(Held On Tuesday 31st January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If $\phi(x)=\frac{1}{\sqrt{x}}\int\limits_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt,\ x>0,$

then $\phi'\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{8}{\sqrt{\pi}}$

(2) $\frac{4}{6+\sqrt{\pi}}$

(3) $\frac{8}{6+\sqrt{\pi}}$

(4) $\frac{4}{6-\sqrt{\pi}}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\phi'(x)=\frac{1}{\sqrt{x}}\left[\left(4\sqrt{2}\sin x-3\phi'(x)\right).1-0\right]-\frac{1}{2}x^{-3/2}$

$\int\limits_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt,$

$\phi'\left(\frac{\pi}{4}\right)=\frac{2}{\sqrt{\pi}}\left[4-3\phi'\left(\frac{\pi}{4}\right)\right]+0$

$\left(1+\frac{6}{\sqrt{\pi}}\right)\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}}$

$\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}+6}$

2. If a point $P(\alpha,\beta,\gamma)$ satisfying $(\alpha\ \beta\ \gamma)\begin{pmatrix}2&10&8\\9&3&8\\8&4&8\end{pmatrix}=(0\ 0\ 0)$ lies on the plane $2x+4y+3z=5,$ then $6\alpha+9\beta+7\gamma$ is equal to:

(1) -1

(2) $\frac{11}{5}$

(3) $\frac{5}{4}$

(4) 11

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $2\alpha+4\beta+3\gamma=5$ ……..(1)

$2\alpha+9\beta+8\gamma=0$ …….. (2)

$10\alpha+3\beta+4\gamma=0$ ……. (3)

$8\alpha+8\beta+8\gamma=0$ ……..(4)

Subtract (4) from (2)

$-6\alpha+\beta=0$

$\beta=6\alpha$ …… (5)

From equation (4)

$8\alpha+48\alpha+8\gamma=0$

$\gamma=-7\alpha$ …….. (6)

From equation (1)

$2\alpha+24\alpha-21\alpha=5$

$5\alpha=5$

$\alpha=1$

$\beta=+6,\quad \gamma=-7$

$\therefore 6\alpha+9\beta+7\gamma$

$=6+54-49$

$=11$

3. Let $a_1, a_2, a_3, \ldots\ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

(1) 24

(2) $\frac{33}{4}$

(3) $\frac{381}{4}$

(4) 9

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. a+6d=3, ……..(1)

Z= a (a+3d)

= (3-6d) (3-3d)

$=18d^2-27d+9$

Differentiating with respect to d

$\Rightarrow$ 36d-27=0

$\Rightarrow d=\frac{3}{4}$, from (1) $a = \frac{-3}{2}$, (Z = minimum)

Now, $S_n=\frac{n}{2}\left(-3+(n-1)\frac{3}{4}\right)=0$

$\Rightarrow$ n=5

Now,

$n!-4a_{n(n+2)} = 120-4(a_{35})$

$= 120-4\left(a+(35-1)d\right)$

$= 120-4\left(\frac{-3}{2}+34.\left(\frac{3}{4}\right)\right)$

$= 120-4\left(\frac{-6+102}{4}\right)$

$= 120-96 = 24$

4. Let (a, b) $\subset (0, 2\pi)$ be the largest interval for which

$\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0, \theta\in(0,2\pi)$, holds. If

$\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}$

$\left(x^2-6x+10\right)=0$

and $\alpha-\beta=b-a$, then α is equal to :

(1) $\frac{\pi}{48}$

(2) $\frac{\pi}{16}$

(3) $\frac{\pi}{8}$

(4) $\frac{\pi}{12}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\sin^{-1}\sin\theta-\left(\frac{\pi}{2}-\sin^{-1}\sin\theta\right)>0$

$\Rightarrow \sin^{-1}\sin\theta>\frac{\pi}{4}$

$\Rightarrow \sin\theta>\frac{1}{\sqrt{2}}$

So, $\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

$\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)=(a,b)$

$b-a=\frac{\pi}{2}=\alpha-\beta$

$\Rightarrow \beta=\alpha-\frac{\pi}{2}$

$\Rightarrow \alpha x^2+\beta x+\sin^{-1}\left[(x-3)^2+1\right]+\cos^{-1}\left[(x-3)^2+1\right]=0$

$x=3,\ 9\alpha+3\beta+\frac{\pi}{2}+0=0$

$\Rightarrow 9\alpha + 3\left(\alpha - \frac{\pi}{2}\right) + \frac{\pi}{2} = 0$

$\Rightarrow 12\alpha - \pi = 0$

$\alpha = \frac{\pi}{12}$

5. Let $y = y(x)$ be the solution of the differential equation $\left(3y^2 - 5x^2\right)y\,dx + 2x\left(x^2 - y^2\right)dy = 0$ such that $y(1) = 1$. then $\left|\left(y(2)\right)^3 - 12y(2)\right|$ is equal to:

(1) $32\sqrt{2}$

(2) 64

(3) $16\sqrt{2}$

(4) 32

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\left(3y^2 - 5x^2\right)y.\,dx + 2x\left(x^2 - y^2\right)dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y\left(5x^2 - 3y^2\right)}{2x\left(x^2 - y^2\right)}$

Put $y = mx$

$\Rightarrow m + x.\frac{dm}{dx} = \frac{m\left(5 - 3m^2\right)}{2\left(1 - m^2\right)}$

$x.\frac{dm}{dx} = \frac{\left(5 - 3m^2\right)m - 2m\left(1 - m^2\right)}{2\left(1 - m^2\right)}$

$\Rightarrow \frac{dx}{x} = \frac{2\left(m^2 - 1\right)}{m\left(m^2 - 3\right)}dm$

$\Rightarrow \frac{dx}{x} = \left(\frac{2}{m} - \frac{\frac{4}{3}}{m} + \frac{\frac{4m}{3}}{m^2 - 3}\right)dm$

$\Rightarrow \int\frac{dx}{x} = \int\frac{\left(\frac{2}{3}\right)}{m} + \int\frac{2}{3}\left(\frac{2m}{m^2 - 3}\right)dm$

$\Rightarrow \ln|x| = \frac{2}{3}\ln|m| + \frac{2}{3}\ln\left|m^2 - 3\right| + C$

Or, $\ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| + C$

Put $(x = 1, y = 1)$: we get $c = -\frac{2}{3}\ln(2)$

$\Rightarrow \ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| - \frac{2}{3}\ln(2)$

$\Rightarrow \left(\frac{y}{x}\right)\left|\left(\frac{y}{x}\right)^2 - 3\right| = 2.(x^{3/2})$

Put x = 2 to get y (2)

