

(Held On Tuesday 24th January, 2023)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS	TEST PAPER WITH SOLUTION
SECTION-A 1. The distance of the point (7, -3, -4) from the plane passing through the points (2, -3, 1), (-1, 1, -2) and (3, -4, 2) is : (1) 4 (2) 5 (3) $5\sqrt{2}$ (4) $4\sqrt{2}$ Official Ans. by NTA (3) QuestPix Ans. (3) Sol. Equation of Plane is $= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$ $x - z - 1 = 0$ Distance of P (7, -3, -4) from Plane is $d = \left \frac{7 + 4 - 1}{\sqrt{2}} \right = 5\sqrt{2}$ 2. $\lim_{t \rightarrow 0} \left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \dots + n^{\frac{1}{\sin^2 t}} \right)^{\sin^2 t}$ is equal to (1) $n^2 + n$ (2) n (3) $\frac{n(n+1)}{2}$ (4) n^2 Official Ans. by NTA (2) QuestPix Ans. (2) Sol. $\lim_{t \rightarrow 0} \left(1^{\csc^2 t} + 2^{\csc^2 t} + \dots + n^{\csc^2 t} \right)^{\sin^2 t}$ $= \lim_{t \rightarrow 0} n^{\left(\left(\frac{1}{n} \right)^{\csc^2 t} + \left(\frac{2}{n} \right)^{\csc^2 t} + \dots + 1 \right)^{\sin^2 t}}$ $= n$	3. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to (1) 1 (2) $\frac{3}{2}$ (3) 2 (4) $-\frac{2}{3}$ Official Ans. by NTA (1) QuestPix Ans. (1) Sol. $\vec{u} = (1, -1, -2)$, $\vec{v} = (2, 1, -1)$, $\vec{v} \cdot \vec{w} = 2$ $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v} \dots \dots \dots (1)$ Taking dot with $\vec{w}$ in (1) $\vec{w} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{w} + \lambda \vec{v} \cdot \vec{w}$ $\Rightarrow 0 = \vec{u} \cdot \vec{w} + 2\lambda$ Taking dot with $\vec{v}$ in (1) $\vec{v} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{v} + \lambda \vec{v} \cdot \vec{v}$ $\Rightarrow 0 = (2 - 1 + 2) + \lambda(6)$ $\lambda = -\frac{1}{2}$ $\Rightarrow \vec{u} \cdot \vec{w} = -2\lambda = 1$ 4. The value $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$ is (1) ${}^{45}C_{23}$ (2) ${}^{44}C_{23}$ (3) ${}^{45}C_{24}$ (4) ${}^{44}C_{22}$ Official Ans. by NTA (1) QuestPix Ans. (1) Sol. $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{23-r}$ $= {}^{45}C_{23}$

5. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

- (1) directrix $4x = 3$
 (2) directrix $4x = -3$
 (3) Length of latus rectum $\frac{3}{2}$
 (4) Length of latus rectum 2

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $y^2 = 24x$

$a = 6$

$xy = 2$

$AB \equiv ty = x + 6t^2 \dots\dots\dots(1)$

$AB \equiv T = S_1$

$kx + hy = 2hk \dots\dots\dots(2)$

From (1) and (2)

$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$

$\Rightarrow$ then locus is $y^2 = -3x$

Therefore directrix is $4x = 3$

6. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

- (1) 18 (2) 19
 (3) 20 (4) 21

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$= (N-2)(N-3)$

For unique solution $\Delta \neq 0$

So $N \neq 2, 3$

$\Rightarrow$ P (system has unique solution) $= \frac{4}{6}$

So $k = 4$

Therefore sum $= 4 + 1 + 4 + 5 + 6 = 20$

7. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$
 (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$
 $= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$

8. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that

$\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is

equal to

- (1) 4 (2) 3
 (3) 2 (4) $\frac{5}{2}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q} + \vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of ΔPQR is $= \frac{1}{2} |\vec{q} \times \vec{r}|$

Area of ΔABC is $= \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$\vec{AB} = \frac{\vec{r} - 2\vec{q}}{3}$, $\vec{AC} = \frac{-\vec{r} - \vec{q}}{3}$

Area of $\Delta ABC = \frac{1}{6} |\vec{q} \times \vec{r}|$

$\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} = 3$

9. If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2 B$, then

- (1) $AB = I$
 (2) $A^2 B = I$
 (3) $A^2 = I$ or $B = I$
 (4) $A^2 B = BA^2$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $A^2 + B = A^2 B$

$$(A^2 - 1)(B - 1) = 1 \dots\dots(1)$$

$$A^2 + B = A^2 B$$

$$A^2(B - 1) = B$$

$$A^2 = B(B - 1)^{-1}$$

$$A^2 = B(A^2 - 1)$$

$$A^2 = BA^2 - B$$

$$A^2 + B = BA^2$$

$$A^2 B = BA^2$$

- 10.** Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1) dx = 0$, $x > 0$,

$$y\left(\frac{1}{2}\right) = 3 - e. \text{ Then } y(1) \text{ is equal to}$$

- (1) 1
(2) e
(3) 2-e
(4) 3

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{dy}{dx} = \frac{1 - xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}$

$$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$$

$$\text{If } e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$$

$$y \cdot e^{-\frac{1}{x}} = \int e^{-\frac{1}{x}} \cdot \frac{1}{x^3} dx \text{ (put } -\frac{1}{x} = t)$$

$$y \cdot e^{-\frac{1}{x}} = -\int e^t \cdot t dt$$

$$y = \frac{1}{x} + 1 + Ce^{\frac{1}{x}}$$

Where C is constant

$$\text{Put } x = \frac{1}{2}$$

$$3 - e = 2 + 1 + Ce^2$$

$$C = -\frac{1}{e}$$

$$y(1) = 1$$

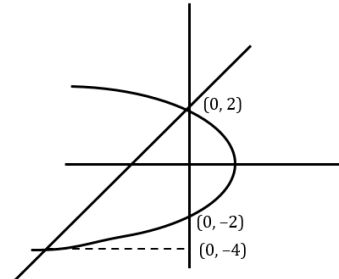
- 11.** The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is :

- (1) $\frac{25}{3}$ (2) $\frac{22}{3}$ (3) 9 (4) $\frac{23}{3}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



$$y^2 + 4x = 4$$

$$y^2 = -4(x - 1)$$

$$A = \int_{-4}^2 \left(\frac{4 - y^2}{4} - \frac{y - 2}{2} \right) dy = 9$$

- 12.** Let α be a root of the equation

$(a - c)x^2 + (b - a)x + (c - b) = 0$ where a, b, c are distinct real numbers such that the matrix

$$\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

is singular. Then the value of

$$\frac{(a - c)^2}{(b - a)(c - b)} + \frac{(b - a)^2}{(a - c)(c - b)} + \frac{(c - b)^2}{(a - c)(b - a)} \text{ is}$$

- (1) 6 (2) 3
(3) 9 (4) 12

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$

$$\Rightarrow \alpha^2(c - b) - \alpha(c - a) + (b - a) = 0$$

It is singular when $\alpha = 1$

$$\frac{(a - c)^2}{(b - a)(c - b)} + \frac{(b - a)^2}{(a - c)(c - b)} + \frac{(c - b)^2}{(a - c)(b - a)}$$

$$\begin{aligned} & \frac{(a - b)^3 + (b - c)^3 + (c - a)^3}{(a - b)(b - c)(c - a)} \\ &= 3 \frac{(a - b)(b - c)(c - a)}{(a - b)(b - c)(c - a)} = 3 \end{aligned}$$

13. The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line

$$\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12} \text{ is}$$

- (1) $13\sqrt{2}$ (2) 31
(3) 26 (4) $20\sqrt{2}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Equation of line

$$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$$

G.P on line $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$

point of intersection of line & plane

$$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$$

$$\lambda = 2$$

Point $(5, 1, 8)$

$$\text{Distance} = \sqrt{36 + 64 + 576} = 26$$

14. For three positive integers p, q, r , $x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3 \log_y x, 3 \log_{zy} y, 7 \log_{xz} z$ are in A.P. with common difference $\frac{1}{2}$. Then

