**(Held On Wednesday 25th January, 2023)** **TIME : 9 : 00 AM to 12 : 00 NOON**

## MATHEMATICS — TEST PAPER WITH SOLUTION

### SECTION-A

**1.** Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S=\left\{x \in Z : x(66-x) \geq \frac{5}{9}M\right\}$ and the event $A=\{x \in S : x \text{ is a multiple of } 3\}$. Then P(A) is equal to

(1) $\frac{15}{44}$ (2) $\frac{1}{3}$

(3) $\frac{1}{5}$ (4) $\frac{7}{22}$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** M = 33 × 33

$x(66-x) \geq \frac{5}{9} \times 33 \times 33$

$11 \leq x \leq 55$

A : {12, 15, 18, .... 54}

$P(A)=\frac{15}{45}=\frac{1}{3}$

**2.** Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}\cdot\vec{c}=0$ and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}-\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$, then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$ is equal to

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$

(3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** $(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{\vec{b}-\vec{c}}{2}$

$\vec{a}\cdot\vec{c}=\frac{1}{2}, \vec{a}\cdot\vec{b}=\frac{1}{2}$

$\therefore \vec{b}\cdot\vec{d}=\frac{1}{2}$

$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\vec{a}\cdot(\vec{b}\times(\vec{c}\times\vec{d}))$

$=\vec{a}\cdot((\vec{b}\cdot\vec{d})\vec{c}-(\vec{b}\cdot\vec{c})\vec{d})$

$=(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})=\frac{1}{4}$

**3.** Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}=\frac{y}{x}(1+xy^2(1+\log_e x))$, x > 0, y(1) = 3. Then $\frac{y^2(x)}{9}$ is equal to :

(1) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

(2) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$

(3) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$

(4) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** $\frac{dy}{dx}-\frac{y}{x}=y^3(1+\log_e x)$

$\frac{1}{y^3}\frac{dy}{dx}-\frac{1}{xy^2}=1+\log_e x$

Let $-\frac{1}{y^2}=t \Rightarrow \frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \frac{dt}{dx}+\frac{2t}{x}=2(1+\log_e x)$

I.F. = $e^{\int\frac{2}{x}dx}=x^2$

$\frac{-x^2}{y^2}=\frac{2}{3}\left((1+\log_e x)x^3-\frac{x^3}{3}\right)+C$

$y(1)=3$

$\frac{y^2}{9}=\frac{x^2}{5-2x^3(2+\log_e x^3)}$

**OR**

$xdy = ydx + xy^3(1+\log_e x)dx$

$$\frac{xdy-ydx}{y^3} = x(1+\log_e x)dx$$

$$-\frac{x}{y}d\left(\frac{x}{y}\right) = x^2(1+\log_e x)dx$$

$$-\left(\frac{x}{y}\right)^2 = 2\int x^2(1+\log_e x)dx$$

**4.** The value of

$$\lim_{n\to\infty}\frac{1+2-3+4+5-6+...+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

is :

(1) $\frac{\sqrt{2}+1}{2}$

(2) $3(\sqrt{2}+1)$

(3) $\frac{3}{2}(\sqrt{2}+1)$

(4) $\frac{3}{2\sqrt{2}}$

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** $$\lim_{n\to\infty}\frac{0+3+6+9+.....n \text{ terms}}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty}\frac{3n(n-1)}{2\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

$$= \frac{3}{2\left(\sqrt{2}-1\right)} = \frac{3}{2}\left(\sqrt{2}+1\right)$$

**5.** The points of intersection of the line ax + by = 0, $(a \neq b)$ and the circle $x^2+y^2-2x=0$ are $A(\alpha,0)$ and $B(1,\beta)$. The image of the circle with AB as a diameter in the line x + y + 2 = 0 is :

(1) $x^2+y^2+5x+5y+12=0$

(2) $x^2+y^2+3x+5y+8=0$

(3) $x^2+y^2+3x+3y+4=0$

(4) $x^2+y^2-5x-5y+12=0$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** Only possibility $\alpha=0, \beta=1$

$\therefore$ equation of circle $x^2+y^2-x-y=0$

Image of circle in x + y + 2 = 0 is

$x^2+y^2+5x+5y+12=0$

**6.** The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2. then their new variance is equal to :

(1) 4.04

(2) 4.08

(3) 3.96

(4) 3.92

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** $\sum_{i=1}^{n} x_i = 10n$

$$\sum_{i=1}^{n} x_i - 8 + 12 = (10.2)n \qquad \therefore n = 20$$

Now $\frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$

$$\frac{\sum_{i=1}^{20} x_i^2 - 8^2 + 12^2}{20} - (10.2)^2$$

= 108 – 104.04 = 3.96

**7.** Let

$y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$.

Then $y'-y''$ at $x=-1$ is equal to

(1) 976 (2) 464

(3) 496 (4) 944

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** $y = \frac{1-x^{32}}{1-x} \Rightarrow y - xy = 1-x^{32}$

$y'-xy'-y = -32x^{31}$

$y''-xy''-y'-y' = -(32)(31)x^{30}$

at $x=-1 \Rightarrow y'-y''=496$

**8.** The vector $\vec{a}=-\hat{i}+2\hat{j}+\hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a}+\sqrt{2}\vec{b}$ on $\vec{c}=5\hat{i}+4\hat{j}+3\hat{k}$ is

(1) $3\sqrt{2}$ (2) 1

(3) $\sqrt{6}$ (4) $2\sqrt{3}$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** $\vec{b}=\lambda\vec{a}\times(\vec{a}\times\hat{j})$

$\Rightarrow\vec{b}=\lambda(-2\hat{i}-2\hat{j}+2\hat{k})$

$|\vec{b}|=|\vec{a}| \quad \therefore \sqrt{6}=\sqrt{12}|\lambda|\Rightarrow\lambda=\pm\frac{1}{\sqrt{2}}$

$\left(\lambda=\frac{1}{\sqrt{2}}\text{ rejected }\because \vec{b}\text{ makes acute angle with y axis}\right)$

$\vec{b}=-\sqrt{2}(-\hat{i}-\hat{j}+\hat{k})$

$\frac{(3\vec{a}+\sqrt{2}\vec{b})\cdot\vec{c}}{|\vec{c}|}=3\sqrt{2}$

**9.** The minimum value of the function $f(x)=\int_0^2 e^{|x-t|}\,dt$ is

(1) $2(e-1)$ (2) $2e-1$

(3) 2 (4) $e(e-1)$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** For $x\le 0$

$f(x)=\int_0^2 e^{t-x}dt=e^{-x}\left(e^2-1\right)$

For $0<x<2$

$f(x)=\int_0^x e^{x-t}dt+\int_x^2 e^{t-x}dt=e^x+e^{2-x}-2$

For $x\ge 2$

$f(x)=\int_0^2 e^{x-t}dt=e^{x-2}(e^2-1)$

For $x\le 0$, f(x) is $\downarrow$ and $x\ge 2$, f(x) is $\uparrow$

$\therefore$ Minimum value of f(x) lies in $x\in(0,2)$

Applying $A.M\ge G.M$,

minimum value of f(x) is 2(e – 1)

**10.** Consider the lines $L_1$ and $L_2$ given by

$L_1:\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$

$L_2:\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

A line $L_3$ having direction ratios 1, –1, –2, intersects $L_1$ and $L_2$ at the points P and Q respectively. Then the length of line segment PQ is

(1) $2\sqrt{6}$

(2) $3\sqrt{2}$

(3) $4\sqrt{3}$

(4) 4

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** Let $P=(2\lambda+1,\lambda+3,2\lambda+2)$

Let $Q=(\mu+2,2\mu+2,3\mu+3)$

$\Rightarrow\frac{2\lambda-\mu-1}{1}=\frac{\lambda-2\mu+1}{-1}=\frac{2\lambda-3\mu-1}{-2}$

$\Rightarrow\lambda=\mu=3\Rightarrow P(7,6,8)\text{ and }Q(5,8,12)$

$PQ=2\sqrt{6}$

**11.** Let x = 2 be a local minima of the function $f(x)=2x^4-18x^2+8x+12$, $x\in(-4,4)$. If M is local maximum value of the function f in (–4, 4), then M =

(1) $12\sqrt{6}-\frac{33}{2}$ (2) $12\sqrt{6}-\frac{31}{2}$

(3) $18\sqrt{6}-\frac{33}{2}$ (4) $18\sqrt{6}-\frac{31}{2}$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** $f'(x)=8x^3-36x+8=4(2x^3-9x+2)$

$f'(x)=0$

$\therefore x=\frac{\sqrt{6}-2}{2}$

Now

$f(x)=\left(x^2-2x-\frac{9}{2}\right)\left(2x^2+4x-1\right)+24x+7.5$

$\therefore f\left(\frac{\sqrt{6}-2}{2}\right)=M=12\sqrt{6}-\frac{33}{2}$

**12.** Let $z_1 = 2+3i$ and $z_2 = 3+4i$. The set

$$S = \left\{ z \in C : |z-z_1|^2 - |z-z_2|^2 = |z_1-z_2|^2 \right\}$$

represents a

(1) straight line with sum of its intercepts on the coordinate axes equals 14

(2) hyperbola with the length of the transverse axis 7

(3) straight line with the sum of its intercepts on the coordinate axes equals –18

(4) hyperbola with eccentricity 2

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** $\left((x-2)^2+(y-3)^2\right) - \left((x-3)^2-(y-4)^2\right) = 1+1$

$\Rightarrow x+y=7$

**13.** The distance of the point $(6,-2\sqrt{2})$ from the common tangent y = mx + c, m > 0, of the curves $x = 2y^2$ and $x = 1+y^2$ is