$\Rightarrow y\left(y^2 - 12\right) = 4 \times 2 \times 2 \times 2\sqrt{2}$

$\Rightarrow y^3 - 12y = 32\sqrt{2}$

$\Rightarrow \left|y^3(2) - 12y(2)\right| = 32\sqrt{2}$

6. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$ is equal to:

(1) $(8, \infty)$

(2) $(4, \infty)$

(3) $(0, 4]$

(4) $(2, 12]$

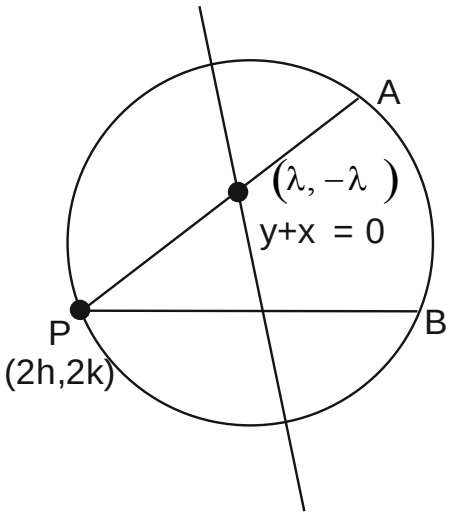
Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $x^2+y^2-\frac{(1+a)x}{2}-\frac{(1-a)y}{2}=0$

Centre $\left(\frac{1+a}{4},\frac{1-a}{4}\right)\Rightarrow(h,k)$

$P\left(\frac{1+a}{2},\frac{1-a}{2}\right)\Rightarrow(2h,2k)$



Equation of chord $\Rightarrow T=S_1$

$\Rightarrow(x-y)\lambda-\frac{2h(x+\lambda)}{2}-\frac{(2k)(y-\lambda)}{2}$

$=2\lambda^2-2h(\lambda)+2k\lambda$

Now, $\lambda(2h,2k)$ satisfies the chord

$\therefore\ (2h-2k)\lambda-h(x+\lambda)-k(y-\lambda)$

$\Rightarrow 2\lambda^2+4k\lambda-4h\lambda+h\lambda-k\lambda+hx+ky=0$

$\Rightarrow 2\lambda^2+\lambda(3k-3h)+ky+hx=0$

$\Rightarrow D>0$

$\Rightarrow 9(k-h)^2-8(ky+hx)>0$

$\Rightarrow 9(k-h)^2-8(2k^2+2h^2)>0$

$\Rightarrow -7k^2-7h^2-18kh>0$

$\Rightarrow 7k^2+7h^2+18kh<0$

$\Rightarrow 7\left(\frac{1-a}{4}\right)^2+7\left(\frac{1+a}{4}\right)^2+18\left(\frac{1-a^2}{16}\right)<0$

$\Rightarrow 7\left[\frac{2(1+a^2)}{16}\right]+\frac{18(1-a^2)}{16}<0,\quad a^2=t$

$\Rightarrow \frac{7}{8}(1+t)+\frac{18(1-t)}{16}<0$

$\Rightarrow \frac{14+14t+18-18t}{16}<0$

$\Rightarrow 4t>32$

$t>8 \qquad a^2>8$

7. Among the relations

$S=\left\{(a,b):a,b\in\mathbb{R}-\{0\},2+\frac{a}{b}>0\right\}$

And $T=\{(a,b):a,b\in\mathbb{R},a^2-b^2\in Z\}$,

(1) S is transitive but T is not

(2) T is symmetric but S is not

(3) Neither S nor T is transitive

(4) Both S and T are symmetric

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. For relation $T=a^2-b^2=-I$

Then, (b, a) on relation R

$\Rightarrow b^2-a^2=-I$

$\therefore$ T is symmetric

$S=\left\{(a,b):a,b\in R-\{0\},2+\frac{a}{b}>0\right\}$

$2+\frac{a}{b}>0\Rightarrow\frac{a}{b}>-2\ ,\Rightarrow\frac{b}{a}<\frac{-1}{2}$

If $(b,a)\in S$ then

$2+\frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

8. The equation

$e^{4x}+8e^{3x}+13e^{2x}-8e^{x}+1=0,x\in R$ has:

(1) two solutions and both are negative

(2) no solution

(3) four solutions two of which are negative

(4) two solutions and only one of them is negative

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $e^{4x}+8e^{3x}+13e^{2x}-8e^{x}+1=0$

Let $e^{x}=t$

Now, $t^4+8t^3+13t^2-8t+1=0$

Dividing equation by t^2,

$$t^2+8t+13-\frac{8}{t}+\frac{1}{t^2}=0$$

$$t^2+\frac{1}{t^2}+8\left(t-\frac{1}{t}\right)+13=0$$

$$\left(t-\frac{1}{t}\right)^2+2+8\left(t-\frac{1}{t}\right)+13=0$$

Let $t-\frac{1}{t}=z$

$$z^2+8z+15=0$$

$$(z+3)(z+5)=0$$

$z=-3$ or $z=-5$

So, $t-\frac{1}{t}=-3$ or $t-\frac{1}{t}=-5$

$t^2+3t-1=0$ or $t^2+5t-1=0$

$$t=\frac{-3\pm\sqrt{13}}{2} \text{ or } t=\frac{-5\pm\sqrt{29}}{2}$$

as $t=e^{x}$ so t must be positive,

$$t=\frac{\sqrt{13}-3}{2} \text{ or } \frac{\sqrt{29}-5}{2}$$

So, $x=\ln\left(\frac{\sqrt{13}-3}{2}\right)$ or $x=\ln\left(\frac{\sqrt{29}-5}{2}\right)$

Hence two solution and both are negative.

9. The number of values of $r\in\{p, q, \sim p, \sim q\}$ for which $((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$ is a tautology, is:

(1) 3

(2) 2

(3) 1

(4) 4

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$

We know, $p\Rightarrow q$ is equivalent to $\sim p\vee q$

$$(\sim(p\wedge q)\vee(r\vee q))\wedge(\sim(p\wedge r)\vee q))$$

$$\Rightarrow(\sim p\vee\sim q\vee r\vee q)\wedge(\sim p\vee\sim r\vee q)$$

$$\Rightarrow(\sim p\vee r\vee t)\wedge(\sim p\vee\sim r\vee q)$$

$$\Rightarrow(t)\wedge(\sim p\vee\sim r\vee q)$$

For this to be tautology, $(\sim p\vee\sim r\vee q)$ must be always true which follows for $r=\sim p$ or $r=q$.

