(Held On Monday 30th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

(1) $((\sim P)\vee \sim Q)\wedge((\sim P)\vee R)$

(2) $((\sim P)\vee \sim Q)\wedge((\sim P)\vee \sim R)$

(3) $(P\vee Q)\wedge((\sim P)\vee R)$

(4) $(P\vee \sim Q)\wedge(P\vee \sim R)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $P\rightarrow(\sim Q\wedge R)$

$\sim P\vee(\sim Q\wedge R)$

$(\sim P\vee \sim Q)\wedge(\sim P\vee R)$

2. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 64 (2) 76

(3) 81 (4) 72

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $y = mx + \frac{4}{m}$

$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = 2\sqrt{2} \therefore m = \pm 1$

$y = \pm x \pm 4$. Point of contact on parabola

Let $m = 1, \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

R (4, 8)

Point of contact on circle Q (–2, 2)

$\therefore (QR)^2 = 36 + 36 = 72$

3. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y) : 0 \le y \le (x - q)^2, 0 \le x \le q\}$ is

(1) 243 (2) 25

(3) $\frac{125}{3}$ (4) 164

Official Ans. by NTA (1)

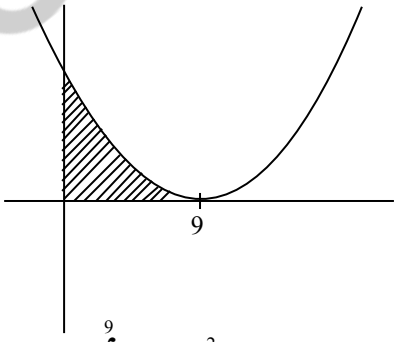
QuestPix Ans. (1)

Sol. $x^2 - px + \frac{5p}{4} = 0$

$D = p^2 - 5p = p(p-5)$

$\therefore q = 9$

$0 \le y \le (x-9)^2$



$\text{Area} = \int_0^9 (x-9)^2\,dx = 243$

4. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \ne 2b$ have a common extreme point, then a + 2b + 7 is equal to

(1) 4

(2) $\frac{3}{2}$

(3) 3

(4) 6

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $f'(x) = x^2 + 2b + ax$

$g'(x) = x^2 + a + 2bx$

$(2b - a) - x(2b - a) = 0$

$\therefore x = 1$ is the common root

Put $x = 1$ in $f'(x) = 0$ or $g'(x) = 0$

$1 + 2b + a = 0$

$7 + 2b + a = 6$

5. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is

(1) $\left[\sqrt{5}, \sqrt{10}\right]$

(2) $\left[2\sqrt{2}, \sqrt{11}\right]$

(3) $\left[\sqrt{5}, \sqrt{13}\right]$

(4) $\left[\sqrt{2}, \sqrt{7}\right]$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $y^2 = 3 - x + 2 + x + 2\sqrt{(3-x)(2+x)}$

$= 5 + 2\sqrt{6 + x - x^2}$

$y^2 = 5 + 2\sqrt{\frac{25}{4} - \left(x - \frac{1}{2}\right)^2}$

$y_{max} = \sqrt{5+5} = \sqrt{10}$

$y_{min} = \sqrt{5}$

6. The solution of the differential equation

$\frac{dy}{dx} = -\left(\frac{x^2 + 3y^2}{3x^2 + y^2}\right)$, $y(1) = 0$ is

(1) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$

(2) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

(3) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(4) $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Put $y = vx$

$v + x\frac{dv}{dx} = -\left(\frac{1+3v^2}{3+v^2}\right)$

$x\frac{dv}{dx} = -\frac{(v+1)^3}{3+v^2}$

$\frac{(3+v^2)dv}{(v+1)^3} + \frac{dx}{x} = 0$

$\int\frac{4dv}{(v+1)^3} + \int\frac{dv}{v+1} - \int\frac{2dv}{(v+1)^2} + \int\frac{dx}{x} = 0$

$\frac{-2}{(v+1)^2} + \ln(v+1) + \frac{2}{v+1} + \ln x = c$

$\frac{-2x^2}{(x+y)^2} + \ln\left(\frac{x+y}{x}\right) + \frac{2x}{x+y} + \ln x = c$

$\frac{2xy}{(x+y)^2} + \ln(x+y) = c$

$\therefore c = 0$, as $x = 1, y = 0$

$\therefore \frac{2xy}{(x+y)^2} + \ln(x+y) = 0$

7. Let $x = \left(8\sqrt{3} + 13\right)^{13}$ and $y = \left(7\sqrt{2} + 9\right)^9$. If [t] denotes the greatest integer $\le$ t, then

(1) [x] + [y] is even

(2) [x] is odd but [y] is even

(3) [x] is even but [y] is odd

(4) [x] and [y] are both odd

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $x = \left(8\sqrt{3} + 13\right) = {}^{13}C_0 \cdot \left(8\sqrt{3}\right)^{13} + {}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1 + \ldots$

$x' = \left(8\sqrt{3} - 13\right)^{13} = {}^{13}C_0\left(8\sqrt{3}\right)^{13} - {}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1 + \ldots$

$x - x' = 2\left[{}^{13}C_1 \cdot \left(8\sqrt{3}\right)^{12}(13)^1 + {}^{13}C_3\left(8\sqrt{3}\right)^{10} \cdot (13)^3 \ldots.\right]$

therefore, $x - x'$ is even integer, hence [x] is even

Now, $y = \left(7\sqrt{2} + 9\right)^9 = {}^9C_0\left(7\sqrt{2}\right)^9 + {}^9C_1\left(7\sqrt{2}\right)^8(9)^1$

$+ {}^9C_2\left(7\sqrt{2}\right)^7(9)^2 \ldots..$

$y' = \left(7\sqrt{2} - 9\right)^9 = {}^9C_0\left(7\sqrt{2}\right)^9 - {}^9C_1\left(7\sqrt{2}\right)^8(9)^1$

$+ {}^9C_2\left(7\sqrt{2}\right)^7(9)^2 \ldots..$

$y - y' = 2\left[{}^9C_1\left(7\sqrt{2}\right)^8(9)^1 + {}^9C_3\left(7\sqrt{2}\right)^6(9)^3 + \ldots\right]$

$y - y' =$ Even integer, hence [y] is even

8. A vector $\vec{v}$ in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2},-1,1\right)$ and (a, b, c), is normal to $\vec{v}$, then

(1) $\sqrt{2}a+b+c=1$

(2) $a+b+\sqrt{2}c=1$

(3) $a+\sqrt{2}b+c=1$

(4) $\sqrt{2}a-b+c=1$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\hat{v}=\cos 60°\hat{i}+\cos 45°\hat{j}+\cos\gamma\hat{k}$

$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1 \quad (\gamma \to \text{Acute})$

$\Rightarrow \cos\gamma=\frac{1}{2}$

$\Rightarrow \boxed{\gamma=60°}$

Equation of plane is

$\frac{1}{2}\left(x-\sqrt{2}\right)+\frac{1}{\sqrt{2}}(y+1)+\frac{1}{2}(z-1)=0$

$\Rightarrow x+\sqrt{2}y+z=1$

(a, b, c) lies on it.

