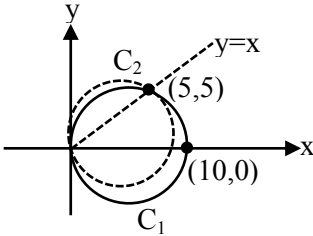
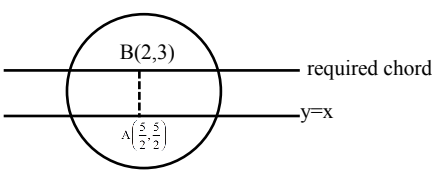


(HELD ON WEDNESDAY 28th JANUARY 2026)
TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS	TEST PAPER WITH SOLUTION
SECTION-A 1. If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then $f(g(2))$ is equal to: (1) $\frac{25}{6}$ (2) $-\frac{25}{6}$ (3) $\frac{7}{2}$ (4) $-\frac{7}{2}$ Ans. (3) Sol. $g(2) = 13$ $f(g(2)) = f(13)$ Now $4g(f(x)) = 3x^2 - 32x + 72$ $4[3f^2(x) + 2f(x) - 3] = 3x^2 - 32x + 72$ Let $f(x) = t$ $12t^2 + 8t - (3x^2 - 32x + 84) = 0$ $f(x) = \frac{-8 \pm \sqrt{64 + 48(3x^2 - 32x + 84)}}{24}$ $f(x) = \frac{-8 \pm 4(3x - 16)}{24}$ $\therefore f(0) = -3 \therefore$ we take +ve sign $\therefore f(x) = \frac{-8 + 4(3x - 16)}{24}$ $\therefore f(13) = \frac{-8 + 4 \times 23}{24} = \frac{84}{24} = \frac{7}{2}$ 2. The value of $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k(k+1)}{k!} \right)$ is : (1) $2/e$ (2) $1/e$ (3) $\sqrt{e}$ (4) $e/2$ Ans. (2) Sol. $T_k = (-1)^{k+1} \cdot \frac{k(k+1)}{k!} = (-1)^{k+1} \left(\frac{k(k-1) + 2k}{k!} \right)$ $\therefore \text{sum} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k-2} + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{k-1}$	$= \left(\frac{1}{-1} - \frac{1}{0} + \frac{1}{1} - \frac{1}{2} + \frac{1}{3} \dots \right) + \left(\frac{2}{0} - \frac{2}{1} + \frac{2}{2} - \frac{2}{3} \dots \right)$ $= \frac{1}{e}$ 3. Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2, which passes through the point $(2, 3)$ and is farthest from the center of C_2, is $x + ay + b = 0$, then $a - b$ is equal to : (1) 10 (2) -6 (3) -2 (4) 6 Ans. (3) Sol. Equation of circle C_2 is $x^2 + y^2 - 5x - 5y = 0$ its centre is $\left(\frac{5}{2}, \frac{5}{2} \right)$   $m_{AB} = -1$ $\therefore$ Slope of required chord = 1 $\therefore$ equation of required chord is $x - y + 1 = 0$ $\therefore a = -1, b = 2$ $\therefore a - b = -2$



4. If $\frac{\tan(A-B)}{\tan A} + \frac{\sin^2 C}{\sin^2 A} = 1$, $A, B, C \in \left(0, \frac{\pi}{2}\right)$,

then

- (1) $\tan A, \tan C, \tan B$ are in G.P.
- (2) $\tan A, \tan B, \tan C$ are in G.P.
- (3) $\tan A, \tan C, \tan B$ are in A.P.
- (4) $\tan A, \tan B, \tan C$ are in A.P.

Ans. (1)

Sol.
$$\frac{\tan A - \tan B}{(1 + \tan A \tan B) \tan A} + \frac{1 + \cot^2 A}{1 + \cot^2 C} = 1$$

Put $\tan A = x, \tan B = y, \tan C = z$

$$\therefore \frac{x-y}{(1+xy)x} + \frac{(x^2+1)z^2}{x^2(z^2+1)} = 1$$

$$\therefore x(x-y)(z^2+1) + z^2(1+x^2)(1+xy) = (1+xy)x^2(1+z^2)$$

after solving we get

$$z^2 = xy \quad \therefore 1 + x^2 \neq 0$$

$$\therefore \tan^2 C = \tan A \cdot \tan B$$

$\tan A, \tan C, \tan B$ are in G.P.

5. Let z be a complex number such that $|z - 6| = 5$ and $|z + 2 - 6i| = 5$.

Then the value of $z^3 + 3z^2 - 15z + 141$ is equal to

- (1) 42
- (2) 37
- (3) 50
- (4) 61

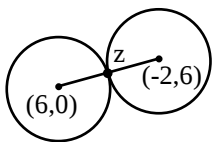
Ans. (3)

Sol. Center of first circle $C_1(6,0), r_1 = 5$

Center of second circle $C_2(-2,6), r_2 = 5$

$$\therefore C_1C_2 = r_1 + r_2$$

$\therefore$ common point Z is mid point of C_1 & C_2



$$\therefore z = 2 + 3i$$

$$\therefore z^2 = 4z - 13$$

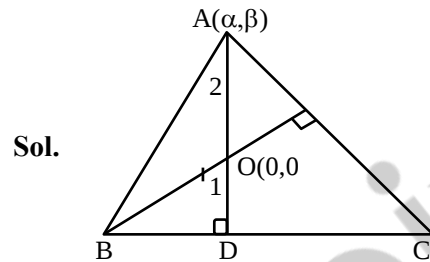
$$\therefore z^3 = 3z - 52$$

$$\therefore z^3 + 3z^2 - 15z + 141 = 50$$

6. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x + 2\sqrt{2}y = 4$. If the co-ordinates of the vertex A are (α, β) , then the greatest integer less than or equal to $|\alpha + \sqrt{2}\beta|$ is

- (1) 2
- (2) 3
- (3) 5
- (4) 4

Ans. (4)



Sol.

$$\therefore m_{BC} \cdot m_{AD} = -1$$

$$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right) \left(\frac{\beta}{\alpha}\right) = -1$$

$$\Rightarrow \beta = 2\sqrt{2}\alpha \quad \dots(1)$$

$$\therefore OD = \left| \frac{-4}{\sqrt{1+8}} \right| = \frac{4}{3} \Rightarrow AO = \frac{8}{3}$$

$$\text{So } AD = \frac{8}{3} + \frac{4}{3} = 4$$

$$\Rightarrow \frac{|\alpha + 2\sqrt{2}\beta - 4|}{3} = 4 \Rightarrow \alpha = \frac{16}{9} \text{ or } -\frac{8}{9}$$

$$\left\{ \begin{array}{l} \therefore A(\alpha, \beta) \text{ \& } (0,0) \text{ lies on same} \\ \text{side of given line} \end{array} \right\}$$

$$\therefore (\alpha, \beta) = \left(\frac{16}{9}, \frac{32\sqrt{2}}{9} \right); \text{ (Rejected)}$$

$$\text{so } (\alpha, \beta) = \left(-\frac{8}{9}, -\frac{16\sqrt{2}}{9} \right)$$

$$= \left\lfloor \alpha + \sqrt{2}\beta \right\rfloor = \left\lfloor \frac{-8-32}{9} \right\rfloor = 4$$



7. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9-digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then,

$$(1) 29x = 5y \quad (2) 45x = 7y$$

$$(3) 21x = 4y \quad (4) 56x = 9y$$

Ans. (3)

Sol. $S = \{1, 2, 3, \dots, 9\}$

$$x = {}^9C_1 \cdot {}^8C_7 \times \frac{9!}{2} = \frac{9 \times 8 \times 9!}{2}$$

$$y = {}^9C_2 \cdot {}^7C_5 \times \frac{9!}{2! \times 2!} = \frac{9 \times 8}{2} \times \frac{7 \times 6}{2} \times \frac{9!}{2! \times 2!}$$

$$\Rightarrow \frac{x}{y} = \frac{4}{21}$$

$$21x = 4y$$

8. Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in \mathbb{N} \text{ and } a, b, c \leq 20\}$ be a set of polynomials. Then the number of polynomials in S , which are divisible by $x^2 + 2$, is

$$(1) 20 \quad (2) 6$$

$$(3) 120 \quad (4) 10$$

Ans. (4)

Sol. $x^3 + ax^2 + bx + c = (x^2 + 2)\left(x + \frac{c}{2}\right)$

$$x^2 : a = \frac{c}{2}$$

$$x : b = 2$$

$$b = 2, a = \frac{c}{2} \quad c \in \{2, 4, \dots, 20\}$$

Number of polynomials in 'S' will be 10.