10. Let $f:\mathbb{R}-\{2,6\}\to\mathbb{R}$ be real valued function defined as $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty,-\frac{21}{4}\right]\cup[0,\infty)$

(2) $\left(-\infty,-\frac{21}{4}\right)\cup(0,\infty)$

(3) $\left(-\infty,-\frac{21}{4}\right]\cup\left[\frac{21}{4},\infty\right)$

(4) $\left(-\infty,-\frac{21}{4}\right]\cup[1,\infty)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Let $y=\frac{x^2+2x+1}{x^2-8x+12}$

By cross multiplying

$$yx^2-8xy+12y-x^2-2x-1=0$$

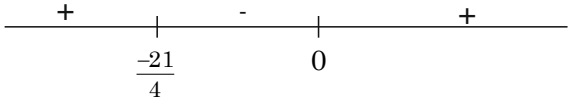
$$x^2(y-1)-x(8y+2)+(12y-1)=0$$

Case 1, $y\neq1$

$D\geq0$

$\Rightarrow (8y+2)^2-4(y-1)(12y-1)\geq0$

$\Rightarrow y(4y+21)\geq0$



$$y\in\left(-\infty,\frac{-21}{4}\right]\cup[0,\infty)-\{1\}$$

Case 2, $y=1$

$$x^2+2x+1=x^2-8x+12$$

$$10x=11$$

$x = \frac{11}{10}$ So, y can be 1

Hence $y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$

11. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6 + \left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6} x^3$

(1) is equal to 9

(2) is equal to 27

(3) does not exist

(4) is equal to $\frac{27}{2}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6 + \left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6} x^3$

$$\lim_{x\to\infty} x^3 \times \left\{ \frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6 + \left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6 + \left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right\}} \right\}$$

$$= \frac{\left(2\sqrt{3}\right)^6 + 0}{2^6 + 0} = 3^3 = (27)$$

12. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is

(1) 6

(2) 4

(3) 5

(4) 7

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Equation of Plane :

$2(x-1) - 3(y+1) - 6(z+5) = 0$

Or $2x - 3y - 6z = 35$

$\Rightarrow$ Required distance $= \frac{|2(3) - 3(-2) - 6(2) - 35|}{\sqrt{4+9+36}}$

$= 5$

13. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + \lfloor x^2 - x + 1 \rfloor$, where [t] denotes the greatest integer function, in the interval [-1, 2], is :

(1) $\frac{3}{4}$

(2) $\frac{3}{2}$

(3) $\frac{1}{4}$

(4) $\frac{5}{4}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $f(x) = |x^2 - x + 1| + \lfloor x^2 - x + 1 \rfloor;\ x \in \lfloor -1, 2 \rfloor$

Let $g(x) = x^2 - x + 1$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because |x^2 - x + 1|$ and $\lfloor x^2 - x + 2 \rfloor$

Both have minimum value at $x = 1/2$

$\Rightarrow$ Minimum $f(x) = \frac{3}{4} + 0$

$= \frac{3}{4}$

14. Let the plane $P: 8x+\alpha_1 y+\alpha_2 z+12=0$ be parallel to the line $L: \frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is :

(1) $\sqrt{14}$

(2) $\frac{6}{\sqrt{14}}$

(3) $\sqrt{\frac{2}{7}}$

(4) $\sqrt{\frac{7}{2}}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. P: $8x+\alpha_1 y+\alpha_2 z+12=0$

L: $\frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$

$\because$ P is parallel to L

$\Rightarrow 8(2)+\alpha_1(3)+5(\alpha_2)=0$

$\Rightarrow 3\alpha_1+5(\alpha_2)=-16$

Also y-intercept of plane P is 1

$\Rightarrow \alpha_1=-12$

And $\alpha_2=4$

$\Rightarrow$ Equation of plane P is $2x-3y+z+3=0$

$\Rightarrow$ Distance of line L from Plane P is

$=\left|\frac{0-3(6)+1+3}{\sqrt{4+9+1}}\right|$

$=\sqrt{14}$

15. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is (2, a, 4), $a \in N$. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is NOT on P?

(1) (2, 2, 4)

(2) (0, 4, 4)

(3) (3, 0, 4)

(4) (0, 6, 3)

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Equation of Plane:

$\left(2\hat{i}+a\hat{j}+4\hat{k}\right).\left[(x-2)\hat{i}+(y-a)\hat{j}+(z-4)\hat{k}\right]=0$

$\Rightarrow 2x+ay+4z=20+a^2$

$\Rightarrow A\equiv\left(\frac{20+a^2}{2},0,0\right)$

$B\equiv\left(0,\frac{20+a^2}{a},0\right)$

$C\equiv\left(0,0,\frac{20+a^2}{4}\right)$

$\Rightarrow$ Volume of tetrahedron

$=\frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$=\frac{1}{6}\vec{a}.\left(\vec{b}\times\vec{c}\right)$

$\Rightarrow\frac{1}{6}\left(\frac{20+a^2}{2}\right).\left(\frac{20+a^2}{a}\right).\left(\frac{20+a^2}{4}\right)=144$

$\Rightarrow\left(20+a^2\right)^3=144\times48\times a$

$\Rightarrow a=2$

$\Rightarrow$ Equation of plane is $2x+2y+4z=24$

Or $x+y+2z=12$

$\Rightarrow(3,0,4)$ Not lies on the Plane $x+y+2z=12$

16. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha(>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of 100 + n students are respectively 50 and 350, then the sum of variances of classes A and B is:

(1) 500 (2) 650

(3) 450 (4) 900

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

A	B	A+B
$\bar{x}_1 = 40$	$\bar{x}_2 = 55$	$\bar{x} = 50$
$\sigma_1 = \alpha$	$\sigma_2 = 30-\alpha$	$\sigma^2 = 350$
$n_1 = 100$	$n_2 = n$	$100+n$

$$\bar{x} = \frac{100\times 40 + 55n}{100+n}$$

$$5000 + 50n = 4000 + 55n$$

$$1000 = 5n$$

$$n = 200$$

$$\sigma_1^{\,2} = \frac{\sum x_i^{\,2}}{100} - 40^2$$

$$\sigma_2^{\,2} = \frac{\sum x_j^{\,2}}{100} - 55^2$$

$$350 = \sigma^{\,2} = \frac{\sum x_i^{\,2} + \sum x_j^{\,2}}{300} - \left(\bar{x}\right)^2$$

$$350 = \frac{\left(1600+\alpha^2\right)\times 100 + \left\lfloor \left(30-\alpha\right)^2 + 3025 \right\rfloor \times 200}{300} - (50)^2$$

$$2850\times 3 = \alpha^2 + 2\left(30-\alpha\right)^2 + 1600 + 6050$$

$$8550 = \alpha^2 + 2\left(30-\alpha\right)^2 + 7650$$

$$\alpha^2 + 2\left(30-\alpha\right)^2 = 900$$

$$\alpha^2 - 40\alpha + 300 = 0$$

$$\alpha = 10, 30$$

$$\sigma_1^{\,2} + \sigma_2^{\,2} = 10^2 + 20^2 = 500$$

17. Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$. Then $25\left|\vec{r}\right|^2$ is equal to

