(HELD ON THURSDAY 02nd APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let α, β be the roots of the equation $x^2-3x+r=0$, and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2+3x+r=0$. If the roots of the equation $x^2+6x=m$ are $2\alpha+\beta+2r$ and $\alpha-2\beta-\frac{r}{2}$, then m is equal to :-

(1) –135 (2) –567

(3) 135 (4) 567

Ans. (4)

Sol. $\alpha+\beta=3$

$$\frac{\alpha}{2}+2\beta=-3$$

On solving, we get $\alpha=6; \beta=-3$

Product of roots $=\alpha\beta=r \Rightarrow r=-18$

Now for $x^2+6x-m=0$

Product of roots $=-m$

$$=(2\alpha+\beta+2r)(\alpha-2\beta-\frac{r}{2})$$

$\Rightarrow -m=(-27)(21)$

$\Rightarrow m=567$

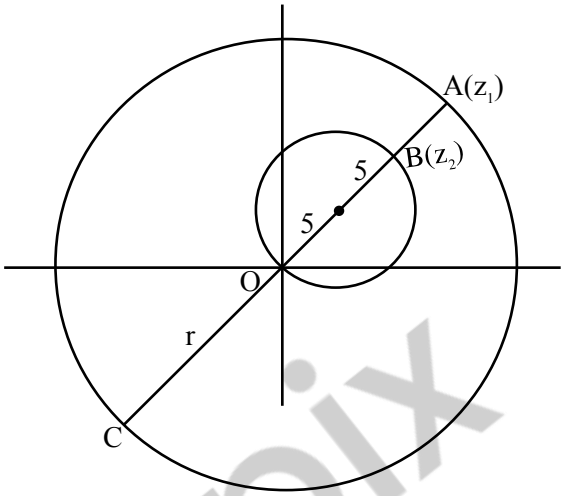
2. Let the circle $C_1: |z|=r$ and $C_2: |z-3-4i|=5$, $z\in\mathbb{C}$, be such that C_2 lies within C_1. If z_1 moves on C_1, z_2 moves on C_2 and $\min|z_1-z_2|=2$, then $\max|z_1-z_2|$ is equal to :-

(1) 12 (2) 17

(3) 22 (4) 24

Ans. (3)

Sol.



$|z|=r$

$OA=r$

minimum $|z_1-z_2|=2$

$\Rightarrow r-10=2 \Rightarrow r=12$

maximum value of CB

$|z_1-z_2|=r+10=12+10=22$

3. If the system of equations

$x+5y+6z=4$,

$2x+3y+4z=7$,

$x+6y+az=b$

has infinitely many solutions, then the point (a, b) lies on the line

(1) $y-x=3$ (2) $x-y=3$

(3) $x+y=11$ (4) $x+y=12$

Ans. (2)

Sol. $D=\begin{vmatrix}1&5&6\\2&3&4\\1&6&a\end{vmatrix}=0$

$$\Rightarrow a=\frac{50}{7}$$

$$D_z=0 \Rightarrow \begin{vmatrix}1&5&4\\2&3&7\\1&6&b\end{vmatrix}=0$$

$$\Rightarrow b=\frac{29}{7}$$

$\therefore$ (a, b) lies on $x-y=3$

4. Let $a_1, a_2, a_3, \ldots\ldots$ be an A.P. and $g_1 = a_1, g_2, g_3, \ldots\ldots$ be an increasing G.P. If $a_1 = a_2 + g_2 = 1$ and $a_3 + g_3 = 4$, then $a_{10} + g_5$ is equal to :

(1) 81 (2) 76

(3) 62 (4) 55

Ans. (4)

Sol. A.P. : $1, a_2, a_3, \ldots$

G.P. : $1, g_2, g_3, \ldots$

$a_2 + g_2 = 1 \Rightarrow 1 + d + r = 1 \Rightarrow d + r = 0$

$1 + 2d + r^2 = 4 \Rightarrow r^2 - 2r - 3 = 0$

$\Rightarrow r = 3, -1$ (reject)

$\therefore d = -3$

$\therefore a_{10} = 1 + 9d = 1 + 9(-3) = -26$

$g_5 = 1.r^4 = 3^4 = 81$

$\therefore a_{10} + g_5 = 55$

5. The sum $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ up to 8 terms, is :

(1) 70 (2) 71

(3) 72 (4) 73

Ans. (2)

Sol. $T_r = \frac{1^3+2^3+3^3+\ldots\ldots r^3}{1+3+5+\ldots\ldots+(2r-1)} = \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}$

$= \frac{r^2+2r+1}{4}$

$S_n = \sum_{r=1}^{n} T_r$

$S_n = \frac{1}{4}\sum\left(r^2+2r+1\right)$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2} + n\right]$

$S_8 = \frac{1}{4}\left[\frac{8\times9\times17}{6} + 8\times9 + 8\right]$

$= \frac{1}{4}[204 + 72 + 8] = 71$

6. If for $3 \le r \le 30$,

$({}^{30}C_{30-r}) + 3({}^{30}C_{31-r}) + 3({}^{30}C_{32-r}) + ({}^{30}C_{33-r}) = {}^{m}C_r$,

then m equals :

(1) 31 (2) 32

(3) 33 (4) 34

Ans. (3)

Sol. ${}^3C_3 \cdot {}^{30}C_{30-r} + {}^3C_2 \cdot {}^{30}C_{31-r} + {}^3C_1 \cdot {}^{30}C_{32-r} + {}^3C_0 \cdot {}^{30}C_{33-r}$

$= {}^{33}C_{33-r} \equiv {}^{m}C_r \equiv {}^{m}C_{m-r}$

$\Rightarrow m = 33$

7. Let p_n denote the total number of triangles formed by joining the vertices of an n-side regular polygon. If $p_{n+1} - p_n = 66$, then the sum of all distinct prime divisors of n is :

(1) 7 (2) 8

(3) 5 (4) 6

Ans. (3)

Sol. $P_n = {}^nC_3$

$P_{n+1} - P_n = 66$

${}^{n+1}C_3 - {}^nC_3 = 66$

$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 66$

$\Rightarrow \frac{n(n-1)}{6}[n+1-n+2] = 66$

$\Rightarrow n(n-1) = 132$

$n = 12$

Prime divisors of 12 are 2 & 3

sum = 2 + 3 = 5

8. A man throws a fair coin repeatedly. He gets 10 points for each head he throws and 5 points for each tail he throws. If the probability that he gets exactly 30 points is $\frac{m}{n}$, gcd (m, n) = 1, then m + n is equal to :

(1) 53 (2) 55

(3) 107 (4) 105

Ans. (3)

Sol. P = (6tail)+(4tail+1head) + (2tail+2head) + (3head)

$P = \left(\frac{1}{2}\right)^6 + \frac{5!}{1!4!}\left(\frac{1}{2}\right)^5 + \frac{4!}{2!2!}\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^3$

$P = \frac{1}{64} + \frac{5}{32} + \frac{3}{8} + \frac{1}{8}$

$P = \frac{1+10+24+8}{64} = \frac{43}{64} = \frac{m}{n}$

m + n = 107

9. The mean and variance of n observations are 8 and 16, respectively. If the sum of the first (n – 1) observations is 48 and the sum of squares of the first (n – 1) observations is 496, then the value of n is :

(1) 21 (2) 16

(3) 13 (4) 7

Ans. (4)

Sol. $x_1 + x_2 + \ldots + x_{n-1} + x_n = 8n$

$48 + x_n = 8n$

$\Rightarrow x_n = 8n - 48 \quad \ldots(1)$

$\Rightarrow 16 = \frac{496 + x_n^2}{n} - (8)^2$

$\Rightarrow 80n = 496 + x_n^2$

$\Rightarrow x_n^2 = 80n - 496 \quad \ldots(2)$

$\Rightarrow (8n - 48)^2 = 80n - 496$

$\Rightarrow 64(n - 6)^2 = 8(10n - 62)$

$\Rightarrow 8(n - 6)^2 = 10n - 62$

$\Rightarrow 4(n - 6)^2 = 5n - 31$

$\Rightarrow 4(n^2 - 36 - 12n) = 5n - 31$

$\Rightarrow 4n^2 + 144 - 48n = 5n - 31$

$\Rightarrow 4n^2 - 53n + 175 = 0$

$\Rightarrow 4n^2 - 28n - 25n + 175 = 0$

$\Rightarrow 4n(n - 7) - 25(n - 7) = 0$

$n = 7$

10. Let a circle pass through the origin and its centre be the point of intersection of two mutually perpendicular lines $x + (k - 1)y + 3 = 0$ and $2x + k^2y - 4 = 0$. If the line $x - y + 2 = 0$ intersects the circle at the points A and B, then $(AB)^2$ is equal to :

(1) 10 (2) 27

(3) 18 (4) 34

Ans. (3)

Sol. $x + (k - 1)y + 3 = 0$

$2x + k^2y - 4 = 0$

$\left(\frac{1}{1-k}\right)\left(\frac{2}{k^2}\right) = 1$

$2 = k^2 - k^3$

$k^3 - k^2 + 2 = 0$

$k = -1$

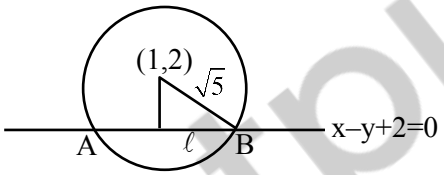
Solving :

$2(x - 2y + 3) = 0$

$2x + y - 4 = 0$

$-5y + 10 = 0$

$y = 2$



$x = 1$

Centre (1, 2)

