

(Held On Sunday 25th July, 2021)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

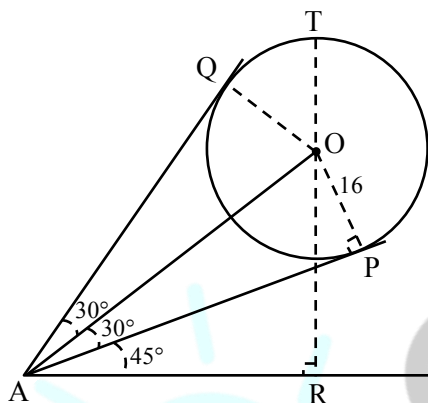
1. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75° . Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(1) $8(2+2\sqrt{3}+\sqrt{2})$ (2) $8(\sqrt{6}+\sqrt{2}+2)$

(3) $8(\sqrt{2}+2+\sqrt{3})$ (4) $8(\sqrt{6}-\sqrt{2}+2)$

Official Ans. by NTA (2)

Sol.



O $\rightarrow$ centre of sphere

P, Q $\rightarrow$ point of contact of tangents from A

Let T be top most point of balloon & R be foot of perpendicular from O to ground.

From triangle OAP, $OA = 16 \operatorname{cosec} 30^\circ = 32$

From triangle ABO, $OR = OA \sin 75^\circ = 32 \frac{(\sqrt{3}+1)}{2\sqrt{2}}$

So level of top most point = $OR + OT$

$$= 8(\sqrt{6} + \sqrt{2} + 2)$$

2. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is :

(1) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(2) decreasing in $\left(0, \frac{\pi}{2}\right)$

(3) increasing in $\left(-\frac{\pi}{6}, 0\right)$

(4) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Official Ans. by NTA (4)

Sol. $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

$$f'(x) = 12\sin^3 x \cos x + 30\sin^2 x \cos x + 12\sin x \cos x$$

$$= 6\sin x \cos x (2\sin^2 x + 5\sin x + 2)$$

$$= 6\sin x \cos x (2\sin x + 1)(\sin x + 2)$$



Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

3. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

(1) 6 (2) 4 (3) 2 (4) 8

Official Ans. by NTA (1)

- Sol.** Let a be first term and d be common diff. of this A.P.

Given $S_{3n} = 3S_{2n}$

$$\Rightarrow \frac{3n}{2} [2a + (3n-1)d] = 3 \frac{2n}{2} [2a + (2n-1)d]$$

$$\Rightarrow 2a + (3n-1)d = 4a + (4n-2)d$$

$$\Rightarrow 2a + (n-1)d = 0$$

$$\text{Now } \frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2} [2a + (4n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]} = \frac{2 \left[\underbrace{2a + (n-1)d}_{=0} + 3nd \right]}{\left[\underbrace{2a + (n-1)d}_{=0} + nd \right]}$$

$$= \frac{6nd}{nd} = 6$$

4. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

- (1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$
 (2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$
 (3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$
 (4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Official Ans. by NTA (1)

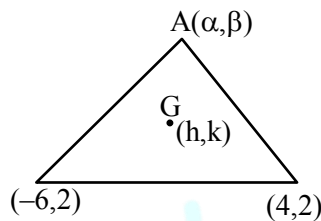
Sol. Given hyperbola is

$$16(x+1)^2 - 9(y-2)^2 = 164 + 16 - 36 = 144$$

$$\Rightarrow \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$$

$$\text{Eccentricity, } e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$$

$\Rightarrow$ foci are (4, 2) and (-6, 2)



Let the centroid be (h, k)

& A(α, β) be point on hyperbola

$$\text{So } h = \frac{\alpha - 6 + 4}{3}, k = \frac{\beta + 2 + 2}{3}$$

$$\Rightarrow \alpha = 3h + 2, \beta = 3k - 4$$

(α, β) lies on hyperbola so

$$16(3h + 2 + 1)^2 - 9(3k - 4 - 2)^2 = 144$$

$$\Rightarrow 144(h + 1)^2 - 81(k - 2)^2 = 144$$

$$\Rightarrow 16(h^2 + 2h + 1) - 9(k^2 - 4k + 4) = 16$$

$$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$$

5. Let the vectors

$$(2+a+b)\hat{i} + (a+2b+c)\hat{j} - (b+c)\hat{k}, (1+b)\hat{i} + 2b\hat{j} - b\hat{k}$$

and $(2+b)\hat{i} + 2b\hat{j} + (1-b)\hat{k}$ a, b, c, $\in \mathbf{R}$

be co-planar. Then which of the following is true?

- (1) $2b = a + c$ (2) $3c = a + b$
 (3) $a = b + 2c$ (4) $2a = b + c$

Official Ans. by NTA (1)

Sol. If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix} = 0$$

Now $R_3 \rightarrow R_3 - R_2, R_1 \rightarrow R_1 - R_2$

$$\text{So } \begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$$= (a+1)2b - (a+c)(2b+1) - c(-2b)$$

$$= 2ab + 2b - 2ab - a - 2bc - c + 2bc$$

$$= 2b - a - c = 0$$

6. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \frac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to :

- (1) $e(-e + 1)$ (2) $e(e - 2)$
 (3) 1 (4) $2e - 1$

Official Ans. by NTA (1)

$$\text{Sol. } \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{x-2}} = e^1$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)} = -\frac{\lambda}{\mu}$$

$$\text{For continuity } \mu = e = -\frac{\lambda}{\mu} \Rightarrow \mu = e, \lambda = -e^2$$

$$\lambda + \mu = e(-e + 1)$$

7. The value of the definite integral

$$\int_{\pi/24}^{5\pi/24} \frac{dx}{1 + \sqrt[3]{\tan 2x}} \text{ is :}$$

- (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{18}$

Official Ans. by NTA (3)

$$\text{Sol. Let } I = \int_{\pi/24}^{5\pi/24} \frac{(\cos 2x)^{1/3}}{(\cos 2x)^{1/3} + (\sin 2x)^{1/3}} dx \dots(i)$$

$$\Rightarrow I = \int_{\pi/24}^{5\pi/24} \frac{\left(\cos\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}}}{\left(\cos\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}} + \left(\sin\left\{2\left(\frac{\pi}{4}-x\right)\right\}\right)^{\frac{1}{3}}} dx$$

$$\left\{ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right\}$$

$$\text{So } I = \int_{\pi/24}^{5\pi/24} \frac{(\sin 2x)^{1/3}}{(\sin 2x)^{1/3} + (\cos 2x)^{1/3}} dx \dots (ii)$$

$$\text{Hence } 2I = \int_{\pi/24}^{5\pi/24} dx \quad [(i) + (ii)]$$

$$\Rightarrow 2I = \frac{4\pi}{24} \Rightarrow I = \frac{\pi}{12}$$

8. If b is very small as compared to the value of a , so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3,$$

then the value of γ is :

$$(1) \frac{a^2+b}{3a^3} \quad (2) \frac{a+b}{3a^2} \quad (3) \frac{b^2}{3a^3} \quad (4) \frac{a+b^2}{3a^3}$$

Official Ans. by NTA (3)

Sol. $(a-b)^{-1} + (a-2b)^{-1} + \dots + (a-nb)^{-1}$

$$= \frac{1}{a} \sum_{r=1}^n \left(1 - \frac{rb}{a}\right)^{-1}$$

$$= \frac{1}{a} \sum_{r=1}^n \left\{ \left(1 + \frac{rb}{a} + \frac{r^2 b^2}{a^2}\right) + (\text{terms to be neglected}) \right\}$$

$$= \frac{1}{a} \left[n + \frac{n(n+1)}{2} \cdot \frac{b}{a} + \frac{n(n+1)(2n+1)}{6} \cdot \frac{b^2}{a^2} \right]$$

$$= \frac{1}{a} \left[n^3 \left(\frac{b^2}{3a^2} \right) + \dots \right]$$

So $\gamma = \frac{b^2}{3a^3}$

9. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + x e^{y-x}$, $-\sqrt{2} < x < \sqrt{2}$, $y(0) = 0$

then, the minimum value of $y(x)$, $x \in (-\sqrt{2}, \sqrt{2})$ is equal to :

- (1) $(2-\sqrt{3}) - \log_e 2$
 (2) $(2+\sqrt{3}) + \log_e 2$
 (3) $(1+\sqrt{3}) - \log_e (\sqrt{3}-1)$
 (4) $(1-\sqrt{3}) - \log_e (\sqrt{3}-1)$

Official Ans. by NTA (4)

Sol. $\frac{dy}{dx} - x = x e^{y-x}$

$$\Rightarrow \frac{dy}{dx} - x = x e^{y-x}$$

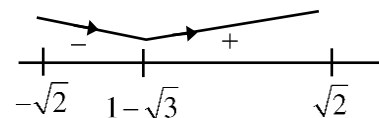
$$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$$

$$\text{At } x=0, y=0 \Rightarrow c=-1$$

$$\Rightarrow e^{x-y} = \frac{2-x^2}{2}$$

$$\Rightarrow y = x - \ln \left(\frac{2-x^2}{2} \right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2-x^2} = \frac{2+2x-x^2}{2-x^2}$$



So minimum value occurs at $x = 1 - \sqrt{3}$

$$y(1-\sqrt{3}) = (1-\sqrt{3}) - \ln \left(\frac{2-(4-2\sqrt{3})}{2} \right)$$

$$= (1-\sqrt{3}) - \ln(\sqrt{3}-1)$$

10. The Boolean expression

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

- (1) $\sim q$ (2) q (3) p (4) $\sim p$

Official Ans. by NTA (4)

Sol. $(p \rightarrow q) \wedge (q \rightarrow \sim p)$

$$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p) \quad \{p \rightarrow q \equiv \sim p \vee q\}$$

$$\equiv (\sim p \vee q) \wedge (\sim p \vee \sim q) \quad \{\text{commutative property}\}$$

$$\equiv \sim p \vee (q \wedge \sim q) \quad \{\text{distributive property}\}$$

$$\equiv \sim p$$

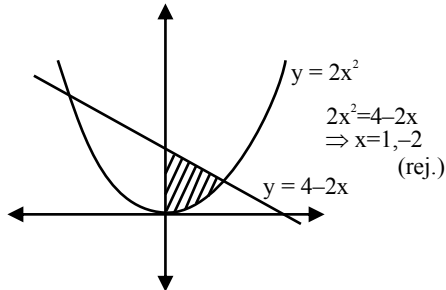


11. The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

- (1) $\frac{8}{3}$ (2) $\frac{17}{3}$ (3) $\frac{13}{3}$ (4) $\frac{7}{3}$

Official Ans. by NTA (4)

Sol.



$$\begin{aligned} \text{Required area} &= \int_0^1 (4 - 2x - 2x^2) dx = 4x - x^2 - \frac{2x^3}{3} \Big|_0^1 \\ &= 4 - 1 - \frac{2}{3} = \frac{7}{3} \end{aligned}$$

12. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

- (1) 8π (2) 11π
(3) 12π (4) 9π

Official Ans. by NTA (4)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$$\Rightarrow 2 \sin \frac{5x}{2} \left\{ \cos \frac{3x}{2} + \cos \frac{x}{2} \right\} = 0$$

$$\Rightarrow 2 \sin \frac{5x}{2} \left\{ 2 \cos x \cos \frac{x}{2} \right\} = 0$$

$$2 \sin \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$\cos \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{So sum} = 6\pi + \pi + 2\pi = 9\pi$$

13. Let $g : \mathbf{N} \rightarrow \mathbf{N}$ be defined as

$$g(3n + 1) = 3n + 2,$$

$$g(3n + 2) = 3n + 3,$$

$$g(3n + 3) = 3n + 1, \text{ for all } n \geq 0.$$

Then which of the following statements is true ?