$r - p - q$ is equal to

- (1) 2 (2) 6
(3) 12 (4) -6

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $pq^2 = \log_x \lambda$

$$qr = \log_y \lambda$$

$$p^2 r = \log_z \lambda$$

$$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \dots\dots(1)$$

$$\log_x z = \frac{pq^2}{p^2 r} = \frac{q^2}{pr} \dots\dots(2)$$

$$\log_z y = \frac{p^2 r}{qr} = \frac{p^2}{q} \dots\dots(3)$$

$$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr} \text{ in A.P}$$

$$\frac{3r}{pq} - 3 = \frac{1}{2}$$

$$r = \frac{7}{6}pq \dots\dots(4)$$

$$r = pq + 1$$

$$pq = 6 \dots\dots(5)$$

$$r = 7 \dots\dots(6)$$

$$\frac{3p^2}{q} = 4$$

After solving $p = 2$ and $q = 3$

15. Let $p, q \in \mathbb{R}$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$,

$i = \sqrt{-1}$ Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

$$(1) x^2 + 4x - 1 = 0 \quad (2) x^2 - 4x + 1 = 0$$

$$(3) x^2 + 4x + 1 = 0 \quad (4) x^2 - 4x - 1 = 0$$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$

$$2^{200} \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)^{200} = 2^{199}(p + iq)$$

$$2 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = p + iq$$

$$p = -1, q = -\sqrt{3}$$

$$\alpha = p + q + q^2 = 2 - \sqrt{3}$$

$$\beta = p - q + q^2 = 2 + \sqrt{3}$$

$$\alpha + \beta = 4$$

$$\alpha \cdot \beta = 1$$

$$\text{equation } x^2 - 4x + 1 = 0$$

16. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is: ____

- (1) transitive but not reflexive
(2) symmetric but not transitive
(3) reflexive but not symmetric
(4) neither symmetric nor transitive

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Reflexive : $(a, a) \Rightarrow \gcd \text{ of } (a, a) = 1$

Which is not true for every $a \in \mathbb{Z}$.

Symmetric:

$$\text{Take } a = 2, b = 1 \Rightarrow \gcd(2, 1) = 1$$

$$\text{Also } 2a = 4 \neq b$$

$$\text{Now when } a = 1, b = 2 \Rightarrow \gcd(1, 2) = 1$$

$$\text{Also now } 2a = 2 = b$$

$$\text{Hence } a = 2b$$



$\Rightarrow R$ is not Symmetric

Transitive:

Let $a = 14, b = 19, c = 21$

$\gcd(a, b) = 1$

$\gcd(b, c) = 1$

$\gcd(a, c) = 7$

Hence not transitive

$\Rightarrow R$ is neither symmetric nor transitive.

17. The compound statement

$(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is

equivalent to

(1) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(2) $(\sim Q) \vee P$

(3) $((\sim P) \vee Q) \wedge (\sim Q)$

(4) $(\sim P) \vee Q$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Let $r = (\sim(P \wedge Q)) \vee ((\sim P) \wedge Q); s = ((\sim P) \wedge (\sim Q))$

P	Q	$\sim(P \wedge Q)$	$(\sim P) \wedge Q$	r	s	$r \rightarrow s$
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

Option (A) : $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

is equivalent to (not of only P) $\wedge$ (not of only Q)

= (Both P, Q) and (neither P nor Q)

18. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$; Then at $x = 0$

(1) f is continuous but not differentiable

(2) f is continuous but f' is not continuous

(3) f and f' both are continuous

(4) f' is continuous but not differentiable

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Continuity of $f(x)$: $f(0^+) = h^2 \cdot \sin \frac{1}{h} = 0$

$$f(0^-) = (-h)^2 \cdot \sin\left(\frac{-1}{h}\right) = 0$$

$f(0) = 0$

$f(x)$ is continuous

$$f'(0^+) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \frac{h^2 \cdot \sin\left(\frac{1}{h}\right) - 0}{h} = 0$$

$$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \frac{h^2 \cdot \sin\left(\frac{-1}{h}\right) - 0}{-h} = 0$$

$f(x)$ is differentiable.

$$f'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$$

$$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\Rightarrow f'(x)$ is not continuous (as $\cos\left(\frac{1}{x}\right)$ is highly

oscillating at $x = 0$)

19. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has:

(1) exactly two solutions in $(-\infty, \infty)$

(2) no solution

(3) a unique solution in $(-\infty, 1)$

(4) a unique solution in $(-\infty, \infty)$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $x^2 - 4x + [x] + 3 = x[x]$

$$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$$

$$\Rightarrow (x-1)(x-3) = [x] \cdot (x-1)$$

$$\Rightarrow x = 1 \text{ or } x - 3 = [x]$$

$$\Rightarrow x - [x] = 3$$

$$\Rightarrow \{x\} = 3 \text{ (Not Possible)}$$

Only one solution $x = 1$ in $(-\infty, \infty)$

20. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements :

(S1) : If $P(A) = 0$, then $A = \phi$

(S2) : If $P(A) = 1$, then $A = \Omega$

Then

(1) only (S1) is true

(2) only (S2) is true

(3) both (S1) and (S2) are true

(4) both (S1) and (S2) are false

Official Ans. by NTA (4)

QuestPix Ans. (3)

Sol. Ω = sample space
 A = be an event
 If $P(A) = 0 \Rightarrow A = \phi$
 If $P(A) = 1 \Rightarrow A = \Omega$
 Then both statement are true

SECTION-B

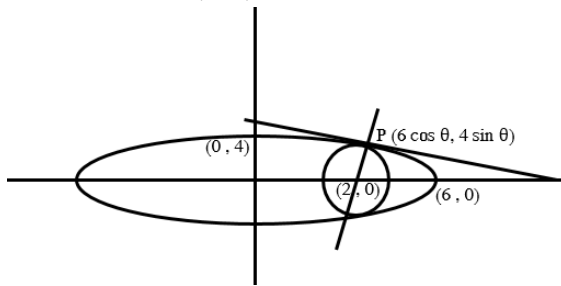
- 21.** Let C be the largest circle centred at $(2, 0)$ and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

If $(1, \alpha)$ lies on C , then $10\alpha^2$ is equal to _____

Official Ans. by NTA (118)

QuestPix Ans. (118)

Sol.



Equation of normal of ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ at any point $P(6 \cos \theta, 4 \sin \theta)$ is

$3 \sec \theta x - 2 \operatorname{cosec} \theta y = 10$ this normal is also the normal of the circle passing through the point $(2, 0)$ So,

$6 \sec \theta = 10$ or $\sin \theta = 0$ (Not possible)

$\cos \theta = \frac{3}{5}$ and $\sin \theta = \frac{4}{5}$ so point $P = \left(\frac{18}{5}, \frac{16}{5}\right)$

So the largest radius of circle

$$r = \frac{\sqrt{320}}{5}$$

So the equation of circle $(x-2)^2 + y^2 = \frac{64}{5}$

Passing it through $(1, \alpha)$

$$\text{Then } \alpha^2 = \frac{59}{5}$$

$$10\alpha^2 = 118$$

- 22.** Suppose $\sum_{r=0}^{2023} r^2 {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then

the value of α is _____

Official Ans. by NTA (1012)

QuestPix Ans. (1012)

Sol. using result

$$\sum_{r=0}^n r^2 {}^nC_r = n(n+1) \cdot 2^{n-2}$$

$$\text{Then } \sum_{r=0}^{2023} r^2 {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$$

$$= 2023 \times \alpha \times 2^{2022} \text{ So,}$$

$$\Rightarrow \alpha = 1012$$

- 23.** The value of $12 \int_0^3 |x^2 - 3x + 2| dx$ is _____

Official Ans. by NTA (22)

QuestPix Ans. (22)

$$\text{Sol. } 12 \int_0^3 |x^2 - 3x + 2| dx$$

$$= 12 \int_0^3 \left| \left(x - \frac{3}{2}\right)^2 - \frac{1}{4} \right| dx$$

$$\text{If } x - \frac{3}{2} = t$$

$$dx = dt$$

$$= 24 \int_0^{3/2} \left| t^2 - \frac{1}{4} \right| dt$$

$$= 24 \left[-\int_0^{1/2} \left(t^2 - \frac{1}{4} \right) dt + \int_{1/2}^{3/2} \left(t^2 - \frac{1}{4} \right) dt \right] = 22$$