$r = \sqrt{5}$

So circle is $(x - 1)^2 + (y - 2)^2 = 5$

Chord $x - y + 2 = 0$

$p = \frac{1}{\sqrt{2}}$

$\ell = \sqrt{5 - \frac{1}{2}} = \frac{3}{\sqrt{2}}$

$AB = \frac{6}{\sqrt{2}} \quad \therefore AB^2 = 18$

11. Let O be the origin, and P and Q be two points on the rectangular hyperbola $xy = 12$ such that the mid point of the line segment PQ is $\left(\frac{1}{2}, -\frac{1}{2}\right)$. Then the area of the triangle OPQ equals :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Ans. (3)

Sol. Equation of chord whose midpoint is $\left(\frac{1}{2}, \frac{-1}{2}\right)$

$$\frac{x}{x_1} + \frac{y}{y_1} = 2$$

$2x - 2y = 2$

$x - y = 1$

solving with $xy = 12$

$P(-3, -4)$, $Q(4, 3)$

$$\text{Area } (\Delta OPQ) = \frac{1}{2}|-3 \times 3 - (-4)(4)| = \frac{7}{2}$$

12. Let the parabola $y = x^2 + px + q$ passing through the point $(1, -1)$ be such that the distance between its vertex and the x-axis is minimum. Then the value of $p^2 + q^2$ is :

(1) 2 (2) 4

(3) 5 (4) 8

Ans. (2)

Sol. parabola passes through $(1, -1)$ so

$-1 = 1 + p + q$

$p + q = -2$

$q = -2 - p$ …(1)

Distance from x-axis

$$\frac{-D}{4a} = \frac{-(p^2 - 4q)}{4(1)} = \frac{4q - p^2}{4}$$

$$= \frac{4(-2-p) - p^2}{4} \quad \text{from (1)}$$

$$= \frac{-8 - 4p - p^2}{4} = \frac{-(p^2 + 4p + 8)}{4} = \frac{-((p+2)^2 + 4)}{4}$$

Minimum at $p = -2 \Rightarrow q = 0$

$p^2 + q^2 = 4 + 0 = 4$

13. Let $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1\}$ and $S = \{a \in Z : 2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2, \theta \in P\}$. Then n(S) is :

(1) 0 (2) 1

(3) 2 (4) 4

Ans. (1)

Sol. $2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2$

$2(\cos^4\theta + \sin^4\theta)(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)\sec 2\theta = a^2$

$2(\cos^4\theta + \sin^4\theta) = a^2$

$2(1 - 2\sin^2\theta\cos^2\theta) = a^2$

$$2\left(1 - \frac{\sin^2 2\theta}{2}\right) = a^2, \; a \in Z$$

$a^2 = 2 - \sin^2 2\theta \in [1, 2]$

$a^2 = 1$ at $\sin^2 2\theta = 1$

$$2\theta = (2n + 1)\frac{\pi}{2}$$

$$\theta = (2n + 1)\frac{\pi}{4} \notin P$$

$n(S) = 0$

14. Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some $\lambda, \mu \in \mathbf{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$. If $\vec{c}.(3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c}.(\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is equal to :

(1) 8 (2) 12

(3) 14 (4) 15

Ans. (2)

Sol. $\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\vec{c} = (\mu - \lambda)\hat{i} + (\lambda + 3\mu)\hat{j} + (3\lambda + \mu)\hat{k}$

$\vec{c}.(3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$

$3(\mu - \lambda) - 6(\lambda + 3\mu) + 2(3\lambda + \mu) = 10$

$-3\lambda - 13\mu = 10$ …(1)

$\vec{c}.(\hat{i} + \hat{j} + \hat{k}) = -2$

$5\mu + 3\lambda = -2$ …(2)

Solving (1) & (2)

$\lambda = 1, \mu = -1$

$\vec{c} = -2\hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{c}|^2 = 12$

15. Let the point A be the foot of perpendicular drawn from the point P(a, b, 0) on the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-\alpha}{3}$. If the midpoint of the line segment PA is $\left(0,\frac{3}{4},\frac{-1}{4}\right)$, then the value of $a^2 + b^2 + \alpha^2$ is equal to :

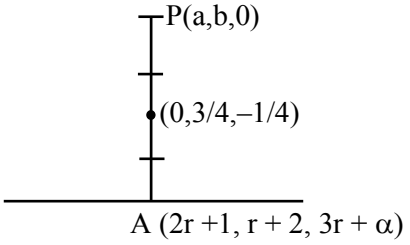
(1) 1 (2) 2

(3) 6 (4) 9

Ans. (1)

Sol. $\frac{2r+1+a}{2}=0 \Rightarrow 2r+a=-1$

$\frac{r+2+b}{2}=\frac{3}{4} \Rightarrow r+b=-\frac{1}{2}$



$\frac{3r+\alpha}{2}=\frac{-1}{4} \Rightarrow 3r+\alpha=\frac{-1}{2}$

$2\cdot a+1\cdot(b-3/4)+3\cdot\frac{1}{4}=0 \Rightarrow a+b=0$

$\Rightarrow 2(-1-2r)+(-r-1/2)=0$

$\Rightarrow -3/2=3r=1 \Rightarrow r=-1/2,\ a=0,\ b=0,\ \alpha=1$

$\therefore a^2+b^2+\alpha^2=1$

16. Two adjacent sides of a parallelogram PQRS are given by $\overrightarrow{PQ}=\hat{j}+\hat{k}$ and $\overrightarrow{PS}=\hat{i}-\hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{4}$ (4) $\frac{2\sqrt{3}}{5}$

Ans. (2)

Sol. Let angle between $\overrightarrow{PQ}$ & $\overrightarrow{PS}$ is θ

$\cos\theta=\frac{\overrightarrow{PQ}.\overrightarrow{PS}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PS}\right|}$

$\cos\theta=\frac{0-1+0}{\sqrt{2}\sqrt{2}}$

$\cos\theta=\frac{-1}{2}$

$\theta=\frac{2\pi}{3}$

$\alpha=\frac{2\pi}{3}-\frac{\pi}{2}$ So $\alpha=\frac{\pi}{6}$

$\sin^2\frac{5\alpha}{2}-\sin^2\frac{\alpha}{2}=\sin\left(\frac{5\alpha}{2}+\frac{\alpha}{2}\right)\sin\left(\frac{5\alpha}{2}-\frac{\alpha}{2}\right)$

$=\sin\frac{6\alpha}{2}\times\sin\frac{4\alpha}{2}$

$=\sin3\alpha\times\sin2\alpha$

$=\sin\frac{\pi}{2}\times\sin\frac{\pi}{3}$

$=1\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$

17. The value of $\int_0^{20\pi}(\sin^4 x+\cos^4 x)dx$ is equal to:

(1) $\frac{15\pi}{2}$ (2) 25π

(3) 15π (4) $\frac{25\pi}{2}$

Ans. (3)

Sol. $I=\int_0^{20\pi}(1-2\sin^2 x\cos^2 x)\cdot dx$

$=20\pi-\frac{1}{2}\int_0^{20\pi}\sin^2 2x\ dx$

$20\pi-\frac{1}{2}(40)\int_0^{\frac{\pi}{2}}\sin^2 2x$

$20\pi-\frac{20}{2}\left(x-\frac{\sin 2x}{2}\right)_0^{\frac{\pi}{2}}$

$20\pi-10\left(\frac{\pi}{2}\right)=15\pi$

18. Let f(x) be a polynomial of degree 5, and have extrema at x = 1 and x = –1. If $\lim_{x\to0}\left(\frac{f(x)}{x^3}\right)=-5$, then f(2) – f(–2) is equal to :

(1) 0 (2) 50

(3) 92 (4) 112

Ans. (4)

Sol. Given f(x) is a polynomial of degree 5

Also, f′(1) = 0; f′(–1) = 0.

Also $\because \lim_{x\to0}\frac{f(x)}{x^3}=-5$ (fixed and finite)

$\therefore$ f(0) = 0; f′(0) = 0; f″(0) = 0

$\frac{f'''(0)}{6}=-5 \Rightarrow f''(0)=-30$

Hence, let f′(x) = (ax + b)(x – 0)(x – 1)(x + 1)

$\Rightarrow f'(x)=ax^4+bx^3-ax^2-bx$

$f''(x)=4ax^3+3bx^2-2ax-b$

$\because f''(0)=0 \Rightarrow b=0$

$f''(x)=12ax^2+6bx-2a$

$\because f''(0)=-2a=-30$

$\Rightarrow a=15$

$\therefore f'(x)=15x.x.(x-1)(x+1)$

$\Rightarrow f'(x)=15x^4-15x^2$

$\Rightarrow f(x)=3x^5-5x^3+C$

$\because f(0)=0 \Rightarrow C=0$

$\therefore f(x)=3x^5-5x^3$

$\therefore f(2)-f(-2)=112$

19. Let $f(x)=\int\left(\frac{16x+24}{x^2+2x-15}\right)dx$. If f(4) = $14\log_e(3)$ and f(7) = $\log_e(2^\alpha.3^\beta)$, α, β ∈ **N**, then α + β is equal to:

(1) 31 (2) 37

(3) 39 (4) 41

Ans. (3)