- (1) There exists an onto function $f : \mathbf{N} \rightarrow \mathbf{N}$ such that $f \circ g = f$
(2) There exists a one-one function $f : \mathbf{N} \rightarrow \mathbf{N}$ such that $f \circ g = f$
(3) $g \circ g \circ g = g$
(4) There exists a function $f : \mathbf{N} \rightarrow \mathbf{N}$ such that $g \circ f = f$

Official Ans. by NTA (1)

Sol. $g : \mathbf{N} \rightarrow \mathbf{N}$ $g(3n + 1) = 3n + 2$

$$g(3n + 2) = 3n + 3$$

$$g(3n + 3) = 3n + 1$$

$$g(x) = \begin{cases} x+1 & x=3k+1 \\ x+1 & x=3k+2 \\ x-2 & x=3k+3 \end{cases}$$

$$g(g(x)) = \begin{cases} x+2 & x=3k+1 \\ x-1 & x=3k+2 \\ x-1 & x=3k+3 \end{cases}$$

$$g(g(g(x))) = \begin{cases} x & x=3k+1 \\ x & x=3k+2 \\ x & x=3k+3 \end{cases}$$

If $f : \mathbf{N} \rightarrow \mathbf{N}$, f is a one-one function such that

$$f(g(x)) = f(x) \Rightarrow g(x) = x, \text{ which is not the case}$$

If $f : \mathbf{N} \rightarrow \mathbf{N}$ f is an onto function

$$\text{such that } f(g(x)) = f(x),$$

one possibility is

$$f(x) = \begin{cases} n & x=3n+1 \\ n & x=3n+2 \\ n & x=3n+3 \end{cases} \quad n \in \mathbf{N}_0$$

Here $f(x)$ is onto, also $f(g(x)) = f(x) \forall x \in \mathbf{N}$

14. Let $f : [0, \infty) \rightarrow [0, \infty)$ be defined as

$$f(x) = \int_0^x [y] dy$$

where $[x]$ is the greatest integer less than or equal to x . Which of the following is true?

- (1) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.
(2) f is both continuous and differentiable except at the integer points in $[0, \infty)$.



- (3) f is continuous everywhere except at the integer points in $[0, \infty)$.
 (4) f is differentiable at every point in $[0, \infty)$.

Official Ans. by NTA (1)

Sol. $f : [0, \infty) \rightarrow [0, \infty), f(x) = \int_0^x [y] dy$

Let $x = n + f, f \in (0, 1)$

So $f(x) = 0 + 1 + 2 + \dots + (n-1) + \int_n^{n+f} n dy$

$$f(x) = \frac{n(n-1)}{2} + nf$$

$$= \frac{[x]([x]-1)}{2} + [x]\{x\}$$

Note $\lim_{x \rightarrow n^+} f(x) = \frac{n(n-1)}{2}, \lim_{x \rightarrow n^-} f(x) = \frac{(n-1)(n-2)}{2} + (n-1)$

$$= \frac{n(n-1)}{2}$$

$$f(x) = \frac{n(n-1)}{2} \quad (n \in \mathbb{N}_0)$$

so $f(x)$ is cont. $\forall x \geq 0$ and diff. except at integer points

15. The values of a and b , for which the system of equations

$$2x + 3y + 6z = 8$$

$$x + 2y + az = 5$$

$$3x + 5y + 9z = b$$

has no solution, are :

- (1) $a = 3, b \neq 13$ (2) $a \neq 3, b \neq 13$
 (3) $a \neq 3, b = 3$ (4) $a = 3, b = 13$

Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If $a = 3, b \neq 13$, no solution.

16. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4 . If the probability that B_3

contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

- (1) $\{x \in \mathbb{R} : |x-3| < 1\}$ (2) $\{x \in \mathbb{R} : |x-2| \leq 1\}$
 (3) $\{x \in \mathbb{R} : |x-1| < 1\}$ (4) $\{x \in \mathbb{R} : |x-5| \leq 1\}$

Official Ans. by NTA (1)

Sol. required probability = $\frac{{}^9C_3 \cdot 3^6}{4^9}$

$$= \frac{{}^9C_3 \cdot \left(\frac{3}{4}\right)^9}{27}$$

$$= \frac{28}{9} \cdot \left(\frac{3}{4}\right)^9 \Rightarrow k = \frac{28}{9}$$

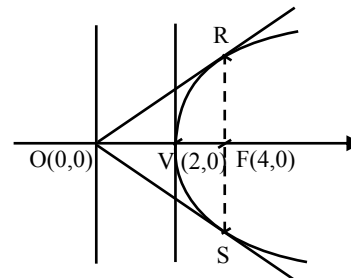
Which satisfies $|x-3| < 1$

17. Let a parabola P be such that its vertex and focus lie on the positive x -axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from $O(0, 0)$ to the parabola P which meet P at S and R , then the area (in sq. units) of ΔSOR is equal to :

- (1) $16\sqrt{2}$ (2) 16
 (3) 32 (4) $8\sqrt{2}$

Official Ans. by NTA (2)

Sol.



Clearly RS is latus-rectum

$$\therefore VF = 2 = a$$

$$\therefore RS = 4a = 8$$

$$\text{Now } OF = 2a = 4$$

$$\Rightarrow \text{Area of triangle } ORS = 16$$

18. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

- (1) 2 (2) 4
 (3) 6 (4) 1

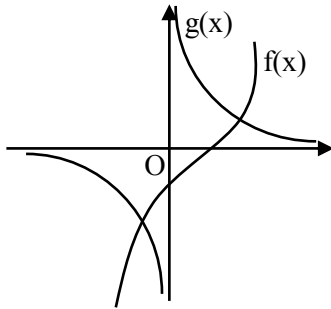
Official Ans. by NTA (1)

Sol. $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$

$$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) = 12e^{2x}$$

$$(e^{3x} - 1)^2 (e^x - e^{-x} - e^{-2x}) = 12$$

$$\Rightarrow \underbrace{e^x - e^{-x} - e^{-2x}}_{\text{increasing (let } f(x))} = \underbrace{\frac{12}{e^{3x} - 1}}_{\text{decreasing (let } g(x))}$$



$\Rightarrow$ No. of real roots = 2

19. Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0)$, $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q , then PQ^2 is equal to :

- (1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) 3

Official Ans. by NTA (3)

Sol. $\frac{3}{2a^2} + \frac{1}{b^2} = 1$ and $1 - \frac{b^2}{a^2} = \frac{1}{3}$
 $\Rightarrow a^2 = 3b^2 = 3$
 $\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1$ (i)

Its focus is (1,0)

Now, eqn of circle is

$$(x-1)^2 + y^2 = \frac{4}{3}$$
(ii)

Solving (i) and (ii) we get

$$y = \pm \frac{2}{\sqrt{3}}, x = 1$$

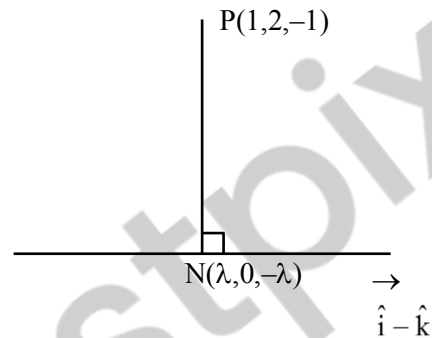
$$\Rightarrow PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

20. Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L: \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N . Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q . If α is the acute angle between the lines PN and PQ , then $\cos \alpha$ is equal to _____.

- (1) $\frac{1}{\sqrt{5}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2\sqrt{3}}$

Official Ans. by NTA (3)

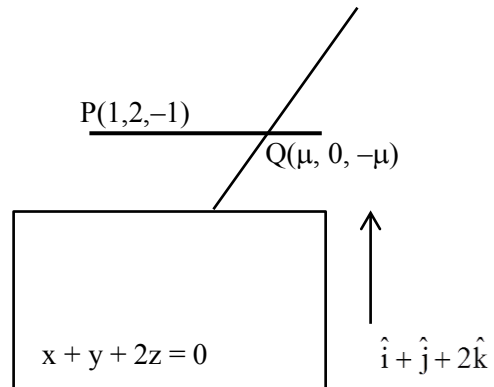
Sol.



$$\overrightarrow{PN} \cdot (\hat{i} - \hat{k}) = 0$$

$$\Rightarrow N(1, 0, -1)$$

Now,



$$\overrightarrow{PQ} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \mu = -1$$

$$\Rightarrow Q(-1, 0, 1)$$

$$\overrightarrow{PN} = 2\hat{j} \text{ and } \overrightarrow{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

SECTION-B

21. Let $y = y(x)$ be solution of the following differential equation

$$e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0, \quad y\left(\frac{\pi}{2}\right) = 0$$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to _____.