(1) $\frac{1}{3}$

(2) 5

(3) $\frac{14}{3}$

(4) $5\sqrt{3}$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** For

$$y^2 = \frac{x}{2},\ T : y = mx + \frac{1}{8m}$$

For tangent to $y^2+1=x$

$$\Rightarrow \left(mx+\frac{1}{8m}\right)^2 + 1 = x$$

$$D = 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$$

$$\therefore T : x - 2\sqrt{2}y + 1 = 0$$

$$d = \left|\frac{6+8+1}{\sqrt{9}}\right| = 5$$

**14.** Let $S_1$ and $S_2$ be respectively the sets of all $a \in R-\{0\}$ for which the system of linear equations

$$ax+2ay-3az=1$$

$$(2a+1)x+(2a+3)y+(a+1)z=2$$

$$(3a+5)x+(a+5)y+(a+2)z=3$$

has unique solution and infinitely many solutions. Then

(1) $n(S_1) = 2$ and $S_2$ is an infinite set

(2) $S_1$ is an infinite set an $n(S_2) = 2$

(3) $S_1 = \Phi$ and $S_2 = \mathbb{R}-\{0\}$

(4) $S_1 = \mathbb{R}-\{0\}$ and $S_2 = \Phi$

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** $$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$= a(15a^2+31a+36) = 0 \Rightarrow a = 0$

$\Delta \neq 0$ for all $a \in R-\{0\}$

Hence $S_1 = R-\{0\}$ $\quad S_2 = \Phi$

**15.** Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$.

If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then f(4) is equal to

(1) $\frac{1}{2}(\log_e 17 - \log_e 19)$

(2) $\log_e 17 - \log_e 18$

(3) $\frac{1}{2}(\log_e 19 - \log_e 17)$

(4) $\log_e 19 - \log_e 20$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** Put $x^2 = t$

$$\int \frac{dt}{(t+1)(t+3)} = \frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$$

$$f(x) = \frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right)+C$$

$$f(3) = \frac{1}{2}(\ln 10 - \ln 12)+C$$

$$\Rightarrow C = 0$$

$$f(4) = \frac{1}{2}\ln\left(\frac{17}{19}\right)$$

**16.** The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

(1) equivalent to $(\sim p) \vee (\sim q)$

(2) a tautology

(3) equivalent to $p \vee q$

(4) a contradiction

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$

$\equiv (\sim (p \wedge \sim q)) \vee (\sim p \vee \sim q)$

$\equiv (\sim p \vee q) \vee (\sim p \vee \sim q)$

$\equiv \sim p \vee t \equiv t$

**17.** Let $f : (0,1) \rightarrow \mathbb{R}$ be a function defined by

$f(x) = \frac{1}{1-e^{-x}}$ , and

$g(x) = (f(-x) - f(x))$ . Consider two statements

(I) g is an increasing function in (0, 1)

(II) g is one-one in (0, 1)

Then,

(1) Only (I) is true

(2) Only (II) is true

(3) Neither (I) nor (II) is true

(4) Both (I) and (II) are true

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** $g(x) = f(-x) - f(x) = \frac{1+e^x}{1-e^x}$

$\Rightarrow g'(x) = \frac{2e^x}{(1-e^x)^2} > 0$

$\Rightarrow$ g is increasing in (0, 1)

$\Rightarrow$ g is one-one in (0, 1)

**18.** The distance of the point P(4, 6, –2) from the line passing through the point (–3, 2, 3) and parallel to a line with direction ratios 3, 3, –1 is equal to :

(1) 3

(2) $\sqrt{6}$

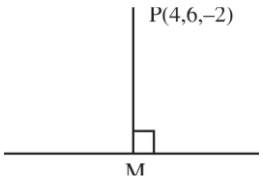
(3) $2\sqrt{3}$

(4) $\sqrt{14}$

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.**




Equation of line is $\frac{x+3}{3} = \frac{y-2}{3} = \frac{z-3}{-1} = \lambda$

$M(3\lambda - 3, 3\lambda + 2, 3 - \lambda)$

D.R of $PM(3\lambda - 7, 3\lambda - 4, 5 - \lambda)$

Since PM is perpendicular to line

$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$

$\Rightarrow \lambda = 2$

$\Rightarrow M(3,8,1) \Rightarrow PM = \sqrt{14}$

**19.** Let $x, y, z > 1$ and

$$A = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{vmatrix}.$$

Then $|\text{adj}(\text{adj } A^2)|$ is equal to

(1) $6^4$

(2) $2^8$

(3) $4^8$

(4) $2^4$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $|A| = \frac{1}{\log x . \log y . \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = 2$

$\Rightarrow |\text{adj}(\text{adj} A^2)| = |A^2|^4 = 2^8$

**20.** If $a_r$ is the coefficient of $x^{10-r}$ in the Binomial expansion of $(1 + x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(1) 4895 (2) 1210

(3) 5445 (4) 3025

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $a_r = {}^{10}C_{10-r} = {}^{10}C_r$

$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2 = \sum_{r=1}^{10} r^3 \left(\frac{11-r}{r}\right)^2 = \sum_{r=1}^{10} r(11-r)^2$

$= \sum_{r=1}^{10} (121r + r^3 - 22r^2) = 1210$

## SECTION-B

**21.** Let S = {1, 2, 3, 5, 7, 10, 11}. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is ____ .

**Official Ans. by NTA (43)**

**QuestPix Ans. (43)**

**Sol.** Elements of the type 3k = 3

Elements of the type 3k + 1 = 1, 7, 9

Elements of the type 3k + 2 = 2, 5, 11

Subsets containing one element $S_1 = 1$

Subsets containing two elements

$S_2 = {}^3C_1 \times {}^3C_1 = 9$

Subsets containing three elements

$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$

Subsets containing four elements

$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$

Subsets containing five elements

$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$

Subsets containing six elements $S_6 = 1$

Subsets containing seven elements $S_7 = 1$

$\Rightarrow$ sum = 43

**22.** For some $a, b, c \in \mathbb{N}$, let f(x) = ax – 3 and $g(x) = x^b + c, x \in \mathbb{R}$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$, then (fog) (ac) + (gof) (b) is equal to ____ .

**Official Ans. by NTA (2039)**

**QuestPix Ans. (2039)**

**Sol.** Let fog(x) = h(x)

$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow$ h(x) = fog(x) = $2x^3 + 7$

$fog(x) = a(x^b + c) - 3$

$\Rightarrow$ a = 2, b = 3, c = 5

$\Rightarrow$ fog(ac) = fog(10) = 2007

$g(f(x)) = (2x - 3)^3 + 5$

$\Rightarrow$ gof(b) = gof(3) = 32

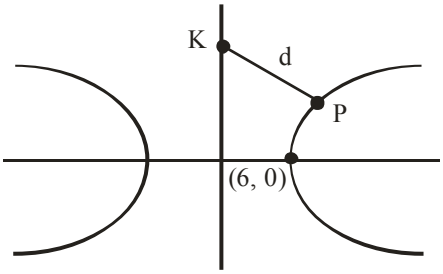
$\Rightarrow$ sum = 2039

**23.** The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then $d^2$ is equal to ____ .

**Official Ans. by NTA (216)**

**QuestPix Ans. (216)**

**Sol.**




$H : \frac{x^2}{36} - \frac{y^2}{9} = 1$

equation of normal is $6x\cos\theta + 3y\cot\theta = 45$

slope = $-2\sin\theta = -\sqrt{2}$

$\Rightarrow \theta = \frac{\pi}{4}$

Equation of normal is $\sqrt{2}x + y = 15$

P : (a secθ, b tanθ)

$\Rightarrow P(6\sqrt{2}, 3)$ and K(0,15)

$d^2 = 216$

**24.** Let

$$S=\left\{\alpha:\log_2(9^{2\alpha-4}+13)-\log_2\left(\frac{5}{2}.3^{2\alpha-4}+1\right)=2\right\}.$$

Then the maximum value of $\beta$ for which the equation $x^2-2\left(\sum_{\alpha\in S}\alpha\right)^2 x+\sum_{\alpha\in S}(\alpha+1)^2\beta=0$ has real roots, is ____ .

**Official Ans. by NTA (25)**

**QuestPix Ans. (25)**

**Sol.** $\log_2(9^{2\alpha-4}+13)-\log_2\left(\frac{5}{2}.3^{2\alpha-4}+1\right)=2$

$$\Rightarrow \frac{9^{2\alpha-4}+13}{\frac{5}{2}3^{2\alpha-4}+1}=4$$

$\Rightarrow \alpha=2$ or $3$

$\sum_{\alpha\in S}\alpha=5$ and $\sum_{\alpha\in S}(\alpha+1)^2=25$

$\Rightarrow x^2-50x+25\beta=0$ has real roots

$\Rightarrow \beta\le 25$

$\Rightarrow \beta_{max}=25$

**25.** The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is ____ .

**Official Ans. by NTA (1080)**

**QuestPix Ans. (1080)**

**Sol.** General term is $\sum\frac{5!(2x)^{n_1}(x^{-7})^{n_2}(3x^2)^{n_3}}{n_1!\,n_2!\,n_3!}$

For constant term,

$n_1+2n_3=7n_2$

& $n_1+n_2+n_3=5$

Only possibility $n_1=1, n_2=1, n_3=3$

$\Rightarrow$ constant term = 1080

**26.** Let $A_1$, $A_2$, $A_3$ be the three A.P. with the same common difference d and having their first terms as A, A + 1, A + 2, respectively. Let a, b, c be the $7^{th}$, $9^{th}$, $17^{th}$ terms of $A_1$, $A_2$, $A_3$, respectively such that $\begin{vmatrix}a & 7 & 1\\ 2b & 17 & 1\\ c & 17 & 1\end{vmatrix}+70=0$

If a = 29, then the sum of first 20 terms of an AP whose first term is c – a – b and common difference is $\frac{d}{12}$, is equal to _____ .

