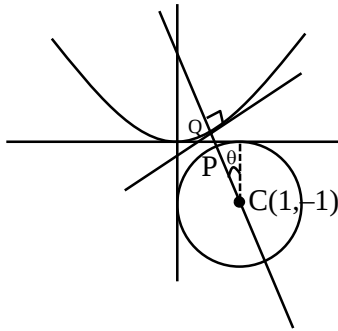


**(Held On Tuesday 26<sup>th</sup> July, 2022)**
**TIME : 3 : 00 PM to 6 : 00 PM**

| MATHEMATICS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | TEST PAPER WITH SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| <p align="center"><b>SECTION-A</b></p> <p>1. The minimum value of the sum of the squares of the roots of <math>x^2 + (3-a)x + 1 = 2a</math> is:</p> <p>(A) 4 (B) 5<br/>(C) 6 (D) 8</p> <p><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> <p><b>Sol.</b> <math>\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta</math><br/>         let <math>f(a) = (3-a)^2 - 2(1-2a)</math><br/> <math>f(a) = a^2 - 2a + 7</math><br/> <math>f(a) = (a-1)^2 + 6</math><br/> <math>f(a)_{\min} = 6</math></p> <p>2. If <math>z = x + iy</math> satisfies <math> z  - 2 = 0</math> and <math> z-i  -  z+5i  = 0</math>, then</p> <p>(A) <math>x + 2y - 4 = 0</math> (B) <math>x^2 + y - 4 = 0</math><br/>         (C) <math>x + 2y + 4 = 0</math> (D) <math>x^2 - y + 3 = 0</math></p> <p><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> <p><b>Sol.</b> <math> z-i  -  z+5i  = 0</math><br/> <math>\Rightarrow  x + (y-1)i  =  x + (y+5)i </math><br/> <math>x^2 + (y-1)^2 = x^2 + (y+5)^2</math><br/> <math>(y-1)^2 - (y+5)^2 = 0</math><br/> <math>(2y+4)(-6) = 0</math><br/> <math>y = -2</math><br/> <math>\therefore x^2 + (-2)^2 = 4</math><br/> <math>x = 0</math><br/> <math>Z \equiv (0, -2)</math>, check options</p> <p>3. Let <math>A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}</math> and <math>B = \begin{bmatrix} 9^2 &amp; -10^2 &amp; 11^2 \\ 12^2 &amp; 13^2 &amp; -14^2 \\ -15^2 &amp; 16^2 &amp; 17^2 \end{bmatrix}</math>, then the value of <math>A'BA</math> is:</p> <p>(A) 1224 (B) 1042 (C) 540 (D) 539</p> <p><b>Official Ans. by NTA (D)</b><br/><b>QuestPix Ans. (D)</b></p> | <p><b>Sol.</b> <math>A'BA = [1 \ 1 \ 1] \begin{bmatrix} 9^2 &amp; -10^2 &amp; 11^2 \\ 12^2 &amp; 13^2 &amp; -14^2 \\ -15^2 &amp; 16^2 &amp; 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}</math></p> <p><math>= [9^2 + 12^2 - 15^2 \quad -10^2 + 13^2 + 16^2 \quad 11^2 - 14^2 + 17^2] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}</math></p> <p><math>= [9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2]</math><br/> <math>= [539]</math></p> <p>4. <math>\sum_{\substack{i,j=0 \\ i \neq j}}^n {}^nC_i {}^nC_j</math> is equal to</p> <p>(A) <math>2^{2n} - {}^{2n}C_n</math> (B) <math>2^{2n-1} - {}^{2n-1}C_{n-1}</math><br/>         (C) <math>2^{2n} - \frac{1}{2} {}^{2n}C_n</math> (D) <math>2^{n-1} + {}^{2n-1}C_n</math></p> <p><b>Official Ans. by NTA (B)</b><br/><b>QuestPix Ans. (B)</b></p> <p><b>Sol.</b> <math>\sum_{\substack{i,j=0 \\ i \neq j}}^n {}^nC_i {}^nC_j</math></p> <p><math>= \sum_{i=0}^n {}^nC_i \cdot \sum_{j=0}^n {}^nC_j - \sum_{i=j=0}^n ({}^nC_i)^2</math></p> <p><math>= (2^n)(2^n) - {}^{2n}C_n</math><br/> <math>= 2^{2n} - {}^{2n}C_n</math></p> <p>5. Let P and Q be any points on the curves <math>(x-1)^2 + (y+1)^2 = 1</math> and <math>y = x^2</math>, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval</p> <p>(A) <math>\left(0, \frac{1}{4}\right)</math> (B) <math>\left(\frac{1}{2}, \frac{3}{4}\right)</math><br/>         (C) <math>\left(\frac{1}{4}, \frac{1}{2}\right)</math> (D) <math>\left(\frac{3}{4}, 1\right)</math></p> <p><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> |

Sol.



$$Q = (t, t^2)$$

$$m_{CQ} = m_{\text{normal}}$$

$$\frac{t^2 + 1}{t - 1} = -\frac{1}{2t}$$

$$\text{Let } f(t) = 2t^3 + 3t - 1$$

$$f\left(\frac{1}{4}\right)f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$$P \equiv (1 + \cos(90 + \theta), -1 + \sin(90 + \theta))$$

$$P = (1 - \sin \theta, -1 + \cos \theta)$$

$$m_{\text{normal}} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos \theta}{-\sin \theta} \Rightarrow \tan \theta = 2t$$

$$x = 1 - \sin \theta = 1 - \frac{2t}{\sqrt{1 + 4t^2}} = g(t) \quad (\text{let})$$

$$\Rightarrow g'(t) < 0$$

$g(t) \downarrow$  function

$$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

6. If the maximum value of  $a$ , for which the function

$f_a(x) = \tan^{-1} 2x - 3ax + 7$  is non-decreasing in

$\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ , is  $\bar{a}$ , then  $f_{\bar{a}}\left(\frac{\pi}{8}\right)$  is equal to

$$(A) 8 - \frac{9\pi}{4(9 + \pi^2)} \quad (B) 8 - \frac{4\pi}{9(4 + \pi^2)}$$

$$(C) 8 \left( \frac{1 + \pi^2}{9 + \pi^2} \right) \quad (D) 8 - \frac{\pi}{4}$$

Official Ans. by NTA (A)

QuestPix Ans. (A)

Sol.  $f_a(x) = \tan^{-1} 2x - 3ax + 7$

$$f'_a(x) = \frac{2}{1 + 4x^2} - 3a \geq 0$$

$$a \leq \left( \frac{2}{3(1 + 4x^2)} \right)_{\min.} \quad \text{at } x = \pm \frac{\pi}{6}$$

$$a_{\max} = \bar{a} = \frac{6}{9 + \pi^2}$$

$$f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1} \frac{\pi}{4} - 3 \frac{6}{9 + \pi^2} \cdot \frac{\pi}{8} + 7 = \tan^{-1} \frac{\pi}{4} - \frac{9\pi}{4(\pi^2 + 9)} + 7$$

7. Let  $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$  for some  $\alpha \in \mathbb{R}$ . Then

the value of  $\alpha + \beta$  is :

$$(A) \frac{14}{5} \quad (B) \frac{3}{2} \quad (C) \frac{5}{2} \quad (D) \frac{7}{2}$$

Official Ans. by NTA (C)

QuestPix Ans. (C)

Sol.  $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$

$$\beta = \lim_{x \rightarrow 0} \frac{1 + \alpha x - \left[ 1 + 3x + \frac{9x^2}{2!} + \dots \right]}{(\alpha x) \frac{(e^{3x} - 1)}{3x}}$$

$$\beta = \lim_{x \rightarrow 0} \frac{(\alpha x - 3x) - \frac{9x^2}{2!} - \dots}{3\alpha x^2}$$

For existence of limit  $\alpha - 3 = 0$

$$\alpha = 3$$

$$\text{Limit } \beta = \frac{-3}{2\alpha}$$

$$\beta = -\frac{1}{2}$$

Now,

$$\alpha + \beta = \frac{5}{2}$$

8. The value of  $\log_e 2 \frac{d}{dx} (\log_{\cos x} \operatorname{cosec} x)$  at  $x = \frac{\pi}{4}$  is

$$(A) -2\sqrt{2} \quad (B) 2\sqrt{2} \quad (C) -4 \quad (D) 4$$

Official Ans. by NTA (D)

QuestPix Ans. (D)

**Sol.**  $\log_e 2 \frac{d}{dx} (\log_{\cos x} \operatorname{cosec} x)$

Let,

$$y = \log_{\cos x} \operatorname{cosec} x$$

$$y = -\frac{\ln(\sin x)}{\ln(\cos x)}$$

$$\frac{dy}{dx} = -\frac{[\cot x \cdot \ln(\cos x) + \tan x \cdot \ln(\sin x)]}{(\ln(\cos x))^2}$$

$$\left. \frac{dy}{dx} \right|_{x=\frac{\pi}{4}} = \frac{4}{\ln 2}$$

Now,

$$\Rightarrow \log_e 2 \cdot \frac{4}{\ln 2} = 4$$

9.  $\int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$  is equal to :-

(A)  $10(\pi + 4)$  (B)  $10(\pi + 2)$

(C)  $20(\pi - 2)$  (D)  $20(\pi + 2)$

**Official Ans. by NTA (D)**

**QuestPix Ans. (D)**

**Sol.**  $I = \int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$  ; (Jack property)

$$I = 40 \int_0^{\pi/2} (\sin x + \cos x)^2 dx$$

$$I = 40 \int_0^{\pi/2} (1 + \sin 2x) dx$$

$$I = 20[\pi + 2]$$

10. Let the solution curve  $y = f(x)$  of the differential

equation  $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ ,  $x \in (-1, 1)$  pass

through the origin. Then  $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx$  is equal to

(A)  $\frac{\pi}{3} - \frac{1}{4}$  (B)  $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

(C)  $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$  (D)  $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$

$$I.F = e^{\int \frac{x}{x^2 - 1} dx}$$

$$I.F = \sqrt{1 - x^2}$$

Solution of D.E.