$\Rightarrow a+\sqrt{2}b+c=1$

9. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases}$,

$g(x)=\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x\neq -1\\ 1, & x=-1\end{cases}$ and $h(x)=2[x]-f(x)$,

where [x] is the greatest integer ≤ x. Then the value of $\lim\limits_{x\to 1} g\left(h\left(x-1\right)\right)$ is

(1) 1

(2) sin(1)

(3) –1

(4) 0

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $LHL=\lim\limits_{k\to 0} g\left(h\left(-k\right)\right) \quad ,k>0$

$=\lim\limits_{k\to 0} g\left(-2+1\right) \quad \because f(x)=-1\ \forall\ x<0$

$= g(-1) = 1$

$RHL=\lim\limits_{k\to 0} g\left(h\left(k\right)\right) \quad ,k>0$

$=\lim\limits_{k\to 0} g\left(-1\right) \quad ,\because f(x)=1,\ \forall\ x>0$

$= 1$

10. The number of ways of selecting two numbers a and b, $a\in\{2,4,6,.....,100\}$ and $b\in\{1,3,5,.....,99\}$ such that 2 is the remainder when a + b is divided by 23 is

(1) 186 (2) 54

(3) 108 (4) 268

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $a\in\{2,4,6,8,10,....,100\}$

$b\in\{1,3,5,7,9,.....,99\}$

Now, $a+b\in\{25,71,117,163\}$

(i) a + b = 25, no. of ordered pairs (a, b) is 12

(ii) a + b = 71, no. of ordered pairs (a, b) is 35

(iii) a + b = 117, no. of ordered pairs (a, b) is 42

(iv) a + b = 163, no. of ordered pairs (a, b) is 19

$\therefore$ total = 108 pairs

11. If P is a 3 × 3 real matrix such that $P^T = aP + (a - 1)I$, where a > 1, then

(1) P is a singular matrix

(2) $|\text{Adj } P|>1$

(3) $|\text{Adj } P|=\frac{1}{2}$

(4) $|\text{Adj } P|=1$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $P^T=aP+(a-1)I$

$\Rightarrow P=aP^T+(a-1)I$

$\Rightarrow P^T-P=a\left(P-P^T\right)$

$\Rightarrow P=P^T,$ as $a\neq -1$

Now, $P=aP+(a-1)I$

$\Rightarrow P=-I\Rightarrow |P|=1$

$\Rightarrow |\text{Adj } P|=1$

12. Let $\lambda \in \mathbb{R}, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.

If $\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\times(\vec{a}-\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$,

then $\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2$ is equal to

(1) 140 (2) 132

(3) 144 (4) 136

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

$\Rightarrow (\vec{b}-\vec{a})\times\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left((\vec{a}-\vec{b})\cdot(\vec{a}+\vec{b})\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40j - 24\hat{k}$

$\Rightarrow 8(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$

$= (4 - 3\lambda)\hat{i} - (2\lambda + 3)\hat{j} + (-\lambda^2 - 2)\hat{k}$

$\Rightarrow \lambda = 1$

$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{a} - \vec{b} = 3\hat{j} - 5\hat{k}$

$\Rightarrow (\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2\hat{i} + 10\hat{j} + 6\hat{k}$

$\therefore$ required answer = 4 + 100 + 36= 140

13. Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2$. If $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is

(1) –24

(2) –48

(3) –84

(4) –60

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$

$\vec{b}\cdot\vec{c} = \vec{b}\cdot(2\vec{a}\times\vec{b}) - 3\vec{b}\cdot\vec{b}$

$= -3|b|^2$

$= -48$

14. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\frac{\pi}{4} - \cot^{-1}(2022)$ (2) $\cot^{-1}(2022) - \frac{\pi}{4}$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$ (4) $\frac{\pi}{4} - \tan^{-1}(2022)$

Official Ans. by NTA (3)

QuestPix Ans. (1,3)

Sol. $a_2 - a_1 = a_3 - a_2 = \ldots = a_{2022} - a_{2021} = 1.$

$\therefore \tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$

$= \left[(\tan^{-1}a_2) - \tan^{-1}a_1\right] + \left[\tan^{-1}a_3 - \tan^{-1}a_2\right] + \ldots$

$+ \left[\tan^{-1}a_{2022} - \tan^{-1}a_{2021}\right]$

$= \tan^{-1}a_{2022} - \tan^{-1}a_1$

$= \tan^{-1}(2022) - \tan^{-1}1 = \tan^{-1}2022 - \frac{\pi}{4}$ **(option 3)**

$= \left(\frac{\pi}{2} - \cot^{-1}(2022)\right) - \frac{\pi}{4}$

$= \frac{\pi}{4} - \cot^{-1}(2022)$ **(option 1)**

15. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line y = 1. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) d, e, f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 \left(\because b^2 = ac\right)$

$\Rightarrow \left(x\sqrt{a} + \sqrt{c}\right)^2 = 0$

$x^2 - \frac{\sqrt{c}}{\sqrt{a}}$ ..... (1)

Now, $dx^2 + 2ex + f = 0$

$\Rightarrow d\left(\frac{c}{a}\right) + 2e\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$

$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$

$$\Rightarrow \frac{d}{a}+\frac{f}{c}=2e\sqrt{\frac{1}{ac}}$$

$$\Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{b}\ \left[\text{as } b=\sqrt{ae}\right]$$

$\therefore \frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P.

16. If a plane passes through the points (–1, k, 0), (2, k, –1), (1, 1, 2) and is parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{17}{5}$ (2) $\frac{5}{17}$

(3) $\frac{6}{13}$ (4) $\frac{13}{6}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$

$$\frac{x-1}{1}=\frac{y+\frac{1}{2}}{1}=\frac{z+1}{-1}$$

Points : $A(-1,k,0),B(2,k,-1),C(1,1,2)$

$$\overrightarrow{CA}=-2\hat{i}+(k-1)\hat{j}-2\hat{k}$$

$$\overrightarrow{CB}=\hat{i}+(k-1)\hat{j}-3\hat{k}$$

$$\overrightarrow{CA}\times\overrightarrow{CB}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & k-1 & -2\\ 1 & k-1 & -3\end{vmatrix}$$

$$=\hat{i}(-3k+3+2k-2)-\hat{j}(6+2)+\hat{k}(-2k+2-k+1)$$

$$=(1-k)\hat{i}-8\hat{j}+(3-3k)\hat{k}$$

The line $\frac{x-1}{1}=\frac{y+\frac{1}{2}}{1}=\frac{z+1}{-1}$ is perpendicular to normal vector.

$$\therefore 1\cdot(1-k)+1(-8)+(-1)(3-3k)=0$$

$$\Rightarrow 1-k-8-3+3k=0$$

$$\Rightarrow 2k=10\Rightarrow \boxed{k=5}$$

$$\therefore \frac{k^2+1}{(k-1)(k-2)}=\frac{26}{4\cdot 3}=\frac{13}{6}$$

17. Let a, b, c > 1, a^3, b^3 and c^3 be in A.P., and $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is – 444, then abc is equal to

(1) 343 (2) 216

(3) $\frac{343}{8}$ (4) $\frac{125}{8}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. As a^3, b^3, c^3 be in A.P. $\rightarrow \boxed{a^3+c^3=2b^3}$ .... (1)

$\log_a^b, \log_c^a, \log_b^c$ are in G.P.

$$\therefore \frac{\log b}{\log a}\cdot\frac{\log c}{\log b}=\left(\frac{\log a}{\log c}\right)^2$$

$$\therefore (\log a)^3=(\log c)^3\Rightarrow \boxed{a=c} \quad \text{..... (2)}$$

From (1) and (2)