Sol. $f(x)=\int\frac{8(2x+2)+8}{x^2+2x-15}dx$

$\Rightarrow f(x)=8\ell n|x^2+2x-15|+\ell n\left|\frac{x-3}{x+5}\right|+C$

$\Rightarrow f(4)=14\ell n3+C$

$\Rightarrow 14\ell n3=14\ell n3+C$

$\therefore C=0$

Now, $f(7)=8\ell n48-\ell n3$

$=\ell n\left(\frac{(48)^8}{3}\right)$

$\Rightarrow f(7)=\ell n(2^{32}\times3^7)$

α = 32, β = 7 $\Rightarrow$ α + β = 39

20. Let x = x(y) be the solution of the differential equation $2y^2\frac{dx}{dy}-2xy+x^2=0$, y > 1 , x(e) = e. Then $x(e^2)$ is equal to:

(1) $\frac{3}{2}e^2$ (2) $\frac{2}{3}e^2$

(3) e^2 (4) $2e^2$

Ans. (2)

Sol. $2y(ydx-xdy)+x^2dy=0$

$\Rightarrow -2yx^2\ d\left(\frac{y}{x}\right)+x^2dy=0$

$\Rightarrow -2y\ d\left(\frac{y}{x}\right)+dy=0$

$\Rightarrow -2d\left(\frac{y}{x}\right)+\frac{1}{y}dy=0$

$\Rightarrow \frac{-2y}{x}+\log_e y=C$

Given x(e) = e

$\Rightarrow C=-1$

$\therefore \frac{-2y}{x}+\log_e y=-1$

$\Rightarrow \frac{2y}{x}-\log_e y=1$

Put $y=e^2$, we get

$\Rightarrow \frac{2e^2}{x}-2=1 \Rightarrow \frac{2e^2}{x}=3 \Rightarrow x=\frac{2e^2}{3}$

$\Rightarrow x(e^2)=\frac{2e^2}{3}$

SECTION-B

21. Let A = {2, 3, 4, 5, 6}. Let R be a relation on the set A × A given by (x, y)R(z, w) if and only if x divides z and y ≤ w. Then the number of elements in R is _____.

Ans. **(120)**

Sol. For x = 2, z = 2, 4, 6

x = 3 z = 3, 6

x = 4 z = 4

x = 5 z = 5

x = 6 z = 6

Total case = 8

and no. of combination of y and w satisfying y ≤ w

= 1 +2 + 3 + 4 + 5 = 15

No. of relations satisfying (x,y) R(z,w) is 8 × 15 = 120.

22. Consider the matrices $A = \begin{bmatrix} 2 & -2 \\ 4 & -2 \end{bmatrix}$ and $B\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$. If matrices P and Q are such that PA = B and AQ = B, then the absolute value of the sum of the diagonal elements of 2(P + Q) is _____.

Ans. **(34)**

Sol. $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -4 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$

$\because PA = B$

$\Rightarrow P = BA^{-1}$

$= \frac{1}{2}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -21 & 12 \\ -7 & 4 \end{bmatrix}$

$\because AQ = B$

$\Rightarrow Q = A^{-1}B$

$= \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 & -6 \\ -5 & -15 \end{bmatrix}$

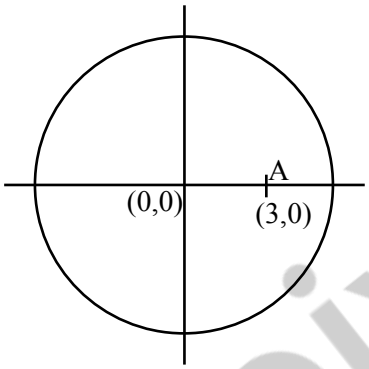
$\therefore |\text{tr}(2(P + Q))| = |(-21) + (4) + (-2) + (-15)|$

$= |-34| = 34$

23. Let A be the point (3, 0) and circles with variable diameter AB touch the circle $x^2 + y^2 = 36$ internally. Let the curve C be the locus of the point B. If the eccentricity of C is e, then $72e^2$ is equal to ____.

Ans. **(18)**

Sol.



Let B(h, k)

Equation of circle with AB as diameter

$(x - h)(x - 3) + (y - k)(y - 0) = 0$

$x^2 + y^2 - (h + 3)x - ky + 3h = 0$

Centre $\left(\frac{h+3}{2}, \frac{k}{2}\right)$

Circle touches internally

$\therefore C_1C_2 = |R - r|$

$$\sqrt{\left(\frac{h+3}{2}\right)^2 + \left(\frac{k}{2}\right)^2} = \left|6 - \frac{1}{2}\left(\sqrt{(h-3)^2 + k^2}\right)\right|$$

$$\sqrt{(x+3)^2 + y^2} + \sqrt{(x-3)^2 + y^2} = 12$$

$2a = 12 \qquad \therefore a = 6$

(–3, 0) and (3, 0) are foci

$2ae = 6$

$12e = 6$

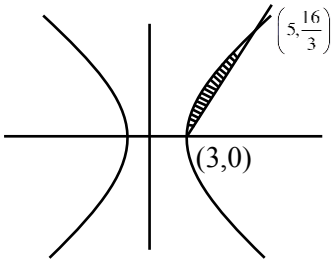
$e = \frac{1}{2}$

$\therefore 72e^2 = 72\left(\frac{1}{4}\right) = 18$

24. If the area of the region bounded by $16x^2 - 9y^2 = 144$ and $8x - 3y = 24$ is A, then $3(A + 6\log_e(3))$ is equal to_____.

Ans. (24)

Sol.



$16x^2 - 9y^2 = 144$

$16x^2 - (8x - 24)^2 = 144$

$16x^2 - 64(x - 3)^2 = 144 \Rightarrow x^2 - 4(x - 3)^2 = 9$

$3x^2 - 24x + 45 \Rightarrow x^2 - 8x + 15 = 0$

$x = 3,5$

$$\text{Area} = \int_3^5 \sqrt{\frac{16x^2 - 144}{9}} - \frac{1}{2}\cdot 2\cdot\frac{16}{3}$$

$$= \frac{4}{3}\int_3^5 \sqrt{x^2 - 9} - \frac{16}{3}$$

$$= \frac{4}{3}\left(\frac{x}{2}\sqrt{x^2 - 9} - \frac{9}{2}\log_e\left(x + \sqrt{x^2 - 9}\right)\right)_3^5 - \frac{16}{3}$$

$$= \frac{4}{3}\left(\frac{5}{2}\cdot 4 - \frac{9}{2}\log_e 9 - \frac{3}{2}\cdot 0 + \frac{9}{2}\log_e 3\right) - \frac{16}{3}$$

Area = $8 - 6\log_e 3 = A$

$\therefore A + 6\ell n3 = 8$

$\Rightarrow 3(A + 6\ell n3) = 24$

25. The number of points in the interval [2, 4], at which the function $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function, is discontinuous, is_____.

Ans. (10)

Sol. In $x \in [2, 4]$, range of $x^2 - x - \frac{1}{2}$ is [1.5, 11.5]

f(x) is discontinuous at points where $x^2 - x - \frac{1}{2}$ becomes integers.

$\therefore$ Integral values of $x^2 - x - \frac{1}{2}$ are 2, 3, 4, 5, 6, 7, 8, 9 10, 11

$\therefore$ Number of points where is discontinuous is 10

(HELD ON THURSDAY 02nd APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

26. Dimensions of universal gravitational constant (G) in terms of Planck's constant (h), distance (L), mass (M) and time (T) are____.

(1) $[hTLM^{-2}]$ (2) $[hT^{-1}LM^{-2}]$

(3) $[hTL^{2}M^{-2}]$ (4) $[h^{-1}T^{-1}LM^{-2}]$

Ans. **(2)**

Sol. $\text{Energy} = \frac{hc}{\lambda} = \frac{GM_1M_2}{r}$

$[ML^2T^{-2}] = [h]\,[T^{-1}] = [G]\,[M^2L^{-1}]$

$[G] = [M^{-2}L^{+1}T^{-1}h^{+1}]$

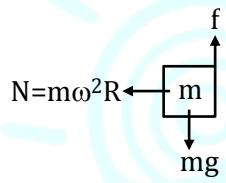
27. A 0.5 kg mass is in contact against the inner wall of a cylindrical drum of radius 4m rotating about its vertical axis. The minimum rotational speed of the drum to enable the mass to remain stuck to the wall (without falling) is 5 rad/s. The coefficient of friction between the drum's inner wall surface and mass is_____.(Take g = 10 m/s^2)

(1) 0.1 (2) 0.5

(3) 0.7 (4) 0.3

Ans. **(1)**

Sol.



$N = m\omega^2R$

$f_s = mg$

$f_s \le f_\ell$

$mg \le \mu N$

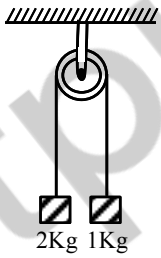
$mg \le \mu m\omega^2R$

$\mu \ge \frac{g}{\omega^2R}$

$\mu_{min} = \frac{g}{\omega^2R} = \frac{10}{10^2 \times 1} = 0.1$

28. Two blocks of masses 2 kg and 1 kg respectively, are tied to the ends of a string which passes over a light frictionless pulley as shown in the figure below. The masses are held at rest at the same horizontal level and then released. The distance traversed by the centre of mass in 2 s is____m.

