

JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

MATHEMATICS

TEST PAPER WITH ANSWER AND SOLUTION

PAPER-1

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

1. Let $f(x)$ be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and

$$\lim_{t \rightarrow x} \frac{t^{10} f(x) - x^{10} f(t)}{t - x^9} = 1 \text{ for each } x > 0. \text{ Then, for all } x > 0, f(x) \text{ is equal to :}$$

(A) $\frac{31}{11x} - \frac{9}{11} x^{10}$ (B) $\frac{9}{11x} + \frac{13}{11} x^{10}$ (C) $\frac{-9}{11x} + \frac{31}{11} x^{10}$ (D) $\frac{13}{11x} + \frac{9}{11} x^{10}$

Ans. (B)

Sol. $\lim_{t \rightarrow x} \frac{t^{10} f(x) - x^{10} f(t)}{t - x^9} = 1$

$$\lim_{t \rightarrow x} \frac{10t^9 f(x) - x^{10} f'(t)}{9t^8} = 1$$

$$\Rightarrow 10xf(x) - x^2 f'(x) = 9$$

$$\Rightarrow x^2 f'(x) = 10xf(x) - 9$$

$$\Rightarrow f'(x) = \frac{10f(x)}{x} - \frac{9}{x^2}$$

$$\Rightarrow \frac{dy}{dx} - \frac{10}{x} y = -\frac{9}{x^2}$$

$$\Rightarrow y \cdot \frac{1}{x^{10}} = \int -\frac{9}{x^2} \cdot \frac{1}{x^{10}} dx$$

$$\Rightarrow \frac{y}{x^{10}} = \frac{1}{11x^{11}} + c \quad \dots(1)$$

$$\because f(1) = 2 \Rightarrow \frac{2}{1} = \frac{9}{11} + c \Rightarrow c = \frac{13}{11}$$

$$\therefore f(x) = \frac{9}{11x} + \frac{13}{11} x^{10}$$

$\Rightarrow$ Option (B) is correct.

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is :

- (A) $\frac{1}{12}$
(B) $\frac{1}{7}$
(C) $\frac{5}{7}$
(D) $\frac{5}{12}$

Ans. (C)
Sol. C → Correct
 G → Guess
 K → Knows

$$P\left(-\right)=\frac{1}{2} \quad , \quad P\left(\frac{C}{K}\right)=$$

$$P\left(-\right)=\frac{1}{6}$$

Let required probability = x

$$\therefore P\left(\frac{G}{C}\right)=\frac{(1-x)P\left(\frac{C}{G}\right)}{(1-x)P\left(\frac{C}{G}\right)+x.P\left(-\right)}$$

$$\frac{1}{6}=\frac{(1-x)\left(\frac{1}{2}\right)}{(1-x)\left(\frac{1}{2}\right)+(x)(1)}$$

$$\Rightarrow x=\frac{5}{7} \Rightarrow \text{Option (C) is correct.}$$

3. Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then

$$\left(\sin \frac{11 x}{2}(\sin 6 x-\cos 6 x)+\left(\cos \frac{11 x}{2}(\sin 6 x+\cos 6 x)\right.\right.$$

is equal to :

- (A) $\frac{\sqrt{11}-1}{2 \sqrt{\quad}}$
(B) $\frac{\sqrt{11}+1}{2 \sqrt{\quad}}$
(C) $\frac{\sqrt{11}+1}{3 \sqrt{\quad}}$
(D) $\frac{\sqrt{11}-1}{3 \sqrt{\quad}}$

Ans. (B)

Sol. $x \in \left(\frac{\pi}{2}, \pi \right)$

$$\left(\sin \frac{11x}{2} \right) (\sin 6x - \cos 6x) + \left(\cos \frac{11x}{2} \right) (\sin 6x + \cos 6x)$$

$$\begin{aligned} & \left\{ \sin 6x \sin \frac{11x}{2} + \cos \frac{11x}{2} \cos 6x \right\} \\ &= \cos \left(6x - \frac{11x}{2} \right) + \sin \left(6x - \frac{11x}{2} \right) \\ &= \cos \frac{x}{2} \sin - \\ &= \frac{1}{2\sqrt{3}} \frac{\sqrt{11}}{2\sqrt{3}} \\ &= \frac{\sqrt{11}+1}{2\sqrt{3}} \Rightarrow \text{Option (B) is correct.} \end{aligned}$$

$$\cot x = -\frac{5}{\sqrt{11}}$$

$$\frac{1 - \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2}} = -\frac{5}{\sqrt{11}}$$

$$\tan \frac{x}{2} = \sqrt{11}, \frac{1}{\sqrt{11}}$$

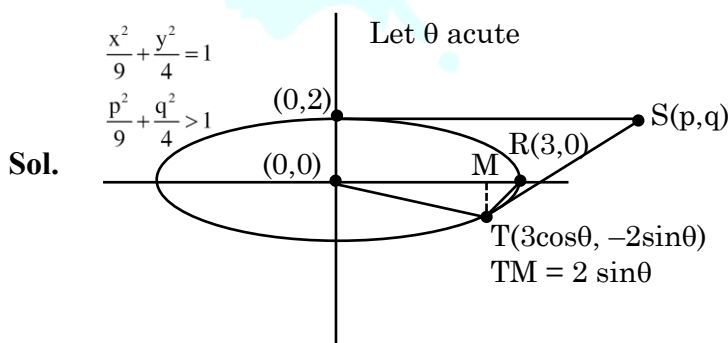
$$\tan \frac{x}{2} = \sqrt{11}, \text{ As } \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

4. Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let $S(p, q)$ be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} = 1$.

Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x -coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct ?

- (A) $q = 2, p = 3\sqrt{3}$ (B) $q = 2, p = 4\sqrt{3}$
(C) $q = 1, p = 5\sqrt{3}$ (D) $q = 1, p = 6\sqrt{3}$

Ans. (A)



$$\text{Ar}(\triangle ORT) = \frac{3}{2}$$

$$\left| \frac{1}{2} \times 3 \times 2 \sin \theta \right| = -$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{11\pi}{2}$$

$$T\left(\frac{3\sqrt{2}}{2}, -1\right)$$

$$\text{Tangent at } (0, 2) \quad \frac{x(0)}{9} + \frac{y(2)}{1} = 1 \Rightarrow y = 2 \quad \dots(1)$$

$$\text{Tangent at } \left(\frac{3\sqrt{2}}{2}, -1\right) \quad \frac{x\left(\frac{3\sqrt{2}}{2}\right)}{9} + \frac{y(-1)}{1} = 1 \quad \dots(2)$$

$$\therefore \text{By solving (1) \& (2)} \Rightarrow p = 3\sqrt{3}, q = 2$$

$\Rightarrow$ Option (A) is Correct.

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

<i>Full Marks</i>	: +4	ONLY if (all) the correct option(s) is(are) chosen;
<i>Partial Marks</i>	: +3	If all the four options are correct but ONLY three options are chosen;
<i>Partial Marks</i>	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
<i>Partial Marks</i>	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
<i>Zero Marks</i>	: 0	If none of the options is chosen (i.e. the question is unanswered);
<i>Negative Marks</i>	: -2	In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 - choosing **ONLY** (A), (B) and (D) will get +4 marks;
 - choosing **ONLY** (A) and (B) will get +2 marks;
 - choosing **ONLY** (A) and (D) will get +2 marks;
 - choosing **ONLY** (B) and (D) will get +2 marks;
 - choosing **ONLY** (A) will get +1 marks;
 - choosing **ONLY** (B) will get +1 marks;
 - choosing **ONLY** (D) will get +1 marks;
 - choosing no option (i.e. the question is unanswered) will get 0 marks; and
 - choosing any other combination of options will get -2 marks.



5. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \{(-1 + \sqrt{2})^n : n \in \mathbb{N}\}$ and $T_2 = \{(1 + \sqrt{2})^n : n \in \mathbb{N}\}$. Then which of the following statements is (are) TRUE ?
- (A) $\mathbb{Z} \cup T_1 \cup T_2 \subset S$
- (B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.
- (C) $T_2 \cap (2024, \infty) \neq \phi$
- (D) For any given $a, b \in \mathbb{Z}$, $\cos(\pi(a + b\sqrt{2})) + i \sin(\pi(a + b\sqrt{2})) \in \mathbb{Z}$ if and only if $b = 0$, where $i = \sqrt{-1}$.

Ans. (A,C,D)

- Sol.** (A) $(-1 + \sqrt{2})^n = m + \sqrt{2}n, m, n \in \mathbb{Z}$
 $(1 + \sqrt{2})^n = m_1 + \sqrt{2}n_1, m_1, n_1 \in \mathbb{Z}$
 $\Rightarrow \mathbb{Z} \cup T_1 \cup T_2 \subseteq S$
 but $b\sqrt{2} \in S$ for negative $b \in \mathbb{Z}$.
 So $\mathbb{Z} \cup T_1 \cup T_2 \subset S$
- (B) $(\sqrt{2} - 1)^n = \frac{1}{(\sqrt{2} + 1)^n} < \frac{1}{2024}$
 $\Rightarrow 2024 < (\sqrt{2} + 1)^n, \exists n \in \mathbb{N}$
 $\Rightarrow T_1 \cap \left(0, \frac{1}{2024}\right) \neq \phi$
- (C) $(1 + \sqrt{2})^n > 2024, \exists n \in \mathbb{N}$
 $\Rightarrow T_2 \cap (2024, \infty) \neq \phi$
- (D) $\sin(\pi(a + b\sqrt{2})) = 0 \Rightarrow b = 0, a \in \mathbb{Z}$.
 $\Rightarrow$ Options (A), (C), (D) are Correct.

6. Let $\mathbb{R}^2$ denote $\mathbb{R} \times \mathbb{R}$. Let
 $S = \{(a, b, c) : a, b, c \in \mathbb{R} \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x, y) \in \mathbb{R}^2 - \{(0, 0)\}\}$.
 Then which of the following statements is (are) TRUE ?
- (A) $\left(2, \frac{7}{2}, 6\right) \in S$
- (B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.
- (C) For any given $(a, b, c) \in S$, the system of linear equations
 $ax + by = 1$
 $bx + cy = -1$
 has a unique solution.
- (D) For any given $(a, b, c) \in S$, the system of linear equations
 $(a + 1)x + by = 0$
 $bx + (c + 1)y = 0$
 has a unique solution

Ans. (B,C,D)

- Sol.** (A) $ax^2 + 2bxy + cy^2 > 0 \quad \forall (x, y) \in \mathbb{R}^2 - \{(0, 0)\}$
 $\Rightarrow ax^2 + 2bxy + cy^2$ must represent pair of imaginary lines and $a, c > 0$.
 $\Rightarrow b^2 < ac$
- (B) $b^2 < 3 \times \frac{1}{12} \Rightarrow |2b| < 1$
- (C) since $b^2 \neq ac$
 $\Rightarrow ax + by = 1$ and $bx + cy = -1$
 are not parallel lines.
- (D) $ac + a + c > b^2 \Rightarrow$ lines are not parallel.
 $\Rightarrow$ Options (B), (C), (D) are Correct.

7. Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$.
 Let $\text{dist}(X, Y)$ denote the distance between two points X and Y in $\mathbb{R}^3$. Let
 $S = \{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\}$ and
 $T = \{Y \in \mathbb{R}^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\}$.
 Then which of the following statements is (are) TRUE ?
- (A) There is a triangle whose area is 1 and all of whose vertices are from S .
- (B) There are two distinct points L and M in T such that each point on the line segment LM is also in T .
- (C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T .
- (D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T .

Ans. (A,B,C,D)

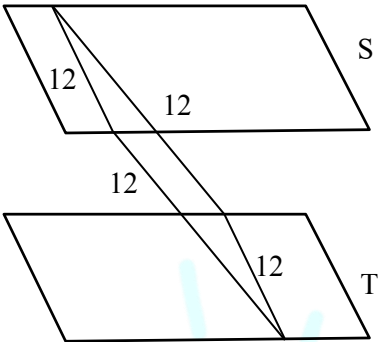
Sol. $S = \{X : (XP)^2 - (XQ)^2 = 50\}$
 $T = \{Y : (YQ)^2 - (YP)^2 = 50\}$
for finding $S \equiv X(x, y, z)$ and for $T \equiv Y(x, y, z)$
 $((x - 1)^2 + (y - 1)^2 + (z - 1)^2) - ((x - 4)^2 + (y - 2)^2 + (z - 7)^2) = 50$
 $\Rightarrow S = \{(x, y, z) : 6x + 8z = 105\}$
 $T = \{(x, y, z) : 6x + 8z = 5\}$

Since S and T both are plane ;

- (A) There exist a triangle in plane S whose area = 1 (always)
(B) L & M lies on plane T, hence line segment joining L & M will lie on plane T.
(C) Distance between S & T

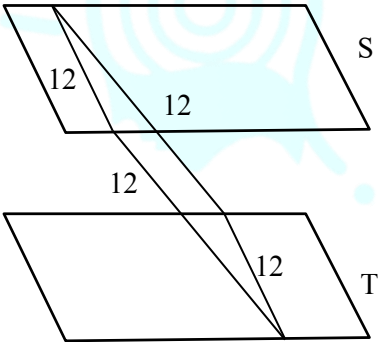
$$d = \left| \frac{105 - 5}{10} \right| = 10$$

Hence for rectangle of perimeter 48 can exist.