$\boxed{a=b=c}$

$$T_1=\frac{a+4b+c}{3}=2a; d=\frac{a-8b+c}{10}=\frac{-6a}{10}=\frac{-3}{5}a$$

$$\therefore S_{20}=\frac{20}{2}\left[4a+19\left(-\frac{3}{5}a\right)\right]$$

$$=10\left[\frac{20a-57a}{5}\right]$$

$= -74a$

$\therefore -74a=-444\Rightarrow \boxed{a=6}$

$\therefore abc=6^3=216$

18. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is

(1) ϕ (2) {99}

(3) $\mathbb{N}$ (4) {9}

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. let a_1 be any natural number

$a_1, a_1+1, a_1+2, \ldots, a_1+99$ are values of a_i'S

$$\bar{x}=\frac{a_1+(a_1+1)+(a_1+2)+\ldots+a_1+99}{100}$$

$$=\frac{100a_1+(1+2+\ldots+99)}{100}=a_1+\frac{99\times 100}{2\times 100}$$

$$=a_1+\frac{99}{2}$$

Mean deviation about mean $= \dfrac{\sum_{1=1}^{100}|x_i - \bar{x}|}{100}$

$= \dfrac{2\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+....+\frac{1}{2}\right)}{100}$

$= \dfrac{1+3+.....+99}{100}$

$= \dfrac{\frac{50}{2}[1+99]}{100}$

$= 25$

So, it is true for every natural no. 'a_1'

19. $\lim\limits_{n\to\infty}\dfrac{3}{n}\left\{4+\left(2+\dfrac{1}{n}\right)^2+\left(2+\dfrac{2}{n}\right)^2+...+\left(3-\dfrac{1}{n}\right)^2\right\}$ is equal to

(1) 12 (2) $\dfrac{19}{3}$

(3) 0 (4) 19

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\lim\limits_{n\to\infty}\dfrac{3}{n}\sum_{r=0}^{n-1}\left(2+\dfrac{r}{n}\right)^2$

$= 3\int_0^1 (2+x)^2\,dx = 27 - 8 = 19$

20. For $\alpha,\beta\in\mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(1) $x^2 - 10x + 16 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$ (4) $x^2 + 14x + 24 = 0$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\begin{vmatrix}1 & -1 & 1\\ 2 & 2 & \alpha\\ 3 & -1 & 4\end{vmatrix} = 0; 8 + \alpha - 2(-4+1) + 3(-\alpha - 2) = 0$

$8 + \alpha + 6 - 3\alpha - 6 = 0$

$\alpha = 4$

SECTION-B

21. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing (x + y) by 25 is _____.

Official Ans. by NTA (23)

QuestPix Ans. (23)

Sol. $x + y = 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25}$

$= 25\,K - (6^{25} + 1) = 25\,K - ((5+1)^{25} + 1)$

$= 25K_1 - 2$ Remainder = 23

22. Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions $f: A \to A$ such that $f(m\cdot n) = f(m)\cdot f(n)$ for every m, n $\in$ A with $m\cdot n \in A$ is equal to _____.

Official Ans. by NTA (432)

QuestPix Ans. (432)

Sol. f(1) = 1; f(9) = f(3) × f(3)

i.e., f(3) = 1 or 3

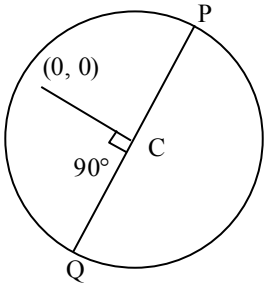
Total function = 1 × 6 × 2 × 6 × 6 × 1 = 432

23. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\dfrac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____.

Official Ans. by NTA (24)

QuestPix Ans. (24)

Sol. $\dfrac{1}{2}\times PC\times\sqrt{5} = \dfrac{\sqrt{35}}{2}; PC = \sqrt{7}$



$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$

$= 2(5 + 7) = 24$

24. The 8th common term of the series

$S_1 = 3+7+11+15+19+.....,$

$S_2 = 1+6+11+16+21+....$

is _____.

Official Ans. by NTA (151)

QuestPix Ans. (151)

Sol. $T_8 = 11 + (8-1) \times 20$

$= 11 + 140 = 151$

25. Let a line L pass through the point P(2, 3, 1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _____.

Official Ans. by NTA (158)

QuestPix Ans. (158)

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$

$\therefore$ Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, $R \equiv (k+2, -k+3, -k+1)$

DR of QR are $(k-3, -k, -k-7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

26. If $\int \sqrt{\sec 2x - 1}\,dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x\right)} \right|$ + constant, then $\beta - \alpha$ is equal to _____.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\int \sqrt{\sec 2x - 1}\,dx = \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}\,dx$

$= \sqrt{2} \int \frac{\sin x}{\sqrt{2\cos^2 x - 1}}\,dx$

put $\cos x = t \quad \Rightarrow -\sin x\,dx = dt$

$= -\sqrt{2} \int \frac{dt}{\sqrt{2t^2 - 1}}$

$= -\ln \left| \sqrt{2}\cos x + \sqrt{\cos 2x} \right| + c$

$= -\frac{1}{2} \ln \left| 2\cos^2 x + \cos 2x + 2\sqrt{\cos 2x} \cdot \sqrt{2}\cos x \right| + c$

$= -\frac{1}{2} \ln \left| \cos 2x + \frac{1}{2} + \sqrt{\cos 2x} \cdot \sqrt{1 + \cos 2x} \right| + c$

$\because \beta = \frac{1}{2}, \alpha = -\frac{1}{2} \Rightarrow \beta - \alpha = 1$

27. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____.

Official Ans. by NTA (13)

QuestPix Ans. (13)

Sol. Two equations have common root

$\therefore (4a)(26a) = (-6)^2 = 36$

$\Rightarrow a^2 = \frac{9}{26} \quad \therefore a = \frac{3}{\sqrt{26}} \Rightarrow \beta = 13$

28. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.

Official Ans. by NTA (240)

QuestPix Ans. (240)

Sol. Digits are 1, 2, 2, 2, 3, 3, 5

If unit digit 5, then total numbers = $\frac{6!}{3!2!}$

If unit digit 3, then total numbers = $\frac{6!}{3!}$

If unit digit 1, then total numbers = $\frac{6!}{3!2!}$

$\therefore$ total numbers = 60 + 60 + 120 = 240

29. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is q. If p : q = m : n, where m and n are coprime, then m + n is equal to _____.

Official Ans. by NTA (14)

QuestPix Ans. (14)

Sol. $p = \dfrac{{}^6C_1}{6\times 6} = \dfrac{1}{6}$

$q = \dfrac{{}^6C_1 \times {}^5C_1 \times 4}{6\times 6\times 6\times 6} = \dfrac{5}{54}$

$\therefore p:q = 9:5 \Rightarrow m+n = 14$

30. Let A be the area of the region

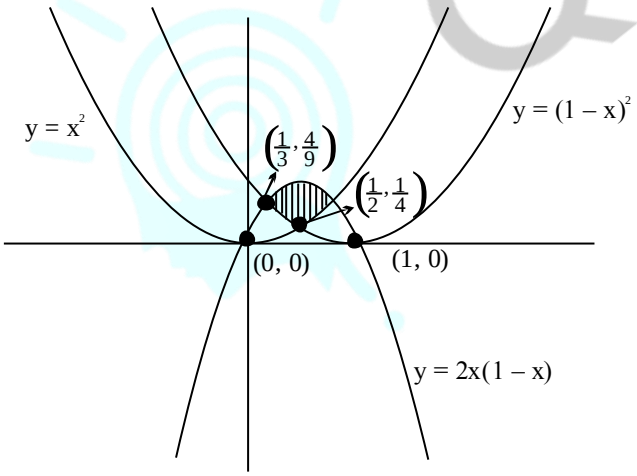
$\left\{(x,y): y \ge x^2, y \ge (1-x)^2, y \le 2x(1-x)\right\}$.

