(Held On Tuesday 31st January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$, $b < 2$, from the origin is 1, then the eccentricity of the ellipse is:

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{4}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Equation of normal is

$2x\sec\theta - by\,\text{cosec}\theta = 4 - b^2$

Distance from (0, 0) $= \frac{4-b^2}{\sqrt{4\sec^2\theta + b^2\,\text{cosec}^2\theta}}$

Distance is maximum if

$4\sec^2\theta + b^2\,\text{cosec}^2\theta$ is minimum

$\Rightarrow \tan^2\theta = \frac{b}{2}$

$\Rightarrow \frac{4-b^2}{\sqrt{4.\frac{b+2}{2}+b^2.\frac{b+2}{b}}}=1$

$\Rightarrow 4-b^2 = b+2 \Rightarrow b=1 \Rightarrow e=\frac{\sqrt{3}}{2}$

2. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z+\frac{1}{z}$ be the curve C_2. Then

(1) the curves C_1 and C_2 intersect at 4 points

(2) the curves C_1 lies inside C_2

(3) the curves C_1 and C_2 intersect at 2 points

(4) the curves C_2 lies inside C_1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Let $w = z+\frac{1}{z} = 4e^{i\theta}+\frac{1}{4}e^{-i\theta}$

$\Rightarrow w = \frac{17}{4}\cos\theta + i\frac{15}{4}\sin\theta$

So locus of w is ellipse $\frac{x^2}{\left(\frac{17}{4}\right)^2}+\frac{y^2}{\left(\frac{15}{4}\right)^2}=1$

Locus of z is circle $x^2 + y^2 = 16$

So intersect at 4 points

3. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1 and the other piece of length ℓ_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi\ell_1) : \ell_2$ is equal to:

(1) 6 : 1

(2) 3 : 1

(3) 1 : 6

(4) 4 : 1

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\ell_1 + \ell_2 = 20 \Rightarrow \frac{d\ell_2}{d\ell_1} = -1$

$A_1 = \left(\frac{\ell_1}{4}\right)^2$ and $A_2 = \pi\left(\frac{\ell_2}{2\pi}\right)^2$

Let $S = 2A_1 + 3A_2 = \frac{\ell_1^2}{8}+\frac{3\ell_2^2}{4\pi}$

$\frac{ds}{d\ell} = 0 \Rightarrow \frac{2\ell_1}{8}+\frac{6\ell_2}{4\pi}.\frac{d\ell_2}{d\ell_1}=0$

$\Rightarrow \frac{\ell_1}{4}=\frac{6\ell_2}{4\pi} \Rightarrow \frac{\pi\ell_1}{\ell_2}=6$

4. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$,

which of the following is NOT true ?

(1) If $\alpha = \beta = 7$, then the system has no solution

(2) If $\alpha = \beta$ and $\alpha \neq 7$ then the system has a unique solution.

(3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

(4) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. By equation 1 and 3 $y + 2z = 8$

$y = 8 - 2z$

And $x = -2 + z$

Now putting in equation 2

$\alpha(z-2)+\beta(-2z+8)+7z=3$

$\Rightarrow(\alpha-2\beta+7)z=2\alpha-8\beta+3$

So equations have unique solution if

$\alpha-2\beta+7\neq 0$

And equations have no solution if

$\alpha-2\beta+7=0$ and $2\alpha-8\beta+3\neq 0$

And equations have infinite solution if

$\alpha-2\beta+7=0$ and $2\alpha-8\beta+3=0$

5. Let the shortest distance between the lines $L: \frac{x-5}{-2}=\frac{y-\lambda}{0}=\frac{z+\lambda}{1}$, $\lambda \geq 0$ and L_1: $x + 1 = y - 1 = 4 - z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?

(1) $\alpha + 2\gamma = 24$

(2) $2\alpha + \gamma = 7$

(3) $2\alpha - \gamma = 9$

(4) $\alpha - 2\gamma = 19$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&0&1\\1&1&-1\end{vmatrix}=-\hat{i}-\hat{j}-2\hat{k}$

$\vec{a_2}-\vec{a_1}=6\hat{i}+(\lambda-1)\hat{j}+(-\lambda-4)\hat{k}$

$2\sqrt{6}=\left|\frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}}\right|$

$|\lambda+3|=12\Rightarrow\lambda=9,-15$

$\alpha=-2k+5$, $\gamma=k-\lambda$ where $k\in R$

$\Rightarrow\alpha+2\gamma=5-2\lambda=-13,35$

6. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2},0\right)$ and directrix $y=-\frac{1}{2}$.

Then

$S=\left\{x\in\mathbb{R}:\tan^{-1}\left(\sqrt{f(x)}+\sin^{-1}\left(\sqrt{f(x)+1}\right)\right)=\frac{\pi}{2}\right\}$:

(1) contains exactly two elements

(2) contains exactly one element

(3) is an infinite set

(4) is an empty set

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\left(x+\frac{1}{2}\right)^2=\left(y+\frac{1}{4}\right)$

$y=\left(x^2+x\right)$

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\pi/2$

$0\leq x^2+x+1\leq 1$

$x^2+x\leq 0$ .....(1)

Also $x^2+x\geq 0$ .....(2)

$\therefore x^2+x=0\Rightarrow x=0,-1$

S contains 2 element.

7. Let $A=\begin{pmatrix}1&0&0\\0&4&-1\\0&12&-3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to:

(1) 6144 (2) 4094

(3) 4097 (4) 2050

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$\Rightarrow A^3 = A^4 = \ldots\ldots = A$

$(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \ldots.. {}^{11}C_{10} A + {}^{11}C_{11} I$

$= \left({}^{11}C_0 + {}^{11}C_1 + \ldots.. {}^{11}C_{10}\right) A + I$

$= \left(2^{11} - 1\right) A + I = 2047A + I$

$\therefore$ Sum of diagonal elements = 2047(1 + 4 – 3) + 3

= 4094 + 3 = 4097

8. Let R be a relation on N × N defined by (a, b) R (c, d) if and only if ad(b – c) = bc(a – d). Then R is

(1) symmetric but neither reflexive nor transitive

(2) transitive but neither reflexive nor symmetric

(3) reflexive and symmetric but not transitive

(4) symmetric and transitive but not reflexive

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. (a, b) R (c, d) $\Rightarrow$ ad(b – c) = bc(a – d)

Symmetric:

(c, d) R (a, b) $\Rightarrow$ cb(d – a) = da(c – b) $\Rightarrow$

Symmetric

Reflexive:

(a, b) R (a, b) $\Rightarrow$ ab(b – a) $\neq$ ba(a – b) $\Rightarrow$

Not reflexive

Transitive: (2,3) R (3,2) and (3,2) R (5,30) but

$((2,3),(5,30)) \notin R \Rightarrow$ Not transitive

9. Let

$$y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)\right)$$

. Then, at x = 1,

(1) $2y' + \sqrt{3}\pi^2 y = 0$

(2) $2y' + 3\pi^2 y = 0$

(3) $\sqrt{2}y' - 3\pi^2 y = 0$

(4) $y' + 3\pi^2 y = 0$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $y = \sin^3(\pi/3\cos g(x))$

$$g(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{3/2}$$

$g(1) = 2\pi/3$

$$y' = 3\sin^2\left(\frac{\pi}{3}\cos g(x)\right) \times \cos\left(\frac{\pi}{3}\cos g(x)\right) \times \frac{\pi}{3}\left(-\sin g(x)\right) g'(x)$$

$$y'(1) = 3\sin^2\left(-\frac{\pi}{6}\right).\cos\left(\frac{\pi}{6}\right).\frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right) g'(1)$$

$$g'(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{1/2}\left(-12x^2 + 10x\right)$$

$$g'(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2) = -\pi$$

$$y'(1) = \frac{\cancel{2}}{4}.\frac{\sqrt{3}}{2}.\frac{\pi}{\cancel{2}}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$$

$$y(1) = \sin^3\left(\pi/3\cos 2\pi/3\right) = -\frac{1}{8}$$

$$2y'(1) + 3\pi^2 y(1) = 0$$

10. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such GPs is

(1) 7 (2) $\frac{9}{2}$

(3) 3 (4) 14

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. a, ar, ar^2, ar^3 $(a, r > 0)$

$a^4r^6 = 1296$

$a^2r^3 = 36$

$$a = \frac{6}{r^{3/2}}$$

$a + ar + ar^2 + ar^3 = 126$

$$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6} = 21$$

$$\left(r^{-3/2} + r^{3/2}\right) + \left(r^{1/2} + r^{-1/2}\right) = 21$$

$r^{1/2} + r^{-1/2} = A$

$r^{-3/2} + r^{3/2} + 3A = A^3$

$A^3 - 3A + A = 21$

$A^3 - 2A = 21$

$A = 3$

$$\sqrt{r} + \frac{1}{\sqrt{r}} = 3$$

$r + 1 = 3\sqrt{r}$

$r^2 + 2r + 1 = 9r$

$r^2 - 7r + 1 = 0$

11. The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is:

(1) 0

(2) 1

(3) 3

(4) 2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$

$$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$$

$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3$ which is in domain

or

$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$

$2\sqrt{(x-1)(x+3)} = 2x - 4$

$x^2 + 2x - 3 = x^2 - 4x + 4$

$6x = 7$

$x = 7/6$ (rejected)