There will be infinite such rectangle possible.

(D) For square



Hence Answers A,B,C,D are correct

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 **ONLY** If the correct integer is entered;
Zero Marks : 0 In all other cases.

8. Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{2}}$. If $x, y \in \mathbb{R}$ are such that

$$3x + 2y = \log_a(18)^{\frac{5}{4}} \text{ and}$$

$$2x - y = \log_b(\sqrt{1080}),$$

then $4x + 5y$ is equal to

Ans. (8)

Sol. $3x + 2y = \log_{3\sqrt{2}}(3\sqrt{2})^{\frac{5}{2}} = \frac{5}{2}$

$$\Rightarrow 6x + 4y = 5 \quad \dots\dots(1)$$

$$2x - y = \log_{\frac{1}{5^{1/6}\sqrt{2}}}(5^{\frac{1}{6}}\sqrt{6})^3 = -3$$

$$\Rightarrow 2x - y = -3 \quad \dots\dots(2)$$

$$\text{equation (1) - (2)}$$

$$\Rightarrow 4x + 5y = 8$$

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to

Ans. (20)

Sol. $f(1) = 1 + a + b + c = -9 \Rightarrow a + b + c = -10 \quad \dots\dots(1)$

$$4x^3 + 3ax^2 + 2bx = 0 \text{ roots are } \sqrt{3}i, -\sqrt{3}i, 0$$

$$\Rightarrow 4x^2 + 3ax + 2b = 0 \begin{cases} \sqrt{3}i \\ -\sqrt{3}i \end{cases}$$

$$\Rightarrow a = 0 \text{ \& } \frac{2b}{4} = (\sqrt{3}i)(-\sqrt{3}i)$$

$$b = 6 \text{ use } a, b \text{ in (1)} \Rightarrow c = -16$$

$$\Rightarrow f(x) = x^4 + 6x^2 - 16 = 0$$

$$(x^2 + 8)(x^2 - 2) = 0$$

$$\Rightarrow x = \pm\sqrt{8}i, \pm\sqrt{2} \Rightarrow |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$$

10. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where $|A|$ denotes the determinant of

A. Then the number of elements in S is _____.

Ans. (16)

10. $|A| = 0(ae - bd) - 1(e - d) + c(b - a)$

$$= c(b - a) + (d - e)$$

$$|A| \in \{-1, 1\} \text{ and } a, b, c, d, e \in \{0, 1\}$$

Case-I

$$c = 0 \quad d = 1, e = 0, a, b \in \{0, 1\}$$

$$d = 0, e = 1$$

$$a \quad b \quad c \quad d \quad e$$

↓

$$2 \quad 2 \quad 1 \quad 2 \rightarrow 8 \text{ cases}$$

Case-II

$$c = 1 \quad b = 1, a = 0, \quad d = 0, e = 0, d = 1, e = 1$$

$$b = 0, a = 1, \quad d = 0, e = 0, d = 1, e = 1$$

$$b = 0, a = 0, \quad d = 1, e = 0$$

$$d = 0, e = 1$$

$$b = 1, a = 1, \quad d = 1, e = 0$$

$$d = 0, e = 1$$

→ 8 cases

⇒ Total 16 cases

11. A group of 9 students, $s_1, s_2, \dots, s_9$, is to be divided to form three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is _____.

Ans. (665)

Sol.

x	y	z
2	3	4
$\bar{S}_1$	$\bar{S}_2$	

C-i) when x does not contain S_1 , but contains S_2

$${}^7C_1 \times \frac{7!}{3!4!} = 245$$

for x for y, z

C-ii) When x does not contain S_1, S_2 and y does not contain S_2

$$\text{i.e. } {}^7C_2 \times \frac{6!}{3!3!} = 420$$

for x for y, z

so total No. of ways 665

12. Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is

Ans. (5)

Sol. $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$

$$\begin{vmatrix} \alpha-1 & 1 & 1 \\ \alpha & \frac{\beta-1}{\beta} & \frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\alpha + \beta + 1 = 0 \quad \dots(1)$$

Also $(\alpha, \beta, 2)$ lies on $3x + 3y - z + l = 0$

$$\Rightarrow 3\alpha + 3\beta - 2 + l = 0 \Rightarrow l = 2 - 3(\alpha + \beta)$$

use (1) in it $\Rightarrow l = 5$

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x . Suppose that the points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy -plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α , then

the value of 24α is

Ans. (42)

Sol. Let equation of line is $y = mx + c$

x	0	1	2	3	4	$R - \{0, 1, 2, 3, 4\}$
$P(x)$	c	$m + c$	$2m + c$	$3m + c$	$4m + c$	0

$$\sum_{x=0}^4 (mx + c) = 1 \Rightarrow 10m + 5c = 1 \Rightarrow 2m + c = \frac{1}{5} \quad \dots(1)$$

$$\text{mean} = \sum x_i P_i = \sum_{i=0}^4 (mx_i + c) \cdot x_i = 30m + 10c = \frac{5}{2}$$

$$\therefore 3m + c = \frac{1}{4} \quad \dots(2)$$

$$\text{from (1) and (2)} \quad m = \frac{1}{20}, \quad c = \frac{1}{10}$$

$$\sum P_i x_i = \sum_{i=0}^4 (mx_i + c) x_i^2$$

$$= \sum_{i=0}^4 (mx_i^3 + cx_i^2) \Rightarrow 100m + 30c \quad (\text{Now putting } m \text{ and } c)$$

$$\Rightarrow \sum P_i x_i^2 = 5 + 3 = 8$$

$$\text{Variance} = \sum P_i x_i^2 - (\sum P_i x_i)^2 = 8 - \left(\frac{5}{2}\right)^2 = -$$

$$\therefore 24\alpha = 42$$

SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

14. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3 \times 3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	The number of matrices $M = (a_{ij})_{3 \times 3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j , is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3 \times 3}$ with all entries in T such that $C_j = 0$ for all j , is	(2)	12
(R)	Let $M = (a_{ij})_{3 \times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix} \right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3 \times 3}$ be a matrix with all entries in T such that $R_i = 0$ for all i . Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct options is

- (A) $(P) \rightarrow (4) \quad (Q) \rightarrow (2) \quad (R) \rightarrow (5) \quad (S) \rightarrow (1)$
 (B) $(P) \rightarrow (2) \quad (Q) \rightarrow (4) \quad (R) \rightarrow (1) \quad (S) \rightarrow (5)$
 (C) $(P) \rightarrow (2) \quad (Q) \rightarrow (4) \quad (R) \rightarrow (3) \quad (S) \rightarrow (5)$
 (D) $(P) \rightarrow (1) \quad (Q) \rightarrow (5) \quad (R) \rightarrow (3) \quad (S) \rightarrow (4)$

Ans. (C)

Sol. α, β are roots of $x^2 + x - 1 = 0$
 $\therefore \alpha + \beta = -1 \Rightarrow 1 + \alpha + \beta = 0$

$$M = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(P) $M = \begin{vmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ 1 & \alpha & \beta \end{vmatrix} \Rightarrow 3! \times 2 = 12$

For one arrangement of row 1 we can arrange other two rows exactly in two ways and row 1 can be arranged in $3!$ ways

$\therefore 3! \times 2 = 12$ ways

(Q) $M = \begin{vmatrix} x & a & b \\ a & y & c \\ b & c & z \end{vmatrix} \Rightarrow$ Consider one such arrangement with $a = \alpha, b = \beta, c = 1$

$$M = \begin{vmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ 1 & \alpha & \beta \end{vmatrix}$$

a, b, c can be arranged in $3!$ ways and corresponding entries can be arranged in 1 way.

(R) $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ -c \end{bmatrix}$

$$ay + bz = a$$

$$-ax + cz = 0$$

$$-bx - cy = -c$$

It is observed that $D = D_x = D_y = D_z = 0$

$\therefore$ infinite solution

(S) $\begin{vmatrix} 1 & \alpha & \beta \\ \beta & \alpha & 1 \\ \alpha & 1 & \end{vmatrix}$

$$\Rightarrow \alpha\beta - 1 - \alpha\beta^2 + \alpha^2 + \beta^2 - \alpha^2\beta = 0 \quad (\text{since } \alpha\beta = \alpha + \beta = -1)$$

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1 . Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

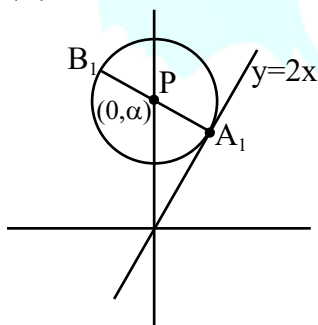
List-I		List-II	
(P)	α equals	(1)	$(-2, 4)$
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	$(-2, 6)$
(S)	B_1 equals	(4)	5
		(5)	$(2, 4)$

The correct option is

- (A) $(P) \rightarrow (4)$ $(Q) \rightarrow (2)$ $(R) \rightarrow (1)$ $(S) \rightarrow (3)$
 (B) $(P) \rightarrow (2)$ $(Q) \rightarrow (4)$ $(R) \rightarrow (1)$ $(S) \rightarrow (3)$
 (C) $(P) \rightarrow (4)$ $(Q) \rightarrow (2)$ $(R) \rightarrow (5)$ $(S) \rightarrow (3)$
 (D) $(P) \rightarrow (2)$ $(Q) \rightarrow (4)$ $(R) \rightarrow (3)$ $(S) \rightarrow (5)$

Ans. (C)

Sol.



Consider centre as $P(0, \alpha)$, $\alpha > 0$

$$\left| \frac{2(0) - \alpha}{\sqrt{5}} \right| = r$$

$$|-\alpha| = \sqrt{5}r$$

$$\alpha = \sqrt{5}r$$

$$\therefore \alpha + r = 5 + \sqrt{5}$$

$$\sqrt{5}r + r = \sqrt{5}(\sqrt{5} + 1)$$

$$r = \sqrt{5}, \alpha = 5$$

$$\therefore P(0, 5)$$

Foot of perpendicular from P to line $2x - y = 0$

$$\frac{x-0}{2} = \frac{y-5}{-1} = \frac{-(2(0)-5)}{5} = 1$$

$$x = 2, y = 4 \quad A_1(2, 4)$$

$$\text{Let } B(p, q) \quad \therefore \frac{p+2}{2} = 0, \frac{q+4}{2} = 5$$

$$\therefore p = -2, q = 6 \quad B(-2, 6)$$

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1: \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2: \frac{x+16}{3} = \frac{y+11}{\gamma} = \frac{z+4}{\gamma}$

intersect. Let R_1 be the point of intersection of L_1 and L_2 . Let $O = (0, 0, 0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2 .

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\frac{\sqrt{3}}{\sqrt{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

(A) $(P) \rightarrow (3) \quad (Q) \rightarrow (4) \quad (R) \rightarrow (1) \quad (S) \rightarrow (2)$

(B) $(P) \rightarrow (5) \quad (Q) \rightarrow (4) \quad (R) \rightarrow (1) \quad (S) \rightarrow (2)$

(C) $(P) \rightarrow (3) \quad (Q) \rightarrow (4) \quad (R) \rightarrow (1) \quad (S) \rightarrow (5)$

(D) $(P) \rightarrow (3) \quad (Q) \rightarrow (1) \quad (R) \rightarrow (4) \quad (S) \rightarrow (5)$

Ans. (C)

Sol. $L_1: \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = a$

$$L_2: \frac{x+16}{2} = \frac{y+11}{2} = \frac{z+4}{\gamma} = b$$

$$x = a - 11 = 3b - 16 \Rightarrow a - 3b = -5 \quad \dots(1)$$

$$y = 2a - 21 = 2b - 11 \Rightarrow 2a - 2b = 10 \quad \dots(2)$$

$$z = 3a - 29 = b\gamma - 4 \Rightarrow 3a - b\gamma = 25 \quad \dots(3)$$

from (1) & (2)

$$a = 10, b = 5$$

Now from (3)

$$3(10) - 5\gamma = 25 \quad \therefore \gamma = 1$$

$$R_1 \equiv (-1, -1, 1)$$

$$\vec{OR}_1 = -\hat{i} - \hat{j} + \hat{k}$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} - (-8)\hat{j} - 4\hat{k}$$

$$\vec{n} = -4\hat{i} + 8\hat{j} - 4\hat{k} = -4(\hat{i} - 2\hat{j} + \hat{k})$$

$$\hat{n} = \pm \frac{4(\hat{i} - 2\hat{j} + \hat{k})}{4\sqrt{6}} = \pm \frac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$$

$$\vec{OR}_1 \cdot \hat{n} = \pm (-\hat{i} - \hat{j} + \hat{k}) \cdot \frac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}} = \pm \frac{2}{\sqrt{6}} = \pm \sqrt{\frac{4}{6}} = \pm \sqrt{\frac{2}{3}}$$

17. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \text{ and } g(x) = \begin{cases} 1-2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = af(x) + b\left(g(x) + g\left|\frac{1}{2} - x\right|\right) + c(x - g(x)) + dg(x), \quad x \in \mathbb{R}$$

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	If $a = 0, b = 1, c = 0$ and $d = 0$, then	(1)	h is one-one.
(Q)	If $a = 1, b = 0, c = 0$ and $d = 0$, then	(2)	h is onto.
(R)	If $a = 0, b = 0, c = 1$ and $d = 0$, then	(3)	h is differentiable on $\mathbb{R}$.
(S)	If $a = 0, b = 0, c = 0$ and $d = 1$, then	(4)	the range of h is $[0, 1]$.
		(5)	the range of h is $\{0, 1\}$.

