(Held On Monday 30th January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A=\begin{pmatrix} m & n \\ p & q \end{pmatrix}, d=|A|\neq 0$ $|A-d(\text{Adj}\,A)|=0$. Then

(1) $(1+d)^2=(m+q)^2$

(2) $1+d^2=(m+q)^2$

(3) $(1+d)^2=m^2+q^2$

(4) $1+d^2=m^2+q^2$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $A=\begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $|A-d(\text{adj}\,A)|=0$

$\Rightarrow$ $|A-d(\text{adj}\,A)|=\left|\begin{bmatrix} m & n \\ p & q \end{bmatrix}-d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}\right|$

$=\begin{vmatrix} m-qd & n(1+d) \\ p(1+d) & q-md \end{vmatrix}=0$

$\Rightarrow$ $(m-qd)(q-md)-np(1+d)^2=0$

$\Rightarrow$ $mq-m^2d-q^2d+mqd^2-np(1+d)^2=0$

$\Rightarrow$ $(mq-np)+d^2(mq-np)-d(m^2+q^2+2np)=0$

$\Rightarrow$ $d+d^3-d((m+q)^2-2d)=0$

$\Rightarrow$ $1+d^2=(m+q)^2-2d$

$\Rightarrow$ $(1+d)^2=(m+q)^2$

$\therefore$ Option (1) is correct.

2. The line l_1 passes through the point (2,6,2) and is perpendicular to the plane $2x+y-2z=10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is :

(1) 7 (2) $\frac{19}{3}$

(3) $\frac{19}{3}$ (4) 9

Official Ans. by NTA (4)

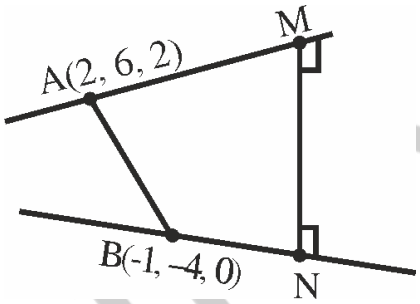
QuestPix Ans. (4)

Sol. Line ℓ, is given by

$L_1:\frac{x-2}{2}=\frac{y-6}{1}=\frac{z-2}{-2}$

Given,

$L_2:\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$



Shortest distance $=\left|\frac{\overrightarrow{AB}\cdot\overrightarrow{MN}}{MN}\right|$

$\overrightarrow{AB}=3\hat{i}+10\hat{j}+2\hat{k}$

$\overrightarrow{MN}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}=-4\hat{i}-8\hat{j}-8\hat{k}$

$MN=\sqrt{16+64+64}=12$

$\therefore$ Shortest distance $=\left|\frac{-12-80-16}{12}\right|=9$

$\therefore$ Option (4) is correct.

3. If an unbiased die, marked with –2, –1, 0, 1, 2, 3 on its faces, is through five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{881}{2592}$

(2) $\frac{521}{2592}$

(3) $\frac{440}{2592}$

(4) $\frac{27}{288}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Either all outcomes are positive or any two are negative.

Now, p = P (positive) $= \frac{3}{6} = \frac{1}{2}$

q = p (negative) $= \frac{2}{6} = \frac{1}{3}$

Required probability

$$= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$$

$$= \frac{521}{2592}$$

$\therefore$ Option (2) is correct.

4. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k+1)x + (2k-1)y = 7$

$(2k+1)x + (k+5)y = 10$ has :

(1) infinitely many solutions

(2) unique solution satisfying x – y = 1

(3) no solution

(4) unique solution satisfying x + y = 1

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$

$\Rightarrow$ 1(10) – 1(7) + k(–1) – 0

$\Rightarrow$ k = 3

For k = 3, 2nd system is

4x + 5y = 7 .....(1)

and 7x + 8y = 10 .....(2)

Clearly, they have a unique solution

(2) – (1) $\Rightarrow$ 3x + 3y = 3

$\Rightarrow$ x + y = 1

5. If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 4 (2) $4 - 2\sqrt{3}$

(3) 2 (4) $5 - \frac{3}{2}\sqrt{3}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Option (1)

$\tan 15° = 2 - \sqrt{3}$

$\frac{1}{\tan 75°} = \cot 75° = 2 - \sqrt{3}$

$\frac{1}{\tan 105°} = \cot(105°) = -\cot 75° = \sqrt{3} - 2$

$\tan 195° = \tan 15° = 2 - \sqrt{3}$

$\therefore$ $2(2 - \sqrt{3}) = 2a \Rightarrow a = 2 - \sqrt{3}$

$\Rightarrow$ $a + \frac{1}{a} = 4$

6. Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) \cdot f(y), \forall x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6875 (2) 6575

(3) 6825 (4) 6528

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Option (3)

$5f(x+y) = f(x) \cdot f(y)$

$5f(0) = f(0)^2 \Rightarrow f(0) = 5$

$5f(x+1) = f(x) \cdot f(1)$

$\Rightarrow$ $\frac{f(x+1)}{f(x)} = \frac{f(1)}{5}$

$\Rightarrow$ $\frac{f(1)}{f(0)} \cdot \frac{f(2)}{f(1)} \cdot \frac{f(3)}{f(2)} = \left(\frac{f(1)}{5}\right)^3$

$\Rightarrow$ $\frac{320}{5} = \frac{(f(1))^3}{5^3} \Rightarrow f(1) = 20$

$\therefore$ $5f(x+1) = 20 \cdot f(x) \Rightarrow f(x+1) = 4f(x)$

$$\sum_{n=0}^{5} f(n) = 5 + 5.4 + 5.4^2 + 5.4^3 + 5.4^4 + 5.4^5$$

$$= \frac{5\left\lfloor 4^6 - 1 \right\rfloor}{3} = 6825$$

7. If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \ldots\ldots + a_{25}$ is equal to :

(1) $\frac{51}{144}$ (2) $\frac{49}{138}$

(3) $\frac{50}{141}$ (4) $\frac{52}{147}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Option (3)

If $a_n = \frac{-2}{4n^2-16n+15}$ then $a_1 + a_2 + \ldots\ldots a_{25}$

$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2-16n+15}$$

$$= \sum \frac{-2}{4n^2-6n-10n+15}$$

$$= \sum \frac{-2}{2n(2n-3)-5(2n-3)}$$

$$= \sum \frac{-2}{(2n-3)(2n-5)}$$

$$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$$

$$= \frac{1}{47} - \frac{1}{(-3)}$$

$$= \frac{50}{141}$$

8. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :

(1) a = b (2) ab = 1

(3) a = 3b (4) ab = 3

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Option (2)

Coefficient Of x^{15} in $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^3\right)^{15-r} \left(\frac{1}{bx^{1/3}}\right)^r$$

$$45 - 3r - \frac{r}{3} = 15$$

$$30 = \frac{10r}{3}$$

$$r = 9$$

Coefficient of $x^{15} = {}^{15}C_9\, a^6\, b^{-9}$

Coefficient of x^{-15} in $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^{1/3}\right)^{15-r} \left(-\frac{1}{bx^3}\right)^r$$

$$5 - \frac{r}{3} - 3r = -15$$

$$\frac{10r}{3} = 20$$

$$r = 6$$

Coefficient $= {}^{15}C_6\, a^9 \times b^{-6}$

$$\Rightarrow \frac{a^9}{b^6} = \frac{a^6}{b^9}$$

$$\Rightarrow a^3b^3 = 1 \Rightarrow ab = 1$$

9. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b}.\vec{c} = 12$, then $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right|$ is equal to :

(1) 15

(2) 9

(3) 12

(4) 6

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\hat{n} \perp \vec{c}$ $\quad \vec{a} = \alpha\vec{b} - \vec{n}$

$\vec{b}\cdot\vec{c} = 12$

$\vec{a}\cdot\vec{c} = \alpha(\vec{b}\cdot\vec{c}) - \vec{n}\cdot\vec{c}$

$\vec{a}\cdot\vec{c} = \alpha(\vec{b}\cdot\vec{c})$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right| = \left|(\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}\right|$

$= \left|(\vec{c}\cdot\vec{b})\vec{a} - \alpha(\vec{b}\cdot\vec{c})\vec{b}\right|$

$= \left|(\vec{c}\cdot\vec{b})\right| \left|\vec{a} - \alpha\vec{b}\right|$

$= 12 \times \left(\left|\vec{n}\right|\right)$

$= 12 \times 1$

$= 12$

10. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(1) 2 (2) 3

(3) 4 (4) 0

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Normal of line is parallel to line $x + 90y + 2 = 0$

$m_N = -\frac{1}{90}$

$-\left(\frac{dx}{dy}\right)_{(x_1y_1)} = -\frac{1}{90} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1y_1)} = 90$

Now,

$\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210 = 90$

$\Rightarrow x = 1, 2, \frac{-2}{3}, \frac{-1}{3}$

(4) normals

11. Let $y = x + 2, 4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x-h)^2 + (y-k)^2 = r^2$. Then $h + k$ is equal to :

(1) 5

(2) $5\left(1+\sqrt{2}\right)$

(3) 6

(4) $5\sqrt{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $L_1 : y = x + 2,\ L_2 : 4y = 3x + 6,\ L_3 : 3y = 4x + 1$

Bisector of lines L_2 & L_3

$\frac{4x - 3y + 1}{5} = \pm\left(\frac{3x - 4y + 6}{5}\right)$

(+) $4x - 3y + 1 = 3x - 4y + 6$

$x + y = 5$

Centre lies on Bisector of $4x - 3y + 1 = 0$ &

(0) $3x - 4y + 6 = 0$

$\Rightarrow h + k = 5$

12. Let the solution curve $y = y(x)$ of the differential equation $\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x$

$\exp\frac{x^3 - \tan^{-1}x^3}{\sqrt{(1+x)^6}}$ pass through the origin. Then $y(1)$ is equal to :

(1) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

(3) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{dy}{dx} + \left(\frac{-3x^5 \tan^{-1}x^3}{(1+x^6)^{3/2}}\right) y = 2e^{\left\{\frac{x - \tan x}{\sqrt{1+x^6}}\right\}}$

$\text{I.F.} = e^{\int \frac{-3x^5 \tan^{-1}x^3}{(1+x^6)^{3/2}} dx}$

$= e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}}$

Solution of differential equation

$y \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = \int 2xe^{\left(\frac{x^3 - \tan^{-1}x^3}{\sqrt{1+x^6}}\right)} \cdot e^{\left(\frac{\tan^{-1}(x^3) - x^3}{\sqrt{1+x^6}}\right)} dx$

$= \int 2x\,dx + c$

$y \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = x^2 + c$

Also it passes through origin

$c = 0$

$y(1) \cdot e^{\frac{\tan^{-1}(1) - 1}{\sqrt{2}}} = 1$

$y(1) \cdot e^{\frac{\frac{\pi}{4} - 1}{\sqrt{2}}} = 1$

$y(1) \cdot e^{\frac{\pi - 4}{4\sqrt{2}}} = 1$

$y(1) = \frac{1}{e^{\frac{\pi-4}{4\sqrt{2}}}} = e^{\frac{4-\pi}{4\sqrt{2}}}$

13. Let a unit vector OP make angle α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0, \frac{\pi}{2}\right)$ OP is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true ?