12. Let a differentiable function f satisfy $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}, x \geq 3$. Then 12f(8) is equal to:

(1) 34

(2) 19

(3) 17

(4) 1

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Differentiate w.r.t. x

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

I.F. $= e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$$xf(x) = \int \frac{x}{2\sqrt{x+1}} dx$$

$x + 1 = t^2$

$$= \int \frac{t^2 - 1}{2t} 2t dt$$

$$xf(x) = \frac{t^3}{3} - t + c$$

$$xf(x) = \frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + c$$

Also putting x = 3 in given equation

$f(3) + 0 = \sqrt{4}$

$f(3) = 2$

$$\Rightarrow C = 8 - \frac{8}{3} = \frac{16}{3}$$

$$f(x) = \frac{\frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + \frac{16}{3}}{x}$$

$$f(8) = \frac{9 - 3 + \frac{16}{3}}{8} = \frac{34}{24}$$

$\Rightarrow$ 12 f(8) = 17

13. If the domain of the function $f(x)=\frac{[x]}{1+x^2}$, where [x] is greatest integer $\le x$, is (2, 6), then its range is

(1) $\left(\frac{5}{26},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

(2) $\left(\frac{5}{26},\frac{2}{5}\right]$

(3) $\left(\frac{5}{37},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

(4) $\left(\frac{5}{37},\frac{2}{5}\right]$

Official Ans. by NTA (4)

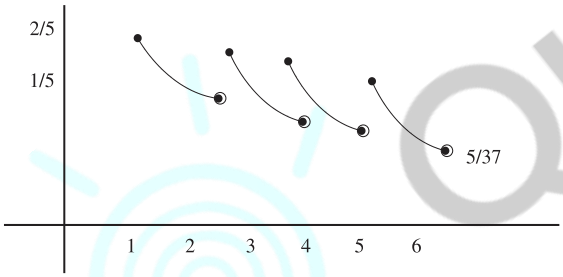
QuestPix Ans. (4)

Sol. $f(x)=\frac{2}{1+x^2}$ $x\in[2,3)$

$f(x)=\frac{3}{1+x^2}$ $x\in[3,4)$

$f(x)=\frac{4}{1+x^2}$ $x\in[4,5)$

$f(x)=\frac{5}{1+x^2}$ $x\in[5,6)$



$\left(\frac{5}{37},\frac{2}{5}\right]$

14. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}|=|\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}.\vec{c}=0$. Consider the following two statement:

(A) $|\vec{a}+\lambda\vec{c}|\ge|\vec{a}|$ for all $\lambda\in\mathbb{R}$.

(B) $\vec{a}$ and $\vec{c}$ are always parallel

(1) only (B) is correct

(2) neither (A) nor (B) is correct

(3) only (A) is correct

(4) both (A) and (B) are correct.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}+\vec{b}-\vec{c}|^2$

$2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a}=2\vec{a}.\vec{b}-2\vec{b}.\vec{c}-2\vec{c}.\vec{a}$

$4\vec{a}.\vec{c}=0$

B is incorrect

$|\vec{a}+\lambda\vec{c}|^2\ge|\vec{a}|^2$

$\lambda^2c^2\ge0$

True $\forall\lambda\in R$ (A) is correct.

15. Let $\in$ α (0, 1) and $\beta=\log_e(1-\alpha)$. Let

$P_n(x)=x+\frac{x^2}{2}+\frac{x^3}{3}+.....+\frac{x^n}{n}$, $x\in(0,1)$.

Then the integral $\int_0^{\alpha}\frac{t^{50}}{1-t}dt$ is equal to

(1) $\beta-P_{50}(\alpha)$

(2) $-(\beta+P_{50}(\alpha))$

(3) $P_{50}(\alpha)-\beta$

(4) $\beta+P_{50}(\alpha)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\int_0^{\alpha}\frac{t^{50}-1+1}{1-t}=-\int_0^{\alpha}\left(1+t+.....+t^{49}\right)+\int_0^{\alpha}\frac{1}{1-t}dt$

$=-\left(\frac{\alpha^{50}}{50}+\frac{\alpha^{49}}{49}+.....+\frac{\alpha^1}{1}\right)+\left(\frac{\ln(1-f)}{-1}\right)_0^{\alpha}$

$=-P_{50}(\alpha)-\ln(1-\alpha)$`

$=-P_{50}(\alpha)-\beta$

16. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0$, $0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) π

(2) 16

(3) 0

(4) $16-5\pi$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4}$

$$\therefore \sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{8}{15}=\sin^{-1}\frac{8}{17}$$

$$\Rightarrow \frac{\alpha}{17}=\frac{8}{17}\Rightarrow \alpha=8$$

$$\therefore \sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)$$

$$=3\pi-8+8-2\pi$$

$$=\pi$$

17. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3, 2). Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is :

(1) $2\left(2+\sqrt{2}\right)$

(2) $4\left(1+\sqrt{2}\right)$

(3) $3+2\sqrt{2}$

(4) $2\left(1+\sqrt{2}\right)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $C=(2,3),\ r=\sqrt{2}$

Centre of $G=A=2+4\frac{1}{\sqrt{2}}$,

$$3+\frac{4}{\sqrt{2}}=\left(2+2\sqrt{2},3+2\sqrt{2}\right)$$

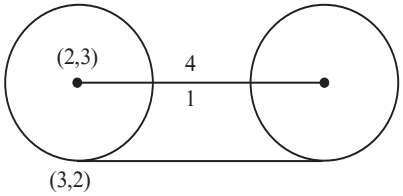
$$A\left(2+2\sqrt{2},3+2\sqrt{2}\right)$$

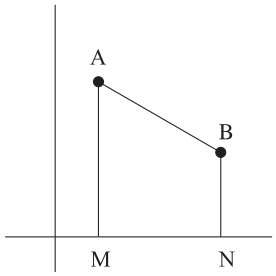
$$B\left(4+2\sqrt{2},1+2\sqrt{2}\right)$$

$$\frac{x-\left(2+2\sqrt{2}\right)}{1}=\frac{y-\left(3+2\sqrt{2}\right)}{-1}=2$$

$\therefore$ area of trapezium:

$$\frac{1}{2}\left(4+4\sqrt{2}\right)2=4\left(1+\sqrt{2}\right)$$





18. $(S1)(p\Rightarrow q)\vee(p\wedge(\sim q))$ is a tautology

$(S2)((\sim p)\Rightarrow(\sim q))\wedge((\sim p)\vee q)$ is a

Contradiction. Then

(1) only (S2) is correct

(2) both (S1) and (S2) are correct

(3) both (S1) and (S2) are wrong

(4) only (S1) is correct

Official Ans. by NTA (2)

QuestPix Ans. (4)

Sol.

p	q	p⇒q	~q	p∧~q	(p⇒q)∨(p∧~q)
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

~p	~q	~p⇒~q	~p∨q	((~p)⇒(~q))∧(~p)∨ q)
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

19. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)}dx$ is equal to

(1) $\frac{7}{2}-\sqrt{3}-\log_e\sqrt{3}$

(2) $-2+3\sqrt{3}+\log_e\sqrt{3}$

(3) $\frac{10}{3}-\sqrt{3}+\log_e\sqrt{3}$

(4) $\frac{10}{3}-\sqrt{3}-\log_e\sqrt{3}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\int_{\pi/3}^{\pi/2}\left(\frac{2+3\sin x}{\sin x(1+\cos x)}\right)dx = 2\int_{\pi/3}^{\pi/2}\frac{dx}{\sin x+\sin x\cos x}+3$

$3\int_{\pi/3}^{\pi/2}\frac{dx}{1+\cos x}$

$\int_{\pi/3}^{\pi/2}\frac{dx}{1+\cos x}=\int_{\pi/3}^{\pi/2}\frac{1-\cos x}{\sin^2 x}dx$

$=\int_{\pi/3}^{\pi/2}\left(\operatorname{cosec}^2 x-\cot x\operatorname{cosec} x\right)dx$

$=\left(\operatorname{cosec} x-\cot x\right)\int_{\pi/3}^{\pi/2}=(1)-\left(\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{3}}\right)=1-\frac{1}{\sqrt{3}}$

$\int_{\pi/3}^{\pi/2}\frac{dx}{\sin x(1+\cos x)}=$

$\int\frac{dx}{(2\tan x/2)\left(1+1-\tan^2 x/2\right)}$

$=\int\frac{\left(1+\tan^2 x/2\right)\sec^2 x/2\,dx}{2\tan x/2\,2}$

$\tan x/2=t \qquad \sec x/2\,\frac{1}{2}dx=dt$

$\frac{1}{2}\int\left(\frac{1+t^2}{t}\right)dt=\frac{1}{2}\left[\ell nt+\frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$

$=\frac{1}{2}\left[\left(0+\frac{1}{2}\right)-\left(\ell n\frac{1}{\sqrt{3}}+\frac{1}{6}\right)\right]=\left(\frac{1}{3}+\ell n\sqrt{3}\right)\frac{1}{2}$

$=\left(\frac{1}{6}+\frac{1}{2}\ell n\sqrt{3}\right)$

$2\left(\frac{1}{6}+\frac{1}{2}\ell n\sqrt{3}\right)+3\left(1-\frac{1}{\sqrt{3}}\right)$

$=\frac{1}{3}+\ell n\sqrt{3}+3-\sqrt{3}=\frac{10}{3}+\ell n\sqrt{3}-\sqrt{3}$

20. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{5}{7}$ (2) $\frac{2}{7}$

(3) $\frac{3}{7}$ (4) $\frac{5}{6}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{{}^5C_2+{}^6C_2}{{}^2C_2+{}^3C_2+{}^4C_2+{}^5C_2+{}^8C_2}=\frac{10+15}{1+3+6+10+15}$

$=\frac{25}{35}=\frac{5}{7}$

SECTION-B

21. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____.