The correct option is :

(A) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (1) (S) $\rightarrow$ (2)

(B) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)

(C) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (2) (S) $\rightarrow$ (4)

(D) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (1) (S) $\rightarrow$ (3)

Ans. (C)

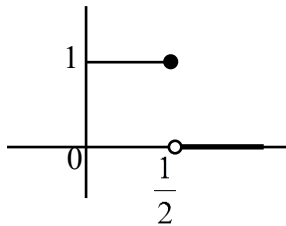
Sol. $f(x) = \begin{cases} x|x|\sin\frac{1}{x} & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$ $g(x) = \begin{cases} 1-2x & ; \quad 0 \leq x \leq \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$

$$g\left(\frac{1}{2} - x\right) = \begin{cases} 2x & ; \quad 0 \leq -x \leq \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases} = \begin{cases} 2x & ; \quad 0 \leq x \leq \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1 & ; \quad 0 \leq x \leq \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(P) Now $a = 0, b = 1, c = 0, d = 0$

$$\therefore h(x) = g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1 & ; 0 \leq x \leq \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$



Hence Range of $h(x)$ is $\{0, 1\}$

(Q) $a = 1, b = 0, c = 0, d = 0$

$$h(x) = f(x) = \begin{cases} x|x|\sin\frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

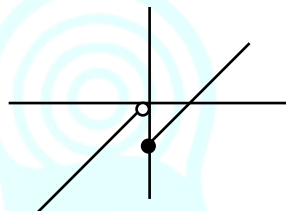
$$\text{RHD} = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - 0}{x} = 0$$

$$\text{LHD} = \lim_{x \rightarrow 0} \frac{-x^2 \sin \frac{1}{x} - 0}{x} = 0$$

Hence $h(x)$ is differentiable on $\mathbb{R}$

(R) $a = 0, b = 0, c = 1, d = 0$

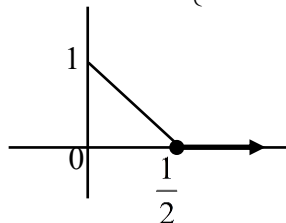
$$h(x) = x - g(x) = \begin{cases} 3x - 1 & ; 0 \leq x \leq \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$



$\therefore h(x)$ is ONTO

(S) $a = 0, b = 0, c = 0, d = 1$

$$h(x) = g(x) = \begin{cases} 1 - 2x & ; 0 \leq x \leq \frac{1}{2} \\ 0 & ; \text{otherwise} \end{cases}$$



Range of $h(x)$ is $[0, 1]$

JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

PHYSICS

TEST PAPER WITH ANSWER AND SOLUTION

PAPER-1

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

- 18.** A dimensionless quantity is constructed in terms of electronic charge e , permittivity of free space ϵ_0 , Planck's constant h , and speed of light c . If the dimensionless quantity is written as $e^\alpha \epsilon_0^\beta h^\gamma c^\delta$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by
- (A) $(2n, -n, -n, -n)$ (B) $(n, -n, -2n, -n)$
 (C) $(n, -n, -n, -2n)$ (D) $(2n, -n, -2n, -2n)$

Ans. (A)

Sol. For the quantity to be dimensionless

$$e^\alpha \epsilon_0^\beta h^\gamma c^\delta = M^0 L^0 T^0 A^0$$

$$(AT)^\alpha (M^{-1} L^{-3} T^4 A^2)^\beta (ML^2 T^{-1})^\gamma (LT^{-1})^\delta = A^0 M^0 L^0 T^0$$

$$\therefore \alpha + 2\beta = 0, \alpha + 4\beta - \gamma - \delta = 0, -\beta + \gamma = 0 \text{ \& } -3\beta + 2\gamma + \delta = 0$$

$$\therefore \alpha = -2\beta, \beta = \gamma \text{ \& } \gamma = \delta$$

$\therefore$ Option (A) satisfies the given condition

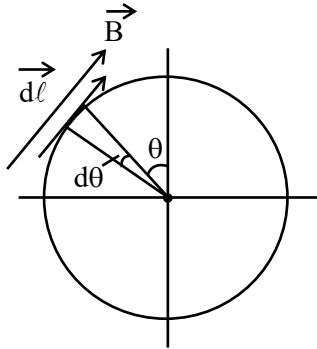
- 19.** An infinitely long wire, located on the z -axis, carries a current I along the $+z$ -direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B} \cdot d\vec{l}$ along a straight line from the point $(-\sqrt{3}a, a, 0)$ to $(a, a, 0)$ is given by

$[\mu_0$ is the magnetic permeability of free space.]

- (A) $7\mu_0 I/24$ (B) $7\mu_0 I/12$ (C) $\mu_0 I/8$ (D) $\mu_0 I/6$

Ans. (A)

Sol.



$$\Rightarrow |d\vec{\ell}| = r d\theta$$

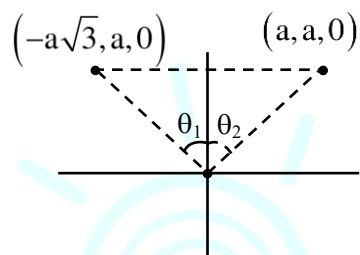
$$\Rightarrow |\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \int \vec{B} \cdot d\vec{\ell} = \int |\vec{B}| |d\vec{\ell}| \cos 0^\circ$$

$$= \int \left(\frac{\mu_0 I}{2\pi r} \right) \times (r d\theta)$$

$$= \int_{\theta_1}^{\theta_2} \frac{\mu_0 I}{2\pi} d\theta = \frac{\mu_0 I}{2} [\theta_2 - (-\theta_1)]$$

[θ_1 is anticlockwise hence taken negative]



$$\Rightarrow \tan \theta_1 = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

$$\theta_1 = \frac{\pi}{3}$$

$$\Rightarrow \tan \theta_2 = \frac{a}{a} = 1$$

$$\theta_2 = \frac{\pi}{4}$$

$$\Rightarrow \int \vec{B} \cdot d\vec{\ell} = \frac{\mu_0 I}{2\pi} \left[\frac{\pi}{3} + \frac{\pi}{4} \right]$$

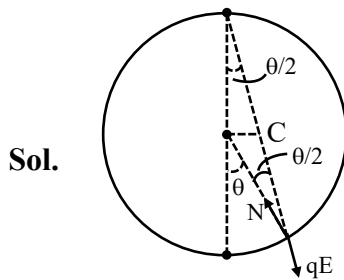
$$= \frac{7\mu_0 I}{24}$$

$\Rightarrow$ Ans. Option (A)

20. Two beads, each with charge q and mass m , are on a horizontal, frictionless, non-conducting, circular hoop of radius R . One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [ϵ_0 is the permittivity of free space]

- (A) $q^2 / (4\pi\epsilon_0 R^3 m)$ (B) $q^2 / (32\pi\epsilon_0 R^3 m)$
 (C) $q^2 / (8\pi\epsilon_0 R^3 m)$ (D) $q^2 / (16\pi\epsilon_0 R^3 m)$

Ans. (B)



$$\text{Restoring force} = qE \sin\left(\frac{\theta}{2}\right)$$

$$\therefore \tau = qE \sin\left(\frac{\theta}{2}\right) R = I\alpha$$

$$E = \frac{Kq}{\left(2R \cos \frac{\theta}{2}\right)^2} = \frac{1}{4\pi\epsilon_0} \frac{q}{4R \cos^2\left(\frac{\theta}{2}\right)}$$

$$\therefore \frac{1}{4\pi\epsilon_0} \frac{qR}{4R \cos^2\left(\frac{\theta}{2}\right)} \sin\left(\frac{\theta}{2}\right) q = mR^2 \alpha$$

For θ very small,

$$\frac{-q^2}{32\pi\epsilon_0 R^3 m} \theta = \alpha$$

$$\therefore \omega^2 = \frac{q^2}{32\pi\epsilon_0 m R^3}$$

Hence option (B)

21. A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are
 (A) -0.5 m, 5 kg m/s (B) 0.5 m, 0 kg m/s (C) 0.5 m, -5 kg m/s (D) -1 m, 5 kg m/s

Ans. (C)

Sol. $F = -20\left(x - \frac{1}{2}\right) = -20X \quad \left(X = x - \frac{1}{2}\right)$

$\therefore$ Particle will perform SHM about $x = \frac{1}{2}$ with

$$\omega = 2 \text{ rad/sec} \Rightarrow T = \pi \text{ sec.}$$

$\therefore$ Phase covered in $t = \frac{\pi}{4}$ second = 90° .

Given particle is at rest at $x = 1 \text{ m} \Rightarrow x = 1$ is extreme position.

$\therefore$ In $\frac{\pi}{4}$ sec, it will be at equilibrium

$\therefore x = 0.5$ m and momentum = $m\omega A = 5 \times 2 \times 0.5 = 5$ kg m/s

Direction will be towards $-ve$ x.

Hence option (C)

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

<i>Full Marks</i>	: +4	ONLY if (all) the correct option(s) is(are) chosen;
<i>Partial Marks</i>	: +3	If all the four options are correct but ONLY three options are chosen;
<i>Partial Marks</i>	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
<i>Partial Marks</i>	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
<i>Zero Marks</i>	: 0	If none of the options is chosen (i.e. the question is unanswered);
<i>Negative Marks</i>	: -2	In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
 choosing **ONLY** (A), (B) and (D) will get +4 marks;
 choosing **ONLY** (A) and (B) will get +2 marks;
 choosing **ONLY** (A) and (D) will get +2 marks;
 choosing **ONLY** (B) and (D) will get +2 marks;
 choosing **ONLY** (A) will get +1 marks;
 choosing **ONLY** (B) will get +1 marks;
 choosing **ONLY** (D) will get +1 marks;
 choosing no option (i.e. the question is unanswered) will get 0 marks; and
 choosing any other combination of options will get -2 marks.

22. A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = kr^2 / 2$, where k is a positive force constant and r is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, h is the Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

(A) $r^2 = n\hbar \sqrt{\frac{1}{mk}}$

(C) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$

(B) $v^2 = n\hbar \sqrt{\frac{k}{m^3}}$

(D) $E = \frac{n\hbar}{2} \sqrt{\frac{k}{m}}$

Ans. (A,B,C)

Sol. The central force will provide necessary centripetal force

$$kr = \frac{mv^2}{r}$$

or, $kr^2 = mv^2$ (1)

By quantisation rule

$$n\hbar = mvr$$

or, $\frac{n\hbar}{r} = mv$... (2)

$$\frac{(1)}{(2)^2} \Rightarrow \frac{kr^2}{\frac{n^2\hbar^2}{r^2}} = \frac{mv}{m v^2}$$

$$\Rightarrow \frac{k}{n^2\hbar^2} r^4 = \frac{1}{m}$$

$$\Rightarrow r = \left(\frac{n^2\hbar^2}{km} \right)^{\frac{1}{4}} \Rightarrow r^2 = \frac{n\hbar}{\sqrt{mk}}$$

(B) Using (1), $K \cdot \frac{n\hbar}{\sqrt{mk}} = mv^2$

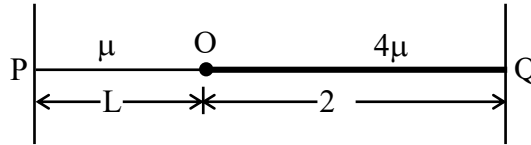
$$\Rightarrow v^2 = n\hbar \sqrt{\frac{k}{m^3}}$$

(C) $\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ from (1)

(D) $E = \frac{1}{2}mv^2 + \frac{1}{2}kr = \frac{n\hbar}{2} \sqrt{\frac{k}{m}} + \frac{1}{2}k \frac{n\hbar}{\sqrt{mk}}$

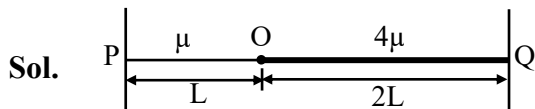
$$E = n\hbar \sqrt{\frac{k}{m}}$$

23. Two uniform string of mass per unit length μ and 4μ , and length L and $2L$, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $\nu_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct ?