(1) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

(2) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(3) $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ and $\gamma \in \left(0, \frac{\pi}{2}\right)$

(4) $\alpha \in \left(0, \frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2}, \pi\right)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Equation of plane :-

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow [x-1]-4[y-2]+3[z-3]=0$$

$$\Rightarrow x-4y+3z=2$$

D.R's of normal of plane <1, –4, 3>

D.C's of $\left\langle \pm\frac{1}{\sqrt{26}}, \mp\frac{4}{\sqrt{26}}, \pm\frac{3}{\sqrt{26}} \right\rangle$

$$\cos\beta = \frac{4}{\sqrt{26}}$$

$$\cos\alpha = \frac{-1}{\sqrt{26}} \qquad \frac{\pi}{2} < \alpha < \pi$$

$$\cos\gamma = \frac{-3}{\sqrt{26}} \qquad \frac{\pi}{2} < \gamma < \pi$$

Ans. : (1)

14. If [t denotes the greatest integer ≤ 1, then the value of $\frac{3(e-1)^2}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is :

(1) $e^9 - e$ (2) $e^8 - e$

(3) $e^7 - 1$ (4) $e^8 - 1$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\int_1^2 x^2 e^{[x^3]+1}dx$

$x^3 = t$

$3x^2\,dx = dt$

$$= \frac{e}{3}\int_1^8 e^{[t]}dt$$

$$= \frac{e}{3}\left\{\int_1^2 e\,dt + \int_2^3 e^2\,dt + ... + \int_7^8 e^7\,dt\right\}$$

$$= \frac{e}{3}(e + e^2 + ... + e^7)$$

$$= \frac{e^2}{3}(1 + e + ... + e^6) = \frac{e^2}{3}\frac{(e^7-1)}{(e-1)}$$

$$\frac{3(e-1)}{e}\int_1^2 x^2 \times e^{[x]+[x^3]}dx = \frac{3}{e}(e-1)\times\frac{e^2}{3}\frac{(e^7-1)}{(e-1)}$$

$$= e(e^7-1)$$

$$= e^8 - e$$

Ans. : (2)

15. If P(h,k) be point on the parabola $x = 4y^2$, which is nearest to the point Q(0,33), then the distance of P from the directrix of the parabola $y^2 = 4(x+y)$ is equal to :

(1) 2 (2) 4

(3) 8 (4) 6

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Equation of normal

$y = -tx + 2at + at^3$

$$y = -tx + \frac{2}{16}t + \frac{1}{16}t^3$$

It passes through (0, 33)

$$33 = \frac{t}{8} + \frac{t^3}{16}$$

$t^3 + 2t - 528 = 0$

$(t - 8)(t^2 + 8t + 66) = 0$

$t = 8$

$$P(at^2, 2at) = \left(\frac{1}{16}\times 64,\ 2\times\frac{1}{16}\times 8\right) = (4, 1)$$

Parabola :

$y^2 = 4(x + y)$

$\Rightarrow y^2 - 4y = 4x$

$\Rightarrow (y-2)^2 = 4(x+1)$

Equation of directix :-

$x + 1 = -1$

$x = -2$

Distance of point = 6

Ans. : (4)

16. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of x–axis and y–axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y–axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to:

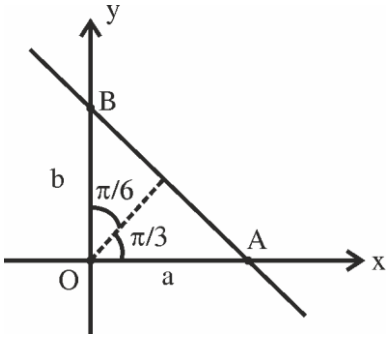
(1) $\frac{392}{3}$ (2) 196

(3) $\frac{196}{3}$ (4) 98

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



Equation of straight line : $\frac{x}{a} + \frac{y}{b} = 1$

Or $x\cos\frac{\pi}{3} + y\sin\frac{\pi}{3} = p$

$\frac{x}{2} + \frac{y\sqrt{3}}{2} = p$

$\frac{x}{3p} + \frac{y}{2p} = 1$

Comparing both : $a = 2p, b = \frac{2p}{\sqrt{3}}$

Now area of $\Delta OAB = \frac{1}{2}\cdot ab = \frac{98}{3}\cdot\sqrt{3}$

$\frac{1}{2}\cdot 2p\cdot\frac{2p}{\sqrt{3}} = \frac{98}{3}\cdot\sqrt{3}$

$p^2 = 49$

$a^2 - b^2 = 4p^2 - \frac{4p^2}{3} = \frac{2}{3}4p^2$

$= \frac{8}{3}\cdot 49 = \frac{392}{3}$

17. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$ is:

(1) ${}^{501}C_{302}$ (2) ${}^{500}C_{301}$

(3) ${}^{500}C_{300}$ (4) ${}^{501}C_{200}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$

$= (1+x)^{500}\cdot\left\{\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}\right\}$

$= (1+x)^{500}\frac{\left((1+x)^{501} - x^{501}\right)}{(1+x)^{501}}\cdot(1+x)$

$= (1+x)^{501} - x^{501}$

Coefficient of x^{301} in $(1+x)^{501} - x^{501}$ is given by

${}^{501}C_{301} = {}^{501}C_{200}$

18. Among the statements:

(S1) $((p\vee q)\Rightarrow r)\Leftrightarrow(p\Rightarrow r)$

(S2) $((p\vee q)\Rightarrow r)\Leftrightarrow((p\Rightarrow r)\vee(q\Rightarrow r))$

(1) Only (S1) is a tautology

(2) Neither (S1) nor (S2) is a tautology

(3) Only (S2) is a tautology

(4) Both (S1) and (S2) are tautologies

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $S_1 \equiv ((p\vee q)\Rightarrow r)\Leftrightarrow(p\Rightarrow r)$

p	q	r	$p\vee q$	$(p\vee q)\Rightarrow r$	$p\Rightarrow r$	$((p\vee q)\Rightarrow r)\Leftrightarrow(p\Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

$S_2 \equiv (p \vee q) \Rightarrow r \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p	q	r	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r \vee (q \Rightarrow r))$	S2
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F
F	T	F	F	F	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

S2 $\rightarrow$ not a tautology

19. The minimum number of elements that must be added to the relation $R = \{(a,b),(b,c)\}$ on the set $\{a,b,c\}$ so that it becomes symmetric and transitive is:

(1) 4 (2) 7

(3) 5 (4) 3

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. *For Symmetric* $(a,b),(b,c) \in R$

$\Rightarrow (b,a),(c,b) \in R$

For Transitive $(a,b),(b,c) \in R$

$\Rightarrow (a,c) \in R$

Now

1. *Symmetric*

$\therefore (a,c) \in R \Rightarrow (c,a) \in R$

2. *Transitive*

$\therefore (a,b),(b,a) \in R$

$\Rightarrow (a,a) \in R \ \& \ (b,c),(c,b) \in R$

$\Rightarrow (b,b) \ \& \ (c,c) \in R$

$\therefore$ *Elements to be added*

$$\{(b,a),(c,b),(a,c),(c,a),(a,a),(b,b),(c,c)\}$$

Number of elements to be added $= 7$

20. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to:

(1) 3

(2) 5

(3) 6

(4) 4

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$

$\Rightarrow \frac{\ln \cos x - \ln \sin x}{\ln \cos x} + 4\frac{\ln \sin x - \ln \cos x}{\ln \sin x} = 1$

$\Rightarrow (\ln \sin x)^2 - 4(\ln \sin x)(\ln \cos x) + 4(\ln \cos x)^2 = 1$

$\Rightarrow \ln \sin x = 2 \ln \cos x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\therefore \alpha + \beta = 4$

Correct option (4)

SECTION-B

21. Let $S = \{1,2,3,4,5,6\}$. Then the number of one–one functions $f : S \rightarrow P(S)$, where $P(S)$ denote the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is _____.

Official Ans. by NTA (3240)

QuestPix Ans. (3240)

Sol. Let S={1,2,3,4,5,6}, then the number of one-one functions, f:S→P(S), where P(S) denotes the power set of S, such that f(n)<f(m) where n<m is

$n(S) = 6$

$$P(S) = \{\phi, \{1\}, \ldots \{6\}, \{1,2\}, \ldots, \{5,6\}, \ldots, \{1,2,3,4,5,6\}\}$$

– 64 *elements*

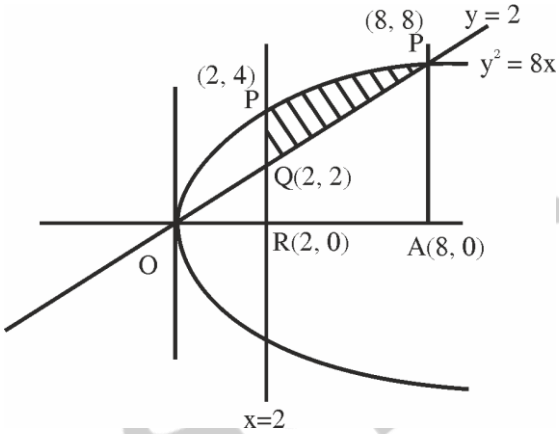
case – 1
f (6) = S i.e. 1 option,
f (5) = any 5 element subset A of S i.e. 6 options,
f (4) = any 4 element subset B of A i.e. 5 options,
f (3) = any 3 element subset C of B i.e. 4 options,
f (2) = any 2 element subset D of C i.e. 3 options,
f (1) = any 1 element subset E of D or empty subset i.e. 3 options,
Total functions = 1080
Case – 2
f (6) = any 5 element subset A of S i.e. 6 options,
f (5) = any 4 element subset B of A i.e. 5 options,
f' (4) = any 3 element subset C of B i.e. 4 options,
f (3) = any 2 element subset D of C i.e. 3 options,
f' (2) = any 1 element subset E of D i.e. 2 options,
f (1) = empty subset i.e. 1 option
Total functions = 720
Case – 3
f(6)= S
f(5) = any 4 element subset A of' S i.e. 15 options,
f(4) = any 3 element subset B of A i.e. 4 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 4
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 3 element subset B of A i.e. 10 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 5
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 2 element subset C of B i.e. 6 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
Case – 6
f(6) = S
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 3 element subset C of B i.e. 4 options,
f(2) = any 1 element subset D of C i.e. 3 options,
f(1) = empty subset i.e. 1 option
Total functions = 360
$\therefore$ Number of such functions = 3240

22. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to _____.