Official Ans. by NTA (2997)

QuestPix Ans. (2997)

Sol. $\underset{6}{2}\underset{6}{+}\underset{6}{+}\underset{6}{+}\underset{6}{+}=1296$

$\underset{6}{3}\underset{6}{+}\underset{6}{+}\underset{6}{+}\underset{6}{+}=1296$

$\underset{6}{4}\,0\underset{6}{+}\underset{6}{+}\underset{6}{+}=216$

$\underset{6}{4}\,2\,0\,\underset{6}{+}\underset{6}{+}=36$

$\underset{6}{4}\,\underline{2}\,\underline{2}\underset{6}{+}\underset{6}{+}=36$

$\underset{6}{4}\,2\,3\,\underset{6}{+}\underset{6}{+}=36$

$\underset{6}{4}\,2\,4\,\underset{6}{+}\underset{6}{+}=36$

$\underset{6}{4}\,2\,7\,\underset{6}{+}\underset{6}{+}=36$

$\underset{6}{4}\,2\,9\,\underset{6}{\underline{0}}+=6$

$429\,2\,0=1$

$429\,2\,2=1$

$429\,2\,3=1$

$=2997$

22. Let $a_1, a_2, \ldots\ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}}+\frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}}+\ldots..\frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$$

is equal to _____.

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. $2a_7 = a_5$ (given)

$2(a_1+6d) = a_1+4d$

$a_1+8d=0 \qquad \ldots\ldots.(1)$

$a_1+10d=18 \qquad \ldots\ldots.(2)$

By (1) and (2) we get $a_1 = -72$, $d = 9$

$a_{18} = a_1+17d = -72+153 = 81$

$a_{10} = a_1+9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}+\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}+\ldots..\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

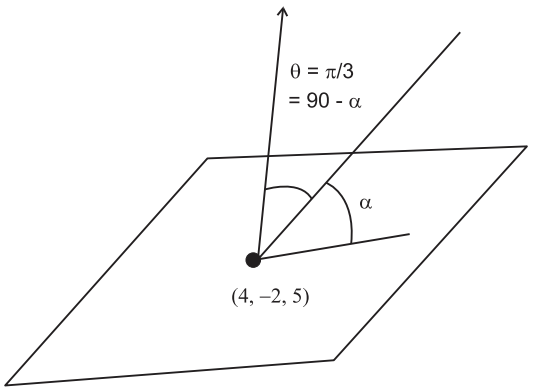
$$12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right)=\frac{12(9-3)}{9}=\frac{12\times 6}{6}=8$$

23. Let θ be the angle between the planes $P_1 = \vec{r}.(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2 = \vec{r}.(2\hat{i}-\hat{j}+\hat{k}) = 15$.

Let L be the line that meets P_2 at the point (4, –2, 5) and makes an angle θ with the normal of P_2. If α is the angle between L and P_2 then $(\tan^2\theta)(\cot^2\alpha)$ is equal to _____.

Official Ans. by NTA (9)

QuestPix Ans. (9)



$$\cos\theta = \frac{(\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+\hat{k})}{6} = \frac{2-1+2}{6} = \frac{1}{2}$$

$\theta = \pi/3 \qquad \alpha = \pi/6$

$(\tan^2\theta)(\cot^2\alpha)$

(3) (3) = 9

24. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}}+\frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in \mathbb{N}$.

Then α is equal to _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $T_{r+1} = {}^{30}C_r\left(x^{2/3}\right)^{30-r}\left(\frac{2}{x^3}\right)^r$

$= {}^{30}C_r.2^r.x^{\frac{60-11r}{3}}$

$\frac{60-11r}{3} < 0 \Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6$

$T_7 = {}^{30}C_6.2^6\,x^{-2}$

We have also observed $\beta = {}^{30}C_6\,(2)^6$ is a natural number.

$\therefore \alpha = 2$

25. Let $\vec{a}$ and $\vec{b}$ be two vector such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a}\times\vec{b}| = \sqrt{48}$. Then $(\vec{a}.\vec{b})^2$ is equal to _____.

Official Ans. by NTA (36)

QuestPix Ans. (36)

Sol. $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ $\quad |\vec{a} \times \vec{b}| = \sqrt{48}$

$|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2 \times |\vec{b}|^2$

$\Rightarrow (\vec{a}.\vec{b})^2 = 84 - 48 = 36$

26. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point R(1, –1, –3) on the line L. If α is the area of triangle PQR. then α^2 is equal to _____.

Official Ans. by NTA (180)

QuestPix Ans. (180)

Sol. Any point on $L((2\lambda+1),(-\lambda-1),(\lambda+3))$

$2(2\lambda+1)+(-\lambda-1)+3(\lambda+3)=16$

$6\lambda + 10 = 16 \Rightarrow \lambda = 1$

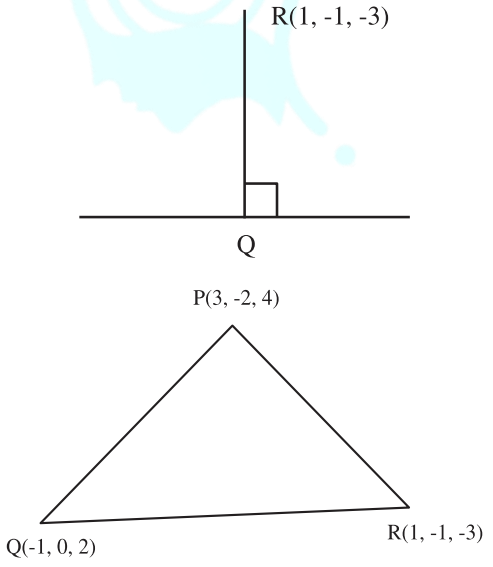
$\therefore$ P = (3, –2, 4)

DR of QR = $\langle 2\lambda, -\lambda, \lambda+6 \rangle$

DR of L = $\langle 2, -1, 1 \rangle$

$4\lambda + \lambda + \lambda + 6 = 0$ $\quad 6\lambda + 6 = 0 \Rightarrow \lambda = -1$

Q = (–1, 0, 2)



$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$ $\quad \overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$\alpha = \frac{1}{2} \times \sqrt{144 + 576} \Rightarrow \alpha^2 = \frac{720}{4} = 180$

27. The remainder on dividing 5^{99} by 11 is _____.

Official Ans. by NTA (9)

QuestPix Ans. (9)

Sol. $5^{99} = 5^4.5^{95}$

$= 625[5^5]^{19}$

$= 625[3125]^{19}$

$= 625[3124+1]^{19}$

$= 625[11k \times 19 + 1]$

$= 625 \times 11k \times 19 + 625$

$= 11\ k_1 + 616 + 9$

$= 11(k_2) + 9$

Remainder = 9

28. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol.

x_i	f_i	$d_i = x_i - 5$	$f_i d_i^2$	$f_i d_i$
2	3	-3	27	-9
3	6	-2	24	-12
4	16	-1	16	-16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_x^2 = \sigma_d^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$= \frac{150}{45+\alpha} - 0 = 3$

$\Rightarrow 150 = 135 + 3\alpha$

$\Rightarrow 3\alpha = 15 \Rightarrow \alpha = 5$

29. Let for $x \in R$

$$f(x)=\frac{x+|x|}{2} \text{ and } g(x)=\begin{cases} x, & x<0 \\ x^2 & x\geq 0 \end{cases}.$$

Then area bounded by the curve y = (fog)(x) and the lines y = 0, 2y – x = 15 is equal to _____.

Official Ans. by NTA (72)

QuestPix Ans. (72)

Sol. $$f(x)=\frac{x+|x|}{2}=\begin{cases} x & x\geq 0 \\ 0 & x<0 \end{cases}$$

$$g(x)=\begin{cases} x^2 & x\geq 0 \\ x & x<0 \end{cases}$$

$$fog(x)=f[g(x)]=\begin{cases} g(x) & g(x)\geq 0 \\ 0 & g(x)<0 \end{cases}$$

$$fog(x)=\begin{cases} x^2 & x\geq 0 \\ 0 & x<0 \end{cases}$$

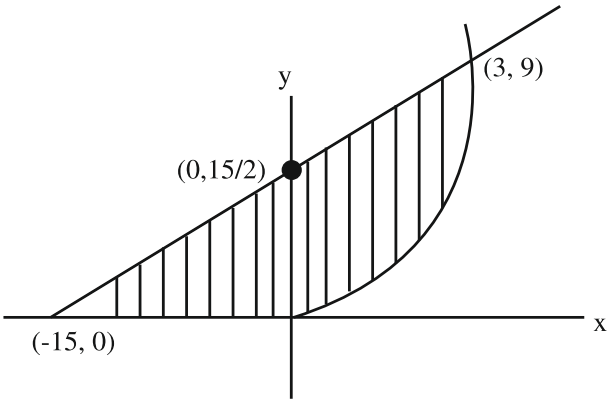
$2y - x = 15$

$$A=\int_0^3\left(\frac{x+15}{2}-x^2\right)dx+\frac{1}{2}\times\frac{15}{2}\times 15$$

$$\frac{x^2}{4}+\frac{15x}{2}-\frac{x^3}{3}\Big|_0^3+\frac{225}{4}$$

$$=\frac{9}{4}+\frac{45}{2}-9+\frac{225}{4}=\frac{99-36+225}{4}$$

$$=\frac{288}{4}=72$$



30. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to _____.