$$y \cdot \sqrt{1 - x^2} = \int \frac{x^4 + 2x}{\sqrt{1 - x^2}} \cdot \sqrt{1 - x^2} dx$$

$$y \cdot \sqrt{1 - x^2} = \int (x^4 + 2x) dx$$

$$y \cdot \sqrt{1 - x^2} = \frac{x^5}{5} + x^2 + C$$

At  $x = 0$ ,  $y = 0$ , get  $C = 0$

$$y = \frac{x^5}{5\sqrt{1 - x^2}} + \frac{x^2}{\sqrt{1 - x^2}}$$

Now,

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5}{5\sqrt{1 - x^2}} dx + \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1 - x^2}} dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = 0 + 2 \int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1 - x^2}} dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

11. The acute angle between the pair of tangents drawn to the ellipse  $2x^2 + 3y^2 = 5$  from the point  $(1, 3)$  is

(A)  $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$  (B)  $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C)  $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$  (D)  $\tan^{-1}\left(\frac{3 + 8\sqrt{5}}{35}\right)$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.** Equation of tangent to the ellipse  $2x^2 + 3y^2 = 5$  is

$$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

It pass through  $(1, 3)$

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let  $\theta$  be the angle between the tangents

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{3\sqrt{320}}{-35} \right|$$

$$\theta = \tan^{-1} \left( \frac{24}{7\sqrt{5}} \right)$$

12. The equation of a common tangent to the parabolas  $y = x^2$  and  $y = -(x-2)^2$  is

- (A)  $y = 4(x-2)$  (B)  $y = 4(x-1)$   
(C)  $y = 4(x+1)$  (D)  $y = 4(x+2)$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.** Equation of tangent of  $y = x^2$  be

$$tx = y + at^2 \quad \dots\dots\dots(1)$$

$$y = tx - \frac{t^2}{4}$$

Solve with  $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

$$D = 0$$

$$(t-4)^2 - 4 \cdot \left( 4 - \frac{t^2}{4} \right) = 0$$

$$t^2 - 4t = 0$$

$$t = 0 \text{ or } t = 4$$

From eq. (1), required common tangent is

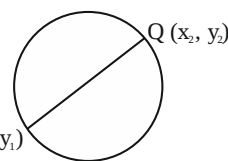
$$y = 4(x-1)$$

13. Let the abscissae of the two points P and Q on a circle be the roots of  $x^2 - 4x - 6 = 0$  and the ordinates of P and Q be the roots of  $y^2 + 2y - 7 = 0$ . If PQ is a diameter of the circle  $x^2 + y^2 + 2ax + 2by + c = 0$ , then the value of  $(a+b-c)$  is

- (A) 12 (B) 13 (C) 14 (D) 16

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**



**Sol.**

Equation of circle diameter form

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(where  $x_1, x_2$  are the roots of  $x^2 - 4x - 6 = 0$  and  $y_1, y_2$  are the roots of  $y^2 + 2y - 7 = 0$ )

$$x^2 + y^2 - 4x + 2y - 13 = 0$$

Now,

Compare it with the given equation, we get

$$a = -2, b = 1, c = -13$$

Now

$$a + b - c = 12$$

14. If the line  $x-1 = 0$ , is a directrix of the hyperbola  $kx^2 - y^2 = 6$ , then the hyperbola passes through the point

- (A)  $(-2\sqrt{5}, 6)$  (B)  $(-\sqrt{5}, 3)$   
(C)  $(\sqrt{5}, -2)$  (D)  $(2\sqrt{5}, 3\sqrt{6})$

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

**Sol.**  $\frac{x^2}{6/k} - \frac{y^2}{6} = 1 \quad \dots\dots\dots(1)$

$$e^2 = 1 + \frac{6}{6/k}$$

$$e = \sqrt{1+k}$$

$$a = \sqrt{\frac{6}{k}}$$

$$\text{Eq. of directrix } x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$$

$$\frac{6}{k(k+1)} = 1$$

$$k = 2$$

From eq. (1), we get  $2x^2 - y^2 = 6$

Check options

15. A vector  $\vec{a}$  is parallel to the line of intersection of the plane determined by the vectors  $\hat{i}, \hat{i} + \hat{j}$  and the plane determined by the vectors  $\hat{i} - \hat{j}, \hat{i} + \hat{k}$ . The obtuse angle between  $\vec{a}$  and the vector  $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$  is

- (A)  $\frac{3\pi}{4}$  (B)  $\frac{2\pi}{3}$   
(C)  $\frac{4\pi}{5}$  (D)  $\frac{5\pi}{6}$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**  $\vec{n}_1 = \hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$

$$\vec{n}_2 = (\hat{i} + \hat{k}) \times (\hat{i} - \hat{j})$$

$$= \hat{i} + \hat{j} - \hat{k}$$

Line of intersection along  $\vec{n}_1 \times \vec{n}_2$

$$= \hat{k} \times (\hat{i} + \hat{j} - \hat{k}) = -\hat{i} + \hat{j}$$

D.R of  $\vec{a} = -\hat{i} + \hat{j}$

D.R of  $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \cdot \vec{b} = -3 \text{ and } (\vec{a} \wedge \vec{b}) = \theta$$

$$\cos \theta = \frac{-3}{\sqrt{2} \times 3}$$

$$\theta = \frac{3\pi}{4}$$

16. If  $0 < x < \frac{1}{\sqrt{2}}$  and  $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$ , then a value

of  $\sin\left(\frac{2\pi\alpha}{\alpha + \beta}\right)$  is

- (A)  $4\sqrt{(1-x^2)}(1-2x^2)$   
(B)  $4x\sqrt{(1-x^2)}(1-2x^2)$   
(C)  $2x\sqrt{(1-x^2)}(1-4x^2)$   
(D)  $4\sqrt{(1-x^2)}(1-4x^2)$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$$\sin^{-1} x = k\alpha$$

$$\cos^{-1} x = k\beta$$

$$k = \frac{\pi}{2(\alpha + \beta)} \quad \dots(i)$$

$$\sin\left(\frac{2\pi\alpha}{\alpha + \beta}\right) = \sin(4\sin^{-1} x)$$

$$= 2\sin(2\sin^{-1} x) \cos(2\sin^{-1} x)$$

$$= 4x\sqrt{1-x^2}(1-2x^2)$$

17. Negation of the Boolean expression  $p \leftrightarrow (q \Rightarrow p)$  is

- (A)  $(\sim p) \wedge q$  (B)  $p \wedge (\sim q)$   
(C)  $(\sim p) \vee (\sim q)$  (D)  $(\sim p) \wedge (\sim q)$

**Official Ans. by NTA (D)**

**QuestPix Ans. (D)**

**Sol.**  $\sim(p \leftrightarrow (q \rightarrow p))$

$$\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$$

$$\sim(p \leftrightarrow (q \rightarrow p)) = (p \wedge \sim(q \rightarrow p)) \vee ((q \rightarrow p) \wedge \sim p)$$

$$(p \wedge \sim(q \rightarrow p)) = p \wedge (q \wedge \sim p) = (p \wedge \sim p) \wedge q = c$$

$$(q \rightarrow p) \wedge \sim p = (\sim q \vee p) \wedge \sim p = \sim p \wedge (\sim q \vee p)$$

$$= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q$$

$$\sim(p \leftrightarrow (q \rightarrow p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$$

18. Let X be a binomially distributed random variable with mean 4 and variance  $\frac{4}{3}$ . Then  $54 P(X \leq 2)$  is equal to

- (A)  $\frac{73}{27}$  (B)  $\frac{146}{27}$   
(C)  $\frac{146}{81}$  (D)  $\frac{126}{81}$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $np = 4$

$$npq = 4/3$$

$$n = 6, p = 2/3, q = 1/3$$

$$54(P(X = 2) + P(X = 1) + P(X = 0))$$

$$54\left({}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right)$$

$$= \frac{146}{27}$$

19. The integral  $\int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$  is equal to

(A)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right| + C$

(B)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\left(\frac{x}{2} + \frac{\pi}{3}\right)} \right| + C$

(C)  $\log_e \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)} \right| + C$

(D)  $\frac{1}{2} \log_e \left| \frac{\tan\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} - \frac{\pi}{6}\right)} \right| + C$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.** 
$$I = \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1 + \frac{2}{\sqrt{3}} \sin 2x\right)} dx$$

$$\frac{\sqrt{3}}{2} \int \frac{\left(1 - \frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(\frac{\sqrt{3}}{2} + \sin 2x\right)} dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)(\cos x - \sin x)}{\sin 60^\circ + \sin 2x} dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \sin x\right)}{2 \sin\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right)} dx$$

$$\int \frac{\left(\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x + \frac{\pi}{6}\right)\right)}{2 \sin\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right)} dx$$

$$\frac{1}{2} \left( \int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)} - \int \frac{dx}{\cos\left(x - \frac{\pi}{6}\right)} \right)$$

$$\frac{1}{2} \ln \left| \frac{\tan\left(\frac{x}{2} + \frac{\pi}{12}\right)}{\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)} \right|$$

20. The area bounded by the curves  $y = |x^2 - 1|$  and  $y = 1$  is

(A)  $\frac{2}{3}(\sqrt{2} + 1)$

(B)  $\frac{4}{3}(\sqrt{2} - 1)$

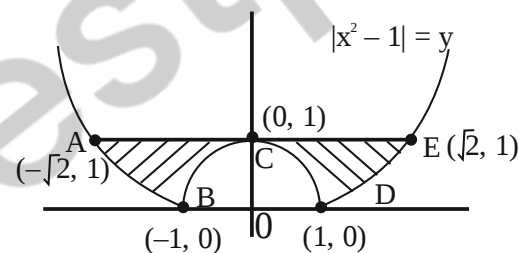
(C)  $2(\sqrt{2} - 1)$

(D)  $\frac{8}{3}(\sqrt{2} - 1)$

**Official Ans. by NTA (D)**

**QuestPix Ans. (D)**

**Sol.**  $y = |x^2 - 1|$



Area = ABCDEA

$$= 2 \left( \int_0^1 (1 - (1 - x^2)) dx + \int_1^{\sqrt{2}} (1 - (x^2 - 1)) dx \right)$$

$$= \frac{8}{3}(\sqrt{2} - 1)$$

## SECTION-B

21. Let  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and  $B = \{3, 6, 7, 9\}$ . Then the number of elements in the set  $\{C \subseteq A : C \cap B \neq \emptyset\}$  is \_\_\_\_\_

**Official Ans. by NTA (112)**

**QuestPix Ans. (112)**

**Sol.**  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and

$B = \{3, 6, 7, 9\}$

Total subset of  $A = 2^7 = 128$

$C \cap B = \emptyset$  when set  $C$  contains the element 1, 2, 4, 5



$$\therefore S = \{C \subseteq A; C \cap B \neq \emptyset\}$$

$$= \text{Total} - (C \cap B = \emptyset)$$

$$= 128 - 2^4 = 112$$

22. The largest value of  $a$ , for which the perpendicular distance of the plane containing the lines  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$  and  $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$  from the point  $(2, 1, 4)$  is  $\sqrt{3}$ , is \_\_\_\_\_.