- 24.** The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _____

Official Ans. by NTA (60)

QuestPix Ans. (60)

Sol. Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

25. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S =$

$\{x + \lambda : x \text{ is an integer solution of E}\}$ is _____

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $|x|^2 - 2|x| + |\lambda - 3| = 0$
 $|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$
 $(|x| - 1)^2 + |\lambda - 3| = 1$
 At $\lambda = 3$, $x = 0$ and 2 ,
 at $\lambda = 4$ or 2 , then
 $x = 1$ or -1

So maximum value of $x + \lambda = 5$

26. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Official Ans. by NTA (546)

QuestPix Ans. (546)

Sol. For at most two language courses
 $= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$

27. Let a tangent to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _____

Official Ans. by NTA (7)

QuestPix Ans. (7)

Sol. Equation of tangent at point $P(4 \cos \theta, 3 \sin \theta)$ is
 $\frac{x \cos \theta}{4} + \frac{y \sin \theta}{3} = 1$ So A is $(4 \sec \theta, 0)$ and
 point B is $(0, 3 \operatorname{cosec} \theta)$

$$\begin{aligned} \text{Length AB} &= \sqrt{16 \sec^2 \theta + 9 \operatorname{cosec}^2 \theta} \\ &= \sqrt{25 + 16 \tan^2 \theta + 9 \cot^2 \theta} \geq 7 \end{aligned}$$

28. The value of $\frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx \dots\dots\dots(1)$

$$\text{Using } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx \dots\dots\dots(2)$$

Adding (1) & (2)

$$2I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} 1 dx$$

$$I = 2$$

29. The shortest distance between the lines

$$\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2} \text{ and } \frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$$

is equal to _____

Official Ans. by NTA (14)

QuestPix Ans. (14)

Sol. Shortest distance between the lines

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}} = \frac{16 + 12 + 168}{-4\hat{i} + 6\hat{j} - 12\hat{k}}$$

$$= \frac{16 + 12 + 168}{14} = \frac{196}{14} = 14$$



30. The 4th term of GP is 500 and its common ratio is

$\frac{1}{m}$, $m \in \mathbb{N}$. Let S_n denote the sum of the first n

terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$,

then the number of possible values of m is _____

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $T_4 = 500$ where a = first term,

r = common ratio = $\frac{1}{m}$, $m \in \mathbb{N}$

$$ar^3 = 500$$

$$\frac{a}{m^3} = 500$$

$$S_n - S_{n-1} = ar^{n-1}$$

$$S_6 > S_5 + 1 \quad \text{and} \quad S_7 - S_6 < \frac{1}{2}$$

$$S_6 - S_5 > 1 \quad \frac{a}{m^6} < \frac{1}{2}$$

$$ar^5 > 1 \quad m^3 > 10^3$$

$$\frac{500}{m^2} > 1 \quad m > 10 \dots\dots(2)$$

$$m^2 < 500 \dots\dots(1)$$

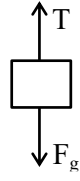
From (1) and (2)

$$m = 11, 12, 13, \dots\dots, 22$$

So number of possible values of m is 12

(Held On Tuesday 24th January, 2023)

TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS	TEST PAPER WITH SOLUTION
SECTION-A 31. Two long straight wires P and Q carrying equal current 10A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire P is F_1. If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire P will be : (1) $8 F_1$ (2) $10 F_1$ (3) $F_1/8$ (4) $F_1/10$ Official Ans. by NTA (1) QuestPix Ans. (1) Sol. Force per unit length between two parallel straight wires = $\frac{\mu_0 i_1 i_2}{2\pi d}$ $\frac{F_1}{F_2} = \frac{\mu_0 (10)^2}{2\pi (5\text{cm})} = \frac{1}{8}$ $\frac{F_1}{F_2} = \frac{\mu_0 (20)^2}{2\pi \left(\frac{5\text{cm}}{2}\right)}$ $\Rightarrow F_2 = 8F_1$ 32. Given below are two statements : Statement-I : An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable. Statement-II : Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed. In the light of the above statements, choose the correct answer from the options given below : (1) Both statement I and statement II are false (2) Statement I is true but Statement II is false (3) Both Statement I and Statement II are true (4) Statement I is false but Statement II is true Official Ans. by NTA (2) QuestPix Ans. (2) Sol. Statement-1 When elevator is moving with uniform speed $T = F_g$  Statement-2 When elevator is going down with increasing speed, its acceleration is downward. Hence	$W - N = \frac{W}{g} \times a$ $N = W \left(1 - \frac{a}{g}\right) \text{ i.e. less than weight.}$ 33. From the photoelectric effect experiment, following observations are made. Identify which of these are correct A. The stopping potential depends only on the work function of the metal. B. The saturation current increases as the intensity of incident light increases. C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light. D. Photoelectric effect can be explained using wave theory of light. Choose the correct answer from the options given below: (1) B, C only (2) A, C, D only (3) B only (4) A, B, D only Official Ans. by NTA (3) QuestPix Ans. (3) Sol. (A) Stopping potential depends on both frequency of light and work function. (B) Saturation current $\propto$ intensity of light (C) Maximum KE depends on frequency (D) Photoelectric effect is explained using particle theory 34. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $R_e = 6400$ km) (1) 9.8 N (2) 4.9 N (3) 19.6 N (4) 8 N Official Ans. by NTA (4) QuestPix Ans. (4) Sol. Acceleration due to gravity at height h $g' = \frac{g}{\left[1 + \frac{h}{R}\right]^2}$ So weight at given height $mg' = \frac{mg}{\left[1 + \frac{h}{R}\right]^2} = \frac{18}{\left[1 + \frac{3200}{6400}\right]^2} = 8\text{N}$



35. A 100 m long wire having cross-sectional area $6.25 \times 10^{-4} \text{ m}^2$ and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be :

- (1) $6.25 \times 10^{-3} \text{ m}$ (2) $4 \times 10^{-4} \text{ m}$
(3) $6.25 \times 10^{-6} \text{ m}$ (4) $4 \times 10^{-3} \text{ m}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Elongation in wire $\delta = \frac{F\ell}{AY}$

$$\delta = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}}$$

$$\delta = 4 \times 10^{-3} \text{ m}$$

36. 1g of a liquid is converted to vapour at $3 \times 10^5 \text{ Pa}$ pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be :

- (1) 4320 J (2) 432000 J
(3) 4800 J (4) $4.32 \times 10^8 \text{ J}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Work done = $P\Delta V$
 $= 3 \times 10^5 \times 1600 \times 10^{-6}$
 $= 480 \text{ J}$

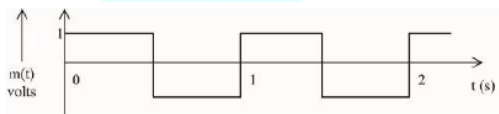
Only 10% of heat is used in work done.

Hence $\Delta Q = 4800 \text{ J}$

The rest goes in internal energy, which is 90% of heat.

Change in internal energy = $0.9 \times 4800 = 4320 \text{ J}$

37. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as $c(t) = 2 \sin(8\pi t)$ volts, the modulation index is :

- (1) 1/4 (2) 1
(3) 1/3 (4) 1/2

Official Ans. by NTA (4)

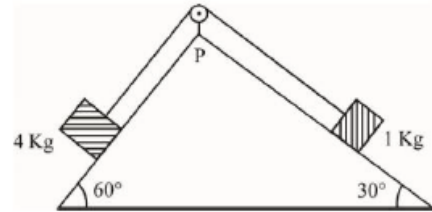
QuestPix Ans. (4)

Sol. Modulation index

$$= \frac{\text{Amplitude of modulating signal}}{\text{Amplitude of carrier wave}}$$

$$\mu = \frac{1}{2}$$

38. As per given figure, a weightless pulley P is attached on a double inclined frictionless surface. The tension in the string (massless) will be (if $g = 10 \text{ m/s}^2$)

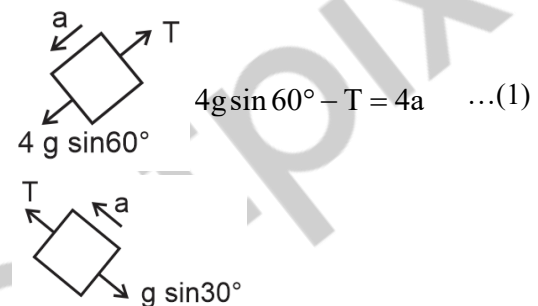


- (1) $(4\sqrt{3} + 1)\text{N}$ (2) $4(\sqrt{3} + 1)\text{N}$
(3) $4(\sqrt{3} - 1)\text{N}$ (4) $(4\sqrt{3} - 1)\text{N}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$$4g \sin 60^\circ - T = 4a \quad \dots(1)$$

$$T - g \sin 30^\circ = a \quad \dots(2)$$

Solving (1) and (2) we get.

$$20\sqrt{3} - T = 4T - 20$$

$$T = 4(\sqrt{3} + 1)\text{N}$$

39. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A : Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

Reason R : The current in the forward bias is more than the current in the reverse bias for a p - n junction diode.