Official Ans. by NTA (710)

QuestPix Ans. (710)

Sol. 1000 – 2799

Divisible by 3

1002 + (n – 1) 3 = 2799

n = **600**

Divisible by 11

$$1-2799\rightarrow\left\lfloor\frac{2799}{11}\right\rfloor=[254]=254$$

$$1-999=\left\lfloor\frac{999}{11}\right\rfloor=90$$

1000 – 2799 = 254 – 90 = 164

Divisible by 33

$$1-2799\rightarrow\left\lfloor\frac{2799}{33}\right\rfloor=84$$

$$1-999\rightarrow\left\lfloor\frac{999}{33}\right\rfloor=30$$

$1000-2799\rightarrow 54$

$\therefore$ n(3) + n(11) – n(33)

600 + 164 – 54 = 710

(Held On Tuesday 31st January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet form parallel to antiparallel position relative to the field direction is _____.

(1) 4 J (2) 1 J

(3) 2 J (4) Zero

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $u = -MB\cos\theta$

$W = \Delta u$

$W = -MB\cos180^\circ (-mB\cos0^\circ)$

$W = 2\,MB = 2 \times 5 \times 0.4 = 4J$

Option 1

32. If a source of electromagnetic radiation having power 15 kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.

(Take Planck constant $h = 6 \times 10^{-34}$ Js)

(1) Micro waves

(2) Ultraviolet rays

(3) Gamma rays

(4) Radio waves

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Energy of one photon $= \dfrac{\text{Power}}{\text{Photon frequency}}$

$$E = h\nu = \frac{15\times10^{3}}{10^{16}}$$

$$\nu = \frac{15\times10^{-13}}{6\times10^{-34}} = 2.5 \times 10^{21}$$

So gamma Rays. Option 3

33. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency(ies) of

(A) 500 Hz

(B) 2 Hz

(C) 250 Hz

(D) 498 Hz

(E) 502 Hz

Choose the correct answer from the options given below:

(1) A only

(2) A, D and E only

(3) B only

(4) A and B only

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Carrier wave frequency

$$V_C = \frac{100\pi}{2\pi} = 500\,\text{Hz}$$

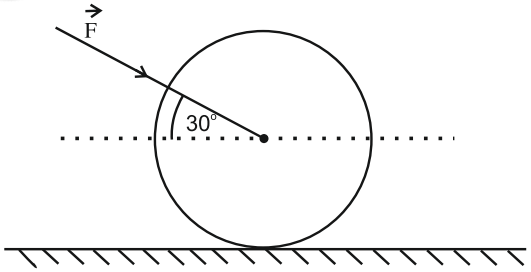
Modulating wave frequency

$$V_m = \frac{4\pi}{2\pi} = 2\,\text{Hz}$$

$\therefore \quad V_C - V_m, V_C, V_C + V_m$

$= 498$ Hz. 500 Hz, 502 Hz.

34. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200\,N$ at an angle of 30° with horizontal. The normal reaction on the roller is (Given g = 10 m s^{-2})



(1) $800\sqrt{2}\,N$

(2) $600\,N$

(3) $800\,N$

(4) $200\sqrt{3}\,N$

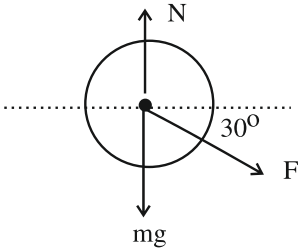
Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.

$N = mg + F\sin30^\circ$

$= 700 + 200 \times \dfrac{1}{2} = 800$ newton.



35. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}$u. The time of flight of the projectile is:

(1) $\frac{u}{2g}$ (2) $\frac{u}{g}$

(3) $\frac{2u}{g}$ (4) $\frac{\sqrt{3}u}{g}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $u\cos\theta = \frac{\sqrt{3}u}{2} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^o$

$T = \frac{2u\sin 30^o}{g} = \frac{u}{g}$

Option 2.

36. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air friction}

(1) Time taken by them to reach the earth's surface will be independent of the properties of their materials

(2) Insulating ball will reach the earth's surface earlier than the metal ball

(3) Both will reach the earth's surface simultaneously

(4) Metal ball will reach the earth's surface earlier than the insulating ball.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. When metal is passing through magnetic field, eddy current will produce and it will oppose the motion, so it will take more time.

37. A free neutron decays into a proton but a free proton does not decay into neutron. This is because

(1) neutron is an uncharged particle

(2) proton is a charged particle

(3) neutron is a composite particle made of a proton and an electron

(4) neutron has larger rest mass than proton

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. As neutron has more rest mass than proton it will require energy to decay proton into neutron.

Option 4.

38. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as:

(1) Both n_e and resistance decrease

(2) Both n_e and resistance increase

(3) n_e increases, resistance decreases

(4) n_e decreases, resistance increases

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. As temperature increases, more electron excite to conduction band and hence conductivity increases, therefore resistance decreases.

39. The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $A/2$, the kinetic energy of the block is

(1) 37.5 J (2) 9.75 J

(3) 18.75 J (4) 12.5 J

Official Ans. by NTA (3)

QuestPix Ans. (3)

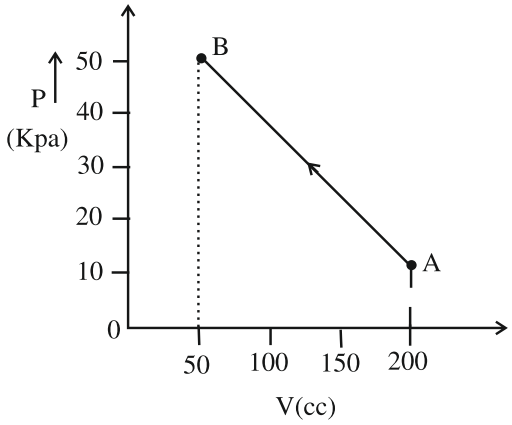
Sol. $u_{max} = \frac{1}{2} m\omega^2 A^2 = 25$ J

$KE \text{ at } \frac{A}{2} = \frac{1}{2} m v_1^2 = \frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{4}\right)$

$KE = \frac{1}{2} m\omega^2 \frac{3A^2}{4} = \frac{3}{4}\left(\frac{1}{2} m\omega^2 A^2\right)$

$KE = \frac{3}{4} \times 25 = 18.75J$

40. The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



(1) 6 J (2) Zero

(3) –4.5 J (4) 4.5 J

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. As $\Delta q = 0$

$\Delta u = -W$

$W = \int PdV$

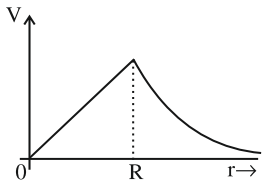
$\Delta u = -W = 30 \times 10^3 \times 150 \times 10^{-6}$

$= 4500 \times 10^{-3}$

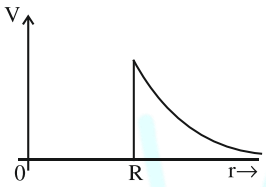
$= 4.5$ J

41. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the centre?

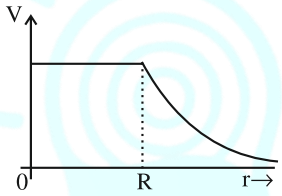
(1)



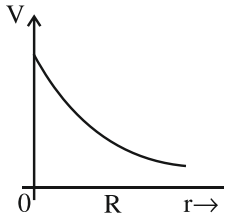
(2)



(3)



(4)



Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Conceptual

42. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The beam of electrons shows wave nature and exhibit interference and diffraction.

