}\ \vec{d}\right]$ is equal to :

(1) – 42 (2) – 40 (3) – 29 (4) – 38

Official Ans. by NTA (1)

Sol. $\vec{a}=\lambda\vec{b}+\mu\vec{c}=\hat{i}(2\lambda+\mu)+\hat{j}(\lambda-\mu)+\hat{k}(\lambda+\mu)$

$\vec{a}\cdot\vec{d}=0=3(2\lambda+\mu)+2(\lambda-\mu)+6(\lambda+\mu)$

$\Rightarrow 14\lambda+7\mu=0\Rightarrow\mu=-2\lambda$

$\Rightarrow \vec{a}=(0)\hat{i}-3\lambda\hat{j}+(-\lambda)\hat{k}$

$\Rightarrow |\vec{a}|=\sqrt{10}|\lambda|=\sqrt{10}\Rightarrow|\lambda|=1$

$\Rightarrow \lambda=1$ or -1

$\left[\vec{a}\vec{b}\vec{c}\right]=0$

$\left[\vec{a}\vec{b}\vec{c}\right]+\left[\vec{a}\vec{b}\vec{d}\right]+\left[\vec{a}\vec{c}\vec{d}\right]=\left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{d}\right]$

$$=\begin{vmatrix}0 & -3\lambda & \lambda\\ 3 & 0 & 2\\ 3 & 2 & 6\end{vmatrix}$$

$=3\lambda(12)+\lambda(6)=42\lambda=-42$

7. If $\displaystyle\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2},\alpha\in\mathbf{R}$ where $[x]$ is the greatest integer less than or equal to x, then the value of α is :

(1) $200\ (1-e^{-1})$ (2) $100\ (1-e)$

(3) $50\ (e-1)$ (4) $150\ (e^{-1}-1)$

Official Ans. by NTA (1)

Sol. $I=\displaystyle\int_0^{100\pi}\frac{\sin^2 x}{e^{\{x/\pi\}}}dx=100\int_0^{\pi}\frac{\sin^2 x}{e^{x/\pi}}dx$

$100\displaystyle\int_0^{\pi}e^{-x/\pi}\frac{(1-\cos 2x)}{2}dx$

$=50\left\{\displaystyle\int_0^{\pi}e^{-x/\pi}dx-\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx\right\}$

$I_1=\displaystyle\int_0^{\pi}e^{-x/\pi}dx=\left[-\pi e^{-x/\pi}\right]_0^{\pi}=\pi(1-e^{-1})$

$I_2=\displaystyle\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx$

$=-\pi e^{-x/\pi}\cos 2x\Big]_0^{\pi}-\displaystyle\int-\pi e^{-x/\pi}(-2\sin 2x)dx$

$=\pi(1-e^{-1})-2\pi\displaystyle\int_0^{\pi}e^{-x/\pi}\sin 2x\,dx$

$=\pi(1-e^{-1})-2\pi\left\{-\pi e^{-x/\pi}\sin 2x\Big]_0^{\pi}-\displaystyle\int_0^{\pi}-\pi e^{-x/\pi}2\cos 2x\,dx\right\}$

$=\pi(1-e^{-1})-4\pi^2 I_2$

$\Rightarrow I_2=\dfrac{\pi(1-e^{-1})}{1+4\pi^2}$

$\therefore I=50\left\{\pi(1-e^{-1})-\dfrac{\pi(1-e^{-1})}{1+4\pi^2}\right\}$

$=\dfrac{200(1-e^{-1})\pi^3}{1+4\pi^2}$

8. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c},\vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ?

(1) $\vec{a}\times\left((\vec{b}+\vec{c})\times(\vec{b}-\vec{c})\right)=\vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(3) $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+\left[\vec{c}\ \ \vec{a}\ \ \vec{b}\right]=8$

(4) $\left|3\vec{a}+\vec{b}-2\vec{c}\right|^2=51$

Official Ans. by NTA (4)

Sol. (1) $\vec{a}\times\left((\vec{b}+\vec{c})\times(\vec{b}-\vec{c})\right)$

$=\vec{a}\left(-\vec{b}\times\vec{c}+\vec{c}\times\vec{b}\right)=-2\left(\vec{a}\times(\vec{b}\times\vec{c})\right)$

$=-2(\vec{a}\times\vec{a})=\vec{0}$

(2) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}$

$=\dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}=\dfrac{\vec{a}\cdot\vec{a}}{|\vec{a}|}=|\vec{a}|=2$

(3) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]+\left[\vec{c}\ \vec{a}\ \vec{b}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]=2\vec{a}\cdot(\vec{b}\times\vec{c})$

$=2\vec{a}.\vec{a}=2|\vec{a}|^2=8$

(4) $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are mutually $\perp$ vectors.

$\therefore\ |\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\Rightarrow|\vec{b}|=|\vec{c}|/2$

Also, $|\vec{b}\times\vec{c}|=|\vec{a}|\Rightarrow|\vec{b}||\vec{c}|=2\Rightarrow|\vec{c}|=2\ \&\ |\vec{b}|=1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2=(3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$=9|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2$

$=(9\times4)+1+(4\times4)$

$=36+1+16=53$

9. The values of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solution, are :

(1) $\lambda = 3, \mu = 5$ (2) $\lambda = 3, \mu \neq 10$

(3) $\lambda \neq 2, \mu = 10$ (4) $\lambda = 2, \mu \neq 10$

Official Ans. by NTA (4)

Sol. $x + y + z = 6$ …(i)

$3x + 5y + 5z = 26$ …(ii)

$x + 2y + \lambda z = \mu$ …(iii)

$5 \times (i) - (ii) \Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ from (i) and (iii)

$y + z = 4$ …(iv)

$2y + \lambda z = \mu - 2$ …(v)

(v) – 2 × (iv)

$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \dfrac{\mu - 10}{\lambda - 2}$ & $y = 4 - \dfrac{\mu - 10}{\lambda - 2}$

$\therefore$ For no solution $\lambda = 2$ and $\mu \neq 10$.

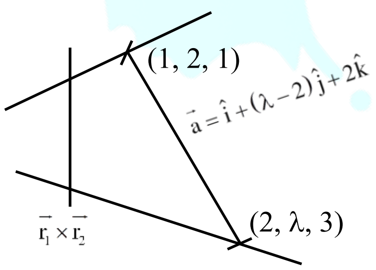
10. If the shortest distance between the straight lines $3(x - 1) = 6(y - 2) = 2(z - 1)$ and $4(x - 2) = 2(y - \lambda) = (z - 3)$, $\lambda \in \mathbf{R}$ is $\dfrac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

(1) 3 (2) 2 (3) 5 (4) –1

Official Ans. by NTA (1)

Sol. $L_1: \dfrac{(x-1)}{2} = \dfrac{(y-2)}{1} = \dfrac{(z-1)}{3}$ $\vec{r}_1 = 2\hat{i} + \hat{j} + 3k$

$L_2: \dfrac{(x-2)}{1} = \dfrac{y-\lambda}{2} = \dfrac{z-3}{4}$ $\vec{r}_2 = \hat{i} + 2\hat{j} + 4\hat{k}$



Shortest distance = Projection of $\vec{a}$ on $\vec{r}_1 \times \vec{r}_2$

$$= \frac{\left|\vec{a}.\left(\vec{r}_1 \times \vec{r}_2\right)\right|}{\left|\vec{r}_1 \times \vec{r}_2\right|}$$

$$\left|\vec{a}\cdot\left(\vec{r}_1 \times \vec{r}_2\right)\right| = \begin{vmatrix} 1 & \lambda - 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$$

$$\left|\vec{r}_1 \times \vec{r}_2\right| = \sqrt{38}$$

$$\therefore \frac{1}{\sqrt{38}} = \frac{|14 - 5\lambda|}{\sqrt{38}}$$

$\Rightarrow |14 - 5\lambda| = 1$

$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \dfrac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$.

11. Which of the following Boolean expressions is **not** a tautology ?

(1) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(3) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(4) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Official Ans. by NTA (4)

Sol. (1) $(p \rightarrow q) \vee (\sim q \rightarrow p)$

$= (\sim p \vee q) \vee (q \vee p)$

$= (\sim p \vee p) \vee q$

$= t \vee q = t$

(2) $(q \rightarrow p) \vee (\sim q \rightarrow p)$

$= (\sim q \vee p) \vee (q \vee p)$

$= (\sim q \vee q) \vee p$

$= t \vee p = t$

(3) $(p \rightarrow \sim q) \vee (\sim q \rightarrow p)$

$= (\sim p \vee \sim q) \vee (q \vee p)$

$= (\sim p \vee p) \vee (\sim q \vee q)$

$= t \vee t = t$

(4) $(\sim q \rightarrow q) \vee (\sim q \rightarrow p)$

$= (p \vee q) \vee (q \vee p)$

$= (p \vee p) \vee (q \vee p)$

$= p \vee q$

Which is not a tautology.

12. Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to :

(1) 2 (2) 1 (3) 3 (4) 9

Official Ans. by NTA (3)

Sol. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Let $x = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$$AX = \begin{bmatrix}a_{11}+a_{12}+a_{13}\\a_{21}+a_{22}+a_{23}\\a_{31}+a_{32}+a_{33}\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\Rightarrow AX = X$

Replace X by AX

$A^2X = AX = X$

Replace X by AX

$A^3X = AX = X$

Let $A^3 = \begin{bmatrix}x_1 & x_2 & x_3\\y_1 & y_2 & y_3\\z_1 & z_2 & z_3\end{bmatrix}$

$$A^3\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

Sum of all the element = 3

13. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(1) $\left[0, \frac{1}{e}\right)$ (2) $[\log_e 2, \log_e 3)$

(3) $[1, e)$ (4) $[0, \log_e 2)$

Official Ans. by NTA (4)

Sol. $\left[e^x\right]^2 + \left[e^x + 1\right] - 3 = 0$

$\Rightarrow \left[e^x\right]^2 + \left[e^x\right] + 1 - 3 = 0$

Let $\left[e^x\right] = t$

$\Rightarrow t^2 + t - 2 = 0$

$\Rightarrow t = -2, 1$

$\left[e^x\right] = -2$ (Not possible)

or $\left[e^x\right] = 1 \quad \therefore\ 1 \le e^x < 2$

$\Rightarrow \quad \ln(1) \le x < \ln(2)$

$\Rightarrow \quad 0 \le x < \ln(2)$

$\Rightarrow \quad x \in [0, \ln 2)$

14. Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$

(3) $81 < C < 156$ (4) $100 < C < 156$

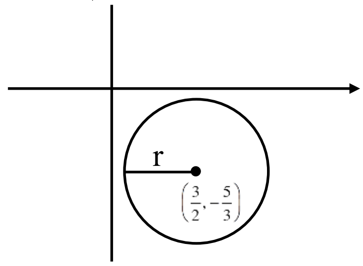
Official Ans. by NTA (4)

Sol. $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$

Centre $\equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$

radius $= r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$



Now,

$\Rightarrow r < \frac{3}{2}$

$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow C > 100$ .....(1)

Now point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$

$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$

$\boxed{C < 156}$ .....(2)

From (1) & (2)

$\boxed{100 < C < 156}$ Ans.

15. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) 2

Official Ans. by NTA (2)

Sol. $z^2 + 3\bar{z} = 0$

Put $z = x + iy$

$\Rightarrow x^2 - y^2 + 2ixy + 3(x - iy) = 0$

$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0 + i0$

$\therefore\ x^2 - y^2 + 3x = 0$ .....(1)

$2xy - 3y = 0$ .....(2)

$x = \frac{3}{2}, y = 0$

Put $x = \frac{3}{2}$ in equation (1)

$$\frac{9}{4}-y^2+\frac{9}{2}=0$$

$$y^2=\frac{27}{4}\Rightarrow y=\pm\frac{3\sqrt{3}}{2}$$

$$\therefore\ (x,y)=\left(\frac{3}{2},\frac{3\sqrt{3}}{2}\right),\left(\frac{3}{2},\frac{-3\sqrt{3}}{2}\right)$$

Put y = 0 $\Rightarrow x^2 - 0 + 3x = 0$

x= 0, –3

$\therefore$ (x, y) = (0, 0), (–3, 0)

$\therefore$ No of solutions = n = 4

$$\sum_{K=0}^{\infty}\left(\frac{1}{n^k}\right)=\sum_{K=0}^{\infty}\left(\frac{1}{4^k}\right)$$

$$=\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\ldots$$

$$=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$$

16. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to

(1) 11 (2) 7 (3) 5 (4) 9

Official Ans. by NTA (3)

Sol. $\sin^7 x \le \sin^2 x \le 1$ ...(1)

and $\cos^7 x \le \cos^2 x \le 1$ ...(2)

also $\sin^2 x + \cos^2 x = 1$

$\Rightarrow$ equality must hold for (1) & (2)

$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 = \cos^2 x$

$\Rightarrow$ sinx = 0 & cosx = 1

or

cosx = 0 & sinx = 1

$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$ 5 solutions

17. If the domain of the function $f(x)=\dfrac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

Official Ans. by NTA (1)

Sol. $O \le x^2 - x + 1 \le 1$

$\Rightarrow x^2 - x \le 0$

$\Rightarrow x \in [0, 1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right)\le\frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\le 1$

$\Rightarrow 0 < 2x - 1 \le 2$

$1 < 2x \le 3$

$\frac{1}{2}<x\le\frac{3}{2}$

Taking intersection

$x\in\left(\frac{1}{2},1\right]$

$\Rightarrow \alpha=\frac{1}{2},\ \beta=1$

$\Rightarrow \alpha+\beta=\frac{3}{2}$

18. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , x\ne 0\\ \alpha & , x=0\end{cases}.$$

If f is continuous at x = 0, then α is equal to :

(1) 1 (2) 3 (3) 0 (4) 2

Official Ans. by NTA (1)

Sol. For continuity

$$\lim_{x\to 0}\frac{x^3}{4\sin^4 x}\left(\ell n\left(1+2xe^{-2x}\right)-2\ell n\left(1-xe^{-x}\right)\right)$$

$= \alpha$

$$\lim_{x\to 0}\frac{1}{4x}\left[2xe^{-2x}+2xe^{-x}\right]=\alpha$$

$$=\frac{1}{4}(4)=\alpha=1$$

19. Let a line L : 2x + y = k, k > 0 be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

(1) 12 (2) –12 (3) 24 (4) –24

Official Ans. by NTA (4)

Sol. Tangent to hyperbola of

Slope m = –2 (given)

$y=-2x\pm\sqrt{3(3)}$

$\left(y=mx\pm\sqrt{a^2m^2-b^2}\right)$

$\Rightarrow y + 2x = \pm 3 \Rightarrow 2x + y = 3\quad (k > 0)$

For parabola $y^2 = \alpha x$

$y = mx + \dfrac{\alpha}{4m}$

$\Rightarrow y = -2x + \dfrac{\alpha}{-8}$

$\Rightarrow \dfrac{\alpha}{-8} = 3$

$\Rightarrow \alpha = -24.$

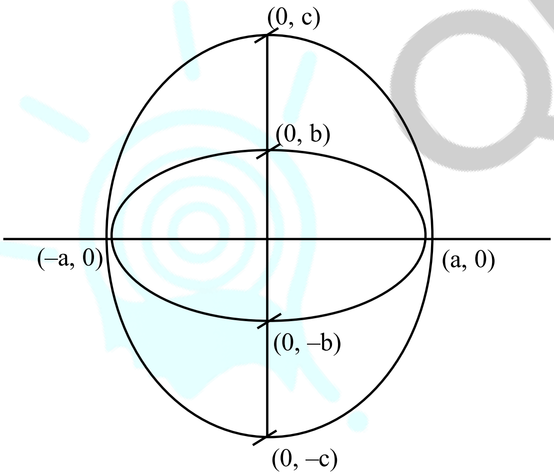
20. Let $E_1 : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

(1) $\dfrac{-1+\sqrt{5}}{2}$ (2) $\dfrac{-1+\sqrt{8}}{2}$

(3) $\dfrac{-1+\sqrt{3}}{2}$ (4) $\dfrac{-1+\sqrt{6}}{2}$

Official Ans. by NTA (1)

Sol.



$e^2 = 1 - \dfrac{b^2}{a^2}$

$e^2 = 1 - \dfrac{a^2}{c^2}$

$\Rightarrow \dfrac{b^2}{a^2} = \dfrac{a^2}{c^2}$

$\Rightarrow \mathbf{c^2} = \dfrac{a^4}{b^2} \Rightarrow c = \dfrac{a^2}{b}$

Also b = ce

$\Rightarrow c = \dfrac{b}{e}$

$\dfrac{b}{e} = \dfrac{a^2}{b}$

$\Rightarrow e = \dfrac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \dfrac{-1+\sqrt{5}}{2}$

SECTION-B

21. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Official Ans. by NTA (720)

Sol. $f(1)+f(2)=3-f(3)$

$\Rightarrow f(1)+f(2)=3+f(3)=3$

The only possibility is : 0 + 1 + 2 = 3

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

So number of bijective functions.

$= \underline{|3} \times \underline{|5} = 720$

22. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______.

Official Ans. by NTA (96)

Sol.

2,4,6,8				
4	4	3	2	1

$= 4 \times 4 \times 3 \times 2 = 96$

23. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _______.

Official Ans. by NTA (3125)

Sol. Let matrix $B=\begin{bmatrix} a & b & c \\ d & e & f \\ g & n & i \end{bmatrix}$

$\because$ AB = BA

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}=\begin{bmatrix} b & a & c \\ e & d & f \\ h & g & i \end{bmatrix}$$

$\Rightarrow$ d = b, e = a, f = c, g=h

$\therefore$ Matrix $B=\begin{bmatrix} a & b & c \\ b & a & c \\ g & g & i \end{bmatrix}$

No. of ways of selecting a, b, c, g, i

$= 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5 = 3125$

$\therefore$ No. of Matrices B = 3125

24. Consider the following frequency distribution :

Class :	$0-6$	$6-12$	$12-18$	$18-24$	$24-30$
Frequency :	a	b	12	9	5

If mean $=\frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to ________.