**Official Ans. by NTA (20)**

**QuestPix Ans. (20)**

**Sol.**  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$

$$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1-a^2) - \hat{j}(-a-1) + \hat{k}(1+a)$$

$$\vec{n} = (1-a)\hat{i} + \hat{j} + \hat{k}$$

One point in plane :  $(1, 1, 0)$

$\therefore$  equation of plane is

$$(1-a)(x-1) + (y-1) + (z-0) = 0$$

$$(1-a)x + y + z + a - 2 = 0$$

$$\therefore D = \frac{|(1-a)2 + 1 + 4 + a - 2|}{\sqrt{(1-a)^2 + 1 + 1}}$$

$$\Rightarrow |5-a| = \sqrt{3} \cdot \sqrt{a^2 - 2a + 3}$$

$$\Rightarrow a^2 + 2a - 8 = 0$$

$$\Rightarrow a = 2, -4$$

$\therefore$  largest value of  $a = 2$

23. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is \_\_\_\_\_.

**Official Ans. by NTA (30)**

**QuestPix Ans. (30)**

**Sol.** Here 1<sup>st</sup> digit is 1 or 2 only

**Case-I**

If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64

$$\begin{array}{|c|c|c|c|} \hline 1 & & & \\ \hline \end{array} \quad \begin{array}{c} \downarrow \\ 1 \end{array} \times \begin{array}{c} \downarrow \\ 3 \end{array} \times \begin{array}{c} \downarrow \\ 6 \end{array} = 18 \text{ ways}$$

**Case - II**

If first digit is 2 then last two digit can be 16, 36, 56, 64

$$\begin{array}{|c|c|c|c|} \hline 2 & & & \\ \hline \end{array} \quad \begin{array}{c} \downarrow \\ 1 \end{array} \times \begin{array}{c} \downarrow \\ 3 \end{array} \times \begin{array}{c} \downarrow \\ 4 \end{array} = 12 \text{ ways}$$

Total ways =  $12 + 18 = 30$  ways

24. If  $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$ , where  $m$  and  $n$  are co-prime, then  $m + n$  is equal to

**Official Ans. by NTA (166)**

**Allen Ans. (166)**

**Sol.**  $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$

$$\Rightarrow \frac{1}{2} \sum_{k=1}^{10} \frac{(k^2 + k + 1) - (k^2 - k + 1)}{(k^2 + k + 1)(k^2 - k + 1)}$$

$$\Rightarrow \frac{1}{2} \left( \sum_{k=1}^{10} \left( \frac{1}{(k^2 - k + 1)} - \frac{1}{(k^2 + k + 1)} \right) \right)$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n}$$

$$m + n = 166$$

25. If the sum of solutions of the system of equations  $2\sin^2 \theta - \cos 2\theta = 0$  and  $2\cos^2 \theta + 3\sin \theta = 0$  in the interval  $[0, 2\pi]$  is  $k\pi$ , then  $k$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**  $2\sin^2 \theta - \cos 2\theta = 0$

$$2\sin^2 \theta - (1 - 2\sin^2 \theta) = 0$$

$$\Rightarrow \sin^2 \theta = \left(\frac{1}{2}\right)^2$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$2\cos^2 \theta + 3\sin \theta = 0$$

$$\Rightarrow 2\sin^2 \theta - 3\sin \theta - 2 = 0$$

$$\therefore \sin \theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

So, the common solution is

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{Sum} = \frac{7\pi + 11\pi}{6} = 3\pi = k\pi$$

$$K = 3$$

26. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If  $\sigma$  is the standard deviation of the data after omitting the two wrong observations from the data, then  $38\sigma^2$  is equal to\_\_\_\_\_.

**Official Ans. by NTA (238)**

**QuestPix Ans. (238)**

**Sol.** Wrong mean =  $\mu_1 = 30$

Wrong S.D =  $\sigma_1 = 5$

$$\frac{\sum x_i}{40} = 30$$

$$\Rightarrow \sum x_i = 1200$$

$$\sigma_1^2 = 25$$

$$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$$

$$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$$

$$\text{New sum} = \sum x'_i = 1200 - 10 - 12 = 1178$$

$$\text{New mean} = \mu'_1 = \frac{1178}{38} = 31$$

$$\text{New } \sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$$

$$\text{New S.D, } \sigma'_1 = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$$

$$36756 - (31)^2 \times 38 = 38\sigma^2$$

$$\Rightarrow 38\sigma^2 = 238$$

27. The plane passing through the line L:  $\ell x - y + 3(1 - \ell)z = 1$ ,  $x + 2y - z = 2$  and perpendicular to the plane  $3x + 2y + z = 6$  is  $3x - 8y + 7z = 4$ . If  $\theta$  is the acute angle between the line L and the y-axis, then  $415 \cos^2 \theta$  is equal to\_\_\_\_\_.

**Official Ans. by NTA (125)**

**QuestPix Ans. (125)**

**Sol.**  $\vec{n}_1 = \ell \hat{i} - \hat{j} + 3(1 - \ell)\hat{k}$

$$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Direction ratio of line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1 - \ell) \\ 1 & 2 & -1 \end{vmatrix}$$

$$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

$3x - 8y + 7z = 4$  will contain the line

$$(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

Normal of  $3x - 8y + 7z = 4$  will be perpendicular to the line

$$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$$

$$\Rightarrow \ell = \frac{2}{3}$$

$$\therefore \text{direction ratio of line} \left( -1, \frac{5}{3}, \frac{7}{3} \right)$$

Angle with y axis

$$\cos \theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$$

$$\cos \theta = \frac{5}{\sqrt{83}}$$

$$\therefore 415 \cos^2 \theta = \frac{25}{83} \times 415 = 125$$

28. Suppose  $y = y(x)$  be the solution curve to the differential equation  $\frac{dy}{dx} - y = 2 - e^{-x}$  such that  $\lim_{x \rightarrow \infty} y(x)$  is finite. If  $a$  and  $b$  are respectively the  $x$ - and  $y$ - intercepts of the tangent to the curve at  $x=0$ , then the value of  $a-4b$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**  $\frac{dy}{dx} - y = 2 - e^{-x}$

I.F. =  $e^{-\int dx} = e^{-x}$

$\therefore$  solution of D.E

$$y \cdot e^{-x} = \int (2e^{-x} - e^{-2x}) dx$$

$$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C \cdot e^x$$

$\therefore \lim_{x \rightarrow \infty} y$  is finite

$$\therefore \lim_{x \rightarrow \infty} \left( -2 + \frac{e^{-x}}{2} + C \cdot e^x \right) \rightarrow \text{finite}$$

This is possible only when  $C = 0$

$$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}e^{-x}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = -\frac{1}{2} = m, \quad y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$$

$\therefore$  equation of tangent

$$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$$

$$\Rightarrow x + 2y = -3$$

$$a = -3, \quad b = \frac{-3}{2}$$

$$a - 4b = -3 + 6 = 3$$

29. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

**Official Ans. by NTA (53)**

**QuestPix Ans. (53)**

**Sol.** 1<sup>st</sup> term = 100 =  $a$

Last term = 199 =  $\ell$

If 3 term

$$a, a + d, a + 2d$$

$$a_n = \ell = a + (n - 1)d$$

$$d_i = \frac{\ell - a}{n - 1}$$

$n \rightarrow$  number of terms

$$n=3, \quad d_1 = \frac{199 - 100}{2}$$

$$= \frac{99}{2} \notin I$$

$$n = 4, \quad d_2 = \frac{99}{3} = 33 \in I$$

$$n = 10, \quad d_3 = \frac{99}{9} = 11 \in I$$

$$n = 12, \quad d_4 = \frac{99}{11} = 9 \in I$$

$$\therefore \sum d_i = 33 + 11 + 9 = 53$$

30. The number of matrices  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , where  $a, b, c, d \in \{-1, 0, 1, 2, 3, \dots, 10\}$ , such that  $A = A^{-1}$ , is \_\_\_\_\_.