In the light of the above statement, choose the correct answer from the options given below :

- (1) **A** is false but **R** is true
(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
(3) **A** is true but **R** is false
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Photodiodes are operated in reverse bias as fractional change in current due to light is more easy to detect in reverse bias.



40. If $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by : (ω - angular frequency) :

(1) $\frac{1}{\omega}(\vec{K} \times \vec{E})$ (2) $\omega(\vec{E} \times \vec{K})$

(3) $\omega(\vec{K} \times \vec{E})$ (4) $\vec{K} \times \vec{E}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

- Sol.** Magnetic field vector will be in the direction of $\hat{K} \times \hat{E}$

magnitude of $B = \frac{E}{C} = \frac{K}{\omega} E$

Or $\vec{B} = \frac{1}{\omega}(\vec{K} \times \vec{E})$

41. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the centre of circular loop and at a distance r from the center of the loop on its axis is :

(1) $1:3\sqrt{2}$ (2) $3\sqrt{2}:2$

(3) $2\sqrt{2}:1$ (4) $1:\sqrt{2}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

- Sol.** Magnetic field due to current carrying circular loop on its axis is given as

$$\frac{\mu_0 i r^2}{2(r^2 + x^2)^{3/2}}$$

At centre, $x = 0$, $B_1 = \frac{\mu_0 i}{2r}$

At $x = r$, $B_2 = \frac{\mu_0 i}{2 \times 2\sqrt{2}r}$

$$\frac{B_1}{B_2} = 2\sqrt{2}$$

42. A travelling wave is described by the equation $y(x, t) = [0.05 \sin(8x - 4t)]$ m. The velocity of the wave is : [all the quantities are in SI unit]

(1) 4 ms^{-1} (2) 2 ms^{-1}

(3) 0.5 ms^{-1} (4) 8 ms^{-1}

Official Ans. by NTA (3)

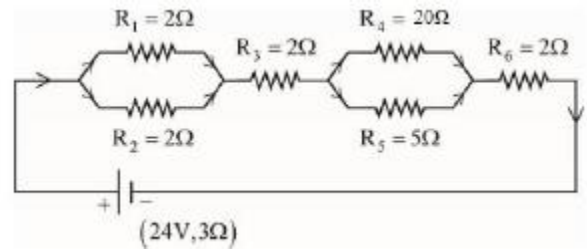
QuestPix Ans. (3)

- Sol.** From the given equation $k = 8 \text{ m}^{-1}$ and $\omega = 4 \text{ rad/s}$

Velocity of wave = $\frac{\omega}{k}$

$$v = \frac{4}{8} = 0.5 \text{ m/s}$$

43. As shown in the figure, a network of resistors is connected to a battery of 24 V with an internal resistance of 3Ω . The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are :



(1) $I_4 = \frac{8}{5} \text{ A}$ and $I_5 = \frac{2}{5} \text{ A}$

(2) $I_4 = \frac{24}{5} \text{ A}$ and $I_5 = \frac{6}{5} \text{ A}$

(3) $I_4 = \frac{6}{5} \text{ A}$ and $I_5 = \frac{24}{5} \text{ A}$

(4) $I_4 = \frac{2}{5} \text{ A}$ and $I_5 = \frac{8}{5} \text{ A}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

- Sol.** Equivalent resistance of circuit

$$R_{eq} = 3 + 1 + 2 + 4 + 2 = 12\Omega$$

Current through battery $i = \frac{24}{12} = 2 \text{ A}$

$$I_4 = \frac{R_5}{R_4 + R_5} \times 2 = \frac{5}{20 + 5} \times 2 = \frac{2}{5} \text{ A}$$

$$I_5 = 2 - \frac{2}{5} = \frac{8}{5} \text{ A}$$

44. Given below are two statements :

Statement I : If the Brewster's angle for the light propagating from air to glass is θ_B , then Brewster's angle for the light propagating from

glass to air is $\frac{\pi}{2} - \theta_B$.

Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statements I and Statement II are true.

(2) Statement I is true but Statement II is false.

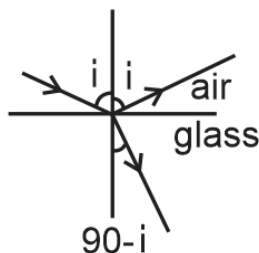
(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$$\mu_a \sin i_1 = \mu_g \sin(90 - i_1)$$

$$\tan i_1 = \frac{\mu_g}{\mu_a}$$

When going from glass to air

$$\tan i_2 = \frac{\mu_a}{\mu_g} = \cot i_1$$

Hence

$$i_2 = \frac{\pi}{2} - i_1$$

45. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force ?

- (1) $d\sqrt{k}$ (2) $k\sqrt{d}$
(3) $1.5d\sqrt{k}$ (4) $2d\sqrt{k}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{kd^2}$ (in medium)

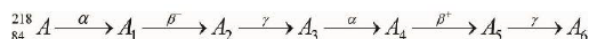
$$F_{\text{Air}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d'^2}$$

$$F = F_{\text{Air}}$$

$$\frac{q_1 q_2}{4\pi\epsilon_0 kd^2} = \frac{q_1 q_2}{4\pi\epsilon_0 d'^2}$$

$$d' = d\sqrt{k}$$

46. Consider the following radioactive decay process

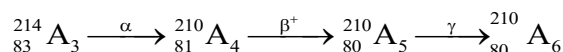
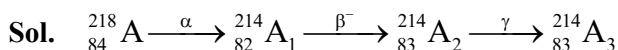


The mass number and the atomic number A_6 are given by :

- (1) 210 and 82 (2) 210 and 84
(3) 210 and 80 (4) 211 and 80

Official Ans. by NTA (3)

QuestPix Ans. (3)



47. Given below are two statements :

Statements I : The temperature of a gas is -73°C . When the gas is heated to 527°C , the root mean square speed of the molecules is doubled.

Statement II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the **correct** answer from the options given below :

- (1) Both statement I and Statement II are true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Official Ans. by NTA (2)

QuestPix Ans. (2)

- Sol. **Statement-I**

$$T_1 = -73^\circ\text{C} = 200\text{ K}$$

$$T_2 = 527^\circ\text{C} = 800\text{ K}$$

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_2}{M}}} = \sqrt{\frac{T_1}{T_2}}$$

$$= \sqrt{\frac{200}{800}} = \frac{1}{2}$$

$$V_2 = 2V_1 \text{ (True)}$$

Statement-II

$$PV = nRT$$

$$\text{Translational KE} = \frac{3}{2}nRT \text{ (False)}$$

48. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is

- (1) 192 m (2) 136 m
(3) 272 m (4) 68 m

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $H_{\text{max}} = \frac{v^2}{2g} = 136\text{m}$

$$R_{\text{max}} = \frac{v^2}{g} = 2H_{\text{max}}$$

$$= 2(136)$$

$$= 272\text{ m}$$

49. A conducting loop of radius $\frac{10}{\sqrt{\pi}}\text{cm}$ is placed

perpendicular to a uniform magnetic field of 0.5T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