Official Ans. by NTA (4)

Sol. Let $e^y = t$

$$\Rightarrow \frac{dt}{dx} - (2 \sin x)t = -\sin x \cos^2 x$$

$$\text{I.F.} = e^{2 \cos x}$$

$$\Rightarrow t \cdot e^{2 \cos x} = \int e^{2 \cos x} \cdot (-\sin x \cos^2 x) dx$$

$$\Rightarrow e^y \cdot e^{2 \cos x} = \int e^{2z} \cdot z^2 dz, \quad z = e^{2 \cos x}$$

$$\Rightarrow e^y \cdot e^{2 \cos x} = \frac{1}{2} \cdot \cos^2 x \cdot e^{2 \cos x} - \frac{1}{2} \cos x \cdot e^{2 \cos x} + \frac{e^{2 \cos x}}{4} + C$$

$$\text{at } x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{4}$$

$$\Rightarrow e^y = \frac{1}{2} \cos^2 x - \frac{1}{2} \cos x + \frac{1}{4} + \frac{3}{4} \cdot e^{-2 \cos x}$$

$$\Rightarrow y = \log \left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4} e^{-2 \cos x} \right]$$

Put $x = 0$

$$\Rightarrow y = \log \left[\frac{1}{4} + \frac{3}{4} e^{-2} \right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$$

22. If the value of

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \text{upto } \infty \right)^{\log_{(0.25)} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \text{upto } \infty \right)}$$

is l , then l^2 is equal to _____.

Official Ans. by NTA (3)

Sol.
$$l = \left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \right)^{\log_{(0.25)} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots \right)}$$

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots$$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \dots$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$\frac{2S}{3} = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$S = \frac{3}{2} \left(\frac{4/3}{1-1/3} \right) = 3$$

$$\text{Now } l = (3)^{\log_{(1/4)} \left(\frac{1/3}{1-1/3} \right)}$$

$$l = 3^{\log_{(1/4)} \left(\frac{1}{2} \right)} = 3^{1/2} = \sqrt{3}$$

$$\Rightarrow l^2 = 3$$

23. Consider the following frequency distribution :

class :	10-20	20-30	30-40	40-50	50-60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _____.

Official Ans. by NTA (164)

Sol. $\therefore$ Sum of frequencies = 584

$$\Rightarrow \alpha + \beta = 390$$

$$\text{Now, Median is at } \frac{584}{2} = 292^{\text{th}}$$

$\therefore$ Median = 45 (lies in class 40 – 50)

$$\Rightarrow \alpha + 110 + 54 + 15 = 292$$

$$\Rightarrow \alpha = 113, \beta = 277$$

$$\Rightarrow |\alpha - \beta| = 164$$

24. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors.

If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.

Official Ans. by NTA (3)

Sol. $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ (Given)

$$\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\text{Now } (\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -2\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \vec{r} = \pm \sqrt{3} \frac{((\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}))}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|} = \pm \frac{\sqrt{3}(-2\hat{i} - 2\hat{j} - 2\hat{k})}{\sqrt{2^2 + 2^2 + 2^2}}$$

$$\vec{r} = \pm(-\hat{i} - \hat{j} - \hat{k})$$

According to question

$$\vec{r} = \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k}$$

$$\text{So } |\alpha| = 1, |\beta| = 1, |\gamma| = 1$$

$$\Rightarrow |\alpha| + |\beta| + |\gamma| = 3$$

25. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is _____.

Official Ans. by NTA (1)

Sol. Coeff. of middle term in $(1+x)^{20} = {}^{20}C_{10}$

& Sum of Coeff. of two middle terms in

$$(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$$

$$\text{So required ratio} = \frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$$

26. Let $M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}$.

Define $f: M \rightarrow \mathbf{Z}$, as $f(A) = \det(A)$, for all $A \in M$, where $\mathbf{Z}$ is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to _____.

Official Ans. by NTA (16)

Sol. $|A| = ad - bc = 15$

where $a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}$

Case I $ad = 9$ & $bc = -6$

For ad possible pairs are $(3,3), (-3,-3)$

For bc possible pairs are $(3,-2), (-3,2), (-2,3), (2,-3)$

So total matrix $= 2 \times 4 = 8$

Case II $ad = 6$ & $bc = -9$

Similarly total matrix $= 2 \times 4 = 8$

$\Rightarrow$ Total such matrices are $= 16$

27. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is $100k$, then k is equal to _____.

Official Ans. by NTA (238)

Sol. Class	10 th	11 th	12 th	
Total student	5	6	8	
	2	3	5	$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$
Number of selection	2	2	6	$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$
	3	2	5	$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\Rightarrow$ Total number of ways $= 23800$

According to question

$$100K = 23800$$

$$\Rightarrow K = 238$$

28. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n , then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$ is equal to _____.

Official Ans. by NTA (1)

Sol. $x^2 + 5\sqrt{2}x + 10 = 0$

& $p_n = \alpha^n - \beta^n$ (Given)

$$\text{Now } \frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \frac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})}$$

$$\frac{P_{17}(\alpha^{20} - \beta^{20} + 5\sqrt{2}(\alpha^{19} - \beta^{19}))}{P_{18}(\alpha^{19} - \beta^{19} + 5\sqrt{2}(\alpha^{18} - \beta^{18}))}$$

$$\frac{P_{17}(\alpha^{19}(\alpha + 5\sqrt{2}) - \beta^{19}(\beta + 5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha + 5\sqrt{2}) - \beta^{18}(\beta + 5\sqrt{2}))}$$

$$\text{Since } \alpha + 5\sqrt{2} = -10 / \alpha \dots\dots\dots(1)$$

$$\& \beta + 5\sqrt{2} = -10 / \beta \dots\dots\dots(2)$$

Now put there values in above expression

$$= -\frac{10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$$

29. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$, where $x \neq 0, 1$ is equal to _____.

Official Ans. by NTA (210)

Sol. $\left((x^{1/3} + 1) - \left(\frac{x^{1/2} + 1}{x^{1/2}} \right) \right)^{10}$

$$= \left(x^{1/3} - \frac{1}{x^{1/2}} \right)^{10}$$

Now General Term

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} \cdot \left(-\frac{1}{x^{1/2}} \right)^r$$

For independent term

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$$

$$\Rightarrow T_5 = {}^{10}C_4 = 210$$

30. Let

$$S = \left\{ n \in \mathbf{N} \left| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in \mathbf{R} \right\},$$

where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is _____.

Official Ans. by NTA (11)

Sol. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ & $A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n$

$$\Rightarrow AX = IX$$

$$\Rightarrow A = I$$

$$\Rightarrow \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n = I$$

$$\Rightarrow A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

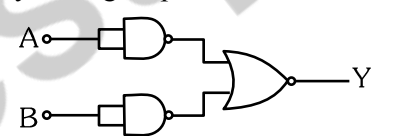
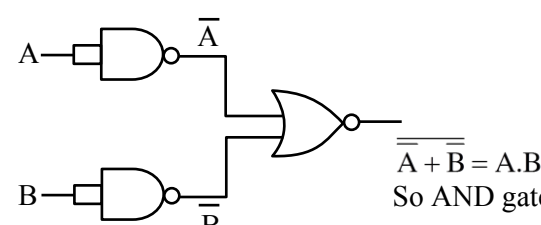
$\Rightarrow n$ is multiple of 8

So number of 2 digit numbers in the set

$$S = 11 \text{ (16, 24, 32,96)}$$

(Held On Sunday 25th July, 2021)

TIME : 9 : 00 AM to 12 : 00

PHYSICS	TEST PAPER WITH SOLUTION
SECTION-A 31. For a gas $C_p - C_v = R$ in a state P and $C_p - C_v = 1.10 R$ in a state Q, T_p and T_Q are the temperatures in two different states P and Q respectively. Then (1) $T_p = T_Q$ (2) $T_p < T_Q$ (3) $T_p = 0.9 T_Q$ (4) $T_p > T_Q$ Official Ans. by NTA (4) Sol. $C_p - C_v = R$ for ideal gas and gas behaves as ideal gas at high temperature so $T_p > T_Q$ 32. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R. Assertion A : Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y and I_z respectively. The respective radii of gyration about all the three axes will be the same. Reason R : A rigid body making rotational motion has fixed mass and shape. In the light of the above statements, choose the most appropriate answer from the options given below : (1) Both A and R are correct but R is NOT the correct explanation of A. (2) A is not correct but R is correct. (3) A is correct but R is not correct. (4) Both A and R are correct and R is the correct explanation of A. Official Ans. by NTA (2) Sol. $I_z = I_x + I_y$ (using perpendicular axis theorem) & $I = mk^2$ (K : radius of gyration) so $mK_z^2 = mK_x^2 + mK_y^2$ $K_z^2 = K_x^2 + K_y^2$ so radius of gyration about axes x, y & z won't be same hence assertion A is not correct reason R is correct statement (property of a rigid body)	33. What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α-particle (λ_α) and proton (λ_p) given that all have the same kinetic energy ? (1) $\lambda_e = \lambda_p = \lambda_\alpha$ (2) $\lambda_e < \lambda_p < \lambda_\alpha$ (3) $\lambda_e > \lambda_p > \lambda_\alpha$ (4) $\lambda_e = \lambda_p > \lambda_\alpha$ Official Ans. by NTA (3) Sol. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \propto \frac{1}{\sqrt{m}}$ $m_\alpha > m_p > m_e$ so $\lambda_e > \lambda_p > \lambda_\alpha$ 34. Identify the logic operation carried out.  (1) OR (2) AND (3) NOR (4) NAND Official Ans. by NTA (2) Sol.  $\overline{A + B} = A.B$ So AND gate 35. A particle of mass $4M$ at rest disintegrates into two particles of mass M and $3M$ respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass $3M$ will be : (1) $1 : 3$ (2) $3 : 1$ (3) $1 : \sqrt{3}$ (4) $1 : 1$ Official Ans. by NTA (4) Sol. $\lambda = \frac{h}{p}$ both the particles will move with momentum same in magnitude & opposite in direction. So De-Broglie wavelength of both will be same i.e. ratio $1 : 1$

36. Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as :

(where λ is the decay constant)

- (1) $\frac{1}{2} \frac{\ln 2}{\lambda}$ (2) $\frac{\ln 2}{\lambda}$
 (3) $\frac{2 \ln 2}{\lambda}$ (4) $\frac{\ln \frac{3}{2}}{\lambda}$

Official Ans. by NTA (4)