9. A bag contains 10 balls out of which k are red and $(10 - k)$ are black, where $0 \leq k \leq 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is :

$$(1) \frac{7}{11} \quad (2) \frac{7}{55}$$

$$(3) \frac{7}{110} \quad (4) \frac{14}{55}$$

Ans. (4)

$$\text{Sol. Probability} = \frac{{}^1C_0 \cdot {}^9C_3}{\sum_{k=0}^{10} {}^kC_0 \cdot {}^{10-k}C_3}$$

$$= \frac{{}^9C_3}{{}^{10}C_3 + {}^9C_3 + {}^8C_3 + \dots + {}^3C_3}$$

$$= \frac{{}^9C_3}{{}^{11}C_4} = \frac{14}{55}$$

10. Ans. ()

$$\text{Sol. } \frac{\beta - \alpha}{\alpha\beta} = \frac{1}{3}, \quad \alpha + \beta = \frac{\lambda + 3}{\lambda}, \quad \alpha\beta = \frac{3}{\lambda}$$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \quad \dots\dots\dots(1)$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda + 3)^2}{\lambda^2} \quad \dots\dots\dots(2)$$

$$(2) - (1) \quad 4\alpha\beta = \frac{(\lambda + 3)^2 - 1}{\lambda^2}$$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$$

$$\Rightarrow \lambda = 0, 2, 4$$

Sum of possible values of λ is = 6



10. If α, β , where $\alpha < \beta$, are the roots of the equation

$$\lambda x^2 - (\lambda + 3)x + 3 = 0 \text{ such that } \frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}, \text{ then}$$

the sum of all possible values of λ is :

- (1) 6 (2) 2
(3) 4 (4) 8

Ans. (1)

Sol. $\frac{\beta - \alpha}{\alpha\beta} = \frac{1}{3}, \alpha + \beta = \frac{\lambda + 3}{\lambda}, \alpha\beta = \frac{3}{\lambda}$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \dots\dots\dots(1)$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda + 3)^2}{\lambda^2} \dots\dots\dots(2)$$

$$(2) - (1) \quad 4\alpha\beta = \frac{(\lambda + 3)^2 - 1}{\lambda^2}$$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$$

$$\Rightarrow \lambda = 0, 2, 4$$

Sum of possible values of λ is = 6

11. If $\int \left(\frac{1 - 5\cos^2 x}{\sin^5 x \cos^2 x} \right) dx = f(x) + C$ where C is the

constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to

- (1) $\frac{1}{\sqrt{3}}(26 + \sqrt{3})$ (2) $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$
(3) $\frac{1}{\sqrt{3}}(26 - \sqrt{3})$ (4) $\frac{2}{\sqrt{3}}(4 + \sqrt{6})$

Ans. (2)

Sol. $\int \frac{dx}{\sin^5 \cos^2 x} - 5 \int \frac{dx}{\sin^5 x}$

$$\int \frac{\sec^2 x dx}{\sin^5 x} - 5 \int \frac{dx}{\sin^5 x}$$

By IBP

$$= \frac{\tan x}{\sin^5 x} - \int -\frac{5}{\sin^6 x} \cdot \cos x \cdot \tan x dx - 5 \int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + c$$

$$f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{2^5}{\sqrt{3}} - (\sqrt{2})^5 = 4\sqrt{2} - \frac{32}{\sqrt{3}}$$

$$= \frac{32}{\sqrt{3}} - 4\sqrt{2}$$

$$= \frac{4}{\sqrt{3}}(8 - \sqrt{6})$$

12. Let f be a polynomial function such that $f(x^2 + 1) = x^4 + 5x^2 + 2$, for all $x \in \mathbb{R}$.

Then $\int_0^3 f(x) dx$ is equal to

- (1) $\frac{41}{3}$ (2) $\frac{33}{2}$
(3) $\frac{27}{2}$ (4) $\frac{5}{3}$

Ans. (2)

Sol. $\because f(x^2 + 1) = x^4 + 5x^2 + 2$
{put $x^2 + 1 = t$ }

$$\Rightarrow f(t) = (t - 1)^2 + 5(t - 1) + 2$$

$$\Rightarrow f(t) = t^2 + 3t - 2$$

$$\text{Now, } \int_0^3 f(t) dt = \int_0^3 (t^2 + 3t - 2) dt$$

$$\left[\frac{t^3}{3} + \frac{3t^2}{2} - 2t \right]_0^3$$

$$\left[\frac{27}{3} + \frac{27}{2} - 6 \right]$$

$$= \frac{33}{2}$$

13. The area of the region

$$R = \{(x, y) : xy \leq 8, 1 \leq y \leq x^2, x \geq 0\} \text{ is}$$

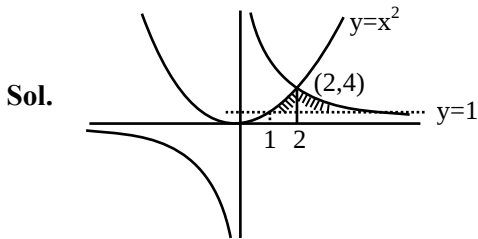
(1) $\frac{1}{3}(49 \log_e(2) - 15)$

(2) $\frac{2}{3}(20 \log_e(2) + 9)$

(3) $\frac{2}{3}(24 \log_e(2) - 7)$

(4) $\frac{1}{3}(40 \log_e(2) + 27)$

Ans. (3)



$$A = \int_1^2 (x^2 - 1) dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx$$

$$A = 8 \log_e 4 - \frac{14}{3} = 16 \log_e 2 - \frac{14}{3}$$

$$= \frac{2}{3}(24 \log_e 2 - 7)$$

14. The value of

$$\lim_{x \rightarrow 0} \frac{\log_e(\sec(ex) \cdot \sec(e^2x) \cdot \dots \cdot \sec(e^{10}x))}{e^2 - e^{2 \cos x}}$$

is equal to

(1) $\frac{(e^{10} - 1)}{2e^2(e^2 - 1)}$

(2) $\frac{(e^{20} - 1)}{2e^2(e^2 - 1)}$

(3) $\frac{(e^{20} - 1)}{2(e^2 - 1)}$

(4) $\frac{(e^{10} - 1)}{2(e^2 - 1)}$

Ans. (3)

Sol. $\Rightarrow \lim_{x \rightarrow 0} \frac{\ln(\sec(ex)) + \ln(\sec(e^2x)) + \dots + \ln(\sec(e^{10}x))}{e^{2 \cos x} \left(\frac{e^{2-2 \cos x} - 1}{2 - 2 \cos x} \right) \times \frac{2 - 2 \cos x}{x^2} \times x^2}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\ell n(\sec(ex)) + \ell n(\sec(e^2x)) + \dots + \ell n(\sec(e^{10}x))}{e^2 x^2}$$

Using L'H rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{e \tan ex + e^2 \tan e^2x + \dots + e^{10} \tan e^{10}x}{2e^2x}$$

$$\Rightarrow \frac{1}{2e^2} [e^2 + e^4 + e^6 + \dots + e^{20}]$$

$$\Rightarrow \frac{1}{2} \frac{e^2((e^2)^{10} - 1)}{e^2(e^2 - 1)}$$

$$\Rightarrow \frac{1}{2} \frac{(e^{20} - 1)}{(e^2 - 1)}$$

15. The mean and variance of 10 observations are 9 and 34.2, respectively. If 8 of these observations are 2, 3, 5, 10, 11, 13, 15, 21, then the mean deviation about the median of all the 10 observations is

(1) 5 (2) 4

(3) 6 (4) 7

Ans. (1)

Sol. $\frac{2+3+5+10+11+13+15+21+a+b}{10} = 9$

$$\frac{80+a+b}{10} = 9 \Rightarrow a+b = 10 \dots (1)$$

$$\frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10} \right)^2 = 34.2$$

$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10} - (9)^2 = 34.2$$

$$1094 + a^2 + b^2 - 810 = 342$$

$$a^2 + b^2 = 58 \dots (2)$$

$$a = 7, b = 3$$

$$\text{or } a = 3, b = 7$$

$$\text{Number} \rightarrow 2, 3, 5, 7, 10, 11, 13, 15, 21$$

$$\text{Mean} = \frac{7+10}{2} = 8.5$$

$$\text{M.D} = \frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}$$

$$= \frac{50}{5}$$

$$= 5$$



16. Let A, B and C be three 2×2 matrices with real entries such that $B = (I + A)^{-1}$ and $A + C = I$. If

$$BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \text{ and } CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}, \text{ then}$$

$x_1 + x_2$ is

- (1) 2 (2) 0
(3) -2 (4) 4

Ans. (2)

Sol. $B = (I + A)^{-1}$, $A + C = I$

$$\Rightarrow B(I + A) = (I + A)B = I$$

$$\Rightarrow B + BA = B + AB$$

$$\Rightarrow B + B(I - C) = B + (I - C)B$$

$$\Rightarrow 2B - BC = 2B - CB$$

$$\Rightarrow BC = CB$$

$$\therefore CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ -6 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 32 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \therefore x_1 + x_2 = 0$$

17. The common difference of the A.P.: $a_1, a_2, \dots, a_n$ is 13 more than the common difference of the A.P.: $b_1, b_2, \dots, b_n$. If $b_{31} = -277$, $b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

- (1) 21 (2) 24
(3) 19 (4) 16

Ans. (3)

Sol. Let common difference of A.P.'s are d_1 & d_2

$$\therefore d_1 = 13 + d_2$$

$$b_1 + 30d_2 = -277 \quad \dots(1)$$

$$b_1 + 42d_2 = -385 \quad \dots(2)$$

$$\text{By (2) - (1)}$$

$$12d_2 = -108$$

$$d_2 = -9$$

$$\therefore \boxed{d_1 = 4}$$

$$\text{Now } a_{78} = 327$$

$$\Rightarrow a_1 + 77d_1 = 327$$

$$\Rightarrow a_1 + 308 = 327$$

$$a_1 = 19$$

18. If the distances of the point $(1, 2, a)$ from the line

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1} \text{ along the lines}$$

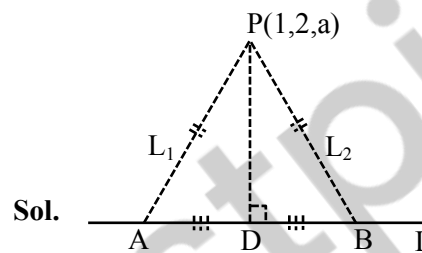
$$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} \text{ and}$$

$$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} \text{ are equal,}$$

then $a + b + c$ is equal to

- (1) 7 (2) 5
(3) 6 (4) 4

Ans. (1)



Sol.

$$L: \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$$

$$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$$

$$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$$

$$\text{Let } A(3\lambda + 1, 4\lambda + 2, b\lambda + a)$$

It lies on L

$$\therefore \frac{3\lambda}{1} = \frac{4\lambda + 2}{2} = \frac{b\lambda + a - 1}{1}$$

$$\Rightarrow \boxed{\lambda = 1} \text{ and } a + b - 1 = 3$$

$$\Rightarrow A(4, 6, 4), a + b = 4 \quad \dots(1)$$

$$\text{Let } B(\mu + 1, 4\mu + 2, c\mu + a)$$

It also lies on L

$$\frac{\mu}{1} = \frac{4\mu + 2}{2} = \frac{c\mu + a - 1}{1}$$

$$\Rightarrow 2\mu = 4\mu + 2$$

$$\Rightarrow \boxed{\mu = -1}$$

$$a - c - 1 = -1$$

$$\Rightarrow \boxed{a = c} \quad \dots(2) \text{ \& } B(0, -2, 0)$$



also $PA = PB$, $P(1, 2, a)$, $A(4, 6, 4)$

$$\Rightarrow 9 + 16 + (a - 4)^2 = 1 + 16 + a^2$$

$$\Rightarrow 16 + 8 = 8a$$

$$\boxed{a=3} \quad \therefore c = 3, b = 1$$

$$\therefore \boxed{a + b + c = 7}$$

19. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9 \text{ and}$$

$$|2\vec{a} + k\vec{b} + k\vec{c}| = 3,$$

the positive value of k is :

$$(1) 3 \quad (2) 6$$

$$(3) 4 \quad (4) 5$$

Ans. (4)