**Official Ans. by NTA (495)**

**QuestPix Ans. (495)**

**Sol.** $\begin{vmatrix}A+6d & 7 & 1\\ 2(A+1+8d) & 17 & 1\\ A+2+16d & 17 & 1\end{vmatrix}+70=0$

$\Rightarrow A=-7$ and $d=6$

$\therefore c-a-b=20$

$S_{20}=495$

**27.** If the sum of all the solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3},$$

$-1<x<1, x\neq 0,$ is $\alpha-\frac{4}{\sqrt{3}}$, then $\alpha$ is equal to ____ .

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** **Case I :** $x>0$

$$\tan^{-1}\frac{2x}{1-x^2}+\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$$

$x=2-\sqrt{3}$

**Case II :** $x<0$

$$\tan^{-1}\frac{2x}{1-x^2}+\tan^{-1}\frac{2x}{1-x^2}+\pi=\frac{\pi}{3}$$

$x=\frac{-1}{\sqrt{3}}\Rightarrow\alpha=2$

**28.** Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is ____ .

**Official Ans. by NTA (9)**

**QuestPix Ans. (9)**

**Sol.** Equation of plane is

$(x-2y-z-5)+b(x+y+3z-5)=0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$

$\therefore$ plane is $13x + 10y + 35z = 65$

Distance from given point to plane = 9

**29.** Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that x + y is divisible by 5, is _____ .

**Official Ans. by NTA (120)**

**QuestPix Ans. (120)**

**Sol.** $x + y = 5\lambda$

**Cases :**

| x | y | Number of ways |
|---|---|---|
| $5\lambda$ | $5\lambda$ | 20 |
| $5\lambda + 1$ | $5\lambda + 4$ | 25 |
| $5\lambda + 2$ | $5\lambda + 3$ | 25 |
| $5\lambda + 3$ | $5\lambda + 2$ | 25 |
| $5\lambda + 4$ | $5\lambda + 1$ | 25 |

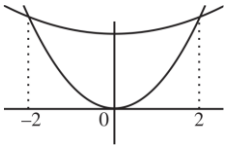
Total = 120

**30.** It the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by $P_1$ and $y = \alpha x, \alpha > 0$, then $\alpha^3$ is equal to _____ .

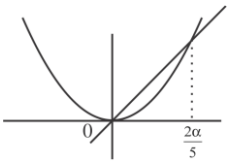
**Official Ans. by NTA (600)**

**QuestPix Ans. (600)**

**Sol.**




Abscissa of point of intersection of $2y = 5x^2$ and $y = x^2 + 6$ is $\pm 2$




$$\text{Area} = 2\int_0^2 \left(x^2 + 6 - \frac{5x^2}{2}\right) dx = \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right) dx$$

$$\Rightarrow \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right) dx = 16$$

$$\Rightarrow \alpha^3 = 600$$

**(Held On Wednesday 25th January, 2023)** **TIME : 9 : 00 AM to 12 : 00 NOON**

| PHYSICS | TEST PAPER WITH SOLUTION |
|---|---|

**SECTION-A**

**31.** Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV. has a de-Broglie wavelength of $\lambda_o$. If the voltage is increased to 40 kV. then the de-Broglie wavelength associated with the electron beam would be:

(1) $3\lambda_0$ (2) $9\lambda_0$

(3) $\frac{\lambda_0}{2}$ (4) $\frac{\lambda_0}{\sqrt{2}}$

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** When electron is accelerated through potential difference V, then

K.E. = eV

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m(KE)}} = \frac{h}{\sqrt{2meV}}$$

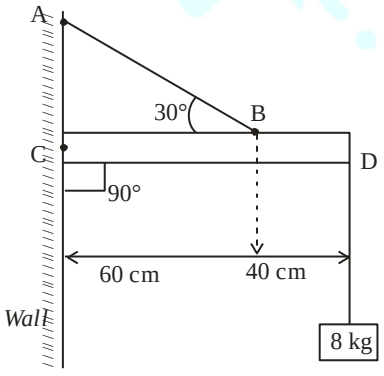
$$\therefore \quad \lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \quad \frac{\lambda}{\lambda_0} = \sqrt{\frac{20}{40}}$$

$$\therefore \quad \lambda = \frac{\lambda_0}{\sqrt{2}}$$

**32.** An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1 m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is :

(Take g = 10 m/s$^2$)



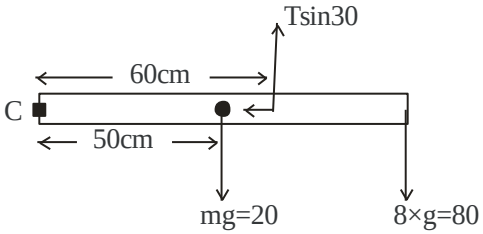

(1) 240 N (2) 90 N

(3) 300 N (4) 30 N

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**




Taking torque about point C

$$\frac{T}{2} \times 60 = 20 \times 50 + 80 \times 100$$

$$\Rightarrow 3T = 100 + 800$$

$$\Rightarrow = 300 \text{ N}$$

**33.** A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

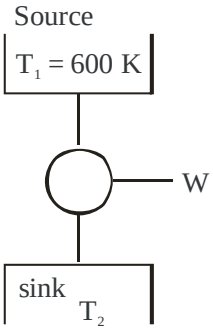
(1) 360 K (2) 1000 K

(3) 900 K (4) 300 K

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.**




Initially $\eta = \frac{1}{2}$

**But** $\eta = 1 - \frac{T_2}{T_1}$

$$\therefore \quad \frac{1}{2} = 1 - \frac{T_2}{600}$$

$$\Rightarrow \frac{T_2}{600} = \frac{1}{2} \qquad \Rightarrow T_2 = 300 \text{ K}$$

Now efficiency is increased to 70% and $T_2$ = 300 K, Let temp of source $T_1$ = T

$$\Rightarrow \frac{7}{10} = 1 - \frac{300}{T}$$

$$\Rightarrow \frac{300}{T} = 1 - \frac{7}{10}$$

$$\Rightarrow \frac{300}{T} = \frac{3}{10} \qquad \therefore T = 1000 \text{ K}$$

**34.** T is the time period of simple pendulum on the earth's surface. Its time period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(1) 4 (2) 2

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

**Official Ans. by NTA (2)**
**QuestPix Ans. (2)**

**Sol.** At surface of earth time period

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

At height h = R

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{4}$$

$$\therefore \quad xT = 2\pi\sqrt{\frac{\ell}{(g/4)}}$$

$$\Rightarrow xT = 2\times 2\pi\sqrt{\frac{\ell}{g}}$$

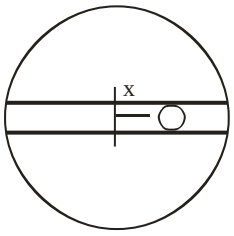
$$\Rightarrow xT = 2T \Rightarrow x = 2$$

**35.** Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)
(take g = 10 $ms^{-2}$,radius of earth = 6400 km )

(1) 24 hours
(2) 1 hour 24 minutes
(3) 1 hour 40 minutes
(4) 12 hours

**Official Ans. by NTA (2)**
**QuestPix Ans. (2)**

**Sol.**




Let at some time particle is at a distance x from centre of Earth, then at that position field

$$E = \frac{GM}{R^3}x$$

$\therefore$ Acceleration of particle

$$\vec{a} = -\frac{GM}{R^3}\vec{x}$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{g}{R}}$$

$$\text{Now } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

$$\Rightarrow T = 2\times 3.14\times\sqrt{\frac{6400\times 10^3}{10}}$$

$$= 2\times 3.14\times 800\,\text{sec} \approx 1 \text{ hour } 24 \text{ minutes}$$

**36.** A car travels a distance of 'x' with speed $V_1$ and then same distance 'x' with speed $V_2$ in the same direction. The average speed of the car is:

(1) $\frac{v_1v_2}{2(v_1+v_2)}$ (2) $\frac{v_1+v_2}{2}$

(3) $\frac{2x}{v_1+v_2}$ (4) $\frac{2v_1v_2}{v_1+v_2}$

**Official Ans. by NTA (4)**
**QuestPix Ans. (4)**

**Sol.**


$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{x+x}{\frac{x}{v_1}+\frac{x}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$

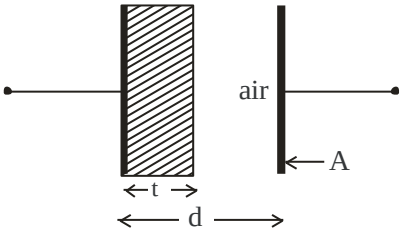
**37.** A parallel plate capacitor has plate area $40cm^2$ and plates separation 2 mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is :

(1) $24\varepsilon_0$ F (2) $\frac{3}{10}\varepsilon_0$ F

(3) $\frac{10}{3}\varepsilon_0$ F (4) $10\varepsilon_0$ F

**Official Ans. by NTA (3)**
**QuestPix Ans. (3)**

**Sol.**




This can be seen as two capacitors in series combination so

$$\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}$$

$$= \frac{1}{\frac{K\epsilon_0 A}{t}} + \frac{1}{\frac{\epsilon_0 A}{d-t}}$$

$$= \frac{t}{K \in_0 A} + \frac{d-t}{\in_0 A}$$

$$= \frac{1\times10^{-3}}{5\in_0 \times 40\times10^{-4}} + \frac{1\times10^{-3}}{\in_0 40\times10^{-4}}$$

$$\frac{1}{C_{eq}} = \frac{1}{20\in_0} + \frac{1}{4\in_0}$$

$$C_{eq} = \frac{20\times4\in_0}{24} = \frac{10\in_0}{3} F$$

**38.** The root mean square velocity of molecules of gas is

(1) Proportional to square of temperature ($T^2$).