Sol. $\frac{3N_0}{4} = N_0 e^{-\lambda t_1}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_2}$$

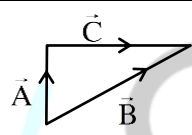
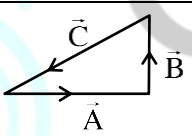
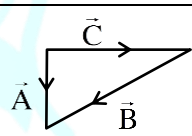
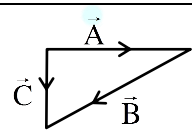
$$\ln(3/4) = -\lambda t_1 \quad \dots (i)$$

$$\ln(1/2) = -\lambda t_2 \quad \dots (ii)$$

$$\ln(3/4) - \ln(1/2) = \lambda(t_2 - t_1) \quad \dots (iii)$$

$$\Delta t = \frac{\ln(3/2)}{\lambda}$$

37. Match List I with List II.

List I		List II	
(a)	$\vec{C} - \vec{A} - \vec{B} = 0$	(i)	
(b)	$\vec{A} - \vec{C} - \vec{B} = 0$	(ii)	
(c)	$\vec{B} - \vec{A} - \vec{C} = 0$	(iii)	
(d)	$\vec{A} + \vec{B} = -\vec{C}$	(iv)	

Choose the correct answer from the options given below :

- (1) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (ii)
 (2) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)
 (3) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)
 (4) (a) $\rightarrow$ (i), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (iii)

Official Ans. by NTA (2)

Sol. (a) $\vec{C} = \vec{A} + \vec{B}$

Option (iv)

(b) $\vec{A} = \vec{B} + \vec{C} = \vec{C} + \vec{B}$

Option (iii)

(c) $\vec{B} = \vec{A} + \vec{C}$

Option (i)

(d) $\vec{A} + \vec{B} + \vec{C} = 0$

Option (ii)

38. A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

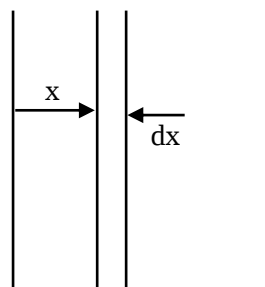
$$\epsilon(x) = \epsilon_0 + kx, \text{ for } \left(0 < x \leq \frac{d}{2}\right)$$

$$\epsilon(x) = \epsilon_0 + k(d - x), \text{ for } \left(\frac{d}{2} \leq x \leq d\right)$$

- (1) $\left(\epsilon_0 + \frac{kd}{2}\right)^{2/kA}$ (2) $\frac{kA}{2 \ln\left(\frac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$
 (3) 0 (4) $\frac{kA}{2} \ln\left(\frac{2\epsilon_0}{2\epsilon_0 - kd}\right)$

Official Ans. by NTA (2)

Sol.



Taking an element of width dx at a distance x ($x < d/2$) from left plate

$$dc = \frac{(\epsilon_0 + kx)A}{dx}$$

Capacitance of half of the capacitor

$$\frac{1}{C} = \int_0^{d/2} \frac{1}{dc} = \frac{1}{A} \int_0^{d/2} \frac{dx}{\epsilon_0 + kx}$$

$$\frac{1}{C} = \frac{1}{kA} \ln\left(\frac{\epsilon_0 + kd/2}{\epsilon_0}\right)$$

Capacitance of second half will be same

$$C_{eq} = \frac{C}{2} = \frac{kA}{2 \ln\left(\frac{2\epsilon_0 + kd}{2\epsilon_0}\right)}$$

39. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be :

(1) $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$ (2) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$
 (3) $\frac{l_2}{l_1}$ (4) $\frac{l_1}{l_2}$

Official Ans. by NTA (2)

Sol. $PV^\gamma = \text{const.}$
 $TV^{\gamma-1} = \text{const.}$
 $T(\ell)^{\frac{5}{3}-1} = \text{const.}$

$$\frac{T_1}{T_2} = \left(\frac{\ell_2}{\ell_1}\right)^{2/3}$$

40. A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option.

- (1) angle of refraction is 24.41°
 (2) angle of refraction is 30°
 (3) refraction is not possible
 (4) angle of refraction is 53.4°

Official Ans. by NTA (3)

Sol. $\sin \theta_c = \frac{1}{\mu} = \frac{1}{2\mu_2} < \sin \theta$

$$\sin \theta > \sin \theta_c$$

$$\theta > \theta_c$$

Total internal reflection will happen

41. Two wires of same length and radius are joined end to end and loaded. The Young's moduli of the materials of the two wires are Y_1 and Y_2 . The combination behaves as a single wire then its Young's modulus is :

(1) $Y = \frac{2Y_1Y_2}{3(Y_1 + Y_2)}$ (2) $Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$
 (3) $Y = \frac{Y_1Y_2}{2(Y_1 + Y_2)}$ (4) $Y = \frac{Y_1Y_2}{Y_1 + Y_2}$

Official Ans. by NTA (2)

Sol. In series combination $\Delta l = \ell_1 + \ell_2$

$$Y = \frac{F/A}{\Delta \ell / \ell} \Rightarrow \Delta \ell = \frac{F\ell}{AY}$$

$$\Rightarrow \Delta \ell \propto \frac{\ell}{Y}$$

Equivalent length of rod after joining is $= 2\ell$

As, lengths are same and force is also same in series

$$\Delta \ell = \Delta \ell_1 + \Delta \ell_2$$

$$\frac{\ell_{eq}}{Y_{eq}} = \frac{\ell}{Y_1} + \frac{\ell}{Y_2} \Rightarrow \frac{2\ell}{Y} = \frac{\ell}{Y_1} + \frac{\ell}{Y_2}$$

$$\therefore Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

42. The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg of ^{198}Au is [in disintegration/second] :

- (1) 2.67×10^{12} (2) 6.06×10^{18}
 (3) 32.36×10^{12} (4) 16.18×10^{12}

Official Ans. by NTA (4)

Sol. $A = \lambda N$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{3 \times 24 \times 60 \times 60} \text{sec}^{-1} = 2.67 \times 10^{-6} \text{sec}^{-1}$$

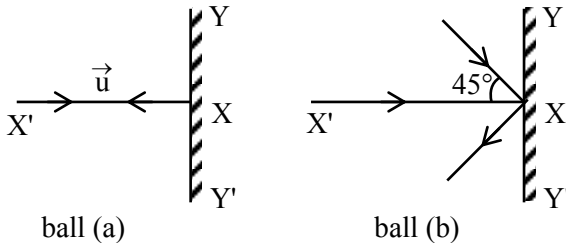
N = Number of atoms in 2 mg Au

$$= \frac{2 \times 10^{-3}}{198} \times 6 \times 10^{23} = 6.06 \times 10^{15}$$

$$A = \lambda N = 1.618 \times 10^{13} = 16.18 \times 10^{12} \text{ dps}$$



43. Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is :



- (1) 1 : 1 (2) $\sqrt{2} : 1$ (3) 2 : 1 (4) 1 : $\sqrt{2}$

Official Ans. by NTA (2)

Sol. Impulse = change in momentum

$$\text{Ball (a)} \quad |\Delta \vec{p}| = 2mu = J_1$$

$$\text{Ball (b)} \quad |\Delta \vec{p}| = 2mu \cos 45^\circ = J_2$$

$$\frac{J_1}{J_2} = \frac{1}{\cos 45^\circ} = \sqrt{2}$$

44. In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin \omega t$; where d_0 , ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as :

- (1) $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$ (2) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$
 (3) $\frac{\lambda D}{d_0^2} a_0$ (4) $\frac{\lambda D}{d_0 + a_0}$

Official Ans. by NTA (2)

Sol. Fringe Width, $\beta = \frac{\lambda D}{d}$

$$\beta_{\max} \Rightarrow d_{\min} \text{ and } \beta_{\min} \Rightarrow d_{\max}$$

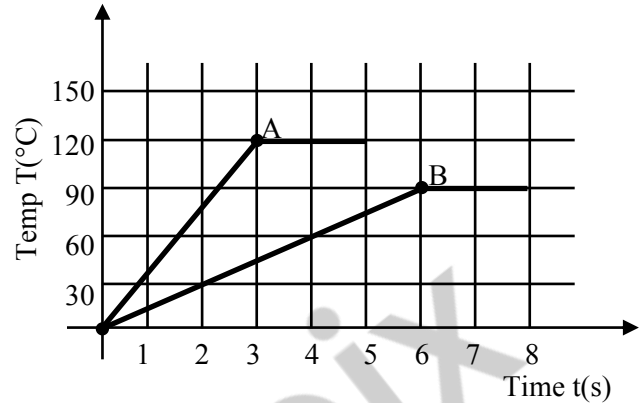
$$d = d_0 + a_0 \sin \omega t$$

$$d_{\max} = d_0 + a_0 \text{ and } d_{\min} = d_0 - a_0$$

$$\therefore \beta_{\min} = \frac{\lambda D}{d_0 + a_0} \text{ and } \therefore \beta_{\max} = \frac{\lambda D}{d_0 - a_0}$$

$$\beta_{\max} - \beta_{\min} = \frac{\lambda D}{d_0 - a_0} - \frac{\lambda D}{d_0 + a_0} = \frac{2\lambda D a_0}{d_0^2 - a_0^2}$$

45. Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is :



- (1) $\frac{8}{3}$ (2) $\frac{3}{8}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

Official Ans. by NTA (2)

Sol. $\left(\frac{\Delta Q}{\Delta t}\right)_A = \left(\frac{\Delta Q}{\Delta t}\right)_B$

$$m S_A \left(\frac{\Delta T}{\Delta t}\right)_A = m S_B \left(\frac{\Delta T}{\Delta t}\right)_B$$

$$\frac{S_A}{S_B} = \frac{\left(\frac{\Delta T}{\Delta t}\right)_B}{\left(\frac{\Delta T}{\Delta t}\right)_A} = \frac{90/6}{120/3} = \frac{15}{40} = \frac{3}{8}$$

46. A linearly polarized electromagnetic wave in vacuum is

$$E = 3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$$

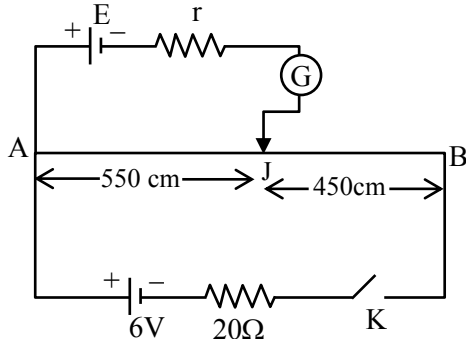
is incident normally on a perfectly reflecting wall at $z = a$. Choose the correct option

- (1) The wavelength is 5.4 m
 (2) The frequency of electromagnetic wave is 54×10^4 Hz.
 (3) The transmitted wave will be $3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$
 (4) The reflected wave will be $3.1 \cos[(1.8)z + (5.4 \times 10^6)t] \hat{i} \text{ N/C}$

Official Ans. by NTA (4)

Sol. Reflected wave will have direction opposite to incident wave.

