**(HELD ON SATURDAY 04th APRIL 2026)** **TIME : 3:00 PM TO 6:00 PM**

| MATHEMATICS | TEST PAPER WITH SOLUTION |
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### SECTION-A

**1.** For the function $f : [1, \infty) \to [1, \infty)$ defined by $f(x) = (x-1)^4 + 1$, among the two statements :

(I) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x)\}$ contains exactly two elements, and

(II) The set $S = \{x \in [1, \infty) : f(x) = f^{-1}(x+1)\}$ is an empty set,

(1) only (I) is TRUE

(2) only (II) is TRUE

(3) both (I) and (II) are TRUE

(4) neither (I) nor (II) is TRUE

**Ans. (1)**

**Sol.** $f(x) = (x-1)^4 + 1$

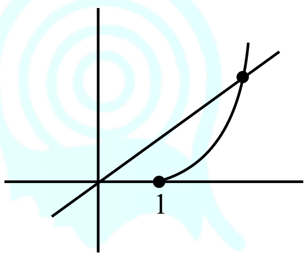
$f'(x) = 4(x-1)^3$ ; $f'(x) \geq 0$

$\therefore$ f(x) is increasing

$\therefore (x-1)^4 + 1 = x$

$(x-1)[(x-1)^3 - 1] = 0$

x = 1, x = 2 are two solutions




Now

$f^{-1}(x) = (x-1)^{1/4} + 1$

$f^{-1}(x+1) = x^{1/4} + 1$

$f^{-1}(x+1) = f(x)$

$x^{1/4} + 1 = (x-1)^4 + 1$

$x = (x-1)^{16}$

Above equation has one solution using graph.

**2.** Let $S = \{z \in \mathbb{C} : z^2 + 4z + 16 = 0\}$.

Then $\sum_{z \in S} \left|z + \sqrt{3}i\right|^2$ is equal to :

(1) 42

(2) 23

(3) 27

(4) 38

**Ans. (4)**

**Sol.** $z^2 + 4z + 16 = 0$

$\Rightarrow (z+2)^2 = -12$

$\Rightarrow z = -2 \pm 2\sqrt{3}i$

$\Rightarrow S = \left\{-2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i\right\}$

$\therefore \sum_{z \in S} \left|z + \sqrt{3}i\right|^2$

$= \left|-2 + 2\sqrt{3}i + \sqrt{3}i\right|^2 + \left|-2 - 2\sqrt{3}i + \sqrt{3}i\right|^2$

$= \left|-2 + 3\sqrt{3}i\right|^2 + \left|-2 - \sqrt{3}i\right|^2$

$= 38$

**3.** If the system of equations :

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \lambda z = \mu$

has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 16

(2) 18

(3) 19

(4) 21

**Ans. (2)**

**Sol.** $D = \lambda - 5$

$D_1 = \lambda + \mu - 18$

$D_2 = 4\lambda - 2\mu + 6$

$D_3 = \mu - 13$

Here $x = \frac{D_1}{D}$; $y = \frac{D_2}{D}$; $z = \frac{D_3}{D}$

For infinitely many solutions

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow \lambda = 5$; $\mu = 13$

$\therefore \lambda + \mu = 18$

**4.** If $\alpha = 1$ and $\beta = 1 + i\sqrt{2}$, where $i = \sqrt{-1}$ are two roots of the equation $x^3 + ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, then $\int_{-1}^{1}\left(x^3 + ax^2 + bx + c\right)dx$ is equal to :

(1) –2 (2) –4

(3) –8 (4) –10

**Ans.** **(3)**

**Sol.** Roots are $1 \pm i\sqrt{2}$ & 1

$-a$ = sum of roots

$a = -3$

$-c = 1\left(1+\sqrt{2}i\right)\left(1-i\sqrt{2}\right)$

$c = -3$

$$I = \int_{-1}^{1}\left(x^3 - 3x^2 + bx - 3\right)dx$$

$$= 2\int_{0}^{1}\left(-3x^2 - 3\right)dx$$

$$= 2(-x^3 - 3x)_0^1 = -8$$

**5.** If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$, $\lambda \neq -2$, has two positive roots, then the number of possible integral values of $\lambda$ is :

(1) 1 (2) 2

(3) 3 (4) 4

**Ans.** **(2)**

**Sol.** $f(x) = (\lambda + 2)x^2 - 3\lambda x + 4\lambda$

**C-1** $af(0) > 0$

$(\lambda + 2)4\lambda > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

**C-2** $-\frac{b}{2a} > 0$

$\frac{3\lambda}{2(\lambda+2)} > 0$

$\Rightarrow \lambda < -2$ or $\lambda > 0$

**C-3** $D \geq 0$

$(-3\lambda)^2 - 4(\lambda + 2) \times 4\lambda \geq 0$

$\lambda(7\lambda + 32) \leq 0$

$\Rightarrow \lambda \in \left[\frac{-32}{7}, 0\right]$

Intersection of C-1, C-2 and C-3

$\Rightarrow \lambda \in \left[\frac{-32}{7}, -2\right)$

$\lambda \in \left[-4.57, -2\right)$

$\Rightarrow \lambda = -4, -3$

$\therefore$ Number of values of $\lambda = 2$

**6.** Let $A = \begin{bmatrix} 1 & 2 & 7 \\ 4 & -2 & 8 \\ 3 & 8 & -7 \end{bmatrix}$ and $\det(A - \alpha I) = 0$, where $\alpha$ is a real number. If the largest possible value of $\alpha$ is p, then the circle $(x-p)^2 + (y-2p)^2 = 320$, intersects the co-ordinate axes at

(1) 1 point

(2) 2 points

(3) 3 points

(4) 4 points

**Ans.** **(3)**

**Sol.** $\begin{vmatrix} 1-\alpha & 2 & 7 \\ 4 & -2-\alpha & 8 \\ 3 & 8 & -7-\alpha \end{vmatrix} = 0$

$\Rightarrow$ (1 – α) [(α + 2)(α + 7) – 64] – 2[–28 – 4α – 24] + 7[32 + 6 + 3α] = 0

$\Rightarrow \alpha^3 + 8\alpha^2 - 88\alpha - 320 = 0$

$(\alpha - 8)(\alpha^2 + 16\alpha + 40) = 0$

$\alpha = 8,\ \alpha = -8 \pm 2\sqrt{6}$

So $p = -8$

$\Rightarrow (x-8)^2 + (y-16)^2 = 320 = 8^2 + 16^2$

put $y = 0 \Rightarrow x = 16, 0$

put $x = 0 \Rightarrow y = 32, 0$

$\Rightarrow$ (16, 0), (0, 32), (0, 0)

$\Rightarrow$ 3 points of intersection

**7.** Let $\alpha = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\infty$ and $\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\infty$. Then the value of $(0.2)^{\log_{\sqrt{5}}(\alpha)} + (0.04)^{\log_5(\beta)}$ is equal to :

(1) 4 (2) 5

(3) 8 (4) 25

**Ans.** **(3)**

**Sol.** $\alpha = \frac{1/4}{1-\frac{1}{2}} = \frac{1}{2}$

$\beta = \frac{1/3}{1-\frac{1}{3}} = \frac{1}{2}$

$(0.2)^{\log_{\sqrt{5}}\alpha} = \alpha^{\log_{\sqrt{5}}\frac{1}{5}} = \left(\frac{1}{2}\right)^{-2} = 4$

$(0.04)^{\log_5 \beta} = 5^{-2\log_5 \beta} = \left(\frac{1}{2}\right)^{-2} = 4$

$\Rightarrow 4 + 4 = 8$

**8.** For 10 observations $x_1, x_2, \ldots, x_{10}$, if $\sum_{i=1}^{10}(x_i+2)^2 = 180$ and $\sum_{i=1}^{10}(x_i-1)^2 = 90$, then their standard deviation is :

(1) 2 (2) $\sqrt{3}$

(3) $2\sqrt{2}$ (4) 3

**Ans.** **(4)**

**Sol.** $\sum_{i=1}^{10}(x_i+2)^2 = 180$

$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 4 = 180$

$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i = 180 - 40 \quad \ldots(1)$

Also $\sum_{i=1}^{10}(x_i - 1)^2 = 90$

$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i + \sum_{i=1}^{10} 1 = 90$

$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i = 90 - 10 \quad \ldots(2)$

$\sum_{i=1}^{10} x_i^2 + 4\sum_{i=1}^{10} x_i = 140$

$\sum_{i=1}^{10} x_i^2 - 2\sum_{i=1}^{10} x_i = 80$

$\sum_{i=1}^{10} x_i^2 = 100$ and $\sum_{i=1}^{10} x_i = 10$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{N} - \left(\frac{\sum_{i=1}^{10} x_i}{N}\right)^2$$

$$= \frac{100}{10} - \left(\frac{10}{10}\right)^2$$

$\sigma^2 = 10 - 1 = 9$

$\Rightarrow \sigma = 3$

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**9.** In the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x>0$, if the term independent of x is (221)k. then k is equal to:

(1) 84 (2) 78

(3) 168 (4) 198

**Ans.** **(1)**

**Sol.** General term in above expression is

$$T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$$

$$=\left(-\frac{1}{3}\right)^r {}^{18}C_r 9^{18-r}\cdot x^{18-\frac{3r}{2}}$$

For term independent of x, $18-\frac{3r}{2}=0 \Rightarrow r=12$

$\therefore$ Coefficient of term independent of x

$$=\left(-\frac{1}{3}\right)^{12} {}^{18}C_{12} 9^{18-12}$$

$= 18564$

$= 221$ k (given)