Official Ans. by NTA (4)

Sol.

Class	**Frequency**	$\mathbf{x_i}$	$\mathbf{f_i\,x_i}$
0-6	a	3	3a
6-12	b	9	9b
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135
	N=(26+a+b)		**(504+3a+9b)**

Mean $=\frac{3a+9b+180+189+135}{a+b+26}=\frac{309}{22}$

$\Rightarrow 66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow 243a + 111b = 3054$

$\Rightarrow \boxed{81a+37b=1018} \rightarrow (1)$

Now, Median $= 12 + \dfrac{\frac{a+b+26}{2}-(a+b)}{12}\times 6=14$

$\Rightarrow \frac{13}{2}-\left(\frac{a+b}{4}\right)=2$

$\Rightarrow \frac{a+b}{4}=\frac{9}{2}$

$\Rightarrow \boxed{a+b=18} \rightarrow (2)$

From equation (1) & (2)

a = 8, b = 10

$\therefore (a-b)^2 = (8-10)^2$

25. The sum of all the elements in the set $\{n \in \{1, 2, \ldots, 100\} \mid$ H.C.F. of n and 2040 is 1$\}$ is equal to ________.

Official Ans. by NTA (1251)

Sol. $2040 = 2^3 \times 3 \times 5 \times 17$

n should not be multiple of 2, 3, 5 and 17.

Sum of all n = (1 + 3 + 5 ….. + 99) – (3 + 9 + 15 + 21 + ….. + 99) – (5 + 25 + 35 + 55 + 65 + 85 + 95) – (17)

$= 2500-\frac{17}{2}(3+99)-365-17$

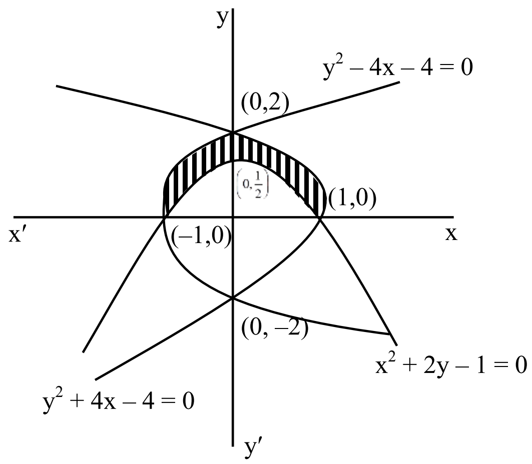
= 2500 – 867 – 365 – 17

= 1251

26. The area (in sq. units) of the region bounded by the curves $x^2+2y-1=0$, $y^2+4x-4=0$ and $y^2-4x-4=0$, in the upper half plane is ________.

Official Ans. by NTA (2)

Sol.



Required Area (shaded)

$$= 2\left[\int_0^2\left(\frac{4-y^2}{4}\right)dy-\int_0^1\left(\frac{1-x^2}{2}\right)dx\right]$$

$$=2\left[\frac{4}{3}-\frac{1}{3}\right]=(2)$$

27. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x)=\begin{cases}3\left(1-\frac{|x|}{2}\right) & \text{if } |x|\le 2\\ 0 & \text{if } |x|>2\end{cases}$$

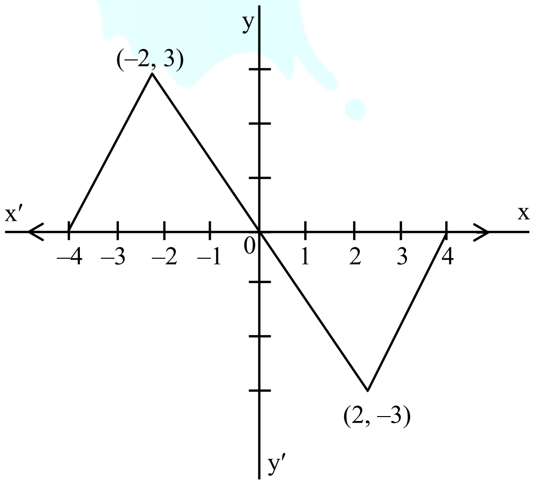
Let $g : \mathbf{R} \to \mathbf{R}$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to _______.

Official Ans. by NTA (4)

Sol. $$f(x-2)\begin{cases}\frac{3x}{2} & -4\le x\le -2\\ -\frac{3x}{2} & -2<x\le 0\\ 0 & x\in(-\infty,-4)\cup(0,+\infty)\end{cases}$$

$$f(x-2)\begin{cases}\frac{3x}{2} & 0\le x\le 2\\ -\frac{3x}{2}+6 & 2\le x\le 4\\ 0 & x\in(-\infty,0)\cup(4,+\infty)\end{cases}$$

$$g(x)=f(x+2)-f(x-2)\begin{cases}\frac{3x}{2}+6 & -4\le x\le -2\\ -\frac{3x}{2} & -2<x<2\\ \frac{3x}{2}-6 & 2\le x\le 4\\ 0 & x\in(-\infty,-4)\cup(4,\infty)\end{cases}$$



n = 0

$m = 4 \Rightarrow (n + m = 4)$

28. If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _______.

Official Ans. by NTA (8)

Sol.

$$\left(2x^r+\frac{1}{x^2}\right)^{10}$$

General term $= {}^{10}C_R\left(2x^2\right)^{10-R}x^{-2R}$

$\Rightarrow 2^{10-R}\,{}^{10}C_R = 180$ ……. (1)

& (10 – R) r – 2R = 0

$$r=\frac{2R}{10-R}$$

$$r=\frac{2(R-10)}{10-R}+\frac{20}{10-R}$$

$\Rightarrow r = -2 + \frac{20}{10-R}$ …… (2)

R = 8 or 5 reject equation (1) not satisfied

At R = 8

$2^{10-R}\,{}^{10}C_R = 180 \Rightarrow \boxed{r=8}$

29. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx = (x + 2)\,dy$, y(1) = 1. If the domain of y = y(x) is an open interval (α, β), then |α + β| is equal to _______.

Official Ans. by NTA (4)

Sol. $y + 1 = Y \Rightarrow dy = dY$

$x + 2 = X \Rightarrow dx = dX$

$\Rightarrow \left(Xe^{\frac{Y}{X}}+Y\right)dX = XdY$

$\Rightarrow XdY - YdX = Xe^{Y/X}dX$

$\Rightarrow d\left(\frac{Y}{X}\right)e^{-X} = \frac{dX}{X}$

$-e^{-Y/X} = \ell|X| + c$

$(3, 2) \rightarrow -e^{-2/3} = \ell|3| + c$

$-e^{-\frac{Y}{X}} = \ell n|X| - e^{-\frac{2}{3}} - \ell n3$

$e^{-\frac{Y}{X}} = e^{2/3} + \ell n3 - \ell n|X| > 0$

$\ell n|X| < \left(e^{2/3} + \ell n3\right)$

Let $\lambda = \left(e^{2/3} + \ell n3\right)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$\underbrace{-e^{\lambda} - 2}_{\alpha} < x < \underbrace{e^{\lambda} - 2}_{\beta}$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although x = –2 should be excluded from domain but according to the given problem it will the most appropriate solution.

30. The number of elements in the set $\{n \in \{1, 2, 3, \ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ________.

Official Ans. by NTA (96)

Sol. $11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10 + 1)^n - (10 - 1)^n > 10^n$

$\Rightarrow \left\{{}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-0} + {}^nC_5 10^{n-5} + \cdots\cdots\right\} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\left\{{}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\cdots\right\} > 10^n$ ..... (1)

For n = 5

$10^5 + 2\left\{{}^5C_3 10^2 + {}^5C_5\right\} > 10^5$ (True)

For n = 6, 7, 8, ... 100

$2n10^{n-1} > 10^n$

$\Rightarrow 2n10^{n-1} + 2\left\{{}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\cdots\right\} > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ For n = 5, 6, 7, ... 100

For n = 4, Inequality (1) is not satisfied

$\Rightarrow$ Inequality does not hold good for

N = 1, 2, 3, 4

So, required number of elements

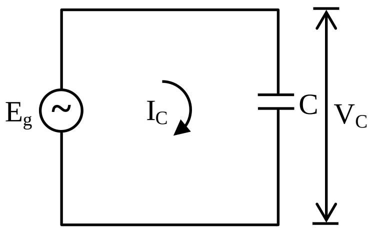
= 96

(Held On Thursday 22nd July, 2021) **TIME : 3: 00 PM to 6: 00 PM**

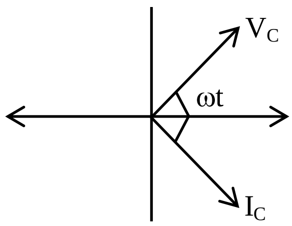
PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0} \sin \omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is :



(1)