47. In the given figure, there is a circuit of potentiometer of length AB = 10 m. The resistance per unit length is 0.1Ω per cm. Across AB, a battery of emf E and internal resistance 'r' is connected. The maximum value of emf measured by this potentiometer is :



- (1) 5 V (2) 2.25 V (3) 6 V (4) 2.75 V

Official Ans. by NTA (1)

- Sol.** Max. voltage that can be measured by this potentiometer will be equal to potential drop across AB

$$R_{AB} = 10 \times 0.1 \times 100 = 100 \text{ ohm.}$$

$$\therefore V_{AB} = \frac{6}{20 + 100} \times 100 = 6 \times \frac{100}{120} = 5V$$

48. In amplitude modulation, the message signal $V_m(t) = 10 \sin(2\pi \times 10^5 t)$ volts and Carrier signal

$$V_c(t) = 20 \sin(2\pi \times 10^7 t) \text{ volts}$$

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz. The value of α is :

- (1) 200 kHz (2) 50 kHz
(3) 100 kHz (4) 0

Official Ans. by NTA (1)

- Sol.** Bandwidth = $2 \times f_m$
= $2 \times 10^5 \text{ HZ} = 200 \text{ KHZ}$

49. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4th second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap ? (Take $g = 9.8 \text{ m/s}^2$)

- (1) 3 drops / 2 seconds
(2) 2 drops / second
(3) 1 drop / second
(4) 1 drop / 7 seconds

Official Ans. by NTA (3)

- Sol.** In 4 sec. 1st drop will travel

$$\Rightarrow \frac{1}{2} \times (9.8) \times (4)^2 = 78.4 \text{ m}$$

$\therefore$ 2nd drop would have travelled

$$\Rightarrow 78.4 - 34.3 = 44.1 \text{ m.}$$

Time for 2nd drop

$$\frac{1}{2} (9.8) t^2 = 44.1$$

$$\boxed{t = 3 \text{ sec}}$$

$\therefore$ each drop have time gap of 1 sec

$\therefore$ 1 drop per sec

50. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2 . If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

- (1) $\frac{v_0 x_1^2}{x_2^2}$ (2) $\frac{v_0 x_2^2}{x_1^2}$ (3) $\frac{v_0 x_1}{x_2}$ (4) $\frac{v_0 x_2}{x_1}$

Official Ans. by NTA (4)

- Sol.** Angular momentum conservation equation

$$v_0 x_2 = v_1 x_1$$

$$v_1 = \frac{v_0 x_2}{x_1}$$

SECTION-B

51. A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is

$$\frac{x}{10} \text{ m/s. Then the value of } x \text{ is } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (25)

- Sol.** $p_i = p_f$
 $2 \times 4 = 2 \times 1 + m_2 \times v_2$
 $m_2 v_2 = 6 \dots (i)$

by coefficient of restitution

$$1 = \frac{v_2 - 1}{4} \Rightarrow v_2 = 5 \text{ m/s}$$

by (i)

$$m_2 \times 5 = 6$$

$$m_2 = 1.2 \text{ kg}$$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

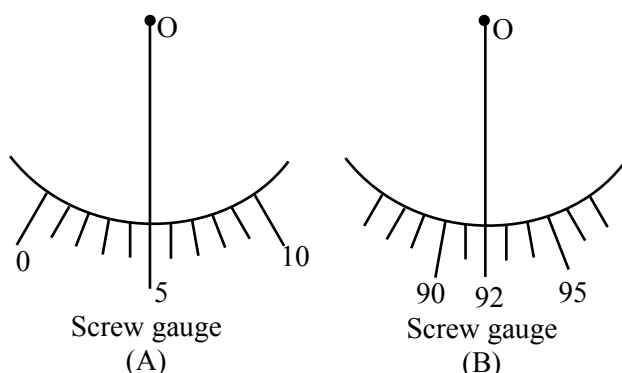
$$v_{cm} = \frac{2 \times 1 + 1.2 \times 5}{2 + 1.2} = \frac{8}{3.2} = \frac{25}{10}$$

$$\boxed{x = 25}$$

52. Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____.

[Figure shows position of reference 'O' when jaws of screw gauge are closed]

Given pitch = 0.1 cm.



Official Ans. by NTA (13)

Sol. For (A)

$$\text{Reading} = \text{MSR} + \text{CSR} + \text{Error}$$

$$0.322 = 0.300 + \text{CSR} + 5 \times \text{LC}$$

$$0.322 = 0.300 + \text{CSR} + 0.005$$

$$\text{CSR} = 0.017$$

For B

$$\text{Reading} = \text{MSR} + \text{CSR} + \text{Error}$$

$$0.322 = 0.200 + \text{CSR} + 0.092$$

$$\text{CSR} = 0.030$$

$$\text{Difference} = 0.030 - 0.017 = 0.013 \text{ cm}$$

$$\text{Division on circular scale} = \frac{0.013}{0.001} = 13$$

53. An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to _____. (Take $e^{-1} = 0.37$)

Official Ans. by NTA (74)

Sol. $I_{\text{max}} = \frac{V}{R} = \frac{20\text{V}}{10\text{k}\Omega} = 2\text{mA}$

For LR – decay circuit

$$I = I_{\text{max}} e^{-Rt/L}$$

$$I = 2\text{mA} e^{\frac{-10 \times 10^3 \times 1 \times 10^{-6}}{10 \times 10^{-3}}}$$

$$I = 2\text{mA} e^{-1}$$

$$I = 2 \times 0.37 \text{ mA}$$

$$I = \frac{74}{100} \text{ mA}$$

$$\boxed{x = 74}$$

54. A circular conducting coil of radius 1 m is being heated by the change of magnetic field $\vec{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is 2 μΩ. The magnetic field is slowly switched off such that its magnitude changes in time as

$$B = \frac{4}{\pi} \times 10^{-3} \text{ T} \left(1 - \frac{t}{100} \right)$$

The energy dissipated by the coil before the magnetic field is switched off completely is $E = \underline{\hspace{1cm}}$ mJ.

Official Ans. by NTA (80)

Sol. $\phi = \vec{B} \cdot \vec{S}$

$$\phi = \frac{4}{\pi} \times 10^{-3} \left(1 - \frac{t}{100} \right) \cdot \pi R$$

$$\phi = 4 \times 10^{-3} \times (1)^2 \left(1 - \frac{t}{100} \right)$$

$$\varepsilon = \frac{-d\phi}{dt}$$

$$\varepsilon = \frac{-d}{dt} \left(4 \times 10^{-3} \left(1 - \frac{t}{100} \right) \right)$$

$$\varepsilon = 4 \times 10^{-3} \left(\frac{1}{100} \right) = 4 \times 10^{-5} \text{ V}$$

When $B = 0$

$$1 - \frac{t}{100} = 0$$

$$t = 100 \text{ sec}$$

$$\text{Heat} = \frac{\varepsilon^2}{R} t$$

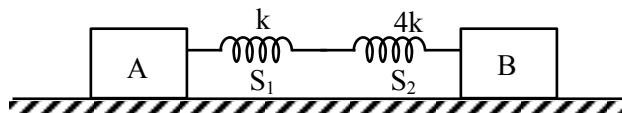
$$\text{Heat} = \frac{(4 \times 10^{-5})^2}{2 \times 10^{-6}} \times 100 \text{ J}$$

$$\text{Heat} = \frac{16 \times 10^{-10} \times 100}{2 \times 10^{-6}} \text{ J}$$

$$\text{Heat} = 0.08 \text{ J}$$

$$\text{Heat} = 80 \text{ mJ}$$

55. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be _____ rad/s when $k = 20 \text{ N/m}$.



Official Ans. by NTA (10)

Sol. $\omega = \sqrt{\frac{k_{eq}}{\mu}}$

μ = reduced mass

springs are in series connection

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$$k_{eq} = \frac{k \times 4k}{5k} = \frac{4k}{5}$$

$$k_{eq} = \frac{4 \times 20}{5} \text{ N/m} = 16 \text{ N/m}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{0.2 \times 0.8}{0.2 + 0.8} = 0.16 \text{ kg}$$

$$\omega = \sqrt{\frac{16}{0.16}} = \sqrt{100} = 10$$

56. The value of aluminium susceptibility is 2.2×10^{-5} . The percentage increase in the magnetic field if space within a current carrying toroid is filled with aluminium is $\frac{x}{10^4}$. Then the value of x is _____.

Official Ans. by NTA (22)

Sol. $B = \mu(H+I)$

$$B = \mu_0 H \left(1 + \frac{I}{H}\right)$$

$$B = B_0(1+x)$$

$$B - B_0 = B_0 x$$

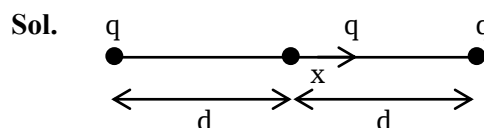
$$\frac{B - B_0}{B_0} = x$$

$$\frac{B - B_0}{B_0} \times 100 = 100x$$

$$= 2.2 \times 10^{-3} = \frac{22}{10^4}$$

57. A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance '2 m' when each is carrying same charge 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' ($x \ll 1 \text{ m}$). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times 10^5 \text{ rad/s}$ if $q^2 = 10 \text{ C}^2$.