- (A) With a node at O, the minimum frequency of vibration of the composite string is ν_0 .
 (B) With an antinode at O, the minimum frequency of vibration of the composite string is $2\nu_0$.
 (C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.
 (D) No vibrational mode with an antinode at O is possible for the composite string.

Ans. (A,C,D)



$$C_1 = \sqrt{\frac{T}{\mu}}, C_2 = \sqrt{\frac{T}{4\mu}} = \frac{C_1}{2}$$

For node at O :

$$L = \frac{n\lambda_1}{2}, 2L = \frac{m\lambda_2}{2} \quad (n, m \text{ are integers})$$

$$\lambda_1 = \frac{2L}{n}, \lambda_2 = \frac{4L}{m}$$

$$\frac{C_1}{\lambda_1} = \frac{C_2}{\lambda_2}$$

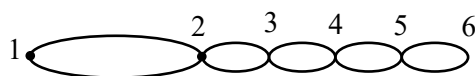
$$\Rightarrow \frac{C_1}{\frac{2L}{n}} = \frac{\frac{C_1}{2}}{\frac{4L}{m}}$$

$$\Rightarrow 4n = m$$

For minimum frequency, $n = 1, m = 4$

$$\therefore \nu_{\min} = \frac{C_1 \times 1}{2L} = \frac{1}{2L} \sqrt{\frac{T}{\mu}} = \nu_0$$

The string will look like



Total no. of nodes = 6 including the end nodes

For antinode at O :

$$L = (2n - 1) \frac{\lambda_1}{4}; 2L = (2m - 1) \frac{\lambda_2}{4} \quad (n, m \text{ are integers})$$

$$\lambda_1 = \frac{4L}{(2n+1)}; \lambda_2 = \frac{8L}{(2m+1)}$$

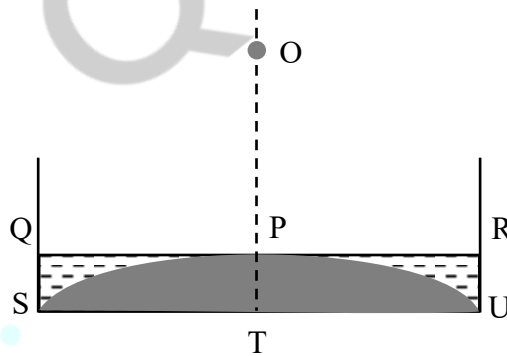
$$\frac{C_1}{\lambda_1} = \frac{C_2}{\lambda_2}$$

$$\frac{C_1}{C_2} = \frac{\lambda_2}{\lambda_1}$$

$$2 = \frac{\frac{4L}{(2n+1)}}{\frac{8L}{(2m+1)}}$$

$$4 = \frac{(2m+1)}{(2n+1)} \Rightarrow \text{even} = \frac{\text{odd}}{\text{odd}} \Rightarrow \text{This node is not possible}$$

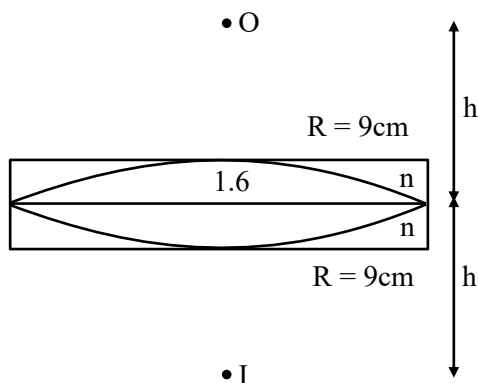
24. A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct?



- (A) For $n = 1.42$, $h = 50$ cm.
 (B) For $n = 1.35$, $h = 36$ cm.
 (C) For $n = 1.45$, $h = 65$ cm.
 (D) For $n = 1.48$, $h = 85$ cm.

Ans. (A,B)

Sol. Since STU is a plane mirror, we can take mirror image of the whole situation about it and final image can be assumed to be at a distance h below the base.



Since object and image are at same distance from equivalent lens, hence $h = 2F_{eq}$

$$\frac{1}{F_{eq}} = \left(\frac{1.6-1}{1} \right) \left(\frac{2}{9} \right) + \frac{(n-1)}{1} \left(\frac{2}{9} \right)$$

$$\frac{1}{\frac{h}{2}} = \frac{1.2}{9} + \frac{2(1-n)}{9}$$

$$\frac{2}{h} = \frac{3.2-2n}{9}$$

$$h = \frac{9}{1.6-n} \text{ cm}$$

(A) for $n = 1.42$, $h = 50 \text{ cm}$

(B) for $n = 1.35$, $h = 36 \text{ cm}$

(C) for $n = 1.45$, $h = 60 \text{ cm}$

(D) for $n = 1.48$, $h = 75 \text{ cm}$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	ONLY If the correct integer is entered;
Zero Marks	: 0	In all other cases.

25. The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from -73°C to 27°C is nk , the value of n is _____.

[Given : $0 \text{ K} = -273^\circ\text{C}$.]

Ans. (25000)

Sol. $T_i = -73^\circ\text{C} = 200 \text{ K}$

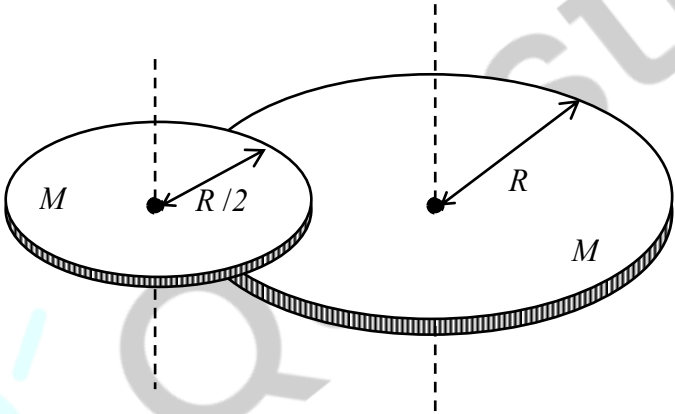
$T_f = 27^\circ\text{C} = 300 \text{ K}$

$$\begin{aligned}
 Q &= \int msdT \\
 &= \int 1 \cdot kT \, dT \\
 &= \int kT \, dT = K \int_{200}^{300} T \, dT \\
 &= \frac{K}{2} \left[T^2 \right]_{200}^{300} = \frac{K}{2} \left[300^2 - 200^2 \right]
 \end{aligned}$$

$$Q = 25000 \text{ K}$$

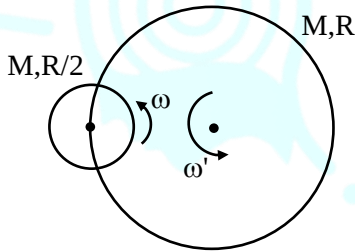
$$\text{Hence } \eta = 25000$$

26. A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius $R/2$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω . If the angular speed at which the large disc rotates is ω/n , then the value of n is _____.



Ans. (12)

Sol.



On applying conservation of angular momentum about axis of larger disc.

$$\begin{aligned}
 \frac{1}{2} \cdot M \left(\frac{R}{2} \right)^2 \cdot \omega - M (\omega' R) \cdot R - \frac{MR^2}{2} \cdot \omega' &= 0 \\
 \Rightarrow \frac{3\omega'}{8} &= \frac{\omega}{2} \\
 \Rightarrow \omega' &= \frac{\omega}{12} \quad \text{Hence, } n = 12
 \end{aligned}$$

27. A point source S emits unpolarized light uniformly in all directions. At two points A and B, the ratio $r = I_A / I_B$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of r will be _____.

Ans. (8)

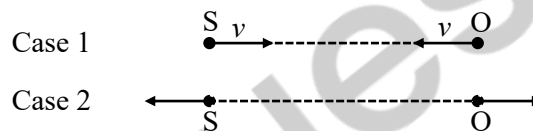
Sol. New intensity at B

$$I'_B = \left(\frac{I_B}{2} \right) \cos^2 45^\circ = \frac{I_B}{4}$$

$$\text{New value of } \alpha = \frac{I_A}{\frac{I_B}{4}} = \frac{4I_A}{I_B}$$

$$= 4 \times 2; \alpha = 8$$

28. A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is _____.



Ans. (200)

Sol. Frequency received by observer

$$f_0 = \left(\frac{C \pm V_0}{C \pm V_s} \right) f, \text{ C is speed of sound}$$

Case-1:

$$f_1 = \left(\frac{C + V}{C - V} \right) f$$

$$288 = \left(\frac{C + V}{C - V} \right) 240$$

Case-2:

$$f_2 = \left(\frac{C - V}{C + V} \right) f$$

$$n = \left(\frac{C - V}{C + V} \right) 240$$

multiply the two equations, we get.

$$(288)(n) = (240)(240)$$

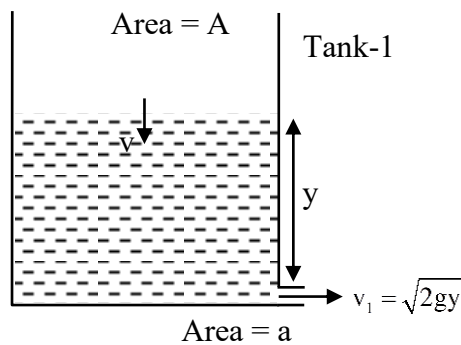
$$N = 200$$

29. Two large, identical water tanks, 1 and 2, kept on the top of a building of height H , are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2 , respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio

t_1/t_2 is _____.

Ans. (3)

Sol.

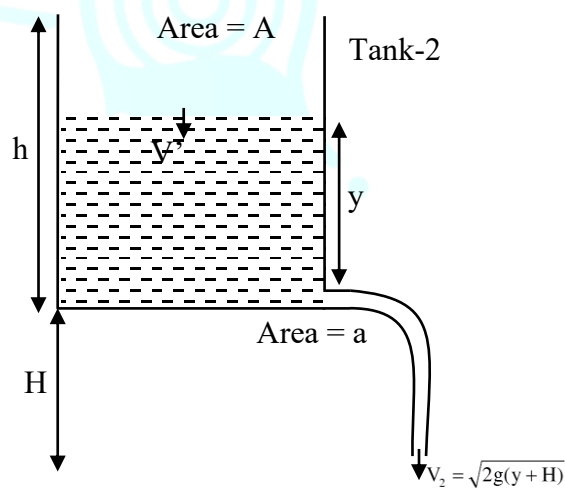


$$Av = av_1$$

$$A\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}; \quad dt = \frac{A}{a\sqrt{2g}} \cdot \frac{-dy}{\sqrt{y}}$$

$$\int_0^{t_1} dt = \frac{A}{a\sqrt{2g}} \int_h^0 \frac{dy}{\sqrt{y}}$$

$$t_1 = \frac{A}{a\sqrt{2g}} 2\sqrt{h}; \quad t_1 = \frac{A}{a} \sqrt{\frac{2h}{g}}$$



$$Av' = av_2$$

$$A \left(-\frac{dy}{dt} \right) = a \sqrt{2g(H+y)}$$

$$dt = -\frac{A}{a\sqrt{2g}} \frac{dy}{\sqrt{H+y}}$$

$$\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}} \int_H^{H+h} \frac{dy}{\sqrt{H+y}}$$

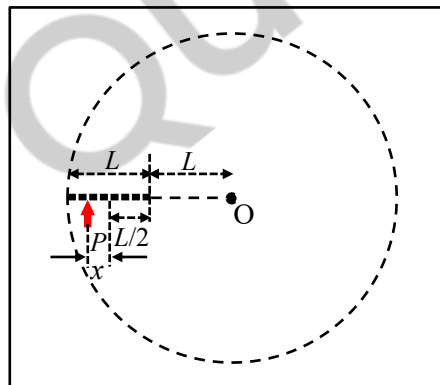
$$t_2 = \frac{A}{a\sqrt{2g}} (2)(\sqrt{H+h} - \sqrt{H}) \quad \& \quad H = \frac{16h}{9}$$

$$= \frac{A}{a} \sqrt{\frac{2h}{g}} \left(\frac{5}{3} - \frac{4}{3} \right)$$

$$t_2 = \frac{A}{a} \sqrt{\frac{2h}{g}} \left(\frac{1}{3} \right)$$

$$\text{ratio } \frac{t_1}{t_2} = 3$$

30. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance $x = L/n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is _____.