(Take g = 10 m/s^2)



(1) 3.33

(2) 3.12

(3) 2.22

(4) 1.42

Ans. **(3)**

Sol. $a = \frac{(m_2 - m_1)}{m_1 + m_2}g = \frac{(2-1)}{3} \times 10 = \frac{10}{3}\,m/s^2$

$a_{cm} = \frac{(1)(10/3) - (2)(10/3)}{3} = \frac{-10}{9}\,m/s^2(\downarrow)$

$S_{cm} = u_{cm}t + \frac{1}{2}a_{cm}t^2$

$= 0 + \frac{1}{2}\left(\frac{10}{9}\right)(4) = \frac{20}{9}\,m\ (\text{downward})$

$S_{COM} = \frac{20}{9}\ (\text{Downward}) = 2.22$

29. A particle having charge 10^{-9} C moving in x–y plane in fields of $0.4\ \hat{j}\ N/C$ and 4×10^{-3} k T experiences a force of $\left(4\hat{i}+2\hat{j}\right) \times 10^{-10}$ N. The velocity of the particle at that instant is_____m/s.

(1) $50\hat{i}+100\hat{j}$ (2) $100\hat{i}+50\hat{j}$

(3) $-50\hat{i}+100\hat{j}$ (4) $50\hat{i}-100\hat{j}$

Ans. **(1)**

Sol. $\vec{F}=q\vec{E}+q\left(\vec{V}\times\vec{B}\right)$

$(4\hat{i}+2\hat{j})\times10^{-10}$

$= 10^{-9}[0.4\,\hat{j} + (v_x\,\hat{i} + v_y\,\hat{j}) \times (4 \times 10^{-3})\,\hat{k}\,]$

$(4\,\hat{i} + 2\,\hat{j}) \times 10^{-10}$

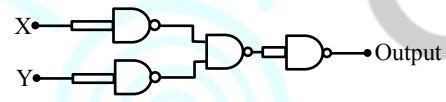
$= 10^{-9}\,(0.4\,\hat{j} - 4 \times 10^{-3}\,v_x\,\hat{j} + 4 \times 10^{-3}\,v_y\,\hat{i}\,)$

On comparing

$v_y = 100,\ v_x = 50$

$\vec{V}=50\hat{i}+100\hat{j}$

30. If X and Y are the inputs, the given circuit works as_____.



(1) OR gate

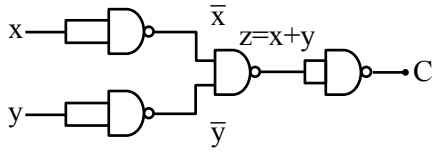
(2) AND gate

(3) NAND gate

(4) NOR gate

Ans. **(4)**

Sol.



$C = \overline{x+y}$

31. If a body of mass 1 kg falls on the earth from infinity, it attains velocity (v) and kinetic energy (k) on reaching the surface of earth. The values of v and k respectively are_____.

(Take radius of earth to be 6400 km and $g = 9.8 m/s^2$)

(1) 11.2 km/s; 6.27×10^7 J

(2) 11.2 km/s; 12.54×10^7J

(3) 8.8 km/s; 6.27×10^7 J

(4) 8.8 km/s; 12.54×10^7 J

Ans. **(1)**

Sol. M.E.C

$K_i + U_i = K_f + U_f$

$\Rightarrow O - \frac{GM_e m}{\infty} = \frac{1}{2}mv^2 - \frac{GMm}{Re}$

$V = \sqrt{\frac{2GM}{Re}} = \sqrt{2gRe} = \sqrt{2\times9.8\times6400\times1000}$

$= 11.2$ km/s

Kinetic energy $= \frac{1}{2}mv^2$

$= 6.27 \times 10^7$ J

32. In a screw gauge the zero of main scale reference line coincides with the fifth division of the circular scale when two studs are in contact. There are 100 divisions in circular scale and pitch of screw gauge is 0.1 mm. When diameter of a sphere is measured, the reading of main scale is 5 mm and 50^{th} division of circular scale coincides with the reference line of main scale. The diameter of sphere is _____mm.

(1) 5.045 (2) 5.055

(3) 5.450 (4) 5.550

Ans. **(1)**

Sol. Least count : $\frac{0.1}{100}\text{mm} = 0.001\text{mm}$

Reading = MSD + (CSD) × LC – zero error

= 5mm + 50 × 0.001 – 5 × 0.001 = 5.045 mm

33. The surface tension of a soap bubble is 0.03 N/m. The work done in increasing the diameter of bubble from 2 cm to 6 cm is $\alpha\pi \times 10^{-4}$ J. The value of α is_____. (Take $\pi = 3.14$)

(1) 0.86 (2) 0.64

(3) 1.92 (4) 7.68

Ans. (3)

Sol. $W = \Delta U = 2T\,[4\pi R_2^2 - 4\pi R_1^2]$

$W = 2 \times 3 \times 10^{-2} \times 4\pi\,[9 \times 10^{-4} - 1 \times 10^{-4}]$

$W = 1.92\,\pi \times 10^{-4}$ J

$\alpha = 1.92$

34. A mixture of carbon dioxide and oxygen has volume 8310 cm^3, temperature 300K, pressure 100 kPa and mass 13.2 g. The number of moles of carbon dioxide and oxygen gases in the mixture respectively are_____.

(Assume both carbondioxide and oxygen gases behave like ideal gases) [R = 8.31J/mol.K]

(1) 0.15 and 0.18 (2) 0.25 and 0.08

(3) 0.21 and 0.12 (4) 0.13 and 0.20

Ans. (3)

Sol. First, we determine the total number of moles (n_{total}) in the mixture using the Ideal Gas Law: PV = nRT. Given the values:

- Pressure (P): 100 kPa = 10^5 Pa
- Volume (V): 8310 cm3 = 8.31×10^{-3} m^3
- Temperature (T): 300 K
- Gas Constant (R): 8.31 J/mol·K

$$\frac{PV}{RT} = \frac{10^5 \times 8.31 \times 10^{-3}}{8.31 \times 300} = \frac{100}{300} \approx 0.333 \text{ mole}$$

1. Mole equation: $n_1 + n_2 = 0.333$
2. Mass equation: $44n_1 + 32n_2 = 13.2$ (where 44 and 32 are the molar masses of CO_2 and O_2 respectively)
3. Solve for individual moles

   Using the first equation to express n_2 as $0.333 - n_1$, we substitute this into the mass equation:

   $44n_1 + 32(0.333 - n_1) = 13.2$

   $44n_1 + 10.667 - 32n_1 = 13.2$

   $12n_1 = 2.533$

   $n_1 \approx 0.211$ mol

   Then, calculate n_2 :

   $n_2 = 0.333 - 0.211 = 0.122$ mol

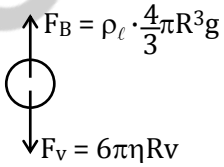
35. If an air bubble of diameter 2 mm rises steadily through a liquid of density 2000 kg/m^3 at a rate of 0.5 cm/s, then the coefficient of viscosity of liquid is_____Poise. (Take g = 10 m/s^2)

(1) 0.88 (2) 8.8

(3) 88.8 (4) 0.088

Ans. (2)

Sol.



For constant speed

$F_B = F_v$

$$\Rightarrow \rho_\ell \frac{4}{3}\pi R^3 g = 6\pi\eta Rv$$

$$\frac{2}{9}\frac{\rho_\ell R^2 g}{v} = \eta$$

$$\Rightarrow \frac{2}{9} \times \frac{2000 \times 10^{-6} \times 10}{5 \times 10^{-3}} = \eta$$

$$\Rightarrow \eta = \frac{8}{9} \times 10$$

$$\Rightarrow \eta = \frac{80}{9} \text{ Poise} = 8.88$$

36. A spherical ball of mass 2 kg falls from a height of 10 m and is brought to rest after penetrating 10 cm into sand. The average force exerted by sand on the ball is _____N.

(1) 1980 (2) 2020

(3) 2000 (4) 1000

Ans. **(2)**

Sol. $(W.D.)_g + (W.D.)_{sand} = \Delta K$

$$2\times10\times\left(10+\frac{10}{100}\right)-F_{sand}\times\left(\frac{10}{100}\right)=0 \ =0$$

$$F_{sand} = 2(10 + 0.1)\times100 = 2020$$

37. An electromagnetic wave travels in free space along the *x*-direction. At a particular point in space and time, $\vec{B}=2\times10^{-7}\hat{j}$ T is associated with this wave. The value of corresponding electric field $\vec{E}$ at this point is _____V/m.

(1) $60\,\hat{k}$ (2) $-60\,\hat{k}$

(3) $30\,\hat{k}$ (4) $-600\,\hat{k}$

Ans. **(2)**

Sol. $\vec{E}=\vec{B}\times\vec{C}$

$$=2\times10^{-7}\hat{j}\times3\times10^{8}\hat{i}$$

$$\vec{E}=-60\hat{k}\ V/m$$

38. Two resistors of 200 Ω and 400 Ω are connected in series with a battery of 100 V. A bulb rated at 200 V, 100 W is connected across the 400 Ω resistance. The potential drop across the bulb is_____V.

(1) 25 (2) 50

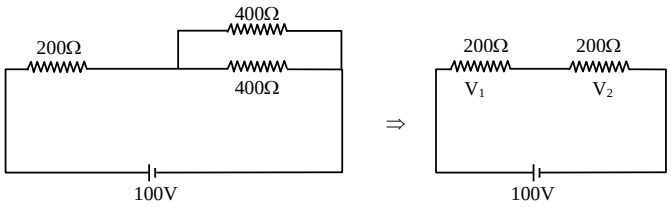
(3) 66.6 (4) 100

Ans. **(2)**

Sol. For resistance of bulb

$$R=\frac{V^2}{P}=\frac{200\times200}{100}=400\Omega$$

Now circuit will look like this



V_1 & V_2 will be same as resistances are same

$V_1 = V_2 = 50V$

$\boxed{V_{bulb} = V_2 = 50V}$

39. Two metal plates (A,B) are kept horizontally with separation of $\left(\frac{12}{\pi}\right)$ cm, with plate A on the top. An atomizer jet sprays oil (density 1.5 g/cm^3) droplets of radius 1mm horizontally. All oil droplets carry a charge 5 nC. The potentials V_A and V_B are required on plates A and B respectively in order to ensure the droplets do not descend. The values of V_A and V_B are_____.