$\Rightarrow k = 84$

**10.** Let P(3cos α, 2sin α), α ≠ 0, be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, Q be a point on the circle $x^2 + y^2 - 14x - 14y + 82 = 0$ and R be a point on the line x + y = 5 such that the centroid of the triangle PQR is $\left(2+\cos\alpha, 3+\frac{2}{3}\sin\alpha\right)$. Then the sum of the ordinates of all possible points R is:

(1) 6 (2) 2

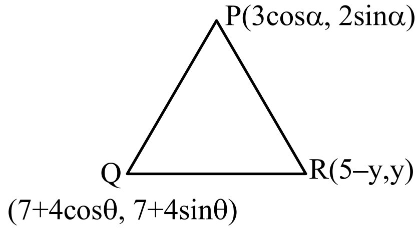
(3) 4 (4) 8

**Ans.** **(4)**

**Sol.** Centroid :

$$\left(\frac{3\cos\alpha+7+4\cos\theta+5-y}{3}, \frac{2\sin\alpha+7+4\sin\theta+y}{3}\right)$$

$$=\left(\cos\alpha+2, \frac{2}{3}\sin\alpha+3\right)$$




On comparison

$$\cos\alpha = \frac{y-6}{4} \;\&\; \sin\alpha = \frac{2-y}{4}$$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (y-6)^2 + (y-2)^2 = 16$

$y^2 - 8y + 12 = 0$

$\Rightarrow$ Sum of ordinates = 8

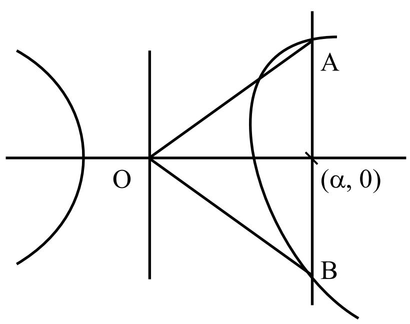
**11.** Let $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be a hyperbola such that the distance between its foci is 6 and the distance between its directrices is $\frac{8}{3}$. If the line x = α intersects the hyperbola H at the points A and B such that the area of the triangle AOB is $4\sqrt{15}$, where O is the origin, then $\alpha^2$ equals

(1) 12 (2) 16

(3) 24 (4) 25

**Ans.** **(2)**

**Sol.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$




Given:

$2ae = 6 \Rightarrow ae = 3$

$$\frac{2a}{e}=\frac{8}{3} \Rightarrow \frac{a}{e}=\frac{4}{3}$$

$a^2 = 4 \quad e = \frac{3}{2} \quad b^2 = 5$

$\frac{x^2}{4} - \frac{y^2}{5} = 1$ solve with $x = \alpha$

$y = \pm\sqrt{\frac{5(\alpha^2 - 4)}{4}}$

$A\left(\alpha, \sqrt{\frac{5(\alpha^2-4)}{4}}\right) \quad B\left(\alpha, -\sqrt{\frac{5(\alpha^2-4)}{4}}\right)$

Ar. of $\Delta OAB = \frac{1}{2}|\alpha|\sqrt{5(\alpha^2-4)} = 4\sqrt{15}$

$\Rightarrow 16 \times 15 = \frac{1}{4}\alpha^2(5)(\alpha^2 - 4)$

$\Rightarrow \alpha^2 = 16$

**12.** $\max\limits_{0 \le x \le \pi}\left(16\sin\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)\right)$ is equal to :

(1) $\frac{3\sqrt{3}}{2}$ (2) $3\sqrt{3}$

(3) $4\sqrt{3}$ (4) $6\sqrt{3}$

**Ans. (2)**

**Sol.** $E = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$

$E = 4\sin x\,[1 + \cos x]$

$\frac{dE}{dx} = 4[\cos x + \cos 2x]$

$= 8\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow x = \left\{\frac{\pi}{3}, \pi\right\}$ are critical points of the function

$E(0) = 0$

$E(\pi) = 0$

$E\left(\frac{\pi}{3}\right) = 3\sqrt{3}$

$\therefore$ maximum value of $E = 3\sqrt{3}$

**13.** The shortest distance between the lines

$\vec{r} = \left(\frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} + 6\hat{k}\right)$

and $\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu\left(\hat{j} - \hat{k}\right)$, $\lambda, \mu \in \mathbb{R}$, is :

(1) $\sqrt{5}$ (2) 3

(3) $2\sqrt{3}$ (4) $\sqrt{15}$

**Ans. (2)**

**Sol.** $\vec{a}_1 = \frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}$

$\vec{a}_2 = \frac{-2}{3}\hat{i} - \frac{1}{3}\hat{k}$

$\vec{b}_1 = 2\hat{i} - 5\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{j} - \hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_1 - \vec{a}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$

$S.D = \left|\frac{(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$S.D = \left|\frac{(\hat{i} + 2\hat{j} + 3\hat{k})\cdot(-\hat{i} + 2\hat{j} + 2\hat{k})}{3}\right|$

$= \frac{9}{3} = 3$

**14.** If $\left(2\alpha + 1, \alpha^2 - 3\alpha, \frac{\alpha - 1}{2}\right)$ is the image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{1}$, then the possible value(s) of $\alpha$ is(are) :

(1) Only 3

(2) Only 3 and –1

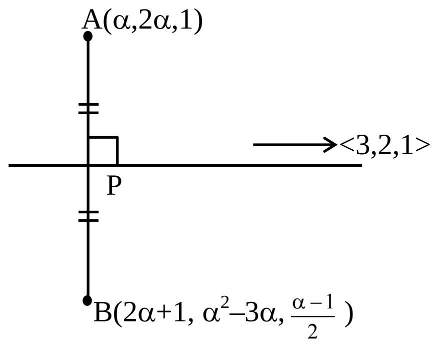
(3) Only 3, $\frac{1}{4}$ and –1

(4) Only 3 and $\frac{1}{4}$

**Ans. (1)**

**Sol.** $\therefore$ P is mid point of AB and lies on the given line

$$\Rightarrow P\left(\frac{3\alpha+1}{2}, \frac{\alpha^2-\alpha}{2}, \frac{\alpha+1}{4}\right)$$




Now put coordinates of P in the line

$$\Rightarrow \frac{\frac{3\alpha+1}{2}-2}{3} = \frac{\frac{\alpha^2-\alpha}{2}-1}{2} = \frac{\alpha+1}{4}$$

$$\Rightarrow \frac{\alpha-1}{2} = \frac{\alpha+1}{4} = \frac{\alpha^2-\alpha-2}{4}$$

$\Rightarrow \alpha = 3$

Now for $\alpha = 3$, clearly AB is perpendicular to $3\hat{i}+2\hat{j}+k$

for $\alpha = 3$ point B is image of point A

**15.** Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u}\times\hat{v}| = \frac{\sqrt{3}}{2}$. If $\vec{A} = \lambda\hat{u}+\hat{v}+(\hat{u}\times\hat{v})$. Then $\lambda$ is equal to :

(1) $\frac{4}{3}(\vec{A}.\hat{u}) - \frac{2}{3}(\vec{A}.\hat{v})$

(2) $\frac{2}{3}(\vec{A}.\hat{u}) - \frac{1}{3}(\vec{A}.\hat{v})$

(3) $\frac{4}{3}(\vec{A}.\hat{u}) + \frac{2}{3}(\vec{A}.\hat{v})$

(4) $(\vec{A}.\hat{u}) - \frac{1}{2}(\vec{A}.\hat{v})$

**Ans. (1)**

**Sol.** $|\hat{u}\times\hat{v}| = \frac{\sqrt{3}}{2}$

$$|\hat{u}||\hat{v}|\sin\theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = \frac{\pi}{3}$$

and $\hat{u}\cdot\hat{v} = |\hat{u}||\hat{v}|\cos\frac{\pi}{3} = \frac{1}{2}$

$\vec{A} = \lambda\hat{u}+\hat{v}+\hat{u}\times\hat{v} \quad \ldots(1)$

Dot with $\hat{u}$

$\vec{A}\cdot\hat{u} = \lambda(1)+\hat{u}.\hat{v}+\hat{u}\cdot(\hat{u}\times\hat{v})$

$\vec{A}\cdot\hat{u} = \lambda+\frac{1}{2}$

$\Rightarrow 2\vec{A}\cdot\hat{u} = 2\lambda+1 \quad \ldots(2)$

Dot equation (1) with $\hat{v}$

$\vec{A}\cdot\hat{v} = \lambda(\hat{u}\cdot\hat{v})+\hat{v}\cdot\hat{v}+\hat{v}\cdot(\hat{u}\times\hat{v})$

$\vec{A}\cdot\hat{v} = \frac{\lambda}{2}+1$

$\vec{A}\cdot\hat{v} - \frac{\lambda}{2} = 1 \quad \ldots(3)$

From (2) and (3)

$$2\vec{A}\cdot\hat{u} - 2\lambda = \vec{A}.\hat{v} - \frac{\lambda}{2}$$

$$\therefore \lambda = \frac{4}{3}\vec{A}\cdot\hat{u} - \frac{2}{3}\vec{A}.\hat{v}$$

**16.** Let for some $\alpha\in\mathbb{R}$, $f : \mathbb{R}\to\mathbb{R}$ be a function satisfying $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ for all $x, y \in \mathbb{R}$. If $f(0) = -1$ and $f(1) = 2$, then the value of $\sum_{n=1}^{5}(\alpha+f(n))$ is :