Ans. (18)

Sol. Linear impulse $\int F dt = \Delta \text{ momentum}$

$$= m (V_{\text{cm}} - 0)$$

$$P = m (\omega r_{\text{cm}})$$

$$= m \omega \left(L + \frac{L}{2} \right)$$

$$P = m \left(\frac{3L}{2} \right) \quad \dots(i)$$

Angular impulse $\int \tau dt = \Delta \text{ angular momentum}$

$$\int r \times F dt = \Delta L$$

$r \times \int F dt = I(\omega - 0)$, and I is moment of inertia about axis of rotation.

$$\left(L + \frac{L}{2} + x \right) \times P = (I_{cm} + md^2)\omega$$

$$= \left(\frac{mL^2}{12} + m \left(L + \frac{L}{2} \right)^2 \right) \omega$$

$$\left(\frac{3L}{2} + x \right) P = mL^2 \left(\frac{1}{12} + \left(\frac{3}{2} \right)^2 \right) \omega$$

$$\left(\frac{3L}{2} + x \right) P = mL^2 \left(\frac{7}{4} \right) \omega \quad \dots(ii)$$

Divide eq.-(i) & (ii)

$$\left(\frac{3L}{2} + x \right) = \frac{L \left(\frac{7}{4} \right)}{\left(\frac{3}{2} \right)}$$

$$\frac{3L}{2} + x = L \left(\frac{14}{8} \right)$$

$$x = \frac{L}{18}$$

SECTION-4 : (Maximum Marks : 12)

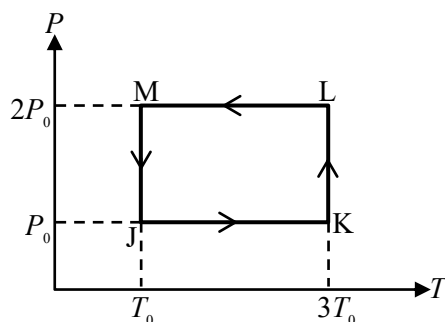
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

31. One mole of a monatomic ideal gas undergoes the cyclic process $J \rightarrow K \rightarrow L \rightarrow M \rightarrow J$, as shown in the P-T diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.
[R is the gas constant.]

List-I

- (P) Work done in the complete cyclic process
(Q) Change in the internal energy of the gas in the process JK
(R) Heat given to the gas in the process KL
(S) Change in the internal energy of the gas in the process MJ

List-II

- (1) $RT_0 - 4RT_0 \ln 2$
(2) 0
(3) $3RT_0$
(4) $-2RT_0 \ln 2$
(5) $-3RT_0 \ln 2$

(A) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 5; S \rightarrow 4$

(B) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 5; S \rightarrow 2$

(C) $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 2$

(D) $P \rightarrow 2; Q \rightarrow 5; R \rightarrow 3; S \rightarrow 4$

Ans. (B)

Sol. J (P_0, V_0, T_0)

K ($P_0, 3V_0, 3T_0$)

M ($2P_0, \frac{V_0}{2}, T_0$)

L ($2P_0, \frac{3V_0}{2}, 3T_0$)

$$P_0 V_0 = nRT_0$$

$$JK \rightarrow \text{isobaric} \Rightarrow W = P_0 (2V_0) = 2nRT_0$$

$$\Delta U = \frac{3}{2} nR(2T_0) = 3nRT$$

$$KL \rightarrow \text{isothermal} \rightarrow W = nR(3T) \ln\left(\frac{1}{2}\right) = -3nRT_0 \ln 2$$

$$\Delta U = 0 \Rightarrow Q = -3nRT_0 \ln 2$$

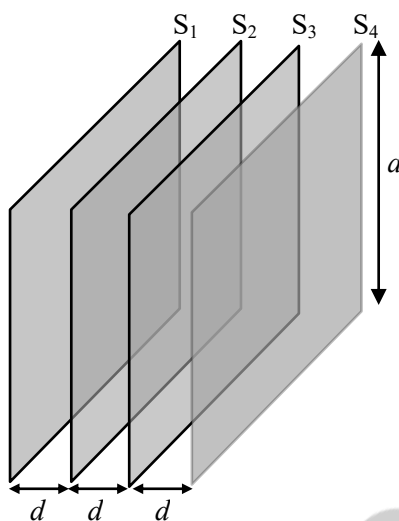
$$LM \rightarrow \text{isobaric} = 2P_0 (-V_0) = -2nRT_0$$

$$MJ \rightarrow \text{isothermal} \Rightarrow nRT_0 \ln 2; \Delta U = 0$$

$$W_{\text{net}} = -2nRT_0 \ln 2$$

$P \rightarrow 4, Q \rightarrow 3, R \rightarrow 5, S \rightarrow 2$

32. Four identical thin, square metal sheets, S_1 , S_2 , S_3 and S_4 , each of side a are kept parallel to each other with equal distance d ($\ll a$) between them, as shown in the figure. Let $C_0 = \epsilon_0 a^2/d$, where ϵ_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I

List-II

- | | |
|---|--------------|
| (P) The capacitance between S_1 and S_4 , with S_2 and S_3 not connected, is | (1) $3C_0$ |
| (Q) The capacitance between S_1 and S_4 , with S_2 shorted to S_3 , is | (2) $C_0/2$ |
| (R) The capacitance between S_1 and S_3 , with S_2 shorted to S_4 , is | (3) $C_0/3$ |
| (S) The capacitance between S_1 and S_2 , with S_3 shorted to S_1 , and S_2 shorted to S_4 , is | (4) $2C_0/3$ |

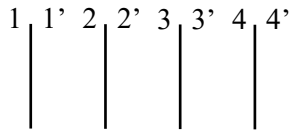
(5) $2C_0$

- (A) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 5$
 (B) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 1$
 (C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 1$
 (D) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 2; S \rightarrow 5$

Ans. (C)



Sol.



(P) $\Rightarrow C = \frac{\epsilon_0 q^2}{3d} \frac{1}{3}$

(Q) $\Rightarrow \left(\frac{C_0}{2} \right)$

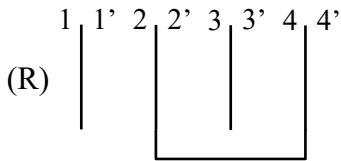
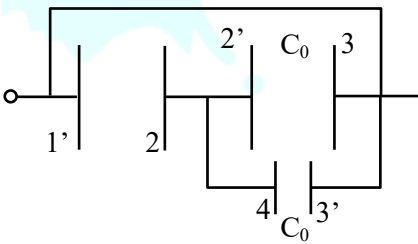
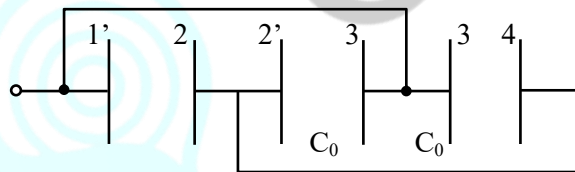
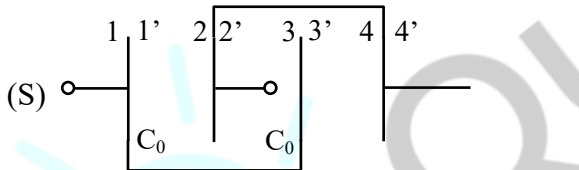


Diagram showing four vertical lines labeled 1, 1', 2, 2', 3, 3', 4, 4' at the top, with a horizontal line connecting 2 and 3, and a horizontal line connecting 4 and 4'. The diagram is labeled $\Rightarrow \frac{(2C_0)(C_0)}{3C_0} \frac{2C_0}{3}$.



$\Rightarrow 3C_0$

33. A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0 . The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α . Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I

List-II

- (P) If $n = 2$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be
- (Q) If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of θ_0 will be
- (R) If $n = \sqrt{3}$ and $\alpha = 180^\circ$, then all the possible values of ϕ_0 will be
- (S) If $n = \sqrt{2}$ and $\theta_0 = 45^\circ$, then all the possible values of α will be

(1) 30° and 0°

(2) 60° and 0°

(3) 45° and 0°

(4) 150°

(5) 0°

(A) $P \rightarrow 5; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

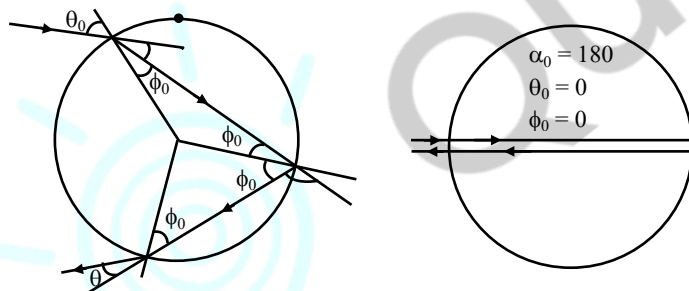
(B) $P \rightarrow 5; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 4$

(C) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$

(D) $P \rightarrow 3; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 5$

Ans. (A)

Sol.



$$\alpha = (\theta_0 - \phi_0) + (180 - 2\phi_0) + (\theta_0 - 2\phi_0)$$

$$\alpha = 180 + 2\theta_0 - 4\phi_0$$

$$(P) \alpha = 180 + 2\theta_0 - 4\phi_0$$

$$180 = 180 + 2\theta_0 - 4\phi_0 \Rightarrow \theta_0 = 2\phi_0 \quad \dots(i)$$

$$\sin\theta_0 = 2\sin\phi_0 \quad \dots(ii)$$

From (i) & (ii)

$$\sin\theta_0 = 2\sin(\theta_0/2) \Rightarrow \cos\left(\frac{\theta_0}{2}\right) = 1$$

$$\frac{\theta_0}{2} = 0$$

$$\Rightarrow \theta_0 = 0$$

$$(Q) \theta_0 = 2\phi_0 \quad \dots(i)$$

$$\sin\theta_0 = \sqrt{3} \sin\phi_0 \quad \dots(ii)$$

From (i) & (ii)

$$\sin\theta_0 = \sqrt{3} \sin\left(\frac{\theta_0}{2}\right)$$

$$\Rightarrow \cos\left(\frac{\theta_0}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\theta_0}{2} = 30, 150$$

$$\theta_0 = 60, 300 \text{ (Rejected)}$$

$$\theta_0 = 60 \text{ \& } 0$$

$$(R) \theta_0 = 2\phi_0$$

$$\sin\theta_0 = \sqrt{3} \sin\phi$$

$$\sin 2\theta_0 = \sqrt{3} \sin\phi$$

$$\cos\phi_0 = \frac{\sqrt{3}}{2}$$

$$\phi_0 = 30, 150 \text{ (Rejected)}$$

$$\phi_0 = 30 \text{ \& } 0 \quad \dots(i)$$

$$(S) \sin 45 = \sqrt{2} \cos\phi_0$$

$$\cos\phi_0 = 1/2$$

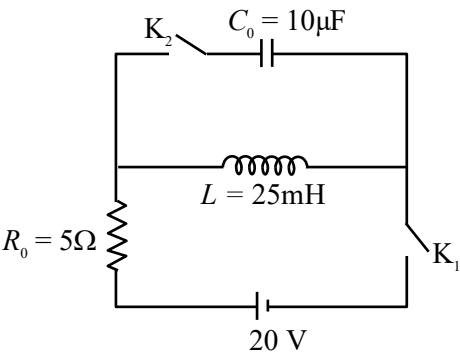
$$\phi_0 = 60$$

$$\alpha = 180 + 2\theta_0 - 4\phi_0$$

$$\alpha = 180 + 90 - 120 \quad \dots(iv)$$

$$= 180 - 30; \alpha = 150^\circ$$

34. The circuit shown in the figure contains an inductor L , a capacitor C_0 , a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2 . Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1 . After a long time, the current attains a steady state value I_2 . Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0 .