(1) 449 (2) 336

(3) 339 (4) 560

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$$

$$\left(\vec{r} - \vec{c}\right)\times\vec{b} = 0, \quad \vec{r}.\vec{a} = 0$$

$$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{b}$$

Also, $\left(\vec{c} + \lambda\vec{b}\right).\vec{a} = 0$

$$\Rightarrow \quad \vec{a}.\vec{c} + \lambda\left(\vec{a}.\vec{b}\right) = 0$$

$$\therefore \lambda = \frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}} = \frac{-8}{5}$$

$$\vec{r} = \frac{5\left(5\hat{i} - 3\hat{i} + 3\hat{k}\right) - 8\left(\hat{i} - \hat{j} + 2\hat{k}\right)}{5}$$

$$\vec{r} = \frac{17\hat{i} - 7\hat{j} + \hat{k}}{5}$$

$$\left|\vec{r}\right|^2 = \frac{1}{25}(289 + 50)$$

$$25\left|\vec{r}\right|^2 = 339$$

18. Let H be the hyperbola, whose foci are $(1\pm\sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its lat us rectum is _____.

(1) 2 (2) 3

(3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $2ae = \left|(1+\sqrt{2}) - (1+\sqrt{2})\right| = 2\sqrt{2}$

$$ae = \sqrt{2}$$

$$a = 1$$

$\Rightarrow b = 1 \quad \because e = \sqrt{2} \Rightarrow$ Hyperbola is rectangular

$$\Rightarrow \text{L.R} = \frac{2b^2}{a} = 2$$

19. Let $\alpha > 0$. If $\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to :

(1) 2

(2) 4

(3) $\sqrt{2}$

(4) $2\sqrt{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. After rationalising

$$\int_0^{\alpha} \frac{x}{\alpha}\left(\sqrt{x+\alpha}+\sqrt{x}\right)$$

$$\int_0^{\alpha} \frac{1}{\alpha}\left[(x+\alpha)^{3/2}-\alpha(x+\alpha)^{1/2}+x^{3/2}\right]$$

$$\frac{1}{\alpha}\left|\frac{2}{5}(x+\alpha)^{5/2}-\alpha\frac{2}{3}(x+\alpha)^{3/2}+\frac{2}{5}x^{5/2}\right|_0^{\alpha}$$

$$=\frac{1}{\alpha}\left(\frac{5}{2}(2\alpha)^{5/2}-\frac{2\alpha}{3}(2\alpha)^{3/2}+\frac{2}{5}\alpha^{5/2}-\frac{2}{5}\alpha^{5/2}+\frac{2}{3}\alpha^{5/2}\right)$$

$$=\frac{1}{\alpha}\left(\frac{2^{7/2}\alpha^{5/2}}{5}\frac{2^{5/2}\alpha^{5/2}}{3}+\frac{2}{3}\alpha^{5/2}\right)$$

$$=\alpha^{3/2}\left(\frac{2^{7/2}}{5}-\frac{2^{5/2}}{3}+\frac{2}{3}\right)$$

$$=\frac{\alpha^{3/2}}{15}\left(24\sqrt{2}-20\sqrt{2}+10\right)=\frac{\alpha^{3/2}}{15}\left(4\sqrt{2}+10\right)$$

Now,

$$\frac{\alpha^{3/2}}{15}\left(4\sqrt{2}+10\right)=\frac{16+20\sqrt{2}}{15}$$

$$\Rightarrow \alpha = 2$$

20. The complex number $z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to:

(1) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(2) $\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}$

(3) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

(4) $\sqrt{2}\,i\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $$Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=\frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$$

$$=\frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}\times\frac{\frac{1}{2}-\sqrt{\frac{3}{2}}i}{\frac{1}{2}-\sqrt{3/2}i}=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$$

Apply polar form,

$$r\cos\theta=\frac{\sqrt{3}-1}{2}$$

$$r\sin\theta=\frac{\sqrt{3}+1}{2}$$

Now, $\tan\theta=\frac{\sqrt{3}+1}{\sqrt{3}-1}$

So, $\theta=\frac{5\pi}{12}$

21. The Coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$, is _____

Official Ans. by NTA (5040)

QuestPix Ans. (5040)

Sol: $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$,

Now, $T_{r+1}= {}^9C_r.\left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$

$= {}^9C_r.\left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r.x^{9-3r}$

Coefficient of x^{-6} i.e. 9-3r = -6 $\Rightarrow r=5$

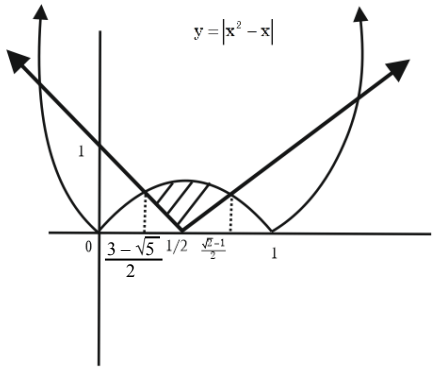
So, Coefficient of $x^{-6} = {}^9C_5\left(\frac{4}{5}\right)^4.\left(\frac{5}{2}\right)^5 = 5040$

22. Let the area of the region $\{(x,y):|2x-1|\le y\le|x^2-x|, 0\le x\le 1\}$ be A. Then $(6A+11)^2$ is equal to _____.

Official Ans. by NTA (125)

QuestPix Ans. (125)

Sol: $y\ge|2x-1|,\ y\le|x^2-x|$



Both curve are symmetric about $x=\frac{1}{2}$ Hence

$$A=2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}\left(\left(x-x^2\right)-\left(1-2x\right)\right)dx$$

$$A=2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}\left(-x^2+3x-1\right)dx=2\left(\frac{-x^3}{3}+\frac{3}{2}x^2-x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$$

On solving $6A+11=5\sqrt{5}$

$(6A+11)^2=125$

23. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$, then n^2+n+15 is equal to:

Official Ans. by NTA (45)

QuestPix Ans. (45)