(2) Inversely proportional to square root of temperature $\sqrt{\frac{1}{T}}$.

(3) Proportional to square root of temperature $\sqrt{T}$.

(4) Proportional to temperature (T).

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** The rms speed of a gas molecule is

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

$$V_{RMS} \ \alpha \ \sqrt{T}$$

**39.** Match List I with List II

| List – I | | List - II | |
|---|---|---|---|
| A | Surface tension | I. | Kg $m^{-1}$ $s^{-1}$ |
| B | Pressure | II. | Kg $ms^{-1}$ |
| C | Viscosity | III. | Kg $m^{-1}$ $s^{-2}$ |
| D | Impulse | IV. | Kg $s^{-2}$ |

Choose the correct answer from the options given below :

(1) A-IV, B-III, C- II, D- I

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-III, D-IV

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** (A) Surface Tension $= \frac{F}{\ell} = \frac{MLT^{-2}}{L} = ML^{-1}T^{-2}$

$$= Kgs^{-2} (IV)$$

(B) Pressure $= \frac{F}{A} = \frac{MLT^{-2}}{L^2}$

$$= kg\ m^{-1}s^{-2} (III)$$

(C) Viscosity $= = \frac{F}{A\left(\frac{dV}{dz}\right)} = \frac{MLT^{-2}}{L^2\left(\frac{LT^{-1}}{L}\right)}$

$$= ML^{-1}T^{-1} = kg\ m^{-1}\ s^{-1} (I)$$

(D) Impulse $= \int Fdt = MLT^{-2} \times T$

$$= MLT^{-1} = Kgms^{-1} (II)$$

So A-(IV), B-(III), C-(I), D- (II)

**40.** In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency $\omega_0$. The value of x is:

(1) 1/4 (2) 16

(3) 1/16 (4) 4

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** The resonance frequency of LC oscillations circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$L \to 2L$

$C \to 8C$

$$\omega = \frac{1}{\sqrt{2L\times8C}} = \frac{1}{4\sqrt{LC}}$$

$$\omega = \frac{\omega_0}{4}$$

So $x = \frac{1}{4}$

**41.** The ratio of the density of oxygen nucleus $\left(^{16}_{8}O\right)$ and helium nucleus $\left(^{4}_{2}He\right)$ is

(1) 4:1 (2) 8:1

(3) 1:1 (4) 2:1

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** Nuclear density is independent of mass number

As nuclear density $= \frac{Au}{\frac{4}{3}\pi R^3}$

Also , $R = R_0 A^{\frac{1}{3}}$

And $R^3 = R_0^3 A$

$\Rightarrow$ Nuclear density $= \frac{Au}{\frac{4}{3}\pi R_0^3 A}$

Nuclear density $= \frac{3u}{4\pi R_0^3}$

$\Rightarrow$ Nuclear density is independent of A

**42.** A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The bandwidth for amplitude modulation is:

(1) 5 kHz (2) 20 kHz

(3) 10 kHz (4) 2.5 kHz

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** Given

Signal frequency $f_m = 5kHz$

Carrier wave frequency $f_c = 2MHz$

$f_c = 2000KHz$

The resultant signal will have band width of frequency given by

$\left|(f_c + f_m) - (f_c - f_m)\right|$

$\Rightarrow \left|(2000+5) - (2000-5)\right| kHz$

$\Rightarrow$ 10 kHz

**43.** All electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at that point and instant?

(1) Positive direction of x

(2) Positive direction of z

(3) Negative direction of x

(4) Negative direction of y

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** As , poynting vector

$\vec{S} = \vec{E} \times \vec{H}$

Given energy transport = negative z direction

Electric field = positive y direction

$\left(-\hat{k}\right) = \left(+\hat{j}\right) \times \left[\hat{i}\right]$

Hence according to vector cross product magnetic field should be positive x direction.

**44.** In Young's double slits experiment, the position of 5$^{th}$ bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1 m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

(1) 60 µm (2) 48 µm

(3) 12 µm (4) 36 µm

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** Given

D = 1m

$\lambda = 600 \times 10^{-9} m$

n = 5

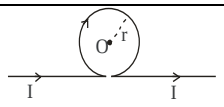
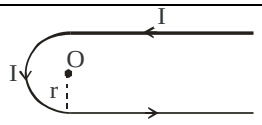
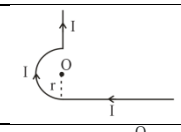
As $y_{nth} = \dfrac{n\lambda D}{d}$

$\Rightarrow \dfrac{5 \times 600 \times 10^{-9} \times 1}{d} = 5 \times 10^{-2}$

$\Rightarrow d = \dfrac{5 \times 600 \times 10^{-9} \times 1}{5 \times 10^{-2}} = 60 \times 10^{-6} m$

$\Rightarrow$ d = 60 µm

**45.** Math List I with List II

| List – I (Current configuration) | | List – II (Magnetic field at point O) | |
|---|---|---|---|
| A |  | I. | $B_0 = \dfrac{\mu_0 I}{4\pi r}[\pi + 2]$ |
| B |  | II. | $B_0 = \dfrac{\mu_0}{4}\dfrac{I}{r}$ |
| C |  | III. | $B_0 = \dfrac{\mu_0 I}{2\pi r}[\pi - 1]$ |
| D |  | IV | $B_0 = \dfrac{\mu_0 I}{4\pi r}[\pi + 1]$ |

Choose the correct answer from the option given below:

(1) A-III, B-IV, C-I, D-II

(2) A-I, B-III, C-IV, D-II

(3) A-III, B-I, C-IV, D-II

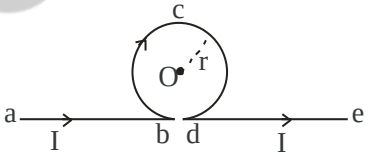
(4) A-II, B-I, C-IV, D-III

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**

(A)




$B_{ab} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r}$ (out of the plane)

$B_{bcd} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r}(2\pi)$ (in the plane)

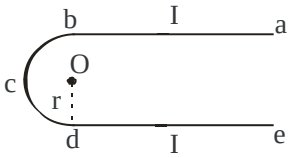
$B_{de} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r}$ (out of the plane)

Hence magnetic field at O is

$B_0 = -\dfrac{\mu_0}{4\pi}\dfrac{I}{r} + \dfrac{\mu_0}{4\pi}\dfrac{I}{r}(2\pi) - \dfrac{\mu_0}{4\pi}\dfrac{I}{r}$

$B_0 = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}(\pi - 1)$ ..... (III)

(B)




$B_{ab} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r}$ (out of the plane)

$B_{bcd} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r}(\pi)$ (out of the plane)

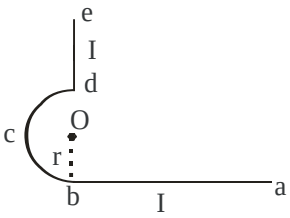
$B_{de} = \frac{\mu_0}{4\pi}\frac{I}{r}$(out of the plane)

Hence magnetic field at O is

$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r} + \frac{\mu_0}{4\pi}\frac{I}{r}(\pi) + \frac{\mu_0}{4\pi}\frac{I}{r}$

$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi + 2) \ldots.$ (I)

(C)




$B_{ab} = \frac{\mu_0}{4\pi}\frac{I}{r}$ (in the plane)

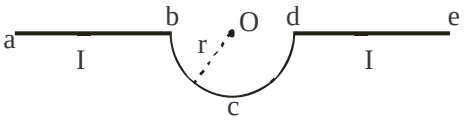
$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi)$(in the plane)

$B_{de} = 0$ (at the axis)

Hence magnetic field at O is

$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r}(1 + \pi) \ldots.$ (IV)

(D)




$B_{ab} = 0$ (at the axis)

$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi)$(out of the plane)

$B_{de} = 0$ (at the axis)

Hence magnetic field at O is

$B_0 = \frac{\mu_0 I}{4r} \ldots.$ (II)

**46.** Given below are two statements : one is labeled as Assertion A and the other is labeled as Reason R

**Assertion A:** Photodiodes are used in forward bias usually for measuring the light intensity.

**Reason R:** For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where $V_o$ is the threshold voltage and $V_z$ is the breakdown voltage.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both A and R are true and R is correct explanation A

(2) Both A and R are true but R is NOT the correct explanation A

(3) A is false but R is true

(4) A is true but R is false

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** Theory based

Photodiodes are operated in reverse bias condition.

For P-N junction current in forward bias $(\text{for } V \geq V_0)$ is always greater than current in reverse bias $(\text{for } V \leq V_z)$.