Then 540 A is equal to

Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol.



$A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} \left(2x - 2x^2 - (1-x)^2\right) dx$

$= 2\left[2x^2 - x^3 - x\right]_{1/3}^{1/2}$

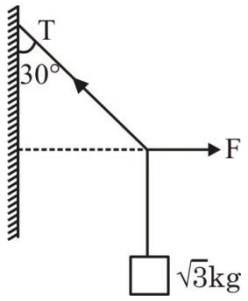
$\therefore A = \dfrac{5}{108} \Rightarrow 540A = \dfrac{5}{108}\times 540 = 25$

(Held On Monday 30th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

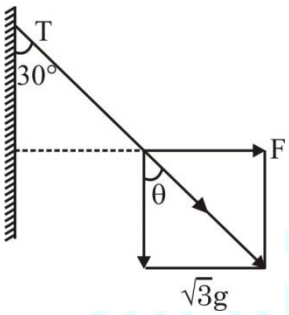
31. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is :
(Given g = 10 ms^{-2})



(1) 20 N (2) 25 N
(3) 10 N (4) 15 N

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol.



$\theta = 30°$

$\cos\theta = \frac{\sqrt{3}g}{T}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}g}{T}$

$\Rightarrow T = 20N$

32. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is :

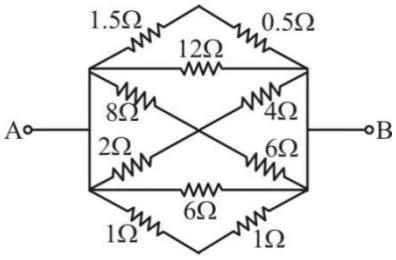
(1) 2 : 1 (2) 1 : 1
(3) 1 : 4 (4) 4 : 1

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. $K_{av} = \frac{5}{2}kT$

Ratio = 1 : 1

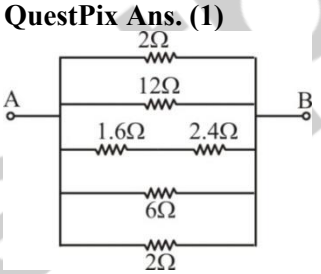
33. The equivalent resistance between A and B is ……



(1) $\frac{2}{3}\Omega$ (2) $\frac{1}{2}\Omega$
(3) $\frac{3}{2}\Omega$ (4) $\frac{1}{3}\Omega$

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol.



$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$

$= \frac{6+1+3+2+6}{12} = \frac{18}{12} = \frac{3}{2}$

$\Rightarrow R_{eq} = \frac{2}{3}\Omega$

34. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : The nuclear density of nuclides $^{10}_{5}B$, $^{6}_{3}Li$, $^{56}_{26}Fe$, $^{20}_{10}Ne$ and $^{209}_{83}Bi$ can be arranged as $\rho^N_{Bi} > \rho^N_{Fe} > \rho^N_{Ne} > \rho^N_{B} > \rho^N_{Li}$.
Reason R : The radius R of nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.
In the light of the above statement, choose the **correct** answer from the options given below :
(1) Both **A** and **R** are true and R is the correct explanation of **A**
(2) **A** is false but **R** is true
(3) **A** is true but **R** is false
(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. Nuclear density is independent of A.

35. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is :

(1) 6° (2) 1.3°

(3) 7.8° (4) 4.5°

Official Ans. by NTA (4)

QuestPix Ans. (4)

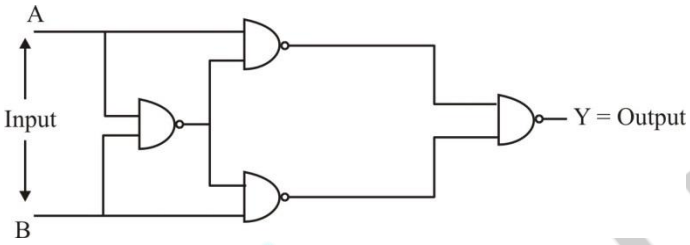
Sol. $\delta_1 = \delta_2$ [for no average deviation]

$\Rightarrow 6°(1.54 - 1) = A(1.72 - 1)$

$\Rightarrow A = \dfrac{6° \times 0.54}{0.72}$

$= \dfrac{18°}{4} = 4.5°$

36. The output Y for the inputs A and B of circuit is given by



Truth table of the shown circuit is :

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Given circuit represent XOR.

37. A vehicle travels 4 km with speed of 3 km/h and another 4 km with speed of 5 km/h, then its average speed is :

(1) 4.25 km/h (2) 3.50 km/h

(3) 4.00 km/h (4) 3.75 km/h

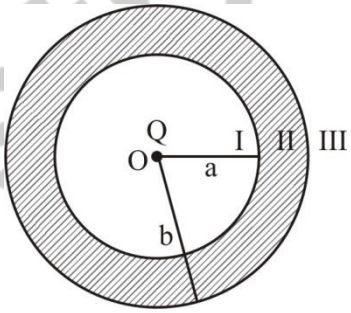
Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\dfrac{2}{V_{av}} = \dfrac{1}{3} + \dfrac{1}{5} = \dfrac{8}{15}$

$\Rightarrow V_{av} = \dfrac{15}{4} = 3.75\,\text{km/h}$

38. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by : (I : r < a, II : a < r < b, III : r > b)



(1) $E_I = 0$, $E_{II} = 0$, $E_{III} \neq 0$

(2) $E_I \neq 0$, $E_{II} = 0$, $E_{III} \neq 0$

(3) $E_I \neq 0$, $E_{II} = 0$, $E_{III} = 0$

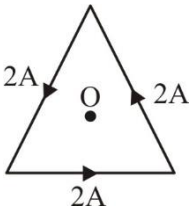
(4) $E_I = 0$, $E_{II} = 0$, $E_{III} = 0$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Electric field inside material of conductor is zero.

39. As shown in the figure, a current of 2A flowing in an equilateral triangle of side $4\sqrt{3}\,\text{cm}$. The magnetic field at the centroid O of the triangle is :



(Neglect the effect of earth's magnetic field.)

(1) $4\sqrt{3} \times 10^{-4}\,\text{T}$ (2) $4\sqrt{3} \times 10^{-5}\,\text{T}$

(3) $\sqrt{3} \times 10^{-4}\,\text{T}$ (4) $3\sqrt{3} \times 10^{-5}\,\text{T}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $d\tan 60° = 2\sqrt{3}$

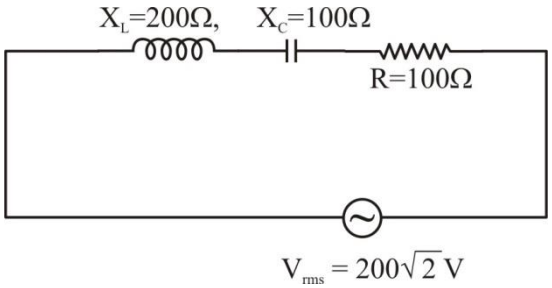
$d = 2$ cm

$$B = 3\times\frac{\mu_0 i}{2\pi d}\sin 60°$$

$$= 3\times\frac{2\times10^{-7}\times2}{2\times10^{-2}}\times\frac{\sqrt{3}}{2}$$

$$= 3\sqrt{3}\times10^{-5}$$

40. In the given circuit, rms value of current (I_{rms}) through the resistor R is :



(1) 2A

(2) $\frac{1}{2}$A

(3) 20 A

(4) $2\sqrt{2}$A

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $z = \sqrt{100^2 + (200-100)^2}$

$= 100\sqrt{2}\,\Omega$

$$i_{rms} = \frac{V_{rms}}{z} = \frac{200\sqrt{2}}{100\sqrt{2}}$$

$= 2$A

41. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of 100 m s^{-1} each. The recoil velocity of the gun is :

(1) 0.02 m/s

(2) 2.5 m/s

(3) 1.5 m/s

(4) 0.6 m/s

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $20\times10^{-3}\times\frac{180}{60}\times100 = 10\,V$

$\Rightarrow$ v = 0.6 m/s

42. Given below are **two** statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Efficiency of a reversible heat engine will be highest at –273°C temperature of cold reservoir.