Official Ans. by NTA (22)

QuestPix Ans. (22)

Sol.



$y = x$

& $y^2 = 8x$

Solving it

$x^2 = 8x$

$\therefore$ $x = 0, 8$

$\therefore$ $y = 0, 8$

$x = 2$ will intersect occur at

$y^2 = 16 \Rightarrow \quad y = \pm 4$

$\therefore$ Area of shaded

$$= \int_2^8 \left(\sqrt{8x} - x\right) dx = \int_2^8 \left(2\sqrt{2}\sqrt{x} - x\right) dx$$

$$= \left[2\sqrt{2} \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^8$$

$$= \left(\frac{4\sqrt{2}}{3} \cdot 2^{9/2} - 32\right) - \left(\frac{4\sqrt{2}}{3} \cdot 2^{9/2} - 2\right)$$

$$= \frac{128}{3} - 32 - \frac{16}{3} + 2 = \frac{112 - 90}{3} = \frac{22}{3} = A$$

$\therefore 3A = 22$

23. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}\left(3\hat{i} - 5\hat{j} + k\right) = 7$ and $P_2 : \vec{r} \cdot \left(\lambda\hat{i} + \hat{j} - 3k\right) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $\left(38\lambda_1, 10\lambda_2, 2\right)$ to the plane P_1 is _____.

Official Ans. by NTA (315)

QuestPix Ans. (315)

Sol. $P_1 = \vec{r} \cdot \left(3\hat{i} - 5\hat{j} + k\right) = 7$

$P_2 = \vec{r} \cdot \left(\lambda\hat{i} + \hat{j} - 3k\right) = 9$

$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$

$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$

$\therefore \cos\theta = \frac{1}{5}\cdot$

$\cos\theta = \frac{\vec{r}_1 \cdot \vec{r}_2}{|\vec{r}_1||\vec{r}_2|}$

$= \frac{(3i - 5j + K)(\lambda i + j - 3K)}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}}$

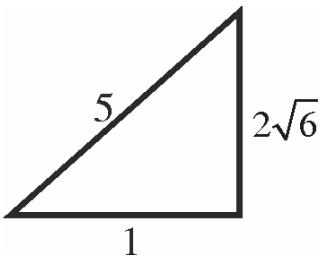
$\frac{1}{5} = \left|\frac{3\lambda - 8}{\sqrt{35} \cdot \sqrt{\lambda^2 + 10}}\right|$

Square $\Rightarrow \frac{1}{25} = \frac{9\lambda^2 + 64 - 48\lambda}{35\left(\lambda^2 + 10\right)}$

$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$

$\Rightarrow x = 5, \frac{25}{19}$



Perpendicular distance of point

$\left(38\lambda_1, 10\lambda_2, 2\right) \equiv (50, 50, 2)$ from plane P_1

$= \frac{|3 \times 50 - 5 \times 50 + 2 - 7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$

Square $= \frac{105 \times 105}{35} = 315$

24. Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}\cdot$ Then $\frac{12}{\pi}$ $\arg(z_1)$ is equal to _____.

Official Ans. by NTA (9)

QuestPix Ans. (9)

Sol. $z = 1 + i$

$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$

$z_1 = \frac{1 + i(1 - i)}{(1 - i)(1 - 1 - i) + \frac{1}{1 + i}}$

$= \frac{1 + i - i^2}{(1 - i)(-i) + \frac{1 - i}{2}}$

$= \frac{2 + i}{\frac{-3i - 1}{2}} = \frac{4 + 2i}{-3i - 1}$

$= \frac{-(4 + 2i)(3i - 1)}{(3i)^2 - (1)^2}$

$\text{Arg}(z_1) = \frac{3\pi}{4}$

$\therefore \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

25. $\lim\limits_{x\to 0}\dfrac{48}{x^4}\int\limits_0^x \dfrac{t^3}{t^6+1}dt$ is equal to _____.

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $48\lim\limits_{x\to 0}\dfrac{\int\limits_0^x \dfrac{t^3}{t^6+1}dt}{x^4}\ \left(\dfrac{0}{0}\right)$

Applying L' Hospitals Rule

$48\lim\limits_{x\to 0}\dfrac{x^3}{x^6+1}\times\dfrac{1}{4x^3}$

$= 12$

26. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted a and b are respectively mean and variance of remaining 6 observation, then $a+3b-5$ is equal to _____.

Official Ans. by NTA (37)

QuestPix Ans. (37)

Sol. $\dfrac{x_1+x_2+....+x_7}{7}=8$

$\dfrac{x_1+x_2+x_3....+x_6+14}{7}=8$

$\Rightarrow x_1+x_2+....+x_6=42$

$\therefore \dfrac{x_1+x_2....+x_6}{6}=\dfrac{42}{6}=7=a$

$\dfrac{\Sigma x_i^2}{7}-8^2=16$

$\Sigma xi^2=560$

$\Rightarrow x_1^2+x_2^2+...+x_6^2=364$

$b=\dfrac{x_1^2+x_2^2+....+x_6^2}{6}-7^2$

$=\dfrac{364}{6}-49$

$b=\dfrac{70}{6}$

$a+3b-5=7+3\times\dfrac{70}{6}-5$

$= 37$

27. If the equation of the plane passing through the point $(1,1,2)$ and perpendicular to the line $x-3y+2z-1=0\ 4x-y+z$ is $Ax+By+Cz=1$, then $140(C-B+A)$ is equal to _____.

Official Ans. by NTA (15)

QuestPix Ans. (15)

Sol. $x-3y+2z-1=0$

$4x-y+z=0$

$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & k\\ 1 & -3 & 2\\ 4 & -1 & 1\end{vmatrix}$

$=-\hat{i}+7\hat{j}+11k$

Dr^s of normal to the plane is –1, 7, 11

Equation of plane :

$-1(x-1)+7(y-1)+11(z-2)=0$

$-x+7y+11z=28$

$\dfrac{-1}{28}x+\dfrac{7y}{28}+\dfrac{11z}{28}=1$

$Ax+By+Cz=1$

$140(C-B+A)=140\left(\dfrac{11}{28}-\dfrac{7}{28}-\dfrac{1}{28}\right)$

$=140\times\dfrac{3}{28}=15$

28. Let $\sum\limits_{n=0}^{\infty}\dfrac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}=ae+\dfrac{b}{e}+c$,

where $a,b,c\in\mathbb{Z}$ and $e=\sum\limits_{n=0}^{\infty}\dfrac{1}{n!}$ Then a^2-b+c is equal to _____.

Official Ans. by NTA (26)

QuestPix Ans. (26)

Sol. $\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}$

$=\sum_{n=0}^{\infty}\frac{1}{(n-3)!}+\sum_{n=0}^{\infty}\frac{3}{(n-2)!}$

$+\sum_{n=0}^{\infty}\frac{1}{(n-1)!}+\sum_{n=0}^{\infty}\frac{1}{(2n-1)!}-\sum_{n=0}^{\infty}\frac{1}{(2n)!}$

$=e+3e+e+\frac{1}{2}\left(e-\frac{1}{e}\right)-\frac{1}{2}\left(e+\frac{1}{e}\right)$

$=5e-\frac{1}{e}$

$\therefore a^2-b+c=26$

29. Number of 4–digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to _____

Official Ans. by NTA (21)

QuestPix Ans. (21)

Sol. For number to be divisible by 15, last digit should be 5 and sum of digits must be divisible by 3.

Possible combinations are

1	2	1	5

Numbers = 3

2	2	3	5

Numbers = 3

3	3	1	5

Numbers = 3

1	1	5	5

Numbers = 3

2	3	5	5

Numbers = 6

3	5	5	5

Numbers = 3

Total Numbers = 21

30. Let $f^1(x)=\frac{3x+2}{2x+3}, x\in R-\left\{\frac{-3}{2}\right\}$

For $n\geq 2$, define $f^n(x)=f^1 0 f^{n-1}(x)$.

If $f^5(x)=\frac{ax+b}{bx+a}, gcd(a,b)=1$, then $a+b$ is equal to _____.

Official Ans. by NTA (3125)

QuestPix Ans. (3125)

Sol. $f^1(x)=\frac{3x+2}{2x+3}$

$\Rightarrow f^2(x)=\frac{13x+12}{12x+13}$

$\Rightarrow f^3(x)=\frac{63x+62}{62x+63}$

$\therefore f^5(x)=\frac{1563x+1562}{1562x+1563}$

$a+b=3125$

(Held On Monday 30th January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. The charge flowing in a conductor changes with time as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. Where α, β and γ are constants. Minimum value of current is :

(1) $\alpha - \frac{3\beta^2}{\gamma}$ (2) $\alpha - \frac{\gamma^2}{3\beta}$

(3) $\beta - \frac{\alpha^2}{3\gamma}$ (4) $\alpha - \frac{\beta^2}{3\gamma}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $Q = (\alpha t - \beta t^2 + \gamma t^3)$

$i = \frac{dQ}{dt} = (\alpha - 2\beta t + 3\gamma t^2)$

$\frac{di}{dt} = (3\gamma t - 2\beta) = 0$

$\Rightarrow t = \frac{\beta}{3\gamma}$

$i = (\alpha - 2\beta t + 3\gamma t^2) = \left(\alpha - \frac{\beta^2}{3\gamma}\right)$

32. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation PT^2 = constant. The volume expansion coefficient of the gas will be:

(1) $3T^2$ (2) $\frac{3}{T^2}$

(3) $\frac{3}{T^3}$ (4) $\frac{3}{T}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. PT^2 = constant, Using $PV = nRT$

$P = \frac{nRT}{V}$

$PT^2 = \frac{nRT}{V} \times T^2 = \text{constant}$

$\Rightarrow T^3 = KV$

So, $\frac{d}{dT}(KV) = 3T^2$

$\Rightarrow \frac{KdV}{dT} = 3T^2$

$\Rightarrow dV = \frac{3T^2}{K} dT$

$dV = V\gamma\, dT$

$\Rightarrow \gamma V = \frac{3T^2}{K}$ $\Rightarrow \gamma = \frac{3T^2}{KV} = \frac{3T^2}{T^3} = \frac{3}{T}$

33. A person has been using spectacles of power-1.0 diopter for distant vision and a separate reading glass of power 2.0 diopters. What is the least distance of distinct vision for this person:

(1) 10 cm (2) 40 cm

(3) 30 cm (4) 50 cm

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$P = 2D = 2m^{-1}$

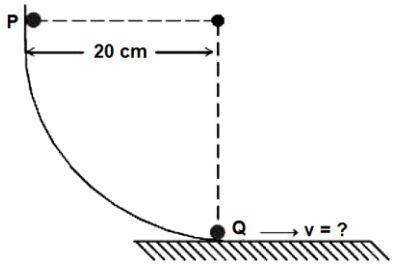
$\Rightarrow \frac{1}{f} = \frac{2}{100} cm^{-1}$

$\frac{1}{V} - \left(-\frac{1}{25}\right) = \frac{2}{100}$

$\Rightarrow \frac{1}{V} = \frac{1}{50} - \frac{1}{25}$

$\Rightarrow V = -50$ cm

34. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be : ($g = 10\ m/s^2$)



(1) 0 (2) 0.25 m/s

(3) 2 m/s (4) 4 m/s

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. The velocities will be interchanged after collision.