Hence Assertion if false but Reason is true

**47.** A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:

(1) $2.4 \times 10^3$ A $m^{-1}$

(2) $1.2 \times 10^3$ A $m^{-1}$

(3) 1 A $m^{-1}$

(4) $2.4 \times 10^{-3}$ A $m^{-1}$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** Magnetic field at centre inside the solenoid is given by

$B = \mu_0 nI$

So magnetic intensity at centre

$H = \frac{B}{\mu_0} = nI = \left(\frac{1200}{2}\right)(2)$

$H = 1.2 \times 10^3$ $Am^{-1}$

**48.** A uniform metallic wire carries a current 2 A. when 3.4 V battery is connected across it. The mass of uniform metallic wire is $8.92 \times 10^{-3}$ kg. density is $8.92 \times 10^3$ kg/$m^3$ and resistivity is $1.7 \times 10^{-8}\ \Omega$ –m. The length of wire is :

(1) $l$ =6.8 m (2) $l$ = 10 m

(3) $l$ = 5 m (4) $l$ = 100 m

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** I = 2A

$\Delta V$ = 3.4 V

Using Ohm's Law

$R = \frac{3.4}{2} = 1.7\Omega$

$1.7 = \frac{\rho L}{A}$

$L = \frac{1.7(A)}{\rho}$

M = (density volume)

$$\text{Volume} = \frac{8.92\times10^{-3}}{8.92\times10^{3}} = 10^{-6}$$

$$L^2 = \frac{1.7}{\rho}\left(10^{-6}\right) = \frac{1.7}{1.7}\times10^{2}$$

L=10m

**49.** A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C ?

(1) 2 minutes (2) 1.4 minutes

(3) 0.5 minute (4) 1 minute

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $\frac{\Delta Q}{\Delta t} = -K\left(T - T_0\right)$

$$\frac{\Delta Q}{\Delta t} = -K\left(T_{avg} - T_0\right)$$

(i) $$\frac{ms\times12}{2} = -K\left(\frac{98+86}{2} - 22\right)$$

$$6 = -\frac{K}{ms}\left[\frac{98+86}{2} - 22\right]$$

$$6 = -\frac{K}{ms}[70] \quad \ldots\ldots\ldots\ldots(i)$$

(ii) $$\frac{ms\times6}{\Delta t} = -K\left(\frac{75+69}{2} - 22\right)$$

$$\frac{6}{\Delta t} = -\frac{K}{ms}(50) \ldots\ldots\ldots\ldots(ii)$$

(ii) ÷ (i)

$$\frac{6}{\Delta t(6)} = \frac{50}{70}$$

$$\Delta t = \frac{7}{5} = 1.4\,\text{min}$$

**50.** A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take g = 10 $m/s^2$)

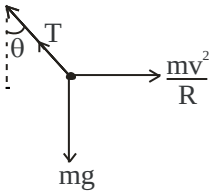
(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**




$T\cos\theta = mg$

$$T\sin\theta = \frac{mv^2}{R}$$

$$\tan\theta = \frac{v^2}{Rg}$$

$$\tan\theta = \frac{20^2}{40\times10}$$

$\tan\theta = 1$

$$\Rightarrow \theta = \frac{\pi}{4}$$

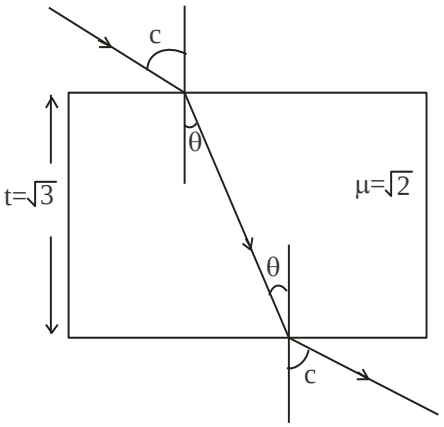
## SECTION-B

**51.** A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface . The lateral displacement of the ray when it passes through the plate is___$\times10^{-2}$cm. (given sin 15° = 0.26)

**Official Ans. by NTA (52)**

**QuestPix Ans. (52)**

**Sol.**




$$\sin c = \frac{1}{\sqrt{2}}$$

$c = 45°$

$\sin c = \mu\sin\theta$

$$\frac{1}{\sqrt{2}} = \sqrt{2}\sin\theta$$

$\theta = 30°$

Lateral displacement:

$x = t\sin(i-r)\sec r$

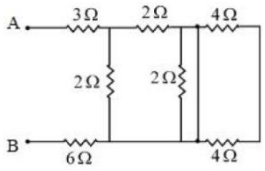
$x = \sqrt{3}\sin(45°-30°)\sec 30°$

$x = \sqrt{3}(0.26)\left(\frac{2}{\sqrt{3}}\right)$

X = 0.52 cm

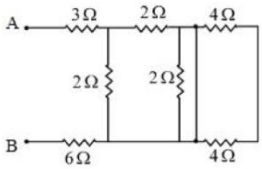
$x = 52\times10^{-2}$ cm

**52.** In the given circuit, the equivalent resistance between the terminal A and B is_____Ω.

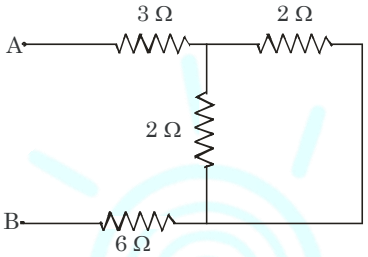



**Official Ans. by NTA (10.00 )**
**QuestPix Ans. (10)**

**Sol.**




Both 4Ω resistance gets short.
Remove the resistors that have no current.

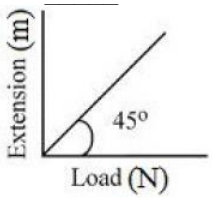



$R_{eq} = 3+(2\|2)+6$

$R_{eq} = 3+1+6$

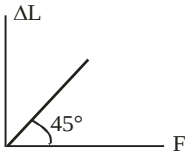
$R_{eq} = 10\Omega$

**53.** As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4 mm. The Young's modulus is found to be x × $10^4$ $Nm^{-2}$.
The value of x is _____.




**Official Ans. by NTA (5 )**
**QuestPix Ans. (5)**

**Sol.**




From graph :
F = ΔL

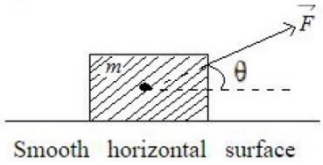
$Y = \frac{FL}{A\Delta L}$

$Y = \frac{L}{A}$

$Y = \frac{62.8\times10^{-2}}{\pi\left(2\times10^{-3}\right)^2}$

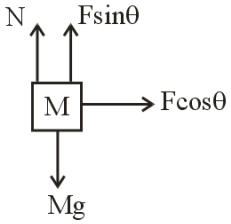
$Y = 5\times10^4\ N/m^2$

**54.** An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as θ = kx, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$. The value of n is___.




**Official Ans. by NTA (2)**
**QuestPix Ans. (2)**

**Sol.**




$F\cos\theta = ma$

$2\cos(kx) = \frac{mvdv}{dx}$

$$\int_0^v v\,dv = 2\int_0^x \cos(kx)\,dx$$

$$\frac{mv^2}{2} = \frac{2}{k}\sin kx$$

$$K.E. = \frac{2}{k}\sin\theta$$

**n = 2**

**55.** The wavelength of the radiation emitted is $\lambda_0$ when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is_____.

**Official Ans. by NTA (27)**
**QuestPix Ans. (27)**

**Sol.** _____ n = 4
_____ n = 3
_____ n = 2
_____ n = 1

Second excited state → first excited state
n = 3 → n =2

$$\frac{hc}{\lambda_0} = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right).....(i)$$

Third excited state → second orbit
n = 4 → n = 2

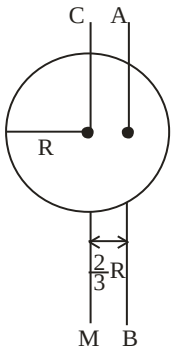
$$\frac{hc}{(20\lambda_0/x)} = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right).....(ii)$$

(ii) ÷ (i)

$$\frac{x}{20} = \frac{\frac{1}{2^2} - \frac{1}{4^2}}{\frac{1}{2^2} - \frac{1}{3^2}}$$

x = 27

**56.** $I_{CM}$ is moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. $I_{AB}$ is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center. Where R is the radius of the disc. The ratio of $I_{AB}$ and $I_{CM}$ is x: 9. The value of x is___.




**Official Ans. by NTA (17)**
**QuestPix Ans. (17)**

**Sol.** $I_{cm} = \frac{mR^2}{2}$

$$I_{AB} = \frac{mR^2}{2} + m\left(\frac{2R}{3}\right)^2 = \frac{17}{18}mR^2$$

$$\frac{I_{AB}}{I_{cm}} = \frac{17}{9} \Rightarrow x = 17$$

**57.** The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m. The velocity with which wave is traveling is_____ km/s

**Official Ans. by NTA (18 )**
**QuestPix Ans. (18)**

**Sol.** $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

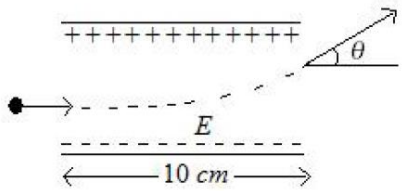
$$\frac{\pi}{3} = \frac{2\pi}{\lambda}(6m)$$

$\Rightarrow$ $\lambda = 36m$

$$V = f\lambda = (500Hz)(36m)$$

= 18000 m/s = 18 km/s

**58.** A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is ___ (in degree).




**Official Ans. by NTA (45)**
**QuestPix Ans. (45 )**

**Sol.** $0.5e = \frac{1}{2}mv_x^2 \Rightarrow v_x = \sqrt{\frac{e}{m}}$

Along x $L = v_x t = \sqrt{\frac{e}{m}}t$

Along y $v_y = \frac{eE}{m}t$

dividing $\frac{v_y}{L} = E\sqrt{\frac{e}{m}} = Ev_x$

$\Rightarrow \text{Tan}\theta = \frac{v_y}{v_x} = E \times L = 10 \times 0.1 = 1$

$\theta = 45°$

**59.** An LCR series circuit of capacitance 62.5 nF and resistance of 50 Ω. is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____mH.
(take $\pi^2 = 10$)