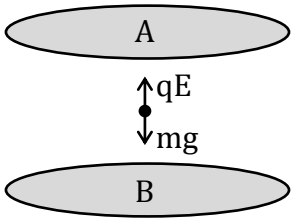
(Neglect the air resistance to the droplets and take g = 10 m/s^2)

(1) 100 V and 580 V (2) 580 V and 100 V

(3) 60 V and 400 V (4) 0 V and –200 V

Ans. **(1)**

Sol.



$$\frac{\Delta V}{(12/\pi)\times10^{-2}}\times5\times10^{-9}=\frac{4}{3}\pi(r^3)\rho(g)$$

$$\Delta V\times5\times10^{-7}=16(r^3)\rho(g)$$

$\Delta V = 480$, $V_A < V_B$

Plate A will be lower potential.

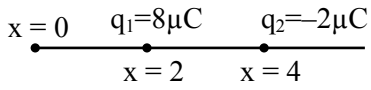
40. Two point charges 8μC and –2μC are located at x = 2 cm and x = 4 cm, respectively on the x-axis. The ratio of electric flux due to these charges through two spheres of radii 3 cm and 5 cm with their centers at the origin is_____.

(1) 4 : 1 (2) 3 : 4

(3) 4 : 3 (4) 4 : 5

Ans. (3)

Sol.



For sphere with radius 3cm and center origin

$$\phi_1 = \frac{q_{en}}{\varepsilon_0} = \frac{8\mu C}{\varepsilon_0}$$

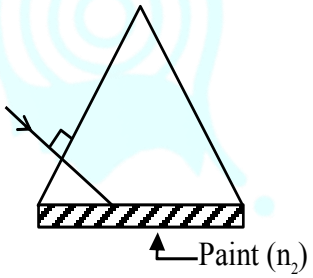
for sphere of radius 5cm

$$\phi_2 = \frac{q_{en}}{\varepsilon_0} = \frac{(8-2)\mu C}{\varepsilon_0} = \frac{6\mu C}{\varepsilon_0}$$

$$\frac{\phi_1}{\phi_2} = \frac{8}{6} = \frac{4}{3} = 1.33$$

$$\frac{\phi_1}{\phi_2} = 1.33$$

41. One side of an equilateral prism is painted by a transparent material of refractive index n_2. The refractive index of prism is 1.6. The minimum value of n_2 required for total internal reflection from painted face is_____.

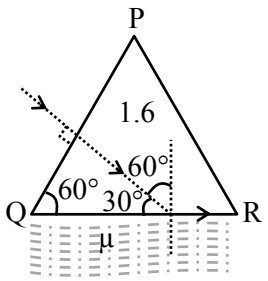


(1) $\frac{3\sqrt{3}}{1.6}$ (2) $\sqrt{3}$

(3) $\frac{3.2}{\sqrt{3}}$ (4) $\frac{4\sqrt{3}}{5}$

Ans. (4)

Sol.



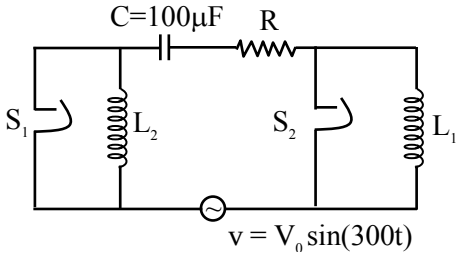
For surface QR

i = 60°

1.6 sin 60° = μ sin 90°

$$\mu = 1.6 \times \frac{\sqrt{3}}{2} = \frac{4}{5} \times \sqrt{3}$$

42. The figure given below shows an LCR series circuit with two switches S_1 and S_2. When switch S_1 is closed keeping S_2 open, the phase difference (ϕ) between the current and source voltage is 30° and phase difference is 60° when S_2 is closed keeping S_1 open. The value of $(3L_1 - L_2)$ is___ H.



(1) $\frac{9}{2}$

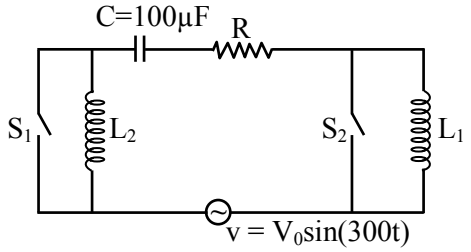
(2) $\frac{2}{9}$

(3) $\frac{1}{3}$

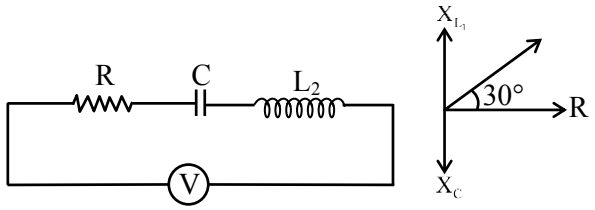
(4) 3

Ans. (2)

Sol.



For S_1 is closed and S_2 open

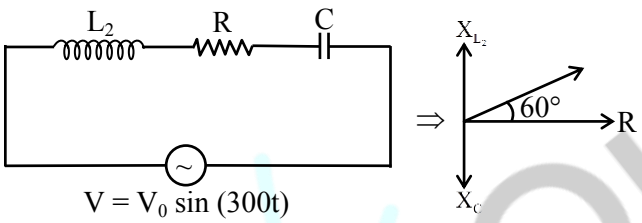


$V = V_0 \sin(300\,t)$

$$\frac{X_{L_1} - X_C}{R} = \tan(30°) = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,(X_{L_1} - X_C) = R \quad .....(1)$$

For S_1 is open and S_2 is closed.



$$\frac{|X_{L_2} - X_C|}{R} = \tan 60° = \sqrt{3}$$

$$\frac{|X_{L_2} - X_C|}{\sqrt{3}} = R \quad ....(2)$$

from Eq. (1) and (2)

$$\sqrt{3}\,(X_{L_1} - X_C) = \frac{(X_{L_2} - X_C)}{\sqrt{3}}$$

$$3\left(\omega L_1 - \frac{1}{\omega C}\right) = \left(\omega L_2 - \frac{1}{\omega C}\right)$$

$$|3L_1 - L_2| = \frac{3}{\omega^2 C} - \frac{1}{\omega^2 C} = \frac{2}{9\times10^4 \times 100 \times 10^{-6}}$$

$$|3L_1 - L_2] = \frac{2}{9}$$

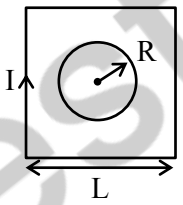
43. A circular current loop of radius R is placed inside square loop of side length L(L>>R) such that they are co-planar and their centers coincide. The permeability of free space is μ_0. The mutual inductance between circular loop and square loop is____.

(1) $2\sqrt{2}\frac{\mu_0 L^2}{R}$ (2) $\sqrt{2}\frac{\mu_0 L^2}{R}$

(3) $\sqrt{2}\frac{\mu_0 R^2}{L}$ (4) $2\sqrt{2}\frac{\mu_0 R^2}{L}$

Ans. (4)

Sol.



Magnetic field due to square

$$B = 4 \times \frac{\mu_0 I}{4\pi\frac{L}{2}}\ \{2\sin 45°\}$$

$$= \frac{4\mu_0 I}{\pi L} \times \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

$$Q = B \times A_{circle}$$

$$Q = \frac{2\sqrt{2}\mu_0 I}{\pi L} \times \pi R^2$$

$$Q = \frac{2\sqrt{2}\mu_0 R^2}{L} I$$

$$\text{Inductance} = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

44. The binding energy per nucleon of $^{209}_{83}Bi$ is _____ MeV.

[Take $m\left(^{209}_{83}Bi\right)$ = 208.980388 u, m_p = 1.007825 u, m_n = 1.008665 u, 1u = 931 MeV/c^2]

(1) 7.48 (2) 7.84

(3) 8.79 (4) 6.94

Ans. (2)

Sol. $83\,{}_1P^1 + 126\,{}_0n^1 \rightarrow {}_{83}Bi^{209}$

$\Delta m = 83m_p + 126\,m_n - M_{Bi}$

$= 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 1.760877$ u

BE = 1.760877 × 931 MeV = 1639.3764

$\frac{BE}{A} = \frac{1639.3764}{209}$ MeV = 7.8439 MeV

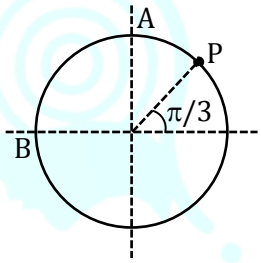
45. The equation of motion of a particle is given by $x = a\sin(50t + \pi/3)$cm. The particle will come to rest at time t_1 and it will have zero acceleration at time t_2. The t_1 and t_2 respectively are_____.

(1) $\frac{\pi}{300}$s, $\frac{\pi}{75}$s (2) $\frac{\pi}{75}$s, $\frac{\pi}{300}$s

(3) $\frac{\pi}{300}$s, $\frac{\pi}{25}$s (4) $\frac{\pi}{50}$s, $\frac{\pi}{100}$s

Ans. (1)

Sol. Phase of the motion of particle is as follows :



V = 0, when particle cross A.