Sol: $\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!}=\frac{11}{21}$

$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n}=\frac{11}{21}$

$\Rightarrow \frac{2n+1}{(n+1)(n+2)}=\frac{11}{42}$

$\Rightarrow n=5$

$\Rightarrow n^2+n+15=25+5+15=45$

24. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^{\ell}}\right)^9$ is – 84 and the Coefficient of $x^{-3\ell}$ is $2^{\alpha}\beta$, where $\beta<0$ is an odd number, Then $|\alpha\ell-\beta|$ is equal to _____

Official Ans. by NTA (98)

QuestPix Ans. (98)

Sol. **In,** $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^{\ell}}\right)^9$

$T_{r+1}={}^9C_r\frac{\left(x^{5/2}\right)^{9-r}}{2^{9-r}}\left(\frac{-4}{x^{\ell}}\right)^r$

$=(-1)^r\frac{{}^9C_r}{2^{9-r}}4^r x^{\frac{45}{2}-\frac{5r}{2}-lr}$

$= 45-5r-2lr=0$

$r=\frac{45}{5+2l}$ ------ (1)

Now, according to the question, $(-1)^r\frac{{}^9C_r}{2^{9-r}}4^r=-84$

$=(-1)^r\ {}^9C_r\,2^{3r-9}=21\times 4$

Only natural value of r possible if $3r-9=0$

$r=3$ and ${}^9C_3=84$

$\therefore l = 5$ from equation (1)

Now, coefficient of $x^{-3l} = x^{\frac{45}{2}-\frac{5r}{2}-lr}$ at $l = 5$, gives $r = 5$

$\therefore \ {}^{9}c_{5}(-1)\frac{4^5}{2^4} = 2^{\alpha} \times \beta$

$= -63 \times 2^7$

$\Rightarrow \alpha = 7, \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

25. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a}.\vec{b}}\right)^2$ is equal to _____.

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2 |2\vec{b} + 3\vec{c}|^2$

$|\vec{a}|^2 = \lambda^2 (4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b}.\vec{c})$

$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b} + 3\vec{c})$

$\frac{|\vec{a} \times \vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2|\vec{b} \times \vec{c}|}{2\vec{b}.\vec{b} + 3\vec{c}.\vec{b}}$

$|\vec{b} \times \vec{c}|^2 = |\vec{b}|^2 |\vec{c}|^2 - (\vec{b}.\vec{c})^2 = \frac{3}{4}$

$\frac{|\vec{a} \times \vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2 \times \frac{\sqrt{3}}{2}}{2.\frac{1}{4} - \frac{3}{2}} = -\sqrt{3}$

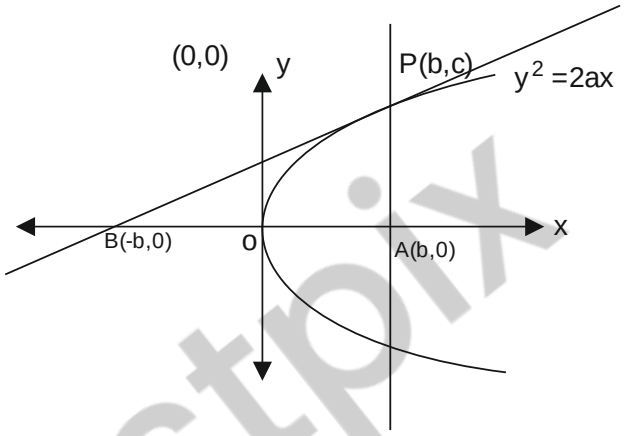
$\left(\frac{\vec{a} \times \vec{c}}{\vec{a}.\vec{b}}\right)^2 = 3$

26. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point P (b, c), $b, c \in N$, on the parabola $y^2 = 2ax$ and the lines x = b, y = 0 is 16 unit^2, then $\sum_{a \in S} a$ is equal to _____.

Official Ans. by NTA (146)

QuestPix Ans. (146)

Sol.



As $P(b, c)$ lies on parabola so $c^2 = 2ab$ ---- (1)

Now equation of tangent to parabola $y^2 = 2ax$ in point form is $yy_1 = 2a\frac{(x + x_1)}{2}$, $(x_1, y_1) = (b, c)$

$\Rightarrow yc = a(x + b)$

For point B, put y = 0, now x= -b

So, area of ΔPBA, $\frac{1}{2} \times AB \times AP = 16$

$\Rightarrow \frac{1}{2} \times 2b \times c = 16$

$\Rightarrow bc = 16$

As b and c are natural number so possible values of (b, c) are (1, 16), (2, 8), (4,4), (8, 2) and (16,1)

Now from equation (1) $a = \frac{c^2}{2b}$ and $a \in N$, so values of (b, c) are $(1,16), (2,8)$ and $(4, 4)$ now values of are 128, 16 and 2.

Hence sum of values of a is 146.

27. The sum

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - ..... + 15.29^2$

is_____.

Official Ans. by NTA (6952)

QuestPix Ans. (6952)

Separating odd placed and even placed terms we get

$S = (1.1^2 + 3.5^2 + ...15.(29)^2) - (2.3^2 + 4.7^2 + ... + 14.(27)^2$

$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2$

Applying summation formula we get

$= 29856 - 22904 = 6952$

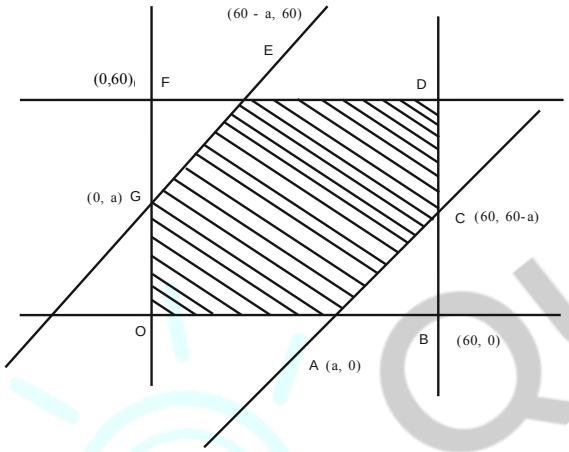
28. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space $[0,60]$ is less than or equal to a . If P (A)= $\frac{11}{36}$, then a is equal to_____.