- (1) emf = 1 mV (2) emf = 10 mV
(3) emf = 100 mV (4) emf = 5 mV

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $EMF = \frac{d\phi}{dt} = \frac{BA - 0}{t}$

$$A = \pi r^2 = \pi \left(\frac{0.1^2}{\pi} \right) = 0.01$$

$$B = 0.5$$

$$EMF = \frac{(0.5)(0.01)}{0.5} = 0.01V = 10 \text{ mV}$$

50. Match List I with List II

LIST I		LIST II	
A.	Planck's constant (h)	I.	$[M^1 L^2 T^{-2}]$
B.	Stopping potential (Vs)	II.	$[M^1 L^1 T^{-1}]$
C.	Work function (ϕ)	III.	$[M^1 L^2 T^{-1}]$
D.	Momentum (p)	IV.	$[M^1 L^2 T^{-3} A^{-1}]$

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-IV, C-III, D-I

(4) A-I, B-III, C-IV, D-II

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (A) Planck's constant

$$h\nu = E$$

$$h = \frac{E}{\nu} = \frac{M^1 L^2 T^{-2}}{T^{-1}} = M^1 L^2 T^{-1} \quad \text{(III)}$$

(B) $E = qV$

$$V = \frac{E}{q} = \frac{M^1 L^2 T^{-2}}{A^1 T^1} = M^1 L^2 T^{-3} A^{-1} \quad \text{(IV)}$$

(C) ϕ (work function) = energy

$$= M^1 L^2 T^{-2} \quad \text{(I)}$$

(D) Momentum (p) = $F \cdot t$

$$= M^1 L^1 T^{-2} T^1$$

$$= M^1 L^1 T^{-1} \quad \text{(II)}$$

SECTION-B

51. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is ____ N.

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. $\frac{1}{2} \times 2 \times v^2 = 10000$

$$\Rightarrow v^2 = 10000$$

$$\Rightarrow v = 100 \text{ m/s}$$

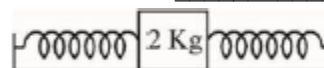
$$\Rightarrow v = at = a \times 5 = 100$$

$$\Rightarrow a = 20 \text{ m/s}^2$$

$$F = ma = 2 \times 20 = 40 \text{ N}$$

52. A block of mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic

motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is _____.



Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $F = -2kx$, $a = -\frac{2kx}{m}$, $\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \times 20}{2}}$

$$= \sqrt{20} \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$$

$$x = 5$$

53. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be _____ if coefficient of linear expansion of the metal is $1.6 \times 10^{-5}/^\circ\text{C}$.

Official Ans. by NTA (12)

QuestPix Ans. (12)

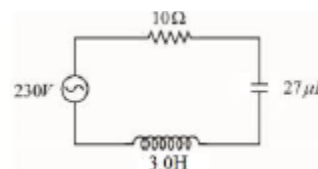
Sol. d_0 at 27°C & d_1 at 177°C

$$d_1 = d_0 (1 + \alpha \Delta T)$$

$$d_1 - d_0 = 5 \times 1.6 \times 10^{-5} \times 150 \text{ cm}$$

$$= 12 \times 10^{-3} \text{ cm}$$

54. In the circuit shown in the figure, the ratio of the quality factor and the band width is ____ s.



Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. $\Delta\omega = \frac{R}{L}$

$$Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \frac{L}{R}$$

$$\omega_0 = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{1}{9 \times 10^{-3}}$$

$$\frac{Q}{\Delta\omega} = \frac{\omega_0 \frac{L}{R}}{\frac{R}{L}} = \omega_0 \frac{L^2}{R^2} = \sqrt{\frac{1}{LC}} \frac{L^2}{R^2}$$

$$= \frac{1}{9 \times 10^{-3}} \times \frac{9}{100} = 10 \text{ s}$$

55. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n \times 10^{-3} \Omega$.

If the resistivity of the material is $2.4 \times 10^{-8} \Omega\text{m}$. The value of n is _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

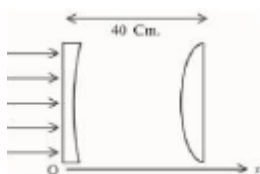
Sol. $R = \rho \frac{\ell}{A}$, the cross-sectional area is $\pi(b^2 - a^2)$

$$R = \rho \frac{\ell}{\pi(b^2 - a^2)} = \frac{2.4 \times 10^{-8} \times 3.14}{3.14 \times (4^2 - 2^2) \times 10^{-6}}$$

$$= 2 \times 10^{-3} \Omega$$

$$\rightarrow n = 2$$

56. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance $x =$ _____ cm, from concave lens.



Official Ans. by NTA (120)

QuestPix Ans. (120)

Sol. $\frac{1}{f_1} = (1.75 - 1) \left(-\frac{1}{30} \right)$

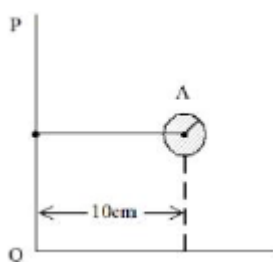
$$\Rightarrow f_1 = -40 \text{ cm}$$

$$\frac{1}{f_2} = (1.75 - 1) \left(\frac{1}{30} \right) \Rightarrow f_2 = 40 \text{ cm}$$

Image from L_1 will be virtual and on the left of L_2 at focal length 40 cm. So the object for L_2 will be 80 cm from L_2 which is $2f$. Final image is formed at 80 cm from L_2 on the right.

So $x = 120$

57. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is _____.



Official Ans. by NTA (110)

QuestPix Ans. (110)

Sol. $I_{cm} = \frac{2}{5} MR^2$

$$I_{PQ} = I_{cm} + mR^2$$

$$I_{PQ} = \frac{2}{5} mR^2 + m(10\text{cm})^2$$

For radius of gyration

$$I_{PQ} = mk^2$$

$$k^2 = \frac{2}{5} R^2 + (10\text{cm})^2$$

$$= \frac{2}{5} (5)^2 + 100$$

$$= 10 + 100 = 110$$

$$k = \sqrt{110} \text{ cm}$$

$$x = 110$$

58. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _____.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. For two perpendicular vectors

$$(a\hat{i} + b\hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$$

$$2a - 3b + 4 = 0$$

On solving, $2a - 3b = -4$

Also given

$$3a + 2b = 7$$

We get $a = 1, b = 2$

$$\frac{a}{b} = \frac{x}{2} \Rightarrow x = \frac{2a}{b} = \frac{2 \times 1}{2}$$

$$\Rightarrow x = 1$$

59. Assume that protons and neutrons have equal masses. Mass of a nucleon is $1.6 \times 10^{-27} \text{ kg}$ and radius of nucleus is $1.5 \times 10^{-15} \text{ A}^{1/3} \text{ m}$. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is _____.

Official Ans. by NTA (11)

QuestPix Ans. (11)

Sol. density of nuclei = $\frac{\text{mass of nuclei}}{\text{volume of nuclei}}$

$$\rho = \frac{1.6 \times 10^{-27} \text{ A}}{\frac{4}{3} \pi (1.5 \times 10^{-15})^3 \text{ A}}$$

$$= \frac{1.6 \times 10^{-27}}{14.14 \times 10^{-45}} = 0.113 \times 10^{18}$$

$$\rho_w = 10^3$$

$$\frac{\rho}{\rho_w} = 11.31 \times 10^{13}$$

Hence ρ_w

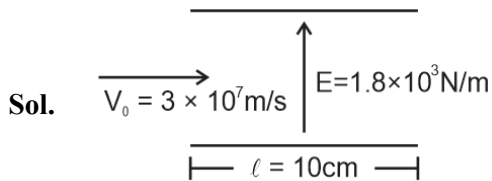


60. A stream of a positively charged particles having $\frac{q}{m} = 2 \times 10^{11} \frac{\text{C}}{\text{kg}}$ and velocity $\vec{v}_0 = 3 \times 10^7 \hat{i} \text{ m/s}$

is deflected by an electric field $1.8\hat{j}\text{kV/m}$. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is _____ mm.