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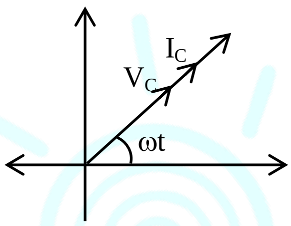

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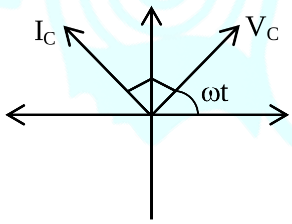

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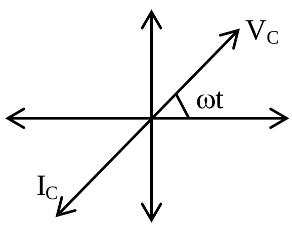
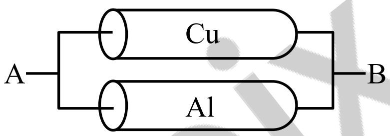


Official Ans. by NTA (3)

Sol. In capacitor, current lead voltage by $\frac{\pi}{2}$

32. A Copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.

(Take Resistivity of Copper = $1.7 \times 10^{-8} \Omega m$
Resistivity of Aluminium = $2.6 \times 10^{-8}\ \Omega m$)



(1) 2.170 mΩ (2) 1.420 mΩ
(3) 0.0858 mΩ (4) 0.858 mΩ

Official Ans. by NTA (4)

Sol. $R = \frac{R_1R_2}{R_1+R_2} = \frac{\ell}{A} \cdot \frac{\rho_1\rho_2}{\rho_1+\rho_2}$

$R = \frac{25\times10^{-2}}{3\times10^{-6}} \times \frac{1.7\times2.6\times10^{\ 16}}{4.3\times10^{-8}}$

$R = 0.858\ m\Omega$

33. What will be the projection of vector $\vec{A} = \hat{i}+\hat{j}+\hat{k}$ on vector $\vec{B} = \hat{i}+\hat{j}$?

(1) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$ (2) $2(\hat{i}+\hat{j}+\hat{k})$
(3) $\sqrt{2}(\hat{i}+\hat{j})$ (4) $(\hat{i}+\hat{j})$

Official Ans. by NTA (4)

Sol. $(A\cos\theta)\hat{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}\hat{B}$

$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \hat{i}+\hat{j}$

34. A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the workdone by the porter in lowering the suitcase.
(take $g = 9.8\ ms^{-2}$)

(1) –62720.0 J (2) –627.2 J
(3) +627.2 J (4) 784.0 J

Official Ans. by NTA (2)

Sol. $W_{Porter} + W_{mg} = \Delta K.E. = 0$
$W_{Porter} = -W_{mg} = -mgh$
$= -80 \times 9.8 \times .8 = -627.2\ J$

35. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

(1) T_0 (2) $8\pi T_0$

(3) $4T_0$ (4) $\frac{1}{4}T_0$

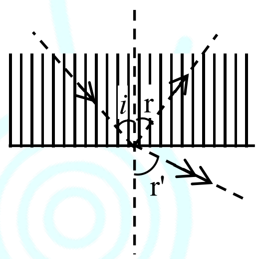
Official Ans. by NTA (4)

Sol. $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$

New time period $T = = 2\pi\sqrt{\frac{\ell/16}{g}} = \frac{2\pi}{4}\sqrt{\frac{\ell}{g}}$

$T = \frac{T_0}{4}$

36. A ray of light passes from a denser medium to a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by :



(1) $\sin^{-1}(\cot r)$

(2) $\tan^{-1}(\sin i)$

(3) $\sin^{-1}(\tan r')$

(4) $\sin^{-1}(\tan r)$

Official Ans. by NTA (4)

Sol. $r + r' + 90° = 180° \Rightarrow r' = 90 - r = 90 - i$

$n_1 \sin i = n_2 \sin r' = n_2 \sin(90 - i)$

$n_1 \sin i = n_2 \cos i \Rightarrow \tan i = \frac{n_2}{n_1}$

Now $\sin C = \frac{n_2}{n_1} = \tan i$

$\Rightarrow C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r)$

37. **Statement I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II : At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (1)

Sol. As temperature increases, domains disintegrate so ferromagnetism decreases and above curie temperature it become paramagnet.

38. A bullet of '4g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

(1) 0.4 kg ms^{-1}, 0.1 ms^{-1}

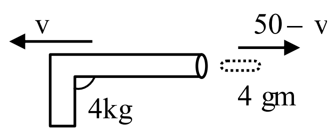
(2) 0.2 kg ms^{-1}, 0.05 ms^{-1}

(3) 0.2 kg ms^{-1}, 0.1 ms^{-1}

(4) 0.4 kg ms^{-1}, 0.05 ms^{-1}

Official Ans. by NTA (2)

Sol.



By momentum conservation

$4 \times 10^{-3}(50 - v) - 4v = 0$

$v = \frac{4\times10^{-3}\times50}{4+4\times10^{-3}} \approx 0.05 ms^{-1}$

Impulse $J = mv = 4 \times .05 = 0.2$ $kgms^{-1}$

39. Choose the correct option :

(1) True dip is not mathematically related to apparent dip.

(2) True dip is less than apparent dip.

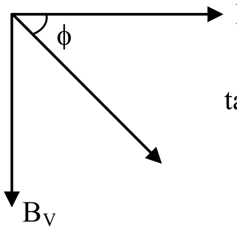
(3) True dip is always greater than the apparent dip.

(4) True dip is always equal to apparent dip.

Official Ans. by NTA (2)

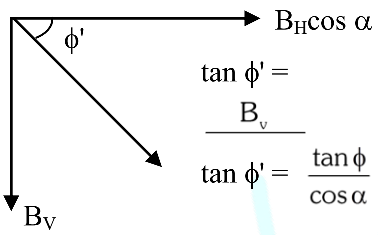
Sol. If apparent dip circle is at an angle α with true dip circle then

True dip circle



$\tan\phi = \frac{B_v}{B_H}$

Apparent dip circle



$\tan\phi' = \frac{B_v}{}$

$\tan\phi' = \frac{\tan\phi}{\cos\alpha}$

As $\cos\alpha < 1$

Hence true dip (ϕ) is less than apparent dip (ϕ')

40. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The **correct** statement for this situation is:-

(1) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

(2) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(3) All of them will have same velocity.

(4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Official Ans. by NTA (1)

Sol. $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$

$I_{ring} > I_{solid\ cylinder} > I_{solid\ sphere}$

$\Rightarrow a_{ring} < a_{solid\ cylinder} < a_{solid\ sphere}$

$\Rightarrow v_{ring} < v_{solid\ cylinder} < v_{solid\ sphere}$

41. Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength $\le$ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

(1) 2 eV (2) 4 eV

(3) 1 eV (4) 0.5 eV

Official Ans. by NTA (1)

Sol. Band gap $= \frac{hc}{\lambda_0}$

λ_0 ; threshold wavelength

Band gap $= \frac{1242\ ev-nm}{621nm} = 2eV$

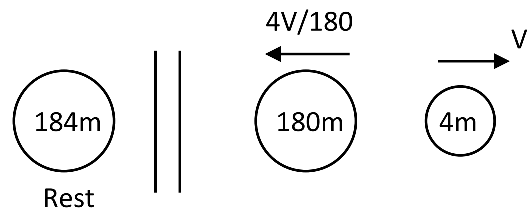
42. A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

(1) 5.0 MeV (2) 5.5 MeV

(3) 0.12 MeV (4) 5.38 MeV

Official Ans. by NTA (4)

Sol.



$$\frac{1}{2}(4m)v^2 + \frac{1}{2}(180m)\left(\frac{4v}{180}\right)^2 \quad 5.5MeV$$

$$\Rightarrow \frac{1}{2}4mv^2\left[1+45\left(\frac{4}{180}\right)^2\right] \quad 5.5MeV$$

$$\Rightarrow K.E_\alpha = \frac{5.5}{1+45.\left(\frac{4}{180}\right)^2}MeV$$

$$K.E_\alpha = 5.38MeV$$

43. An electron of mass m_e and a proton of mass m_P are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is :-

(1) $\frac{m_P}{m_e}$ (2) 1 (3) $\sqrt{\frac{m_P}{m_e}}$ (4) $\frac{m_e}{m_p}$

Official Ans. by NTA (3)

Sol. KE = $e\Delta V$

$$\lambda_e = \frac{h}{\sqrt{2m_e(e\Delta V)}}$$

$$\lambda_P = \frac{h}{\sqrt{2m_p(e\Delta V)}}$$

$$\Rightarrow \frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_P}{m_e}}$$

44. Match **List-I** with **List-II** :

	List–I		List–II
(a)	$\omega L > \frac{1}{\omega C}$	(i)	Current is in phase with emf
(b)	$\omega L = \frac{1}{\omega C}$	(ii)	Current lags behind the applied emf
(c)	$\omega L < \frac{1}{\omega C}$	(iii)	Maximum current occurs
(d)	Resonant frequency	(iv)	Current leads the emf

Choose the **correct** answer from the options given below :

(1) (a) – (ii) ; (b) – (i) ; (c) - (iv) ; (d) – (iii)
(2) (a) – (ii) ; (b) – (i) ; (c) - (iii) ; (d) – (iv)
(3) (a) – (iii) ; (b) – (i) ; (c) - (iv) ; (d) – (ii)
(4) (a) – (iv) ; (b) – (iii) ; (c) - (ii) ; (d) – (i)

Official Ans. by NTA (1)

Sol. (a) For $x_L > x_C$, voltage leads the current

(ii)

(b) For $x_L = x_C$, voltage & current are in same phase

(i)

(c) For $x_L < x_C$, current leads the voltage

(iv)

(d) For resonant frequency $x_L = x_C$, current is maximum

(iii)

45. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km ? The average population density around the tower is 2000/km^2 and the value of $R_e = 6.5 \times 10^6$ m.