(1) 110 (2) 140

(3) 150 (4) 170

**Ans. (2)**

**Sol.** $f(x+y) = f(x) + 2y^2 + y + \alpha xy \quad \ldots(1)$

Put $x = 0$ in eq.(1)

$\Rightarrow f(y) = -1 + 2y^2 + y$

Now put $x = y = 1$ in eq. (1)

$\Rightarrow f(2) = f(1) + 3 + \alpha$

$\Rightarrow 9 = 2+3+\alpha \Rightarrow \alpha = 4$

Now $\sum_{n=1}^{5}(\alpha+f(n)) = \sum_{n=1}^{5}(2y^2+y+3)$

$$= \frac{2\times5\times6\times11}{6} + \frac{5\times6}{2} + 3\times5 = 140$$

**17.** Let A = {(a, b, c) : a, b, c are non-negative integers and a + b + 2c = 22}.

The n(A) is equal to :

(1) 121 (2) 124

(3) 144 (4) 169

**Ans. (3)**

**Sol.** $c = 0 \Rightarrow a + b = 22 \rightarrow {}^{22+2-1}C_{2-1} = {}^{23}C_1$

$c = 1 \Rightarrow a + b = 20 \rightarrow {}^{20+2-1}C_{2-1} = {}^{21}C_1$

$\vdots \qquad \vdots \qquad \vdots$

$c = 10 \Rightarrow a + b = 2 \rightarrow {}^{2+2-1}C_{2-1} = {}^{3}C_1$

$c = 11 \Rightarrow a + b = 0 \rightarrow {}^{0+2-1}C_{2-1} = {}^{1}C_1$

Number of non-negative integral solutions

${}^{1}C_1 + {}^{3}C_1 + {}^{5}C_1 + \ldots + {}^{23}C_1$

$1 + 3 + 5 \ldots\ldots + 23$

$= 144$

**18.** The area of the region bounded by the curves $x + 3y^2 = 0$ and $x + 4y^2 = 1$ is equal to :

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{5}{3}$

**Ans. (3)**

**Sol.** $x = -3y^2$ and $x = 1 - 4y^2$

$$A = \int_{-1}^{1} \left((1-4y^2) + 3y^2\right) dy$$

$$A = \int_{-1}^{1} (1-y^2)dy = 2\int_{0}^{1} (1-y^2)dy$$

$$A = \frac{4}{3}$$

**19.** Let y = y(x) be the solution of the differential equation :

$$\frac{dy}{dx} + \left(\frac{6x^2 + \left(3x^2 + 2x^3 + 4\right)e^{-2x}}{\left(x^3+2\right)\left(2+e^{-2x}\right)}\right) y = 2 + e^{-2x},$$

$x \in (-1, 2)$, satisfying $y(0) = \frac{3}{2}$.

If $y(1) = \alpha(2 + e^{-2})$, than $\alpha$ is equal to :

(1) $\frac{13}{8}$ (2) $\frac{6}{13}$

(3) $\frac{12}{13}$ (4) $\frac{13}{12}$

**Ans. (4)**

**Sol.** $$\text{I.F.} = e^{\int \frac{6x^2 + e^{-2x}(3x^2+2x^3+4)}{(x^3+2)(2+e^{-2x})} dx}$$

$$= e^{\int \frac{6x^2 + e^{-2x}(3x^2-2x^3-4)+(4x^3+8)e^{-2x}}{(x^3+2)(2+e^{-2x})} dx}$$

Let $(x^3 + 2)(2 + e^{-2x}) = t$

$(6x^2 + e^{-2x}(3x^2 - 2x^3 - 4))dx = dt$

$$\text{I.F.} = e^{\ell n|(x^3+2)(2+e^{-2x})| - \int \frac{4e^{-2x}dx}{2+e^{-2x}}}$$

$$= e^{\ell n|(x^3+2)(2+e^{-2x})| - 2\ell n|2+e^{-2x}|}$$

$$= e^{\ell n\left|\frac{x^3+2}{2+e^{-2x}}\right|} = \frac{x^3+2}{2+e^{-2x}}$$

$$\Rightarrow y.\left(\frac{x^3+2}{2+e^{-2x}}\right) = \int \left(\frac{x^3+2}{2+e^{-2x}}\right)(e^{-2x}+2)dx + C$$

$$y.\left(\frac{x^3+2}{2+e^{-2x}}\right) = \frac{x^4}{4} + 2x + C \text{ at } y(0) = \frac{3}{2} \Rightarrow C = 1$$

$$\text{So } y.\left(\frac{x^3+2}{2+e^{-2x}}\right) = \frac{x^4}{4} + 2x + 1$$

at x = 1

$$y.\left(\frac{3}{2+e^{-2}}\right) = \frac{1}{4} + 2 + 1$$

$$\Rightarrow y = \frac{13}{12}(e^{-2}+2)$$

$$\text{So } \alpha = \frac{13}{12}$$

**20.** The integral $\int_0^1 \cot^{-1}\left(1 + x + x^2\right) dx$ is equal to :

(1) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

(2) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

(3) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

(4) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

**Ans. (4)**

**Sol.** $= \int_0^1 \tan^{-1}\left(\frac{1}{1+x+x^2}\right)dx$

$= \int_0^1 \tan^{-1}\left(\frac{(x+1)-x}{1+x(x+1)}\right)dx$

$= \int_0^1 (\tan^{-1}(x+1) - \tan^{-1} x)dx$

$= \left(x\tan^{-1}(x+1) - \frac{1}{2}\ell n|1+(1+x)^2| + \tan^{-1}(x+1)\right)_0^1$

$- \left(x\tan^{-1} x - \frac{1}{2}\ell n|1+x^2|\right)_0^1$

$= \left(2\tan^{-1} 2 - \frac{\pi}{4} - \frac{1}{2}\ell n\frac{5}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\ell n 2\right)$

$= 2\tan^{-1} 2 - \frac{1}{2}\ell n\left(\frac{5}{4}\right) - \frac{\pi}{2}$

## SECTION-B

**21.** From a month of 31 days, 3 different dates are selected at random. If the probability that these dates are in an increasing A.P. is equal to $\frac{a}{b}$, where a, b $\in$ N and *gcd*(a,b) = 1, then a + b is equal to ______.

**Ans. (944)**

**Sol.** Total numbers of ways of selecting

3 numbers $= {}^{31}C_3 = \frac{31\times30\times29}{3\times2\times1} = 4495$

a, b, c are in A.P.

$\Rightarrow$ either a & c both odd numbers

or both even numbers

$= {}^{16}C_2 + {}^{15}C_2$

$= 120 + 105$

$= 225$

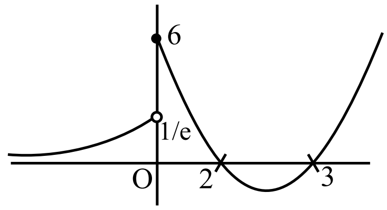
Probability $= \frac{225}{4495} = \frac{45}{899}$

$= \frac{a}{b} \Rightarrow a + b = 944$

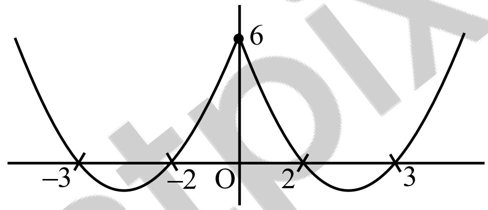
**22.** Let $f(x) = \begin{cases} e^{x-1} & ,x<0 \\ x^2 - 5x + 6 & ,x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$. If the number of points where g is not continuous and is not differentiable are $\alpha$ and $\beta$ respectively, then $\alpha + \beta$ is equal to ______

**Ans. (4)**

**Sol.** Graph of f(x)

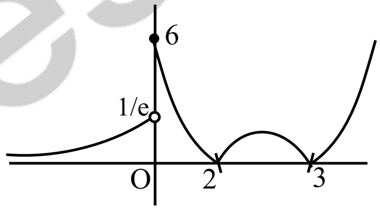



Now graph of f(|x|)




f(|x|) is continuous function and it is non diff. at x = 0

Graph of |f(x)|




|f(x)| is discontinuous at x = 0

|f(x)| is non-diff. at x = 0, 2, 3

g(x) = f(|x|) + |f(x)|

g(x) will be discontinuous at x = 0

g(x) will be non diff at x = 0, 2, 3

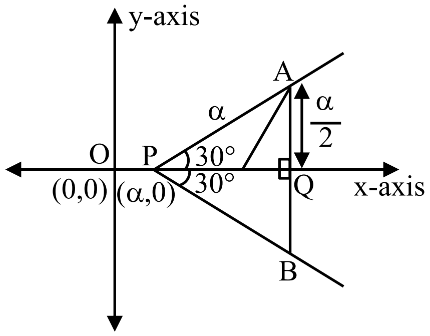
$\alpha = 1, \beta = 3$

$\alpha + \beta = 1 + 3 = 4$

**23.** Let A, B be points on the two half-lines $x - \sqrt{3}|y| = \alpha$, $\alpha > 0$ at a distance of $\alpha$ from their point of intersection P. The line segment AB meets the angle bisector of the given half-lines at the point Q. If PQ = $\frac{9}{2}$ and R is the radius of the circumcircle of $\Delta$PAB, then $\frac{\alpha^2}{R}$ is equal to ______.