**Official Ans. by NTA (50)**

**QuestPix Ans. (50)**

**Sol.**  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given  $A = A^{-1}$

$$\therefore A^2 = A \cdot A^{-1} = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore a^2 + bc = 1 \quad \dots(1)$$

$$ab + bd = 0 \quad \dots(2)$$

$$ac + cd = 0 \quad \dots(3)$$

$$bc + d^2 = 1 \quad \dots(4)$$



(1) – (4) gives

$$a^2 - d^2 = 0$$

$$\Rightarrow (a + d) = 0 \text{ or } a - d = 0$$

## Case – I

$$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$$

(a)  $(a, d) = (-1, 1)$

$\therefore$  from equation (1)

$$1 + bc = 1 \Rightarrow bc = 0$$

$$b = 0 \quad C = 12 \text{ possibilities}$$

$$c = 0 \quad b = 12 \text{ possibilities}$$

but  $(0, 0)$  is repeated

$$\therefore 2 \times 12 = 24$$

$$24 - 1 (\text{repeated}) = 23 \text{ pairs}$$

(b)  $(a, d) = (1, -1) \Rightarrow bc = 0 \rightarrow 23 \text{ pairs}$

(c)  $(a, d) = (0, 0) \Rightarrow bc = 1$

$$\Rightarrow (b, c) = (1, 1) \text{ \& } (-1, -1), 2 \text{ pairs}$$

## Case – II

$$a = d$$

from (2) and (3)

$$a \neq 0 \text{ then } b = c = 0$$

$$a^2 = 1$$

$$a = \pm 1 = d$$

$$(a, d) = (1, 1), (-1, -1) \rightarrow 2 \text{ pairs}$$

$$\therefore \text{Total} = 23 + 23 + 2 + 2$$

$$= 50 \text{ pairs}$$

**(Held On Tuesday 26<sup>th</sup> July, 2022)**
**TIME : 3 : 00 PM to 6 : 00 PM**

| PHYSICS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | TEST PAPER WITH SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
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| <p align="center"><b>SECTION-A</b></p> <p><b>31.</b> Two projectiles are thrown with same initial velocity making an angle of <math>45^\circ</math> and <math>30^\circ</math> with the horizontal respectively. The ratio of their respective ranges will be<br/>           (A) <math>1:\sqrt{2}</math> (B) <math>\sqrt{2}:1</math><br/>           (C) <math>2:\sqrt{3}</math> (D) <math>\sqrt{3}:2</math><br/> <b>Official Ans. by NTA (C)</b><br/> <b>QuestPix Ans. (C)</b><br/> <b>Sol.</b> Let projection speed is <math>u</math><br/> <math display="block">R_1 = \frac{u^2 \sin(90^\circ)}{g}; R_2 = \frac{u^2 \sin(60^\circ)}{g}</math> <math display="block">\frac{R_1}{R_2} = \frac{2}{\sqrt{3}}</math></p> <p><b>32.</b> In a Vernier Calipers. 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4<sup>th</sup> Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6<sup>th</sup> Vernier scale division exactly coincides with the main scale reading. The diameter of the spherical body will be :<br/>           (A) 3.02 cm (B) 3.06 cm<br/>           (C) 3.10 cm (D) 3.20 cm<br/> <b>Official Ans. by NTA (C)</b><br/> <b>QuestPix Ans. (10)</b><br/> <b>Sol.</b> 1 M.S.D = 1mm<br/>           9 M.S.D = 10 V.S.D<br/>           1 V.S.D = 0.9 M.S.D = 0.9 mm<br/>           L.C of vernier caliper = <math>1 - 0.9 = 0.1 \text{ mm} = 0.01 \text{ cm}</math><br/>           zero error = <math>-(10 - 4) \times 0.1 \text{ mm} = -0.6 \text{ mm}</math><br/>           Reading = M.S.R + V.S.R - Zero error<br/> <math>= 3 \text{ cm} + 6 \times 0.01 - [-0.06]</math><br/> <math>= 3 + 0.06 + 0.06</math><br/> <math>= 3.12 \text{ cm}</math><br/>           Nearest given answer in the options is 3.10</p> | <p><b>33.</b> A ball of mass 0.15 kg hits the wall with its initial speed of <math>12 \text{ ms}^{-1}</math> and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is 100 N. calculate the time duration of the contact of ball with the wall.<br/>           (A) 0.018 s (B) 0.036 s<br/>           (C) 0.009 s (D) 0.072 s<br/> <b>Official Ans. by NTA (B)</b><br/> <b>QuestPix Ans. (B)</b><br/> <b>Sol.</b> <math>\vec{P}_i = 0.15 \times 12 (\hat{i})</math><br/> <math>\vec{P}_f = 0.15 \times 12 (-\hat{i})</math><br/> <math> \Delta \vec{P}  = 3.6 \text{ kg-m/s}</math><br/> <math>3.6 = F \Delta t</math><br/> <math>3.6 = 100 \Delta t</math><br/> <math>\Delta t = 0.036 \text{ sec}</math></p> <p><b>34.</b> A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be :<br/>           (A) 1:1 (B) 2:1 (C) 1:4 (D) 4:1<br/> <b>Official Ans. by NTA (B)</b><br/> <b>QuestPix Ans. (B)</b><br/> <b>Sol.</b> <math>K.E = \frac{p^2}{2m}</math><br/> <math>K_1 = \frac{p_1^2}{2(8)}; K_2 = \frac{p_2^2}{2(2)}</math><br/> <math>K_1 = K_2</math><br/>           So,<br/> <math>4p_2^2 = p_1^2</math><br/> <math>\frac{p_1}{p_2} = 2</math></p> <p><b>35.</b> Two uniformly charged spherical conductors A and B of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be :<br/>           (A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4<br/> <b>Official Ans. by NTA (B)</b><br/> <b>QuestPix Ans. (B)</b></p> |

**Sol.**  $V_A = V_B$

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$$

$$E_A = \frac{KQ_A}{R_A^2}; E_B = \frac{KQ_B}{R_B^2}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^2}{R_A^2} = \frac{R_B}{R_A} = \frac{2}{1}$$

- 36.** The oscillating magnetic field in a plane electromagnetic wave is given by  $B_y = 5 \times 10^{-6} \sin 1000\pi (5x - 4 \times 10^8 t)$  T. The amplitude of electric field will be :

- (A)  $15 \times 10^2 \text{ Vm}^{-1}$  (B)  $5 \times 10^{-6} \text{ Vm}^{-1}$   
(C)  $16 \times 10^{12} \text{ Vm}^{-1}$  (D)  $4 \times 10^2 \text{ Vm}^{-1}$

**Official Ans. by NTA (D)**

**QuestPix Ans. (D)**

**Sol.**  $B_0 = 5 \times 10^{-6}$

$$v = \text{Speed of wave} = \frac{4 \times 10^8}{5} = 8 \times 10^7 \left[ \because v = \frac{\omega}{k} \right]$$

$$E_0 = vB_0 = 40 \times 10^1$$

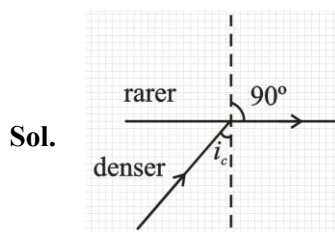
$$= 4 \times 10^2 \text{ V/m}$$

- 37.** Light travels in two media  $M_1$  and  $M_2$  with speeds  $1.5 \times 10^8 \text{ ms}^{-1}$  and  $2.0 \times 10^8 \text{ ms}^{-1}$  respectively. The critical angle between them is:

- (A)  $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$  (B)  $\tan^{-1}\left(\frac{2}{3}\right)$   
(C)  $\cos^{-1}\left(\frac{3}{4}\right)$  (D)  $\sin^{-1}\left(\frac{2}{3}\right)$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**



$$v = \frac{c}{n}$$

$$n_d \sin i_c = n_r \sin 90^\circ$$

$$\sin i_c = \frac{n_r}{n_d} = \frac{v_d}{v_r}$$

$$\sin i_c = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{1.5}{2}$$

$$\sin i_c = \frac{3}{4}$$

$$\tan i_c = \frac{3}{\sqrt{4^2 - 3^2}} \Rightarrow \frac{3}{\sqrt{7}}$$

$$i_c = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$$

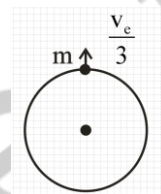
- 38.** A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:

(Take radius of earth = 6400 km and  $g = 10 \text{ ms}^{-2}$ )

- (A) 800 km (B) 1600 km  
(C) 2133 km (D) 4800 km

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**



**Sol.**

$$v_e = \sqrt{\frac{2Gm}{R}}$$

$$\frac{-GMm}{R} + \frac{1}{2}m \frac{v_e^2}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{v_e^2}{18}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R}$$

$$\frac{1}{R+h} = \frac{8}{9R}$$

$$9R = 8R + 8h$$

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800 \text{ km}$$

- 39.** The maximum and minimum voltage of an amplitude modulated signal are 60 V and 20 V respectively. The percentage modulation index will be :

- (A) 0.5% (B) 50% (C) 2% (D) 30%

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $V_{\max} = 60$

$V_{\min} = 20$

% modulation =

$$\left( \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} \right) 100 \Rightarrow \left( \frac{60 - 20}{60 + 20} \right) 100 \Rightarrow \left( \frac{40}{80} \right) 100$$

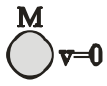
$\Rightarrow 50\%$

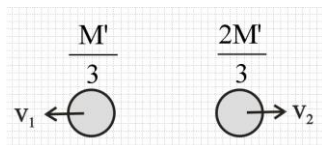
- 40.** A nucleus of mass  $M$  at rest splits into two parts having masses  $\frac{M'}{3}$  and  $\frac{2M'}{3}$  ( $M' < M$ ). The ratio of de Broglie wavelength of two parts will be :

- (A) 1 : 2 (B) 2 : 1  
(C) 1 : 1 (D) 2 : 3

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

**Sol.** 



$|\vec{P}_1| = |\vec{P}_2|$

Here  $\vec{P}$  is momentum

So  $\lambda = \frac{h}{p}$

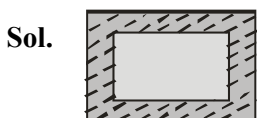
Hence both will have same de broglie wavelength.