(1) Height = 1731 m
Population Covered = 1413×10^5
(2) Height = 1241 m
Population Covered = 7×10^5
(3) Height = 1600 m
Population Covered = 2×10^5
(4) Height = 1800 m
Population Covered = 1413×10^8

Official Ans. by NTA (1)

Sol. Radius covered r = $\sqrt{2RH_T}$

$$150 \text{ km} = \sqrt{2\times(6.5\times10^6 \text{ m})H_T}$$

$(150 \text{ km} \times 10^3)^2 = 2 \times 6.5 \times 10^6 H_T$

$H_T = 1731$m

Population covered = $(\pi r^2)(2000/\text{km}^2)$

$= 3.14 \times (150)^2 \times 2000 = 1413 \times 10^5$

46. What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T ?

(1) $\frac{2}{3}k_BT$ (2) k_BT (3) $\frac{3}{2}k_BT$ (4) $\frac{1}{2}k_BT$

Official Ans. by NTA (3)

Sol. As per Equi-partition law :

Each degree of freedom contributes

$\frac{1}{2}k_BT$ Average Energy

In monoatomic gas D.O.F. = 3

$\Rightarrow$ Average energy = $3\times\frac{1}{2}k_BT = \frac{3}{2}k_BT$

47. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point ?

$(\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2})$

(1) 2.77×10^{-8} T (2) 1.96×10^{-8} T
(3) 8.31 T (4) 5.88 T

Official Ans. by NTA (1)

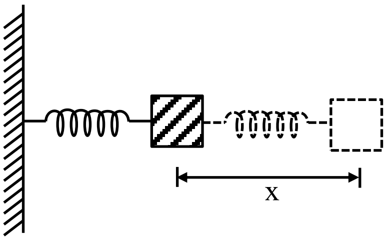
Sol. $I_{avg} = \frac{B_0^2 C}{2\mu_0}$ & $\frac{1}{\mu_0} = \epsilon_0 C^2$

$$I = \frac{B_0^2}{2} \epsilon_0 C^3$$

$$B_0 = \sqrt{\frac{2I}{\epsilon_0 C^3}}$$

$B_0 = 2.77 \times 10^{-8}$ T

48. The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by x(t) = Asinωt + Bcosωt with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time t = 0, the position of mass is x(0) and velocity v (0), then its displacement can also be represented as x(t) = Ccos(ωt – ϕ), where C and ϕ are :

(1) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)}\right)$

(2) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(3) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(4) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)}\right)$

Official Ans. by NTA (4)

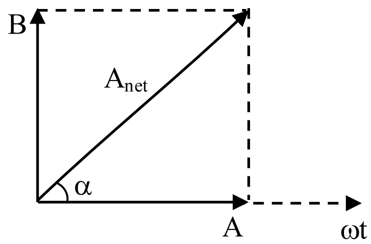
Sol. x = A sin ωt + Bcos ωt

$v = \frac{dx}{dt}$ = Aω cosωt – Bω sinωt

At t = 0, x (0) = B

v (0) = Aω

x = A sin ωt + B sin (ωt + 90°)



$$A_{net} = \sqrt{A^2 + B^2}$$

$$\tan\alpha = \frac{B}{A} \Rightarrow \cot\alpha = \frac{A}{B}$$

$$\Rightarrow x = \sqrt{A^2+B^2}\sin(\omega t + \alpha)$$

$$\Rightarrow x = \sqrt{A^2+B^2}\cos(\omega t - (90 - \alpha))$$

$$x = C\cos(\omega t - \phi)$$

$$\Rightarrow C = \sqrt{A^2+B^2}$$

$$C = \sqrt{\frac{[v(0)]^2}{\omega^2} + [x(0)]^2}$$

$$\phi = 90 - \alpha$$

$$\tan\alpha = \cos\alpha = \frac{A}{B}$$

$$\Rightarrow \tan\phi = \frac{v(0)}{x(0).\omega}$$

$$\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$$

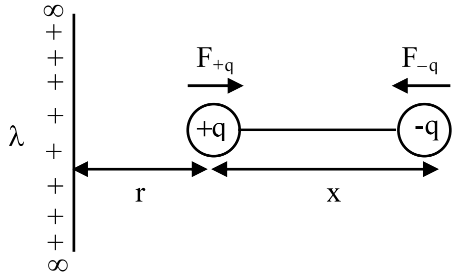
49. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 815.1 nC (2) 8.8 μC

(3) 0.485 mC (4) 4.44 μC

Official Ans. by NTA (4)

Sol.



r = 10 mm, x = 2,,

$$\left|\vec{F}_q\right| = \frac{2k\lambda}{r}.q$$

$$\left|\vec{F}_{-q}\right| = \frac{2k\lambda}{r+x}.q$$

$$\Rightarrow \quad \left|\vec{F}_{net}\right| = \frac{2k\lambda q}{r} - \frac{2k\lambda q}{r+x}$$

$$\left|\vec{F}_{net}\right| = \frac{2k\lambda q.x}{r(r+x)}$$

$$4 = \frac{2\times 9\times 10^9 \times 3\times 10^{-6}\times q\times 2mm}{10mm.12mm}$$

$$\Rightarrow \quad q = 4.44\ \mu C$$

50. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is _______ S.

(1) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - \right]$

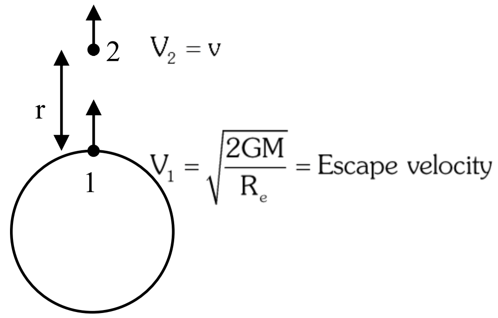
(2) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - \right]$

(3) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - \right]$

(4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - \right]$

Official Ans. by NTA (4)

Sol.



Applying energy conservation from (1) to (2)

$$\frac{1}{2}m\left(\frac{2GM}{R_e}\right) - \frac{GMm}{R_e} = \frac{1}{2}mv^2 - \frac{GMm}{R+r}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{R+r}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R+r}} = \frac{dr}{dt}$$

$$\Rightarrow \quad \sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e+h}\left(\sqrt{R+r}\right)dr$$

$$\sqrt{2GM}\ .\ t = \frac{2}{3}\left[(R+r)^{3/2}\right]_{R_e}^{R_e+h}$$

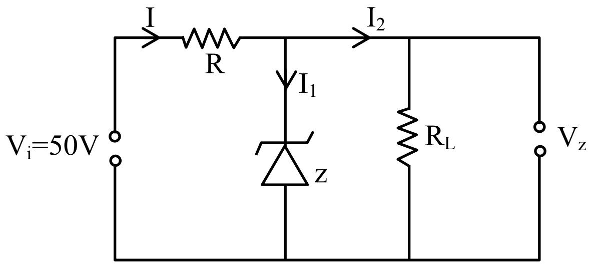
$$t = \frac{2}{3}\sqrt{\frac{R_e^3}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

$$\frac{GM}{R_e^2} = g$$

$$t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

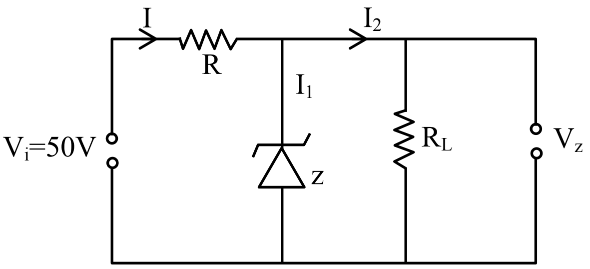
SECTION-B

51. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be _____ Ω.



Official Ans. by NTA (500)

Sol.