**Ans. (9)**

**Sol.** $\Delta PAB$ is equilateral




$$PQ = \frac{\sqrt{3}\alpha}{2} = \frac{9}{2} \qquad \therefore \quad \alpha = \frac{9}{\sqrt{3}}$$

$$\alpha = 3\sqrt{3}$$

$$\frac{\alpha}{2r} = \cos 30^\circ$$

$$\alpha = \frac{\sqrt{3}}{2}\cdot 2R$$

$$\alpha = \sqrt{3}\,R$$

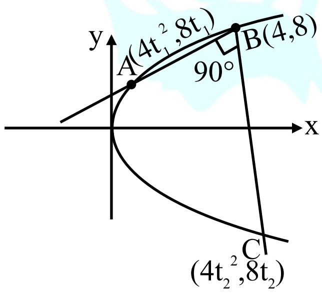
$$\boxed{R=3}$$

$$\frac{\alpha^2}{R} = \frac{(3\sqrt{3})^2}{3} = 9$$

**24.** Let A, B and C be the vertices of a variable right angled triangle inscribed in the parabola $y^2 = 16x$. Let the vertex B containing the right angle be (4, 8) and the locus of the centroid of $\Delta ABC$ be a conic $C_0$. Then three times the length of latus rectum of $C_0$ is _____.

**Ans.** **(16)**

**Sol.**




Variable point A & C are shown in the figure

$\because\ m_{AB}.m_{BC} = -1$

$\Rightarrow t_1 + t_2 + t_1t_2 = -5 \ldots(1)$

Suppose locus of centroid of $\Delta ABC$ is (h,k)

$\therefore\ 3h = 4 + 4t_1^2 + 4t_2^2$ & $3k = 8 + 8t_1 + 8t_2$

by eliminating $t_1$ & $t_2$ using equation (1) also

we get $h = \frac{9}{48}k^2 + \frac{40}{3}$

$\therefore$ locus of centroid of parabola is

$$x = \frac{9}{48}y^2 + \frac{40}{3}$$

$$\therefore\ \ell(L.R.) = \frac{48}{9}$$

$$\therefore\ 3\ell(L.R.) = \frac{48}{3} = 16$$

**25.** Let $f$ be a twice differentiable function such that

$$f(x) = \int_0^x \tan(t-x)dt - \int_0^x f(t)\tan t\,dt,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Then $f''\left(\frac{\pi}{6}\right) + 12f'\left(-\frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$

is equal to _____.

**Ans.** **(5)**

**Sol.** $f(x) = \int_0^x \tan(x-t-x)dt - \int_0^x f(t)\tan t\,dt$

$f(x) = -\int_0^x \tan t dt - \int_0^x f(t)\tan t\,dt$

$f'(x) = -\tan x - f(x)\tan x$

$$\frac{dy}{y+1} = -\tan x\,dx$$

$y + 1 = \cos x.c$

$f(0) = 0 \Rightarrow 1 = c$

$y = \cos x - 1 = f(x)$

$$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - 1$$

$$f'(x) = -\sin x$$

$$f'\left(\frac{-\pi}{6}\right) = \frac{1}{2}$$

$$f''(x) = -\cos x$$

$$f''\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$f''\left(\frac{\pi}{6}\right) + 12f'\left(\frac{-\pi}{6}\right) + f\left(\frac{\pi}{6}\right) = 5$$

**(HELD ON SATURDAY 04th APRIL 2026)** **TIME : 3:00 PM TO 6:00 PM**

## PHYSICS — TEST PAPER WITH SOLUTION

### SECTION-A

**26.** Match the **LIST-I** with **LIST-II**

| | List-I | | List-II |
|---|---|---|---|
| A. | Planck's constant | I. | $ML^2T^{-2}$ |
| B. | Stopping potential | II. | $T^{-1}$ |
| C. | Work function | III. | $ML^2T^{-1}$ |
| D. | Threshold frequency | IV. | $ML^2T^{-3}A^{-1}$ |

Choose the ***correct*** answer from the options given below :

(1) A-III, B-IV, C-I, D-II

(2) A-I, B-II, C-III, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-IV, C-III, D-II

**Ans. (1)**

**Sol.** (i) $[h]=\dfrac{[E]}{[v]}=\dfrac{ML^2T^{-2}}{T^{-1}}=ML^2T^{-1}$

(ii) Stopping potential : $E = q.V_0$

$$[V_0]=[v_0]=\frac{[E]}{[q]}=\frac{ML^2T^{-2}}{AT}=ML^2T^{-3}A^{-1}$$

(iii) Work function ($\phi$) $\rightarrow$ Energy form

$[\phi] = ML^2T^{-2}$

(iv) Threshold freq. $[v_0] = T^{-1}$

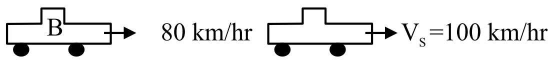
**27.** Two cars A and B are moving in the same direction along a straight line with speeds 100 km/h and 80 km/h, respectively such that car A is moving ahead of car B. A person in car B throws a stone with a speed v so that it hits the car A with a speed of 5m/s. The value of v is ________ km/h.

(1) 18 (2) 28

(3) 38 (4) 48

**Ans. (3)**

**Sol.**




$V = \vec{V}_{SB}$

$V = \vec{V}_S - \vec{V}_B$

$V_B + V = \vec{V}_S$

$\boxed{\vec{V}_S = V + 80}$

$\vec{V}_{SA} = 5m/s = 18km/hr$

$\vec{V}_S - \vec{V}_A = \vec{V}_{SA}$

$V + 80 - 100 = 18$ km/hr

$V - 20 = 18$

$V = 38$ km/hr

**28.** At $t = 0$, a body of mass 100 g starts moving under the influence of a force $(5\hat{i}+10\hat{j})N$. After 2s its position is $(2x\hat{i}+5y\hat{j})m$. The ratio x : y is_____.

(1) 1 : 2 (2) 2 : 5

(3) 5 : 2 (4) 5 : 4

**Ans. (4)**

**Sol.** $\vec{a}=\dfrac{\vec{F}}{m}=\dfrac{5\hat{i}+10\hat{j}}{0.1}=\left(50\hat{i}+100\hat{j}\right)$

$\vec{s}=\vec{u}t+\dfrac{1}{2}\vec{a}t^2$

$\vec{s}=0+\dfrac{1}{2}\left[50\hat{i}+100\hat{j}\right][2]^2$

$\vec{s}=100\hat{i}+200\hat{j} = \vec{r}=2x\hat{i}+5y\hat{j}$

$2x = 100$ $5y = 200$

$\boxed{x=50}$ $\boxed{y=100}$

$\boxed{\dfrac{x}{y}=\dfrac{5}{4}}$

**29.** If x and y coordinates of a projectile as a function of time (t) are given as 24t and 43.6t – 4.9t², respectively, then the angle (in degrees) made by the projectile with horizontal when t = 2 s is ________.

(1) 60 (2) 45

(3) 30 (4) 75

**Ans. (2)**

**Sol.** $x = 24t \quad y = 43.6t - 4.9t^2$

$$v_x = \frac{dx}{dt} = 24$$

$$v_y = \frac{dy}{dt} = 43.6 - 9.8t$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{43.6 - 9.8t}{24}$$

$$\text{At } t = 2 \quad \frac{43.6 - 19.6}{24}$$

$\tan\theta = 1$

$\theta = 45°$

**30.** The height in terms of radius of the earth ($R$), at which the acceleration due to gravity becomes $\frac{g}{9}$, where $g$ is acceleration due to gravity on earth's surface, is ________.

(1) $\sqrt{3}R$ (2) $2\sqrt{2}R$

(3) 2R (4) $\frac{4}{9}R$

**Ans. (3)**

**Sol.** $$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g}{9} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$1+\frac{h}{R} = 3$$

$h = 2R$

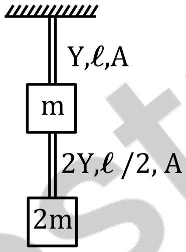
**31.** A metal string A is suspended from a rigid support and its free end is attached to a block of mass M. Second block having mass 2M is suspended at the bottom of the first block using a string B. The area of cross sections of strings A and B are same. The ratio of lengths of strings of A to B is 2 and the ratio of their Young's moduli ($Y_A/Y_B$) is 0.5. The ratio of elongations in A to B is ________.

(1) 1 (2) 4

(3) 8 (4) 6

**Ans. (4)**

**Sol.**




$$\Delta\ell = \frac{T\ell}{YA}$$

$$\Delta\ell_1 = \frac{3mg\ell}{YA}$$

$$\Delta\ell_2 = \frac{2mg(\ell/2)}{2YA} = \frac{mg\ell}{2YA}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = 6$$

**32.** A water spray gun is attached to a hose of cross sectional area 30 cm². The gun comprises of 10 perforations each of cross sectional area of 15 mm². If the water flows in the hose with the speed of 50 cm/s, calculate the speed at which the water flows out from each perforation. (Neglect any edge effects)

(1) 100 m/s (2) 10 m/s

(3) 1000 m/s (4) $15 \times 10^2$ m/s

**Ans. (2)**

**Sol.** Using equation of continuity

$AV = 10av$

$v = \dfrac{AV}{10a} = \dfrac{30 \times 50}{10 \times .15} = 1000 \text{ cm/s} = 10 \text{ m/s}$

**33.** Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

**Assertion A:** If the average kinetic energy of $H_2$ and $O_2$ molecules, kept in two different sized containers are same, then their temperatures will be same.