- 41.** An ice cube of dimensions  $60 \text{ cm} \times 50 \text{ cm} \times 20 \text{ cm}$  is placed in an insulation box of wall thickness  $1 \text{ cm}$ . The box keeping the ice cube at  $0^\circ\text{C}$  of temperature is brought to a room of temperature  $40^\circ\text{C}$ . The rate of melting of ice is approximately: (Latent heat of fusion of ice is  $3.4 \times 10^5 \text{ J kg}^{-1}$  and thermal conducting of insulation wall is  $0.05 \text{ Wm}^{-1}\text{C}^{-1}$ )

- (A)  $61 \times 10^{-1} \text{ kg s}^{-1}$  (B)  $61 \times 10^{-5} \text{ kg s}^{-1}$   
(C)  $208 \text{ kg s}^{-1}$  (D)  $30 \times 10^{-5} \text{ kg s}^{-1}$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**



$$\frac{dQ}{dt} = \frac{KA\Delta T}{\ell}$$

$A = 2(0.6 \times 0.5 + 0.5 \times 0.2 + 0.2 \times 0.6)$

$= 2(0.3 + 0.1 + 0.12)$

$= 2(0.4 + 0.12)$

$= 2(0.52)$

$= 1.04 \text{ m}^2$

$$R_{th} = \frac{\ell}{KA} \Rightarrow \frac{1 \times 10^{-2}}{0.05 \times 1.04} \Rightarrow \frac{10^{-2}}{0.052}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R_{th}} \Rightarrow \frac{40 \times 0.052}{10^{-2}} \Rightarrow 2.08 \times 10^2 \text{ J/s}$$

$2.08 \times 10^2 = m \times 3.4 \times 10^5$

$$m = \frac{2.08}{3.4 \times 10^3} \Rightarrow 0.61 \times 10^{-3} \text{ kg/s}$$

$= 61 \times 10^{-5} \text{ Kg/s}$

- 42.** A gas has  $n$  degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be :

- (A)  $\frac{n}{n+2}$  (B)  $\frac{n+2}{n}$   
(C)  $\frac{n}{2n+2}$  (D)  $\frac{n}{n-2}$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**  $C_v = \frac{nR}{2}$   $C_p = \frac{(n+2)R}{2}$

$$\frac{C_v}{C_p} = \frac{n}{n+2}$$

- 43.** A transverse wave is represented by  $y = 2\sin(\omega t - kx)$  cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be ;

- (A)  $4\pi$  (B)  $2\pi$   
(C)  $\pi$  (D)  $2$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**  $y = 2 \sin(\omega t - kx)$

Maximum particle velocity =  $A \omega$

Wave velocity =  $\frac{\omega}{k}$

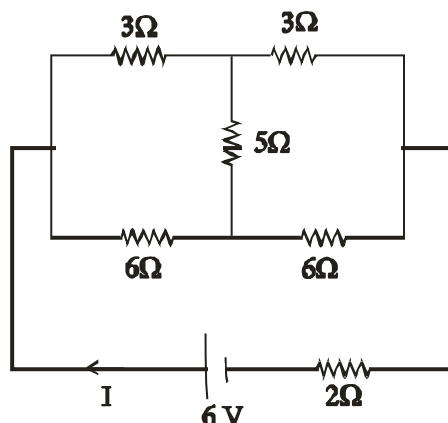
$$\frac{\omega}{k} = A \omega$$

$$k = \frac{1}{A} = \frac{2\pi}{\lambda}$$

$\lambda = 2\pi A$

$= 4 \pi \text{ cm}$

44. A battery of 6 V is connected to the circuit as shown below. The current  $I$  drawn from the battery is :

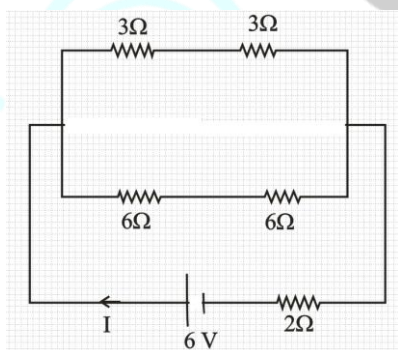


- (A) 1A (B) 2A  
(C)  $\frac{6}{11}$  A (D)  $\frac{4}{3}$  A

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.** Balanced wheat stone bridge in circuit so there is no current in  $5\Omega$  resistor so it can be removed from the circuit.



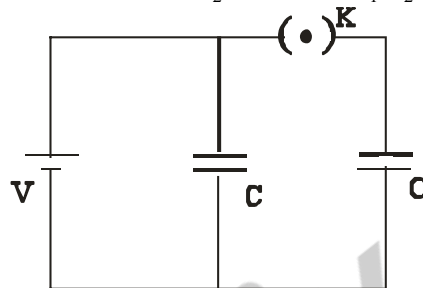
$$R_{eq} = \frac{6 \times 12}{6 + 12} + 2$$

$$= \frac{6 \times 12}{18} + 2$$

$$R_{eq} = 6\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6}{6} = 1 \text{ Amp.}$$

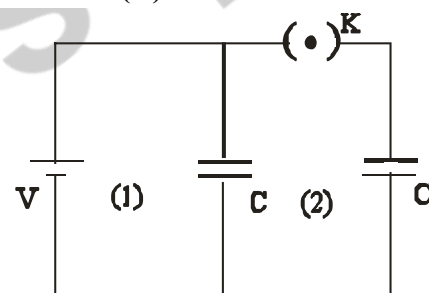
45. A source of potential difference  $V$  is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is  $E_1$ . Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now  $E_2$ . The ratio  $E_1/E_2$  will be :



- (A)  $\frac{1}{10}$  (B)  $\frac{2}{5}$   
(C)  $\frac{5}{13}$  (D)  $\frac{5}{26}$

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**



**Sol.**

(1) Switch is closed

$$C_{eq} = 2C$$

$$\text{Energy } E_1 = \frac{1}{2} C_{eq} V^2$$

$$= \frac{1}{2} 2C \times V^2$$

$$E_1 = CV^2$$

(ii) When switch is opened charge on right capacitor remain  $CV$  while potential on left capacitor remain same

Dielectric  $K = 5$

$$C' = KC$$

$$C' = 5C$$

$$E_2 = \frac{1}{2} (5C) V^2 + \frac{(CV)^2}{2(5C)}$$

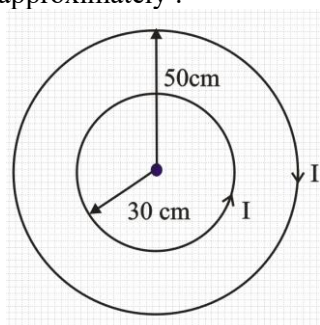
$$E_2 = \frac{5CV^2}{2} + \frac{CV^2}{10}$$

$$E_2 = \frac{13CV^2}{5}$$

$$\frac{E_1}{E_2} = \frac{CV^2}{13CV^2} = \frac{5}{13}$$

$$\frac{E_1}{E_2} = \frac{5}{13}$$

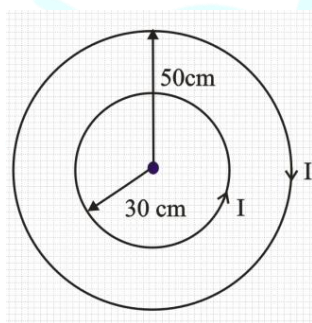
46. Two concentric circular loops of radii  $r_1=30$  cm and  $r_2=50$  cm are placed in X-Y plane as shown in the figure. A current  $I = 7$  A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately :



- (A)  $\frac{7}{2} \hat{k} \text{ Am}^2$  (B)  $-\frac{7}{2} \hat{k} \text{ Am}^2$   
(C)  $7 \hat{k} \text{ Am}^2$  (D)  $-7 \hat{k} \text{ Am}^2$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**



**Sol.**

Magnetic moment

$$\vec{M} = -i\pi(0.5)^2 \hat{k} + i\pi(0.3)^2 \hat{k}$$

$$\vec{M} = -7 \times \frac{22}{7} \left( \frac{25}{100} - \frac{9}{100} \right) \hat{k}$$

$$= -22 \left( \frac{16}{100} \right) \hat{k}$$

$$\vec{M} = -3.52 \hat{k} \text{ Am}^2$$

$$= -\frac{7}{2} \hat{k} \text{ Am}^2$$

47. A velocity selector consists of electric field  $\vec{E} = E\hat{k}$  and magnetic field  $\vec{B} = B\hat{j}$  with  $B=12$  mT. The value  $E$  required for an electron of energy 728 eV moving along the positive x-axis to pass undeflected is :

(Given, mass of electron =  $9.1 \times 10^{-31}$  kg)

- (A) 192 kVm<sup>-1</sup> (B) 192 m Vm<sup>-1</sup>  
(C) 9600 kVm<sup>-1</sup> (D) 16 kVm<sup>-1</sup>

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**  $\vec{E} = E\hat{k}$   $B = 12$  mT

$\vec{B} = B\hat{j}$  Energy = 728 eV

$$\text{Energy} = \frac{1}{2}mv^2$$

$$728 \text{ eV} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

$$728 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

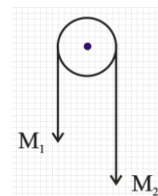
$$v = 16 \times 10^6 \text{ m/s}$$

$$E = vB$$

$$E = 16 \times 10^6 \times 12 \times 10^{-3}$$

$$E = 192 \times 10^3 \text{ V/m}$$

48. Two masses  $M_1$  and  $M_2$  are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass  $M_2$  is twice that of  $M_1$ , the acceleration of the system is  $a_1$ . When the mass  $M_2$  is thrice that of  $M_1$ , the acceleration of the system is  $a_2$ . The ratio  $\frac{a_1}{a_2}$  will be:



- (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$   
(C)  $\frac{3}{2}$  (D)  $\frac{1}{2}$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $a = \frac{m_2g - m_1g}{m_1 + m_2}$

Case 1  $M_2 = 2m_1$

$$a_1 = \frac{2m_1g - m_1g}{3m_1}$$

$$a_1 = g/3$$

Case -2

$$M_2 = 3m_1$$

$$a_2 = \frac{3m_1g - m_1g}{4m_1}$$

$$a_2 = \frac{g}{2}$$

$$\frac{a_1}{a_2} = \frac{\frac{g}{3}}{\frac{g}{2}} = \frac{2}{3}$$

49. Mass numbers of two nuclei are in the ratio of 4:3. Their nuclear densities will be in the ratio of

- (A) 4:3 (B)  $\left(\frac{3}{4}\right)^{\frac{1}{3}}$   
(C) 1 : 1 (D)  $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

**Sol.** Radius of nucleus  $R = R_0 A^{\frac{1}{3}}$

$$\text{Density of nucleus} = \frac{\text{Mass of nucleus}}{\text{volume of nucleus}}$$

$$\rho = \frac{m \times A}{\frac{4}{3}\pi R^3} \quad \text{Where } m : \text{mass of proton or neutron}$$

$$\rho = \frac{m \times A}{\frac{4}{3}\pi R_0^3 A}$$

$$\rho \propto A^0$$

Hence density of nucleus is independent of mass number

50. The area of cross section of the rope used to lift a load by a crane is  $2.5 \times 10^{-4} \text{ m}^2$ . The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, the required area of cross section of the rope should be :  
(take  $g = 10 \text{ ms}^{-2}$ )  
(A)  $6.25 \times 10^{-4} \text{ m}^2$   
(B)  $10 \times 10^{-4} \text{ m}^2$   
(C)  $1 \times 10^{-4} \text{ m}^2$   
(D)  $1.67 \times 10^{-4} \text{ m}^2$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.** Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

$$\text{breaking stress} = \frac{\text{Maximum lifting capacity}}{\text{Area of cross section of rope}}$$

$$\frac{10}{2.5 \times 10^{-4}} = \frac{25}{A}$$

$$A = 625 \times 10^{-6} \\ = 6.25 \times 10^{-4} \text{ m}^2$$

### SECTION-B

51. If  $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})\text{m}$  and  $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})\text{m}$ . The magnitude of component of vector  $\vec{A}$  along vector  $\vec{B}$  will be \_\_\_\_\_ m.