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol: $|x-y| < a \Rightarrow -a < x-y < a$

$\Rightarrow x-y < a \text{ and } x-y > -a$



$P(A) = \frac{ar(OACDEG)}{(OBDF)}$

$= \frac{ar(OBDF) - ar(ABC) - ar(EFG)}{ar(OBDF)}$

$\Rightarrow \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$

$\Rightarrow \quad 1100 = 3600 - (60-a)^2$

$\Rightarrow \quad (60-a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow \quad a = 10$

29. Let $A = [a_{ij}], a_{ij} \in Z \cap [0,4], 1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2,13)$ is _____.

Official Ans. by NTA (196)

QuestPix Ans. (204)

As given a + b + c + d = 3 or 5 or 7 or 11

if sum = 3

$(1 + x + x^2 + ... + x^4)^4 \to x^3$

$(1-x^5)^4(1-x)^{-4} \to x^3$

$\therefore \ {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1-4x^5)(1-x)^{-4} \to x^5$

$\Rightarrow {}^{4+5-1}C_5 - 4x^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1-4x^5)(1-x)^{-4} \to x^7$

$\Rightarrow {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 11

$(1-4x^5+6x^{10})(1-x)^{-4} \to x^{11}$

$\Rightarrow {}^{4+11-1}C_{11} - 4.{}^{4+6-4}C_6 + 6.{}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4.{}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

30. Let A be a $n \times n$ matrix such that |A|=2. If the determinant of the matrix Adj (2. Adj($2A^{-1}$)). is 2^{84}, then n is equal to _____.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $\left|\text{Adj}\left(2\text{Adj}(2A^{-1})\right)\right|$

$= \left|2\text{Adj}\left(\text{Adj}(2A^{-1})\right)\right|^{n-1}$

$= 2^{n(n-1)}\left|\text{Adj}(2A^{-1})\right|^{n-1}$

$= 2^{n(n-1)}\left|(2A^{-1})\right|^{(n-1)(n-1)}$

$= 2^{n(n-1)}2^{n(n-1)(n-1)}\left|A^{-1}\right|^{(n-1)(n-1)}$

$= 2^{n(n-1)+n(n-1)(n-1)}\frac{1}{|A|^{(n-1)^2}}$

$= \frac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}}$

$= 2^{n(n-1)+n(n+1)^2-(n-1)^2}$

$= 2^{(n-1)(n^2-n+1)}$

Now, $2^{(n-1)(n^2-n+1)}$

$2^{(n-1)(n^2-n+1)} = 2^{84}$

So, n = 5

(Held On Tuesday 31st January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4A is passed through it. If the current is increased to 16A, the thermal energy developed by the resistor in 10 s will be:

(1) H (2) 16H

(3) $\frac{H}{4}$ (4) 4H

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $H = i^2Rt = 4^2R \times 10 = 160R$

$H' = I^2Rt = 16^2R \times 10 = 2560R = 16\ H$

32. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4 s. At the end of 3rd second, the displacement of body (in m) from its starting point is:

(1) 30 (2) 15π

(3) 5π (4) $10\sqrt{2}$

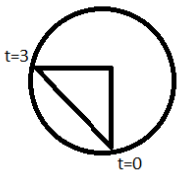
Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.

$$\theta = \frac{3}{4} \times 2\pi = \frac{3\pi}{2}$$

= |Displacement| $= \sqrt{2}R$



$= 10\sqrt{2}$ m

33. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then microscope have to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is :

(1) 75 cm (2) 50 cm

(3) 18 cm (4) 12 cm

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $d = h\left(1 - \frac{1}{\mu}\right)$

$30 = h\left(1 - \frac{3}{5}\right)$

$30 = h \times \frac{2}{5}$

h = 75 cm

34. A stone of mass 1 kg is tied to end of a massless string of length 1 m. If the breaking tension of the string is 400 N, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is:

(1) 20 ms^{-1} (2) 40 ms^{-1}

(3) 400 ms^{-1} (4) 10 ms^{-1}

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $T = \frac{mv^2}{r}$

$V = \sqrt{\frac{Tr}{m}}$

$V = \sqrt{\frac{400 \times 1}{1}} = 20\ ms^{-1}$

35. For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11}\ Nm^{-2}$ and density is $8 \times 10^3\ kg\ m^{-3}$. The velocity of longitudinal wave in the rod will be.

(1) $145.75 \times 10^3\ ms^{-1}$

(2) $3.65 \times 10^3\ ms^{-1}$

(3) $18.96 \times 10^3\ ms^{-1}$

(4) $6.32 \times 10^3\ ms^{-1}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $V = \sqrt{\frac{Y}{\rho}}$

$= \sqrt{\frac{3.2 \times 10^{11}}{8000}}$

$= 6.32 \times 10^3\ ms^{-1}$

36. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

(1) N : n (2) $n^2 : N^2$

(3) $N^2 : n^2$ (4) n : N

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Let the length of wire is *l*, thus the radius of first coil

$\boldsymbol{R_1 = \frac{l}{2\pi N}}$ **and the radius of second coil**

$\boldsymbol{R_2 = \frac{l}{2\pi n}}$

So $\boldsymbol{B_1 = \frac{\mu_0 Ni}{\frac{2l}{2\pi N}} = \frac{\mu_0 N^2 \pi i}{l}}$

And $\boldsymbol{B_2 = \frac{\mu_0 ni}{\frac{2l}{2\pi n}} = \frac{\mu_0 n^2 \pi i}{l}}$

So $\boldsymbol{\frac{B_1}{B_2} = \frac{N^2}{n^2}}$

37. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be:

(1) 525 J (2) 441 J

(3) 572 J (4) 735 J

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $Q = nC_P\Delta T$

$735 = n \times \frac{7R}{2} \times \Delta T$

$105 = \frac{n}{2} R\,\Delta T$

Since diatomic gas has degree of freedom = 5

$525 = \frac{5n}{2} R\,\Delta T$

$\Delta U = 525J$

38. Given below are two statements

Statement I: For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension).

Statement II: In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) **Statement I** is correct but **Statement II** is incorrect

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. In amplitude modulation, amplitude of Carrier wave changes and frequency remains same. Minimum size of antenna is $\frac{\lambda}{4}$

39. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be :

(1) 100% (2) 50%

(3) 75% (4) 0%

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Current sensitivity

$\frac{\theta}{i} = \frac{nAB}{K}$

Voltage sensitivity = $\frac{nAB}{KR}$

As current sensitivity increases by 50%

So number of turns increases by 50%

Resistance also increases by 50%

Therefore, voltage sensitivity remains constant.

40. If the two metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8 eV and 2.2 eV. Then choose the correct option

(1) Metal B will not emit photo-electrons

(2) Both metals A and B will emit photo-electrons

(3) Both metals A and B will not emit photo-electrons

(4) Metal A will not emit photo-electrons

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. for photo-emission

$h\nu > \phi$

E of photon $= \frac{12400}{350 \times 10^{-9}} = \frac{12400}{3500\,\text{Å}} = \frac{124}{35} = 3.54$ eV

for ϕ_1, $h\nu < \phi_1$ 3.54 eV < 4.8 eV

for ϕ_2, $h\nu < \phi_2$ 3.54 eV > 2.2 eV

So, second plate will be able to emit electron.

41. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be:

(1) $\frac{W}{91}$ (2) $\frac{W}{100}$

(3) $\frac{W}{9}$ (4) $\frac{W}{3}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $g' = \frac{g_0}{\left(\frac{r}{R}\right)^2} = \frac{g_0}{10^2} = \frac{g_0}{100}$

$\Rightarrow W' = \frac{w}{100}$

42. Match List-I with List-II.

	List-I		List-II
A.	Angular momentum	I	$[ML^2T^{-2}]$
B	Torque	II	$[ML^{-2}T^{-2}]$
C	Stress	III	$[ML^2T^{-1}]$
D	Pressure gradient	IV	$[ML^{-1}T^{-2}]$

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-III, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-II, B-III, C-IV, D-I

(4) A-IV, B-II, C-I, D-III

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $[\tau] = [r][F] = [L][MLT^{-2}] = [ML^2T^{-2}]$

$[\text{Stress}] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{L^2} = [ML^{-1}T^{-2}]$

$[\text{Pressure gradient}] = \frac{[P]}{[Z]} = \frac{[ML^{-1}T^{-2}]}{L^1} = [ML^{-2}T^{-2}]$

$[L] = [\tau][t] = [ML^2T^{-2}][T] = [ML^2T^{-1}]$

43. An alternating voltage source V = 260 sin (628t) is connected across a pure inductor of 5 mH. Inductive reactance in the circuit is:

(1) 3.14Ω (2) 6.28Ω

(3) 0.5Ω (4) 0.318Ω

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $X_L = \omega L = 200\pi \times 5 \times 10^{-3}$

$= \pi\ \Omega$

$= 3.14\ \Omega$

44. Match List-I with List-II.

	List-I		List-II
A.	Microwaves	I	Physiotherapy
B	UV rays	II	Treatment ofcancer
C	Infra-red rays	III	Lasik eye surgery
D	X-rays	IV	Aircraft navigation

Choose the correct answer from the option given below:

(1) A-II, B-IV, C-III, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-III, B-II, C-I, D-IV

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. UV rays are used for Lasik eye surgery. IR is used for physiotherapy. X-rays are used for cancer treatment. And Microwaves are used for aircraft navigation.

45. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of 3rd orbit will be

(1) $\frac{R}{3}$ (2) 2.25R

(3) 3R (4) 9R

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $R = 0.529\frac{n^2}{z}\text{Å}$

$\frac{R_3}{R_2} = \frac{9}{4}$

$R_3 = \frac{9}{4}R_2$

$R_3 = 2.25R$

46. Under the same load, wire A having length 5.0 m and cross section 2.5×10^{-5} m² stretches uniformly by the same amount as another wire B of length 6.0 m and a cross section of 3.0×10^{-5} m² stretches. The ratio of the Young's modulus of wire A to that of wire B will be:

(1) 1 : 4 (2) 1 : 1

(3) 1 : 10 (4) 1 : 2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\Delta L_1 = \Delta L_2$

$\frac{F_1L_1}{A_1Y_1} = \frac{F_2L_2}{A_2Y_2}$

$\frac{5}{2.5\times10^{-5}\times Y_1} = \frac{6}{3\times10^{-5}\times Y_2}$

$\frac{Y_1}{Y_2} = 1$

47. Considering a group of positive charges, which of the following statements is correct?

(1) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.

(2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.

(3) Both the net potential and the net field can be zero at a point.

(4) Both the net potential and the net electric field cannot be zero at a point.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Since potential of positive charge is always positive & electric field is a vector. So two electric field can be get cancelled if it has same magnitude & opposite direction.

48. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity g = 10 ms^{-2})

(1) 0.2 (2) 0.3

(3) 0.5 (4) 0.4

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $v = u + at$

$\Rightarrow 0 = 20 + (-\mu g)(5)$

$\Rightarrow \mu = 0.4$

49. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{C_P}{C_V}$ will be.

(1) $\frac{4}{3}$ (2) $\frac{3}{1}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. **For** adiabatic process

PV^{γ} = Constant, where $\gamma = \frac{C_P}{C_V}$

$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$

$\frac{81}{16} = \left(\frac{27}{8}\right)^{\gamma}$

$\left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^{3\gamma} \quad \Rightarrow \gamma = \frac{4}{3}$

50. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base and collector have same doping level.

Statement II: In a transistor, collector is the thickest and base is the thinnest segment.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) **Statement I** is correct but **Statement II** is incorrect

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Statement-I is false but Statement-II is true.

SECTION-B

51. A series LCR circuit consists of R=80Ω. X_L=100Ω, and X_C=40Ω. The input voltage is 2500 cos(100 πt) V. The amplitude of current, in the circuit, is_____A.

Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol. $Z=\sqrt{R^2+(X_L-X_C)^2}$

$\sqrt{80^2+(100-40)^2}=100\ \Omega$

$i_\circ=\frac{V_0}{Z}=25A$

52. Two light waves of wavelengths 800 and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from plane of slits. If the two slits are separated by 0.35 mm, then shortest distance from the central bright maximum to the point where the bright fringes of the two wavelength coincide will be _____mm.

Official Ans. by NTA (48)

QuestPix Ans. (48)

Sol. $n_1\frac{\lambda_1 D}{d}=n_2\frac{\lambda_2 D}{d}$

$800n_1=600n_2\Rightarrow\frac{n_1}{n_2}=\frac{3}{4}$

Minimum $n_1=3\ \&\ n_2=4$

Minimum separation $=\frac{3\times800nm\times7m}{0.35\ mm}=48$

53. A water heater of power 2000 W is used to heat water. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}. The efficiency of heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is _____s.

(Assume that the specific heat capacity of water remains constant over the temperature range of the water).

Official Ans. by NTA (300)

QuestPix Ans. (300)

Sol. P × t × η = ms ΔT

2000 × t × 0.7 = 2 × 1000 × 4.2 × 50

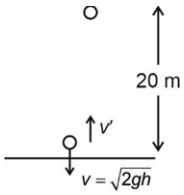
t = 300 s

54. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of_____m.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol.



$v'=ev$

$h'=\frac{(V')^2}{2g}=\frac{e^2V^2}{2g}=e^2h=(0.5)^2\ x\ 20$

$=\frac{20}{4}=5m$

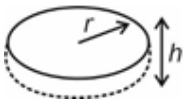
h' = 5m

55. Two discs of same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3:5. The moment of inertia of these discs respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is_____.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol.