**Reason R:** The r.m.s. speed of $H_2$ and $O_2$ molecules are same at same temperature.

Choose the ***correct*** answer from the options given below

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(3) **A** is true but **R** is false

(4) **A** is false but **R** is true

**Ans.** **(3)**

**Sol.** Avg. kinetic energy of molecule will be $\dfrac{f}{2}KT$

But $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

Molar mass m different so $v_{rms}$ is different

**34.** The temperature of a metal strip having coefficient of linear expansion $\alpha$ is increased from $T_1$ to $T_2$ resulting in increase of its length by $\Delta L_1$. The temperature is further increased from $T_2$ to $T_3$ such that the increase in its length is $\Delta L_2$.

Given $T_3 + T_1 = 2T_2$ and $T_2 - T_1 = \Delta T$, the value of $\Delta L_2$ is _________.

(1) $\Delta L_1[1 + 2\alpha^2(\Delta T)^2]$ (2) $\Delta L_1[1 + \alpha^2(\Delta T)^2]$

(3) $\Delta L_1[1 + 2\alpha\Delta T]$ (4) $\Delta L_1[1 + \alpha\Delta T]$

**Ans.** **(4)**

**Sol.** $\Delta L_1 = L_0\,\alpha\,(T_2 - T_1)$

So final length at $T_2 = L_0 + \Delta L_1$

$\Delta L_2 = (L_0 + \Delta L_1)\,\alpha\,(T_3 - T_2)$ and $T_3 - T_2 = T_2 - T_1 = \Delta T$

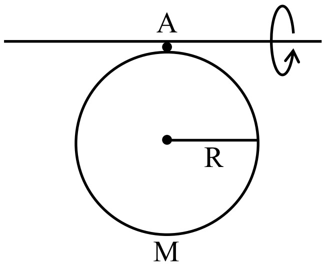
$\Delta L_2 = L_0\,\alpha\,\Delta T + \Delta L_1\,\alpha\,\Delta T$

$= \Delta L_1 + \Delta L_1\,\alpha\,\Delta T$

$= \Delta L_1\,(1 + \alpha\Delta T)$

**35.** A uniform disc of radius R and mass M is free to oscillate about the axis A as shown in the figure. For small oscillations the time period is ________.

(g is acceleration due to gravity)




(1) $2\pi\sqrt{\dfrac{5R}{4g}}$ (2) $2\pi\sqrt{\dfrac{2R}{3g}}$

(3) $2\pi\sqrt{\dfrac{3R}{2g}}$ (4) $2\pi\sqrt{\dfrac{3R}{g}}$

**Ans.** **(1)**

**Sol.** $I_{disc} = \dfrac{mR^2}{4} + mR^2 = \dfrac{5mR^2}{4}$

$\therefore$ For small displacement

$\tau = -mgR\theta$

$\therefore \alpha = -\dfrac{mgR}{\dfrac{5mR^2}{4g}}\theta$

$\therefore \omega = \sqrt{\dfrac{4g}{5R}}$

$\therefore T = 2\pi\sqrt{\dfrac{5R}{4g}}$

**36.** A rigid dipole undergoes a simple harmonic motion about its centre in the presence of an electric field $\vec{E}_1 = E_0\hat{x}$. If another electric field $\vec{E}_2 = 2E_0(\hat{y} + \hat{z})$ is introduced to the system, what will be the percentage change in the frequency of the oscillation (approximate) ?

(1) 73% (2) 63%

(3) 83% (4) 53%

**Ans.** **(1)**

**Sol.** Initial freq. $f_1 = \frac{1}{2\pi}\sqrt{\frac{P(E_0)}{I}}$

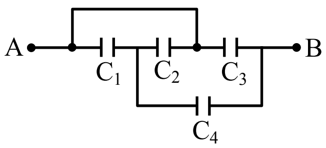
Final EF $= E_0\hat{i} + 2E_0(\hat{j}+\hat{k})$

Magnitude $= 3E_0$

Final Freq. $f_2 = \frac{1}{2\pi}\sqrt{\frac{P(3E_0)}{I}}$

% change $= \frac{\Delta f}{f_1} = (\sqrt{3}-1)\times 180 \ \simeq 73$

**37.** From the circuit given below, the capacitance between terminals A and B shown in the circuit is ____μF. (take $C_1 = C_2 = C_3 = 1$ μF and $C_4 = 2$μF.)




(1) 2

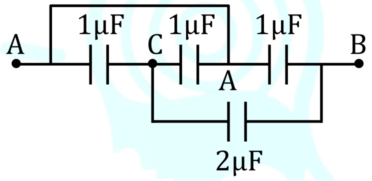
(2) 7/2

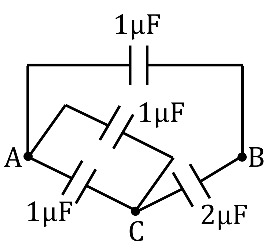
(3) 7/3

(4) 5/2

**Ans.** **(2)**

**Sol.**




$C_{AB} = 2\mu F$

**38.** Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

**Assertion A :** In electrostatics, a conductor does not store any net charge inside.

**Reason R :** Inside the capacitor (with no dielectric medium), the free charge carriers, if placed between the plates of capacitor, experience force and drift.

Choose the ***correct*** answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**.

(3) **A** is true but **R** is false

(4) **A** is false but **R** is true

**Ans.** **(2)**

**Sol.** Theoretical

**39.** A solenoid has a core made of material with relative permeability 400. The magnetic field produced in the interior of solenoid is 1.0T. The magnetic intensity is SI units is $\alpha \times 10^5$. The value of $\alpha$ is _______. (Free space permeability $\mu_0 = 4\pi \times 10^{-7}$ SI units).

(1) $\frac{25}{\pi}$ (2) $\frac{1}{16\pi}$

(3) $\frac{1}{\pi}$ (4) $\frac{1}{4\pi}$

**Ans.** **(2)**

**Sol.** $H = \frac{B}{\mu} = \frac{B}{\mu_0\mu_r} = \frac{1}{400\times 4\pi\times 10^{-7}}$

$H = \frac{1}{16\pi}\times 10^5$

**40.** A magnetic field vector in an electromagnetic wave is represented by, $\vec{B} = B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{j}$. Its associated electric field vector is _______.

(1) $\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}$

(2) $\vec{E} = -v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{i}$

(3) $\vec{E} = v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{k}$

(4) $\vec{E} = v\lambda B_0 \sin\left(2\pi vt - \frac{2\pi x}{\lambda}\right)\hat{i}$

**Ans.** **(1)**

**Sol.** $\hat{E} \times \hat{B} = \hat{C}$

$\hat{B} = \hat{j}$

$\hat{C} = \hat{i}$

$\therefore \hat{E} = -\hat{k}$

$\frac{E_0}{B_0} = C = v\lambda$

$E_0 = B \times v\lambda$

**41.** A convex lens is made from glass material having refractive index of 1.4 with same radius of curvature on both sides. The ratio of its focal length and radius of curvature is ______.

(1) 0.5 (2) 2.5

(3) 0.8 (4) 1.25

**Ans.** **(4)**

**Sol.** $\frac{1}{f} = (1.4 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

$\frac{1}{f} = \frac{0.8}{R}$

$\frac{f}{R} = 1.25$

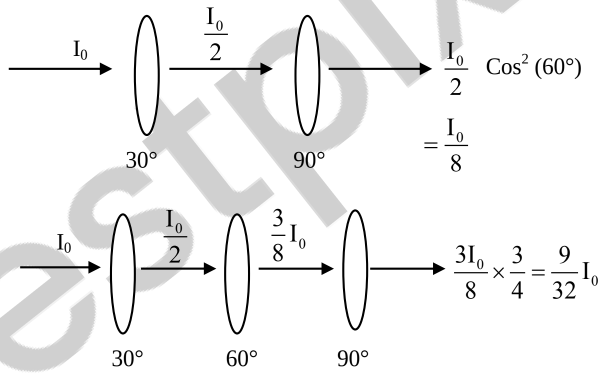
**42.** An unpolarized light of certain intensity passes through a combination of two polarizers whose transmission axes are at 30° and 90°, respectively, with respect to the horizontal axis. A third polarizer with its transmission axis at 60° with the horizontal axis is placed between the two existing polarizers. The ratio of the output intensities with and without the third polarizer is _____.

(1) 3/4 (2) 4/3

(3) 9/4 (4) 4/9

**Ans.** **(3)**

**Sol.**




$\frac{9I_0}{32} \times \frac{8}{I_0} = \frac{9}{4}$

**43.** In Rutherford's alpha-particle scattering experiment, only a few alpha particles rebound back because

(A) The size of gold nucleus is very small as compared to the size of gold atom.

(B) Alpha particle and gold nucleus have equal charge.

(C) The impact parameter is minimum for a few alpha particles.

(D) A few alpha particles have very high kinetic energy.

(E) Only a few alpha particles undergo head-on collision with the nuclei.

Choose the correct answer from the options given below :

(1) A, B Only (2) B, E Only

(3) C,D Only (4) A,C,E Only

**Ans.** **(4)**

**Sol.** A,C, E only

Theoretical

**44.** The de Broglie wavelength associated with an electron accelerated through a potential difference V is $\lambda_e$ and the de Broglie wavelength associated with a proton accelerated through the same potential difference is $\lambda_p$. If their corresponding masses are $m_e$ and $m_p$, respectively, then the ratio of their de Broglie wavelengths $\left(\frac{\lambda_e}{\lambda_p}\right)$ is ______.

(1) $\sqrt{\frac{m_p}{m_e}}$ (2) $\sqrt{\frac{m_e}{m_p}}$

(3) $\frac{m_p}{m_e}$ (4) $\left(\frac{m_p}{m_e}\right)^2$

**Ans. (1)**

**Sol.** $\lambda = \frac{h}{\sqrt{2mqV}} \propto \frac{1}{\sqrt{m}}$

So $\frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_p}{m_e}}$

$\sqrt{\frac{m_e}{m_p}}$

**45.** Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

**Assertion A :** A diode under reverse- biased condition provides very small current which is nearly independent of voltage until a critical limit at which the current increases drastically.