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.**  $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})\text{m}$  and  $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})\text{m}$

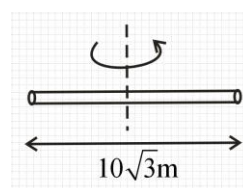
Component of  $\vec{A}$  along  $\vec{B} = \vec{A} \cdot \hat{B}$

$$= \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{2 + 6 - 2}{\sqrt{1^2 + 2^2 + 2^2}} \\ = \frac{6}{3} = 2$$

52. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be \_\_\_\_\_ m.  
Given, the length of the rod is  $10\sqrt{3} \text{ m}$ .

**Official Ans. by NTA (5)**

**QuestPix Ans. (5)**



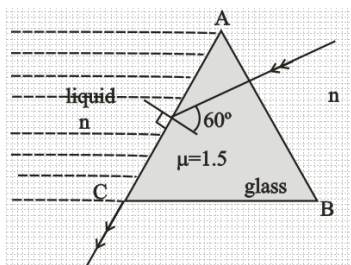
**Sol.**

$$I = \frac{m\ell^2}{12} = mk^2 \Rightarrow k^2 = \frac{\ell^2}{12} \Rightarrow k = \frac{\ell}{\sqrt{12}} = \frac{\ell}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$$

53. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle  $60^\circ$  at the side AC, the refracted light beam just grazes along face AC.

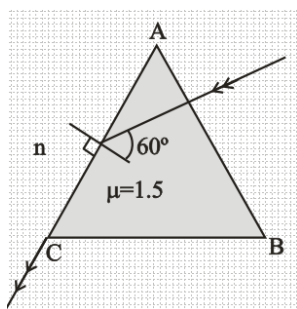
The refractive index of the liquid  $n = \frac{\sqrt{x}}{4}$ . The value of x is \_\_\_\_\_.

(Given refractive index of glass = 1.5)



**Official Ans. by NTA (27)**

**QuestPix Ans. (27)**



**Sol.**

Using Snell's law at face AC

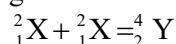
$$1.5 \sin 60^\circ = n \times \sin 90^\circ$$

$$1.5 \times \frac{\sqrt{3}}{2} = n = \frac{\sqrt{x}}{4}$$

$$3\sqrt{3} = \sqrt{x}$$

$$x = 27$$

54. Two lighter nuclei combine to form a comparatively heavier nucleus by the relation given below:



The binding energies per nucleon  ${}^2_1\text{X}$  and  ${}^4_2\text{Y}$  are 1.1 MeV and 7.6 MeV respectively. The energy released in this process is \_\_\_\_\_ MeV.

**Official Ans. by NTA (26)**

**QuestPix Ans. (26)**

**Sol.** Energy released in the given process = Binding

energy of product – Binding energy of reactants

$$= 7.6 \times 4 - (1.1 \times 2) \times 2$$

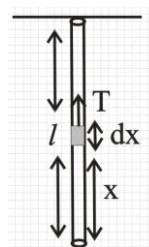
$$= 30.4 - 4.4$$

$$= 26 \text{ MeV}$$

55. A uniform heavy rod of mass 20 kg. Cross sectional area  $0.4 \text{ m}^2$  and length 20 m is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is  $x \times 10^{-9} \text{ m}$ . The value of x is \_\_\_\_\_. : (Given. Young's modulus  $Y = 2 \times 10^{11} \text{ Nm}^{-2}$  and  $g = 10 \text{ ms}^{-2}$ )

**Official Ans. by NTA (25)**

**QuestPix Ans. (25)**



**Sol.**

$$Y = \frac{T}{A} \frac{dx}{dy}$$

$$m = 20 \text{ kg}$$

$$A = 0.4 \text{ m}^2$$

$$l = 20 \text{ m}$$

let extension is dy in length dx

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{T}{A} = \frac{T}{A} \frac{dx}{dy}$$

$$dy = \frac{T dx}{AY}$$

$$\text{Tension at a distance } x \text{ from lower end} = \frac{mg}{\ell} x$$

$$\text{So, } \int_0^l dy = \int_0^l \frac{mg}{\ell} x \frac{dx}{AY}$$

$$\Delta \ell = \frac{mg}{\ell AY} \left[ \frac{x^2}{2} \right]_0^l$$

$$\Delta \ell = \frac{mg\ell}{2AY}$$

$$\Delta \ell = \frac{20 \times 10 \times 20}{2 \times 0.4 \times 2 \times 10^{11}}$$

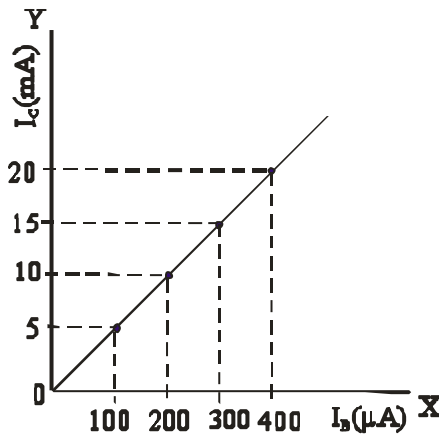
$$2500 \times 10^{-11}$$

$$\Delta \ell = 25 \times 10^{-9}$$

$$= x \times 10^{-9}$$

$$x = 25$$

56. The typical transfer characteristic of a transistor in CE configuration is shown in figure. A load resistor of  $2\text{ k}\Omega$  is connected in the collector branch of the circuit used. The input resistance of the transistor is  $0.50\text{ k}\Omega$ . The voltage gain of the transistor is



Official Ans. by NTA (200)

QuestPix Ans. (200)

- Sol. Current gain in C-E configuration

$$\Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B}$$

$$R_C = 2\text{ k}\Omega, R_B = 0.50\text{ k}\Omega$$

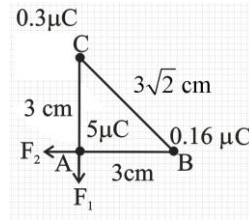
$$\begin{aligned} \text{Voltage gain} &= \frac{\Delta I_C R_C}{\Delta I_B R_B} = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} \times \frac{2}{0.5} \\ &= \frac{10^{-2}}{5 \times 10^{-5}} = \frac{1000}{5} = 200 \end{aligned}$$

57. Three point charges of magnitude  $5\mu\text{C}$ ,  $0.16\mu\text{C}$  and  $0.3\mu\text{C}$  are located at the vertices A, B, C of a right angled triangle whose sides are  $AB = 3\text{ cm}$ ,  $BC = 3\sqrt{2}\text{ cm}$  and  $CA = 3\text{ cm}$  and point A is the right angle corner. Charge at point A experiences \_\_\_\_\_ N of electrostatic force due to the other two charges.

Official Ans. by NTA (17)

QuestPix Ans. (17)

Sol.



$$F_1 = \frac{k \times 5 \times 0.3 \times 10^{-12}}{9 \times 10^{-4}}$$

$$= \frac{9 \times 10^9 \times 5 \times 0.3 \times 10^{-12}}{9 \times 10^{-4}}$$

$$= 1.5 \times 10 = 15\text{ N}$$

$$F_2 = \frac{9 \times 10^9 \times 5 \times 0.16 \times 10^{-12}}{9 \times 10^{-4}} = 8\text{ N}$$

$$\text{force experienced by charge at A} = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{289} = 17\text{ N}$$

58. In a coil of resistance  $8\Omega$ , the magnetic flux due to an external magnetic field varies with time as  $\phi = \frac{2}{3}(9 - t^2)$ . The value of total heat produced in the coil, till the flux becomes zero, will be \_\_\_\_\_ J.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.  $\phi = \frac{2}{3}(9 - t^2) = 0$

$$t = 3\text{ sec}$$

$$e = \frac{-d\phi}{dt} = -\frac{2}{3}(0 - 2t) = \frac{4t}{3}$$

$$\text{Heat produced in 3 sec} = \int_0^3 \frac{e^2}{r} dt = \int_0^3 \frac{16t^2}{9 \times 8} dt = 2\text{ J}$$