Sol. $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a}$$

$$|2\vec{a} + k(\vec{b} + \vec{c})| = 3$$

$$|\vec{a}(2 - k)| = 3$$

$$K = 5 \text{ or } -1$$

Positive value of k is 5

20. Let $y = y(x)$ be the solution of the differential equation

$$x \frac{dy}{dx} - \sin 2y = x^3(2 - x^3) \cos^2 y, \quad x \neq 0.$$

If $y(2) = x$, then $\tan(y(1))$ is equal to

$$(1) \frac{3}{4} \quad (2) \frac{7}{4}$$

$$(3) -\frac{7}{4} \quad (4) -\frac{3}{4}$$

Ans. (2)

Sol. $x \frac{dy}{dx} - \sin 2y = x^3(2 - x^3) \cos^2 y$

$$\sec^2 y \frac{dy}{dx} - 2 \tan y \cdot \frac{1}{x} = x^2(2 - x^3)$$

$$\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - \frac{2t}{x} = x^2(2 - x^3) \quad (\text{LDE})$$

$$\text{I.F.} = e^{\int -\frac{2}{x} dx} = e^{-2 \ln x} = \frac{1}{x^2}$$

$$\therefore \frac{t}{x^2} = \int \frac{1}{x^2} x^2 (2 - x^3) dx + C$$

$$\frac{\tan y}{x^2} = 2x - \frac{x^4}{4} + C$$

$$y(2) = 0 \Rightarrow 0 = 4 - 4 + C \Rightarrow C = 0$$

$$\tan y = 2x^3 - \frac{1}{4}x^6$$

$$x = 1 \Rightarrow \tan y = 2 - \frac{1}{4} = \frac{7}{4} \Rightarrow (2)$$

SECTION-B

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R} - (a, b)$, then $a^2 + b^2$ is equal to ____.

Ans. (90)

Sol. Let first three terms of G.P. are $\frac{A}{r}, A, Ar$

$$\frac{A}{r} \cdot A \cdot Ar = 27$$

$$A = 3$$

$$3\left(\frac{1}{r} + 1 + r\right) = 3 + 3\left(r + \frac{1}{r}\right)$$

$$\text{We know, } r + \frac{1}{r} \geq 2 \text{ or } r + \frac{1}{r} \leq -2$$

$$S \in \mathbb{R} - (-3, 9)$$

$$a^2 + b^2 = 9 + 81 = 90$$

22. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2 \sec^2 \theta = 8$ be e_1 and ℓ_1 , respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2 \sec^2 \theta + y^2 = 6$ be e_2 and ℓ_2 , respectively. If $e_1^2 = e_2^2 (\sec^2 \theta + 1)$, then $\left(\frac{\ell_1 \ell_2}{e_1 e_2}\right) \tan^2 \theta$ is equal to _____.

Ans. (8)

Sol. $\frac{x^2}{8} - \frac{y^2}{8 \cos^2 \theta} = 1$, $e_1 = \sqrt{1 + \frac{8 \cos^2 \theta}{8}}$

$$\ell_1 = \frac{2b^2}{a} = \frac{2 \cdot (8 \cos^2 \theta)}{2\sqrt{2}}$$

$$\frac{x^2}{6} + \frac{y^2}{6 \cos^2 \theta} = 1$$
; $e_2 = \sqrt{1 + \frac{6 \cos^2 \theta}{6}} = \sec \theta$

$$\ell_2 = \frac{2b^2}{a} = \frac{2 \cdot 6 \cos^2 \theta}{\sqrt{6}}$$

$$e_1^2 = e_2^2 (1 + \sec^2 \theta)$$

$$1 + \cos^2 \theta = \sec^2 \theta \left(1 + \frac{1}{\cos^2 \theta}\right)$$

$$1 + \cos^2 \theta = \sec^2 \theta + \tan^2 \theta$$

Solving we get $\theta = \frac{\pi}{4}$

$$\ell_1 = 2\sqrt{2}$$

$$e_1 = \sqrt{\frac{3}{2}}$$

$$\ell_2 = \sqrt{6}$$

$$e_2 = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\ell_1 \ell_2}{e_1 e_2}\right) \tan^2 \theta = 8 \quad \text{(By putting values)}$$

23. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{2}{3}\right)\right)$ then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$ is _____.

Ans. (1)

Sol. Let $\theta = \frac{1}{2} \sin^{-1} \frac{2}{3}$, then $\frac{1}{2} \cos^{-1} \frac{1}{3} = \left(\frac{\pi}{4} - \theta\right)$

$$k = \tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta} = \frac{2}{\sin 2\theta}$$

$$k = \frac{2}{\frac{2}{3}} = 3$$

$$\sin^{-1}(3x - 1) = \sin^{-1} x - \cos^{-1} x$$

$$\sin^{-1}(3x - 1) = \frac{\pi}{2} - 2\cos^{-1} x$$

$$3x - 1 = \sin\left(\frac{\pi}{2} - 2\cos^{-1} x\right)$$

$$3x - 1 = 2x^2 - 1 \Rightarrow x = 0, \frac{3}{2} \text{ (rejected)}$$

No. of solution = 1

24. The value of $\sum_{r=1}^{20} \left(\left| \sqrt{\pi \left(\int_0^r x |\sin \pi x| dx \right)} \right| \right)$ is _____.

Ans. (210)

Sol. Let $I_r = \int_0^r x |\sin \pi x| dx \quad \dots (1)$

Apply King Property

$$= \int_0^r (r - x) |\sin \pi x| dx \quad \dots (2)$$

By (1) + (2)

$$2I_r = \int_0^r r |\sin \pi x| dx \Rightarrow I_r = \frac{r}{2} \int_0^r |\sin \pi x| dx$$

$$I_1 = \frac{1}{2} \int_0^1 |\sin \pi x| dx = \frac{1}{2\pi} \int_0^\pi |\sin t| dt = \frac{1}{2\pi} (2)$$

$$I_2 = \frac{2}{2} \int_0^2 |\sin \pi x| dx = \frac{2}{2\pi} \int_0^{2\pi} |\sin t| dt = \frac{2}{2\pi} (4)$$

$$S = \sqrt{\pi \cdot \frac{1}{2\pi} \cdot 2} + \sqrt{\pi \cdot \frac{2}{2\pi} \cdot 4} + \sqrt{\pi \cdot \frac{3}{2\pi} \cdot 6} + \dots + \sqrt{\pi \cdot \frac{20}{2\pi} \cdot (2 \cdot 20)}$$

$$= 1 + 2 + 3 + \dots + 20$$

$$= \frac{20 \times 21}{2} = 210$$

25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$, $a, b \in \mathbb{Z}$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ____.

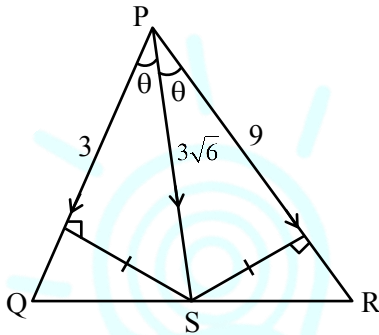
QuestPix Ans. (Bonus)

NTA Ans. (37)

Sol. $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$

$$\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k} \quad (a, b \in \mathbb{Z})$$

$$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$$



$$|\overrightarrow{PR}| = 9$$

$$a^2 + b^2 + 16 = 81$$

$$a^2 + b^2 = 65 \quad \dots(1)$$

$$\cos \theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PS}}{|\overrightarrow{PQ}| |\overrightarrow{PS}|}$$

$$= \frac{-2 + 7 + 4}{3 \cdot 3\sqrt{6}} = \frac{9}{3 \cdot 3\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}$$

$$\frac{1}{\sqrt{6}} = \frac{\overrightarrow{PS} \cdot \overrightarrow{PR}}{|\overrightarrow{PS}| |\overrightarrow{PR}|} = \frac{a - 7b - 8}{3\sqrt{6} \cdot 9}$$

$$a - 7b = 35 \quad \dots(2)$$

From (1) & (2)

$$\Rightarrow a = 7, b = -4$$

$$\therefore 3a - 4b = 21 + 16 = 37$$

(Matched with NTA)

But here PS is internal Angle bisector of $\angle QPR$ hence

$$PS = \frac{2PQ \cdot PR}{PQ + PR} \cos \theta$$

$$\Rightarrow 3\sqrt{6} = \frac{2 \times 3 \times 9}{3 + 9} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{2\sqrt{6}}{3} = \frac{2\sqrt{2}}{\sqrt{3}} > 1 \text{ (Not possible)}$$

Hence NO such triangle is possible

Another way to check

For evaluated value of a & b [$a = 7, b = -4$]

$$\overrightarrow{PR} = 7\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$$

$$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$$

$$\therefore \overrightarrow{QS} = \overrightarrow{PS} - \overrightarrow{PQ} = 3\hat{i} - 6\hat{j}$$

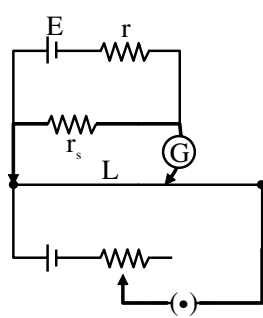
$$\overrightarrow{SR} = \overrightarrow{PR} - \overrightarrow{PS} = 6\hat{i} + 3\hat{j} - 6\hat{k}$$

$$\therefore \overrightarrow{QS} \text{ is not parallel to } \overrightarrow{SR}$$

$$\therefore Q, S, R \text{ are not co-linear}$$

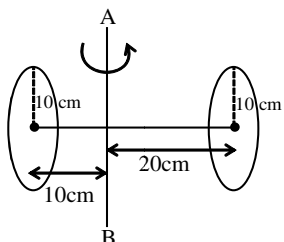
(Bonus)