Reason R : Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements. Choose the **most appropriate** answer from the options given below :

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct but R is Not the correct explanation of A

(4) Both A and R are correct and R is the correct explanation of A

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Conceptual

43. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d. The conductor in now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be

(1) V_d

(2) $\frac{V_d}{2}$

(3) $\frac{V_d}{4}$

(4) $2V_d$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $V_d = \frac{eE}{m}\tau$ that is independent of area

44. At a certain depth "d" below surface of earth. value of acceleration due to gravity becomes four times that of its value at a height 3R above earth surface. Where R is Radius of earth (Take R = 6400 km). The depth d is equal to

(1) 5260 km

(2) 640 km

(3) 2560 km

(4) 4800 km

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\frac{GM}{R^2}\left[1-\frac{d}{R}\right]=\frac{4\times GM}{(4R)^2}$

$1-\frac{d}{R}=\frac{1}{4}\Rightarrow\frac{d}{R}=\frac{3}{4}\Rightarrow d\frac{3}{4}R$

d = 4800 km

45. If 1000 droplets of water of surface tension 0.07 N/m. having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is-

$\left(\text{Take}\,\pi=\frac{22}{7}\right)$

(1) 7.92×10^{-6} J

(2) 7.92×10^{-4} J

(3) 9.68×10^{-4} J

(4) 8.8×10^{-5} J

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. 1000 x $\frac{4\pi}{3}(1)^3=\frac{4\pi}{3}R^3$

R = 10 mm

T x 1000 x 4π $(10^{-3})^2$ – T x 4π $(10 \text{ x } 10^{-3})^2 = \Delta E$

$\Delta E = 4 \text{ x } \pi \text{ x } 7 \text{ x } 10^{-2}\,[1000 - 100] \text{ x } 10^{-6}$

$\Delta E = 7.92 \text{ x } 10^{-4}$ J

Option 2.

46. A rod with circular cross-section area 2 cm^2 and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi\times10^{-6}$ Wb. The relative permeability of the rod is

(Given : Permeability of vacuum $\mu_0=4\pi\times10^{-7}NA^{-2}$)

(1) 12.5

(2) $\frac{32}{5}$

(3) 125

(4) $\frac{5}{16}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\phi = \mu_r\mu_o \frac{N}{\ell} \text{ I x A}$

$\mu_r = 125$

Option 3.

47. The correct relation between $\gamma=\frac{C_p}{c_v}$ and temperature T is :

(1) $\gamma\propto\frac{1}{\sqrt{T}}$ (2) $\gamma\propto T^o$

(3) $\gamma\propto\frac{1}{T}$ (4) $\gamma\propto T$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. γ is independent of temperature

Option 2

48. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarised light is I_0 then intensity of transmitted light after passing through polaroid B will be :

(1) $\frac{I_0}{4}$ (2) $\frac{I_0}{2}$

(3) $\frac{I_0}{8}$ (4) Zero

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $I_A=\frac{I_o}{2}$

$IC=\frac{I_o}{2}\cos^2 45=\frac{I_o}{4}$

$I_B=I_C\cos^2 45=\frac{I_o}{8}$

Option 3.

49. If R, X_L. and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless:

(1) $R X_L X_C$ (2) $\frac{R}{\sqrt{X_L X_C}}$

(3) $\frac{R}{X_L X_C}$ (4) $R\frac{X_L}{X_C}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. All three have same dimension therefore $\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

Option 2

50. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. The total force exerted by the balls on the wall is

(1) $\frac{100mv}{t}$ (2) $\frac{200mv}{t}$

(3) 200 mvt (4) $\frac{mv}{100t}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $P_i = Nmv\,\hat{i}$ $\vec{P}_f = -Nmv\hat{i}$

N is Number of balls strikes with wall

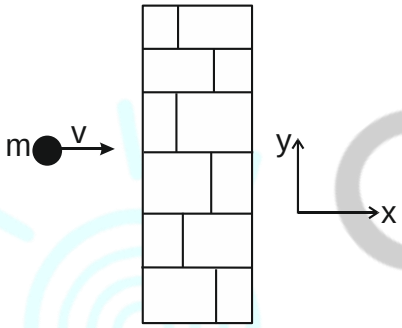
N = 100

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = -2Nmv\hat{i}$

$= -200\ Nmv\hat{i}$

$\vec{F}_{Total} = \frac{\Delta\vec{P}}{\Delta t} = -\frac{200mvt}{t}$

$|\vec{F}| = \frac{200mv}{t}$



SECTION-B

51. A thin rod having a length of 1 m and area of cross-section $3\times10^{-6}\,m^2$ is suspended vertically from one end. The rod is cooled from 210°C to 160°C. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are $2\times10^{11}\,N\,m^{-2}$ and $2\times10^{-5}K^{-1}$, respectively. The value of M is_____ kg. (Take g = 10 m s^{-2})

Official Ans. by NTA (60)

QuestPix Ans. (60)

Sol. If $\Delta\ell$ is decease in length of rod due to decease in temperature

$\Delta\ell = \ell\,\alpha\,\Delta T$

$\alpha = 2\times10^{-5}K^{-1}, \Delta T = (210-160)$

$= 50\ K$

$\Delta\ell = 1\times2\times10^{-5}\times50 = 10^{-3}\,m$

Young Modulus = $Y = \frac{F/A}{\Delta\ell/\ell}$ $A = 3\times10^{-6}\,m^2$

$2\times10^{11} = \frac{Mg/3\times10^{-6}}{10^{-3}/1}$

$Mg = 2\times10^{11}\times3\times10^{-9} = 6\times10^{-2}$

$M = 60\ kg$

Ans is 60.

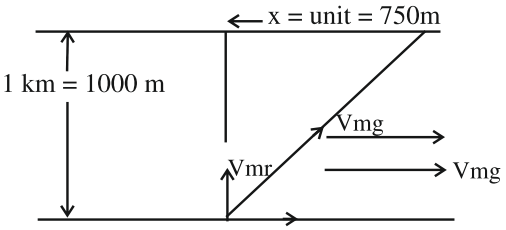
52. The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is _____km h^{-1}.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



time to cross the River width ω = 1000 m

is = $\frac{1\ km}{4\ km/h}$

Drift x = Vm/g × t

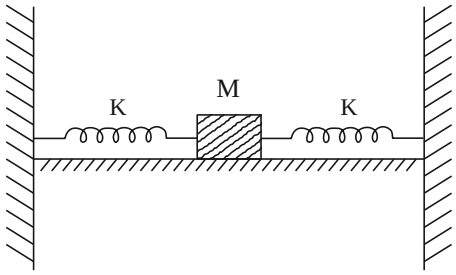
Where Vm/g is velocity of River w.r. to ground.

$x = Vm / g \times \frac{1}{4} = 750m = \frac{3}{4}km$

$Vm / g = 3km / hr$

Ans is 3 km/hr.

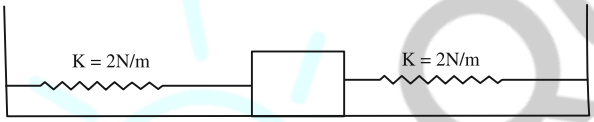
53. In the figure given below. a block of mass M = 490 g placed on a frictionless table is connected with two springs having same spring constant (K = 2 N m^{-1}). If the block is horizontally displaced through 'X'm then the number of complete oscillations it will make in 14π seconds will be_____



Official Ans. by NTA (20)

QuestPix Ans. (20)

Sol.



Keff = K + K as both springs are in use in parallel

= 2k

= 2×2 = 4 N/m m = 490 gm

= 0.49 kg

$T = 2\pi\sqrt{\frac{m}{Keff}} = 2\pi\sqrt{\frac{0.49kg}{4}}$

$= 2\pi\sqrt{\frac{49}{400}} = 2\pi\frac{7}{20} = \frac{7\pi}{10}$

No. of oscillation in the 14π is

$N = \frac{time}{T} = \frac{14\pi}{7\pi/10} = 20$

Ans in 20.

54. In a medium the speed of light wave decreases to 0.2 times to its speed in free space The ratio of relative permittivity to the refractive index of the medium is x : 1. The value of x is _____.

(Given speed of light in free space = 3 × 10^8 m s^{-1} and for the given medium $\mu_r = 1$)

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $V = \frac{C}{\mu} \Rightarrow \mu = \frac{C}{V} = \frac{C}{0.2C}$

$\boxed{\mu = 5}$

$\mu = \sqrt{\epsilon_r \mu_r}$

$\Rightarrow \epsilon_r = \frac{\mu^2}{\mu_r}$

$\therefore \frac{\epsilon_r}{\mu} = \frac{\mu}{\mu_r} = 5$

55. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7 × 10^{-3} J. The speed of the centre of mass of the sphere is _____ cm s^{-1}.

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 7\times10^{-3}$

$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 = 7\times10^{-3}$

$\frac{1}{2}MV^2\left[1+\frac{2}{5}\right] = 7\times10^{-3}$

$\frac{1}{2}(1)(V^2)\left(\frac{7}{5}\right) = 7\times10^{-3}$

$V^2 = 10^{-2}$

$V = 10^{-1} = 0.1m/s = 10cm/s$

Ans : 10

56. An inductor of 0.5 mH, a capacitor of 20 μF and resistance of 20 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is -

Official Ans. by NTA (242)

QuestPix Ans. (242)

Sol. $X_L = X_C$

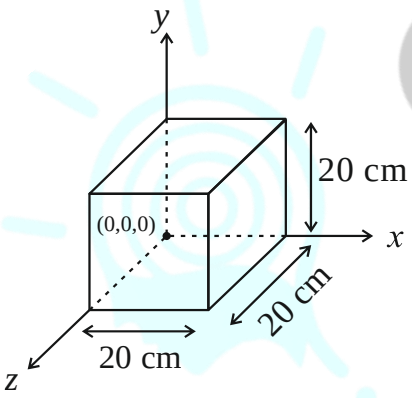
So, $Z = R = 20\ \Omega$

$i_{rms} = \frac{220}{20} = 11$

$i_{max} = 11\sqrt{2} = \sqrt{242}$

Ans : 242

57. Expression for an electric field is given by $\vec{E} = 4000\,x^2\hat{i}\,\frac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is _____ V cm.