**(Held On Tuesday 26<sup>th</sup> July, 2022)**
**TIME : 3 : 00 PM to 6 : 00 PM**

| CHEMISTRY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | TEST PAPER WITH SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| <p align="center"><b>SECTION-A</b></p> <p><b>61.</b> Hemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of hemoglobin is :<br/>(Given : Atomic mass of Fe is 56 u, <math>N_A</math> in <math>6.022 \times 10^{23} \text{ mol}^{-1}</math>)<br/>(A) <math>1.21 \times 10^5</math> (B) <math>12.0 \times 10^{16}</math><br/>(C) <math>1.21 \times 10^{20}</math> (D) <math>3.4 \times 10^{22}</math><br/><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> <p><b>Sol.</b> No. of Fe atoms = <math>\frac{0.34}{100} \times \frac{3.3}{56} \times 6.022 \times 10^{23}</math><br/>= <math>1.206 \times 10^{20}</math></p> <p><b>62.</b> Arrange the following in increasing order of their covalent character.<br/>(A) <math>\text{CaF}_2</math> (B) <math>\text{CaCl}_2</math><br/>(C) <math>\text{CaBr}_2</math> (D) <math>\text{CaI}_2</math><br/>Choose the correct answer from the options given below.<br/>(A) <math>B &lt; A &lt; C &lt; D</math> (B) <math>A &lt; B &lt; C &lt; D</math><br/>(C) <math>A &lt; B &lt; D &lt; C</math> (D) <math>A &lt; C &lt; B &lt; D</math><br/><b>Official Ans. by NTA (B)</b><br/><b>QuestPix Ans. (B)</b></p> <p><b>Sol.</b> According to Fajan's rule,<br/>Covalent character <math>\propto</math> size of Anion</p> <p><b>63.</b> Class XII students were asked to prepare one litre of buffer solution of pH 8.26 by their chemistry teacher. The amount of ammonium chloride to be dissolved by the student in 0.2 M ammonia solution to make one litre of the buffer is (Given <math>pK_b(\text{NH}_3) = 4.74</math>; Molar mass of <math>\text{NH}_3 = 17 \text{ g mol}^{-1}</math>; Molar mass of <math>\text{NH}_4\text{Cl} = 53.5 \text{ g mol}^{-1}</math>)<br/>(A) 53.5 g (B) 72.3 g<br/>(C) 107.0 g (D) 126.0 g<br/><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> <p><b>Sol.</b> <math>\text{POH} = 14 - 8.26</math><br/>= <math>pK_b + \log \frac{[\text{NH}_4^+]}{[\text{NH}_3]}</math><br/>= <math>5.74 = 4.74 + \log \frac{[\text{NH}_4^+]}{0.2} \Rightarrow [\text{NH}_4^+] = 2</math><br/>Hence<br/><math>\text{NH}_4\text{Cl} = 2 \times 53.5 = 107 \text{ g}</math></p> | <p><b>64.</b> At 30°C, the half life for the decomposition of <math>\text{AB}_2</math> is 200 s and is independent of the initial concentration of <math>\text{AB}_2</math>. The time required for 80% of the <math>\text{AB}_2</math> to decompose is (Given: <math>\log 2 = 0.30</math>; <math>\log 3 = 0.48</math>)<br/>(A) 200 s (B) 323 s<br/>(C) 467 s (D) 532 s<br/><b>Official Ans. by NTA (C)</b><br/><b>QuestPix Ans. (C)</b></p> <p><b>Sol.</b> <math>T_{1/2} = 200 \text{ s}</math> and 1<sup>st</sup> order reaction<br/><math display="block">K = \frac{2.303 \log 2}{200} = \frac{2.303}{t} \log \frac{A_0}{0.2A_0}</math><br/><math display="block">\frac{\log 2}{200} = \frac{1}{t} \log 5</math><br/><math display="block">t = \frac{7}{3} \times 200 = 466.67 \text{ s} = 467 \text{ s}</math></p> <p><b>65.</b> Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.<br/><b>Assertion A :</b> Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.<br/><b>Assertion R :</b> The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.<br/>In the light of the above statements, choose the most appropriate answer from the options given below;<br/>(A) Both A and R are true and R is the correct explanation of A<br/>(B) Both A and R are true but R is NOT the correct explanation of A<br/>(C) A is true but R is false<br/>(D) A is false but R is true<br/><b>Official Ans. by NTA (A)</b><br/><b>QuestPix Ans. (3)</b></p> |

66. The metal that has very low melting point and its periodic position is closer to a metalloid is :

(A) Al (B) Ga  
(C) Se (D) In

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol. Melting point**

Al → 933 K

Ga → 303 K

In → 430 K

Se → 490 K

67. The metal that is not extracted from its sulphide ore is :

(A) Aluminium (B) Iron  
(C) Lead (D) Zinc

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.** Al is extracted from  $\text{Al}_2\text{O}_3 \cdot 2\text{H}_2\text{O}$  i.e., Bauxite ore

68. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are

(A)  $\text{Mn}^{4+}$ ,  $\text{H}_2\text{O}$  only (B)  $\text{Mn}^{2+}$ ,  $\text{H}_2\text{O}$  only  
(C)  $\text{Mn}^{4+}$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_2$  only (D)  $\text{Mn}^{2+}$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_2$  only

**Official Ans. by NTA (D)**

**QuestPix Ans. (D)**

**Sol.**  $6\text{H}^+ + 2\text{MnO}_4^- + 5\text{H}_2\text{O}_2 \longrightarrow 2\text{Mn}^{2+} + 8\text{H}_2\text{O} + 5\text{O}_2$

69. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** LiF is sparingly soluble in water.

**Reason R :** The ionic radius of  $\text{Li}^+$  ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below .

(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true but R is NOT the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

**Sol.** Due to high lattice energy LiF is sparingly soluble in water.  $\text{Li}^+$  has high hydration energy among its group members due to smallest size.

70. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** Boric acid is a weak acid

**Reason R :** Boric acid is not able to release  $\text{H}^+$  ion on its own. It receives  $\text{OH}^-$  ion from water and releases  $\text{H}^+$  ion.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A

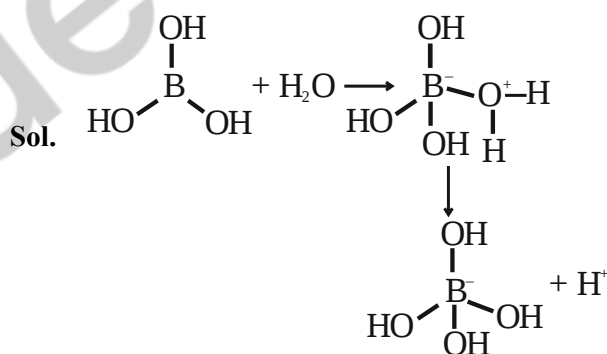
(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**



71. The metal complex that is diamagnetic is (Atomic number : Fe, 26; Cu, 29)

(A)  $\text{K}_3[\text{Cu}(\text{CN})_4]$

(B)  $\text{K}_2[\text{Cu}(\text{CN})_4]$

(C)  $\text{K}_3[\text{Fe}(\text{CN})_4]$

(D)  $\text{K}_4[\text{FeCl}_6]$

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**  $\text{K}_3[\text{Cu}(\text{CN})_4]$

O.N. of copper is  $\text{Cu}^{+1}$

$\text{Cu}^{+1} = [\text{Ar}]3\text{d}^{10} \Rightarrow \text{Diamagnetic}$

72. Match List I with List II

| List I<br>Pollutant   | List II<br>Source        |
|-----------------------|--------------------------|
| A. Microorganisms     | I. Strip mining          |
| B. Plant nutrients    | II. Domestic sewage      |
| C. Toxic heavy metals | III. Chemical fertilizer |
| D. Sediment           | IV. Chemical factory     |

Choose the correct answer from the options given below :

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**

**Sol.**

| List I<br>Pollutant   | List II<br>Source   |
|-----------------------|---------------------|
| A. Microorganisms     | Domestic sewage     |
| B. Plant nutrients    | Chemical fertilizer |
| C. Toxic heavy metals | Chemical factory    |
| D. Sediment           | Strip mining        |

73. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is :

(A)  $-\text{COOH} > -\text{CONH}_2 > -\text{COCl} > -\text{CHO}$

(B)  $-\text{SO}_3\text{H} > -\text{COCl} > -\text{CONH}_2 > -\text{CN}$

(C)  $-\text{COOR} > -\text{COCl} > -\text{NH}_2 > >\text{C}=\text{o}$

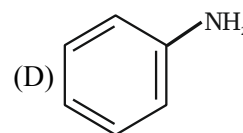
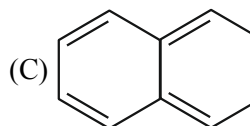
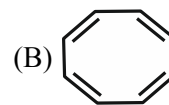
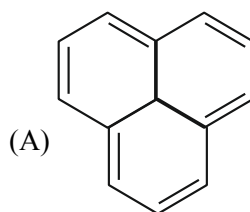
(D)  $-\text{COOH} > -\text{COOR} > -\text{CONH}_2 > -\text{COCl}$

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

**Sol.**  $-\text{SO}_3\text{H} > -\text{COCl} > -\text{CONH}_2 > -\text{CN}$

74. Which of the following is not an example of benzenoid compound ?



**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

75. Hydrolysis of which compound will give carbolic acid ?

(A) Cumene

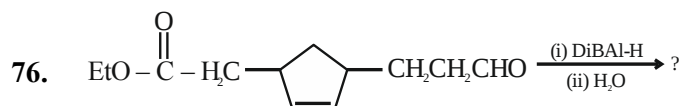
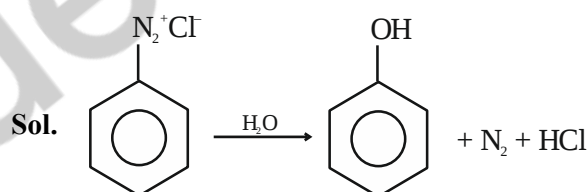
(B) Benzenediazonium chloride

(C) Benzal chloride

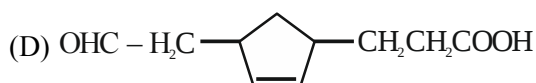
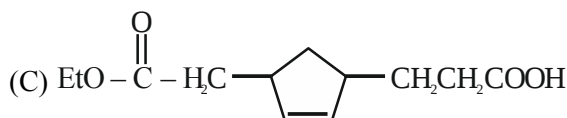
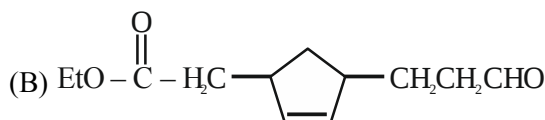
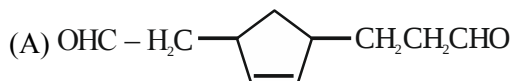
(D) Ethylene glycol ketal

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**

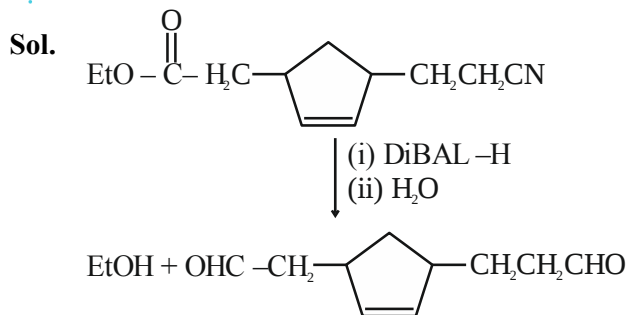


Consider the above reaction and predict the major product.