(HELD ON WEDNESDAY 28th JANUARY 2026)
TIME : 9:00 AM TO 12:00 NOON

PHYSICS	TEST PAPER WITH SOLUTION
SECTION-A 26. 10 kg of ice at -10°C is added to 100 kg of water to lower its temperature from 25°C. Consider no heat exchange to surroundings. The decrement to the temperature of water is _____ $^{\circ}\text{C}$. (specific heat of ice = $2100 \text{ J/Kg}^{\circ}\text{C}$, specific heat of water = $4200 \text{ J/Kg}^{\circ}\text{C}$, latent heat of fusion of ice = $3.36 \times 10^5 \text{ J/Kg}$) (1) 10 (2) 15 (3) 6.67 (4) 11.6 Ans. (1) Sol. $10 \times 3.36 \times 10^5 + 10 \times 2100 \times 10 + 10 \times 4200 \times (T-0)$ $= 100 \times 4200 \times (25 - T)$ $\Rightarrow T = 15^{\circ}\text{C}$ $\Delta T = 25 - 15 = 10^{\circ}\text{C}$ 27. The electric current in the circuit is given as $i = i_0(t/T)$. The r.m.s current for the period $t = 0$ to $t = T$ is _____ (1) $\frac{i_0}{\sqrt{2}}$ (2) i_0 (3) $\frac{i_0}{\sqrt{6}}$ (4) $\frac{i_0}{\sqrt{3}}$ Ans. (4) Sol. $i_{\text{rms}}^2 = \frac{\int_0^T (i_0^2 t^2 / T^2) dt}{\int_0^T dt} = \frac{i_0^2}{T^3} \cdot \frac{T^3}{3} = \frac{i_0^2}{3}$ $i_{\text{rms}} = \frac{i_0}{\sqrt{3}}$	28. In the potentiometer, when the cell in the secondary circuit is shunted with 4Ω resistance, the balance is obtained at the length 120 cm of wire. Now when the same cell in shunted with 12Ω resistance, the balance is shifted to a length of 180 cm. The internal resistance of cell is ____ Ω. (1) 3 (2) 4 (3) 12 (4) 6 Ans. (2) Sol. Let E is emf and r is internal resistance of cell.  $\frac{E \cdot 4}{r + 4} = 120 \text{ K}$ $\frac{E \cdot 12}{r + 12} = 180 \text{ K}$ $\Rightarrow \frac{1}{3} \frac{r + 12}{r + 4} = \frac{2}{3}$ $r + 12 = 2(r + 4)$ $\Rightarrow r = 4$

29. Two circular discs of radius each 10 cm are joined at their centres by a rod of length 30 cm and mass 600 gm as shown in figure.

If the mass of each disc is 600 gm and applied torque between two discs is 43×10^5 dyne cm, the angular acceleration of the discs about the given axis AB is _____ rad/s^2 .



- (1) 22 (2) 11
(3) 100 (4) 27

Ans. (2)

Sol. $\alpha = \frac{\tau}{I}$

$$I = \frac{1}{4}mR^2 + mR^2 + \frac{1}{4}mR^2 + m(2R)^2 + \frac{m(3R)^2}{12} + m\left(\frac{R}{2}\right)^2$$

$$= \left(\frac{3}{2} + 4 + 1\right)mR^2 = \frac{13}{2}mR^2 = \frac{13}{2} \times 600 \times 10^2 = 39 \times 10^4$$

$$\alpha = \frac{43 \times 10^5}{39 \times 10^4} \text{ rad/s}^2 = \frac{430}{39} \text{ rad/s}^2 \approx 11 \text{ rad/s}^2$$

30. Water drops fall from a tap on the floor, 5 m below, at regular intervals of time, the first drop strikes the floor when the sixth drop begins to fall. The height at which the fourth drop will be from ground, at the instant when the first drop strikes the ground is _____ m.

($g = 10 \text{ m/s}^2$)

- (1) 2.5 (2) 4.0
(3) 4.2 (4) 3.8

Ans. (3)

Sol. Time to reach ground $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ sec}$

Five drops per second

Time between each drop = 0.2 sec.

Time of fall for 4th drop is $1 - 0.6 = 0.4 \text{ sec}$

Height fall of 4th drop is $= \frac{1}{2} \times 10 \times 0.4^2 = 0.8 \text{ m}$

Height from ground = $5 - 0.8 = 4.2 \text{ m}$

31. An atom 8_3X is bombarded by shower of fundamental particles and in 10 s this atom absorbed 10 electrons, 10 protons and 9 neutrons. The percentage growth in the surface area of the nucleons is recorded by :

- (1) 250% (2) 150%
(3) 225% (4) 900%

NTA Ans. (3)

Allen Ans. (BONUS)

Sol. Surface area $\propto A^{2/3}$

$$X_i = 8^{2/3} K = 4K$$

$$X_f = (8 + 10 + 9)^{2/3} K = 9K$$

% increase in surface area of nucleus

$$x_i = \frac{9K - 4K}{4K} \times 100 = 125\%$$

32. The electric field of an electromagnetic wave travelling through a medium is given by

$$\vec{E}(x, t) = 25 \sin(2.0 \times 10^{15} t - 10^7 x) \hat{n}$$

then the refractive index of the medium is _____.

(All given measurement are in SI units)

- (1) 1.2 (2) 2
(3) 1.5 (4) 1.7

Ans. (3)

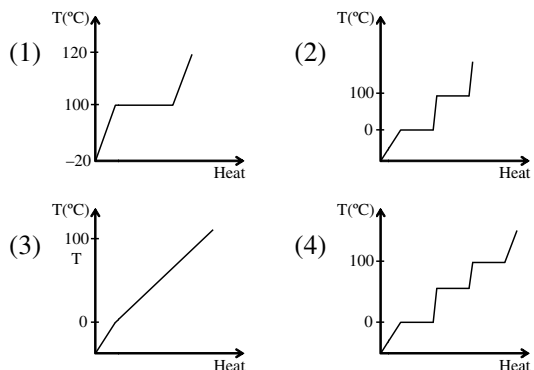
Sol. $\omega = 2 \times 10^{15} \text{ rad/s}$

$$k = 10^7 \text{ m}^{-1}$$

$$v = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k} = \frac{2 \times 10^{15}}{10^7} = 2 \times 10^8 = \frac{c}{1.5}$$

$$\Rightarrow \mu = 1.5$$

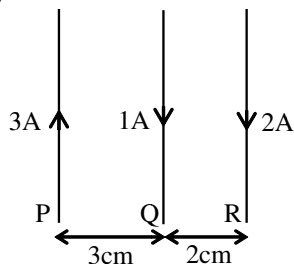
33. Which of the following best represents the temperature versus heat supplied graph for water, in the range of -20°C to 120°C ?



Ans. (2)

Sol. 2

34. Three long straight wires carrying current are arranged mutually parallel as shown in the figure. The force experienced by 15 cm length of wire Q is _____.



$$(\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A})$$

- (1) $6 \times 10^{-7} \text{ N}$ towards P
 (2) $6 \times 10^{-6} \text{ N}$ towards R
 (3) $6 \times 10^{-7} \text{ N}$ towards R
 (4) $6 \times 10^{-6} \text{ N}$ towards P

Ans. (2)

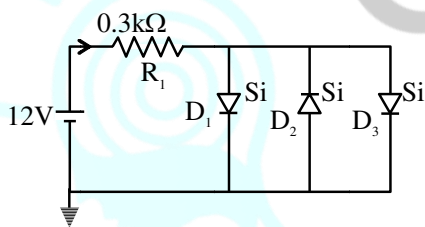
Sol. $F_{\text{net}} = \frac{\mu_0}{2\pi} I_0 \left(\frac{I_1}{d_1} + \frac{I_2}{d_2} \right) \ell$

$$F_{\text{net}} = 2 \times 10^{-7} \times 1 \left(\frac{3}{3} + \frac{2}{2} \right) \times \frac{15 \times 10^{-2}}{10^{-2}}$$

$$= 4 \times 15 \times 10^{-7}$$

$$F_{\text{net}} = 6 \times 10^{-6} \text{ N}$$

35. Assuming in forward bias condition there is a voltage drop of 0.7 V across a silicon diode, the current through diode D_1 in the circuit is _____ mA. (Assume all diodes in the given circuit are identical)



- (1) 20.15
 (2) 11.7
 (3) 17.6
 (4) 18.8

Ans. (4)

Sol. $12 - 0.3 \times 10^3 I - 0.7 = 0$

$$\frac{11.3}{0.3 \times 10^3} = I$$

$$37.66 \times 10^{-3} \text{ A} = I$$

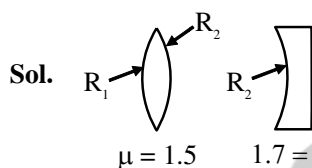
Current through diode D_1 , $I_1 = I/2$

$$I_1 = 18.83 \text{ mA}$$

36. The magnitudes of power of a biconvex lens (refractive index 1.5) and that of a plano-concave lens (refractive index = 1.7) are same. If the curvature of plano-concave lens exactly matches with the curvature of back surface of the biconvex lens, then ratio of radius of curvature of front and back surface of the biconvex lens is _____.