Reason R : The efficiency of Carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $\eta = \left(1-\frac{T_2}{T_1}\right)$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) **A** is false but **R** is true

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Both A and R are true and R is the correct explanation of A

43. Match List I with List II.

	List I		**List II**
A	Torque	I.	kg m^{-1} s^{-2}
B	Energy density	II.	kg ms^{-1}
C	Pressure gradient	III.	kg m^{-2} s^{-2}
D	Impulse	IV.	kg m^2 s^{-2}

Choose the **correct** answer from the options given below :

(1) A-IV, B-III, C-I, D-II

(2) A-I, B-IV, C-III, D-II

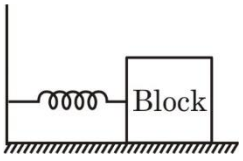
(3) A-IV, B-I, C-II, D-III

(4) A-IV, B-I, C-III, D-II

Official Ans. by NTA (4)

QuestPix Ans. (4)

44. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg the angular frequency is ω_2. The ratio ω_2/ω_1 is :



(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) 2 (4) $\frac{1}{2}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\omega = \sqrt{\frac{k}{m}}$

$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$

45. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to :

(1) 3 (2) $\frac{9}{4}$

(3) $\frac{3}{2}$ (4) 4

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $KE = \frac{P^2}{2m}, \quad P = \frac{h}{\lambda}$

$eV_1 = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$

$eV_2 = \frac{\left(\frac{h}{1.5\lambda}\right)^2}{2m}$

$\frac{V_1}{V_2} = (1.5)^2 = \frac{9}{4}$

46. Match List I with List II :

	List I		List II
A.	Attenuation	I	Combination of a receiver and transmitter.
B.	Transducer	II	Process of retrieval of information from the carrier wave at received
C.	Demodulation	III	Converts one form of energy into another
D.	Repeater	IV	Loss of strength of a signal while propagating through a medium

Choose the correct answer from the options given below :

(1) A-I, B-II, C-III, D-IV

(2) A-II, B-III, C-IV, D-I

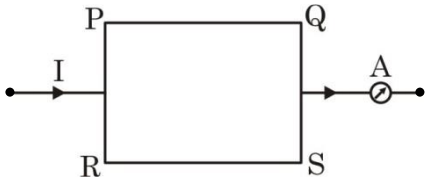
(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-III, C-II, D-I

Official Ans. by NTA (4)

QuestPix Ans. (4)

47. A current carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5 cm and PQ = RS = 100 cm. If ammeter current reading changes from I to 2I, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $f_{PQ}^{I} : f_{PQ}^{2I}$ is :



(1) 1 : 2 (2) 1 : 4

(3) 1 : 5 (4) 1 : 3

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $F \propto I_1 I_2$

$F_I : F_{2I} = 1 : 4$

48. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wires having uniform circular cross sections)

(1) 6.06×10^{-2} mm

(2) 2.77×10^{-2} mm

(3) 3.0×10^{-2} mm

(4) 6.9×10^{-2} mm

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $Y = \frac{F/A}{\frac{\Delta\ell}{\ell}}$

$\Rightarrow F = \frac{YA}{\ell}\Delta\ell$

$\left(\frac{A\Delta\ell}{\ell}\right)_1 = \left(\frac{A\Delta\ell}{\ell}\right)_2$

$\Rightarrow \frac{\Delta\ell_2}{\Delta\ell_1} = \frac{A_1}{A_2} \times \frac{\ell_2}{\ell_1}$

$\Rightarrow \frac{\Delta\ell_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$

$\Rightarrow \Delta\ell_2 = 6.9 \times 10^{-2}$ mm

49. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be :

(1) $2\sqrt{gR}$ (2) $\sqrt{gR}$

(3) $\sqrt{\frac{gR}{2}}$ (4) $\sqrt{2gR}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Loss in PE = Gain in KE

$$\left(-\frac{GMm}{2R}\right)-\left(-\frac{GMm}{R}\right)=\frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2=\frac{GM}{R}=gR$$

$$\Rightarrow \quad v=\sqrt{gR}$$

50. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is :

(1) $\frac{1}{2\pi}\frac{W}{m^2}$ (2) $\frac{1}{40\pi}\frac{W}{m^2}$

(3) $\frac{1}{10\pi}\frac{W}{m^2}$ (4) $\frac{1}{20\pi}\frac{W}{m^2}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $I_{EF}=\frac{1}{2}\times\frac{5}{4\pi\times5^2}$

$$=\frac{1}{40\pi}W/m^2$$

SECTION-B

51. A faulty thermometer reads 5°C in melting ice and 95°C in steam. The correct temperature on absolute scale will be……. K when the faulty thermometer reads 41°C.

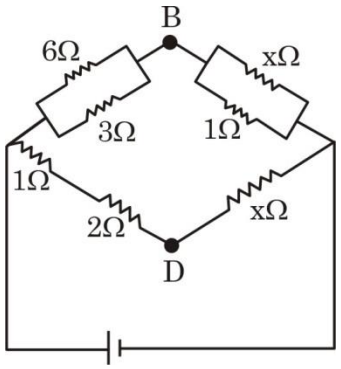
Official Ans. by NTA (313)

QuestPix Ans. (313)

Sol. $\frac{41°-5°}{95°-5°}=\frac{C-0°}{100°-0°}$

$$\Rightarrow \quad C=\frac{36}{90}\times100=40°C=313\,K$$

52. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is ……..



Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\frac{2}{3}=\frac{x}{x+1}$

$$\Rightarrow \quad \frac{2}{3}=\frac{1}{x+1}$$

$$\Rightarrow \quad x=0.5=\frac{1}{2}$$

n = 2

53. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}$s. The value of x is ……. $\left(\text{Take } \pi=\frac{22}{7}\right)$

Official Ans. by NTA (88)

QuestPix Ans. (88)

Sol. $4v^2 = 50 - x^2$

$$\Rightarrow \quad v=\frac{1}{2}\sqrt{50-x^2}$$

$$\omega=\frac{1}{2}$$

$$T=\frac{2\pi}{\omega}=4\pi=\frac{88}{7}$$

x = 88

54. In a Young's double slit experiment, the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0}=$.....

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$I_1 = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = 2I_0$

$I_2 = 4I_0 \cos^2\left(\frac{2\pi}{3}\right) = I_0$

$\Rightarrow \frac{I_1 + I_2}{I_0} = 3$

55. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$ s. The value of α is _____.