$\therefore t_1 = \frac{\pi}{6\omega} = \frac{\pi}{300}$

a = 0, when particle cross B.

$\therefore t_2 = \frac{2\pi}{3\omega} = \frac{\pi}{75}$

SECTION-B

46. In a Young's double slit experiment, the intensity at some point on the screen is found to be $\frac{3}{4}$ times of the maximum of the interference pattern. The path difference between the interfering waves at this point is $\frac{\lambda}{x}$ where λ is wavelength of the incident light. The value of x is _____.

Ans. (6)

Sol. $I_r = I_{max}\cos^2\frac{\phi}{2}$

$\frac{3I_{max}}{4} = I_{max}\cos^2\frac{\phi}{2}$

$\cos\frac{\phi}{2} = \frac{\sqrt{3}}{2}$

$\frac{\phi}{2} = 30° = \frac{\pi}{6}$

$\left(\phi = \frac{\pi}{3}\right)$

Path difference $\Delta x = \frac{\phi}{2\pi} \times \lambda$

$\Rightarrow \frac{\pi}{3 \times 2\pi} \times \lambda$

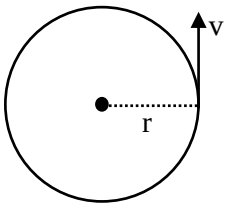
$\Delta x = \frac{\lambda}{6}$

x = 6

47. Using Bohr's model, calculate the ratio of the magnetic fields generated due to the motion of the electrons in the 2nd and 4th orbits of hydrogen atom__.

Ans. (64)

Sol.



$$B = \frac{\mu_0 I}{2\pi r}$$

$$I = \frac{ev}{2\pi r}$$

$$B \propto \frac{V}{r^2}$$

$$V \propto \frac{Z}{n}$$

$$r \propto \frac{n^n}{Z}$$

$$B \propto \frac{\left(\frac{Z}{n}\right)}{\left(\frac{n^2}{Z}\right)^2}$$

$$B \propto \frac{Z^3}{n^5}$$

$$\frac{B_1}{B_2} = \frac{n_2^5}{n_1^5} = \frac{4^5}{2^5} = 64$$

48. 5 moles of unknown gas is heated at constant volume from 10°C to 20°C. The molar specific heat of this gas at constant pressure c_p = 8 cal/mol. °C and R = 8.36 J/mol.°C. The change in the internal energy of the gas is ____ calorie.

Ans. (300)

Sol. $C_P - C_V = R \Rightarrow C_V = (C_P - R)$

$C_V = 6$ cal/mol°C

$\Delta U = nC_V \Delta T$

$= 5 \times 6 \times 10 = 300$ cal

49. If sunlight is focused on a paper using convex lens, it starts burning the paper in shortest time when the lens is kept at 30 cm above the paper. If the radius of curvature of the lens is 60 cm then the refractive index of the lens material is $\frac{\alpha}{10}$. The value of α is___ .

Ans. (20)

Sol. To burn in minimum time distance of paper must be at focus.

So f = 30 cm

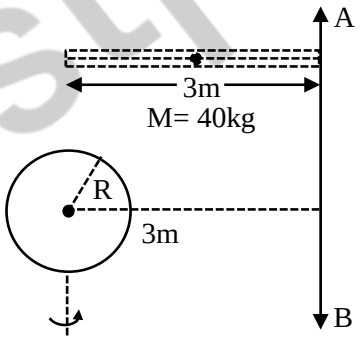
$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{60} - \frac{1}{-60}\right)$$

$$\frac{1}{30} = \frac{\mu - 1}{30}$$

$\mu = 2$

$\boxed{\alpha = 20}$

50. Moment of inertia about an axis AB for a rod of mass 40 kg and length 3 m is same as that of a solid sphere of mass of 10 kg and radius R about an axis parallel to AB axis with separation of 3 m as shown in figure below. The value of R is given as $\sqrt{\frac{\alpha}{2}}$. The value of α is_____.



Ans. (15)

Sol. $I_{Rod} = I_{SS}$

$$\frac{M_1 L^2}{3} = \left(\frac{2}{5} m_2 R^2 + m_2 L^2\right)$$

$$\frac{40 \times (3)^2}{3} = \frac{2}{5} \times 10 \times R^2 + 10 \times (3)^2$$

$120 = 4R^2 + 90$

$4R^2 = 30$

$$R = \sqrt{\frac{15}{2}} = \sqrt{\frac{\alpha}{2}}$$

$\alpha = 15$

(HELD ON THURSDAY 02nd APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

51. The ratio of mass percentage (w/w) of C : H in a hydrocarbon is 12 : 1. It has two carbon atoms. The weight (in g) of $CO_2(g)$ formed when 3.38 g of this hydrocarbon is completely burnt in oxygen is :

(Given : Molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(1) 5.68 (2) 11.44

(3) 22.74 (4) 17.05

Ans. (2)

Sol. Molecular formula of hydrocarbon is C_2H_2

$$\text{Moles of } C_2H_2 = \frac{3.38}{26}$$

$$C_2H_2(g) + \frac{5}{2}O_2(g) \rightarrow 2CO_2(g) + H_2O(l)$$

$$\text{Moles of } CO_2 \text{ produced} = \left(\frac{3.38}{26} \times 2\right)$$

$$\text{Mass of } CO_2 \text{ produced} = \left(\frac{3.38}{26} \times 2\right) \times 44 = 11.44$$

52. The first and second ionization constants of a weak dibasic acid H_2A are 8.1×10^{-8} and 1.0×10^{-13} respectively. 0.1 mol of H_2A was dissolved in 1L of 0.1 M HCl solution. The concentration of HA^- in the resultant solution is :

(1) 0.1 M (2) 9.53×10^{-6} M

(3) 8.1×10^{-8} M (4) 1.0×10^{-13} M

Ans. (3)

Sol.

$$\underset{0.1M}{HCl} \rightarrow \underset{0.1M}{H^+} + \underset{0.1M}{Cl^-}$$

$$\underset{0.1-x}{H_2A} \rightleftharpoons \underset{x+0.1}{H^+} + \underset{x}{HA^-}$$

(Due to common ion effect x is very less)

$$Ka_1 = \frac{[H^+][HA^-]}{[H_2A]}$$

$$Ka_1 = \frac{0.1 \times [HA^-]}{0.1}$$

$$[HA^-] = Ka_1 = 8.1 \times 10^{-8}$$

53. SF_4 is isostructural with :

A. $BrF_4^{\ominus}$

B. CH_4

C. $IF_4^{\oplus}$

D. XeF_4

E. XeO_2F_2

Choose the **correct** answer from the options given below :

(1) C only

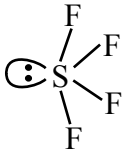
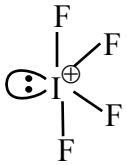
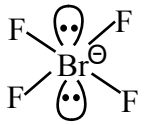
(2) C and E only

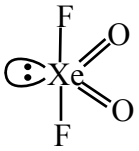
(3) A and D only

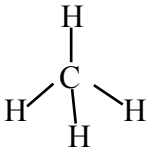
(4) B and E only

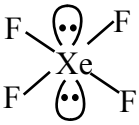
Ans. (2)

Sol.

Species	Structure / Shape
SF_4	 → see-saw
$IF_4^{\oplus}$	 → see-saw
$BrF_4^{\ominus}$	 → square planar

XeO_2F_2  → see-saw

CH_4  → Tetrahedral

XeF_4  → square planar

54. Gas 'A' undergoes change from state 'X' to state 'Y'. In this process, the heat absorbed and work done by the gas is 10 J and 18 J respectively. Now gas is brought back to state 'X' by another process during which 6 J of heat is evolved. In the reverse process of 'Y' to 'X',

(1) 18 J of the work is done by the gas 'A'.

(2) 2 J of the work is done by the gas 'A'.

(3) 12 J of the work is done on the gas 'A' by the surrounding.

(4) 14 J of the work is done on the gas 'A' by the surrounding.

Ans. (4)

Sol. For process X → Y

$w = -18$ J

$q = +10$ J

$\Delta U = q + w$

$\Delta U_{X\to Y} = 10 + (-18) = -8$ J

For reverse process :

$\Delta U_{Y\to X} = 8$ J

$q = -6$ J

$\Delta U = q + w$

$8 = -6 + w$

$w = 14$ J

55. Solution A is prepared by dissolving 1 g of a protein (molar mass = 50000 g mol^{-1}) in 0.5 L of water at 300 K. Its osmotic pressure is x bar. Solution B is made by dissolving 2 g of same protein in 1 L of water at 300 K. Osmotic pressure of solution B is y bar. Entire solution of A is mixed with entire solution of B at same temperature. The osmotic pressure of resultant solution is z bar. x, y and z respectively are :

($R = 0.083$ L bar mol^{-1} K^{-1})

(1) 9.96×10^{-4}; 9.96×10^{-4}; 9.96×10^{-4}

(2) 9.96×10^{-4}; 9.96×10^{-4}; 19.92×10^{-4}

(3) 4.98×10^{-4}; 4.98×10^{-4}; 9.96×10^{-4}

(4) 4.98×10^{-4}; 4.98×10^{-4}; 4.98×10^{-4}

Ans. (1)

Sol. $'x' = \dfrac{1}{50,000 \times 0.5} \times R \times 300 = 9.96 \times 10^{-4}$ bar