- (1) 5 : 2
 (2) 5 : 12
 (3) 12 : 5
 (4) 2 : 5

Ans. (1)



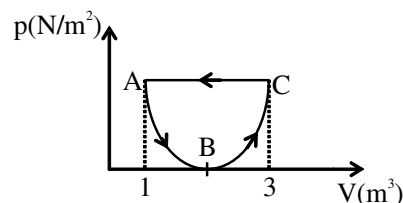
$$|P_A| = |P_B|$$

$$0.5 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{0.7}{R_2}$$

$$\frac{5}{R_1} = \frac{2}{R_2}$$

$$\frac{R_1}{R_2} = \frac{5}{2}$$

37. In the following p - V diagram the equation of state along the curved path is given by $(V - 2)^2 = 4ap$ where a is a constant. The total work done in the closed path is



- (1) $-\frac{1}{a}$
 (2) $+\frac{1}{3a}$
 (3) $\frac{1}{2a}$
 (4) $-\frac{1}{3a}$

Ans. (4)

Sol. $w = \text{Area of parabola}$

$$= \frac{2}{3} (\text{Area of rectangle AC31A})$$

$$= \frac{2}{3} P_0 (3-1) = \frac{4P_0}{3}$$

When $V = 1$

$$(1-2)^2 = 4aP_0$$

$$P_0 = \frac{1}{4a}$$

$$w = \frac{4}{3} P_0 = \frac{4}{3} \frac{1}{4a} = \frac{1}{3a}$$

$$\boxed{w_{\text{gas}} = \frac{-1}{3a}}$$

- 38.** For the two cells having same EMF E and internal resistance r , the current passing through the external resistor 6Ω is same when both the cells are connected either in parallel or in series. The value of internal resistance r is _____ Ω .

- (1) 3 (2) 4
(3) 9 (4) 6

Ans. (4)

Sol. In series, $i_1 = \frac{2E}{6+2r}$

$$\text{In parallel, } i_2 = \frac{E}{6+\frac{r}{2}}$$

$$i_1 = i_2 \Rightarrow \frac{2E}{6+2r} = \frac{E}{6+\frac{r}{2}}$$

$$12+r=6+2r$$

$$\boxed{r=6\Omega}$$

- 39.** Two wires A and B made of different materials of length 6.0 cm and 5.4 cm , respectively and area of cross sections $3.0 \times 10^{-5} \text{ m}^2$ and $4.5 \times 10^{-5} \text{ m}^2$, respectively are stretched by the same magnitude under a given load. The ratio of the Young's modulus of A to that of B is $x : 3$. The value of x is _____.

- (1) 1 (2) 4
(3) 2 (4) 5

Ans. (4)

Sol. $T = \frac{F/A}{\Delta \ell / \ell} \Rightarrow Y = \frac{F\ell}{A\Delta \ell}$

$$\frac{Y_A}{Y_B} = \frac{\ell_A}{\ell_B} \left(\frac{A_B}{A_A} \right)$$

$$= \frac{6}{5.4} \left(\frac{4.5 \times 10^{-5}}{3 \times 10^{-5}} \right) = \frac{9}{5.4} = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3}$$

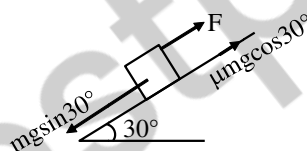
$$\boxed{x=5}$$

- 40.** A block of mass 5 kg is moving on an inclined plane which makes an angle of 30° with the horizontal. Friction coefficient between the block and inclined plane surface is $\frac{\sqrt{3}}{2}$. The force to be applied on the block so that the block will move down without acceleration is _____ N .

- (1) 25 (2) 12.5
(3) 7.5 (4) 15

Ans. (2)

Sol.



$$mgsin30^\circ = F + \mu mgcos30^\circ$$

$$F = 5 \times 10 \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times 5 \times 10 \times \frac{\sqrt{3}}{2}$$

$$F = 25 - \frac{75}{2} = 25 - 37.5$$

$$F = -12.5 \text{ N}$$

$\therefore$ force will be downward on incline of magnitude 12.5 N

- 41.** Given below are two statements :

Statement-I : A plane wave after passing through prism remains as plane wave but passing through small pin hole may become spherical wave.

Statement-II : The curvature of a spherical wave emerging from a slit will increase for increasing slit width.

In the light of the above statements, choose the **correct** answer from the options given below.

- (1) Both Statement-I and Statement-II are false.
(2) Both Statement-I and Statement-II are true.
(3) Statement-I is true but Statement-II is false.
(4) Statement-I is false but Statement-II is true.

Ans. (3)

Sol. Increasing the slit width 'a' decreases the diffraction angle ($\theta = \lambda/a$) and reduces the spreading of the wave. A narrower slit produces a more pronounced spherical wave (high curvature) while a wider slit leads to a flatter, less curved wave.

42. When both jaws of vernier callipers touch each other, zero mark of the vernier scale is right to zero mark of main scale, 4th mark on vernier scale coincides with certain mark on the main scale. While measuring the length of a cylinder, observer observes 15 divisions on main scale and 5th division of vernier scale coincides with a main scale division. Measured length of cylinder is _____ mm. (Least count of Vernier calliper = 0.1 mm)

- (1) 15.4 (2) 15.1
(3) 15.5 (4) 15.9

Ans. (2)

Sol. Reading = MSR + (VSR × LC) – (zero Error)
= 15 mm + (5 × 0.1 mm) – (4 × 0.1 mm)
Reading = 15.1 mm

$$\therefore \ell = 15.1 \text{ mm}$$

43. The magnetic field at the centre of a current carrying circular loop of radius R is $16 \mu\text{T}$. The magnetic field at a distance $x = \sqrt{3}R$ on its axis from the centre is _____ μT .

- (1) $2\sqrt{2}$ (2) 4
(3) 2 (4) 8

Ans. (3)

Sol. $\frac{\mu_0 I}{2R} = 16 \mu\text{T}$
 $\frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{2 \times 8R^3} = 2 \mu\text{T}$

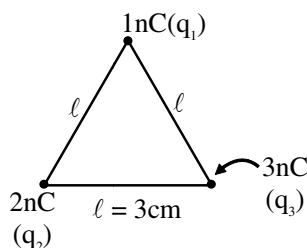
44. Two point charges of 1 nC and 2 nC are placed at the two corners of equilateral triangle of side 3 cm. The work done in bringing a charge of 3 nC from infinity to the third corner of the triangle is _____ μJ .

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N.m}^2 / \text{C}^2$$

- (1) 2.7 (2) 5.4
(3) 3.3 (4) 27

Ans. (1)

Sol.

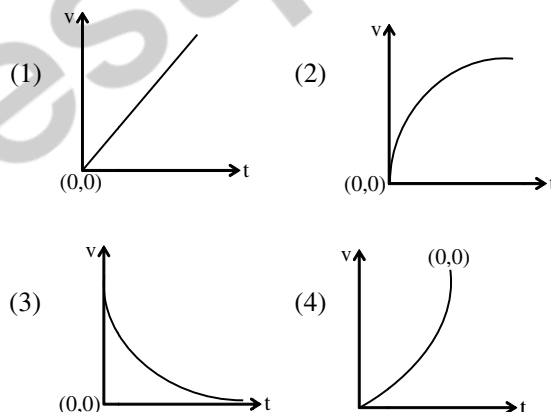


$$W = \left(\frac{kq_1}{\ell} + \frac{kq_2}{\ell} \right) q_3$$

$$= \frac{9 \times 10^9}{3 \times 10^{-2}} (3 \times 10^{-9}) \times 3 \times 10^{-9}$$

$$= 27 \times 10^{-7} \text{ J} = 2.7 \mu\text{J}$$

45. A particle of mass m falls from rest through a resistive medium having resistive force, $F = -kv$, where v is the velocity of the particle and k is a constant. Which of the following graphs represents velocity (v) versus time (t) ?



Ans. (2)

Sol. $m \cdot \frac{dv}{dt} = mg - kv$

$$\int_0^v \frac{dv}{mg - kv} = \int_0^t \frac{dt}{m}$$

$$\frac{-1}{k} \ln \left(\frac{mg - kv}{mg} \right) = \frac{t}{m}$$

$$v = \frac{mg}{k} (1 - e^{-kt/m})$$

SECTION-B

46. The displacement of a particle, executing simple harmonic motion with time period T , is expressed as $x(t) = A \sin \omega t$, where A is the amplitude. The maximum value of potential energy of this oscillator is found at $t = T/2\beta$. The value of β is _____.

Ans. (2)

Sol. Potential energy is maximum at extreme position
The particle starting at mean position reaches extreme position in time $\frac{T}{4}$.

47. The ratio of de Broglie wavelength of a deuteron with kinetic energy E to that of an alpha particle with kinetic energy $2E$, is $n : 1$. The value of n is _____.

(Assume mass of proton = mass of neutron) :

Ans. (2)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m \cdot KE}}$

$$\frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{m_\alpha \cdot KE_\alpha}{m_d \cdot KE_d}} = \sqrt{\frac{4m \cdot 2E}{2m \cdot E}} = 2 : 1$$

48. A solid sphere of radius 10 cm is rotating about an axis which is at a distance 15 cm from its centre. The radius of gyration about this axis is $\sqrt{n}$ cm. The value of n is _____.

Ans. (265)

Sol. Let radius of gyration is k

$$\Rightarrow mk^2 = \frac{2}{3}mR^2 + md^2$$

$$k^2 = \frac{2}{3} \times 10^2 + 15^2 = 265$$

$$(\sqrt{n})^2 = 265 \Rightarrow n = 265$$

49. A convex lens of refractive index 1.5 and focal length $f = 18$ cm is immersed in water. The difference in focal lengths of the given lens when it is in water and in air is $\alpha \times f$. The value of α is _____.

(refractive index of water = $4/3$)

Ans. (3)

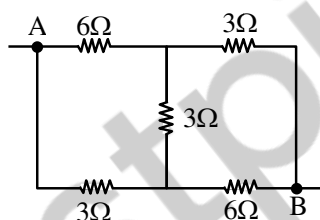
Sol. $\frac{1}{f_{\text{Air}}} = \left(\frac{1.5-1}{1} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$

$$\frac{1}{f_{\text{water}}} = \left(\frac{1.5-4/3}{4/3} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{f_{\text{water}}}{f_{\text{air}}} = \frac{0.5}{0.5/4} = 4$$

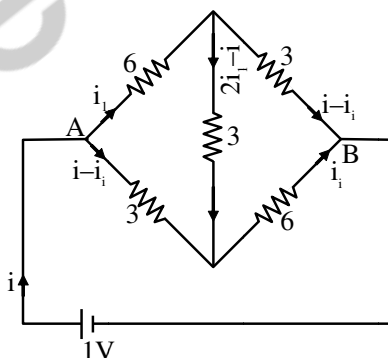
$$\Rightarrow f_{\text{water}} - f_{\text{air}} = 3f$$

50. The equivalent resistance between the points A and B in the following circuit is $\frac{x}{5} \Omega$. The value of x is _____.