Official Ans. by NTA (2)

QuestPix Ans. (2)



$$a = \frac{F}{m} = \frac{qE}{m} = (2 \times 10^{11})(1.8 \times 10^3)$$

$$= 3.6 \times 10^{14} \text{ m/s}^2$$

$$\text{Time to cross plates} = \frac{d}{v}$$

$$t = \frac{0.10}{3 \times 10^7}$$

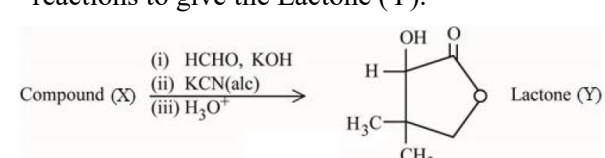
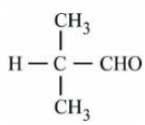
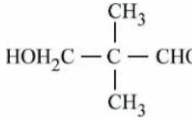
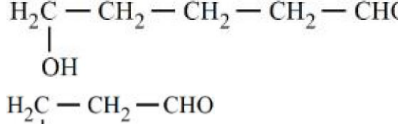
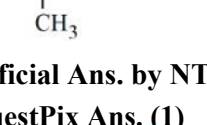
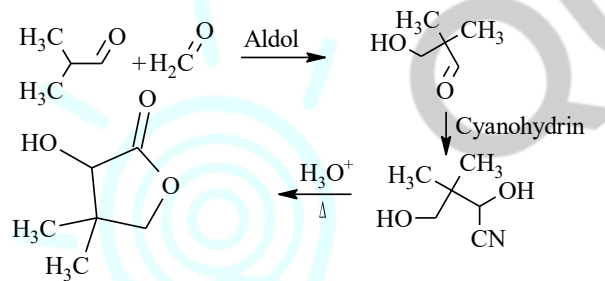
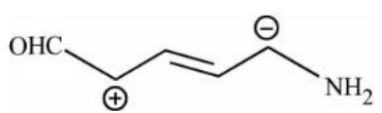
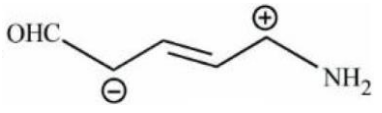
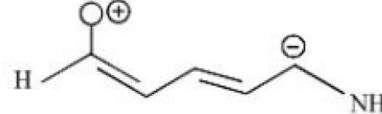
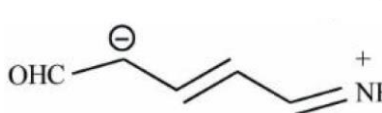
$$y = \frac{1}{2}at^2 = \frac{1}{2}(3.6 \times 10^{14})\left(\frac{0.01}{9 \times 10^{14}}\right)$$

$$= 0.2 \times 0.01$$

$$= 0.002 \text{ m}$$

$$= 2 \text{ mm}$$

(Held On Tuesday 24th January, 2023)
TIME : 9 : 00 AM to 12 : 00 NOON

CHEMISTRY	TEST PAPER WITH SOLUTION
SECTION-A 61. Compound (X) undergoes following sequence of reactions to give the Lactone (Y).  (1)  (2)  (3)  (4)  Official Ans. by NTA (1) QuestPix Ans. (1) Sol.  62. Assertion A : Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases. Reason R : I⁻ is a good nucleophile as well as a good leaving group. In the light of the above statements, choose the correct answer from the options given below. (1) A is false but R is true (2) A is true but R is false (3) Both A and R are true and R is the correct explanation of A (4) Both A and R are true but R is NOT the correct explanation of A Official Ans. by NTA (3) QuestPix Ans. (3)	Sol. The rate of hydrolysis of alkyl chloride improves because of better Nucleophilicity of I⁻. 63. Order of Covalent bond; A. KF > KI; LiF > KF B. KF < KI; LiF > KF C. SnCl₄ > SnCl₂; CuCl > NaCl D. LiF > KF; CuCl < NaCl E. KF < KI; CuCl > NaCl (1) C, E only (2) B, C only (3) B, C, E only (4) A, B only Official Ans. by NTA (3) QuestPix Ans. (3) Sol. According to Fajan's Rule, A. KF > KI – False; LiF > KF – True B. KF < KI – True; LiF > KF – True C. SnCl₄ > SnCl₂ – True; CuCl > NaCl – True D. LiF > KF – True; CuCl < NaCl – False E. KF < KI – True; CuCl > NaCl – True 64. Increasing order of stability of the resonance structure is : A.  B.  C.  D.  (1) C, D, B, A (2) C, D, A, B (3) D, C, A, B (4) D, C, B, A Official Ans. by NTA (2) QuestPix Ans. (BONUS) Sol. No option is matching the correct answer.

Order should be : $C < A < B < D$

65. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is

- (1) Cr^{2+} (2) Mn^{2+}
(3) V^{2+} (4) Ti^{2+}

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\text{Cr}^{2+} : [\text{Ar}], 3d^4, 4s^0$ $n = 4$, $\mu = \sqrt{4(4+2)} = \sqrt{24}$
= 4.89 BM

$\text{Mn}^{2+} : [\text{Ar}], 3d^5, 4s^0$ $n = 5$, $\mu = \sqrt{5(5+2)} = \sqrt{35}$
= 5.91 BM

$\text{V}^{2+} : [\text{Ar}], 3d^3, 4s^0$ $n = 3$, $\mu = \sqrt{3(3+2)} = \sqrt{15}$
= 3.87 BM

$\text{Ti}^{2+} : [\text{Ar}], 3d^2, 4s^0$ $n = 2$, $\mu = \sqrt{2(2+2)} = \sqrt{8}$
= 2.82 BM

66. Match List I with List II.

LIST I		LIST II	
A.	Reverberatory furnace	I.	Pig Iron
B.	Electrolytic cell	II.	Aluminum
C.	Blast furnace	III.	Silicon
D.	Zone Refining furnace	IV.	Copper

- (1) A – IV, B – II, C – I, D – III
(2) A – I, B – IV, C – II, D – III
(3) A – I, B – III, C – II, D – IV
(4) A – III, B – IV, C – I, D – II

Official Ans. by NTA (1)

QuestPix Ans. (1)

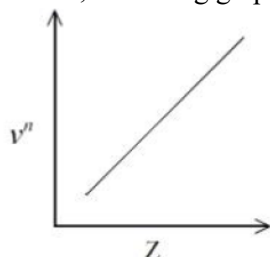
Sol. Reverberatory furnace: Used for roasting of Copper.

Electrolytic cell : For reactive metal : Al

Blast furnace : Hematite to Pig Iron

Zone Refining furnace: For semiconductors : Si

67. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power 'n' i.e. ν^n of X-rays emitted is plotted against atomic number 'Z', following graph is obtained.



The value of 'n' is

- (1) 1 (2) 2

- (3) $\frac{1}{2}$ (4) 3

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. According to Henry Moseley $\sqrt{\nu} \propto Z - b$

$$\text{So } n = \frac{1}{2}$$

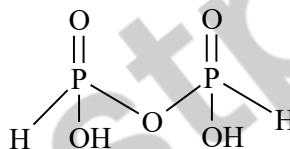
68. Which of the Phosphorus oxoacid can create silver mirror from AgNO_3 solution ?

- (1) $(\text{HPO}_3)_n$
(2) $\text{H}_4\text{P}_2\text{O}_5$
(3) $\text{H}_4\text{P}_2\text{O}_6$
(4) $\text{H}_4\text{P}_2\text{O}_7$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



Oxyacid having P–H bond can reduce AgNO_3 to Ag.

69. The primary and secondary valencies of cobalt respectively in $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$ are :

- (1) 3 and 5
(2) 2 and 6
(3) 2 and 8
(4) 3 and 6

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$

Oxidation number of Co is +3.

So primary valency is 3.

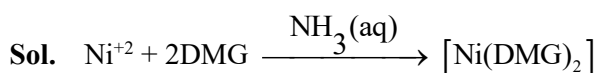
It is an octahedral complex so secondary valency 6 or Co-ordination number 6.

70. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is :

- (1) Cu^{2+}
(2) Co^{2+}
(3) Fe^{2+}
(4) Ni^{2+}

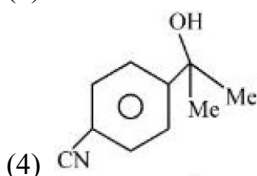
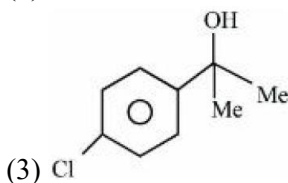
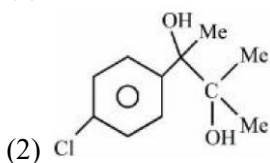
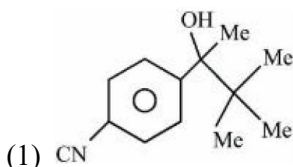
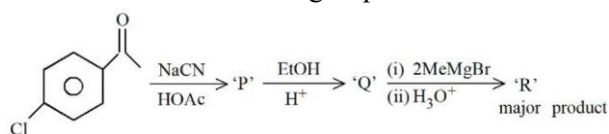
Official Ans. by NTA (4)

QuestPix Ans. (4)



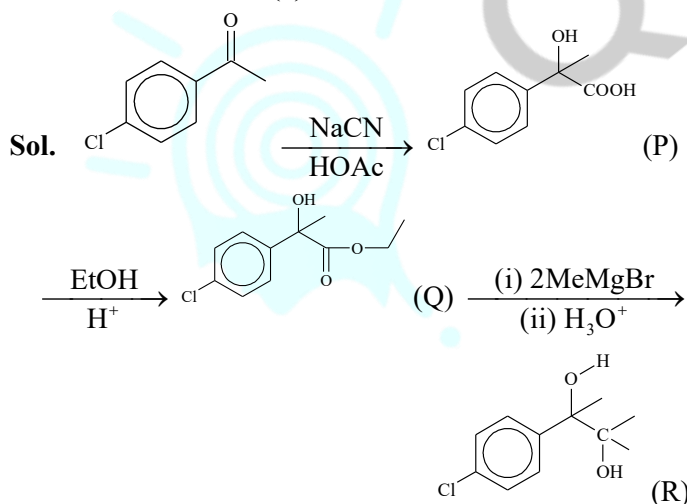
Rosy Red complex

71. 'R' formed in the following sequence of reaction is:



Official Ans. by NTA (2)

QuestPix Ans. (2)



72. Match List I with List II.

LIST I		LIST II	
A.	Chlorophyll	I.	Na_2CO_3
B.	Soda ash	II.	CaSO_4
C.	Dentistry, Ornamental work	III.	Mg^{2+}
D.	Used in white washing	IV.	$\text{Ca}(\text{OH})_2$

Choose the correct answer from the options given below :

(1) A – III, B – I, C – II, D – IV

(2) A – II, B – I, C – III, D – IV

(3) A – III, B – IV, C – I, D – II

(4) A – II, B – III, C – IV, D – I

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Chlorophyll : Mg^{+2} complex

Soda ash : Na_2CO_3

Dentistry, Ornamental work : CaSO_4

Used in white washing : $\text{Ca}(\text{OH})_2$

73. **Statement I :** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.

Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. **Statement I :** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration. : True

Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential. : True

74. Reaction of BeO with ammonia and hydrogen fluoride gives 'A' which on thermal decomposition gives BeF_2 and NH_4F . What is 'A' ?

(1) $(\text{NH}_4)_2\text{BeF}_4$

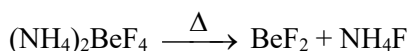
(2) H_3NBeF_3

(3) $(\text{NH}_4)\text{BeF}_3$

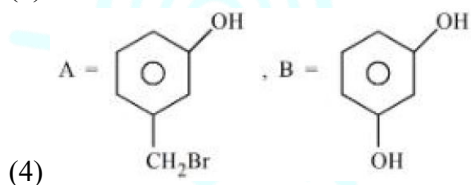
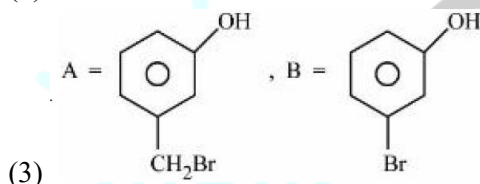
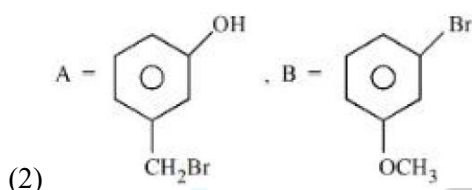
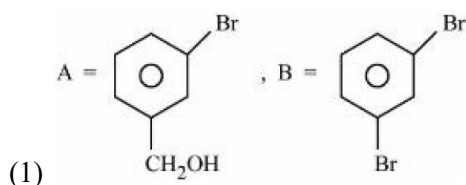
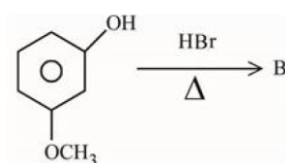
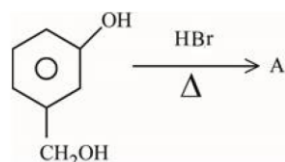
(4) $(\text{NH}_4)\text{Be}_2\text{F}_5$

Official Ans. by NTA (1)

QuestPix Ans. (1)

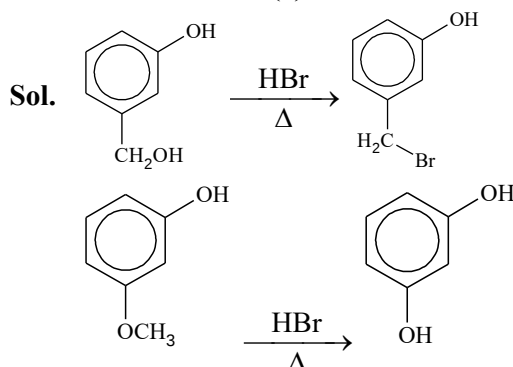


75. 'A' and 'B' formed in the following set of reactions are :

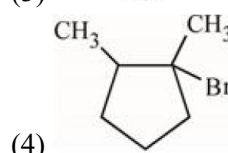
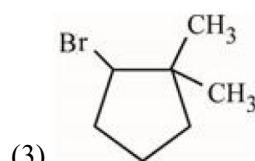
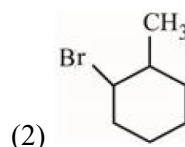
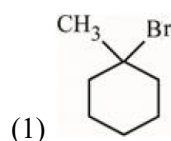
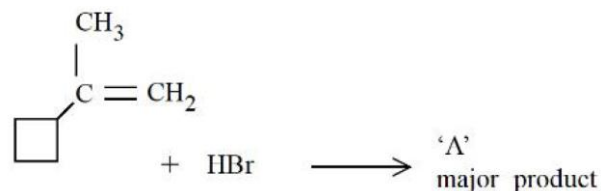


Official Ans. by NTA (4)

QuestPix Ans. (4)

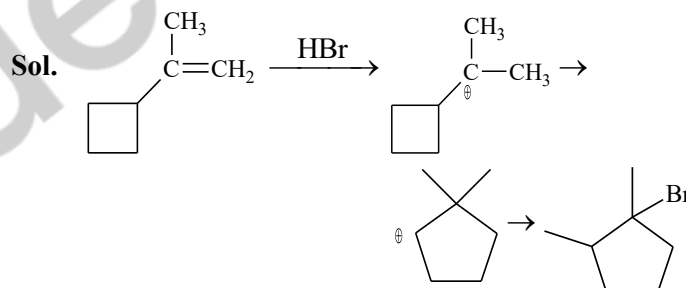


76. In the following given reaction 'A' is



Official Ans. by NTA (4)

QuestPix Ans. (4)



77. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by

- A. Liquid water
- B. Ice
- C. Impure water

- (1) $\text{A} = \text{B} > \text{C}$
- (2) $\text{B} > \text{A} > \text{C}$
- (3) $\text{C} > \text{B} > \text{A}$
- (4) $\text{A} > \text{B} > \text{C}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\text{Ice} > \text{Liquid water} > \text{Impure water}$

Due to impurity extent of H-Bonding decreases.

78. Given below are two statements :

Statement I : Noradrenaline is a neurotransmitter.

Statement II : Low level of noradrenaline is not the cause of depression in human.