Speed of P just before collision $= \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 0.2} = 2$m/s

35. Choose the correct relationship between Poisson ratio (σ). bulk modulus (K) and modulus of rigidity (η) of a given solid object:

(1) $\sigma = \frac{3K-2\eta}{6K+2\eta}$ (2) $\sigma = \frac{6K+2\eta}{3K-2\eta}$

(3) $\sigma = \frac{3K+2\eta}{6K+2\eta}$ (4) $\sigma = \frac{6K-2\eta}{3K-2\eta}$

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol. $Y = 3\eta(1+\sigma)$

$Y = 3K(1-\sigma)$

$\Rightarrow 2\eta(1+\sigma) = 3K(1-2\sigma)$

$\Rightarrow \sigma = \left(\frac{3K-2\eta}{6K+2\eta}\right)$

36. The magnetic moments associated with two closely wound circular coils A and B of radius r_A = 10 cm and r_B = 20 cm respectively are equal if: (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

(1) $2N_AI_A = N_BI_B$ (2) $N_A = 2N_B$

(3) $N_AI_A = 4N_BI_B$ (4) $4N_AI_A = N_BI_B$

Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol. $M = NIA$

$M_A = M_B$

$\therefore$ $N_A I_A A_A = N_B I_B A_B$

$\therefore$ $N_A I_A \pi(0.1)^2 = N_B I_B \pi(0.2)^2$

$\therefore$ $N_AI_A = 4N_BI_B$

37. A small object at rest, absorbs a light pulse of power 20 mW and duration 300 ns. Assuming speed of light as 3×10^8 m/s. the momentum of the object becomes equal to :

(1) 0.5×10^{-17} kg m/s (2) 2×10^{-17} kg m/s

(3) 3×10^{-17} kg m/s (4) 1×10^{-17} kg m/s

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. $\text{Momentum} = \frac{\text{Energy}}{C}$

$= \frac{\text{Power}\times\text{time}}{C}$

$= \frac{(20\times10^{-3}\text{w})(300\times10^{-9}\text{s})}{3\times10^8\text{ m/s}}$

$= 2\times10^{-17}\text{ kg}-\text{m/s}$

38. Speed of an electron in Bohr's 7th orbit for Hydrogen atom is 3.6 × 10^6 m/s. The corresponding speed of the electron in 3rd orbit, in m/s is :

(1) $\left(1.8\times10^6\right)$ (2) $\left(7.5\times10^6\right)$

(3) $\left(3.6\times10^6\right)$ (4) $\left(8.4\times10^6\right)$

Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. $V_n \propto \frac{Z}{n}$

$Z = 1, \therefore V_n \propto \frac{1}{n}$

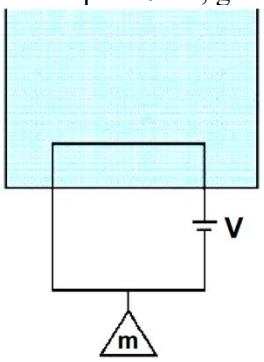
$\therefore \frac{V_3}{V_7} = \frac{7}{3}$

$\therefore V_3 = \frac{7}{3}V_7$

$= \frac{7}{3}\times3.6\times10^6\text{ m/s}$

$= 8.4\times10^6\text{ m/s}$

39. A massless square loop, of wire of resistance 10 Ω. supporting a mass of I g. hangs vertically with one of its sides in a uniform magnetic field of 10^3 G, directed outwards in the shaded region. A dc voltage V is applied to the loop. For what value of V. the magnetic force will exactly balance the weight of the supporting mass of 1g?
(If sides of the loop = 10 cm, g = 10 ms^{-2})



(1) $\frac{1}{10}$V (2) 100 V

(3) 1 V (4) 10 V

Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. $F_m = mg$

$\therefore$ $ILB = mg$

$\therefore$ $\left(\frac{V}{R}\right)LB = mg$

$\therefore$ $V = \frac{mgR}{LB}$

$= \frac{(1\times10^{-3}\text{ kg})(10\text{ m/s}^2)(10\Omega)}{(0.1\text{m})(10^3\times10^{-4}\text{ T})} = 10\text{ V}$

40. Two isolated metallic solid spheres of radii R and 2R are charged such that both have same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is :

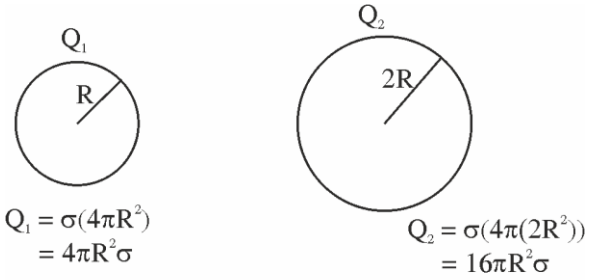
(1) $\frac{9}{4}$ (2) $\frac{4}{3}$

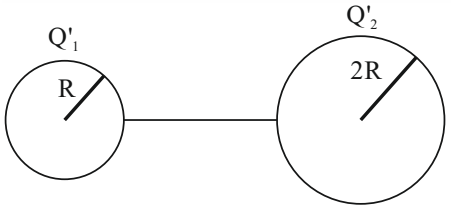
(3) $\frac{5}{3}$ (4) $\frac{5}{6}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.





$$\frac{Q_1'}{4\pi\varepsilon_0 R} = \frac{Q_2'}{4\pi\varepsilon_0 (2R)}$$

$\therefore \quad Q_2' = 2Q_1'$

$$Q_1' + Q_2' = Q_1 + Q_2$$

$\therefore \quad \frac{Q_2'}{2} + Q_2' = 20\pi R^2\sigma$

$$\frac{3}{2}Q_2' = 20\pi R^2\sigma$$

$\therefore \quad \frac{Q_2'}{4\pi(2R)^2} = \frac{2}{3}\cdot\frac{20\pi R^2\sigma}{16\pi R^2}$

$\therefore \quad \frac{\sigma'}{\sigma} = \frac{5}{6}$

41. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.

B. Internal energy of the gas will increase.

C. Internal energy of the gas will not change.

D. The gas will do positive work.

E. The gas will do negative work.

Choose the **correct** answer from the options given below :

(1) A and E only (2) B and D only

(3) C and E only (4) C and D only

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $dQ = dU + dW \Rightarrow dU = nC_V\,dT$

$dU = 0$ (for isothermal)

$\therefore$ U = constant

Also $dQ > 0$ (supplied)

Hence $dW > 0$

42. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are :

(1) $Nm^3C^{-1}; Nm^2C^{-1}$ (2) $Nm^2C^{-1}; Nm^3C^{-1}$

(3) $Nm^3C; Nm^2C$ (4) $Nm^2C; Nm^3C$

Official Ans. by NTA (2)

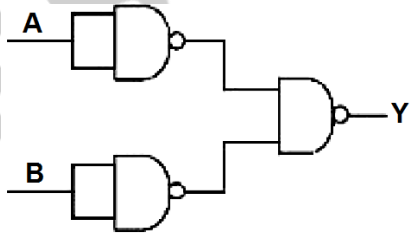
QuestPix Ans. (2)

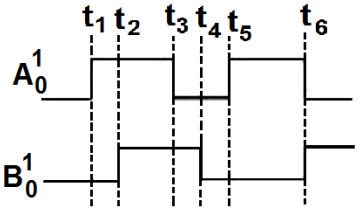
Sol. $\vec{E} = \frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}$

$$\left[\frac{A}{x^2}\right] = NC^{-1} \Rightarrow [A] = Nm^2C^{-1}$$

$$\left[\frac{B}{y^3}\right] = NC^{-1} \Rightarrow [B] = Nm^3C^{-1}$$

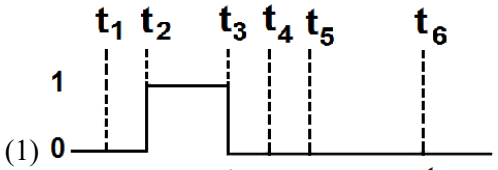
43. The output waveform of the given logical circuit for the following inputs A and B as shown below, is :



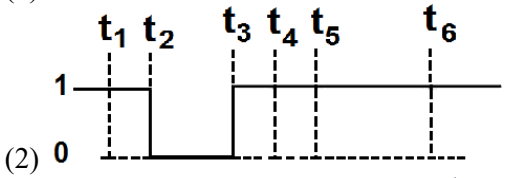


Inputs

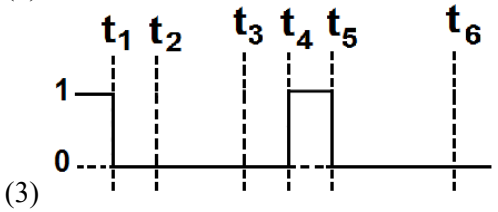
(1)

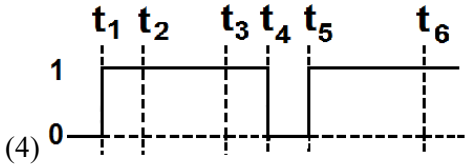


(2)



(3)





(4)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\overline{(A \cdot A)} = \overline{A}$

$\overline{B \cdot B} = \overline{B}$

$\left(\overline{\overline{A} \cdot \overline{B}}\right) = A + B$

OR Gate

44. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5 cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be _____ m.