**Official Ans. by NTA (100)**

**QuestPix Ans. (100)**

**Sol.** $f = \frac{1}{2\pi\sqrt{LC}}$

$2000\,\text{Hz} = \frac{1}{2\pi\sqrt{L \times 62.5 \times 10^{-9}}}$

$L = \frac{1}{4\pi^2 \times 2000^2 \times 62.5 \times 10^{-9}} = 0.1\text{H} = 100\text{mH}$

**60.** If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value of x is

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** $\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix} = \sqrt{3}\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$

$\Rightarrow \frac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|} = \frac{1}{2}\left(\sqrt{3}\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}\right)$

$= \frac{1}{4}\left(\sqrt{3}\,\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right) \quad x = 4$

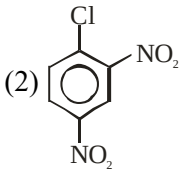
**(Held On Wednesday 25th January, 2023)** **TIME : 9 : 00 AM to 12 : 00 NOON**

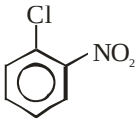
## CHEMISTRY | TEST PAPER WITH SOLUTION

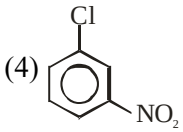
### SECTION-A

**61.** The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with $OH^-$ is

(1) 

(2) 

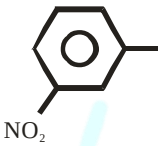
(3) 

(4) 

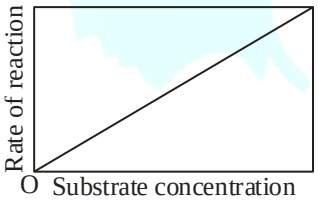
**Official Ans. by NTA (4)**

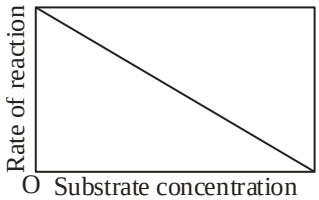
**QuestPix Ans. (4)**

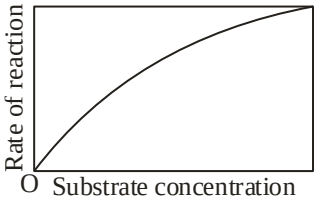
**Sol.** Electron withdrawing groups are highly ineffective at meta position in nucleophilic aromatic substitution reactions.

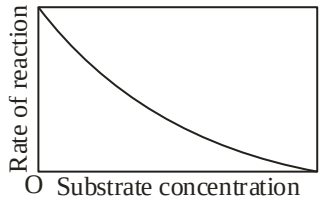
Hence compound  will have lowest rate in nucleophilic aromatic substitution.

**62.** The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph

(a) 


(b) 


(c) 


(d) 

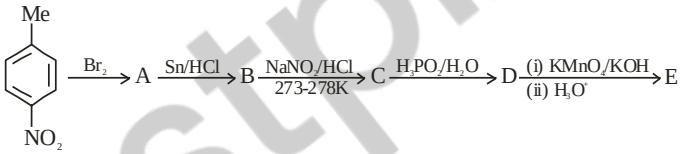

(1) b (2) c (3) d (4) a

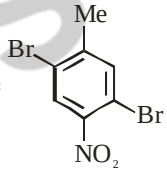
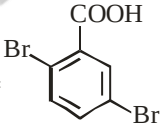
**Official Ans. by NTA (2)**

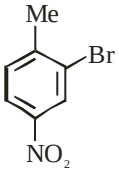
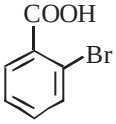
**QuestPix Ans. (2)**

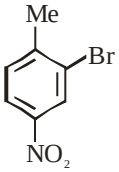
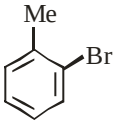
**Sol.** Fact base.

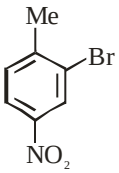
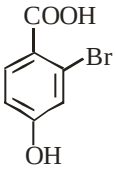
**63.** Identify the product formed (A and E)




(1) A =  , E = 

(2) A =  , E = 

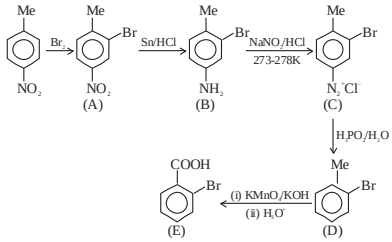
(3) A =  , E = 

(4) A =  , E = 

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.**




**64.** Match List I with List II

| List I | | List II | |
|---|---|---|---|
| Elements | | Colour imparted to the flame | |
| A | K | I | Brick Red |
| B | Ca | II | Violet |
| C | Sr | III | Apple Green |
| D | Ba | IV | Crimson Red |

Choose the correct answer from the options given below:

(1) A-II, B-I, C-III. D-IV

(2) A-II, B-IV, C-I. D-III

(3) A-II, B-I, C-IV. D-III

(4) A-IV, B-III, C-II. D-I

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**

| Element | Colour in flame test |
|---|---|
| K | Violet |
| Ca | Brick red |
| Sr | Crimson red |
| Ba | Apple green |

**65.** Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively

(1) $P_4O_6$ and $H_3PO_3$

(2) $PCl_3$ and $H_3PO_3$

(3) $PCl_5$ and $H_3PO_4$

(4) $POCl_3$ and $H_3PO_4$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $P_4 + 8SOCl_2 \rightarrow \underset{[A]}{4PCl_3} + 4SO_2 + 2S_2Cl_2$

$PCl_3 + 3H_2O \rightarrow \underset{[B]}{H_3PO_3} + 3HCl$

**66.** A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}^{rd}$ of the total faces. The empirical formula of the compound is

(1) $X_2Y_{1.5}$ (2) $X_{2.5}Y$

(3) $XY_{2.5}$ (4) $X_{1.5}Y_2$

**Official Ans. by NTA (2)**

**QuestPix Ans. (Bonus)**

**Sol.** $X_{4\times\frac{1}{8}+1\times1}\ Y_{6\times\frac{1}{3}\times\frac{1}{2}}$

$\Rightarrow X_{\frac{1}{2}+1}\ Y_1$

$\Rightarrow X_{\frac{2}{3}}\ Y_1$

$\Rightarrow X_{1.5}\ Y_1$

$\Rightarrow X_3\ Y_2$

**67.** The radius of the 2nd orbit of $Li^{2+}$ is x. The expected radius of the 3rd orbit of $Be^{3+}$ is

(1) $\frac{9}{4}x$ (2) $\frac{4}{9}x$

(3) $\frac{27}{16}x$ (4) $\frac{16}{27}x$

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

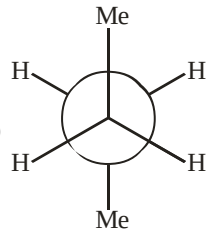
**Sol.** $Li^{2+}$ | $Be^{3+}$

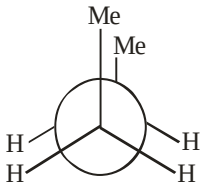
$r_2 = x = k\times\frac{2^2}{3} = \frac{4k}{2}$ | $r_3 = y = k\times\frac{3^2}{4}$

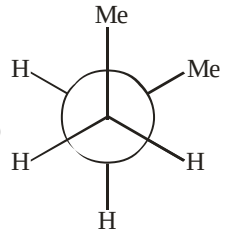
$\frac{y}{x} = \frac{9}{4}\times\frac{3}{4} = \frac{27}{16}$

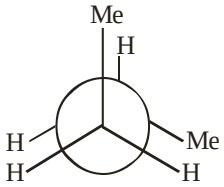
$y = \frac{27}{16}x$

**68.** Which of the following conformations will be the most stable ?

(1) 

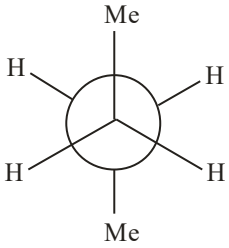
(2) 

(3) 

(4) 

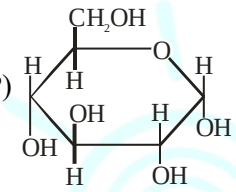
**Official Ans. by NTA (1)**

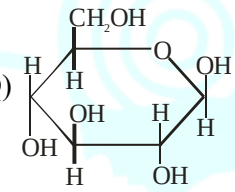
**QuestPix Ans. (1)**

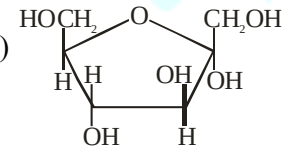
**Sol.** Conformation  has lowest vanderwaal and torsional strain. Hence it must be most stable.

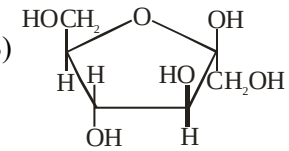
**69.** Match items of Row I with those of Row II.

**Row I :**

(P) 

(Q) 

(R) 

(S) 

**Row II :**

(i) α-D-(–) Fructofuranose.

(ii) β-D-(–) Fructofuranose

(iii) α-D-(-) Glucopyranose.