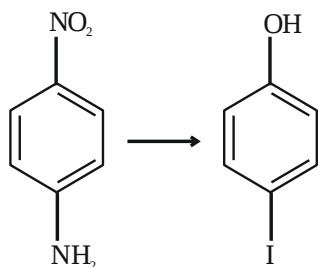


**Official Ans. by NTA (A)**

**QuestPix Ans. (A)**



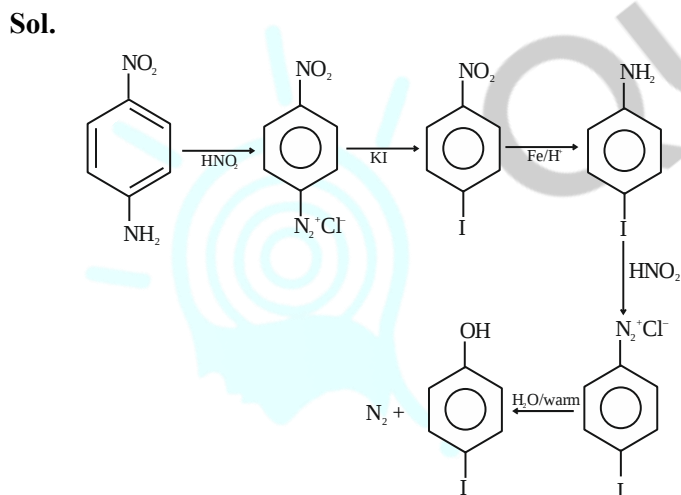
77. The correct sequential order of the reagents for the given reaction is :



- (A) HNO<sub>2</sub>, Fe/H<sup>+</sup>, HNO<sub>2</sub>, KI, H<sub>2</sub>O/H<sup>+</sup>  
 (B) HNO<sub>2</sub>, KI, Fe/H<sup>+</sup>, HNO<sub>2</sub>, H<sub>2</sub>O/warm  
 (C) HNO<sub>2</sub>, KI, HNO<sub>2</sub>, Fe/H<sup>+</sup>, H<sub>2</sub>O/H<sup>+</sup>  
 (D) HNO<sub>2</sub>, Fe/H<sup>+</sup>, KI, HNO<sub>2</sub>, H<sub>2</sub>O/warm

**Official Ans. by NTA (B)**

**QuestPix Ans. (B)**



78. Vulcanization of rubber is carried out by heating a mixture of :

- (A) isoprene and styrene  
 (B) neoprene and sulphur  
 (C) isoprene and sulphur  
 (D) neoprene and styrene

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

- Sol.** Vulcanization of rubber is carried out by heating a mixture of isoprene & sulphur

79. Animal starch is the other name of :  
 (A) amylose (B) maltose  
 (C) glycogen (D) amylopectin

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

- Sol.** Glycogen  
 80. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium

**Reason R :** Phenolphthalein is a weak acid. It doesn't dissociate in basic medium.

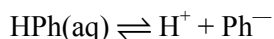
In the light of the above statements, choose the most appropriate answer from the options given below :

- (A) Both A and R are true and R is the correct explanation of A  
 (B) Both A and R are true but R is NOT the correct explanation of A.  
 (C) A is true but R is false  
 (D) A is false but R is true

**Official Ans. by NTA (C)**

**QuestPix Ans. (C)**

- Sol.** Phenolphthalein dissociate in basic medium



(colourless) (Pink)

### SECTION-B

81. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity 0.0125 m<sup>3</sup> at 6 bar and 27°C. The mass of helium in the mixture is \_\_\_\_\_ g. (nearest integer)

Given : R = 8.3 JK<sup>-1</sup>mol<sup>-1</sup> (Atomic masses of H and He are 1u and 4u, respectively)

**Official Ans. by NTA (8)**

**QuestPix Ans. (8)**

- Sol.** PV = n<sub>mix</sub>RT

$$n_{\text{mix}} = \frac{6 \times 12.5}{0.083 \times 300} \approx 3$$

Let mole of He = x

Mole of H<sub>2</sub> = 3 - x

$$4x + 2(3 - x) = 10$$

$$\boxed{x = 2\text{mol}}$$

Mass of He = 8g

82. Consider an imaginary ion  ${}^{48}_{22}\text{X}^{3-}$ . The nucleus contains 'a' % more neutrons than the number of electrons in the ion. The value of 'a' is \_\_\_\_\_. [nearest integer]

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.**  ${}^{48}_{22}\text{X}^{3-}$

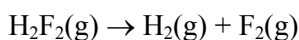
No. of neutrons = 26

No. of electrons = 25

% of extra neutrons

$$\text{than electrons} = \frac{26 - 25}{25} \times 100 = 4$$

83. For the reaction



$$\Delta U = -59.6 \text{ kJ mol}^{-1} \text{ at } 27^\circ\text{C}.$$

The enthalpy change for the above reaction is (–) \_\_\_\_  $\text{kJ mol}^{-1}$  [nearest integer] Given :  $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ .

**Official Ans. by NTA (57)**

**QuestPix Ans. (57)**

**Sol.**  $\Delta H = \Delta U + \Delta n_g RT$

$$\Delta H = -59.6 + 1 \times 8.314 \times 300 \times 10^{-3} = -57.10$$

84. The elevation in boiling point for 1 molal solution of non-volatile solute A is 3K. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of  $K_b$  and  $K_f$  i.e.,  $K_b/K_f$  is 1 : X. The value of X is [nearest integer]

**Official Ans. by NTA (1)**

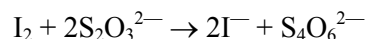
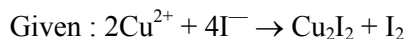
**QuestPix Ans. (1)**

**Sol.**  $\Delta T_b = iK_b m_1$   $\Delta T_f = iK_f m_2$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b \times 1}{K_f \times 2} \Rightarrow \frac{3}{6} = \frac{1}{2} = \frac{K_b}{K_f} \times \frac{1}{2}$$

$$\frac{K_b}{K_f} = \frac{1}{1} \Rightarrow x = 1$$

85. 20 mL of 0.02 M hypo solution is used for the titration of 10 mL of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of  $\text{Cu}^{2+}$  is found to be \_\_\_\_  $\times 10^{-2} \text{ M}$  [nearest integer]



**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.**  $n_{\text{eq. of I}_2} = n_{\text{eq. of Na}_2\text{S}_2\text{O}_3} = 20 \times 0.002 \times 1$

$$2 \times n_{\text{mol of I}_2} = 0.4$$

$$n_{\text{mol of I}_2} = 0.2 \text{ m mol}$$

$$n_{\text{mol of Cu}^{+2}} = 0.2 \times 2 \times 10^{-3}$$

$$[\text{Cu}^{+2}] = \frac{0.4 \times 10^{-3}}{10 \times 10^{-3}} = 0.04 = 4 \times 10^{-2}$$

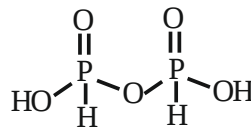
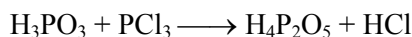
86. The number of non-ionisable protons present in the product B obtained from the following reaction is \_\_\_\_\_.  $\text{C}_2\text{H}_5\text{OH} + \text{PCl}_3 \rightarrow \text{C}_2\text{H}_5\text{Cl} + \text{A}$



**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.**  $\text{C}_2\text{H}_5\text{OH} + \text{PCl}_3 \longrightarrow \text{C}_2\text{H}_5\text{Cl} + \text{H}_3\text{PO}_3$



87. The spin-only magnetic moment value of the compound with strongest oxidizing ability among  $\text{MnF}_4$ ,  $\text{MnF}_3$  and  $\text{MnF}_2$  is \_\_\_\_ B.M. [nearest integer]

**Official Ans. by NTA (5)**

**QuestPix Ans. (5)**

**Sol.**  $\begin{array}{ccc} \text{MnF}_4 & \text{MnF}_3 & \text{MnF}_2 \\ \downarrow +4 & \downarrow +3 & \downarrow +2 \\ \text{E.C} = [\text{Ar}]3\text{d}^3 & [\text{Ar}]3\text{d}^4 & [\text{Ar}]3\text{d}^5 \end{array}$

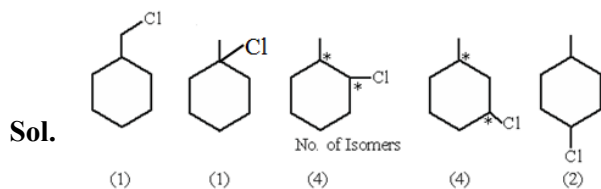
Hence  $\text{MnF}_3 \Rightarrow$  strongest O.A

$$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 = 5$$

88. Total number of isomers (including stereoisomers) obtain on monochlorination of methylcyclohexane is \_\_\_\_\_.

**Official Ans. by NTA (12)**

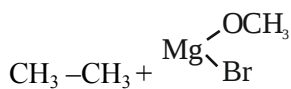
**QuestPix Ans. (12)**



89. A 100 mL solution of  $\text{CH}_3\text{CH}_2\text{MgBr}$  on treatment with methanol produces 2.24 mL of a gas at STP. The weight of gas produced is \_\_\_\_\_ mg. [nearest integer]

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**



$$n = \frac{2.24 \times 10^{-3}}{22.4} = 10^{-4}$$

$$W = n \times M$$

$$= 10^{-4} \times 30 = 3 \text{ mg}$$

90. How many of the following drugs is/are example(s) of broad spectrum antibiotic ?  
Ofloxacin, Penicillin G, Terpineol, Salvarsan

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