Official Ans. by NTA (640)

QuestPix Ans. (640)

Sol. Flux = $\vec{E}\cdot\vec{A}$

$= 4000\ (0{\cdot}2)^2\ \frac{V}{m} \cdot (0{\cdot}2)^2 m^2$

$= 4000 \times 16 \times 10^{-4}\ Vm$

$= 640\ Vcm$

Ans. 640

58. A lift of mass M = 500 kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2}. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be ____kJ.

Official Ans. by NTA (7)

QuestPix Ans. (7)

Sol. $v^2 = u^2 + 2as$

$= 2^2 + 2\,(2)(6)$

$= 4 + 24 = 28$

$KE = \frac{1}{2}mv^2$

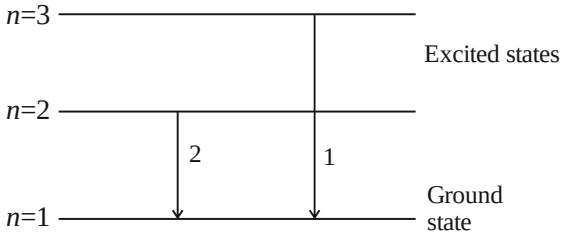
$= \frac{1}{2}(500)28$

$= 7000\ J$

$= 7\ kJ$

Ans. 7

59. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _____.



Official Ans. by NTA (27)

QuestPix Ans. (27)

Sol. $\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda_1} = Rz^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8}{9}Rz^2$ …….. (1)

$\frac{1}{\lambda_2} = Rz^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}Rz^2$ …….. (2)

$$1/2 \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{8}{9} \times \frac{4}{3} = \frac{32}{27}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{27}{32}$$

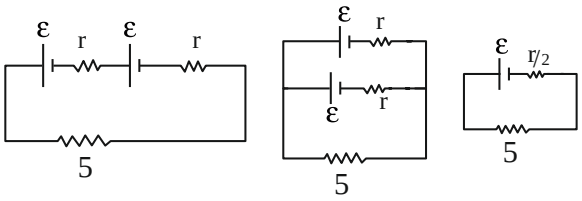
Ans. 27

60. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω. The internal resistance of each cell will be _____ Ω.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. Parallel



$$i = \frac{2\varepsilon}{5+2r} \quad \ldots. (1) \qquad i = \frac{\varepsilon}{\frac{r}{2}+5} \quad \ldots. (2)$$

Equating (1) and (2)

$$\frac{2\varepsilon}{5+2r} = \frac{\varepsilon}{\frac{r}{2}+5} \Rightarrow r+10 = 5+2r$$

$r = 5$ **Ans. 5**

(Held On Tuesday 31st January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

61. Nd^{2+} = _____

(1) $4f^2 6s^2$ (2) $4f^4$

(3) $4f^3$ (4) $4f^4 6s^2$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol Nd(60) = [Xe] $4f^4\ 5d^0\ 6s^2$

Nd^{2+} = [Xe] $4f^4\ 5d^0\ 5s^0$

62. The methods NOT involved in concentration of ore are

(A) Liquation

(B) Leaching

(C) Electrolysis

(D) Hydraulic washing

(E) Froth floatation

Choose the correct answer from the options given below :

(1) B, D and C only

(2) C, D and E only

(3) A and C only

(4) B, D and E only

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Methods involved in concentration of one are

(i) Hydraulic Washing

(ii) Froth Flotation

(iii) Magnetic Separation

(iv) Leaching

63. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow[\substack{(ii)\Delta \\ (iii)NaCN \\ (iv)H_3O^+}]{(i)\text{dil.NaOH}} \underset{(C_5H_8O_3)}{\text{Product B}}$$

The correct statement for product B is. It is

(1) optically active and adds one mole of bromine

(2) racemic mixture and is neutral

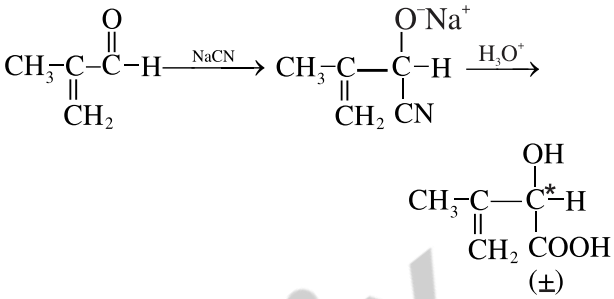
(3) racemic mixture and gives a gas with saturated $NaHCO_3$ solution

(4) optically active alcohol and is neutrall

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $CH_3-CH_2-CHO + HCHO \xrightarrow[\Delta]{OH^-}$



Carboxylic acid will give CO_2 gas, with $NaHCO_3$ solution

64. The correct order of basicity of oxides of vanadium is

(1) $V_2O_3 > V_2O_4 > V_2O_5$

(2) $V_2O_3 > V_2O_5 > V_2O_4$

(3) $V_2O_5 > V_2O_4 > V_2O_3$

(4) $V_2O_4 > V_2O_3 > V_2O_5$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. With increase in % of oxygen acidic nature of oxide of an element increase and basic nature decreases

65. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are

(1) X = Cu_2I_2 Y=$Na_2S_4O_5$

(2) X=Cu_2I_2 Y=$Na_2S_4O_6$

(3) X=CuI_2 Y=$Na_2S_4O_3$

(4) X=CuI_2 Y=$Na_2S_4O_6$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.

$$Cu^{2+} + 2KI \longrightarrow \underset{\text{Unstable}}{CuI_2}\downarrow + 2K^+$$

I^- is strong R.A it reduces Cu^{2+} to Cu^+

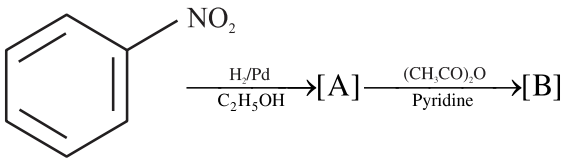
$$2CuI_2 \longrightarrow \underset{(\text{White})\ 'X'}{Cu_2I_2}\downarrow + I_2$$

$$KI + I_2 \longrightarrow K^+I_3^- (\text{Brown solution})$$

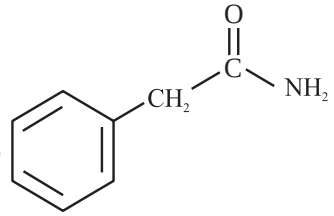
$$I_3^- \rightleftharpoons I_2 + I^-$$

$$KI_3 + Na_2S_2O_3 \rightarrow KI + \underset{(Y)}{Na_2S_4O_6}$$

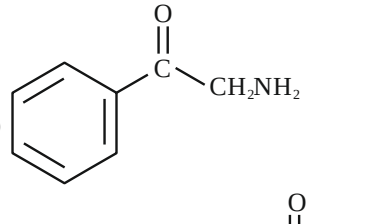
66.



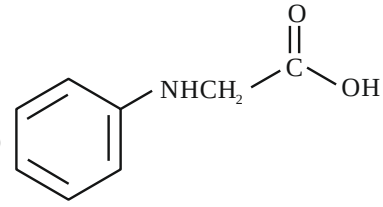
(1)



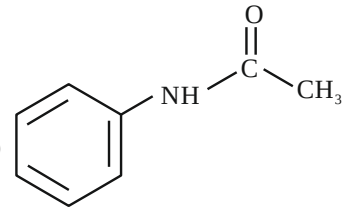
(2)



(3)

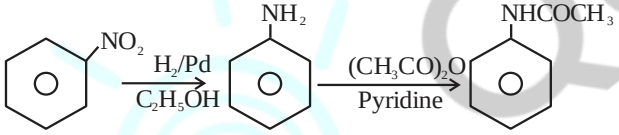


(4)



Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol.