**Reason R :** Below the critical voltage limit, only majority charge carriers flow which increases drastically above critical voltage.

Choose the ***correct*** answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**.

(3) **A** is true but **R** is false

(4) **A** is false but **R** is true

**Ans. (3)**

**45.**

**Sol.** Theoretical

## SECTION-B

**46.** A diode has Zener voltage of 10V and maximum power dissipation of 0.5W, then the minimum resistance to be used in series with this diode for safety when it is connected to a 25V power supply is ______Ω.

**Ans. (300)**

**Sol.** Maximum current from diode

$i = \frac{P}{V} = \frac{0.5}{10} = 0.05A$

Now let resistance is R

$V_B = i\,(R + R_{diode})$

$25 = 0.05 \times R + 10$

$R = \frac{15}{0.05} = 300\ \Omega$

**47.** A gun mounted on the ground fires bullets in all directions with same speed. The farthest distance the bullets could reach is 6.4 m. The speed of the bullets from the gun is ____ m/s. (take $g = 10\ m/s^2$)

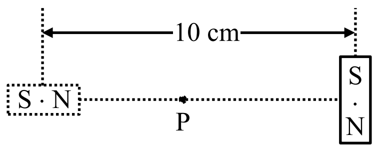
**Ans. (8)**

**Sol.** $R_{max} = \frac{v^2}{g} = 6.4$

$v = \sqrt{64} = 8$ m/s

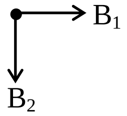
**48.** Two identical small bar magnets each of dipole moment $3\sqrt{5}\ J/T$ are placed at a center to center separation of 10 cm, with their axes perpendicular to each other as shown in figure. The value of magnetic field at the point P midway between the magnets is $\alpha \times 10^{-3}$ T. The value of $\alpha$ is _______.

$(\mu_0 = 4\pi \times 10^{-7}\ Tm/A)$




**Ans. (12)**

**Sol.** $B_{net} = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{4\pi}\frac{M}{(r)^3}\sqrt{5}$




$= 10^{-7} \times 3\sqrt{5} \times \sqrt{5} \times \frac{8}{10^{-3}}$

$= 120 \times 10^{-4} = 12$ mT

**49.** A circular coil of radius 2 cm and 125 turns carries a current of 1 A. The coil is placed in a uniform magnetic field of magnitude 0.4 T. The axis of the coil makes an angle of 30° with the direction of the magnetic field. The torque acting on the coil is $\alpha \times 10^{-4}$ N.m. The value of $\alpha$ is ________.

($\pi = 3.14$)

**Ans.** **(314)**

**Sol.** $\tau = |\vec{M} \times \vec{B}|$

$= |NI\vec{A} \times \vec{B}|$

$\tau = 125 \times 1 \times \pi\left(\frac{2}{100}\right)^2 \times 0.4 \times \sin 30°$

$= 100\,\pi \times 10^{-4}$ N-m

So $\alpha = 100\,\pi = 314$

**50.** In a double slit experiment, when one of the slits is covered by a transparent mica sheet of refractive index 1.56, the central fringe shifts to the position of 7th bright fringe, obtained with both slits uncovered. If the light source wavelength is 450 nm, the thickness of mica sheet is $\alpha \times 10^{-9}$ m. The value of $\alpha$ is_______.

**Ans.** **(5625)**

**Sol.** $\Delta y = (\mu - 1)t\frac{D}{d} = 7\frac{\lambda D}{d}$

$(\mu - 1)t = 7\lambda$

$(1.56 - 1)\,t = 7 \times 450$ m

$t = 5625$ nm

**(HELD ON SATURDAY 04th APRIL 2026)** **TIME : 3:00 PM TO 6:00 PM**

| CHEMISTRY | TEST PAPER WITH SOLUTION |
|---|---|

### SECTION-A

**51.** The correct order of total number of atoms of present in

(A) 2 moles of cyclohexane

(B) 684 g of sucrose

(C) 90.8 L of dihydrogen at STP

(1) C > A > B (2) C > B > A

(3) B > C > A (4) B > A C

**Ans. (3)**

**Sol.** (A) Mole of atoms in $C_6H_{12}$

$= 2 \times 18$

$= 36$

(B) Moles of atoms of sucrose $= \frac{604}{342} \times 45$

(C) $n_{H_2} = \frac{90.8}{22.7} = 4$ moles

Mole fo atoms = 4 × 2 = 8

(B) > (A) > (C)

**52.** The species having identical radii according to the Bohr's theory are :

A. H (first orbit)

B. $He^+$ (first orbit)

C. $He^+$ (second orbit)

D. $Li^{2+}$ (first orbit)

E. $Be^{3+}$ (Second orbit)

Choose the correct answer from the options given below :

(1) A and C Only (2) A and E Only

(3) B and E Only (4) C and D Only

**Ans. (2)**

**Sol.** (A) → same

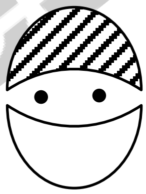
(B) → same

(C) → same

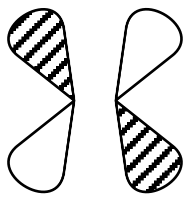
(D) → $r = \frac{90 \times (1)^2}{3} = \frac{90}{3}$

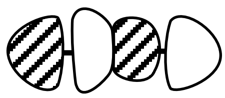
(E) → same

**53.** Which of the following pictorial diagram most correctly represents the π* (π – antibonding) molecular orbital between two atoms if the internuclear axis is taken to be in the z-direction ($\xrightarrow{\text{z-axis}}$) ?

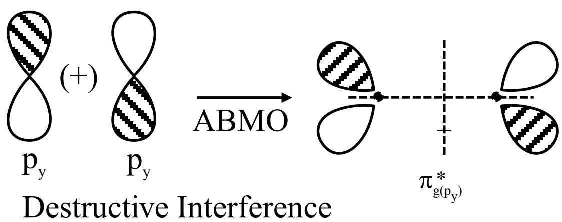
(1) 

(2) 

(3) 

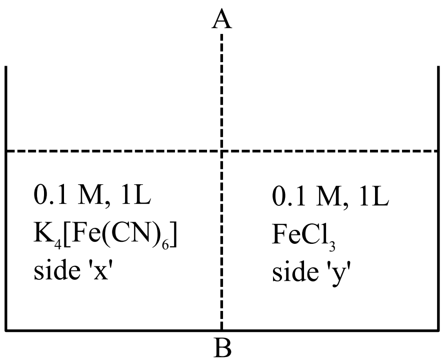
(4) 

**Ans. (3)**

**Sol.** 


Destructive Interference

**54.** At 27° C, 0.1 M, 1 L $K_4[Fe(CN)_6]$ aqueous solution and 0.1 M, 1 L $FeCl_3$ aqueous solution are placed in a container separated by a semi permeable membrane AB. Assume complete dissociation of both the solutes. Which of the following statement is ***correct*** ?




(1) Blue color is formed on both sides.

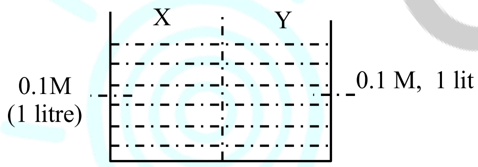
(2) Ionic solutes in aqueous solution can pass through semi-permeable membrane.

(3) Solution on side 'y' is hypotonic.

(4) To cause the reverse flow of solvent during osmosis, external pressure (any value) should be applied to side 'x'.

**Ans.** **(3)**

**Sol.**




**55.** 20 mL of a solution of acetic acid required 28.4 mL of 0.1 M NaOH for its neutralization. A solution (X) was prepared by mixing 20 mL of the above acetic acid and 14.2 mL of 0.1 M NaOH solution. What is the pH of the solution (X) ? ($pK_a$ value of acetic acid is 4.75).

(1) 7.0 (2) 4.45

(3) 3.5 (4) 4.82

**Ans.** **(2)**

**Sol.** In experiment 1, 28.4 ml of NaOH is required for complete neutralisation, therefore for 14.2 ml, half equivalence point will be achieved to form acidic buffer.

**56.** Match the **LIST-I** with **LIST-II**

| | **List-I Reaction** | | **List-II Mechanism** |
|---|---|---|---|
| A. | Williamson Synthesis | I. | Electrophilic addition |
| B. | Friedel Craft Reaction | II. | Free radical substitution |
| C. | Bromination of vinyl benzene | III. | Nucleophilic substitution |
| D. | Chlorination of toluene in light | IV. | Electrophilic substitution |

Choose the ***correct*** answer from the options given below:

(1) A-III, B-I, C-II, D-IV

(2) A-III, B-IV, C-II, D-I

(3) A-III, B-IV, C-I, D-II

(4) A-I, B-III, C-IV, D-II

**Ans.** **(3)**

**Sol.** Theory based

**57.** The 1st ionization enthalpy for Mg is +737 kJ/mol. The most probable estimated value of the 2nd ionization enthalpy of Mg is_____.