Voltage across $R_L = 5V$

$\Rightarrow i_2 = \frac{5}{R_L}$

Also voltage across R = 50 – 5 = 45 volt

By $v = iR \Rightarrow R = \frac{v}{i} = \frac{45}{i_i + i_2}$

$$R = \frac{45}{90mA + \frac{5}{R_L}}$$

Current in zener diode is maximum when $R_L \to \infty$ ($i_2 \to 0$ and $i_i = i$)

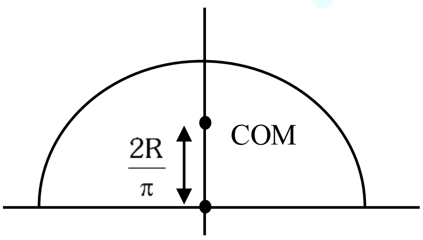
So $R = \frac{45}{90mA} = 500\Omega$

52. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$.

Then, the value of |x| is __________.

Official Ans. by NTA (2)

Sol. COM of semi-circular ring is at $\frac{2R}{\pi}$

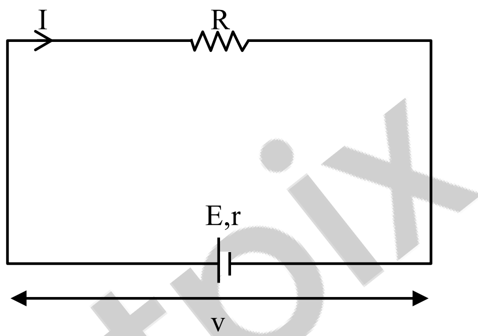


Distance from centre $\Rightarrow x = 2$

53. In an electric circuit, a call of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is ________.

Official Ans. by NTA (15)

Sol.



Terminal voltage $v = iR = \frac{ER}{R+r}$

$1^{st} \to 1.25 = \frac{E(5)}{5+r}$ ..(i)

$2^{nd} \to 1 = \frac{E(2)}{2+r}$ ...(ii)

By (i) and (ii)

$r = 1\Omega, E = \frac{3}{2}V = \frac{15}{10}$ volt

$\Rightarrow x = 15$

54. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is ______ nC, if electric flux density of its field is found as $D = e^{-x} \sin y \,\hat{i} - e^{-x} \cos y \,\hat{j} + 2z\hat{k}$ C/m^2.

Official Ans. by NTA (4)

Sol. Electric flux density

$\left(\vec{D}\right) = \frac{\text{charge}}{\text{Area}} \times \hat{r} = \frac{Q}{4\pi r^2}\hat{r} = \epsilon_0 \left(\frac{Q}{4\pi \epsilon_0 r^2}\hat{r}\right)$

$\Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{e^{-x}\sin y\hat{i} - e^{-x}\cos y\hat{j} \quad 2z\hat{k}}{0}$

Also by Gauss's law

$\frac{\rho}{\epsilon_0} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\frac{\vec{D}}{\epsilon_0}$

$\Rightarrow \rho = \frac{\partial}{\partial x}\left(e^{-x}\sin y\right) + \frac{\partial}{\partial y}\left(-e^{x}\cos y\right) + \frac{\partial}{\partial z}(2z)$

$\rho = -e^{-x}\sin y + e^{-x}\sin y + 2$

At origin $\rho = -e^{-0}\sin 0 + e^{-0}\sin 0 + 2$

$\rho = 2C/m^3$

Charge = $\rho \times$ volume $= 2 \times 2 \times 10^{-9} = 4 \times 10^{-9} =$ 4nC

55. Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is ________.

Official Ans. by NTA (3)

Sol. Direction of P $\hat{v}_1 = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

Direction of Q $\hat{v}_2 = \pm\frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$

Angle between $\hat{v}_1$ and $\hat{v}_2$

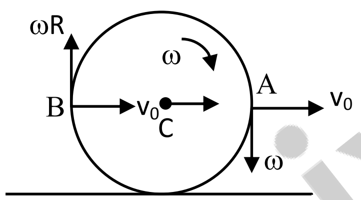
$\frac{\hat{v}_1.\hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\pm 1/\sqrt{3}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow x = 3$

56. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}\, v_0$. Then the value of x is ________.

Official Ans. by NTA (2)

Sol.



For no slipping $v_0 = \omega R$

Now $v_A = v_B = \sqrt{v_0^2 + (\omega R)^2}$

$= \sqrt{2}v_0$

$\Rightarrow$ $x = 2$

57. A ray of light passing through a prism ($\mu = \sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is ________ (in degrees)

Official Ans. by NTA (60)

Sol. At minimum deviation $r_1 = r_2 = \frac{A}{2}$

Also given $i = 2r_1 = A$

Now $1.\sin i = \sqrt{3}\sin r_1$

$1\sin A = \sqrt{3}\sin\frac{A}{2}$

$\Rightarrow$ $2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\frac{A}{2}$

$\Rightarrow$ $\cos\frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30°$

$\Rightarrow$ $A = 60°$

58. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is ________ J/m.
(Young's modulus of material of track is 10^{11} Nm^{-2})
Official Ans. by NTA (5)

Sol. Elastic energy $= \frac{Y}{2}(\text{strain})^2 \times \text{Area} \times \text{length}$

$\Rightarrow$ Elastic energy per unit length $= \frac{Y}{2}(\text{strain})^2 \times \text{Area}$

$$\left(\text{strain} = \frac{\Delta\ell}{\ell} = \alpha\Delta T = 10^{-5} \times 10 = 10^{\ 4}\right.$$

$$= \frac{10^{11}}{2} \times \left(10^{-4}\right)^2 \times 10^{-2} = 5J/m$$

59. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1.	64.0	8	128.0	16.0
2.	64.0	4	64.0	16.0
3.	20.0	4	36.0	9.0

(Least count of length = 0.1 m
least count for time = 0.1 s)
If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no._____.
Official Ans. by NTA (1)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$\Delta T = \frac{\text{least count of time }(\Delta T_0)}{\text{number of oscillations(n)}}$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T_0}{nT}$$

As $\Delta\ell$ and ΔT_0 are same for all observations so

$\frac{\Delta g}{g}$ is minimum for highest value of ℓ, n and T

$\Rightarrow$ Minimum percentage error in g is for student number-1

60. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is ________°C.
Official Ans. by NTA (57)

Sol. By newton's law of cooling (with approximation)

$$\frac{\Delta T}{\Delta t} = -C\left(T_{avg} - T_s\right)$$

$$1^{st} \ \frac{-10°C}{5\,min} = -C(70°C - 25°C)$$

$$\Rightarrow \quad C = \frac{2}{45} min^{-1}$$

$$2^{nd} \ \frac{T-65}{5\,min} = -C\left(\frac{T+65}{2} - 25\right) = -\left(\frac{2}{45}\right)\left(\frac{T+15}{2}\right)$$

$\Rightarrow$ 9 (T – 65) = – (T + 15)

$\Rightarrow$ 10T = 570

$\Rightarrow$ T = 57°C

Alternate Solution :

Newton's law of cooling (without approximation)

$$T_P - T_S = (T_i - T_S)e^{-Ct}$$

1^{st} $65 - 25 = (75 - 25)e^{-5C} \Rightarrow e^{-5C} = \frac{4}{5}$

2^{nd} $T - 25 = (65 - 25)e^{-5C} = 40 \times \frac{4}{5} = 32$

T = 57°C

(Held On Thursday 22nd July, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61. The water having more dissolved O_2 is :

(1) boiling water (2) water at 80°C

(3) polluted water (4) water at 4°C

Official Ans. by NTA (4)

Sol.

On heating concentration of O_2 in water decreases. So boiling water and water at 80°C having less O_2 concentration. Polluted water also having less O_2 concentration. So water at 4°C having maximum O_2 concentration.

62. Which one of the following statements for D.I. Mendeleeff, is **incorrect**?

(1) He authored the textbook – Principles of Chemistry.

(2) At the time, he proposed Periodic Table of elements structure of atom was known.

(3) Element with atomic number 101 is named after him.

(4) He invented accurate barometer.

Official Ans. by NTA (2)

Sol. At the time, he proposed the periodic table but structure of atom was unknown.

63. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)?

(1) Simple distillation

(2) Steam distillation

(3) Fractional distillation

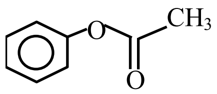
(4) Reduced pressure distillation

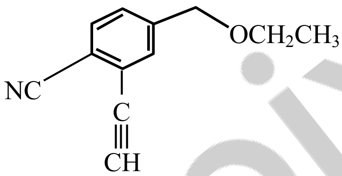
Official Ans. by NTA (4)

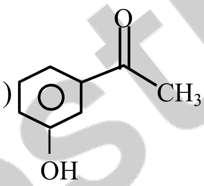
Sol.

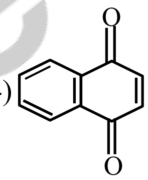
Reduced pressure distillation or vacuum distillation is used for the purification of high boiling organic liquids which decomposes at or below their boiling point.