Official Ans. by NTA (300)

QuestPix Ans. (300)

Sol. $\frac{dN_1}{dt} = -\lambda_1 N \qquad \frac{dN_2}{dt} = -\lambda_2 N$

$\frac{dN}{dt} = -(\lambda_1 + \lambda_2)N$

$\Rightarrow \lambda_{eq} = \lambda_1 + \lambda_2$

$\Rightarrow \frac{1}{t_{1/2}} = \frac{1}{300} + \frac{1}{30} = \frac{11}{300}$

$\Rightarrow t_{1/2} = \frac{300}{11}$

56. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4s is $\frac{1}{3}\alpha^2\sqrt{P}$ m. The value of α will be ……

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\frac{1}{2}mV^2 = Pt$

$V = \sqrt{\frac{2Pt}{m}}$

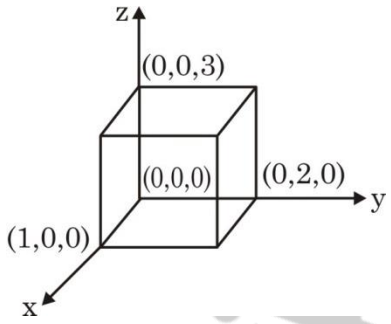
$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$

$x = \sqrt{\frac{2P}{m}}\frac{2}{3}\left[t^{3/2}\right]_0^4$

$x = \frac{16\sqrt{P}}{3} = \frac{1}{3} \times 16\sqrt{P}$

$\alpha = 4$

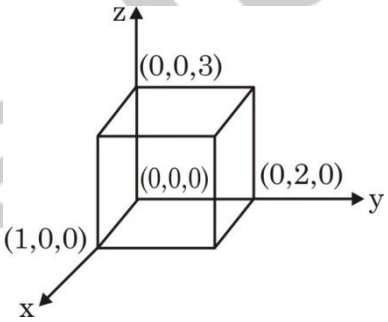
57. As shown in figure, a cuboid lies in a region with electric field $\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$ N/C. The magnitude of charge within the cuboid is $n\epsilon_0$ C. The value of n is _____ (if dimension of cuboid is $1 \times 2 \times 3$ m^3)



Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$



$\phi_{net} = -8 \times 3 + 2 \times 6 = -12$

$-12 = \frac{q}{\epsilon_0}$

$|q| = 12\epsilon_0$

58. In an ac generator, a rectangular coil of 100 turns each having area 14×10^{-2} m^2 is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be _____ V. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Official Ans. by NTA (1584)

QuestPix Ans. (1584)

Sol. $\xi_{max} = NAB\omega$

$= 100 \times 14 \times 10^{-2} \times 3 \times \frac{360 \times 2\pi}{60}$

$= 1584$V

59. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x} ms^{-2}$. The value of x _____.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Official Ans. by NTA (125)

QuestPix Ans. (125)

Sol. $a = \omega^2 R = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$

$= \left(\frac{56}{60} \times \frac{22}{7}\right)^2 \times 1.8$

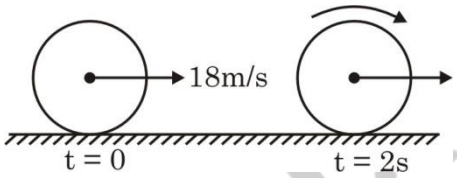
$= \frac{(44)^2}{225} \times 1.8$

$= \frac{1936 \times 1.8}{225}$

$x = 125$

60. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at t = 0 s on a rough horizontal surface. It starts off with a purely sliding motion at t = 0 s. After 2s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2s will be _____ J

(given, coefficient of friction is 0.3 and $g = 10 m/s^2$).



Official Ans. by NTA (54)

QuestPix Ans. (54)

Sol. $a = -\mu_k g = -3$

$V = 18 - 3 \times 2$

$V = 12 m/s$

$KE = \frac{1}{2} mv^2 + \frac{1}{2} \frac{mr^2}{2} \frac{v^2}{r^2}$

$KE = \frac{3}{4} mv^2$

$KE = 3 \times 18 = 54 J$

(Held On Monday 30th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

61. Which of the following reaction is correct ?

(1) $2LiNO_3 \xrightarrow{\Delta} 2LiNO_2 + O_2$

(2) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2N_2O_4 + O_2$

(3) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

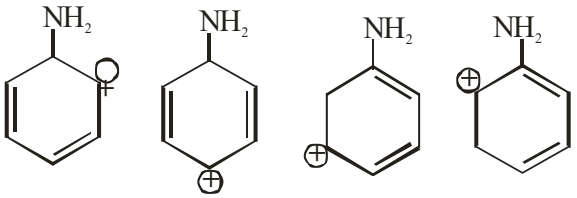
(4) $2LiNO_3 \xrightarrow{\Delta} 2Li + 2NO_2 + O_2$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

62. The most stable carbocation for the following is:



(a) (b) (c) (d)

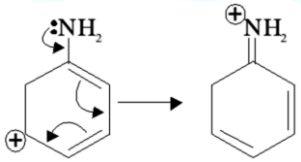
(1) c (2) d

(3) b (4) a

Official Ans. by NTA (1)

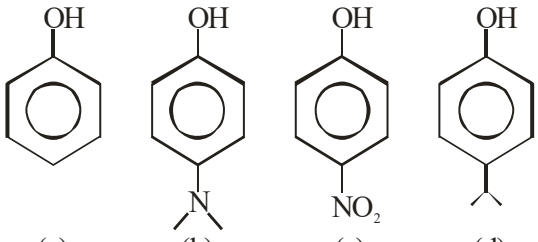
QuestPix Ans. (1)

Sol.



The +M effect of NH_2 is stabilizing the carbocation.

63. The correct order of pK_a values for the following compounds is:



(a) (b) (c) (d)

(1) c > a > d > b (2) b > d > a > c

(3) b > a > d > c (4) a > b > c > d

Official Ans. by NTA (2)

QuestPix Ans. (2)

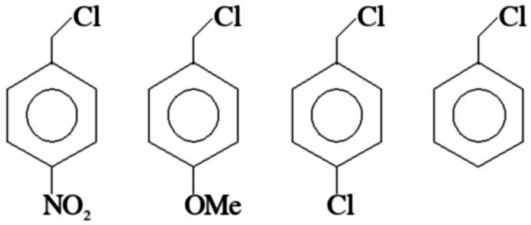
Sol. Due to –M effect of $-NO_2$ group, it increases acidity +M effect of N $(CH_3)_2$ decreases acidity.

Hyperconjugation of isopropyl decrease acidity

$\therefore$ order of acidic strength

(c) > (a) > (d) > (b)

64. Decreasing order towards S_N1 reaction for the following compounds is:



(a) (b) (c) (d)

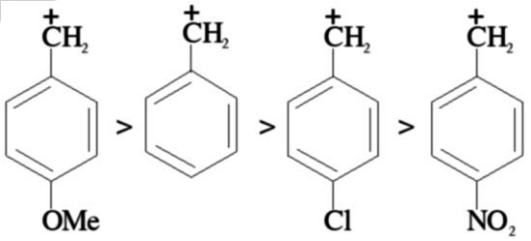
(1) a > c > d > b (2) a > b > c > d

(3) b > d > c > a (4) d > b > c > a

Official Ans. by NTA (3)

QuestPix Ans. (3)

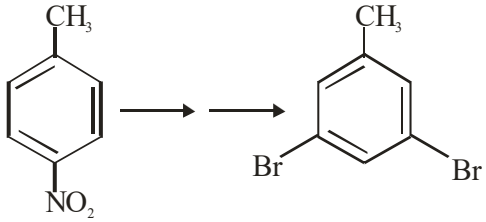
Sol. The rate of S_N1 reaction depends upon stability of carbocation which follows the order



$\therefore$ Reactivity order

(b) > (d) > (c) > (a)

65.