In the light of the above statements, choose the correct answer from the options given below

- (1) Statement I is correct but Statement II is incorrect
- (2) Statement I is incorrect but Statement II is correct
- (3) Both Statement I and Statement II are correct
- (4) Both Statement I and Statement II are incorrect

Official Ans. by NTA (1)

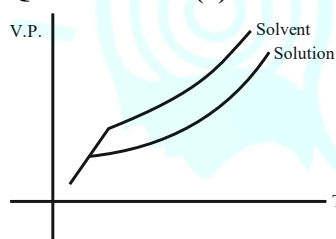
QuestPix Ans. (1)

Sol. Fact

79. In the depression of freezing point experiment
- A. Vapour pressure of the solution is less than that of pure solvent
 - B. Vapour pressure of the solution is more than that of pure solvent
 - C. Only solute molecules solidify at the freezing point
 - D. Only solvent molecules solidify at the freezing point
- (1) A and D only
 - (2) B and C only
 - (3) A and C only
 - (4) A only

Official Ans. by NTA (1)

QuestPix Ans. (1)



Sol.

Vapour pressure (V.P.) of solvent is greater than vapour pressure (V.P.) of solution.

Only solvent freezes.

80. Which of the following is true about freons ?
- (1) These are chlorofluorocarbon compounds
 - (2) These are chemicals causing skin cancer
 - (3) These are radicals of chlorine and chlorine monoxide
 - (4) All radicals are called freons

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Fact

SECTION-B

81. The dissociation constant of acetic is $x \times 10^{-5}$. When 25 mL of 0.2 M CH_3COONa solution is mixed with 25 mL of 0.02 M CH_3COOH solution, the pH of the resultant solution is found to be equal to 5. The value of x is _____.

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. Buffer of HOAc and NaOAc

$$\text{pH} = \text{pK}_a + \log \frac{0.1}{0.01}$$

$$5 = \text{pK}_a + 1$$

$$\text{pK}_a = 4$$

$$\text{K}_a = 10^{-4}$$

$$x = 10$$

82. 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1 M solution ?

Given : Molar Mass of Na, O and H is 23, 16 and 1 g mol⁻¹ respectively

Official Ans. by NTA (180)

QuestPix Ans. (180)

Sol. $M = \frac{5}{40} \times \frac{1000}{450}$

$$M_1V_1 = M_2V_2$$

$$\left(\frac{5}{40} \times \frac{1000}{450} \right) \times V_1 = 0.1 \times 500$$

$$V_1 = 180$$

83. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is _____ nm. (Nearest integer)

Official Ans. by NTA (492)

QuestPix Ans. (492)

Sol. $\frac{1}{(\lambda_1)_p} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{16} \right)$
 $\frac{1}{(\lambda_2)_p} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{25} \right)$
 $\frac{(\lambda_2)_p}{(\lambda_1)_p} = \frac{7}{16 \times 9} = \frac{25 \times 7}{16 \times 16}$
 $(\lambda_2)_p = \frac{25 \times 7}{16 \times 16} \times 720$
 $(\lambda_2)_p = 492 \text{ nm}$

- 84.** The number of correct statement/s from the following is _____.

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperature.

D. A plot of $\ln k$ vs $\frac{1}{T}$ is a straight line with slope

equal to $-\frac{E_a}{R}$

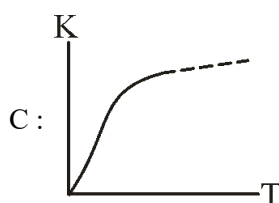
Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. **A :** $k = Ae^{-\frac{E_a}{RT}}$

As E_a increases k decreases

B : Temperature coefficient = $\frac{k_{T+10}}{k_T}$



Option (C) is wrong. Δk may be greater or lesser depending on temperature.

D : $\ln k = \ln A - \frac{E_a}{RT}$

- 85.** At 298 K, a 1 litre solution containing 10 mmol of $\text{Cr}_2\text{O}_7^{2-}$ and 100 mmol of Cr^{3+} shows a pH of 3.0.

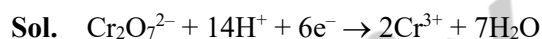
Given : $\text{Cr}_2\text{O}_7^{2-} \rightarrow \text{Cr}^{3+}$; $E^0 = 1.330 \text{ V}$ and

$\frac{2.303 RT}{F} = 0.059 \text{ V}$

The potential for the half cell reaction is $x \times 10^{-3} \text{ V}$. The value of x is _____.

Official Ans. by NTA (917)

QuestPix Ans. (917)



$E = 1.33 - \frac{0.059}{6} \log \frac{(0.1)^2}{(10^{-2})(10^{-3})^{14}}$

$E = 1.33 - \frac{0.059}{6} \times 42 = 0.917$

$E = 917 \times 10^{-3}$

$x = 917$

- 86.** When $\text{Fe}_{0.93}\text{O}$ is heated in presence of oxygen, it converts to Fe_2O_3 . The number of correct statement/s from the following is _____.

A. The equivalent weight of $\text{Fe}_{0.93}\text{O}$ is $\frac{\text{Molecular weight}}{0.79}$.

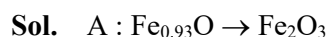
B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $\text{Fe}_{0.93}\text{O}$ is 0.79 and 0.14 respectively.

C. $\text{Fe}_{0.93}\text{O}$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions.

D. The % composition of Fe^{2+} and Fe^{3+} in $\text{Fe}_{0.93}\text{O}$ is 85% and 15% respectively.

Official Ans. by NTA (4)

QuestPix Ans. (4)



$nf = \left(3 - \frac{200}{93} \right) \times 0.93$

$nf = 0.79$

B : $2x + (0.93 - x) \times 3 = 2$

$x = 0.79$

$\text{Fe}^{2+} = 0.79, \text{Fe}^{3+} = 0.21$

C : Fact

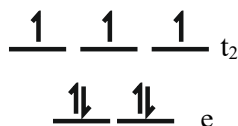
D : $\% \text{Fe}^{2+} = \frac{0.79}{0.93} \times 100 = 85\%; \text{Fe}^{3+} = 15\%$

87. The d-electronic configuration of $[\text{CoCl}_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of 'm' and 'number of unpaired electrons' is _____.

Official Ans. by NTA (7)

QuestPix Ans. (7)

Sol. $\text{Co}^{2+} : 3d^7 4s^0$, $\text{Cl}^- : \text{WFL}$



Configuration $e^4 t_2^3 : m = 4$

Number of unpaired electrons = 3

So, answer = 7

88. For independent process at 300 K.

Process	$\Delta H / \text{kJ mol}^{-1}$	$\Delta S / \text{J K}^{-1}$
A	-25	-80
B	-22	40
C	25	-50
D	22	20

The number of non-spontaneous process from the following is _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\Delta G = \Delta H - T\Delta S$

$$A : \Delta G (\text{J mol}^{-1}) = -25 \times 10^3 + 80 \times 300 : -ve$$

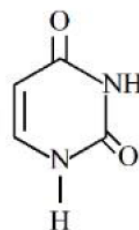
$$B : \Delta G (\text{J mol}^{-1}) = -22 \times 10^3 - 40 \times 300 : -ve$$

$$C : \Delta G (\text{J mol}^{-1}) = 25 \times 10^3 + 300 \times 50 : +ve$$

$$D : \Delta G (\text{J mol}^{-1}) = 22 \times 10^3 - 20 \times 300 : +ve$$

Processes C and D are non-spontaneous.

89. Uracil is base present in RNA with the following structure. % of N in uracil is _____.



Given :

Molar mass N = 14 g mol^{-1} ; O = 16 g mol^{-1} ; C = 12 g mol^{-1} ; H = 1 g mol^{-1} ;

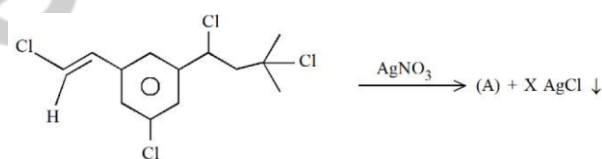
Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol. Mol. Wt of $\text{C}_4\text{N}_2\text{H}_4\text{O}_2 = 112$

$$\% \text{N} = \frac{28}{112} \times 100 = 25\%$$

90. Number of moles of AgCl formed in the following reaction is _____.



Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Benzylic and tertiary carbocations are stable.