Official Ans. by NTA (6)



Net force on free charged particle

$$F = \frac{kq^2}{(d+x)^2} - \frac{kq^2}{(d-x)^2}$$

$$F = -kq^2 \left[\frac{4dx}{(d^2 - x^2)^2} \right]$$

$$a = -\frac{4kq^2 d}{m} \left(\frac{x}{d^4} \right)$$

$$a = -\left(\frac{4kq^2}{md^3} \right) x$$

So, angular frequency

$$\omega = \sqrt{\frac{4kq^2}{md^3}}$$

$$\omega = \sqrt{\frac{4 \times 9 \times 10^9 \times 10}{1 \times 10^{-6} \times 1^3}}$$

$$\omega = 6 \times 10^8 \text{ rad/sec}$$

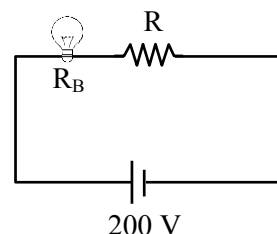
58. An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R' that must be put in series with the bulb so that the bulb delivers the same power is _____ Ω .

Official Ans. by NTA (50)

Sol. Power, $P = \frac{V^2}{R_B}$

$$R_B = \frac{V^2}{P} = \frac{100 \times 100}{200}$$

$$R_B = 50 \Omega$$



To produce same power, same voltage (i.e. 100 V) should be across the bulb

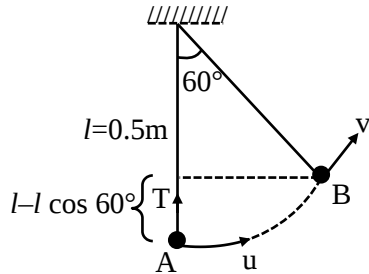
Hence, $R = R_B$

$$R = 50 \Omega$$

59. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be ($g = 10 \text{ m/s}^2$) _____ m/s.

Official Ans. by NTA (2)

Sol.



Applying work energy theorem :

$$W_g + W_T = \Delta K$$

$$-mgl(1 - \cos 60^\circ) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$v^2 = u^2 - 2gl(1 - \cos 60^\circ)$$

$$v^2 = 9 - 2 \times 10 \times 0.5 \left(\frac{1}{2} \right)$$

$$v^2 = 4$$

$$v = 2 \text{ m/s}$$

60. A particle of mass 'm' is moving in time 't' on a trajectory given by

$$\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$$

Where α and β are dimensional constants.

The angular momentum of the particle becomes the same as it was for $t = 0$ at time $t =$ _____ seconds.

Official Ans. by NTA (10)

Sol. $\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$

$$\vec{v} = 20\alpha t \hat{i} + 5\beta \hat{j}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$= m[10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}] \times [20\alpha t \hat{i} + 5\beta \hat{j}]$$

$$\vec{L} = m[50\alpha\beta t^2 \hat{k} - 100\alpha\beta(t^2 - 5t)\hat{k}]$$

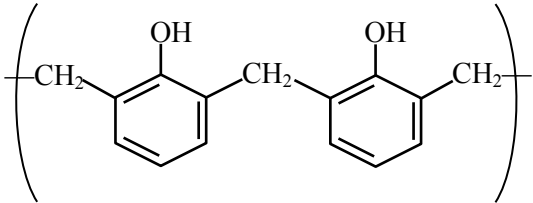
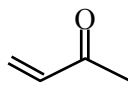
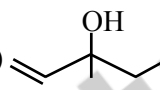
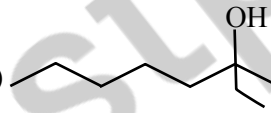
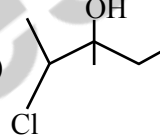
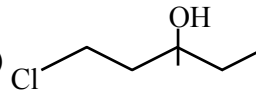
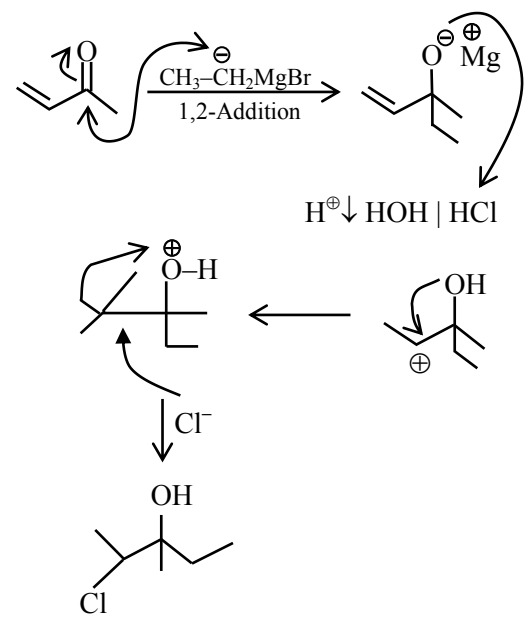
At $t = 0$, $\vec{L} = \vec{0}$

$$50\alpha\beta t^2 - 100\alpha\beta(t^2 - 5t) = 0$$

$$t - 2(t-5) = 0$$

$$t = 10 \text{ sec}$$

(Held On Sunday 25th July, 2021)
TIME : 9 : 00 AM to 12 : 00 NOON

CHEMISTRY	TEST PAPER WITH SOLUTION
SECTION-A 61.  is a repeating unit for : (1) Novolac (2) Buna-N (3) Acrilan (4) Neoprene Official Ans. by NTA (1) Sol. Phenol + $\text{H}-\overset{\text{O}}{\parallel}{\text{C}}-\text{H} \xrightarrow{\text{H}^+}$ Novolac $\xrightarrow{\text{Polymerisation}}$ (Given structure) Linear polymer Bakelite (3D structures) 62. Which one of the following species responds to an external magnetic field? (1) $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ (2) $[\text{Ni}(\text{CN})_4]^{2-}$ (3) $[\text{Co}(\text{CN})_6]^{3-}$ (4) $[\text{Ni}(\text{CO})_4]$ Official Ans. by NTA (1) Sol. 1. $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$ $\text{Fe}^{3+} : [\text{Ar}]3d^5$ Hybridisation : sp^3d^2 Magnetic nature : Paramagnetic (so this complex response to external magnetic field) 2. $[\text{Ni}(\text{CN})_4]^{2-}$ $\text{Ni}^{2+} : [\text{Ar}]3d^8$ Hybridisation : dsp^2 Magnetic nature : diamagnetic 3. $[\text{Co}(\text{CN})_6]^{3-}$ $\text{Co}^{3+} : [\text{Ar}]3d^6$ Hybridisation : d^2sp^3 Magnetic nature : diamagnetic	$[\text{Ni}(\text{CO})_4]$ $\text{Ni} : [\text{Ar}] 3d^8 4s^2$ Hybridisation : sp^3 Magnetic nature : diamagnetic 63.  $\xrightarrow[\text{(ii) H}_2\text{O, HCl}]{\text{(i) C}_2\text{H}_5\text{MgBr, dry ether}}$ P (Major product) Consider the above reaction, the major product 'P' is: (1)  (2)  (3)  (4)  Official Ans. by NTA (3) Sol. 

64. Sodium stearate $\text{CH}_3(\text{CH}_2)_{16}\text{COO}^-\text{Na}^+$ is an anionic surfactant which forms micelles in oil. Choose the **correct** statement for it from the following :

- (1) It forms spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ - group pointing towards the centre of sphere.
- (2) It forms non-spherical micelles with $-\text{COO}^-$ group pointing outwards on the surface.
- (3) It forms spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ - group pointing outwards on the surface of sphere
- (4) It forms non-spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ -group pointing towards the centre.

Official Ans. by NTA (1)

Sol.

Forms spherical micelles with $\text{CH}_3(\text{CH}_2)_{16}$ group pointing towards the centre of sphere

65. The water soluble protein is :

- (1) Fibrin
- (2) Albumin
- (3) Myosin
- (4) Collagen

Official Ans. by NTA (2)

Sol.

Albumin is water soluble.

66. At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_{H}) and its isotope, deuterium (E_{D}), is best described by :

- (1) $E_{\text{H}} = \frac{1}{2} E_{\text{D}}$
- (2) $E_{\text{H}} = E_{\text{D}}$
- (3) $E_{\text{H}} \approx E_{\text{D}} - 7.5$
- (4) $E_{\text{H}} = 2E_{\text{D}}$

Official Ans. by NTA (3)

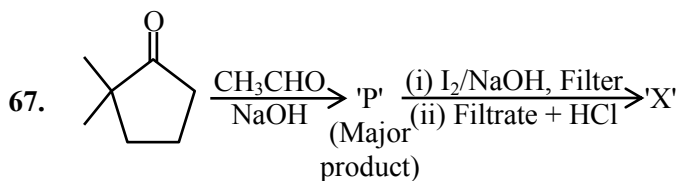
Sol.

Enthalpy of bond dissociation (kJ/mole) at 298.2K

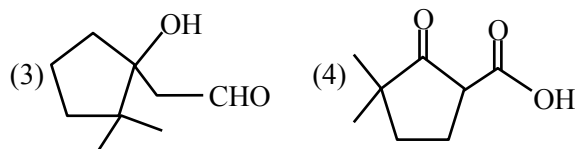
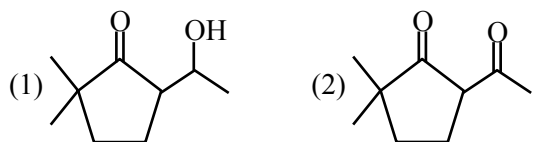
For , hydrogen = 435.88

For , Deuterium = 443.35

$$\therefore E_{\text{H}} \approx E_{\text{D}} - 7.5$$

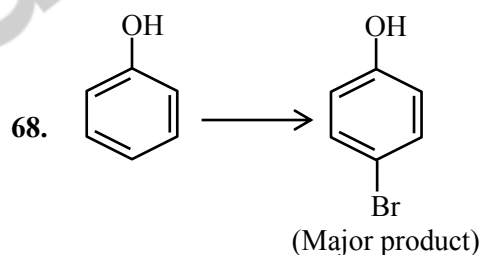
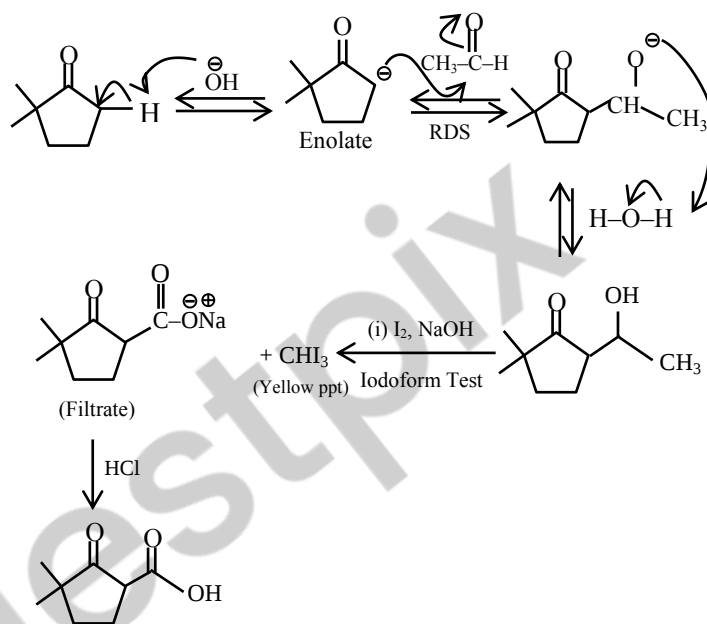


Consider the given reaction, the product 'X' is:



Official Ans. by NTA (4)

Sol.