Ans. (21)

Sol.



$$6i_1 + 3(2i_1 - i) = 3(i - i_1)$$

$$\Rightarrow 15i_1 = 6i \Rightarrow i_1 = \frac{2}{5}i \quad \text{--- (1)}$$

$$3(i - i_1) + 6i_1 = 1$$

$$3i + 3i_1 = 1$$

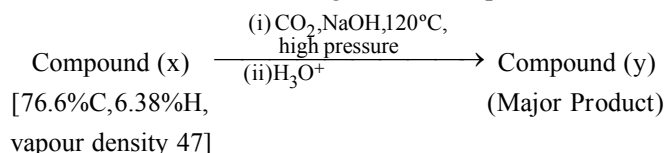
$$\left(3 + \frac{6}{5} \right) i = 1$$

$$\Rightarrow i = \frac{5}{21} A = \frac{1V}{R_{\text{eq}}} \Rightarrow R_{\text{eq}} = \frac{21}{5} \Omega$$

(HELD ON WEDNESDAY 28TH JANUARY 2026)
TIME : 9:00 AM TO 12 :00 NOON

CHEMISTRY	TEST PAPER WITH SOLUTION
SECTION-A 51. 20.0 dm³ of an ideal gas 'X' at 600 K and 0.5 MPa undergoes isothermal reversible expansion until pressure of the gas is 0.2 MPa. Which of the following option is correct ? (Given: log 2 = 0.3010 and log 5 = 0.6989) (1) $w = -9.1$ kJ, $\Delta U = 0$, $\Delta H = 0$, $q = 9.1$ kJ (2) $w = 9.1$ J, $\Delta U = 9.1$ J, $\Delta H = 0$; $q = 0$ (3) $w = +4.1$ kJ, $\Delta U = 0$, $\Delta H = 0$; $q = -4.1$ kJ (4) $w = -3.9$ kJ, $\Delta U = 0$, $\Delta H = 0$; $q = 3.9$ kJ Ans. (1) Sol. For isothermal reversible process $\Delta U = \Delta H = 0$ $w_{\text{iso}} = -p_1 V_1 \ln \frac{p_1}{p_2}$ $w_{\text{iso}} = -0.5 \times 10^6 \times 20 \times 10^{-3} \ln \frac{0.5}{0.2}$ $w_{\text{iso}} = -0.5 \times 10^6 \times 20 \times 10^{-3} \times 2.303 \times (.6989 - .3010)$ $w \approx -9.1 \text{ kJ}$ $q = -w = 9.1 \text{ kJ}$ 52. CORRECT order of stability for the following is $\text{CH}_2 = \text{CH}^-$, $\text{CH}_3 - \text{CH}_2^-$, $\text{CH} \equiv \text{C}^-$ (1) $\text{CH}_3 - \text{CH}_2^- > \text{CH}_2 = \text{CH}^- > \text{CH} \equiv \text{C}^-$ (2) $\text{CH}_2 = \text{CH}^- > \text{CH} \equiv \text{C}^- > \text{CH}_3 - \text{CH}_2^-$ (3) $\text{CH} \equiv \text{C}^- > \text{CH}_2 = \text{CH}^- > \text{CH}_3 - \text{CH}_2^-$ (4) $\text{CH} \equiv \text{C}^- > \text{CH}_3 - \text{CH}_2^- > \text{CH}_2 = \text{CH}^-$ Ans. (3) Sol. $\text{CH}_2 = \text{CH}^-$ $\text{CH}_3 = \text{CH}_2^-$ $\text{CH} \equiv \text{C}^-$ Stability $\propto \%S$ Order of stability $\text{CH} \equiv \text{C}^- > \text{CH}_2 = \text{CH}^- > \text{CH}_3 - \text{CH}_2^-$	53. At T(K), 2 moles of liquid A and 3 moles of liquid B are mixed. The vapour pressure of ideal solution formed is 320 mm Hg. At this stage, one mole of A and one mole of B are added to the solution. The vapour pressure is now measured as 328.6 mm Hg. The vapour pressure (in mm Hg) of A and B are respectively: (1) 300, 200 (2) 600, 400 (3) 400, 300 (4) 500, 200 Ans. (4) Sol. 2 moles of A + 3 moles of B $\downarrow$ $X_A = 2/5$, $X_B = 3/5$ $P_S = X_A P_A^\circ + X_B P_B^\circ$ $320 = P_A^\circ \left(\frac{2}{5}\right) + P_B^\circ \left(\frac{3}{5}\right)$ $2P_A^\circ + 3P_B^\circ = 1600 \dots\dots(I)$ Now 1 mole of A & 1 mole of B is added $X'_A = \frac{3}{7}, X'_B = \frac{4}{7}$ $P'_S = 328.6 = P_A^\circ \left(\frac{3}{7}\right) + P_B^\circ \left(\frac{4}{7}\right)$ $3P_A^\circ + 4P_B^\circ = 2300.2 \dots\dots(II)$ Now eq (I) $\times 3$ - eq (II) $\times 2$ $6P_A^\circ + 9P_B^\circ = 4800$ $6P_A^\circ + 8P_B^\circ = 4600.4$ $P_B^\circ \approx 200 \text{ mm of Hg}$ $P_A^\circ \approx 500 \text{ mm of Hg}$

54. Consider the following reaction sequence

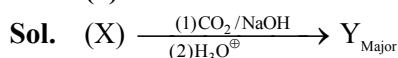


Compound (y) develops characteristic colour with neutral FeCl_3 solution.

Identify the **INCORRECT** statement from the following for the above sequence.

- (1) Both compounds x and y will dissolve in NaOH.
- (2) Compound y will dissolve in NaHCO_3 and evolve a gas.
- (3) Compound x is more acidic than compound y.
- (4) Both compounds x and y will burn with sooty flame.

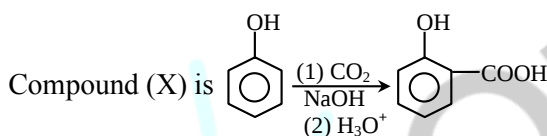
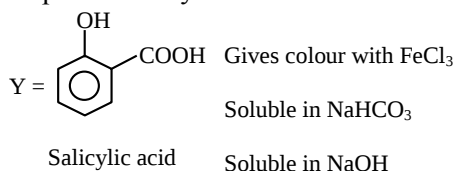
Ans. (3)



76.6% C

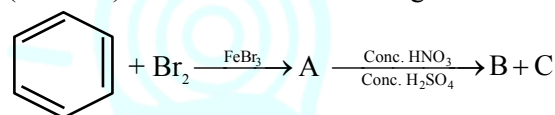
6.38% H

Vapour Density 47



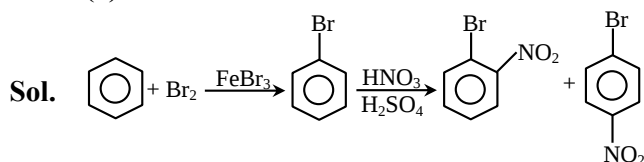
Kolbe Schmitt reaction

55. Method used for separation of mixture of products (B and C) obtained in the following reaction is:



- (1) Simple distillation
- (2) sublimation
- (3) steam distillation
- (4) fractional distillation

Ans. (4)



B & C separate by
Fractional Distillation method
Due to their different boiling point.

56. Consider a weak base 'B' of $\text{pK}_b = 5.699$. 'x' mL of 0.02 M HCl and 'y' mL of 0.02 M weak base 'B' are mixed to make 100 mL of a buffer of pH 9 at 25°C . The values of 'x' and 'y' respectively are:

[Given: $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.699$]

(1)

x	y
11.1	88.9

(2)

x	y
42.7	57.3

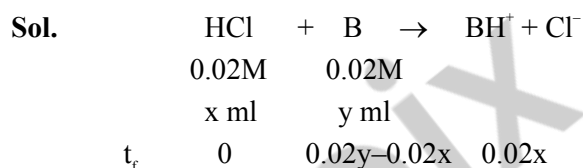
(3)

x	y
14.3	85.7

(4)

x	y
85.7	14.3

Ans. (3)



$$\text{pOH} = \text{pK}_b + \log \left[\frac{\text{Salt}}{\text{Base}} \right]$$

$$5 = 5.699 + \log \left[\frac{\text{Salt}}{\text{Base}} \right]$$

$$\frac{x}{y-x} = \frac{1}{5}$$

$$6x = y$$

$$7x = 100$$

$$x = \frac{100}{7} \text{ ml}$$

$$\& y = \frac{600}{7} \text{ ml}$$

57. An organic compound undergoes first order decomposition. The time taken for decomposition to $\left(\frac{1}{8}\right)^{\text{th}}$ and $\left(\frac{1}{10}\right)^{\text{th}}$ of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively.

What is the value of $\frac{t_{1/8}}{t_{1/10}} \times 10$?

($\log 2 = 0.3$)

- (1) 9
- (2) 0.9
- (3) 3
- (4) 30

Ans. (1)

Sol. $t = \frac{1}{k} \ln \frac{A_0}{A_t}$

$$t_{1/8} = \frac{1}{k} \ln \frac{A_0}{A_0/8} = \frac{1}{k} \ln 8$$

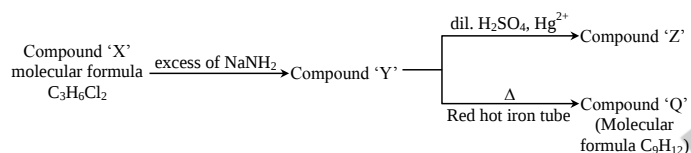
$$t_{1/10} = \frac{1}{k} \ln \frac{A_0}{A_0/10} = \frac{1}{k} \ln 10$$

$$\frac{t_{1/8}}{t_{1/10}} = \frac{\ln 8}{\ln 10} = \frac{\log 8}{\log 10}$$

$$\frac{t_{1/8}}{t_{1/10}} = \log 8 = 3 \log 2 = 0.9$$

$$\frac{t_{1/8}}{t_{1/10}} \times 10 = 9$$

- 58.** Given below are two statements for the following reaction sequence.