(1) – 906 kJ/mol

(2) –856 kJ/mol

(3) +1450 kJ/mol

(4) +590 kJ/mol

**Ans.** **(3)**

**Sol.** Since successive ionization energy increases thus $IE_2$ (Mg) > $IE_1$ (Mg)

**58.** The electronegativity of a group 13 element 'E' is same as that of Ge (on Pauling scale and upto one decimal point). The **CORRECT** statements about $E^{3+}$ are

A. It can act as a reducing agent.
B. It can act as an oxidizing agent.
C. $E^{3+}$ is more stable than $E^{+}$.
D. The standard electrode potential value for $E^{3+}/E$ is positive.

Choose the correct answer from the options given below

(1) A and C Only (2) B and C Only
(3) B and D Only (4) A and D Only

**Ans.** **(3)**

**Sol.** EN of '$T\ell$' is 1.8 that is same as 'Ge'

'E' = $T\ell$

$T\ell^{+3}$ acts as oxidizing agent

Stability order : $T\ell^{+3} < T\ell^{+1}$ (due to inert pair effect)

The standard electrode potential value $E^\circ(T\ell^{3+}/T\ell)$ = +1.26 V

**59.** Pairs of elements with the same number of electrons in their respective 4f orbital are
[Atomic number, Eu-63, Gd-64, Dy-66, Ho-67, Tm-69, Yb-70, Lu-71, Hf-72]

A. (Eu and Gd)
B. (Dy and Ho)
C. (Yb and Hf)
D. (Lu and Tm)

Choose the correct answer from the options given below:

(1) B and C Only (2) A and B Only
(3) A and D Only (4) A and C Only

**Ans.** **(4)**

**Sol.** Gd – $[Xe]4f^7\,5d^1\,6s^2$
Eu – $[Xe]4f^7\,6s^2$
Dy – $[Xe]4f^{10}\,6s^2$
Ho – $[Xe]4f^{11}\,6s^2$
Hf – $[Xe]4f^{14}\,6s^2\,5d^2$
Yb – $[Xe]4f^{14}\,6s^2$
Lu – $[Xe]4f^{14}\,5d^1\,6s^2$
Tm – $[Xe]4f^{13}\,6s^2$

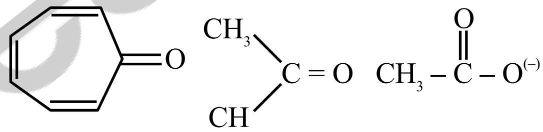
**60.** Consider the metal complexes $[Ni(en)_3]^{2+}$ (A), $[NiCl_4]^{2-}$(B) and $[Ni(NH_3)_6]^{2+}$(C). Choose the **CORRECT** option by considering the number of unpaired electron present in (A), (B) and (C) respectively and the order of frequency of absorption.

(1) 2, 2, 2 and (A) > (C) > (B)
(2) 0, 2, 0 and (A) > (C) > (B)
(3) 2, 2, 0 and (B) > (C) > (A)
(4) 2, 2, 2 and (C) > (A) > (B)

**Ans.** **(1)**

**Sol.** (A) $Ni^{2+} \Rightarrow 3d^8$ , SFL , $t_{2g}^{2,2,2}\,e_g^{1,1} \Rightarrow$ unpaired $e^- = 2$
(B) $Ni^{2+} \Rightarrow 3d^8$ , WFL , tetrahedral, $e^{2,2}\ t_2^{2,1,1}$ $\Rightarrow$ unpaired $e^- = 2$
(C) $Ni^{2+} \Rightarrow 3d^8$ , SFL , $t_{2g}^{2,2,2}\,e_g^{1,1} \Rightarrow$ unpaired $e^- = 2$
As $\Delta_o > \Delta_t$ and Ligand strength : en > $NH_3$ > $Cl^-$
Strength of ligand increases, $\Delta$ increases, hence more is the frequency required of absorption.

**61.** Consider the following molecules/species:



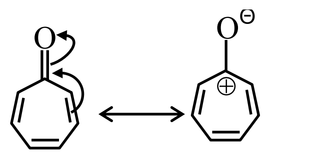
(x) (y) (z)

The correct order of carbon – oxygen double bond length is:

(1) x > y > z (2) y > z > x
(3) z > x > y (4) x > z > y

**Ans.** **(4)**

**Sol.**



RS1 RS2
Quasi aromatic

C–O bond length is maximum in compound (b) because RS2 of this compound has aromatic character.

**62.** Consider |x| is the difference in oxidation states of Mn in highest manganese fluoride and highest manganese oxide. The ions with |x| number of unpaired electrons from the following are:

A. $Sc^{3+}$ B. $Zn^{2+}$

C. $V^{2+}$ D. $Fe^{2+}$

E. $Co^{2+}$

Choose the correct answer from the options given below:

(1) A and B Only (2) C, D and E Only

(3) C and E Only (4) B and E Only

**Ans.** **(3)**

**Sol.** Oxide of Mn in its highest O.S. = $Mn_2O_7$ $[Mn^{+7}]$

Fluoride of Mn in its highest O.S. = $MnF_4$ $[Mn^{+4}]$

$\Rightarrow$ Difference in O.S. = |x| = 3

$Sc^{+3} \Rightarrow$ [Ar] ; 0 unpaired $e^-$

$Zn^{2+} \Rightarrow$ [Ar] $3d^{10}$ ; 0 unpaired electrons

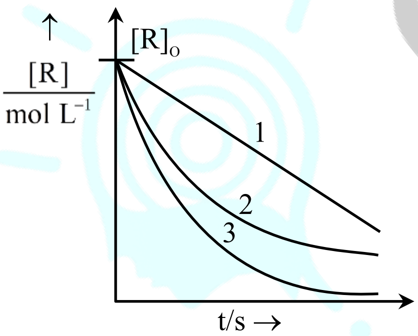
$Fe^{+2} \Rightarrow$ [Ar] $3d^6$ ; 4 unpaired electrons

$Co^{+2} \Rightarrow$ [Ar] $3d^7$ ; 3 unpaired electrons

$V^{+2} \Rightarrow$ [Ar] $3d^3$ ; 3 unpaired electrons

**63.** Consider the given graph showing variation of reactant concentration with time.

Three different reactions were started with identical initial concentration of reactants. Which of the following statement is correct ?




(1) The order of all the three reactions is same.

(2) The rate constant of reaction 3 is larger than the rate constant of reaction 2 if the order of reaction is same for both.

(3) The SI unit of rate constant of reaction 1 is $s^{-1}$.

(4) Thermal decomposition of HI on gold surface is an example of reaction 2.

**Ans.** **(2)**

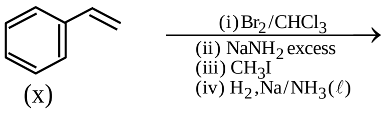
**Sol.** $\Rightarrow$ Curve I[st] represent zero order kinetics

$$[R_t] = [R]_0 - Kt$$

$\Rightarrow K_3 > K_2$

$\Rightarrow$ Reaction-1 is zero order having unit of rate constant same as rate of reaction i.e., molarity / second.

**64.**




Compound (X) is subjected to the sequence of reactions as shown above. Molar mass of the major product (Y) formed is _______ $gmol^{-1}$.

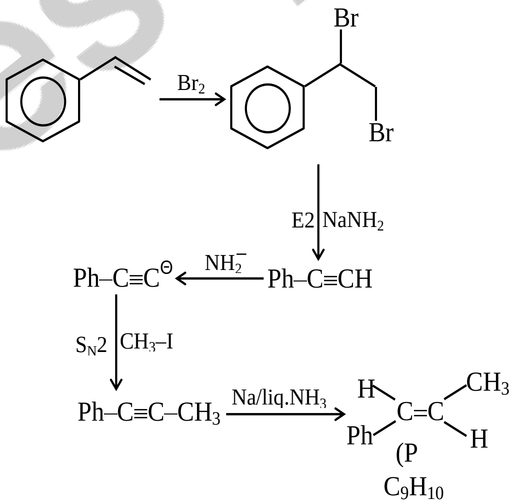
(Given molar mass in g $mol^{-1}$ C : 12, H : 1 , O : 16)

(1) 90 (2) 118

(3) 160 (4) 125

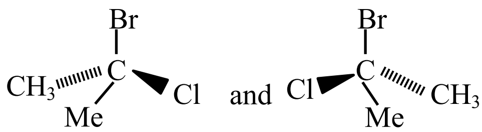
**Ans.** **(2)**

**Sol.**




Molar mass of Y = 118 gm

**65.** The following structures are



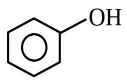

(1) enantiomer (2) Identical molecules

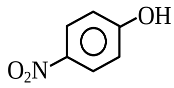
(3) Diastereomers (4) meso compound

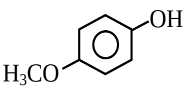
**Ans.** **(2)**

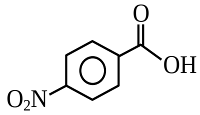
**Sol.** No chiral center (2 methyl groups)

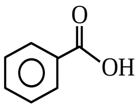
**66.** The descending order of acidity among the following compounds is

A. 

B. 

C. 

D. 

E. 

Choose the correct answer from the option given below options

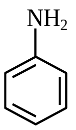
(1) B > D > E > A > C (2) D > B > E > A > C

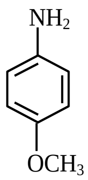
(3) C > A > B > D > E (4) D > E > B > A > C

**Ans.** **(4)**

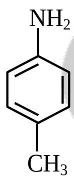
**Sol.** Theory based.
Benzoic acid is more acidic than phenol.
Electron withdrawing group in increases acidicity.