The given reaction can occur in the presence of :

- (a) Bromine water
- (b) Br_2 in CS_2 , 273 K
- (c) $\text{Br}_2/\text{FeBr}_3$
- (d) Br_2 in CHCl_3 , 273 K

Choose the correct answer from the options given below :

- (1) (b) and (d) only
- (2) (a) and (c) only
- (3) (b), (c) and (d) only
- (4) (a), (b) and (d) only

Official Ans. by NTA (3)

Sol.

Bromine water gives tribromo products, other gives monobromo products in which para is major product.

69. Given below are two statements, one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (R) : Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

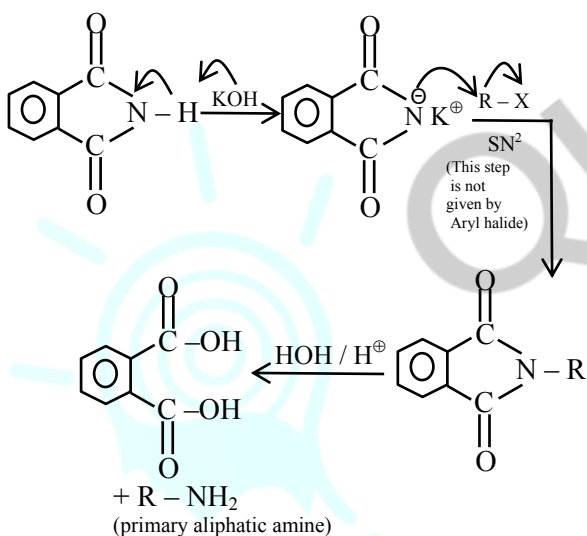
Reason : Aryl halides do not undergo nucleophilic substitution reaction.

In the light of the above statements, choose the **correct** answer from the options given below :

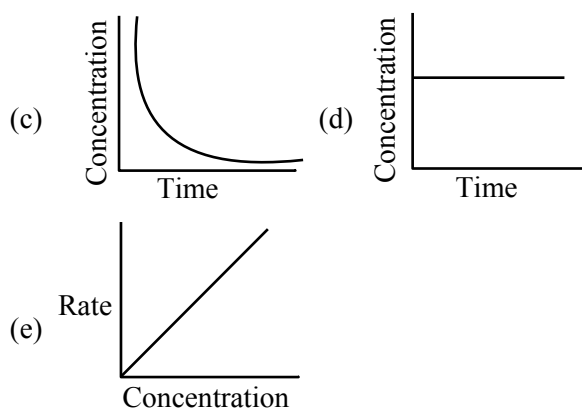
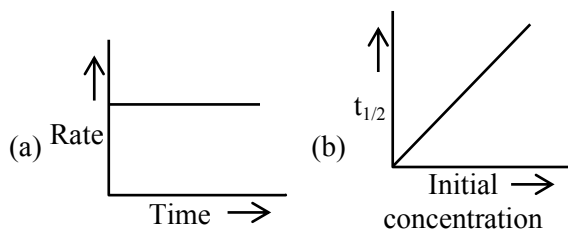
- (1) Both (A) and (R) true but (R) is not the correct explanation of (A).
- (2) (A) is false but (R) is true.
- (3) Both (A) and (R) true and (R) is correct explanation of (A).
- (4) (A) is true but (R) is false.

Official Ans. by NTA (3)

Sol. Gabriel phthalimide synthesis



70. For the following graphs,



Choose from the options given below, the **correct** one regarding order of reaction is :

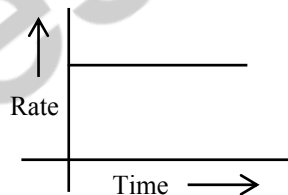
- (1) (b) zero order (c) and (e) First order
- (2) (a) and (b) Zero order (e) First order
- (3) (b) and (d) Zero order (e) First order
- (4) (a) and (b) Zero order (c) and (e) First order

Official Ans. by NTA (1)

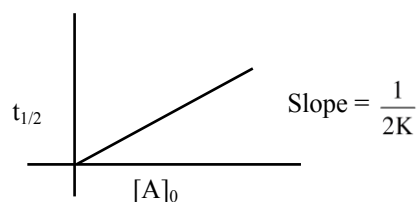
QuestPix Ans. (2)

Sol.

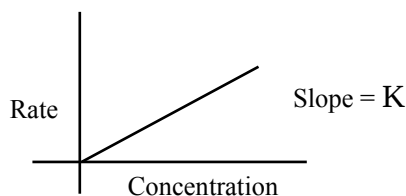
For zero order reaction's
rate = $K [\text{Reactant}]^0$
 $\Rightarrow r = k$



$$t_{1/2} = \frac{[A]_0}{2K} \text{ for zero order}$$

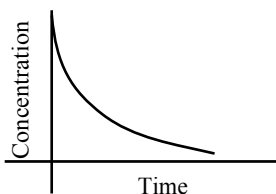


For first order reaction $\rightarrow$
 $r = K [\text{Concentration}]$

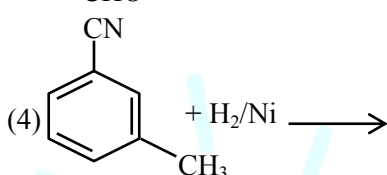
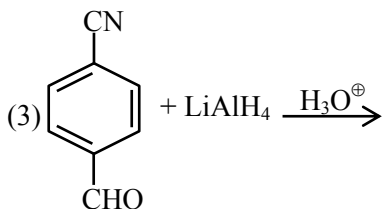
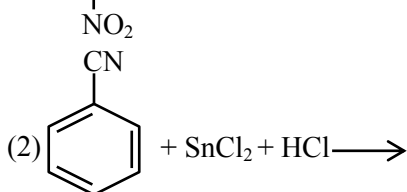
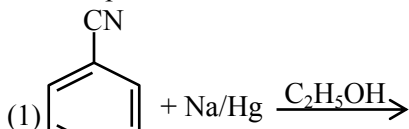


Reactant concentration after time $t \rightarrow$

$$C_t = C_0 e^{-kt}$$

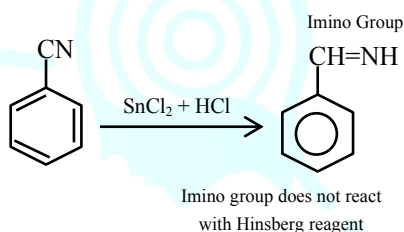


71. Which one of the products of the following reactions **does not** react with Hinsberg reagent to form sulphonamide?



Official Ans. by NTA (2)

Sol.



72. The ionic radii of K^+ , Na^+ , Al^{3+} and Mg^{2+} are in the order :

- (1) $\text{Na}^+ < \text{K}^+ < \text{Mg}^{2+} < \text{Al}^{3+}$
- (2) $\text{Al}^{3+} < \text{Mg}^{2+} < \text{K}^+ < \text{Na}^+$
- (3) $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{K}^+$
- (4) $\text{K}^+ < \text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+$

Official Ans. by NTA (3)

- Sol.** Al^{3+} , Mg^{2+} and Na^+ are isoelectronic ionic species. For monoatomic ionic isoelectronic species as positive charge increases ionic size decreases. The order of size of Na^+ & K^+ is $\text{Na}^+ < \text{K}^+$, $\therefore$ order of ionic radii is : $\text{Al}^{3+} < \text{Mg}^{2+} < \text{Na}^+ < \text{K}^+$

73. Which one of the following compounds of Group-14 elements is **not** known?

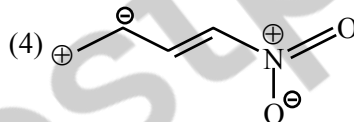
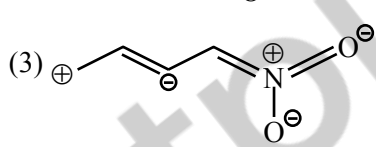
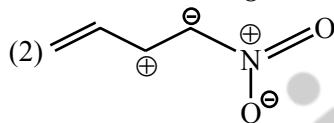
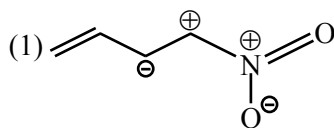
- (1) $[\text{GeCl}_6]^{2-}$
- (2) $[\text{Sn}(\text{OH})_6]^{2-}$
- (3) $[\text{SiCl}_6]^{2-}$
- (4) $[\text{SiF}_6]^{2-}$

Official Ans. by NTA (3)

Sol.

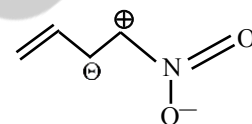
$[\text{SiCl}_6]^{2-}$ does not exist due to steric crowding of surrounding atoms.

74. Which one among the following resonating structures is **not** correct?



Official Ans. by NTA (1)

Sol.



It is unstable RS (due to similar charge on adjacent atom)

75. Given below are two statements :

Statement I : None of the alkaline earth metal hydroxides dissolve in alkali.

Statement II : Solubility of alkaline earth metal hydroxides in water increases down the group.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

- (1) **Statement I** is correct but **Statement II** is incorrect.
- (2) **Statement I** is incorrect but **Statement II** is correct.
- (3) **Statement I** and **Statement II** both are incorrect.
- (4) **Statement I** and **Statement II** both are correct.

Official Ans. by NTA (2)

Sol.

Statement-I is incorrect

$\text{Be}(\text{OH})_2$ dissolve in alkali due to its amphoteric nature.