(X) (Y)

In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be:

(1) (i) Br_2, Fe (ii) Fe, H^+ (iii) $LiAlH_4$

(2) (i) Br_2(aq) (ii) $LiAlH_4$ (iii) H_3O^+

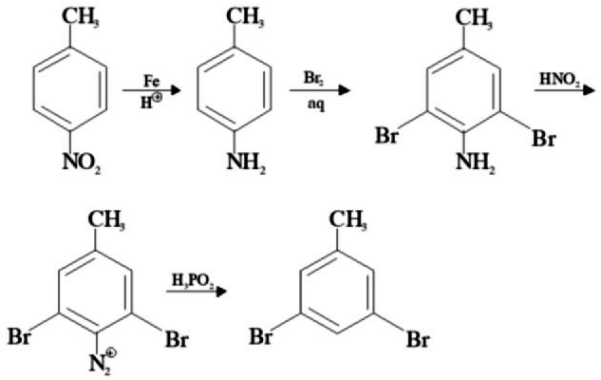
(3) (i) Fe, H^+ (ii) Br_2(aq) (iii) HNO_2 (iv) CuBr

(4) (i) Fe, H^+ (ii) Br_2(aq) (iii) HNO_2 (iv) H_3PO_2

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



66. Maximum number of electrons that can be accommodated in shell with n = 4 are:

(1) 16

(2) 32

(3) 50

(4) 72

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. The number of electrons in the orbitals of sub-shell of n = 4 are

4s	2
4p	6
4d	10
4f	14
(Total)	**32**

67. Match List I with List II:

	List I (Complexes)		List II (Hybridisation)
(A)	$[Ni(CO)_4]$	I	sp^3
(B)	$[Cu(NH_3)_4]^{2+}$	II	dsp^2
(C)	$[Fe(NH_3)_6]^{2+}$	III	sp^3d^2
(D)	$[Fe(H_2O)_6]^{2+}$	IV	d^2sp^3

(1) A – II, B – I, C – III, D – IV

(2) A – I, B – II, C – III, D – IV

(3) A – II, B – I, C – IV, D – III

(4) A – I, B– II, C – IV, D – III

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. For $[Fe(NH_3)_6]^{+2}$, $\Delta_0 < P$, hence the pairing of electrons does not occur in t_{2g}. Therefore complex is outer orbital and its hybridisation is sp^3d^2.

List I (Complexes)	List II (Hybridisation)
$[Ni(CO)_4]$	sp^3
$[Cu(NH_3)_4]^{2+}$	dsp^2
$[Fe(NH_3)_6]^{2+}$	sp^3d^2
$[Fe(H_2O)_6]^{2+}$	sp^3d^2

68. The Cl – Co – Cl bond angle values in a fac-$[Co(NH_3)_3Cl_3]$ complex is/are:

(1) 90° & 180°

(2) 90°

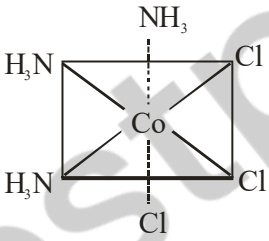
(3) 180°

(4) 90° & 120°

Official Ans. by NTA (2)

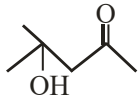
QuestPix Ans. (2)

Sol.



The Cl – Co – Cl bond angle in above octahedral complex is 90°

69. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A :  can be easily reduced using Zn-Hg/ HCl to 

Reason R : Zn-Hg/HCl is used to reduce carbonyl group to $–CH_2–$ group.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

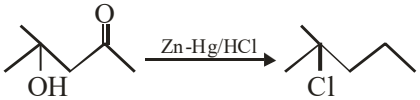
(3) Both A and R are true but R is not the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



The acid sensitive alcohol group reacts with HCl, hence Clemmenson reduction is not suitable for above conversion.

70. Chlorides of which metal are soluble in organic solvents:

(1) Ca (2) Mg

(3) K (4) Be

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $BeCl_2$ having covalent nature is soluble in organic solvent.

71. Given below are two statements : One is labelled as **Assertion A** and the other labelled as **Reason R**.

Assertion A: Antihistamines do not affect the secretion of acid in stomach.

Reason R : Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is not the correct explanation of A.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Antiallergic and antacid drugs work on different receptors

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72. The wave function (Ψ) of 2s is given by

$$\Psi_{2s} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

At $r = r_0$, radial node is formed. Thus, r_0 in terms of a_0

(1) $r_0 = a_0$ (2) $r_0 = 4a_0$

(3) $r_0 = \frac{a_0}{2}$ (4) $r_0 = 2a_0$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. At node $\Psi_{2s} = 0$

$$\therefore 2-\frac{r_0}{a_0} = 0$$

$$\therefore r_0 = 2a_0$$

73. $KMnO_4$ oxidises I^- in acidic and neutral/faintly alkaline solution, respectively to

(1) I_2 & IO_3^-

(2) IO_3^- & I_2

(3) IO_3^- & IO_3^-

(4) I_2 & I_2

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. In acidic medium

$$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$$

In neutral/faintly alkaline solution

$$2MnO_4^- + I^- + H_2O \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

74. Bond dissociation energy of E–H bond of the "H_2E" hydrides of group 16 elements (given below), follows order.

(A) O

(B) S

(C) Se

(D) Te

(1) A > B > C > D

(2) A > B > D > C

(3) B > A > C > D

(4) D > C > B > A

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Bond dissociation energy of E–H bond in hydrides of group 16 follows the order

$H_2O > H_2S > H_2Se > H_2Te$

75. The water quality of a pond was analysed and its BOD was found to be 4. The pond has

(1) Highly polluted water

(2) Water has high amount of fluoride compounds

(3) Very clean water

(4) Slightly polluted water

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Clean water as BOD value of < 5 while polluted water has BOD of 15 or more.

76. Match List I with List II:

	List I (Mixture)		List II (Separation Technique)
(A)	$CHCl_3 + C_6H_5NH_2$	I	Steam distillation
(B)	$C_6H_{14} + C_5H_{12}$	II	Differential extraction
(C)	$C_6H_5NH_2 + H_2O$	III	Distillation
(D)	Organic compound in H_2O	IV	Fractional distillation

(1) A-IV, B-I, C-III, D-II

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-I, C-III, D-IV

(4) A-III, B-I, C-IV, D-II

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

List I (Mixture)	List II (Separation Technique)
$CHCl_3 + C_6H_5NH_2$	Distillation
$C_6H_{14} + C_5H_{12}$	Fractional distillation
$C_6H_5NH_2 + H_2O$	Steam distillation
Organic compound in H_2O	Differential extraction

NCERT (XI) Vol. 2 Page No. 359, 360.

77. Boric acid in solid, whereas BF_3 is gas at room temperature because of

(1) Strong ionic bond in Boric acid

(2) Strong van der Waal's interaction in Boric acid

(3) Strong hydrogen bond in Boric acid

(4) Strong covalent bond in BF_3

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Boric acid has strong hydrogen bonding while BF_3 does not. Therefore boric acid is solid.

78. Given below are two statements:

Statement I: During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II: During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. In Electrolytic refining, the pure metal is used as cathode and impure metal is used as anode.