Statement I: Compound 'Z' will give yellow precipitate with NaOI.

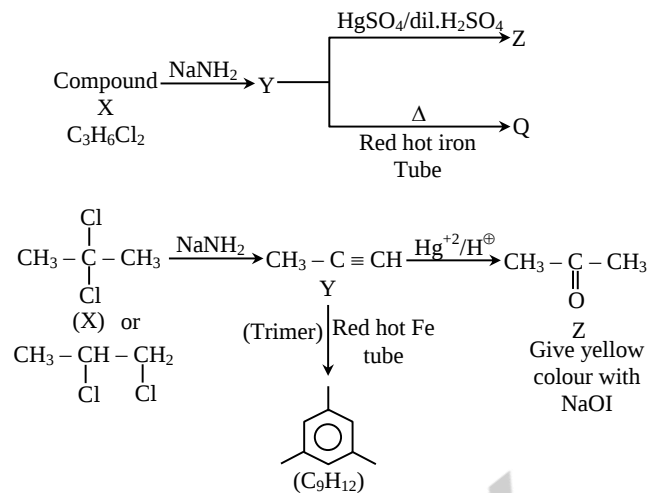
Statement II: Compound 'Q' has two different types of 'H' atoms (aromatic : aliphatic) in the ratio 1 : 3.

In the light of the above statements, choose the **correct** answer from the option given below:

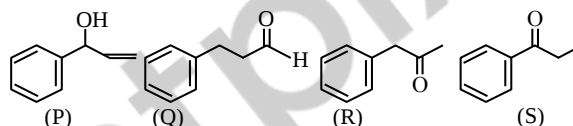
- (1) Statement I is true but Statement II is false
- (2) Both Statement I and Statement II are true
- (3) Statement I is false but Statement II is true
- (4) Both Statement I and Statement II are false

Ans. (2)

Sol.



- 59.** Given below are the four isomeric compounds (P, Q, R, S)



Identify **correct** statements from below.

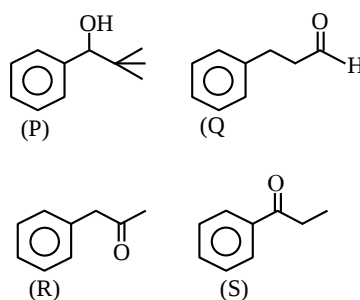
- A. Q, R and S will give precipitate with 2, 4 - DNP.
- B. P and Q will give positive Bayer's test.
- C. Q and R will give sooty flame.
- D. R and S will give yellow precipitate with $I_2/NaOH$.
- E. Q alone will deposit silver with Tollen's reagent

Choose the correct option.

- (1) A, C and E only
- (2) A and E only
- (3) C and E only
- (4) A, B, D and E only

Ans. (1)

Sol.



- (A) Q, R, S all three give 2, 4 DNP test as they have Aldehyde/ketone group
- (C) Q & R gives sooty flame
- (E) Q gives Tollens reagent test

60. Given below are two statements:

Statement I: The number of species among BF_4^- , SiF_4 , XeF_4 and SF_4 , that have unequal E–F bond lengths is two. Here, E is the central atom.

Statement II: Among O_2^- , O_2^{2-} , F_2 and O_2^+ , O_2^- has the highest bond order.

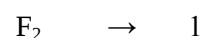
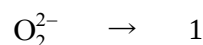
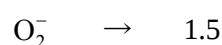
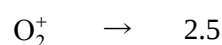
In the light of the above statements, choose the **correct** answer from the options given below

- (1) Both Statement I and Statement II are false
- (2) Both Statement I and Statement II are true
- (3) Statement I is true but Statement II is false
- (4) Statement I is false but Statement II is true

Ans. (1)

Sol. In BF_4^- , SiF_4 and XeF_4 all bond lengths are identical

Molecules B.O.



61. Regarding the hydrides of group 15 elements EH_3 (E = N, P, As, Sb), select the correct statement from the following:

- A. The stability of hydrides decreases down the group.
- B. The basicity of hydrides decreases down the group.
- C. The reducing character increases down the group.
- D. The boiling point increases down the group.

Choose the **correct** answer from the options given below:

- (1) A, B & C only (2) A & D only
- (3) A, B, C & D (4) B & C only

Ans. (1)

Sol. Stability : $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{BiH}_3$

Basicity : $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{BiH}_3$

Reducing character : $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 < \text{BiH}_3$

Boiling point : $\text{PH}_3 < \text{AsH}_3 < \text{NH}_3 < \text{SbH}_3 < \text{BiH}_3$

62. Given below are two statements:

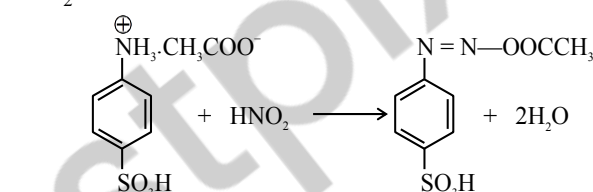
Statement I: Griss–Ilosvay test is used for the detection of nitrite ion, which involves the use of sulphanilic acid and α -naphthylamine reagent.

Statement II: In the above test, sulphanilic acid is diazotized by the acidified nitrite ion, which on further coupling with α -naphthylamine forms an azo-dye.

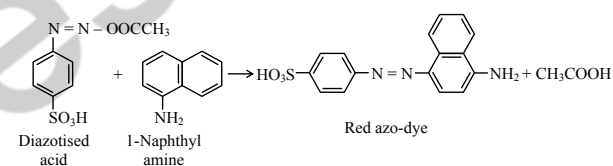
In the light of the above statements, choose the **correct** answer from the options given below

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are true
- (3) Statement I is true but Statement II is false
- (4) Both Statement I and Statement II are false

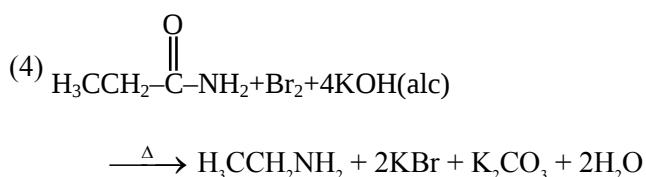
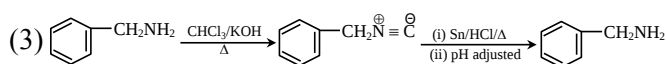
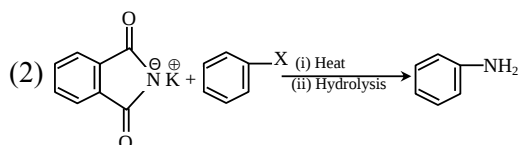
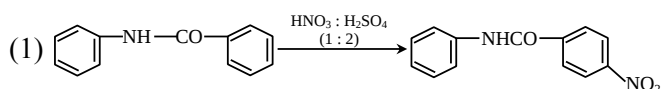
Ans. (2)



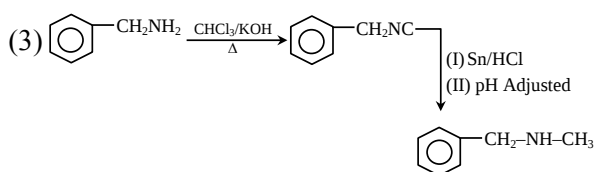
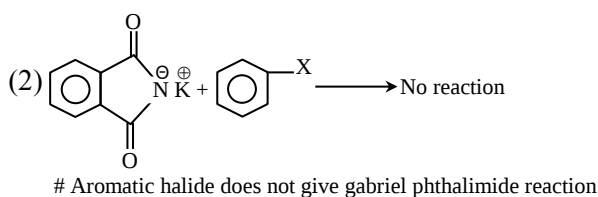
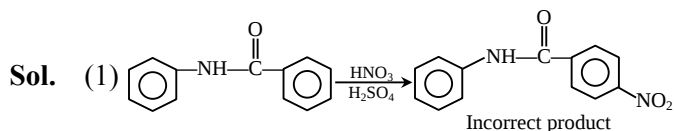
(Sulphanilic acid solution)



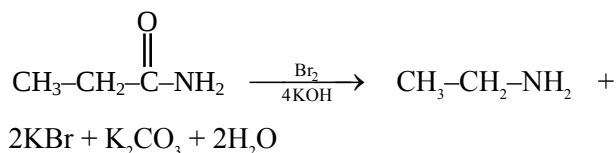
63. Consider the following reactions giving major product. Identify the correct reaction.



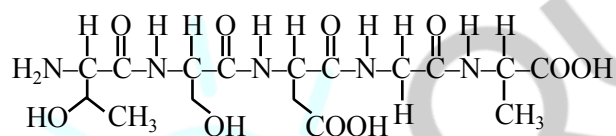
Ans. (4)



(4) Hoffmann bromamide degradation



64. In the given pentapeptide, find out an essential amino acid (Y) and the sequence present in the pentapeptide:



Choose the correct answer from the options given below:

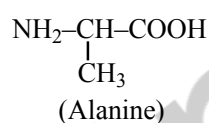
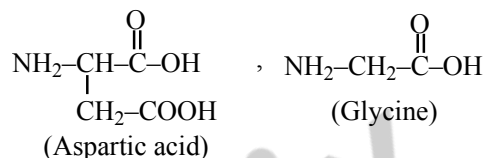
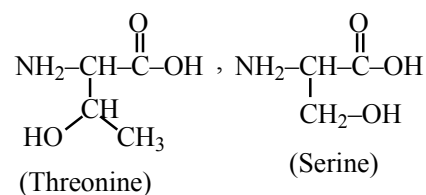
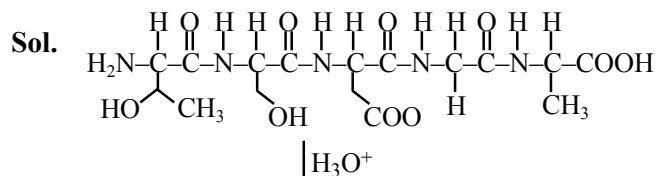
(Y)	(Sequence)
Threonine	Ser – Thr – Asp – Gly – Ala

(Y)	(Sequence)
Serine	Thr – Ser – Asp – Ala – Gly

(Y)	(Sequence)
Threonine	Thr – Ser – Asp – Gly – Ala

(Y)	(Sequence)
Serine	Ser – Asp – Thr – Ala – Gly

Ans. (3)



Consider the above reaction

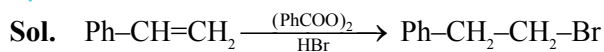
- The reaction proceeds through a more stable radical intermediate.
- The role of peroxide is to generate $\dot{\text{H}}$ (Hydrogen radical).
- During this reaction, benzene is formed as a biproduct.
- 1-Bromo-2-phenylethane is formed as the minor product.