Statement-II is correct

Solubility of alkaline earth metal hydroxide in water increases down the group due to rapid decreases in lattice energy as compared to hydration energy.

76. The correct order of following 3d metal oxides, according to their oxidation numbers is :

(a) CrO_3 (b) Fe_2O_3 (c) MnO_2 (d) V_2O_5 (e) Cu_2O

(1) $(d) > (a) > (b) > (c) > (e)$

(2) $(a) > (c) > (d) > (b) > (e)$

(3) $(a) > (d) > (c) > (b) > (e)$

(4) $(c) > (a) > (d) > (e) > (b)$

Official Ans. by NTA (3)

Sol.

(a) CrO_3^{+6} (d) $\text{V}_2\text{O}_5^{+5}$

(b) $\text{Fe}_2\text{O}_3^{+3}$ (e) Cu_2O^{+1}

(c) MnO_2^{+4}

So order of oxidation state

$a > d > c > b > e$

77. Which one of the following chemical agent is **not** being used for dry-cleaning of clothes?

(1) H_2O_2 (2) CCl_4
(3) Liquid CO_2 (4) $\text{Cl}_2\text{C} = \text{CCl}_2$

Official Ans. by NTA (2)

QuestPix Ans. (1)

Sol.

CO_2 , CCl_4 and $\text{Cl}_2\text{C} = \text{CCl}_2$ are used as dry cleaning agents for clothes.

H_2O_2 is used as bleaching agent in laundry.

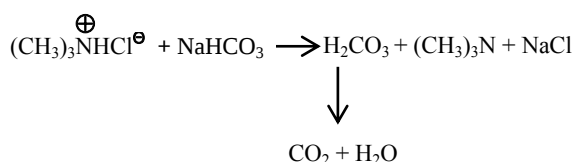
78. Which one of the following compounds will liberate CO_2 , when treated with NaHCO_3 ?

(1) $(\text{CH}_3)_3\text{NHCl}^{\oplus\ominus}$ (2) $(\text{CH}_3)_4\text{NOH}^{\oplus\ominus}$

(3) $\text{CH}_3 - \overset{\text{O}}{\parallel} \text{C} - \text{NH}_2$ (4) CH_3NH_2

Official Ans. by NTA (1)

Sol.



79. In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is :

(1) TiO_2 (2) Fe_2O_3
(3) ZnO (4) SiO_2

Official Ans. by NTA (4)

Sol.

In bauxite impurities of Fe_2O_3 , TiO_2 and SiO_2 are present, Fe_2O_3 and TiO_2 are basic oxides therefore does not react with or dissolve in NaOH whereas SiO_2 is acidic oxide it gets dissolved in NaOH , hence leach out



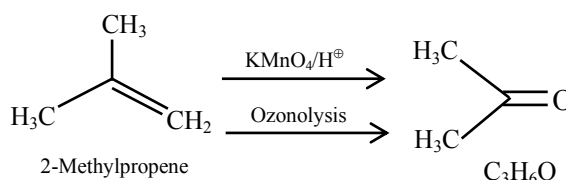
80. An organic compound 'A' C_4H_8 on treatment with KMnO_4/H^+ yields compound 'B' $\text{C}_3\text{H}_6\text{O}$.

Compound 'A' also yields compound 'B' on ozonolysis. Compound 'A' is :

(1) 2-Methylpropene
(2) 1-Methylcyclopropane
(3) But-2-ene
(4) Cyclobutane

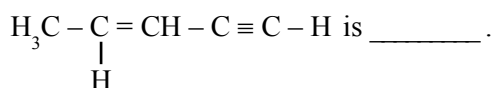
Official Ans. by NTA (1)

Sol.



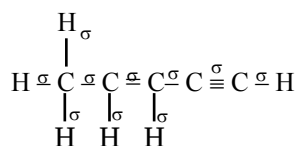
SECTION-B

81. The number of sigma bonds in



Official Ans. by NTA (10)

Sol.



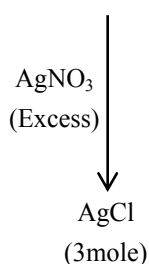
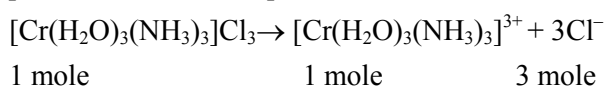
numbers of σ bonds = 10

82. Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $\text{CrCl}_3 \cdot 3\text{NH}_3 \cdot 3\text{H}_2\text{O}$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is _____.

Official Ans. by NTA (0)

Sol.

Mole of AgCl precipitated is equal the mole of Cl^- present in ionization sphere.



Since none of Cl^- is present in the co-ordination sphere. Therefore answer is zero.

83. A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, $x \times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is _____ . (Nearest integer)

$$(h = 6.626 \times 10^{-34} \text{ Js})$$

Official Ans. by NTA (2)

Sol.

Total energy provided by

$$\text{Source per second} = \frac{1000}{10} = 100 \text{ J}$$

$$\text{Energy required to eject electron} = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}}{400 \times 10^{-9}} \times 3 \times 10^8$$

Number of electrons ejected

$$= \frac{100}{\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}}$$

$$\begin{aligned} &= \frac{400 \times 10^{-9} \times 10}{6.626 \times 3} \\ &= \frac{40 \times 10^{-20}}{6.626 \times 3} \\ &= 2.01 \times 10^{20} \end{aligned}$$

84. CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is _____ . (Nearest integer)

(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar)

Official Ans. by NTA (25)

Sol. From Henry's law

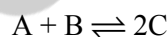
$$P_{\text{gas}} = K_H \cdot X_{\text{gas}}$$

$$0.835 = 1.67 \times 10^3 \times \frac{n(\text{CO}_2)}{\frac{0.9 \times 1000}{18}}$$

$$n(\text{CO}_2) = 0.025$$

$$\text{Millimoles of } \text{CO}_2 = 0.025 \times 1000 = 25$$

85. For the reaction

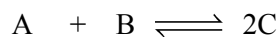


the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is $x \times 10^{-1}$ M. The value of x is _____ .

(Nearest integer)

Official Ans. by NTA (25)

Sol.



$$\begin{array}{ccc} 1 & 1 & 1 \\ -x & -x & 2x \end{array}$$

$$\begin{array}{ccc} 1-x & 1-x & 1+2x \end{array}$$

$$K = \frac{[\text{C}]_{\text{eq}}^2}{[\text{A}]_{\text{eq}}[\text{B}]_{\text{eq}}} = \frac{(1+2x)^2}{(1-x)(1-x)}$$

$$100 = \left(\frac{1+2x}{1-x} \right)^2$$

$$\left(\frac{1+2x}{1-x} \right) = 10$$

$$x = \frac{3}{4}$$

$$[C]e_q = 1 + 2x$$

$$= 1 + 2 \left(\frac{3}{4} \right)$$

$$= 2.5 \text{ M}$$

$$25 \times 10^{-1} \text{ M}$$

86. Consider the cell at 25°C

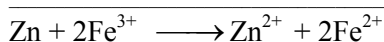
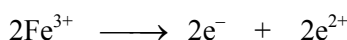


The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)

(Given : $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^0 = 0.77\text{V}$, $E_{\text{Zn}^{2+}/\text{Zn}} = -0.76\text{V}$)

Official Ans. by NTA (24)

Sol.



$$E_{\text{cell}}^0 = 0.77 - (0.76)$$

$$= 1.53 \text{ V}$$

$$1.50 = 1.53 - \frac{0.06}{2} \log \left(\frac{[\text{Fe}^{2+}]^2}{[\text{Fe}^{3+}]^2} \right)$$

$$\log \left(\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} \right) = \frac{0.03}{0.06} = \frac{1}{2}$$

$$\frac{[\text{Fe}^{2+}]}{[\text{Fe}^{3+}]} = 10^{1/2} = \sqrt{10}$$

$$\frac{[\text{Fe}^{3+}]}{[\text{Fe}^{2+}]} = \frac{1}{\sqrt{10}}$$

$$\frac{[\text{Fe}^{3+}]}{[\text{Fe}^{2+}] + [\text{Fe}^{3+}]} = \frac{1}{1 + \sqrt{10}} = \frac{1}{4.16}$$

$$= 0.2402$$

$$= 24 \times 10^{-2}$$

87. At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol⁻¹ and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol⁻¹. The enthalpy of sublimation of the substance (X) in kJ mol⁻¹ is _____. (in nearest integer)

Official Ans. by NTA (101)

Sol.

$$\Delta H_{\text{sub}} = \Delta H_{\text{fus}} + \Delta H_{\text{vap}}$$

$$= 2.8 + 98.2$$

$$= 101 \text{ kJ/mol}$$

88. A home owner uses $4.00 \times 10^3 \text{ m}^3$ of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x \times 10^5 \text{ g}$. The value of x is _____. (Nearest integer)

(Given $R = 0.083 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

Official Ans. by NTA (26)

Sol.

$$n(\text{CH}_4) = \frac{PV}{RT}$$

$$= \frac{1 \times 4 \times 10^3 \times 1000}{0.083 \times 300}$$

Weight of CH_4

$$= \frac{40 \times 16 \times 10^5}{0.083 \times 300} \text{ gm}$$

$$= 25.7 \times 10^5 \text{ gm}$$

89. When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $\text{K}_2\text{Cr}_2\text{O}_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is $x \times 10^{-2} \text{ M}$. The value of x is _____. (Nearest integer)

Official Ans. by NTA (18)

Sol.

milli-equivalents of Fe^{2+} = milli-equivalents of $\text{K}_2\text{Cr}_2\text{O}_7$

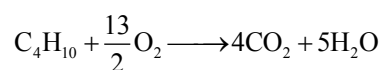
$$M \times 10 \times 1 = 0.02 \times 15 \times 6$$

$$M = 0.18 = 18 \times 10^{-2} \text{ M}$$

90. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1} \text{ g}$. (in nearest integer)

Official Ans. by NTA (464)

Sol.



$$\text{Moles of } \text{H}_2\text{O} = \frac{72}{18} = 4$$

$$\text{Moles of } \text{C}_4\text{H}_{10} \text{ used} = \frac{1}{5} \times 4$$

$$\text{Weight of } \text{C}_4\text{H}_{10} \text{ used} = \frac{4}{5} \times 58$$

$$= 46.4 \text{ gm}$$