67. Cobalt chloride when dissolved in water forms pink colored complex X which has octahedral geometry. This solution on treating with cone HCl forms deep blue complex, Y which has a Z geometry. X, Y and Z, respectively, are

(1) X=$[Co(H_2O)_6]^{2+}$, Y=$[CoCl_4]^{2-}$, Z=Tetrahedral
(2) X=$[Co(H_2O_6)]^{2+}$, Y=$[CoCl_6]^{3-}$, Z=Octahedral
(3) X=$[Co(H_2O)_6]^{3+}$, Y=$[CoCl_6]^{3-}$, Z=Octahedral
(4)X=$[Co(H_2O)_4Cl_2]^{+}$, Y=$[CoCl_4]^{2-}$, Z=Tetrahedral

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol.

$$CoCl_2 + 6H_2O \longrightarrow [Co(H_2O)_6]Cl_2$$

Pink(X)
octahedral
$\downarrow$ +HCl(conc.)
$[CoCl_4]^{2-}$
(Y)Blue solution
(Z)Tetrahedral

68. Identify X, Y and Z in the following reaction. (Equation not balanced)

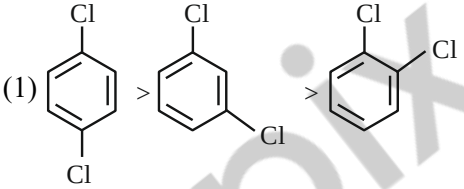
$$\dot{Cl}O + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

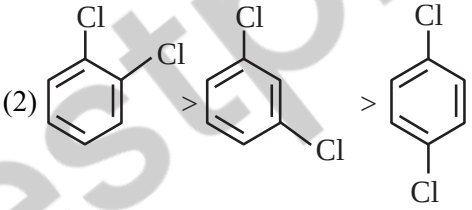
(1) X=$ClONO_2$, Y=HOCl, Z = NO_2
(2) X=$ClNO_2$, Y=HCl, Z=HNO_3
(3) X=$ClONO_2$, Y=HOCl, Z=HNO_3
(4) X=$ClNO_3$, Y=Cl_2, Z=NO_2

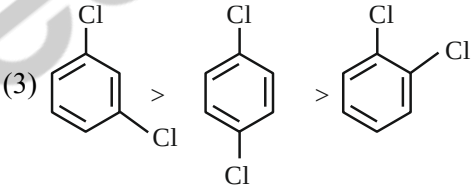
Official Ans. by NTA (3)
QuestPix Ans. (3)

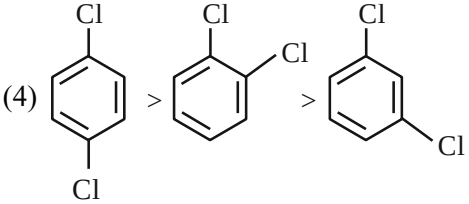
Sol. $\dot{Cl}O + NO_2 \longrightarrow \underset{(X)}{ClONO_2} \xrightarrow{+H_2O} \underset{(Y)}{HOCl} + \underset{(Z)}{HNO_3}$

69. The correct order of melting point of dichlorobenzenes is

(1) 

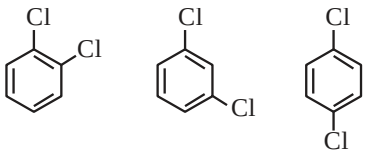
(2) 

(3) 

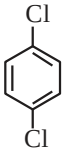
(4) 

Official Ans. by NTA (4)
QuestPix Ans. (4)

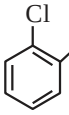
Sol.



b.p / K	453	446	448
m.p / K	256	249	323

 $\Rightarrow$ More Symmetrical & better Crystal fitting

M.P a Packing efficiency

 $\Rightarrow \mu_{max}$

70. A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acid is

(1) $NH_2-CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2CH_2COOH$

(2) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(3) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle NH_2}{\underset{|}{CH}}-CH_2COOH$

(4) $CH_3-\underset{\displaystyle NH_2}{\underset{|}{\overset{\displaystyle CH_3}{\overset{|}{C}}}}-CH_2-CH_2COOH$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Only in option (2) α-Amino acid is given all the other options are not α-Amino acids.

71. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from n=4 to n = 2 of He^+ spectrum

(1) n = 2 to n = 1

(2) n = 1 to n = 3

(3) n = 1 to n = 2

(4) n = 3 to n = 4

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. He^+ ion :

$$\frac{1}{\lambda(H)} = R(1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda(He^+)} = R(2)^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

Given $\lambda(H) = \lambda(He^+)$

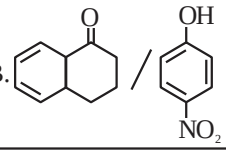
$$R(1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] = R(4)\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$$\frac{1}{n_1^2}-\frac{1}{n_2^2} = \frac{1}{1^2}-\frac{1}{2^2}$$

On comparing n_1=1 & n_2=2

Ans. 1

72. Match items of column I and II

Column I (Mixture of compounds)	Column II (Separation Technique)
A. H_2O/CH_2Cl_2	i. Crystallization
B. 	ii. Differential solvent extraction
C. Kerosene/Naphthalene	iii. Column chromatography
D. $C_6H_{12}O_6$/NaCl	iv. Fractional Distillation

Correct match is :

(1) A-(iii), B-(iv), C-(ii), D-(i)

(2) A-(i), B-(iii), C-(ii), D-(iv)

(3) A-(ii), B-(iii), C-(iv), D-(i)

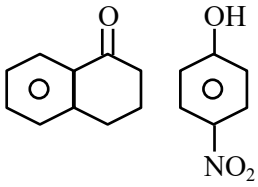
(4) A-(ii), B-(iv), C-(i), D-(iii)

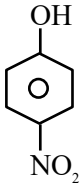
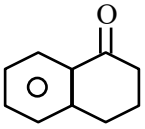
Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. **A.** $H_2O/CH_2Cl_2 \rightarrow$ ii, $CH_2Cl_2 > H_2O$ (density) so they can be separated by differential solvent extraction.

B.

 iii. column chromatography

Due to H-bonding in  it can be separated from  by column chromatography.

C. Kerosene / Naphthalene → iv. Fractional distillation.

Due to different B.P. of kerosene and Naphthalene it can be separated by fractional distillation.

D. $C_6H_{12}O_6$/NaCl → i. Crystallization.

NaCl (ionic compound) can be crystallized.

73. The correct increasing order of the ionic radii is

(1) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

(2) $K^+ < S^{2-} < Ca^{2+} < Cl^-$

(3) $S^{2-} < Cl^- < Ca^{2+} < K^+$

(4) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. In isoelectronic species size $\propto \frac{1}{Z}$

$Ca^{2+} < K^+ < Cl^- < S^{2-}$: Size

Z : 20 19 17 18

74. H_2O_2 acts as a reducing agent in

(1) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

(2) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

(3) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$

(4) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

$$\overset{+1}{NaOCl} + \overset{-1}{H_2O_2} \longrightarrow 2NaCl + H_2O + \overset{0}{O_2}$$

(–1)

75. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar?

(1) Aspartame

(2) Sucralose

(3) Alitame

(4) Saccharin

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Sweetness value order wrt cane sugar

Alitame > Sucralose > Saccharin > Aspartame

77. Match List I with List II

List I	List II
A. XeF_4	I. See – saw
B. SF_4	II. Square planar
C. NH_4^+	III. Bent T – shaped
D. BrF_3	IV. Tetrahedral

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-II, B-I, C-III, D-IV

(3) A-IV, B-I, C-II, D-III

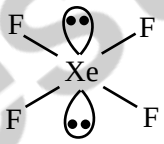
(4) A-II, B-I, C-IV, D-III

Official Ans. by NTA (4)

QuestPix Ans. (4)

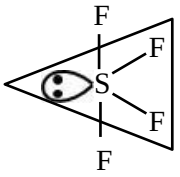
Sol.

(A)



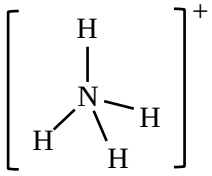
Square planer

(B) SF_4



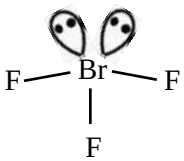
See-Saw

(C) NH_4^+



Tetrahedral

(D) BrF_3



Bent T- Shaped

77. Choose the correct set of reagents for the following conversion

trans (Ph-CH=CH–CH_3) $\rightarrow$ cis (Ph-CH=CH–CH_3)

(1) Br_2, alc KOH, $NaNH_2$, Na(Liq NH_3)

(2) Br_2, alc KOH, $NaNH_2$, H_2 Lindlar Catalyst

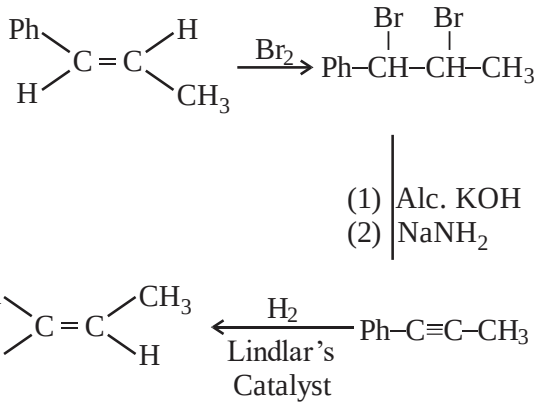
(3) Br_2, aq KOH, $NaNH_2$, H_2 Lindlar Catalyst

(4) Br_2, aq KOH, $NaNH_2$, Na(Liq NH_3)

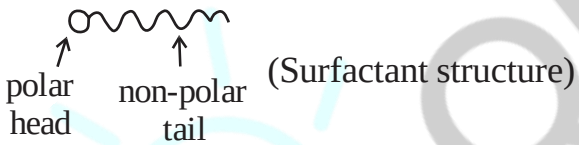
Official Ans. by NTA (2)

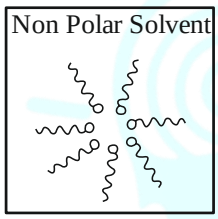
QuestPix Ans. (2)

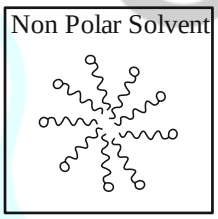
Sol.