64. Which of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

(1) 

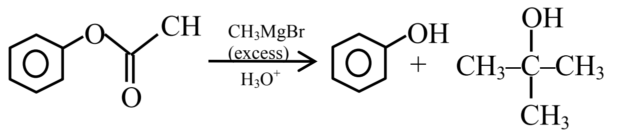
(2) 

(3) 

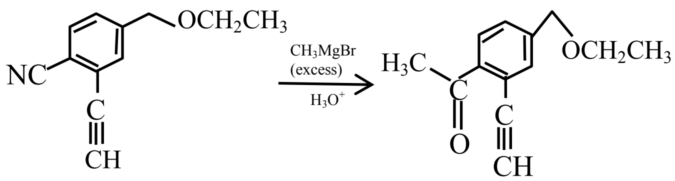
(4) 

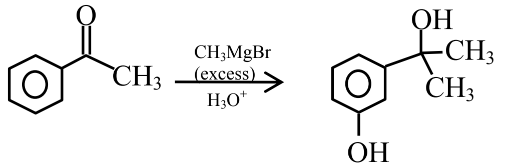
Official Ans. by NTA (1)

Sol.

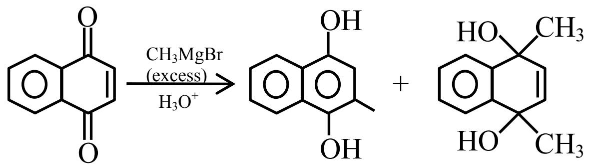


Tertiary alcohol





Phenolic –OH group and tertiary alcohol is present thus two functional groups are present in the product

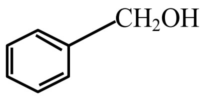


Two 3° alcohol

Since the given question is single correct choice the best appropriate option is (A)

65. Which of the following compounds does not exhibit resonance?

(1) $CH_3CH_2OCH=CH_2$

(2) 

(3) $CH_3CH_2CH_2CONH_2$

(4) $CH_3CH_2CH=CHCH_2NH_2$

Official Ans. by NTA (4)

Sol.

$CH_3–CH_2–CH = CH–CH_2–NH_2$

No conjugation thus resonance is not possible.

66. Match **List-I** with **List-II**

List-I (Elements)	List-II (Properties)
(a) Ba	(i) Organic solvent soluble compounds
(b) Ca	(ii) Outer electronic configuration $6s^2$
(c) Li	(iii) Oxalate insoluble in water
(d) Na	(iv) Formation of very strong monoacidic base

Choose the **correct** answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)

(2) (a)-(iv), (b)-(i), (c)-(ii) and (d)-(iii)

(3) (a)-(iii), (b)-(ii), (c)-(iv) and (d)-(i)

(4) (a)-(i), (b)-(iv), (c)-(ii) and (d)-(iii)

Official Ans. by NTA (1)

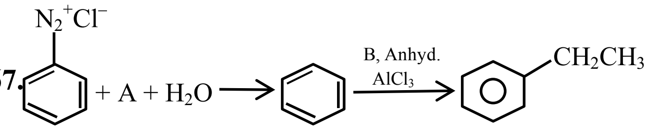
Sol. .

(a) 'Ba' having outer electronic configuration $6s^2$.

(b) CaC_2O_4 is water insoluble

(c) 'Li' is soluble in organic solvents

(d) NaOH is strong Monoacidic base among given.

67. 

Major product

In the chemical reactions given above A and B respectively are :

(1) H_3PO_2 and CH_3CH_2Cl

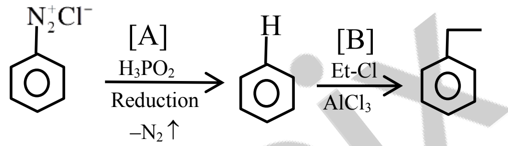
(2) CH_3CH_2OH and H_3PO_2

(3) H_3PO_2 and CH_3CH_2OH

(4) CH_3CH_2Cl and H_3PO_2

Official Ans. by NTA (1)

Sol.



68. Isotope(s) of hydrogen which emits low energy β^- particles with $t_{1/2}$ value > 12 years is/are

(1) Protium

(2) Tritium

(3) Deuterium

(4) Deuterium and Tritium

Official Ans. by NTA (2)

Sol.

^1_1H and ^2_1H are stable while ^3_1H is radioactive.

69. Match **List-I** with **List-II** :

List-I (Species)		List-II (Hybrid Orbitals)
(a) SF_4	(i)	sp^3d^2
(b) IF_5	(ii)	d^2sp^3
(c) NO_2^+	(iii)	sp^3d
(d) NH_4^+	(iv)	sp^3
	(v)	sp

Choose the **correct** answer from the options given below :

(1) (a)-(i), (b)-(ii), (c)-(v) and (d)-(iii)

(2) (a)-(ii), (b)-(i), (c)-(iv) and (d)-(v)

(3) (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)

(4) (a)-(iv), (b)-(iii), (c)-(ii) and (d)-(v)

Official Ans. by NTA (3)

Sol.

(a) SF_4 – sp^3d hybridisation

(b) IF_5 – sp^3d^2 hybridisation

(c) NO_2^+ – sp hybridisation

(d) NH_4^+ – sp^3 hybridisation

70. When silver nitrate solution is added to potassium iodide solution then the sol produced is :

(1) AgI / I^- (2) AgI / Ag^+

(3) KI / NO_3^- (4) $AgNO_3 / NO_3^-$

Official Ans. by NTA (1)

Sol.

$$\underset{\text{(drop by drop)}}{AgNO_3(aq.)} + \underset{\text{excess}}{KI(aq.)} \longrightarrow \underset{\text{Sol}}{AgI / I^-}$$

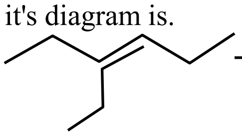
71. Which of the following molecules does not show stereo isomerism ?

(1) 3,4-Dimethylhex-3-ene
(2) 3-Methylhex-1-ene
(3) 3-Ethylhex-3-ene
(4) 4-Methylhex-1-ene

Official Ans. by NTA (3)

11. Sol.

3-Ethylhex-3-ene will not show stereo isomerism it's diagram is.



$\longrightarrow$ (1) Not show geometrical isomerism

(2) Not show optical isomerism

72. Given below are the statements about diborane

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2

(b) Each boron atom is in sp^2 hybridized state

(c) Diborane has one bridged 3 centre-2-electron bond

(d) Diborane is a planar molecule

The option with **correct** statement(s) is -

(1) (c) and (d) only
(2) (a) only
(3) (c) only
(4) (a) and (b) only

Official Ans. by NTA (2)

Sol.

Diborane is prepared by the reaction of $NaBH_4$ with I_2.

$2NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

In diborane, 'B' is sp^3 hybrid, it is Non-planar and two $3c–2e^-$ bonds are present.

73. Which one of the following group-15 hydride is the strongest reducing agent ?

(1) AsH_3 (2) BiH_3 (3) PH_3 (4) SbH_3

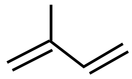
Official Ans. by NTA (2)

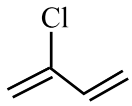
Sol.

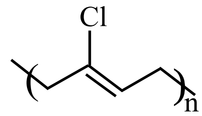
Among 15^{th} group hydrides, BiH_3 is strongest reducing agent.

74. Match **List-I** with **List-II :**

List-I | **List-II**

(a) Chloroprene (i) 

(b) Neoprene (ii) 

(c) Acrylonitrile (iii) 

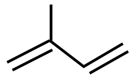
(d) Isoprene (iv) CH_2=CH–CN

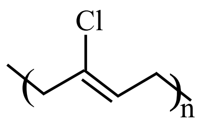
Choose the **correct** answer from the options given below :

(1) (a) - (iii), (b)-(iv), (c) -(ii), (d) -(i)
(2) (a) - (ii), (b)-(iii), (c) -(iv), (d) -(i)
(3) (a) - (ii), (b)-(i), (c) -(iv), (d) -(iii)
(4) (a) - (iii), (b)-(i), (c) -(iv), (d) -(ii)

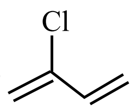
Official Ans. by NTA (2)

Sol.

(a) Chloroprene (i) 

(b) Neoprene (ii) 

(c) Acrylonitrile (iii) CH_2=CH–CN

(d) Isoprene (iv) 

75. The set having ions which are coloured and paramagnetic both is -

(1) Cu^{2+}, Cr^{3+}, Sc^+
(2) $Cu^{2+}, Zn^{2+}, Mn^{4+}$
(3) Sc^{3+}, V^{5+}, Ti^{4+}
(4) $Ni^{2+}, Mn^{7+}, Hg^{2+}$

Official Ans. by NTA (1)

Sol.

$$\left.\begin{array}{l} Cu^{2+} : [Ar]3d^9 4s^0 \\ Cr^{3+} : [Ar]3d^3 4s^0 \\ Sc^{+} : [Ar]3d^1 4s^1 \end{array}\right] \begin{array}{l} \text{All are coloured and} \\ \text{paramagnetic due to} \\ \text{presence of unpaired electrons} \end{array}$$

76. Thiamine and pyridoxine are also known respectively as :

(1) Vitamin B_2 and Vitamin E

(2) Vitamin E and Vitamin B_2

(3) Vitamin B_6 and Vitamin B_2

(4) Vitamin B_1 and Vitamin B_6

Official Ans. by NTA (4)

Sol.