(1) 0.20 (2) 0.5

(3) 0.05 (4) 0.10

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $h = \dfrac{2S\cos\theta}{r\rho g}$

$\therefore \quad \dfrac{h_1}{h_2} = \dfrac{S_1}{S_2}\dfrac{\rho_2}{\rho_1}$

$\dfrac{5}{h_2} = \left[\dfrac{1}{2}\right]\left[\dfrac{2}{1}\right] \Rightarrow h_2 = 5\,\text{cm} = 0.05\ \text{m}$

{Info about angle of contact not there so most appropriate is 3}

45. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120 V and 80 V respectively. The amplitude of each sideband is :

(1) 15 V (2) 10 V

(3) 20 V (4) 5V

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $A_c + A_m = 120$

$A_c - A_m = 80$

$\therefore \quad A_c = 100$

$A_m = 20$

Modulation index $= \dfrac{20}{100} = \dfrac{1}{5}$

Amplitude of each sideband

$= A_c \dfrac{(\text{modulation index})}{2}$

$= 100 \times \dfrac{1}{10} = 10\ \text{volt}$

46. In a series LR circuit with $X_L = R$. power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be :

(1) 1 : 3 (2) $1:\sqrt{2}$

(3) 1 : 1 (4) 1 : 2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $P = \dfrac{R}{Z} \Rightarrow P_1 = \dfrac{R}{\sqrt{R^2 + X_L^2}} = \dfrac{R}{R\sqrt{2}}$ (as $X_L = R$)

$P_1 = \dfrac{1}{\sqrt{2}}$

$P_2 = \dfrac{R}{\sqrt{R^2 + (X_L - X_L)^2}} = P_2 = 1$

$\dfrac{P_1}{P_2} = \dfrac{1}{\sqrt{2}}$

47. If the gravitational field in the space is given as $\left(-\dfrac{K}{r^2}\right)$. Taking the reference point to be at r = 2 cm with gravitational potential V = 10 J/kg. Find the gravitational potential at r = 3 cm in SI unit (Given, that K = 6 J cm/kg)

(1) 9 (2) 11

(3) 12 (4) 10

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $-\dfrac{dV}{dr} = -\dfrac{k}{r^2} \Rightarrow \int_{10}^{V} dV = \int_{2}^{3} \dfrac{k}{r^2} dr$

$V - 10 = k\left[\dfrac{1}{2} - \dfrac{1}{3}\right]$

$V - 10 = \dfrac{k}{6} \Rightarrow V = 11\ \text{volts}$

48. A ball of mass 200 g rests on a vertical post of height 20 m. A bullet of mass 10 g, travelling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30 m and the bullet at a distance of 120 m from the foot of the post. The value of initial velocity of the bullet will be (if g = 10 m/s^2) :

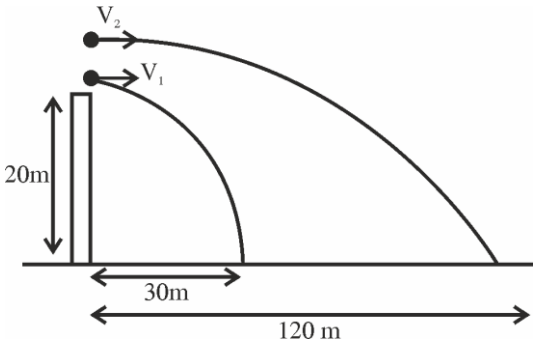
(1) 120 m/s (2) 60 m/s

(3) 400 m/s (4) 360 m/s

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

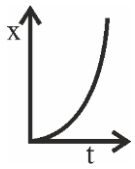
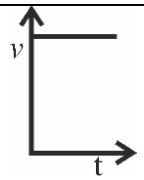
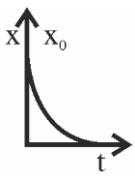
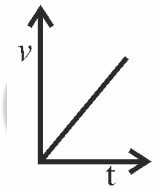
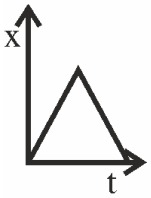
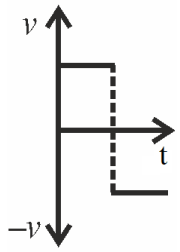
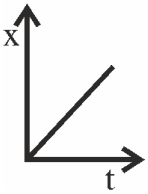
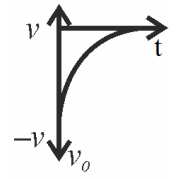


$$v_1 = \frac{30}{\sqrt{\frac{2h}{g}}},\quad v_2 = \frac{120}{\sqrt{\frac{2h}{g}}}$$

$$(0.01)\,u = (0.2)\frac{30\sqrt{g}}{\sqrt{2h}} + (0.01)\frac{120\sqrt{g}}{\sqrt{2h}}$$

$u = 300 + 60 = 360\ ms^{-1}$

49. Match Column-I with Column-II :

Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

Choose the correct answer from the options given below:

(1) A- II B-IV, C-III, D-I

(2) A- I. B-II, C-III, D-IV

(3) A- II B-III, C-IV, D-I

(4) A- I, B-III. C-IV, D-II

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\frac{dx}{dt} = \text{slope} \geq 0$ always increasing

(A - II)

$\frac{dx}{dt} < 0$; and at $t \to \infty \frac{dx}{dt} \to 0$

(B - IV)

$\frac{dx}{dt} > 0$ for first half $\frac{dx}{dt} < 0$ for second half.

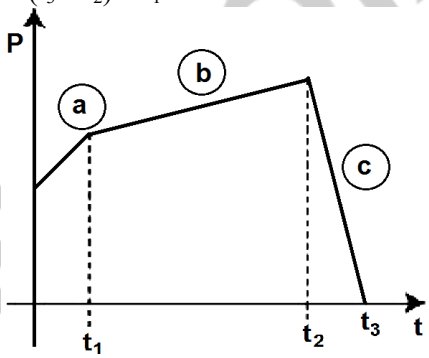
(C - III)

$\frac{dx}{dt} = \text{constant}$

(D - I)

50. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively ?

If $(t_3 - t_2) < t_1$.



(1) c and a (2) b and c

(3) c and b (4) a and b

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\left|\frac{d\vec{p}}{dt}\right| = |\vec{F}| \Rightarrow \frac{d\vec{p}}{dt} = \text{Slope of curve}$

max slope (c), min slope (b)

SECTION-B

51. The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) – time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is _ .

Official Ans. by NTA (8)

QuestPix Ans. (8)

Sol. $x = A\sin(\omega t)$

$$U_{(x)} = \frac{1}{2}kx^2,$$

$$\frac{dU}{dt} = \frac{1}{2}k2x\frac{dx}{dt}$$

$$= kA^2\omega\sin\omega t\cos\omega t \times \frac{2}{2}$$

$$\left(\frac{dU}{dt}\right)_{max} = \frac{kA^2\omega}{2}(\sin 2\omega t)_{max}$$

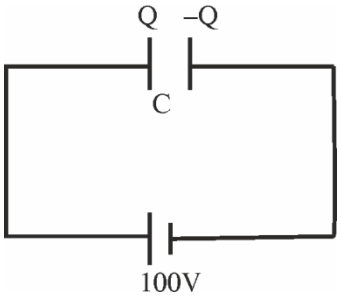
$2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4}\omega = \frac{T}{8} \quad \Rightarrow \beta = 8$

52. A capacitor of capacitance 900 μF is charged by a 100 V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as x × 10^{-2} J. The value of x is _____.

Official Ans. by NTA (225)

QuestPix Ans. (225)

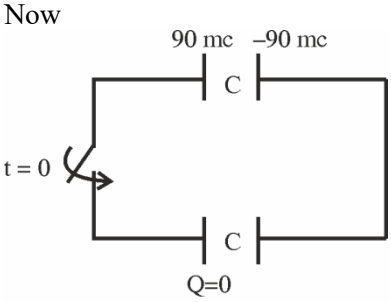
Sol.

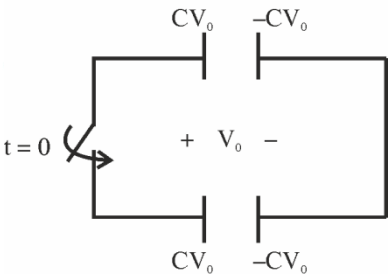


$C = 900\,\mu F$

$Q = CV = 900 \times 10^{-6} \times 100 = 9 \times 10^{-2} = 90$ MC

Now



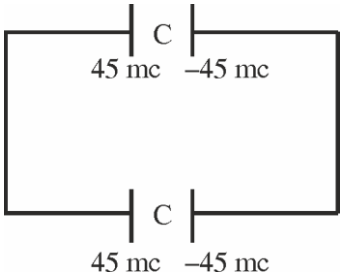


Common potential will be developed across both capacitors by kVL

Total charge on left plates of capacitors should be conserved.

∴ $90\text{ mc} + 0 = 2cv_0$

$cv_0 = 45$ mc



Heat dissipated $= U_i - U_f$ [Change in energy stored in the capacitors]

$$= \frac{1}{2}\frac{(90\,mc)^2}{900\,\mu F} - 2 \times \frac{1}{2}\frac{(45\,mc)^2}{900\,\mu F} \left[U = \frac{Q^2}{2c} \right]$$

$$= \frac{1}{2 \times 900 \times 10^{-6}}(8100 - 4050) \times 10^{-6}$$

$= 2.25$ Joule

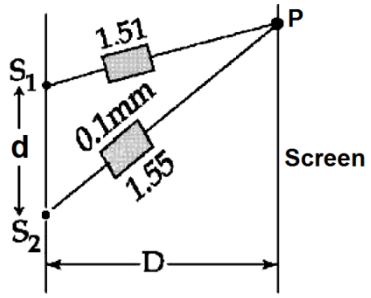
OR

$$\text{Heat} = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\frac{C^2}{2C}(100 - 0)^2$$

$$= \frac{1}{2}\frac{900 \times 10^{-6}}{2} \times 10^4 = \frac{9}{4}\text{Joule} = 2.25\,\text{Joule}$$

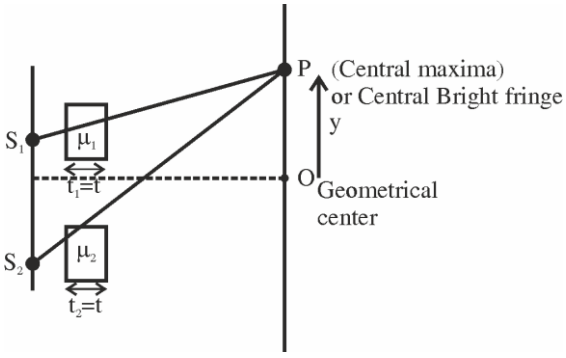
53. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam (λ = 4000Å) from S_1 and S_2 respectively. The central bright fringe spot will shift by _____ number of fringes.



Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol.



Path difference at P be Δx

$\Delta x = (\mu_2 - \mu_1)t$

$= (1.55 - 1.51)\,0.1 \text{ mm}$

$= 0.04 \times 10^{-4}$

$\Delta x = 4 \times 10^{-6} = 4\,\mu m$

$y = \frac{\Delta x D}{d} = 4 \times 10^{-6} \frac{D}{d}$

{y is the distance of central maxima from geometric center}

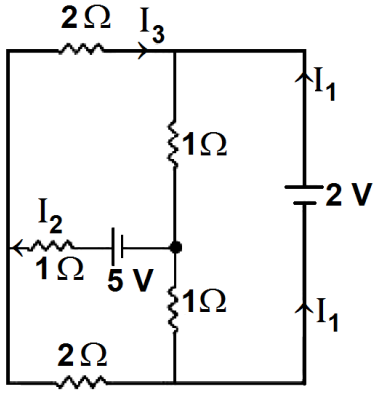
$\text{fringe width } (\beta) = \frac{\lambda D}{d} = 4 \times 10^{-6}\,m \frac{D}{d} = 4\mu m \frac{D}{d}$

$\therefore$ Central bright fringe spot will shift by 'x'

Number of shift $= \frac{y}{\beta}$

$= \frac{4 \times 10^{-6} D/d}{4 \times 10^{-7} D/d} = 10$ Ans

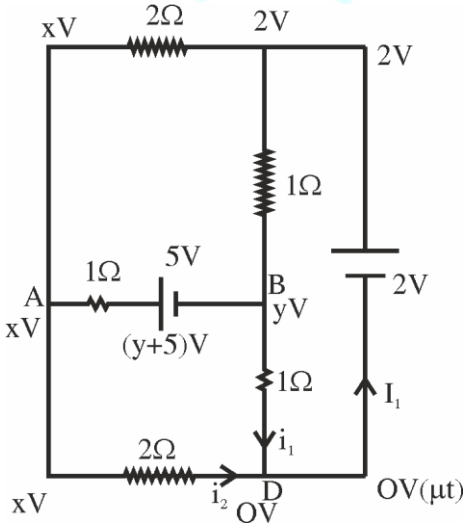
54. In the following circuit, the magnitude of current I_1, is _____ A.



Official Ans. by NTA (1)

QuestPix Ans. (1.5)

Sol.



Junction law at A,

$\frac{x-(y+5)}{1} + \frac{x-2}{2} + \frac{x-0}{2} = 0$ .....(1)

Junction law at B,

$\frac{y+5-x}{1} + \frac{y-0}{1} + \frac{y-2}{1} = 0$ .....(2)

On solving equation (1) and Equation (2)

x = 3

& y = 0

At D junction

$I_1 = i_1 + i_2$

$I_1 = \frac{y-0}{1} + \frac{x-0}{2}$

$= \frac{0-0}{1} + \frac{3-0}{2}$

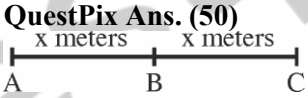
$I_1 = 1.5$ A

55. A horse rider covers half the distance with 5 m/s speed. The remaining part of the distance was travelled with speed 10 m/s for half the time and with speed 15 m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is x/7 m/s. The value of x is

Official Ans. by NTA (50)

QuestPix Ans. (50)

Sol.



$t_{AB} = \frac{x}{5\,m/s}$

In motion BC

$x = d_1 + d_2$

where d_1 & d_2 we the distance travelled with 10 m/s and 15 m/s respectively in equal time intervals $\frac{'t'}{2}$ each

$d_1 = \frac{10t}{2}, \; d_2 = \frac{15t}{2}$

$d_1 + d_2 = x = \frac{t}{2}(10+15) = \frac{25t}{2}$

$< v > = \frac{2x}{\frac{x}{5} + \frac{2x}{25}} = \frac{2 \times 25}{5+2} = \frac{50}{7}\,m/s$

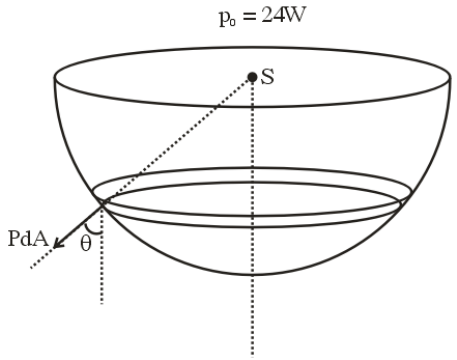
Ans. : 50

56. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W The radius of curvature of hemisphere is 10 cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is _____ $\times 10^{-8}$ N.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

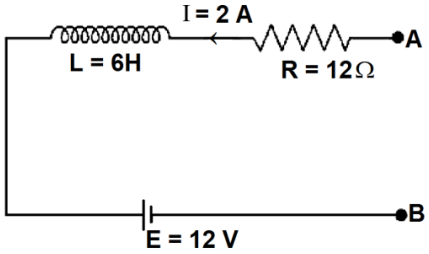


$$\text{Force} = \int PdA\cos\theta$$

$$= \frac{2I}{C}\int dA\cos\theta = \frac{2I}{C}\pi R^2 = 2\frac{p_0}{4\pi R^2}\cdot\frac{\pi R^2}{C}$$

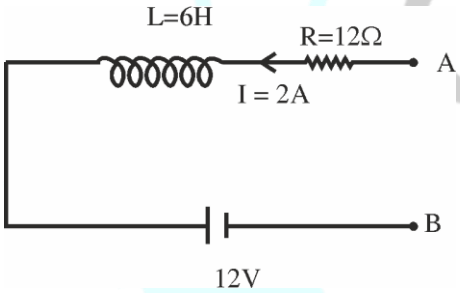
$$= \frac{p_0}{2C} = \frac{24}{2\times3\times10^8} = 4\times10^{-8}\ \text{N (Ans)}$$

57. As per the given figure, if $\frac{dI}{dt} = -1$ A/s then the value of V_{AB} at this instant will be _____ V.



Official Ans. by NTA (30)
QuestPix Ans. (30)

Sol.



$$\frac{dI}{dt} = -1\frac{A}{\sec}$$

$$V_A - IR - L\frac{dI}{dt} - 12 = V_B$$

$$V_A - 2\times12 - 6(-1) - 12 = V_B$$

$$V_A - V_B = 36 - 6 = 30\ \text{volt}$$

58. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5 mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46th division the circular scale coincide with the reference line. The diameter of the wire is _____ × 10^{-2} mm.

Official Ans. by NTA (22)
QuestPix Ans. (220)

Sol. $\text{Least count} = \frac{\text{Pitch}}{\text{No. of circular divisions}}$

$$= \frac{0.5\,\text{mm}}{100}$$

Least count = 5×10^{-3} mm
Positive Error = MSR + CSR (LC)
$= 0\text{ mm} + 6\ (5 \times 10^{-3}\text{ mm})$
Reading of Diameter = MSR + CSR (LC) – Positive zero error
$= 4 \times 0.5\text{ mm} + (46(5 \times 10^{-3})) - 6(5 \times 10^{-3})$ mm
$= 2\text{ mm} + 40 \times 5 \times 10^{-3}\text{ mm} = 2.2$ mm (Ans.)

59. In an experiment for estimating the value of focal length of converging mirror, image of an object placed at 40 cm from the pole of the mirror is formed at distance 120 cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1 cm. The value of error in measurement of focal length of the mirror is 1/K cm. The value of K is _____ .

Official Ans. by NTA (32)
QuestPix Ans. (32)

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{-1}{120} - \frac{1}{40} = \frac{1}{f}, \quad f = -30\,\text{cm}$$

Now,

$$\frac{-1}{v^2}dv - \frac{1}{u^2}du = -\frac{1}{f^2}df$$

Also $dv = du = \frac{1}{20}$ cm

$$\therefore \quad \frac{\frac{1}{20}}{(120)^2} + \frac{\frac{1}{20}}{(40)^2} = \frac{df}{(30)^2}$$

On solving

$$df = \frac{1}{32}\text{cm}$$

$\therefore$ $k = 32$

60. A thin uniform rod of length 2m. cross sectional area 'A' and density 'd' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then the value of α is _____.

Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol. $(KE)_{Rotational} = \frac{1}{2} I\omega^2 = E$

$$E = \frac{1}{2}\frac{m\ell^2}{12}\omega^2$$

$$E = \frac{1}{2}\frac{dA\ell^3}{12}\omega^2$$

$$E = \frac{dA(2)^3}{24}\omega^2$$

$$\sqrt{\frac{3E}{dA}} = \omega$$

$\alpha = 3$ Ans.

(Held On Monday 30th January, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

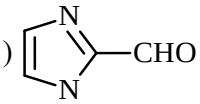
CHEMSITRY	TEST PAPER WITH SOLUTION

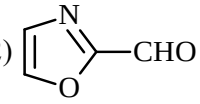
SECTION-A

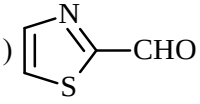
61. Which of the following compounds would give the following set of qualitative analysis ?

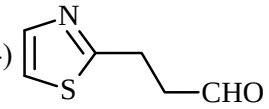
(i) Fehling's Test : Positive

(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not

(1) 

(2) 

(3) 

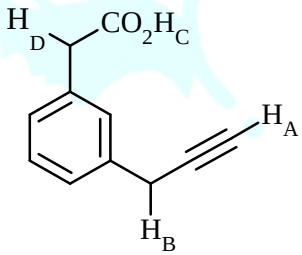
(4) 

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Aromatic aldehydes do not give Fehling's test..
Both nitrogen and sulfur must be present to obtain blood red colour
Sodium nitroprusside gives blood red colour with S & N.