(iv) β-D-(-) Glucopyranose

Correct match is

(1) P→iv, Q→iii, R→i, S→ii

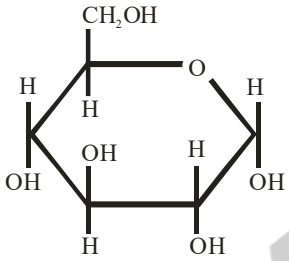
(2) P→i, Q→ii, R→iii, S→iv

(3) P→iii, Q→iv, R→ii, S→i

(4) P→iii, Q→iv, R→i, S→ii

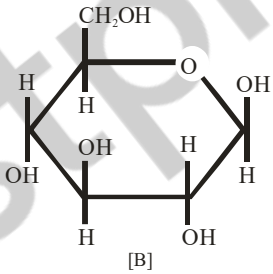
**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** Structure 

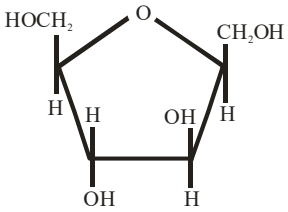
[A]

Represents α-D-(+) Glucopyranose

Structure 

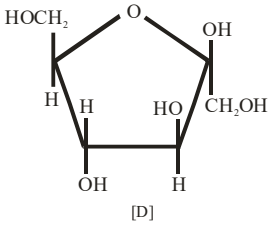
[B]

Represents β-D-(+) Glucopyranose

Structure 

[C]

Represents β-D-(–) Fructofuranose

Structure 

[D]

Represents β-D-(–) Fructofuranose

(from the given options best answer is D)

**70.** Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

**Assertion A** : Acetal/Ketal is stable in basic medium.

**Reason R** : The high leaving tendency of alkoxide ion gives the stability to acetal/ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true and R is the correct explanation of A

(4) Both A and R are true but R is NOT the correct explanation of A

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** For Assertion :Acetal and ketals are basically ethers hence they must be stable in basic medium but should break down in acidic medium.

Hence assertion is correct.

For reason: Alkoxide ion ($RO^-$) is not considered a good leaving group hence reason must be false.

**71.** Inert gases have positive electron gain enthalpy. Its correct order is

(1) Xe < Kr < Ne < He

(2) He < Ne < Kr < Xe

(3) He < Xe < Kr < Ne

(4) He < Kr < Xe < Ne

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**

| Element | ΔegH[KJ/mol] |
|---|---|
| He | +48 |
| Ne | +116 |
| Kr | +96 |
| Xe | +77 |

From NCERT

So, order is Ne > Kr >Xe> He

**72.** Which one of the following reactions does not occur during extraction of copper ?

(1) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(2) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$

(3) $CaO + SiO_2 \rightarrow CaSiO_3$

(4) $FeO + SiO_2 \rightarrow FeSiO_3$

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** $CuFeS_2 + O_2 \xrightarrow{\text{Partial roasting}}$

$Cu_2S + FeO + SO_2 + \underset{\text{very small}}{FeS} + \underset{\text{very small}}{Cu_2O}$

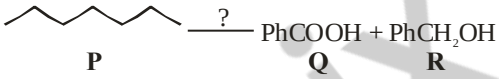
$Cu_2S + O_2 \rightarrow Cu_2O + SO_2$

$FeS + O_2 \rightarrow FeO + SO_2$

$FeO + SiO_2 \rightarrow FeSiO_3$

No formation of calcium silicate ($CaSiO_3$) in extraction of Cu.

**73.**




The correct sequence of reagents for the preparation of Q and R is :

(1) (i) $Cr_2O_3$, 770 K, 20 atm ;
(ii) $CrO_2Cl_2$, $H_3O^+$ ;
(iii) NaOH ;
(iv) $H_3O^+$

(2) (i) $CrO_2Cl_2$, $H_3O^+$ ; (ii) $Cr_2O_3$, 770 K, 20 atm;
(iii) NaOH ; (iv) $H_3O^+$

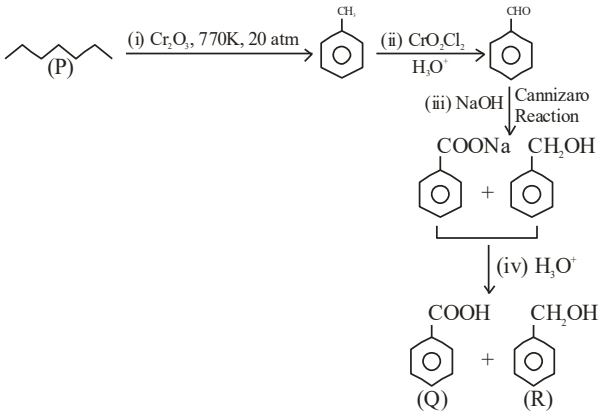
(3) (i) $KMnO_4$, $OH^-$ ; (ii) $Mo_2O_3$, A; (iii) NaOH ;
(iv) $H_3O^+$

(4) (i) $Mo_2O_3$, Δ ; (ii) $CrO_2Cl_2$, $H_3O^+$ ; (iii) NaOH ;
(iv) $H_3O^+$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.**




**74.** The correct order in aqueous medium of basic strength in case of methyl substituted amines is :

(1) $Me_2NH > MeNH_2 > Me_3N > NH_3$

(2) $Me_2NH > Me_3N > MeNH_2 > NH_3$

(3) $NH_3 > Me_3N > MeNH_2 > Me_2NH$

(4) $Me_3N > Me_2NH > MeNH_2 > NH_3$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.** In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting $H^+$. After considering all these factors overall basic strength order is $Me_2NH > MeNH_2 > Me_3N > NH_3$

**75.** '25 volume' hydrogen peroxide means

(1) 1 L marketed solution contains 250 g of $H_2O_2$.

(2) 1 L marketed solution contains 75 g of $H_2O_2$.

(3) 100 mL marketed solution contains 25 g of $H_2O_2$.

(4) 1 L marketed solution contains 25 g of $H_2O_2$.

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.**

Volume = 11.35 × M

Strength

$$M = \frac{25}{11.35} M$$

g/L = 25 × 34/11.35 = 74.889

**76.** Which of the following statements is incorrect for antibiotics?

(1) An antibiotic must be a product of metabolism.

(2) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.

(3) An antibiotic should promote the growth or survival of microorganisms.

(4) An antibiotic should be effective in low concentrations.

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.** An antibiotic should not promote growth or survival of microorganisms. Antibiotics should inhibit growth of microbes.

**77.** Compound A reacts with $NH_4Cl$ and forms a compound B. Compound B reacts with $H_2O$ and excess of $CO_2$ to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A. B and C, are respectively.

(1) $CaCl_2$, $NH_3$, $NH_4HCO_3$

(2) $CaCl_2$, $NH_4^+$, $(NH_4)_2CO_3$

(3) $Ca(OH)_2$, $NH_3$, $NH_4HCO_3$

(4) $Ca(OH)_2$, $NH_4^+$, $(NH_4)_2CO_3$

**Official Ans. by NTA (3)**

**QuestPix Ans. (3)**

**Sol.**

$$\underset{(A)}{Ca(OH)_2} + 2NH_4Cl \xrightarrow{\Delta} \underset{(B)}{2NH_3} + CaCl_2 + 2H_2O$$

$$\underset{(B)}{NH_3} + H_2O + \underset{(exc)}{CO_2} \longrightarrow \underset{(C)}{NH_4HCO_3}$$

$$\underset{(C)}{NH_4HCO_3} + NaCl \longrightarrow NaHCO_3 \downarrow + NH_4Cl$$

**78.** Some reactions of $NO_2$ relevant to photochemical smog formation are

$$NO_2 \xrightarrow{\text{Sunlight}} \begin{matrix} X + Y \\ | \\ A \\ | \\ B \end{matrix}$$

Identify A, B, X and Y

(1) X = [O], Y = NO, A = $O_2$, B = $O_3$

(2) X = $N_2O$, Y = [O], A = $O_3$, B = NO

(3) X = $\frac{1}{2}O_2$, Y = $NO_2$, A = $O_3$, B = $O_2$

(4) X = NO, Y = [O], A = $O_2$, B = $N_2O_3$

**Official Ans. by NTA (1)**

**QuestPix Ans. (1)**

**Sol.**

$$NO_2 \xrightarrow{\text{Sun light}} \underbrace{[O]}_{X} + \underbrace{NO}_{Y}$$

$$\underbrace{O_2}_{A}$$

$$\underbrace{O_3}_{B}$$

**79.** Match the **List-I** with **List-II** :

| Cations | Group reaction |
|---|---|
| P → $Pb^{2+}$, $Cu^{2+}$ | (i) $H_2S$ gas in presence of dilute HCl |
| Q → $Al^{3+}$, $Fe^{3+}$ | (ii) $(NH_4)_2CO_3$ in presence of $NH_4OH$ |
| R → $Co^{2+}$, $Ni^{2+}$ | (iii) $NH_4OH$ in presence of $NH_4CI$ |
| S → $Ba^{2+}$, $Ca^{2+}$ | (iv) $H_2S$ in presence of $NH_4OH$ |

(1) P→i, Q→iii, R→ii, S→iv
(2) P→iv, Q→ii, R→iii, S→i
(3) P→iii, Q→i, R→iv, S→ii
(4) P→i, Q→iii, R→iv, S→ii

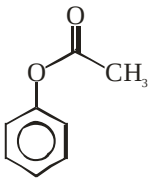
**Official Ans. by NTA (4)**

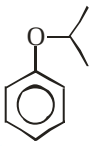
**QuestPix Ans. (4)**

**Sol.**

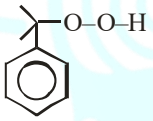
| Cations | Group No. | Group reagent |
|---|---|---|
| $Pb^{+2}$ ,$Cu^{+2}$ | II | $H_2S$(g) in presence of dilHCl |
| $Al^{+3}$, $Fe^{+3}$ | III | $NH_4OH$ in presence of $NH_4Cl$ |
| $CO^{+2}$, $Ni^{+2}$ | IV | $H_2S$ in presence of $NH_4OH$ |
| $Ba^{+2}$, $Ca^{+2}$ | V | $(NH_4)_2CO_3$ in presence of $NH_4OH$ |

**80.** In the cumene to phenol preparation in presence of air, the intermediate is

(1) 

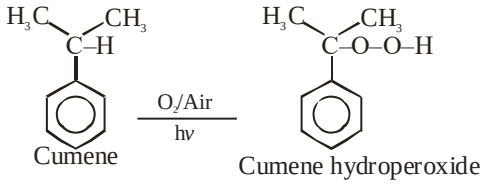
(2) 

(3) 

(4) 

**Official Ans. by NTA (4)**

**QuestPix Ans. (4)**

**Sol.** 


## SECTION-B

**81.** An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is_____g (Nearest integer)

Assume that there is no other way of consuming stored energy.