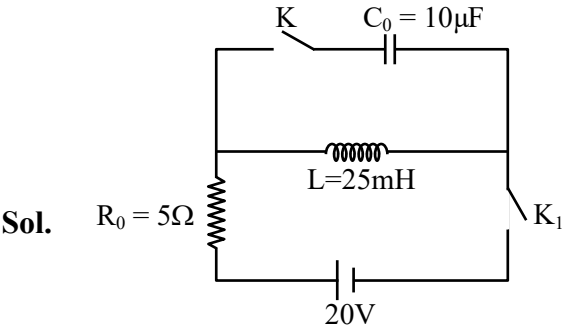


Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

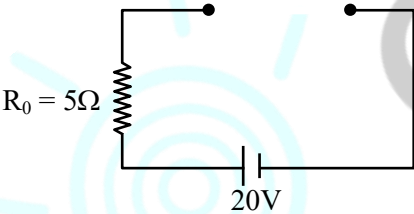
- List-I**
 (P) The value of I_1 in Ampere is
 (Q) The value of I_2 in Ampere is
 (R) The value of ω_0 in kilo-radians/s is
 (S) The value of V_0 in Volt is
- List-II**
 (1) 0
 (2) 2
 (3) 4
 (4) 20
 (5) 200

- (A) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 5$
 (B) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 5$
 (C) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2; S \rightarrow 4$
 (D) $P \rightarrow 2; Q \rightarrow 5; R \rightarrow 3; S \rightarrow 4$

Ans. (A)

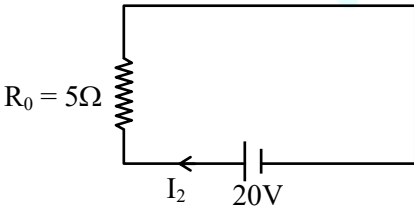


- (P) When K_1 is closed current in R_0 is I_1
 At $t = 0$; circuit will be



- $I_1 = 0$
 $P \rightarrow (1)$

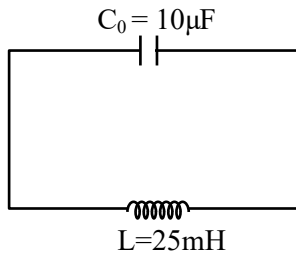
- (Q) After long time inductor behave as a wire so I_2



- $I_2 = \frac{20}{5} = 4\text{A}$
 $Q \rightarrow (3)$



(R) When K_2 is closed and K_1 open



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

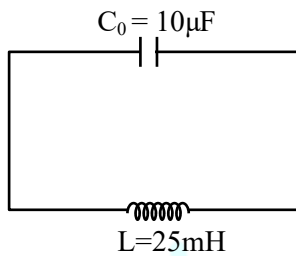
$$\omega_0 = \frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}} = 5 \times 10^4$$

$$\omega_0 = 2 \times 10^3 \text{ rad/s}$$

$$\omega_0 = 2 \text{ kilo-radian/s}$$

R $\rightarrow$ (2)

(S) Now K_2 is closed and K_1 open



$$\frac{1}{2} LI_2^2 = \frac{1}{2} CV$$

$$25 \times 10^{-3} \times (4)^2 = 10 \times 10^{-6} \times V_0^2$$

$$V_0^2 = 2500 \times 16$$

$$V_0 = 50 \times 4 = 200 \text{ V}$$

S $\rightarrow$ (5)

JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

CHEMISTRY

TEST PAPER WITH ANSWER AND SOLUTION

PAPER-1

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

- 35.** A closed vessel contains 10 g of an ideal gas X at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas Y is added to it and the pressure becomes 6 atm. The ratio of root mean square velocities of X and Y at 300 K is

- (A) $2\sqrt{2} : \sqrt{3}$ (B) $2\sqrt{2} : 1$ (C) 1 : 2 (D) 2 : 1

Ans. (D)

Sol. For Ideal Gas

$$PV = nRT$$

$$\therefore n \propto P \text{ at constant } T \text{ \& } V.$$

$$\therefore \text{mole} = \frac{\text{Mass}}{\text{Molar mass}}$$

$$\text{For gas X : } \frac{10}{M_X} \propto 2 \text{ atm} \quad \dots\dots\dots (1)$$

$$\text{For gas X \& Y : } \frac{10}{M_X} + \frac{80}{M_Y} \propto 6 \text{ atm} \quad \dots\dots\dots (2)$$

From (2) – (1)

$$\frac{80}{M_Y} \propto 4 \quad \dots\dots\dots (3)$$

On dividing (1) by (3)

$$\frac{M_Y}{8M_X} = \frac{1}{2}$$

$$\therefore \frac{M_Y}{M_X} = 4 \quad \dots\dots\dots (4)$$

$$\therefore v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$$

$$\therefore \frac{(v_{\text{rms}})}{(v_{\text{rms}})_Y} = \sqrt{\frac{M_Y}{M_X}} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

36. At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid (HNO_2) gives the species

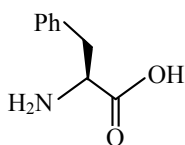
- (A) H_3O^+ , NO_3^- and NO (B) H_3O^+ , NO_3^- and NO_2
(C) H_3O^+ , NO^- and NO_2 (D) H_3O^+ , NO_3^- and N_2O

Ans. (A)

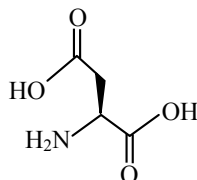
Sol. $3\text{HNO}_2(\text{aq}) \rightleftharpoons \text{H}_3\text{O}^+ + \text{NO}_3^- + 2\text{NO}$

37. Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is

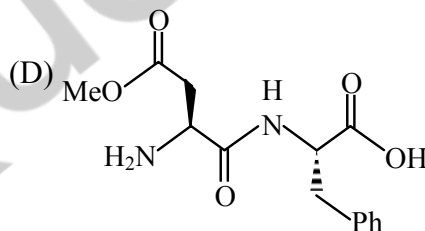
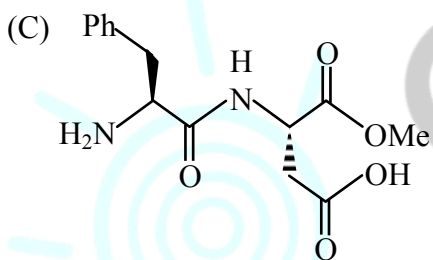
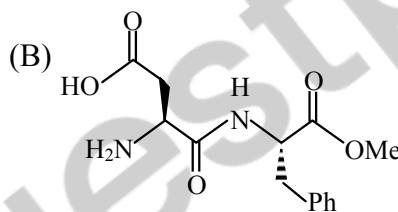
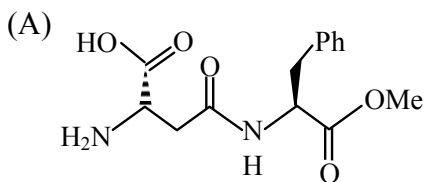
Structures of phenylalanine and aspartic acid are given below.



Phenylalanine

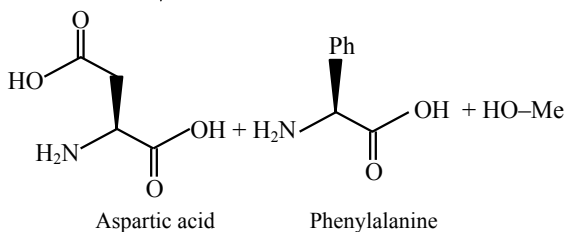
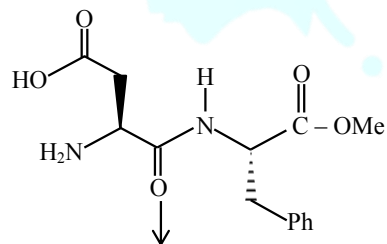


Aspartic acid



Ans. (B)

Sol. Aspartame structure is a dipeptide consisting aspartic acid and methyl ester of phenylalanine



38. Among the following options, select the option in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.

[en = H₂NCH₂CH₂NH₂]

(A) **Set-I** : [Ni(CO)₄] and [PdCl₂(PPh₃)₂]

Set-II : [Co(NH₃)₅Cl] SO₄ and [Co(NH₃)₅(SO₄)]Cl

(B) **Set-I** : [Co(en)(NH₃)₂Cl₂] and [PdCl₂(PPh₃)₂]

Set-II : [Co(NH₃)₆] [Cr(CN)₆] and [Cr(NH₃)₆] [Co(CN)₆]

(C) **Set-I** : [Co(NH₃)₃(NO₂)₃] and [Co(en)₂Cl₂]

Set-II : [Co(NH₃)₅Cl]SO₄ and [Co(NH₃)₅(SO₄)]Cl

(D) **Set-I** : [Cr(NH₃)₅Cl]Cl₂ and [Co(en)(NH₃)₂Cl₂]

Set-II : [Cr(H₂O)₆]Cl₃ and [Cr(H₂O)₅Cl]Cl₂ · H₂O

Ans. (C)

Sol. Set-I : [Co(NH₃)₃(NO₂)₃] shows two geometrical isomers :- facial and meridional

[Co(en)₂Cl₂] shows two geometrical isomers :- cis and trans

Set-II : [Co(NH₃)₅Cl]SO₄ and [Co(NH₃)₅SO₄]Cl are ionization isomers of each other.

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
choosing **ONLY** (A), (B) and (D) will get +4 marks;
choosing **ONLY** (A) and (B) will get +2 marks;
choosing **ONLY** (A) and (D) will get +2 marks;
choosing **ONLY** (B) and (D) will get +2 marks;
choosing **ONLY** (A) will get +1 marks;
choosing **ONLY** (B) will get +1 marks;
choosing **ONLY** (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -2 marks.

39. Among the following, the correct statement(s) for electrons in an atom is(are)
- (A) Uncertainty principle rules out the existence of definite paths for electrons.
- (B) The energy of an electron in $2s$ orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.
- (C) According to Bohr's model, the most negative energy value for an electron is given by $n = 1$, which corresponds to the most stable orbit.
- (D) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n .

Ans. (A,B,C)

Sol. (A) Uncertainty principle talks about probability of finding electrons in different regions around the nucleus rather than definite paths.

(B) With increase in distance of electron from the nucleus, its energy increases.

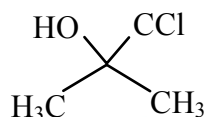
(C) Energy of electron $E_n = -13.6 \times \frac{Z^2}{n^2}$ eV/atom.

(D) Velocity of electron $V_n = 2.19 \times 10^6 \times \frac{Z}{n}$ m/sec.

40. Reaction of *iso*-propylbenzene with O_2 followed by the treatment with H_3O^+ forms phenol and a by-product **P**. Reaction of **P** with 3 equivalents of Cl_2 gives compound **Q**. Treatment of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.

The correct statement(s) regarding **P**, **Q**, **R** and **S** is(are)

(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives

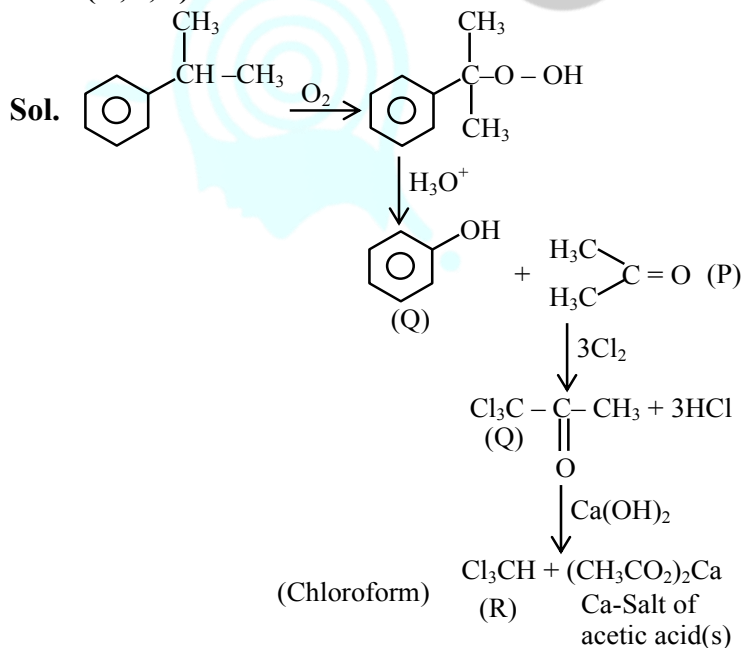


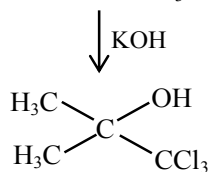
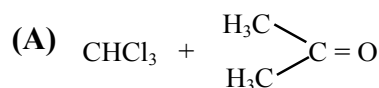
(B) Reaction of **R** with O_2 in the presence of light gives phosgene gas

(C) **Q** reacts with aqueous NaOH to produce Cl_3CCH_2OH and $Cl_3CCOONa$

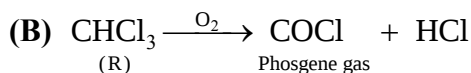
(D) **S** on heating gives **P**

Ans. (A,B,D)

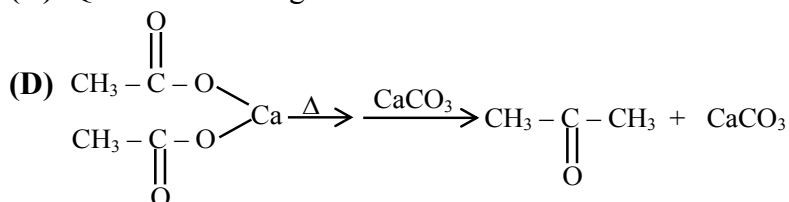




(Chloritone)



(C) Q does not undergo Cannizzaro reaction



41. The option(s) in which at least three molecules follow Octet Rule is(are)

- (A) CO_2 , C_2H_4 , NO and HCl
 (B) NO_2 , O_3 , HCl and H_2SO_4
 (C) BCl_3 , NO , NO_2 and H_2SO_4
 (D) CO_2 , BCl_3 , O_3 and C_2H_4

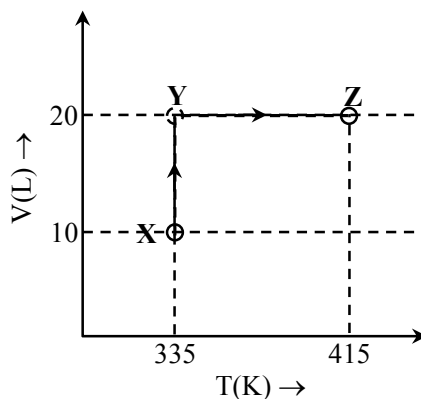
Ans. (A,D)

Sol. NO , NO_2 , BCl_3 and H_2SO_4 do not follow octet rule.