$'y' = \dfrac{2}{50,000 \times 1} \times R \times 300 = 9.96 \times 10^{-4}$ bar

$'z' = \dfrac{'x' \times \frac{1}{2} + 'y' \times 1}{\frac{3}{2}} = 9.96 \times 10^{-4}$ bar

56. At 25°C, 20.0 mL of 0.2 M weak monoprotic acid HX is titrated against 0.2 M NaOH. The pH of the solution (a) at the start of the titration (when NaOH has not been added) and (b) when 10 mL of NaOH is added respectively, are :

Given : $K_a = 5 \times 10^{-4}$, $pK_a = 3.3$, $\alpha << 1$

(1) 0.7, 2.0

(2) 2.0, 3.3

(3) 1.1, 2.2

(4) 3.0, 2.2

Ans. (2)

Sol. For 0.2M, 20ml HX solution,

$$\text{pH of weak acid} = \frac{1}{2}[pk_a - \log c]$$

$$= \frac{1}{2}[4 - \log 5 - \log 0.2]$$

$$= \frac{1}{2}[4 - \log 5 + \log 5]$$

$$= 2$$

$$HX + \overset{(LR)}{NaOH} \longrightarrow NaA + H_2O$$

20 ml, 0.2 M 10 ml, 0.2 M

4m mole 2m mole

2m mole – 2m mole

[HX + NaA] $\Rightarrow$ acidic buffer

$$pH = pK_a + \log \frac{salt}{acid}$$

$$pH = 3.3 + \log \frac{2}{2}$$

$$pH = 3.3$$

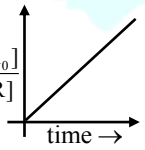
57. Consider the reaction aX $\rightarrow$ bY, for which the rate constant at 30°C is 1×10^{-3} mol^{-1} Ls^{-1}. Which of the following statements are true ?

A. When concentration of 'X' is increased to four times, the rate of reaction becomes 16 times.

B. The reaction is a second order reaction.

C. The half-life period is independent of the concentration of X.

D. Decomposition of N_2O_5 is an example of the above reaction.

E. $\ell n \frac{[R_0]}{[R]}$ 

is valid for the above reaction.

Choose the **correct** answer from the options given below :

(1) A and B only (2) A, B and C only

(3) A, B, D and E only (4) C and D only

Ans. **(1)**

Sol. Rate of constant (k) = 10^{-3} M^{-1} sec^{-1}

$\because$ 1 – n = –1 [unit of k = M^{1-n} sec^{-1}]

n = 2 $\Rightarrow$ 2nd order reaction

rate = $k[A]^2$

(A) If concentration of A is 4 times then reaction will become 16 times.

(B) Order of reaction is 2

(C) Half-life of the reaction is independent of the concentration of the reactant for 1st order, not for 2nd order reaction.

(D) Decomposition of N_2O_5 is example of 1st order reaction

(E) Graph of $\ell n \frac{[R]_0}{[R]_t}$ vs t is straight line for 1st order not for 2nd order reaction.

So statement (A) & (B) are correct

58. The correct set that contains all kinds (basic, acidic, amphoteric and neutral) of oxides is :

(1) Na_2O, K_2O, Al_2O_3 and As_2O_3

(2) Al_2O_3, As_2O_3, CO and NO

(3) K_2O, Cl_2O_7, As_2O_3 and NO

(4) Na_2O, N_2O, Al_2O_3 and CO

Ans. **(3)**

Sol. Basic Oxides : Na_2O , K_2O

Acidic Oxides : Cl_2O_7

Neutral Oxides : N_2O , NO & CO

Amphoteric Oxides : Al_2O_3 , As_2O_3

59. Given below are two statements :

Statement-I : The second ionization enthalpy of B, Al and Ga is in the order of B > Al > Ga.

Statement-II : The correct order in terms of first ionization enthalpy is Si < Ge < Pb < Sn.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. (2)

Sol. Order of II ionization energy for boron family elements is :

$$\underset{(2427\ KJ/mol)}{B} > \underset{(1979\ KJ/mol)}{Ga} > \underset{(1816\ KJ/mol)}{Al}$$

Order of I ionization energy for carbon family elements is :

$$\underset{(786\ KJ/mol)}{Si} > \underset{(761\ KJ/mol)}{Ge} > \underset{(715\ KJ/mol)}{Pb} > \underset{(708\ KJ/mol)}{Sn}$$

60. Given below are two statements :

Statement-I : Among Zn, Mn, Sc and Cu, the energy required to remove the third valence electron is highest for Zn and lowest for Sc.

Statement-II : The correct order of the following complexes in terms of CFSE is

$[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. (1)

Sol. Statement I

	IE_3 (kJ/mol)
Sc	2393
Mn	3260
Cu	3556
Zn	3837

Sc < Mn < Cu < Zn (3^{rd} IE)

Statement II

CFSE $\Rightarrow$ SFL > WFL

(en > H_2O)

CFSE $\propto$ Charge on metal

$Co^{+2} < Co^{+3}$

$[Co(H_2O)_6]^{2+} < [Co(H_2O)_6]^{3+} < [Co(en)_3]^{3+}$

61. Which of the following complexes will show coordination isomerism ?

(A) $[Ag(NH_3)_2][Ag(CN)_2]$

(B) $[Co(NH_3)_6][Cr(CN)_6]$

(C) $[Co(NH_3)_6][Co(CN)_6]$

(D) $[Fe(NH_3)_6][Co(CN)_6]$

(E) $[Co(NH_3)_6][Fe(CN)_6]$

Choose the **correct** answer from the options given below :

(1) B, C and D Only

(2) B, D and E Only

(3) A, C and D Only

(4) C, D and E Only

Ans. (1, 2, 4)

Sol. Compound, $[Ag(NH_3)_2][Ag(CN)_2]$ cannot show coordination isomerism. All other complexes (B, C, D and E) will show co-ordination isomerism.

62. Complete combustion of X g of an organic compound gave 0.25 g of CO_2 and 0.12 g of H_2O. If the % of carbon is 25% and of hydrogen is 4.89%, then X = _____ $\times 10^{-3}$ g. (Nearest integer)

(Molar mass of C, H and O are 12, 1 and 16 g mol^{-1} respectively.)

(1) 273 (2) 27

(3) 2730 (4) 227

Ans. **(1)**

Sol. Mass of carbon = $\frac{0.25}{44}\times 12$

Mass% of carbon = $\frac{\text{Mass of carbon}}{x}\times 100$

$x = \frac{\frac{0.25\times 12}{44}\times 100}{25}$

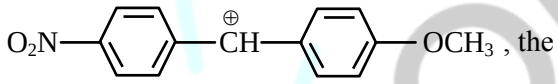
$x = \frac{12}{44}$

$x = 0.2727 = 272.7 \times 10^{-3}$

$\simeq 273 \times 10^{-3}$

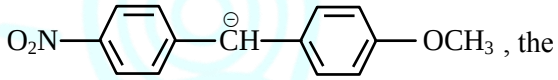
63. Given below are two statements :

Statement I : In



carbocation is stabilised by +R effect of $-OCH_3$ group.

Statement II : In



carbanion is stabilised by –R effect of $-NO_2$ group.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. **(1)**

Sol. Both statements are correct

64. The compound (X) on

(i) on heating in the presence of anhydrous $AlCl_3$ and HCl gas gives 2, 4–dimethyl pentane

(ii) aromatization gives toluene and

(iii) cyclisation gives methyl cyclohexane

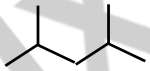
The correct name of compound (X) is :

(1) Hept-2-ene (2) Hept-1,3,5-triene

(3) Heptane (4) Hept-2,4,6-triene

Ans. **(3)**

Sol. n-Heptane $\xrightarrow[Cr_2O_3/773K/10-20atm]{\text{Aromatisation}}$ 

n-Heptane $\xrightarrow[\text{Isomerisation}]{\text{anhyd. } AlCl_3/HCl}$ 

65. Correct statements regarding alkyl halides (R–X) among the following are :

(A) Alcohol being less polar solvent as compared to water, alcoholic KOH favours elimination reaction with R–X.

(B) Order of reactivity towards S_N1 mechanism is $C_6H_5–CH_2–Cl > C_6H_5–CHCl–C_6H_5$.

(C) Non substituted aryl halides exhibit properties similar to alkyl halides.

(D) Vinyl chloride is an example of haloalkene and allyl chloride is an example of haloalkyne.

(E) R–Cl can be prepared by reacting R–OH with $SOCl_2$ but Ar–Cl cannot be prepared by reacting Ar–OH with $SOCl_2$.

Choose the **correct** answer from the options given below :

(1) A, B and C Only

(2) B and D Only

(3) A and E Only

(4) D and E Only

Ans. **(3)**

Sol. Ar-Cl and R–Cl shows different chemical properties

Rate of $S_N^1 \propto$ stability of first formed carbocation

Water is more polar than alcohol

Vinyl alcohol, allyl alcohol, both are alkene unsaturated alcohols

$R–OH \xrightarrow{SOCl_2} R–Cl$

Phenol does not give Ph–Cl with $SOCl_2$.