62. What is the correct order of acidity of the protons marked A–D in the given compounds ?



(1) $H_C > H_D > H_B > H_A$

(2) $H_C > H_D > H_A > H_B$

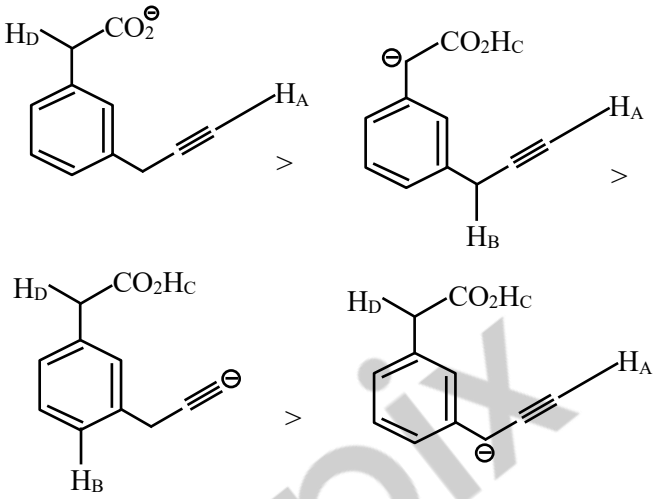
(3) $H_D > H_C > H_B > H_A$

(4) $H_C > H_A > H_D > H_B$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. acidity of an acid depends upon the stability of its conjugate base



63. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R) : Ketoses undergo β-elimination followed by formation of furfural.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) Both (A) and (R) are true and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Seliwanoff 's test is a differentiating test for Ketose and aldose. This test relies on the principle that the keto hexose are more rapidly dehydrated to form 5-hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, Cherry red or brown red coloured complex is formed rapidly indicating a positive test.

64. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to :

(1) separate CuO as $CuSiO_3$

(2) remove calcium as $CaSiO_3$

(3) decrease the temperature needed for roasting of Cu_2S

(4) remove FeO as $FeSiO_3$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. The copper ore contains iron, it is mixed with silica before heating in reverberatory furnace. FeO slags off as $FeSiO_3$.

$$FeO + SiO_2 \longrightarrow FeSiO_3$$

65. Amongst the following compounds, which one is an antacid ?

(1) Ranitidine (2) Meprobamate

(3) Terfenadine (4) Brompheniramine

Official Ans. by NTA (1)

QuestPix Ans. (1)

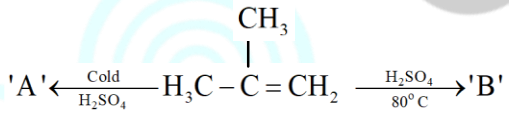
Sol. **1. Ranitidine:** Antacid

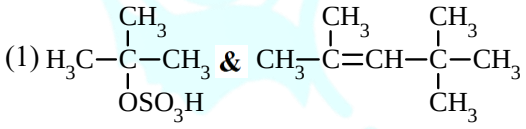
2. Meprobamate: Tranquilizer

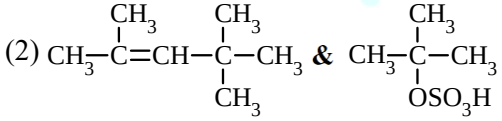
3. Terfenadine: Antihistamine

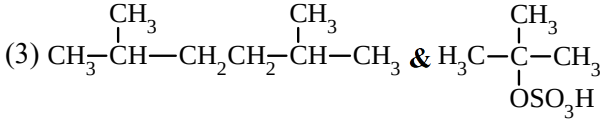
4. Brompheniramine: Antihistamine

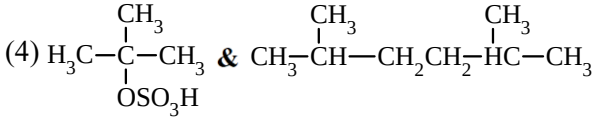
66. The major products 'A' and 'B', respectively, are



(1) 

(2) 

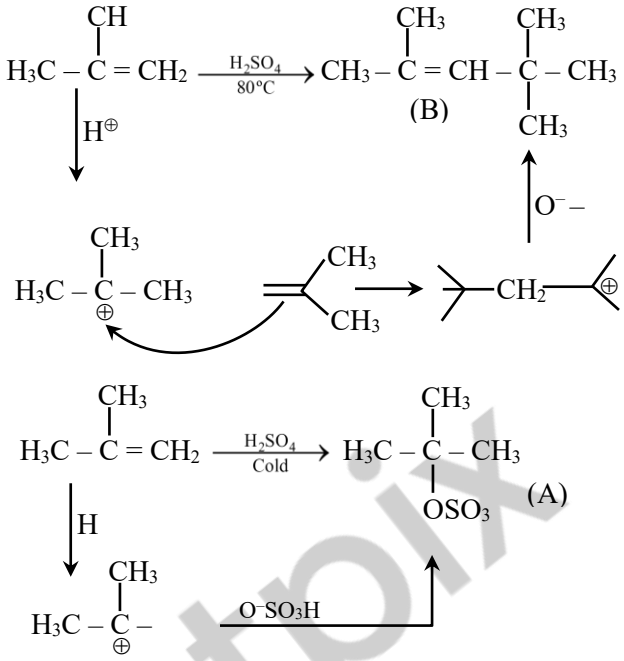
(3) 

(4) 

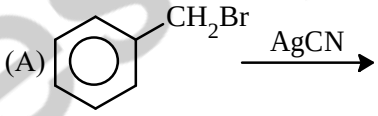
Official Ans. by NTA (1)

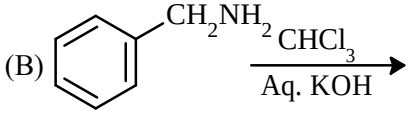
QuestPix Ans. (1)

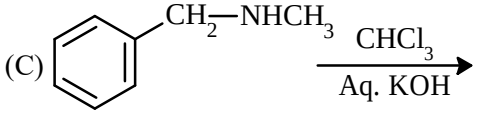
Sol.

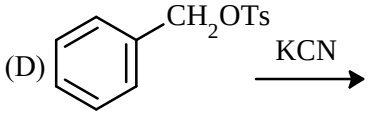


67. Benzyl isocyanide can be obtained by :

(A) 

(B) 

(C) 

(D) 

Choose the correct answer from the options given below :

(1) A and D

(2) Only B

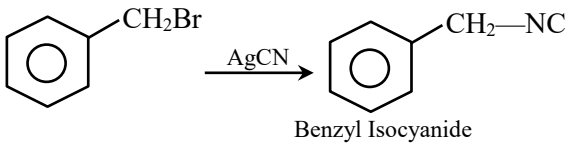
(3) A and B

(4) B and C

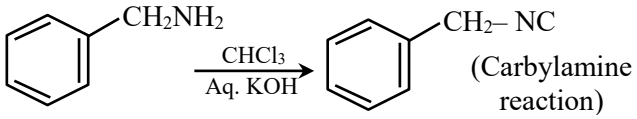
Official Ans. by NTA (3)

QuestPix Ans. (3)

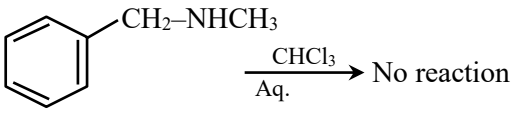
Sol.

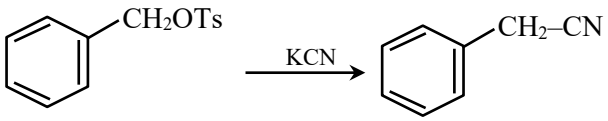


Benzyl Isocyanide



Benzyl Isocyanide





68. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R) : Silica gel adsorbs moisture from air via adsorption, thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

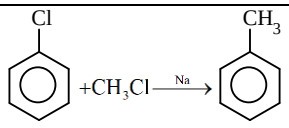
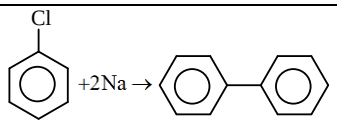
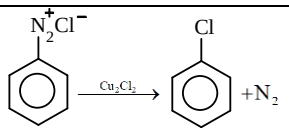
(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Silica gel prevents water corrosion (rusting) and instrument malfunction by adsorbing moisture from the air.

69. Match **List I** with **List II**

List I		List II	
A		I	Fitting reaction
B		II	Wurtz Fitting reaction
C		III	Finkelstein reaction
D	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	IV	Sandmeyer reaction

(1) A – II, B – I, C – III, D – IV

(2) A – III, B – II, C – IV, D – I

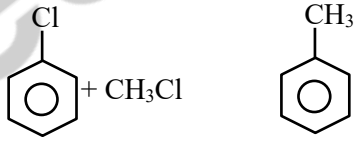
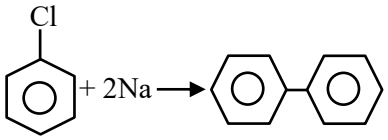
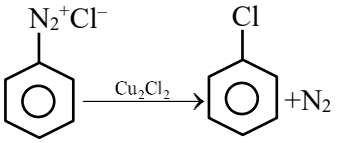
(3) A – IV, B – II, C – III, D – I

(4) A – II, B – I, C – IV, D – III

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

LIST-I		LIST-II
A.		Wurtz-fitting reaction
B.		Fitting reaction
C.		Sandmeyer reaction
D.	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	Finkelstein reaction

70. Caprolactam when heated at high temperature in presence of water, gives

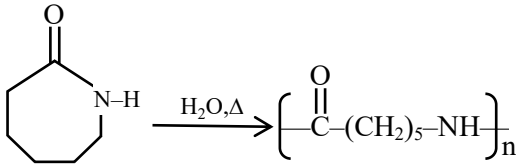
(1) Teflon (2) Dacron

(3) Nylon 6, 6 (4) Nylon 6

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



Caprolactam Nylon -6

71. The alkaline earth metal sulphate(s) which are readily soluble in water is/are:

(A) $BeSO_4$

(B) $MgSO_4$

(C) $CaSO_4$

(D) $SrSO_4$

(E) $BaSO_4$

Choose the **correct** answer from the options given below:

(1) A only (2) B only

(3) A and B (4) B and C

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Due to high hydration energy Be^{2+} and Mg^{2+}, $BeSO_4$ and $MgSO_4$ are readily soluble in water.

72. Which of the following is correct order of ligand field strength?

(1) $CO < en < NH_3 < C_2O_4^{2-} < S^{2-}$

(2) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

(3) $NH_3 < en < CO < S^{2-} < C_2O_4^{2-}$

(4) $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. The increasing order of field strength of ligands (according to spectrochemical series)

$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

73. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.