**67.** The strongest conjugate acid will result from

(1) 

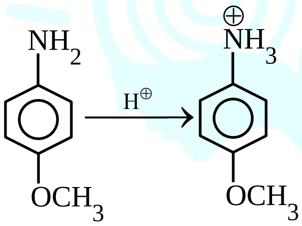
(2) 

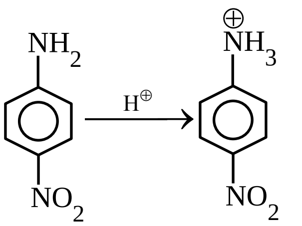
(3) 

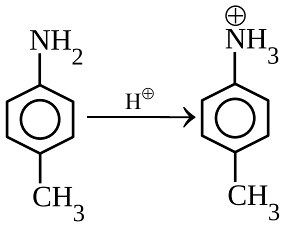
(4) 

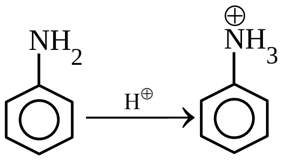
**Ans.** **(3)**

**Sol.**

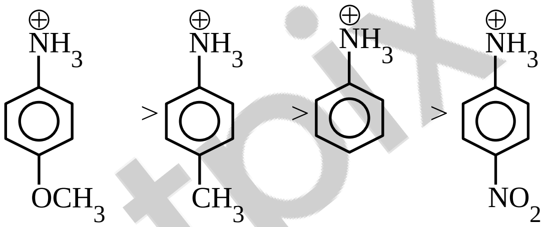






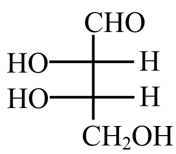


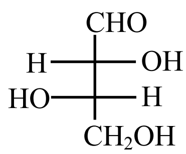
Stability order of conjugated acid :

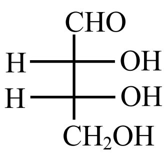


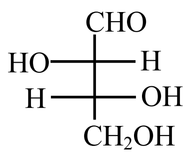
For stronger conjugate acid, we need weaker base.

**68.** A D-aldotetrose on oxidation with concentrated $HNO_3$ resulted in optically inactive dicarboxylic acid. The structure of the D-aldotetrose is :

(1) 

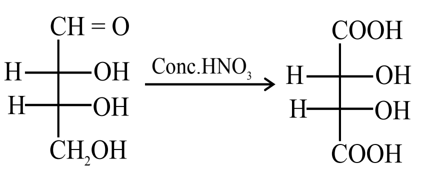
(2) 

(3) 

(4) 

**Ans.** **(3)**

**Sol.**



D-Aldotetrose

Optically inactive
Dicarboxylic acid

**69.** Among $Fe^{3+}$, $Pb^{2+}$, $Cu^{2+}$ and $Mn^{2+}$, identify the one that gets precipitated out while passing $H_2S$ in presence of $NH_4OH$ as group reagent. The highest possible oxidation state of the corresponding metal is

(1) +3 (2) +4

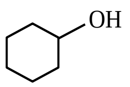
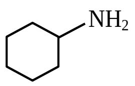
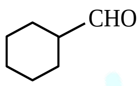
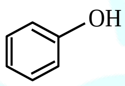
(3) +2 (4) +7

**Ans. (4)**

**Sol.** Group IV reagent : $NH_4OH + H_2S$

$Mn^{+2}$ will be precipitated out and hence highest possible oxidation state of Mn is +7

**70.** Match the LIST-I with LIST-II

| List-I Compound | | List-II Test | |
|---|---|---|---|
| A. |  | I | Hinsberg's reagent test |
| B. |  | II | Phthalein dye test |
| C. |  | III. | Lucas test |
| D. |  | IV. | Tollen's test |

Choose the correct answer from the options given below :

(1) A-III, B-I, C-IV , D-II

(2) A-III, B-IV, C-I , D-II

(3) A-I, B-III, C-II , D-IV

(4) A-I, B-II, C-III , D-IV

**Ans. (1)**

**Sol.** Theory based.

Tollen's test is used for test for aldehydes.

Heinsberg's reagent is used to test for amines.

Lucas test is used to test for alcohols.

Phthalein dye test is used for test of phenols.

## SECTION-B

**71.** If 3.365 g of ethanol ($l$) burnt completely in a bomb calorimeter at 298.15K, the heat produced is 99.472 kJ. The $|\Delta H_f^\circ|$ of ethanol at 298.15 K is ________ $\times 10^2$ kJ $mol^{-1}$. (Nearest integer)

Given :

Standard enthalpy of combustion of graphite $= -393.5$ kJ $mol^{-1}$

Standard enthalpy of formation of water ($l$) $= -285.8$ kJ $mol^{-1}$

Molar mass in $gmol^{-1}$ of C, H , O are 12, 1 and 16 respectively.

**Ans. (282)**

**Sol.** $\Delta U^o_{combustion} = \frac{-q}{moles} = \frac{-99.472}{3.365/46} \Rightarrow 1359.8$ KJ/mole

$C_2H_5OH(\ell) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$ ; $\Delta H_r^o = \Delta H_C^o$

$\Delta H_C^o = \Delta U^o + \Delta n_g RT$

$= -1359.8 + \frac{(-1) \times 8.314 \times 298.15}{1000}$

$= -1362.27$ KJ/mole

$-1362.27 = [2 \times (-393.5) + 3 \times (-285.8)] - \Delta H^o_{f\ C_2H_5OH(\ell)}$

$-1362.27 = -787 - 857.4 - \Delta H^o_{f\ [C_2H_5OH(\ell)]}$

$\Delta H^o_{f\ [C_2H_5OH(\ell)]} = -2.82 \times 10^2$ KJ/mole

$\simeq -3 \times 10^2$

**72.** For the following reaction at 50°C and 2 atm pressure,

$$2N_2O_5\ (g) \rightleftharpoons 2N_2O_4(g) + O_2(g)$$

$N_2O_5$ is 50% dissociated

The magnitude of standard free energy change at this temperature is x.

x = ________ J $mol^{-1}$ [Nearest integer]

Given : R = 8.314 $mol^{-1}K^{-1}$ , log 2 = 0.30, log 3 = 0.48, ln 10 = 2.303 , °C + 273 = K

**Ans. (2474)**

**Sol.** $2N_2O_5\,(g) \rightleftharpoons 2N_2O_4\,(g) + O_2(g)$

| | | | |
|---|---|---|---|
| t=0 | 1 | – | – |
| $t_{eqm.}$ | 1–2x | 2x | x |
| | 1–0.5 | 0.5 | 0.25 |
| | 0.5 | 0.5 | 0.25 |
| | $\frac{0.5}{1.25}\times 2$ | $\frac{0.5}{1.25}\times 2$ | $\frac{0.25}{1.25}\times 2$ |

$$K_P = \frac{P^2_{N_2O_4}.P_{O_2}}{P^2_{N_2O_5}} = 0.4$$

$\Delta G° = -2.303\ RT \log K_P$

$= -2.303 \times 8.314 \times 323 \log (4/10)$

$= 2473.81$ J/mole

**73.** An electrochemical cell. consist of the following two redox couples,

$M^{x+}(aq)/\ M(s)\ \ [E^{\ominus}_{red} = +0.15V]$ and

$Fe^{3+}(aq)\ /\ Fe(s)\ [E^{\ominus}_{red} = -0.036V]$

The cell EMF ($E_{cell}$) is recorded to be 0.2057 V.

If the reaction quotient of the electrochemical reaction is found to be $10^{-2}$, then the value of x is _______(Nearest integer)

[Given : M is a p-block metal and $\frac{2.303RT}{F} = 0.059V$]

**Ans.** **(2)**

**Sol.** $E_{cell} = E^{o}_{cell} - \frac{0.059}{3x}\log 10^{-2}$

$$0.2057 = 0.186 - \frac{0.059}{3x}(-2)$$

$$3x = \frac{0.059\times 2}{0.0197}$$

$x = 1.99 \simeq 2$

**74.** For a first order reaction $A \rightarrow B$

| t/ min | [A]/M |
|---|---|
| 0 | 0.6500 |
| x | 0.0650 |
| 20 | 0.00065 |

x = _______ min. (Nearest integer)

**Ans.** **(7)**

**Sol.** $K = \frac{1}{t}\ln\left[\frac{A_0}{A_t}\right]$

$$K = \frac{1}{20}\ln\left[\frac{0.6500}{0.00065}\right] \Rightarrow \frac{3\ln 10}{20}\text{min}^{-1}$$

$$\text{Also, } x = \frac{1}{K}\ln\left[\frac{A_0}{A_x}\right] \Rightarrow \frac{20}{3\ln 10}\ln\left(\frac{0.6500}{0.0650}\right)$$

$$= \frac{20}{3}\text{min} \Rightarrow 6.7\,\text{min}$$

**75.** In sulphur estimation, $2.0 \times 10^{-3}$ mol of an organic compound (X) (molar mass 76g $mol^{-1}$) gave 0.4813 g of barium sulphate (molar mass 233 $gmol^{-1}$). The percentage of sulphur in the compound (X) is _______ $\times 10^{-1}$% (Nearest integer)

**Ans.** **(435)**

**Sol.** $\%\text{ of S} = \frac{\text{Mass of S}}{\text{Mass of } BaSO_4}\times\frac{m}{w}\times 100$

m = gm of $BaSO_4$ ppt

w = gm of Organic compound

$w = 2 \times 10^{-3} \times 76 = 0.152$ gm

$$\%\text{ of S} = \frac{32}{233}\times\frac{0.4813}{0.152}\times 100 = 43.487$$

In form of $10^{-1}$

$= 43.487 \times 10^{-1} = 434.87 \approx 435$