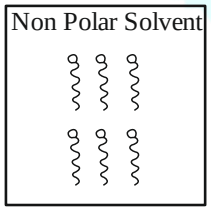


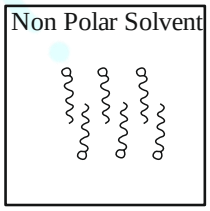
78. Adding surfactants in non polar solvent, the micelles structure will look like



(a) 

(b) 

(c) 

(d) 

(1) b

(2) c

(3) a

(4) d

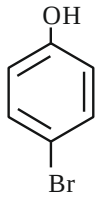
Official Ans. by NTA (3)

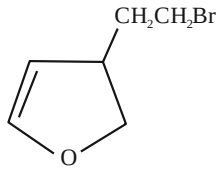
QuestPix Ans. (3)

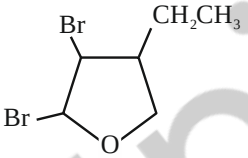
Sol. Non-Polar tail towards non-polar solvent

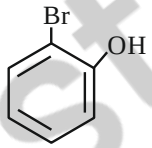
Ans. 3

79. An organic compound 'A' with empirical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is

(1) 

(2) 

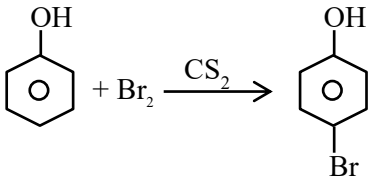
(3) 

(4) 

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Aromatic compounds burns with sooty flame



(Major)

80. Which one of the following statements is correct for electrolysis of brine solution?

(1) Cl_2 is formed at cathode

(2) O_2 is formed at cathode

(3) H_2 is formed at anode

(4) OH^- is formed at cathode

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Electrolysis of brine solution

$$NaCl(aq.) \longrightarrow Na^+_{(aq)} + Cl^+_{(aq)}$$

At anode : $2Cl^+_{(aq.)} \longrightarrow \underset{\text{(Major)}}{Cl_2(g)} + 2e^-$

$$2H_2O_{(\ell)} \longrightarrow \underset{\text{Minor}}{O_{2(g)}} + 4H^+_{(aq)} + 4e^-$$

At Cathode : $2H_2O_{(\ell)} + 2e^- \longrightarrow H_{2(g)} \uparrow + 2OH^-_{(aq)}$

$$2Na^+ + 2OH^- \longrightarrow 2NaOH$$

SECTION-B

81. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is _____ (Nearest integer)

$\text{Given}: \frac{2.303RT}{F} = 0.06V$

$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd(s) \quad E^\circ = 0.83V$

$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq)$

$E^\circ = 0.65V$

Official Ans. by NTA (6)

QuestPix Ans. (6)

Sol. $\Delta G^\circ = -RT\ell n K$

$-nFE^\circ_{cell} = -RT \times 2.303(\log_{10} K)$

$\frac{E^\circ_{Cell}}{0.06} \times n = \log K \quad \ldots(1)$

$Pd^{+2}(aq.) + 2e^- \rightleftharpoons Pd(s)\ ,\ E^\circ_{cat,red^n} = 0.83$

$Pd(s) + 4Cl^-(aq.) \rightleftharpoons PdCl_4^{2-}, (aq) + 2e^-, E^\circ_{Anode,Oxid^n} = 0.65$

Net Reaction $\rightarrow Pd^{2+}(aq.) + 4Cl^-(aq.) \rightleftharpoons PdCl_4^{2-}(aq.)$

$E^\circ_{cell} = E^\circ_{cat,red^n} - E^\circ_{Anode,Oxid^n}$

$E^\circ_{cell} = 0.83 - 0.65$

$E^\circ_{cell} = 0.18 \quad \ldots(2)$

Also n = 2 $\quad \ldots(3)$

Using equation (1), (2) & (3)

logK = 6

82. $A \rightarrow B$

The rate constants of the above reaction at 200 K and 300K are 0.03 min^{-1} and 0.05 min^{-1} respectively. The activation energy for the reaction is _____ J (Nearest integer)

(Given : In 10 = 2.3

$R = 8.3\ J\ K^{-1}\ mol^{-1}$

log5 = 0.70

log3=0.48

log2 = 0.30

Official Ans. by NTA (2520)

QuestPix Ans. (2520)

Sol.

$\log \frac{K_{300}}{K_{200}} = \frac{E_a}{2.3 \times 8.314}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log \frac{0.05}{0.03} = \frac{Ea}{2.305 \times 8.314} \times \left[\frac{1}{200} - \frac{1}{300}\right]$

$E_a = 2519.88\ J \Rightarrow E_a = 2520\ J$

83. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to Cl^- (aq) is (-) _____ kJ mol^{-1} (Nearest integer)

Given : $\Delta_{dis}H^\circ_{Cl_{2(g)}} = 240 kJmol^{-1}$.

$\Delta_{eg}H^\circ_{Cl_{(g)}} = -350 kJmol^{-1}$,

$\Delta_{hyd}H^\circ_{Cl^-_{(g)}} = -380 kJmol^{-1}$

Official Ans. by NTA (610)

QuestPix Ans. (610)

Sol. $\frac{1}{2}Cl_{2(g)} \rightarrow Cl_{(g)} \rightarrow Cl^-_{(g)} \rightarrow Cl^-_{(aq.)}$

$\Delta H^\circ = \frac{1}{2} \times 240 + (-350) + (-380)$

= – 610 ans.

84. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2.

The % of carbon in the organic compound is _____ (Nearest integer)

Official Ans. by NTA (44)

QuestPix Ans. (44)

Sol. weight of C in 0.792 gm CO_2

$= \frac{12}{44} \times 0.792 = 0.216$

% of C in compound = $\frac{0.216}{0.492} \times 100$

= 43.90%

Ans : 44

85. At 27°C, a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is ____ g mol^{-1} (Nearest integer)

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (62250)

QuestPix Ans. (62250)

Sol. : $\pi = CRT$

$\frac{400Pa}{10^5} = \frac{\frac{2.5g}{M_o}}{250/1000L} \times 0.83 \frac{L-bar}{K.mol} \times 300K$

$M_o = 62250$

86. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L (Nearest integer)
(Given : Molar mass of Zn is 65.4g mol^{-1} and Molar volume of H_2 at STP = 22.7L)
Official Ans. by NTA (4)
QuestPix Ans. (4)

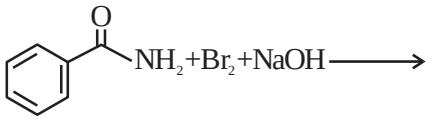
Sol. $Zn + 2HCl \rightarrow ZnCl_2 + H_2\uparrow$

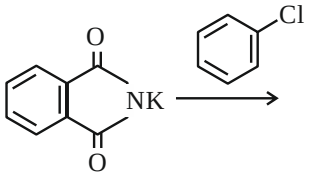
Moles of Zn used = $\frac{11.5}{65.4}$ = Moles of H_2 evolved

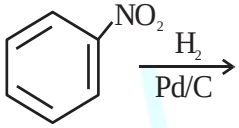
Volume of $H_2 = \frac{11.5}{65.4} \times 22.7L = 3.99L$

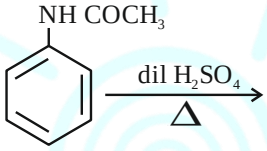
Ans : 4

87. How many of the transformation given below would result in aromatic amines?

(1) 

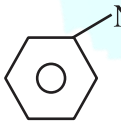
(2) 

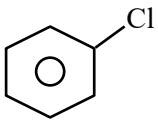
(3) 

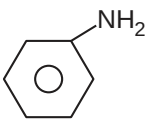
(4) 

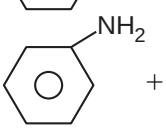
Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol. Product in the given reactions are as follow-

1. 

2. No reactions will be observed as in Gabriel phthalimide synthesis  is poor substrate for SN^2

3. 

4.  + CH_3COOH

Aromatic amines will he formed in 1, 3 & 4
Ans : 3

88. For reaction : $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_P = 2 \times 10^{12}$ at 27°C and 1 atm pressure. The K_c for the same reaction is _____ $\times 10^{13}$. (Nearest integer)
(Given R = 0.082 L atm K^{-1} mol^{-1})
Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

$K_P = 2 \times 10^{12}$ at 300 K

$K_P = K_C \times (RT)^{\Delta n_g}$

$2 \times 10^{12} = K_c \times (0.082 \times 300)^{-1/2}$

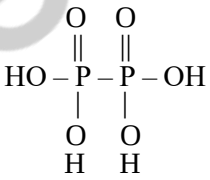
$K_C = 9.92 \times 10^{12}$

$K_C = 0.992 \times 10^{13}$

Ans. 1

89. The oxidation sate of phosphorus in hypophosphoric acid is + _____ .
Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. $H_4P_2O_6$



O.S. of P is +4

90. The total pressure of a mixture of non-reacting gases X (0.6 g) and Y (0.45 g) in a vessel is 740 mm of Hg. The partial pressure of the gas X is _____ mm of Hg. (Nearest Integer)

(Given : molar mass X = 20 and Y = 45 g mol^{-1})

Official Ans. by NTA (555)

QuestPix Ans. (555)

Sol. $P_X = \chi_X P_T$

$= \frac{\frac{0.6}{20}}{\frac{0.6}{20} + \frac{0.45}{45}} \times 740$

P_X = 555 mm Hg