(i) $NO_2 \xrightarrow{h\upsilon} A + B$

(ii) $B + O_2 \rightarrow C$

(iii) $A + C \rightarrow NO_2 + O_2$

Choose the correct answer from the options given below:

(1) $O, NO \& NO_3^-$ (2) $O, N_2O \& NO$

(3) $N, O_2 \& O_3$ (4) $NO, O \& O_3$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $NO_{2g} \xrightarrow{h\nu} \underset{(A)}{NO_g} + \underset{(B)}{O_g}$

$\underset{(B)}{O_g} + O_{2g} \rightleftharpoons \underset{(C)}{O_{3g}}$

$\underset{(A)}{NO_g} + \underset{(C)}{O_{3g}} \longrightarrow NO_{2g} + O_{2g}$

74. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4):

(1) Black (2) Red

(3) Green (4) Blue

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $Cr_2O_7^{2-} + SO_3^{2-} \xrightarrow{H^+} \underset{\text{Green}}{Cr^{3+}} + SO_4^{2-}$

75. To inhibit the growth of tumours, identify the compounds used from the following:

(A) EDTA

(B) Coordination Compounds of Pt

(C) D – Penicillamine

(D) Cis – Platin

Choose the correct answer from the option given below:

(1) B and D Only

(2) C and D Only

(3) A and B Only

(4) A and C Only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Cis – Platin is used in chemotherapy to inhibits the growth of tumors. ($cis[Pt(NH_3)_2Cl_2]$)

76. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis?

(1) Fe^{3+}

(2) Zn^{2+}

(3) Co^{2+}

(4) Ni^{2+}

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $Zn^{2+}, Co^{2+}, Ni^{2+} = IV^{th}$ Group

$Fe^{3+} = III^{rd}$ Group

77. Match **List I** with **List II**

LIST–I (molecules/ions)		**LIST–II (No. of lone pairs of e⁻ on central atom)**	
(A)	IF_7	I.	Three
(B)	ICl_4^-	II.	One
(C)	XeF_6	III.	Two
(D)	XeF_2	IV.	Zero

Choose the **correct** answer from the options given below:
(1) A – II, B – III, C – IV, D – I
(2) A – IV, B – III, C – II, D – I
(3) A – II, B – I, C – IV, D – III
(4) A – IV, B – I, C – II, D – III

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. IF_7 zero lone pair

ICl_4^- two lone pair

XeF_6 one lone pair

XeF_2 three lone pair

78. For OF_2 molecule consider the following:
(A) Number of lone pairs on oxygen is 2.
(B) FOF angle is less than 104.5^o.
(C) Oxidation state of O is –2.
(D) Molecule is bent 'V' shaped.
(E) Molecular geometry is linear.

Correct options are:
(1) C, D, E only
(2) B, E, A only
(3) A, C, D only
(4) A, B, D only

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

- Two lone pair one oxygen
- Molecule is 'v' shaped
- Bond angle is less than 104.5°(102º)
- O·S· of 'O' is + 2

79. Lithium aluminium hydride can be prepared from the reaction of

(1) LiCl and Al_2H_6

(2) LiH and Al_2Cl_6

(3) LiCl, Al and H_2

(4) LiH and $Al(OH)_3$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $8LiH + Al_2Cl_6 \longrightarrow 2LiAlH_4 + 6LiCl$

80. Match **List – I** with **List – II**

LIST–I (Atomic number)		**LIST–II (Block of periodic table)**	
(A)	37	I.	p–block
(B)	78	II.	d–block
(C)	52	III.	f–block
(D)	65	IV.	s–block

Choose the **correct** answer from the options given below:
(1) A – II, B – IV, C – I, D – III
(2) A – I, B – III, C – IV, D – II
(3) A – IV, B – III, C – II, D – I
(4) A – IV, B – II, C – I, D – III

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

Atomic number	**Block**
37 (K)	s-block
78 (Pt)	d-block
52 (Te)	p-block
65 (Tb)	f-block

SECTION-B

81. Consider the cell

$$Pt_{(s)}\left|H_2(g,1atm)\right|H^+(aq,1M)\left|\left|Fe^{3+}(aq),Fe^{2+}(aq)\right|Pt(s)\right.$$

When the potential of the cell is 0.712 V at 298 K, the ratio $\lfloor Fe^{2+} \rfloor / \lfloor Fe^{3+} \rfloor$ is _____.

(Nearest integer)

Given: $Fe^{3+}+e^- = Fe^{2+}, E^oFe^{3+},Fe^{2+}\left|Pt = 0.771\right.$

$$\frac{2.303RT}{F} = 0.06V$$

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol

$$Pt_{(s)}\left|H_2(g,1atm)\right|H^+(aq,1M)\,||\,Fe^{3+}(aq),Fe^{2+}(aq)\,|\,Pt(s)$$

at anode $H_2 \longrightarrow 2H^+ + 2e^-$

At cathode $Fe^{3+}_{aq} + e^- \longrightarrow Fe^{2+}_{aq}$

$$E^\circ = E^\circ_{H_2|H^+} + E^\circ_{Fe^{3+}|Fe^{2+}} = 0\cdot771V$$

$$E = E^\circ - \frac{0\cdot06}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$0\cdot712 = (0+0\cdot771) - \frac{0\cdot06}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$\log\frac{Fe^{2+}}{Fe^{3+}} = \frac{0\cdot059}{0\cdot06} \approx 1$$

$$\boxed{\frac{Fe^{2+}}{Fe^{3+}} = 10}$$

82. A 300 mL bottle of soft drink has 0.2 M CO_2 dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is _____ mL. (Nearest integer)

Given: At STP, molar volume of an ideal gas is $22.7\,L\,mol^{-1}$

Official Ans. by NTA (1362)

QuestPix Ans. (1362 ml)

Sol. Mole of CO_2 = 0.2 M × (300 × 10^{-3}) L

= 0.06 Mole

Volume of 0.06 mole CO_2 at S.T.P

= 0.06 × 22.7

= 1.362 L

83. A solution containing 2 g of a non–volatile solute in 20 g of water boils at 373.52 K. The molecular mass of the solute is _____ $g\,mol^{-1}$. (Nearest integer)

Given, water boils at 373 K, K_b for water $= 0.52\,K\,kg\,mol^{-1}$

Official Ans. by NTA (100g)

QuestPix Ans. (100g)

Sol. $\Delta T_b = 373.52 - 373$

$= 0.52$

$\Delta T_b = Kb\cdot m$

$$0.52 = 0.52 \times \frac{2}{\text{Molar Mass}} \times \frac{1}{20\times10^{-3}}$$

Molar Mass = 100g/mol

84. If compound A reacts with B following first order kinetics with rate constant $2.011\times10^{-3}s^{-1}$. The time taken by A (in seconds) to reduce from 7 g to 2 g will be _____. (Nearest Integer)

$[\log 5 = 0.698, \log 7 = 0.845, \log 2 = 0.301]$

Official Ans. by NTA (623)

QuestPix Ans. (623)

Sol. $A + B \rightarrow P$

t = 0 7g

t = t 2g

at constant volume

$$t = \frac{2.303}{K}\log\frac{[A]_0}{[A]_t}$$

$$= \frac{2\cdot303}{2\cdot011\times10^{-3}}\log\frac{7}{2}$$

$$= \frac{2\cdot303\times0\cdot544}{2\cdot011\times10^{-3}}$$

$= 622.989$

≈ 623

85. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in $J\,mol^{-1}$ is _____. (Nearest integer)

(Given: $h = 6.626\times10^{-34}$ Js

$N_A = 6.022\times10^{23}\,mol^{-1}$)

Official Ans. by NTA (798)

QuestPix Ans. (798)

Sol. For one photon E = hv

For one mole photon,

$E = 6\cdot023\times10^{23}\times6\cdot626\times10^{-34}\times2\times10^{12}$

$= 798\cdot16$ J

≈ 798 J

86. The number of electrons involved in the reduction of permanganate to manganese dioxide in acidic medium is _____.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\overset{+7}{Mn}O_4^- + 4H^+ + 3e^- \longrightarrow \overset{+4}{Mn}O_2 + 2H_2O$

87. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is _____ J. (Nearest integer)

Official Ans. by NTA (0)

QuestPix Ans. (0)

Sol. For ideal gas U = f (T)

and for isothermal process, $\Delta U = 0$

88. 600 mL of 0.01M HCl is mixed with 400 mL of 0.01 M H_2SO_4. The pH of the mixture is _____ $\times10^{-2}$. (Nearest integer)

[Given $\log 2 = 0.30$, $\log 3 = 0.48$

$\log 5 = 0.69$ $\log 7 = 0.84$

$\log 11 = 1.04$]

Official Ans. by NTA (186)

QuestPix Ans. (186)

Sol. Total milimoles of H^+ = (600×0.01) + (400×0.01×2)

= 1 4

$$\left[H^+\right] = \frac{14}{1000} = 14\times10^{-3}$$

pH = 3 – log 14

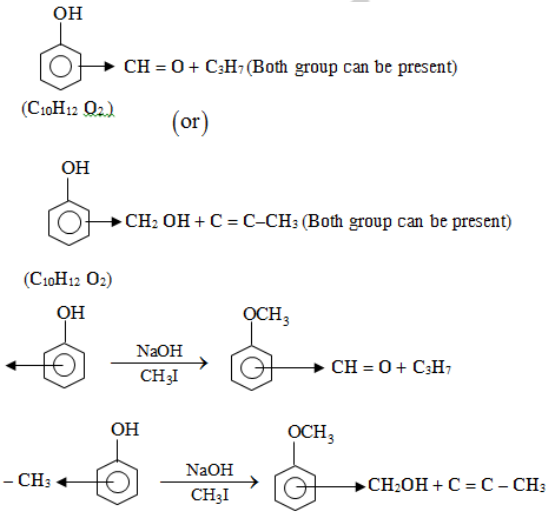
= 1.86

= 186 × 10^{-2}

89. A trisubstituted compound 'A', $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH and CH_3Br gives $C_{11}H_{14}O_2$, with hydroiodic acid gives methyl iodide and with hot conc. NaOH gives a compound B, $C_{10}H_{12}O_2$. Compound 'A' also decolorises alkaline $KMnO_4$. The number of π bond/s present in the compound 'A' is _____.

Sol. **Official Ans. by NTA (4)**

QuestPix Ans. (4)



90. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 mL of chloroform $(CHCl_3)$ to prepare 2.6×10^{-3}M solution of $CH_2Cl_2\,(DCM)$. The concentration of DCM is _____ ppm (by mass).

Given: Atomic mass : C = 12; H : 1; Cl = 35.5

density of $CHCl_3 = 1.49\,g\,cm^{-3}$

Official Ans. by NTA (221)

QuestPix Ans. (148)

Sol. $\text{Molarity} = \frac{\text{mole}}{\text{volume}}$

$$2.6\times10^{-3} = \frac{x/85}{0.67141}$$

x = 0.148 g

$$\text{conc. Fo DCM in ppm} = \frac{0.148}{1.49\times671.141}\times10^6$$

= 148 ppm