E. The same reaction in absence of peroxide proceeds via carbocation intermediate.

Identify the correct statements. Choose the **correct** answer from the options given below:

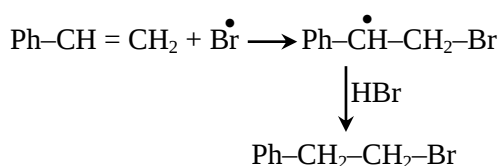
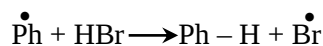
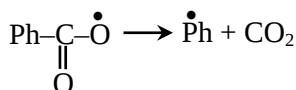
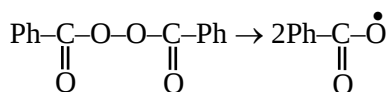
- A & E Only
- A, B & D Only
- C, D & E Only
- A, C & E Only

Ans. (4)



Anti Markovnikov addition

- Reaction follow radical addition in presence of peroxide
- In absence of peroxide follow carbocation mechanism
- Benzene also formed



- 66.** The wave numbers of three spectral lines of H atom are considered. Identify the set of spectral lines belonging to Balmer series.

(R = Rydberg constant)

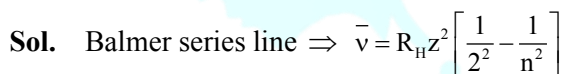
(1) $\frac{5R}{36}, \frac{3R}{16}, \frac{21R}{100}$

(2) $\frac{5R}{36}, \frac{8R}{9}, \frac{15R}{16}$

(3) $\frac{7R}{144}, \frac{3R}{16}, \frac{16R}{255}$

(4) $\frac{3R}{4}, \frac{3R}{16}, \frac{7R}{144}$

Ans. (1)



if $n = 3 \Rightarrow \bar{\nu} = R(1)^2 \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = \frac{5R}{36}$

if $n = 4 \Rightarrow \bar{\nu} = \frac{3R}{16}$

if $n = 5 \Rightarrow \bar{\nu} = \frac{21R}{100}$

67.

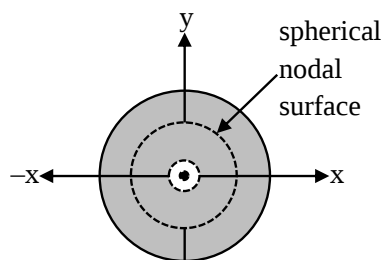


Figure 1. electron probability density for 2s orbital

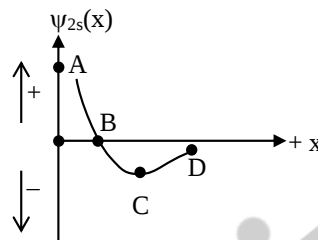


Figure 2. wave function for 2s orbital

Which of the following point in Figure 2 most accurately represents the nodal surface as shown in Figure 1 ?

- (1) B (2) D
(3) C (4) A

Ans. (1)

Sol. At spherical node

$$\psi_r = 0$$

- 68.** Given below are two statements :

Statement I : The number of pairs, from the following, in which both the ions are coloured in aqueous solution is 3.



Statement II : Th^{4+} is the strongest reducing agent among Th^{4+} , Ce^{4+} , Gd^{3+} and Eu^{2+} .

In the light of the above statements, choose the **correct** answer from the options given below

- (1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Ans. (3)

Sol. Sc^{3+} , Ti^{4+} and Zn^{2+} are colourless
 Th^{4+} cannot act as a reducing agent.

69. In period 4 of the periodic table, the elements with highest and lowest atomic radii are respectively.

- (1) Na & Cl
- (2) K & Se
- (3) K & Br
- (4) Rb & Br

Ans. (3)

Sol. In a period moving from left to right atomic size decreases.

70. The correct statement among the following is :

- (1) $[\text{Ni}(\text{CN})_4]^{2-}$ and $[\text{NiCl}_4]^{2-}$ are diamagnetic and $\text{Ni}(\text{CO})_4$ is paramagnetic.
- (2) $\text{Ni}(\text{CO})_4$ and $[\text{NiCl}_4]^{2-}$ are diamagnetic and $[\text{Ni}(\text{CN})_4]^{2-}$ is paramagnetic.
- (3) $\text{Ni}(\text{CO})_4$ and $[\text{Ni}(\text{CN})_4]^{2-}$ are diamagnetic and $[\text{NiCl}_4]^{2-}$ is paramagnetic.
- (4) $\text{Ni}(\text{CO})_4$ is diamagnetic and $[\text{NiCl}_4]^{2-}$ and $[\text{Ni}(\text{CN})_4]^{2-}$ are paramagnetic.

Ans. (3)

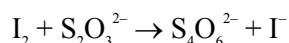
Sol. $[\text{Ni}(\text{CN})_4]^{2-} \rightarrow 3d^8 \rightarrow \text{diamagnetic} \rightarrow \text{dsp}^2$
 $[\text{Ni}(\text{CO})_4] \rightarrow 3d^{10} \rightarrow \text{diamagnetic} \rightarrow \text{sp}^3$
 $[\text{NiCl}_4]^{2-} \rightarrow 3d^8 \rightarrow e^{2,2} t_2^{2,1,1} \rightarrow \text{sp}^3 \rightarrow \text{paramagnetic}.$

SECTION-B

71. 500 mL of 1.2 M KI solution is mixed with 500 mL of 0.2 M KMnO_4 solution in basic medium. The liberated iodine was titrated with standard 0.1 M $\text{Na}_2\text{S}_2\text{O}_3$ solution in the presence of starch indicator till the blue color disappeared. The volume (in L) of $\text{Na}_2\text{S}_2\text{O}_3$ consumed is _____. (Nearest integer)

Ans. (3)

Sol. $\text{MnO}_4^- + \text{I}^- \rightarrow \text{MnO}_2 + \text{I}_2$

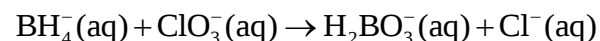


gram eq of KMnO_4 = gram eq of $\text{Na}_2\text{S}_2\text{O}_3$

$$0.2 \times \frac{500}{1000} \times 3 = 0.1 \times V \times 1$$

$$V = 3 \text{ L}$$

72. Consider the following redox reaction taking place in acidic medium



If the Nernst equation for the above balanced reaction is

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{RT}{nF} \ln Q,$$

Then the value of n is _____. (Nearest integer)

Ans. (24)

Sol. $3\text{BH}_4^- + 4\text{ClO}_3^- \longrightarrow 4\text{Cl}^- + 3\text{H}_2\text{BO}_3^- + 3\text{H}_2\text{O}$

n-factor = 8

moles = 3

$$\therefore n = 3 \times 8 = 24$$

73. X is the number of geometrical isomers exhibited by $[\text{Pt}(\text{NH}_3)(\text{H}_2\text{O})\text{BrCl}]$.

Y is the number of optically inactive isomer(s) exhibited by $[\text{CrCl}_2(\text{ox})_2]^{3-}$

Z is the number of geometrical isomers exhibited by $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$

The value of X + Y + Z is _____.

Ans. (6)

Sol. Here

X = 3 (Two cis + one trans isomers)

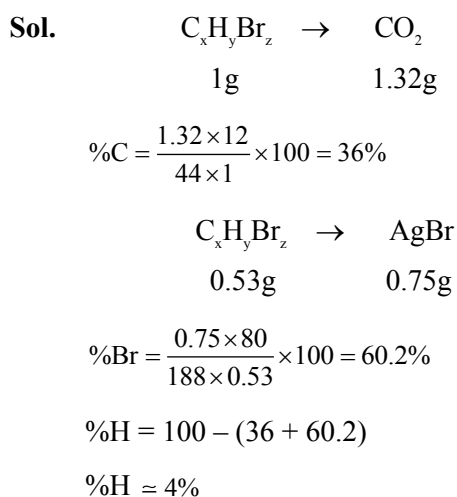
Y = 1 (trans isomer)

Z = 2 (Fac- mer isomer)

$$X+Y+Z = 3 + 1 + 2 = 6$$

74. 0.53 g of an organic compound (x) when heated with excess of nitric acid (concentrated) and then with silver nitrate gave 0.75 g of silver bromide precipitate. 1.0 g of (x) gave 1.32 g of CO_2 gas on combustion. The percentage of hydrogen in the compound (x) is _____. [Nearest Integer]
[Given : Molar mass in g mol^{-1} H : 1, C : 12, Br : 80, Ag : 108, O : 16; Compound (x) : $\text{C}_x\text{H}_y\text{Br}_z$]

Ans. (4)



75. Consider the dissociation equilibrium of the following weak acid $\text{HA} \rightleftharpoons \text{H}^+(\text{aq}) + \text{A}^-(\text{aq})$

If the pK_a of the acid is 4, then the pH of 10 mM HA solution is _____. (Nearest integer)

[Given : The degree of dissociation can be neglected with respect to unity]

Ans. (3)

Sol. $\text{pH} = \frac{1}{2}[\text{pK}_a - \log c]$

$$\text{pH} = \frac{1}{2}[4 - \log 10^{-2}]$$

$$\text{pH} = 3$$