Na_3AlF_6 is added during electrolysis of Al_2O_3 to lower the melting point and increase conductivity.

79. Formulae for Nessler's reagent is:

(1) KHg_2I_2 (2) $KHgI_3$

(3) K_2HgI_4 (4) HgI_2

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Nessler's reagent is K_2HgI_4.

80. 1 L, 0.02 M solution of $[Co(NH_3)_5SO_4]Br$ is mixed with 1L, 0.02 M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below:

1 L Solution (X) + $AgNO_3$ solution (excess)$\rightarrow$Y

1 L Solution (X) + $BaCl_2$ solution (excess)$\rightarrow$Z

The number of moles of Y and Z respectively are

(1) 0.02, 0.02

(2) 0.01, 0.01

(3) 0.02, 0.01

(4) 0.01, 0.02

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\underset{0.01\ \text{mol}}{\left[Co(NH_3)_5SO_4\right]Br} + \underset{\text{excess}}{AgNO_3} \rightarrow \underset{0.01\ \text{Mol}}{AgBr}\downarrow$

$\underset{0.01\ \text{mol}}{\left[Co(NH_3)_5Br\right]SO_4} + \underset{\text{excess}}{BaCl_2} \rightarrow \underset{0.01\ \text{Mol}}{BaSO_4}\downarrow$

SECTION-B

81. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is _____K (Nearest integer). Given C_v=20 J $mol^{-1}K^{-1}$.

Official Ans. by NTA (150)

QuestPix Ans. (150)

Sol. $q = 0$

$\Delta U = w$

$1 \times 20 \times [T_2 - 300] = -3000$

$T_2 - 300 = -150$

$T_2 = 150$ K

82. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0Å. If density of the FeO in the crystal is 4.0 g cm^{-3}, then the number of FeO units present per unit cell is _____ (Nearest integer)

Given : Molar mass of Fe and O is 56 and 16g mol^{-1} respectively.

$N_A = 6.0 \times 10^{23}$ mol^{-1}

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $d = \dfrac{Z \times M}{N_0 \times a^3}$

$4 = \dfrac{Z \times 72}{6 \times 10^{23} \times 125 \times 10^{-24}}$

$Z = 4.166 \simeq 4$

83. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be is _____ s. (Nearest integer).

Given : ln 10 = 2.3; log 2 = 0.3

Official Ans. by NTA (1350)

QuestPix Ans. (1350)

Sol. $\dfrac{t_1}{t_2} = \dfrac{\frac{1}{K}\ln\frac{a_0}{0.4a_0}}{\frac{1}{K}\ln\frac{a_0}{0.1a_0}}$

$\dfrac{540}{t_2} = \dfrac{\ln\frac{10}{4}}{\ln 10}$

$\dfrac{540}{t_2} = \dfrac{\log 10 - \log 4}{\log 10}$

$\dfrac{540}{t_2} = \dfrac{1 - 0.6}{1}$

$\Rightarrow \dfrac{540}{t_2} = 0.4$

$\Rightarrow t_2 = \dfrac{540}{0.4} = 1350$ sec

84. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is ____ (Nearest integer).

Given K_f = 1.8 K kg mol^{-1}

Official Ans. by NTA (243)

QuestPix Ans. (243)

Sol. $\Delta T_f = i.K_f.m$

$\Rightarrow \Delta T_f = 2.67 \times 1.8 \times \dfrac{38}{98} \times \dfrac{1000}{62}$

$\Rightarrow \Delta T_f = 30.05$

$\therefore$ F.P. = 243 K

85. Consider the following equation :

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \Delta H = -190$ kJ

The number of factors which will increase the yield of SO_3 at equilibrium from the following is _____

A. Increasing temperature
B. Increasing pressure
C. Adding more SO_2
D. Adding more O_2
E. Addition of catalyst

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. The yield of SO_3 at equilibrium will be due to :

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

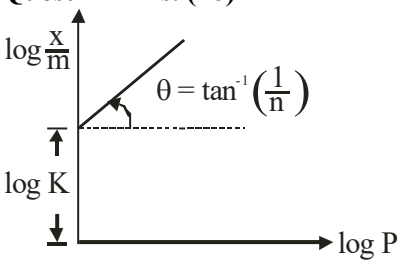
86. The graph of $\log\frac{x}{m}$ vs log p for an adsorption process is a straight line inclined at an angle of 45° with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is _____$\times 10^{-1}$ (Nearest integer)

Given : log 2 = 0.3010

Official Ans. by NTA (16)

QuestPix Ans. (16)

Sol.



$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

$$\frac{1}{n} = \tan 45° = 1$$

$$\log k = 0.6020 = \log 4$$

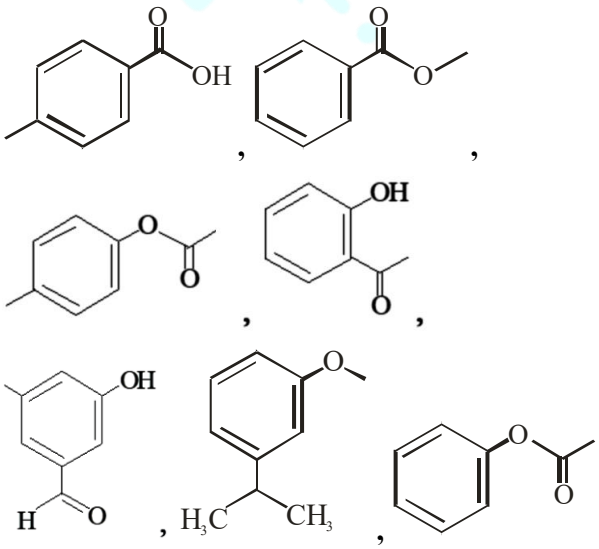
$$\Rightarrow K = 4$$

$$\therefore \frac{x}{m} = K.P^{1/n}$$

$$\frac{x}{m} = 4(0.4) = 1.6$$

$$\frac{x}{m} = 1.6 = 16 \times 10^{-1}$$

87. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is _____ .



Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Compound 2, 3, 7

88. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are _____ .

Official Ans. by NTA (6)

QuestPix Ans. (6)

Sol. Number of peptide linkage = (amino acid – 1)

= 7 – 1 = 6

89. The strength of 50 volume solution of hydrogen peroxide is _____ g/L (Nearest integer).

Given:

Molar mass of H_2O_2 is 34 g mol^{-1}

Molar volume of gas at STP = 22.7 L.

Official Ans. by NTA (150)

QuestPix Ans. (150)

Sol. $\text{Molarity} = \frac{50}{11.35}$

$\therefore \text{Strength in gm/L} = \frac{50}{11.35} \times 34$

90. The electrode potential of the following half cell at 298 K.

$X \mid X^{2+}(0.001\ M) \parallel Y^{2+}(0.01\ M) \mid Y$

is _____$\times 10^{-2}$ V (Nearest integer).

Given : $E^0_{x^{2+}|x} = -2.36V$

$E^0_{Y^{2+}|Y} = +0.36V$

$\frac{2.303RT}{F} = 0.06V$

Official Ans. by NTA (275)

QuestPix Ans. (275)

Sol. $X + Y^{2+} \rightarrow Y + X^{2+}$

$E^0_{Cell} = 0.36 - (-2.36) = 2.72\ V$

$$E_{Cell} = 2.72 - \frac{0.06}{2}\log\frac{0.001}{0.01}$$

$= 2.72 + 0.03 = 2.75\ V$

$= 275 \times 10^{-2}\ V$