Vitamine-B_1 is also known as Thiamine while vitamin B-6 is known as Pyridoxine

77. Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al

(c) Ag (d) Mg

Choose the **correct** answer from the options given below :

(1) (a) and (c) only

(2) (a) and (d) only

(3) (a) and (b) only

(4) (c) and (d) only

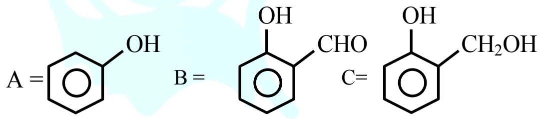
Official Ans. by NTA (1)

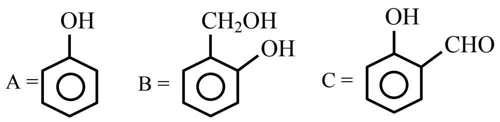
Sol.

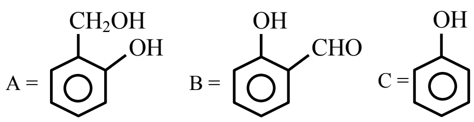
Pb and Ag commonly exist in the form of sulphide ore like PbS (galena) and Ag_2S (Argentite)

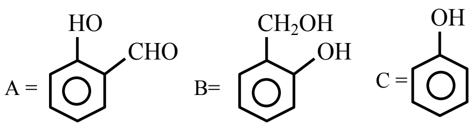
'Al' is mainly found in the form of oxide ore whereas 'Mg' is found in the form of halide ore.

78. An organic compound A (C_6H_6O) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C .

(1) 

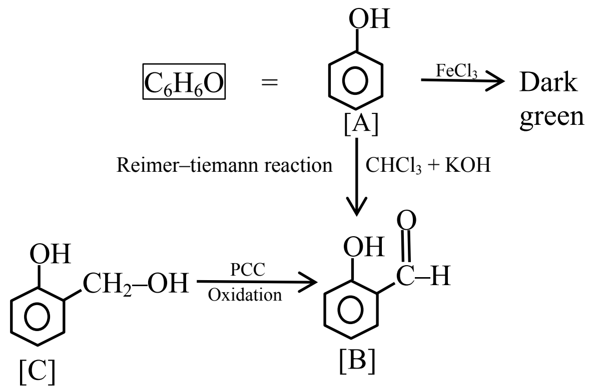
(2) 

(3) 

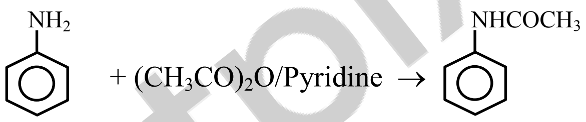
(4) 

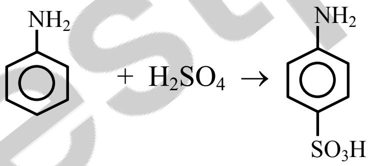
Official Ans. by NTA (1)

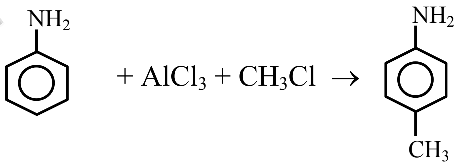
Sol.

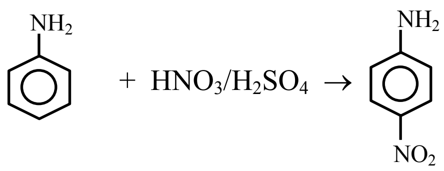


79. Which one of the following reactions does not occur ?

(1) 

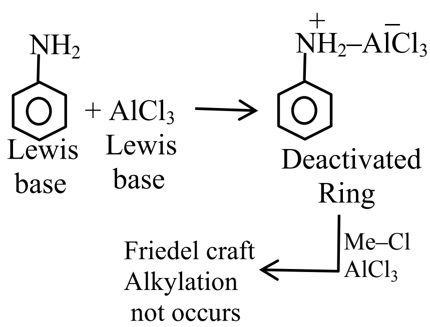
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



(1) Aniline is lewis base give acid base reaction with $AlCl_3$ and form Anilinium ion

(2) Anilinium ion has strongest deactivated ring so further Friedel craft Alkylation not occurs.

80. Which one of the following 0.06 M aqueous solutions has lowest freezing point ?

(1) $Al_2(SO_4)_3$ (2) $C_6H_{12}O_6$

(3) KI (4) K_2SO_4

Official Ans. by NTA (1)

Sol.

$T_f - T_f' = i\,K_f.m$

For minimum T_f'

'i' should be maximum.

$Al_2(SO_4)_3$ i = 5

$C_6H_{12}O_6$ i = 1

KI i = 2

K_2SO_4 i = 3

SECTION-B

81. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is

Official Ans. by NTA (1)

QuestPix Ans. (3)

Sol.

$[Co(NH_3)_6]Cl_2$

Co^{2+} : $[Ar]3d^7 4s^0 4p^0$

For this complex Δ_0 < P.E., so pairing of electron does not take place.

sp^3d^2 hybridisation

Total 3 unpaired electrons are present.

$[Co(NH_3)_6]Cl_3$

Co^{3+} : $[Ar]\ 3d^6\ 4s^0\ 4p^0$

d^2sp^3 hybridisation

NH_3 acts as SFL because Δ_0 > P.E.

So here all electrons becomes paired.

82. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene ______. (Nearest integer)

Official Ans. by NTA (78)

Sol.

$$C_6H_6 + CH_3Cl \longrightarrow C_6H_5CH_3 + HCl$$

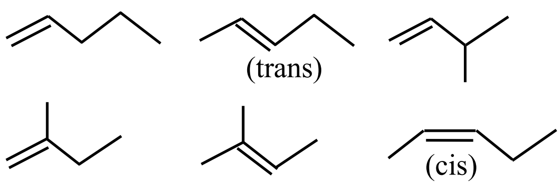
$$\frac{10}{78} \qquad \left(\frac{10}{78}\times 92\right)\text{gm} \Rightarrow$$

$$\frac{A_y}{T_y} = \%\text{yield} = \frac{9.2}{920}\times 78\times 100 \Rightarrow 78\%$$

83. The number of acyclic structural isomers (including geometrical isomers) for pentene are ____

Official Ans. by NTA (6)

Sol.



84. Assume a cell with the following reaction

$Cu_{(s)} + 2Ag^{+}(1\times 10^{-3}\ M) \rightarrow Cu^{2+}(0.250\ M) + 2Ag_{(s)}$

$E^{\ominus}_{cell} = 2.97\,V$

E_{cell} for the above reaction is __________V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K]

Official Ans. by NTA (3)

Sol.

$$E = E^\circ - \frac{0.059}{2}\log\frac{[Cu^{+2}]}{[Ag^+]^2}$$

$$= 2.97 - \frac{0.059}{2}\log\frac{0.25}{(10^{-3})^2} = 2.81V$$

85. Value of K_P for the equilibrium reaction

$N_2O_{4\,(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_C for this reaction at same temperature is _______. (Nearest integer)

$(R = 0.083\ L\ bar\ K^{-1}\ mol^{-1})$

Official Ans. by NTA (2)

Sol.

$$K_C = \frac{K_P}{RT} = \frac{47.9}{0.083\times 288} = 2$$

86. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is _______ kJ. (Nearest integer)

Official Ans. by NTA (21)

Sol.

1 mol graphite = 12 gm C

$$\text{Ans.} = \frac{248}{12} = 20.67\text{kJ / gm heat evolved}$$

87. A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is ______ g mol^{-1}. (Nearest integer)

[Given : $N_A = 6.022 \times 10^{23}$ mol^{-1}]

Official Ans. by NTA (106)

Sol.

$$d\left(\frac{gm}{cc}\right) = \frac{4 \times \frac{M}{N_A}}{(a\ cm)^3}$$

$$7.62 = \frac{4 \times M / 6.022 \times 10^{23}}{(0.4518 \times 10^{-7} cm)^3} \Rightarrow M = 105.8\ g/mol$$

88. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to _______

Official Ans. by NTA (12)

Sol.

$_{23}V : 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^3\, 4s^2$

Number of electrons in p-orbitals is equal to 12.00

89. $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. The concentration of N_2O_5 after 1 hour was 1.60 $\times 10^{-2}$ mol L^{-1}. The rate constant of the reaction at 318 K is _______ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Official Ans. by NTA (7)

Sol.

$$K = \frac{2.303}{t}\log\frac{[N_2O_5]_0}{[N_2O_5]_t}$$

$$= \frac{2.303}{60}\log\frac{2.4}{1.6} = 6.76 \times 10^{-3}\ min^{-1} \approx 7 \times 10^{-3}\ min^{-1}$$

90. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is ______ $\times 10^{-3}$M. (Nearest integer)

[Given : Atomic mass of C = 12, H = 1, O = 16 u]

Official Ans. by NTA (4)

Sol. $[Glucose] = \frac{C(gm/\ell)}{M(gm/mol)} = \frac{0.72}{180} = 4 \times 10^{-3} M$