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 **ONLY** If the correct integer is entered;
Zero Marks : 0 In all other cases.

42. Consider the following volume-temperature ($V - T$) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence $X \rightarrow Y \rightarrow Z$ is _____.

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m} = 12 \text{ J K}^{-1} \text{ mol}^{-1}$ and gas constant, $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$]

Ans. (8120)

Sol. For ideal gas

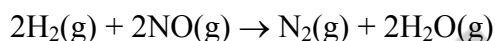
$$\Delta H = nC_p\Delta T$$

$$\therefore C_p = C_v + R = 12 + 8.3 = 20.3 \text{ J/K-mole}$$

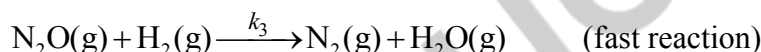
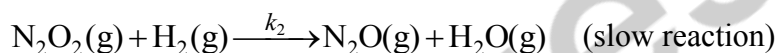
$$\therefore \Delta H = 5 \times 20.3 \times (415 - 335)$$

$$\boxed{\Delta H = 8120 \text{ Joule}}$$

43. Consider the following reaction,



which follows the mechanism given below:



The order of the reaction is _____.

Ans. (3)

Sol. Rate law = $k_2 [\text{N}_2\text{O}_2] [\text{H}_2]$ [$\because$ slowest step of reaction is RDS]

$$\therefore \frac{k_1}{k_{-1}} = \frac{[\text{N}_2\text{O}_2]}{[\text{NO}]^2}$$

$$\therefore [\text{N}_2\text{O}_2] = \frac{k_1}{k_{-1}} [\text{NO}]^2$$

$$\therefore \text{Rate} = k_2 \times \frac{k_1}{k_{-1}} [\text{NO}]^2 [\text{H}_2]$$

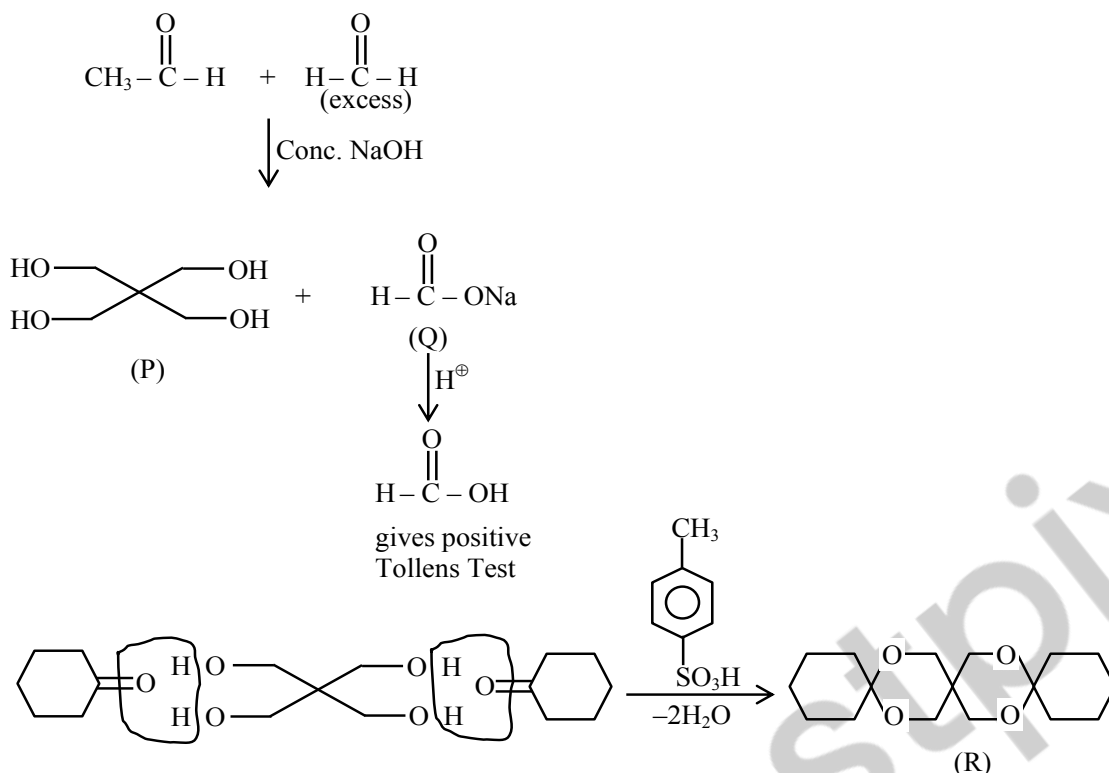
$\therefore$ Order of reaction is (3)

44. Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollens' test, whereas **Q** on acidification gives positive Tollens' test. Treatment of **P** with excess cyclohexanone in the presence of catalytic amount of *p*-toluenesulfonic acid (PTSA) gives product **R**.

Sum of the number of methylene groups ($-\text{CH}_2-$) and oxygen atoms in **R** is _____.

Ans. (18)

Sol.



Total CH_2 in R = 14

and oxygen in R = 4

So $14 + 4 = 18$

45. Among $\text{V}(\text{CO})_6$, $\text{Cr}(\text{CO})_5$, $\text{Cu}(\text{CO})_3$, $\text{Mn}(\text{CO})_5$, $\text{Fe}(\text{CO})_5$, $[\text{Co}(\text{CO})_3]^{3-}$, $[\text{Cr}(\text{CO})_4]^{4-}$, and $\text{Ir}(\text{CO})_3$, the total number of species isoelectronic with $\text{Ni}(\text{CO})_4$ is _____.

[Given, atomic number : V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

Ans. (3)

Sol. In case of complexes, isoelectronic species should be those having same effective atomic number (EAN)

$$\text{Ni}(\text{CO})_4 \Rightarrow 28 + 4 \times 2 = 36$$

$$\text{(i) } \text{V}(\text{CO})_6 \Rightarrow 23 + 2 \times 6 = 35$$

$$\text{(ii) } \text{Cr}(\text{CO})_5 \Rightarrow 24 + 2 \times 5 = 34$$

$$\text{(iii) } \text{Cu}(\text{CO})_3 \Rightarrow 29 + 2 \times 3 = 35$$

$$\text{(iv) } \text{Mn}(\text{CO})_5 \Rightarrow 25 + 2 \times 5 = 35$$

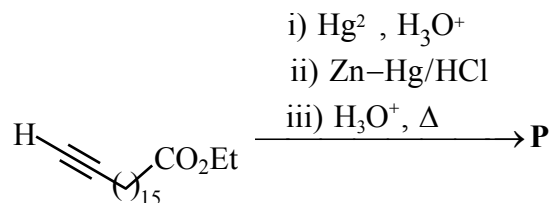
$$\text{(v) } \text{Fe}(\text{CO})_5 \Rightarrow 26 + 2 \times 5 = 36$$

$$\text{(vi) } [\text{Co}(\text{CO})_3]^{3-} \Rightarrow 27 + 3 + 2 \times 3 = 36$$

$$\text{(vii) } [\text{Cr}(\text{CO})_4]^{4-} \Rightarrow 24 + 4 + 2 \times 4 = 36$$

$$\text{(viii) } [\text{Ir}(\text{CO})_3] \Rightarrow 77 + 2 \times 3 = 83$$

46. In the following reaction sequence, the major product **P** is formed.

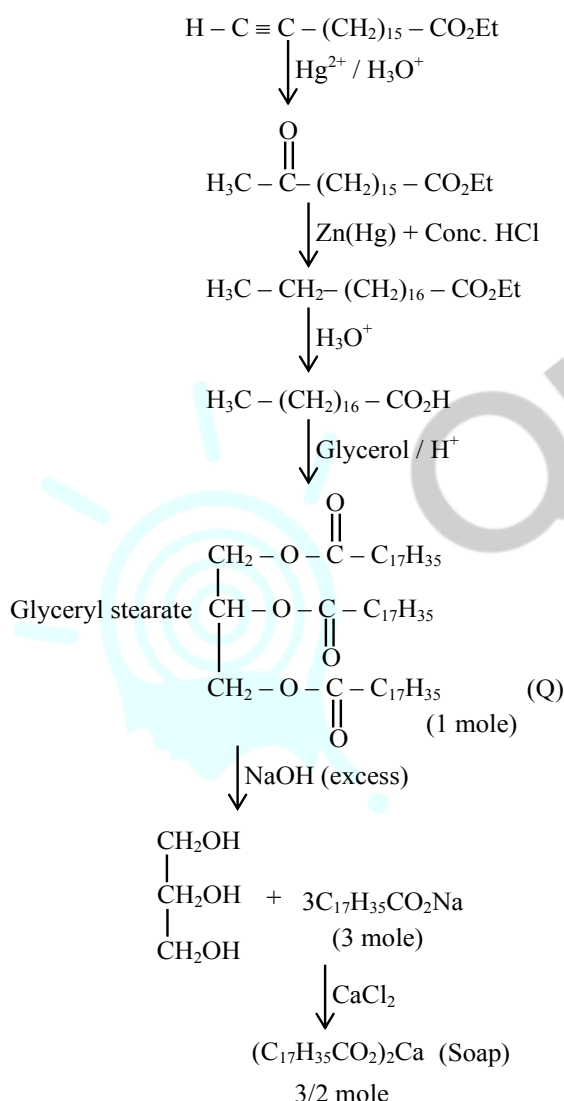


Glycerol reacts completely with excess **P** in the presence of an acid catalyst to form **Q**. Reaction of **Q** with excess NaOH followed by the treatment with CaCl_2 yields Ca-soap **R**, quantitatively. Starting with one mole of **Q**, the amount of **R** produced in gram is _____.

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Ans. (909)

Sol.



$$\frac{3}{2} \text{ mole soap} = \frac{3}{2} \times 606 \text{ gm} = 909 \text{ gm}$$

47. Among the following complexes, the total number of diamagnetic species is _____.
 $[\text{Mn}(\text{NH}_3)_6]^{3+}$, $[\text{MnCl}_6]^{3-}$, $[\text{FeF}_6]^{3-}$, $[\text{CoF}_6]^{3-}$, $[\text{Fe}(\text{NH}_3)_6]^{3+}$, and $[\text{Co}(\text{en})_3]^{3+}$
 [Given, atomic number : Mn = 25, Fe = 26, Co = 27;
 en = $\text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2$]

Ans. (1)

Sol. $\text{Mn}^{3+} \Rightarrow [\text{Ar}]3d^4$

d^4 configuration in t_{2g} and e_g orbitals will always have unpaired electrons irrespective of SFL and WFL.

$\text{Fe}^{3+} \Rightarrow [\text{Ar}]3d^5$

d^5 configuration will also have unpaired electron irrespective of SFL and WFL.

$\text{Co}^{3+} \Rightarrow [\text{Ar}]3d^6$

$d^6 \Rightarrow$ it can be both paramagnetic or diamagnetic based on field of ligands.

In case of $\text{F}^- \Rightarrow$ weak field ligand, configuration will be $t_{2g}^4 e_g^2$ hence it is paramagnetic but in case of

en $\Rightarrow$ strong field ligand, configuration will be $t_{2g}^6 e_g^0$ hence it will be diamagnetic.

SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

48. In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.

The limiting ionic conductivity (Λ_0) values (in $\text{mS m}^2 \text{mol}^{-1}$) for different ions in aqueous solutions are given below :

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
Λ_0	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.

Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

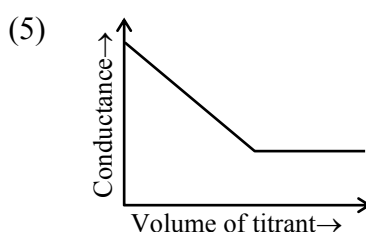
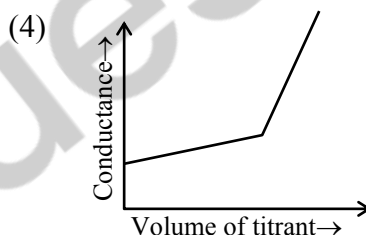
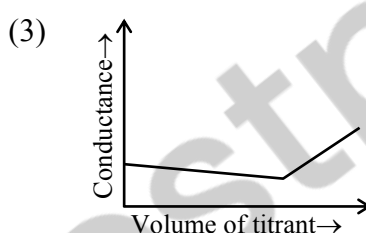
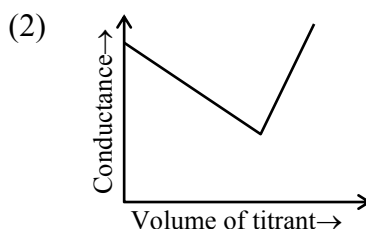
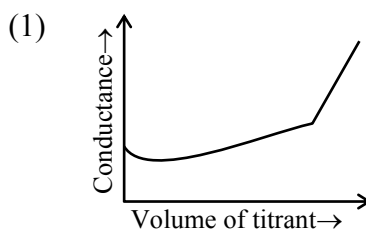
List-I

(P) Titrate : KCl
Titrant : AgNO_3

(Q) Titrate : AgNO_3
Titrant : KCl

(R) Titrate : NaOH
Titrant : HCl

(S) Titrate : NaOH
Titrant : CH_3COOH

List-II


- (A) $P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 5$
 (B) $P \rightarrow 2, Q \rightarrow 4, R \rightarrow 3, S \rightarrow 1$
 (C) $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 5$
 (D) $P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$

Ans. (C)

Sol. Option (P) :

On adding AgNO_3 solution to KCl solution precipitation of AgCl will occur due to which Cl^- already present will be replaced by NO_3^- ions. So conductance of solution will decrease till equivalence point. After complete precipitation of AgCl, further added AgNO_3 will increase the number of ions in resulting solution so conductance will increase.

Option (Q) :

On adding KCl solution to AgNO_3 solution precipitation of AgCl will occur due to which already present Ag^+ ions will be replaced by K^+ ions in solution. So conductance of solution will increase. After complete precipitation of AgCl further added KCl will increase the number of ions in resulting solution so conductance will increase further.

Option (R) :

On adding HCl solution to NaOH solution, OH^- will be replaced by Cl^- ions so conductance of solution decreases. After complete neutralisation further added HCl will increase number of ions in the solution. So conductance will increase further.

Option (S) :

On adding CH_3COOH solution to NaOH solution OH^- will be replaced by CH_3COO^- ions, so conductance of solution decreases. After complete neutralisation further added CH_3COOH will remain undissociated because it is a weak acid and there is also common ion effect on acetate ions. So number of ions in solution will remain almost constant therefore conductance of solution will remain constant.

49. Based on **VSEPR** model, match the xenon compounds given in **List-I** with the corresponding geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

List-I

- (P) XeF_2
(Q) XeF_4
(R) XeO_3
(S) XeO_3F_2

List-II

- (1) Trigonal bipyramidal and two lone pair of electrons
(2) Tetrahedral and one lone pair of electrons
(3) Octahedral and two lone pair of electrons
(4) Trigonal bipyramidal and no lone pair of electrons
(5) Trigonal bipyramidal and three lone pair of electrons

- (A) $\text{P} \rightarrow 5, \text{Q} \rightarrow 2, \text{R} \rightarrow 3, \text{S} \rightarrow 1$
(B) $\text{P} \rightarrow 5, \text{Q} \rightarrow 3, \text{R} \rightarrow 2, \text{S} \rightarrow 4$
(C) $\text{P} \rightarrow 4, \text{Q} \rightarrow 3, \text{R} \rightarrow 2, \text{S} \rightarrow 1$
(D) $\text{P} \rightarrow 4, \text{Q} \rightarrow 2, \text{R} \rightarrow 5, \text{S} \rightarrow 3$

Ans. (B)

Sol. $\text{XeF}_2 \Rightarrow$ 2 sigma bonds and 3 lone pairs on Xe, number of hybrid orbitals = 5, sp^3d hybridisation, geometry will be trigonal bipyramidal.

P-5

$\text{XeF}_4 \Rightarrow$ 4 sigma bonds and 2 lone pairs on Xe, number of hybrid orbitals = 6, sp^3d^2 hybridisation, geometry will be octahedral.

Q-3

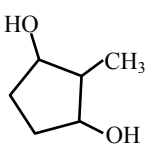
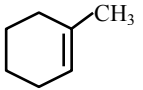
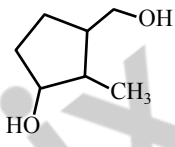
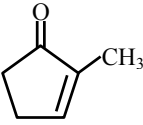
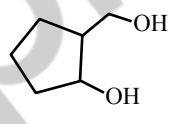
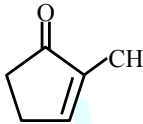
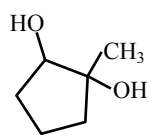
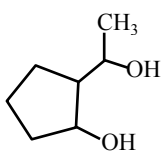
$\text{XeO}_3 \Rightarrow$ 3 sigma bonds and 1 lone pairs on Xe, number of hybrid orbitals = 4, sp^3 hybridisation, geometry will be tetrahedral.

R-2

$\text{XeO}_3\text{F}_2 \Rightarrow$ 5 sigma bonds and 0 lone pairs on Xe, number of hybrid orbitals = 5, sp^3d hybridisation, geometry will be trigonal bipyramidal.

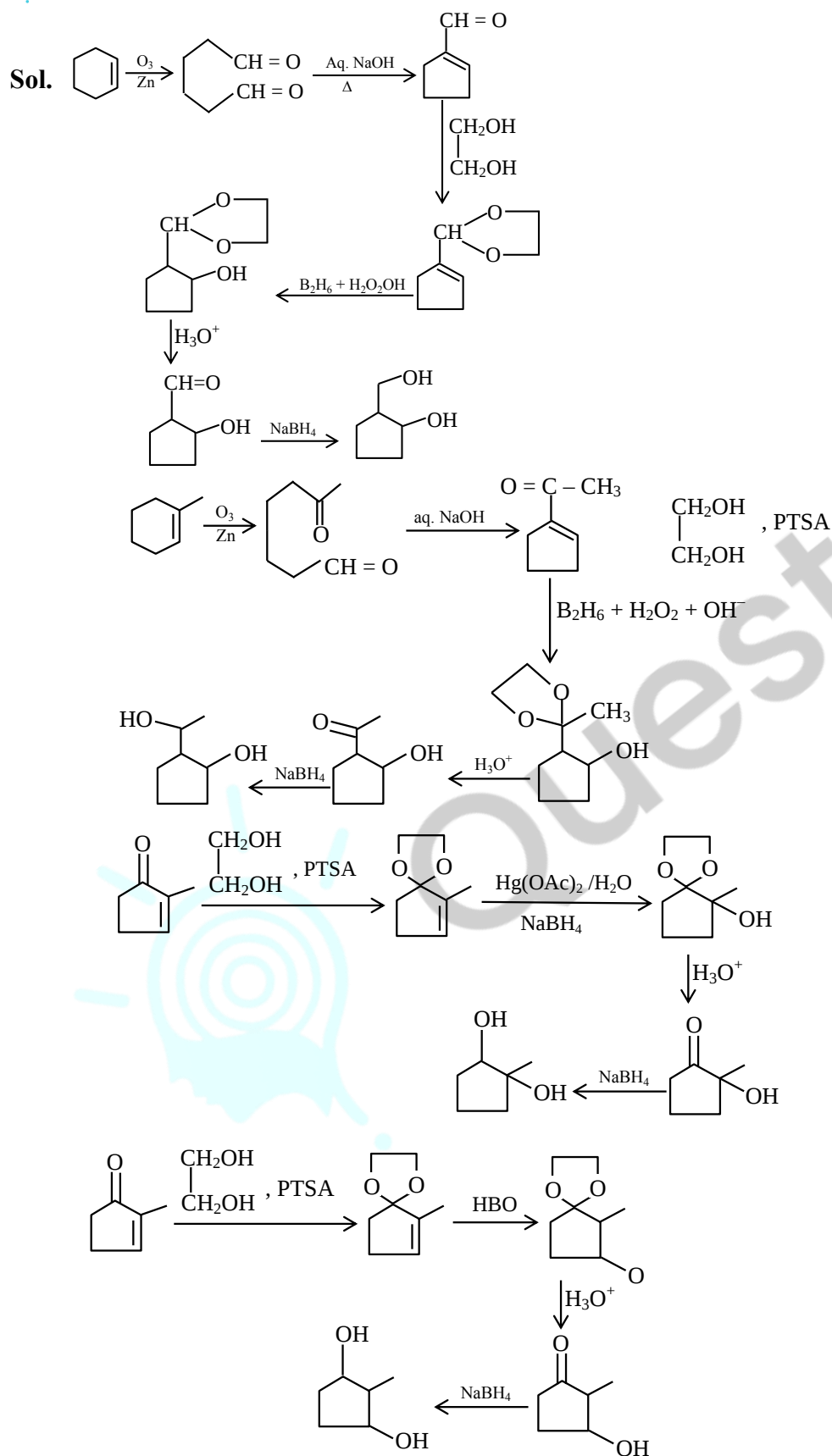
S-4

50. **List-I** contains various reaction sequences and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

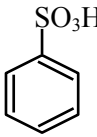
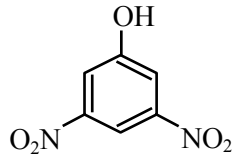
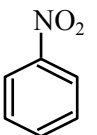
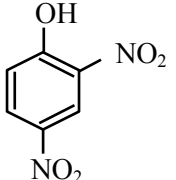
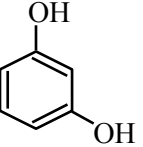
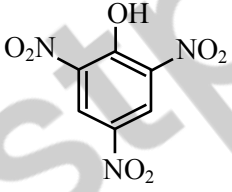
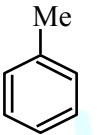
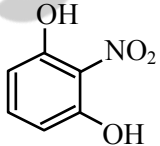
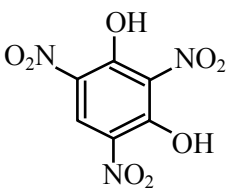
List-I		List-II	
(P) 	i) O_3, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH_3, b) H_2O_2, NaOH v) H_3O^+ vi) $NaBH_4$	(1) 	
(Q) 	i) O_3, Zn ii) aq. NaOH, Δ iii) ethylene glycol, PTSA iv) a) BH_3, b) H_2O_2, NaOH v) H_3O^+ vi) $NaBH_4$	(2) 	
(R) 	i) ethylene glycol, PTSA ii) a) $Hg(OAc)_2$, H_2O, b) $NaBH_4$ iii) H_3O^+ iv) $NaBH_4$	(3) 	
(S) 	i) ethylene glycol, PTSA ii) a) BH_3, b) H_2O_2, NaOH iii) H_3O^+ iv) $NaBH_4$	(4) 	
		(5) 	

- (A) P $\rightarrow$ 3, Q $\rightarrow$ 5, R $\rightarrow$ 4, S $\rightarrow$ 1
 (B) P $\rightarrow$ 3, Q $\rightarrow$ 2, R $\rightarrow$ 4, S $\rightarrow$ 1
 (C) P $\rightarrow$ 3, Q $\rightarrow$ 5, R $\rightarrow$ 1, S $\rightarrow$ 4
 (D) P $\rightarrow$ 5, Q $\rightarrow$ 2, R $\rightarrow$ 4, S $\rightarrow$ 1

Ans. (A)



51. **List-I** contains various reaction sequences and **List-II** contains different phenolic compounds. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

List-I		List-II	
(P)  $\xrightarrow[\text{ii) Conc. HNO}_3]{\text{i) molten NaOH, H}_3\text{O}^+}$	(1)		
(Q)  $\xrightarrow[\text{v) nc. HNO}_3/\text{Conc. H}_2\text{SO}_4]{\text{i) Conc. HNO}_3/\text{Conc. H}_2\text{SO}_4, \text{ii) Sn/HCl, iii) NaNO}_2/\text{HCl, 0-5 } ^\circ\text{C, iv) H}_2\text{O}}$	(2)		
(R)  $\xrightarrow[\text{iii) H}_3\text{O}^+, \Delta]{\text{i) Conc. H}_2\text{SO}_4, \text{ii) Conc. HNO}_3}$	(3)		
(S)  $\xrightarrow[\text{vi) H}_2\text{O}]{\text{i) a) KMnO}_4/\text{KOH, } \Delta; \text{ b) H}_3\text{O}^+, \text{ii) Conc. HNO}_3/\text{Conc. H}_2\text{SO}_4, \Delta, \text{iii) a) SOCl}_2, \text{ b) NH}_3, \text{iv) Br}_2, \text{ NaOH, v) NO}_2/\text{HCl, 0-5 } ^\circ\text{C}}$	(4)		
	(5)		

- (A) P-2, Q-3, R-4, S-5
 (B) P-2, Q-3, R-5, S-1
 (C) P-3, Q-5, R-4, S-1
 (D) P-3, Q-2, R-5, S-4

Ans. (C)