**Given :** The enthalpy of evaporation of water is 45 kJ $mol^{-1}$

Molar mass of C, H & O are 12. 1 and 16 g $mol^{-1}$.

**Official Ans. by NTA (360)**

**QuestPix Ans. (360)**

**Sol.** $C_6H_{12}O_6(s) + 6O_2 \rightarrow 6CO_2(g) + 6H_2O(l)$

Extra energy used to convert $H_2O(l)$ into $H_2O(l)$ into $H_2O(g)$

$$= \frac{1800}{2} = 900\text{kJ}$$

$$\Rightarrow \quad 900 = n_{H_2O} \times 45$$

$$n_{H_2O} = \frac{900}{45} = 20 \text{ mole}$$

$$W_{H_2O} = 20 \times 18 = 360 \text{ g}$$

**82.** A litre of buffer solution contains 0.1 mole of each of $NH_3$ and $NH_4Cl$. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _____$\times 10^{-3}$ (Nearest integer)

[**Given** : $pK_b(NH_3) = 4.745$

log 2 = 0.301

log 3 = 0.477

T = 298 K]

**Official Ans. by NTA (9079)**

**QuestPix Ans. (9079)**

**Sol.** In resultant solution

$$n_{NH_3} = 0.1 - 0.02 = 0.08$$

$$n_{NH_4Cl} = n_{NH_4^+} = 0.1 + 0.02 = 0.12$$

$$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$= 4.745 + \log\frac{0.12}{0.08}$

$= 4.745 + \log\frac{3}{2}$

$= 4.745 + 0.477 - 0.301$

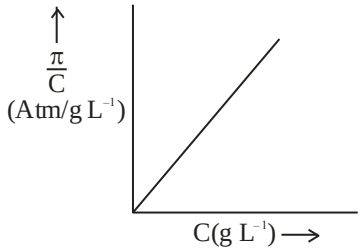
$pOH = 4.921$

$pH = 14 - pH$

$= 9.079$

**83.** The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph.

The molar mass of PVC is_____g $mol^{-1}$ (Nearest integer)




(Given : R = 0.083 L atm $K^{-1}$ $mol^{-1}$)

**Official Ans. by NTA (41500)**

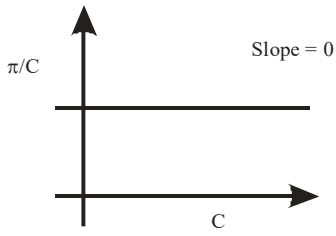
**QuestPix Ans. (Bonus/41500)**

**Sol.** $\pi = M'RT = \left(\frac{W/M}{V}\right)RT$

$\Rightarrow \quad \pi = \left(\frac{W}{V}\right)\left(\frac{1}{M}\right)RT = C\left(\frac{RT}{M}\right)$

$\Rightarrow \quad \frac{\pi}{C} = \frac{RT}{M} \neq f(c)$

If we assume graph between $\frac{\pi}{C}$ and C




Assuming π vs C graph

$$\text{Slope} = \frac{RT}{M} = \frac{0.083 \times 300}{M} = 6 \times 10^{-4}$$

$$\therefore \quad M = \frac{0.083 \times 300}{6 \times 10^{-4}} = \frac{830 \times 300}{6} = 41{,}500$$

gm/mole

**84.** How many of the following metal ions have similar value of spin only magnetic moment in gaseous state ?_____

(Given: Atomic number : V, 23 ; Cr, 24 ; Fe, 26 ; Ni, 28)

$V^{3+}$. $Cr^{3+}$, $Fe^{2+}$, $Ni^{3+}$

**Official Ans. by NTA (2)**

**QuestPix Ans. (2)**

**Sol.** $\mu_s = \sqrt{n(n+2)}BM$ (n=no. of unpaired electrons)

| | n |
|---|---|
| $V^{3+}$ : [Ar] $3d^2 4s^0$ | 2 |
| $Cr^{3+}$ : [Ar] $3d^3 4s^0$ | 3 |
| $Fe^{2+}$ : [Ar] $3d^6 4s^0$ | 4 |
| $Ni^{3+}$ : [Ar] $3d^7 4s^0$ | 3 |

$Cr^{3+}$& $Ni^{3+}$ have same value of $\mu_s$

**85.** The density of a monobasic strong acid (Molar mass 24.2 g mol) is 1.21 kg L. The volume of its solution required for the complete neutralization of 25 mL of 0.24 M NaOH is_____× $10^{-2}$ mL (Nearest integer)

**Official Ans. by NTA (12)**

**QuestPix Ans. (12)**

**Sol.** millimole of NaOH = 0.24 × 25

$\therefore$ millimole of acid = 0.24 × 25

$\Rightarrow$ mass of acid = 0.24 × 25 × 24.2 mg

for pure acid,

$$V = \frac{w}{d};\ (d = 1.21\ kg/L\ = 1.21\ g/ml)$$

$$\therefore V = \frac{0.24 \times 25 \times 24.2}{1.12} \times 10^{-3}$$

$= 120 \times 10^{-3}$ ml

$= 12 \times 10^{-2}$ ml

**86.** For the first order reaction A $\rightarrow$ B Br the half life is 30 mm. The time taken for 75% completion of the reaction is_____mm. (Nearest mteger)

**Given** : log 2 = 0.3010

log 3 = 0.4771

log 5 = 0.6989

**Official Ans. by NTA (60)**

**QuestPix Ans. (60)**

**Sol.** $t_{1/2} = T_{50} = 30$ min

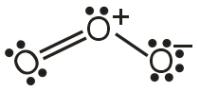
$T_{75} = 2t_{1/2} = 30 \times 2 = 60$ min

**87.** The total number of lone pairs of electrons on oxygen atoms of ozone is_____

**Official Ans. by NTA (6)**

**QuestPix Ans. (6)**

**Sol.** (Total no, of lone pairs on oxygen atoms = 6



**88.** In sulphur estimation. 0.471 g of an organic compound gave 1.4439 g of barium sulphate.

The percentage of sulphur in the compound is_____(Nearest Integer)

(Given: Atomic mass Ba: 137 u: S: 32 u, O: 16 u )

**Official Ans. by NTA (42)**

**QuestPix Ans. (42)**

**Sol**

$$\%\text{sulphur} = \frac{32}{233} \times \frac{\text{weight of } BaSO_4 \text{formed}}{\text{weight of organic compound}} \times 100$$

$$= \frac{32}{233} \times \frac{1.4439}{0.471} \times 100$$

$= 42.10$

Nearest integer 42

**89.** The number of paramagnetic species from the following is_____.

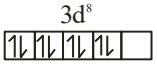
$[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$, $[NiCl_4]^{2-}$

$[Fe(CN)_6]^{4-}$, $[Cu(NH_3)_4]^{2+}$

$[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$

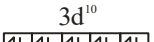
**Official Ans. by NTA (4)**

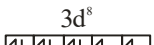
**QuestPix Ans. (4)**

**Sol.** $\left[Ni(CN)_4\right]^{2-} : Ni^{+2} =$  :

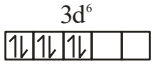
diamagnetic

$^{-}CN$ : strong field ligand

$\left[Ni(CO)_4\right]$ : Ni =  : diamagnetic

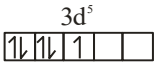
$[NiCl_4]^{2-}$ : $Ni^{2+}$ =  : paramagnetic

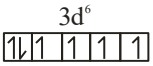
$Cl^-$ : weak field ligand

$[Fe(CN)_6]^{4-}$ : $Fe^{2+}$  : diamagnetic

$^-CN$ : strong field ligand

$[Cu(NH_3)_4]^{+2}$ : $Cu^{+2}$ $\Rightarrow$ one unpaired electron : paramagnetic

$[Fe(CN)_6]^{3-}$ : $Fe^{+3}$ :  : paramagnetic, $^-CN$ : strong field ligand

$[Fe(H_2O)_6]^{2+}$ : $Fe^{2+}$ :  : paramagnetic $H_2O$ : Weak field ligand

**90.** Consider the cell

$$Pt(s)\left|H_2(s)(1atm)\right|H^+(aq,[H^+]=1)\Big\|Fe^{3+}(aq),Fe^{2+}(aq)\left|Pt(s)\right.$$

**Given** : $E^\circ_{Fe^{3+}/Fe^{2+}} = 0.771V$ and $E^\circ_{H^+/\frac{1}{2}H_2} = 0V, T = 298K$

If the potential of the cell is 0.712 V the ratio of concentration of $Fe^{2+}$ to $Fe^{2+}$ is_____(Nearest integer)

**Official Ans. by NTA (10)**

**QuestPix Ans. (10)**

**Sol.** $\frac{1}{2}H_2(g) + Fe^{3+}(aq.) \longrightarrow H^+(aq) + Fe^{2+}(aq.)$

$$E = E^o - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow \quad 0.712 = (0.771 - 0) - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow \quad \log\frac{[Fe^{2+}]}{[Fe^{3+}]} = \frac{(0.771 - 0712)}{0.059} = 1$$

$$\Rightarrow \quad \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$