66. An organic compound "x" where molar ratio of C, O and H are equal, on treatment with 50% KOH under reflux followed by acidification produced "y". The most likely structure of "y" is:

[Molar mass of 'x' is 58 g mol^{-1}]

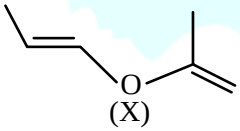
(1) $CH_2=CH–\overset{\overset{O}{\|}}{C}–OH$ (2) $CH_3–CH=CH–CH=O$

(3) $O=\underset{\underset{CH_2–OH}{|}}{C}–OH$ (4) $CH_3–\overset{\overset{O}{\|}}{C}–OH$

Ans. (3)

Sol. $\begin{matrix}CHO\\|\\CHO\end{matrix} \xrightarrow[\text{Intramolecular cannizaro reaction}]{KOH} \begin{matrix}CH_2OH\\|\\COO^{\ominus}\end{matrix} \xrightarrow{H^{\oplus}} \begin{matrix}CH_2OH\\|\\COOH\end{matrix}$

67. A molecule (X) with following structure under mild acidic condition is hydrolysed to produce (Y) and (Z). Identify the correct statements about (Y) and (Z).



(X)

(A) Both (Y) and (Z) have same molar mass

(B) (Y) and (Z) can be distinguished from each other by $NaHCO_3$

(C) (Y) and (Z) react with HCN with same rates

(D) (Y) and (Z) undergo addition reaction with 2, 4-DNP

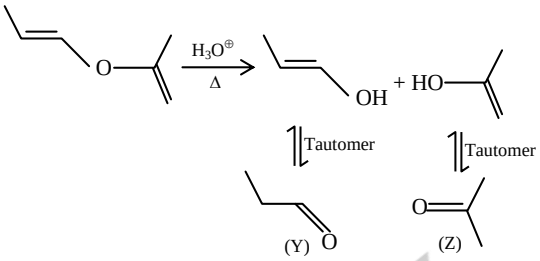
Choose the **correct** answer from the options given below :

(1) A, B and C Only (2) B and C Only

(3) C and D Only (4) A and D Only

Ans. (4)

Sol.

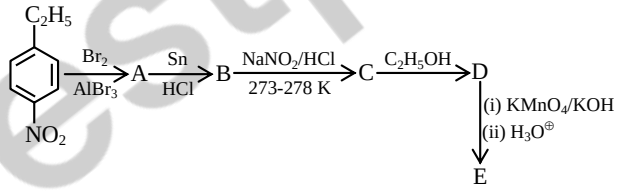


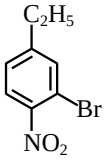
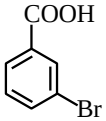
Y will give positive fehling's test

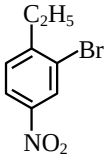
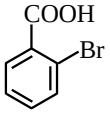
Molar mass of [Z] (C_3H_6O) = 58

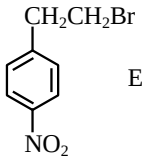
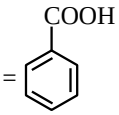
Molar mass of [Y] (C_3H_6O) = 58

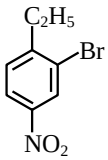
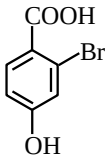
68. Identify compounds A and E in the following reaction sequence.



(1) A =  E = 

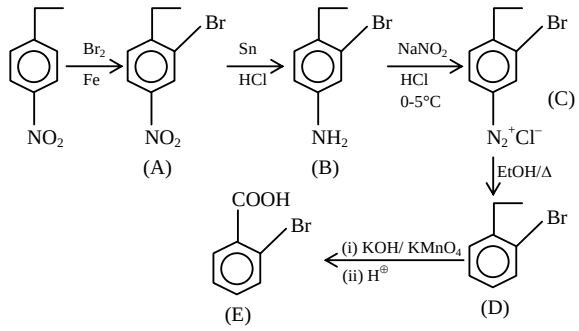
(2) A =  E = 

(3) A =  E = 

(4) A =  E = 

Ans. (2)

Sol.



69. Identify the correct pair having amino acid (A) and the hormone (B) that is iodinated derivative of the amino acid (A).

(T and Y represent one letter code for amino acids)

Amino acid (A)	**Hormone (B)**
(1) T	Insuline
(2) T	Thyroxine
(3) Y	Thyroxine
(4) Y	Insuline

Ans. **(3)**

Sol. Thyroxine is derivative of Tyrosine and single letter symbol of Tyrosine amino acid is Y.

70. Among Fe^{2+}, Fe^{3+}, Cr^{2+} and Zn^{2+}, the ion that shows positive borax bead test and with highest ionisation enthalpy is:

(1) Fe^{2+} (2) Zn^{2+}

(3) Cr^{2+} (4) Fe^{3+}

Ans. **(4)**

Sol. Zn^{2+} does not show Borax Bead Test.

Among remaining ions; Fe^{3+} has maximum ionization energy.

SECTION-B

71. The surface of sodium metal is irradiated with radiation of wavelength x nm. The kinetic energy of ejected electrons is 2.8×10^{-20} J. The work function of sodium is 2.3 eV. The value of x is _____ $\times 10^2$ nm. (Nearest integer)

(Given : h = 6.6×10^{-34} J s; 1eV = $1.6\ 0 \times 10^{-19}$ J; c = 3.0×10^8 ms^{-1})

Ans. **(5)**

Sol. for photoelectric effect :

$$\frac{hc}{\lambda} = W + K.E.$$

$$\frac{6.6\times10^{-34}\times3\times10^{8}}{x} = 2.3\times1.6\times10^{-19} + 2.8\times10^{-20}$$

$$\frac{19.8\times10^{-26}}{x} = 3.96\times10^{-19}$$

$x = 5 \times 10^{-7}$

$x = 500 \times 10^{-9}$ m

$x = 5 \times 10^2$ nm

72. Consider the following gas phase reaction being carried out in a closed vessel at 25°C.

$2A(g) \rightarrow 4B(g) + C(g)$

time (min)	total pressure of the system (mm Hg)
30	300
∞	600

The pressure of C(g) at 30 minutes time interval would be _____mm Hg. (Nearest integer)

Ans. **(20)**

Sol.

$$2A(g) \rightarrow 4B(g) + C(g)$$

			p°
t = 30 min	p°–2x	4x	x
t = ∞	–	2p°	p°/2

$p_\infty = 2p° + p°/2 = 600$

$5p°/2 = 600$

$P° = 240$ mm Hg

At t = 30 min, $p = 300 = p° - 2x + 4x + x$

$300 = 240 + 3x$

$3x = 60$

$x = 20$

Pressure of C(g) = 20 mm Hg

73. Consider the following two half-cell reactions along with the standard reduction potential given :

$CO_2 + 6H^+ + 6e^- \rightarrow CH_3OH + H_2O \quad E^o_{red} = 0.02V$

$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O \quad E^o_{red} = 1.23V$

The fuel cell was set up using the above two reactions such that the cell operates under the standard condition of 1 bar pressure and 298 K temperature. The fuel cell works with 80% efficiency. If the work derived from the cell using 1 mol of CH_3OH is used to compress an ideal gas isothermally against a constant pressure of 1kPa, then the change in the volume of the gas, $\Delta V =$ _____ m^3. (Nearest integer)

Given : F = 96500 C mol^{-1}

Ans. (560)

Sol. $E^o_{cell} = [1.23 - 0.02] = 1.21V$

$\Delta G° = -6 \times 96500 \times 1.21$ J

$W = \frac{80}{100} \times \Delta G° = -P\Delta V$

$0.8 \times 6 \times 96500 \times 1.21 = 1 \times 10^3 (\Delta V)$

$\Delta V = 560.472\ m^3$

74. Number of paramagnetic ions among the following d- and f-block metal ions is _____.

Mn^{2+}, Cu^{2+}, Zn^{2+}, Yb^{2+}, Sc^{3+}, La^{3+}, Gd^{3+}, Lu^{3+}, Ti^{4+}, Ce^{4+}

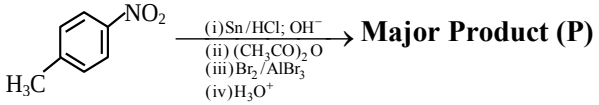
(Atomic number of Mn = 25, Cu = 29, Zn = 30, Yb = 70, Sc = 21, La = 57, Gd = 64, Lu = 71, Ti = 22, Ce = 58)

Ans. (3)

Sol. Paramagnetic : Mn^{+2}, Cu^{+2}, Gd^{+3}

Diamagnetic : Zn^{+2}, Yb^{+2}, Sc^{+3}, La^{+3}, Lu^{+3}, Ti^{+4}, Ce^{+4}

75. Consider the following reactions sequence

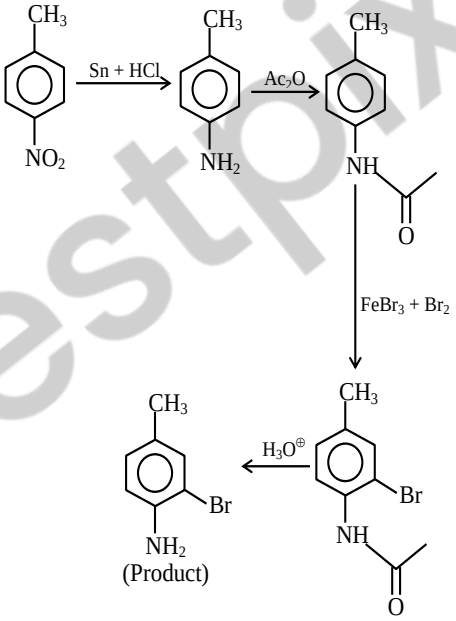


When the product (P) is subjected to Carius analysis using $AgNO_3$, 1.0 g of the product (P) will produce _____ g of the precipitate of AgBr. (Nearest interger)

(Given : molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14, Br : 80, Ag : 108)

Ans. (1)

Sol.



Molar mass of product = 186

Mass of AgBr = $\frac{1}{186} \times 188 \approx 1$