Mass of both disc is equal

So $\Rightarrow M_1=M_2$

$\Rightarrow (\pi r_1^{\,2})h_1\rho_1=(\pi r_1^{\,2})h_2\rho_2$

$\Rightarrow r_1^{\,2}\times\frac{h_1}{h_2}\times\frac{\rho_1}{\rho_2}=r_2^{\,2}$

$\Rightarrow r_1^{\,2}\times2\times\frac{\rho_1}{\rho_2}=r_2^{\,2}\qquad\frac{\rho_1}{\rho_2}=\frac{3}{5}$

$\Rightarrow\frac{\rho_2}{\rho_1}=\frac{5}{3}$

$\Rightarrow\ \frac{r_1^{\,2}}{r_2^{\,2}}=\left(\frac{\rho_2}{2\rho_1}\right)=\left(\frac{5}{6}\right)$

Ratio of M.O.I $=\frac{\frac{1}{4}Mr_1^{\,2}}{\frac{1}{4}Mr_2^{\,2}}=\left(\frac{r_1^{\,2}}{r_2^{\,2}}\right)=\left(\frac{5}{6}\right)$

56. If the binding energy of ground state electron in a hydrogen atom is 13.6 eV, then, the energy required to remove the electron from the second excited state of Li^{2+} will be: $x \times 10^{-1}$ eV. The value of x is _____.

Official Ans. by NTA (136)

QuestPix Ans. (136)

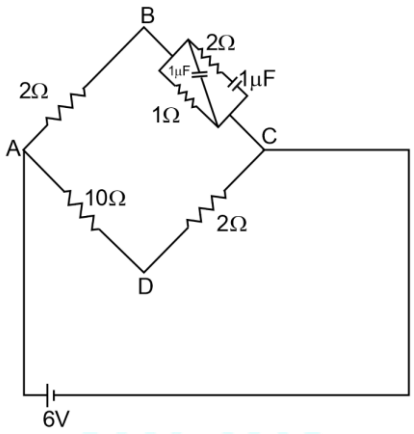
Sol. $E_B = 13.6 \frac{z^2}{n^2}$

For Li^{2+},

$z = 3, n = 3$

$E_B = 13.6 \left(\frac{3^2}{3^2}\right) = 13.6$ eV

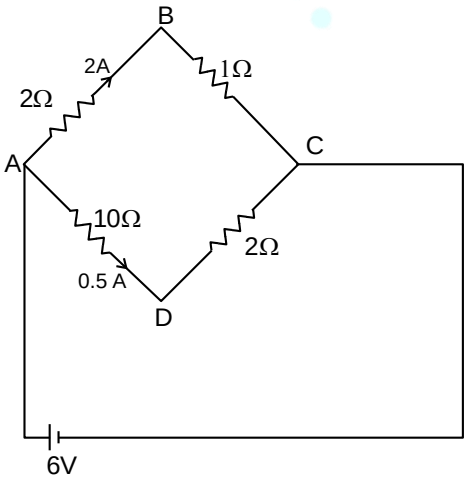
57. For the given circuit, in the steady state, $|V_B - V_D|$ = _____V.



Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.



In steady state the capacitors will act as the open wire. Hence the circuit will be as above. If $V_c = 0V$ then the potential at points A, B, C and D will be

$V_a = 6$ V

$V_a - V_b = 4$ V

$V_b = 2$ V

$V_a - V_d = 10 \times 0.5$

$V_d = 1$ V

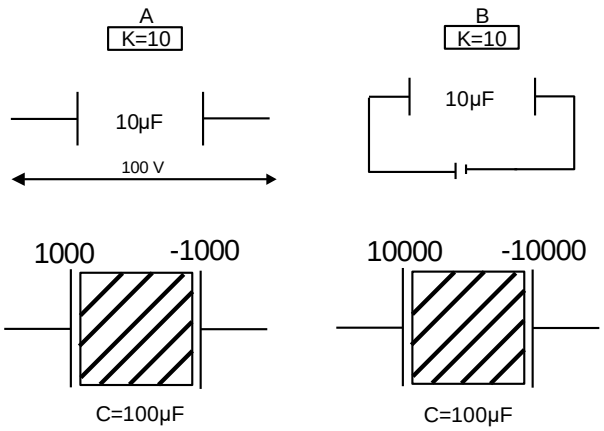
$V_b - V_d = 1$ V

58. Two parallel plate capacitors C_1 and C_2 each having capacitance of 10 μF are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between it plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be _____V. (Assuming Dielectric constant = 10)

Official Ans. by NTA (55)

QuestPix Ans. (55)

Sol.



$\mathbf{V = \frac{1000+10000}{100+100}}$

$\mathbf{= \frac{11000}{200}}$

$\mathbf{= 55V}$

59. The displacement equations of two interfering waves are given by

$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ cm,

$y_2 = 5\left[\sin(\omega t) + \sqrt{3}\cos\omega t\right]$cm respectively. The amplitude of the resultant wave is _____cm.

Official Ans. by NTA (20)

QuestPix Ans. (20)

Sol. $y_1 = 5\left[\sin(\omega t) + \sqrt{3}\cos(\omega t)\right]$

$= 10\sin\left(\omega t + \frac{\pi}{3}\right)$

$y_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$

$y_1 + y_2 = y_{resultant} = 10\sin\left(\omega t + \frac{\pi}{3}\right) + 10\sin\left(\omega t + \frac{\pi}{3}\right)$

$y_{resultant} = 20\sin\left(\omega t + \frac{\pi}{3}\right)$

Amplitude = 20

60. Two bodies are projected from ground with same speeds 40 ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles, is _____m. (Given g=10ms^{-2})

Official Ans. by NTA (80)

QuestPix Ans. (80)

Sol. If angle of projection for projectile 1 is 60° thenangle of projectile 2 is 30° for same range

$H_1 = \frac{u^2 sin^2 60°}{2g}$, $H_2 = \frac{u^2 sin^2 30°}{2g}$

$H_1 + H_2 = \frac{u^2}{2g} = \frac{40^2}{2\times 10} = 80$ m

(Held On Tuesday 31ST January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

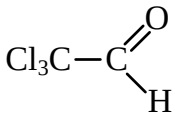
61. In the following halogenated organic compounds the one with maximum number of chlorine atoms in its structure is :

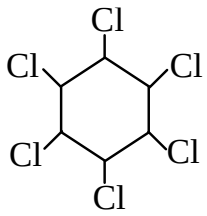
(1) Chloral (2) Gammaxene

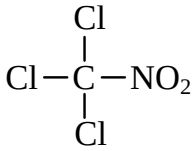
(3) Chloropicrin (4)Freon -12

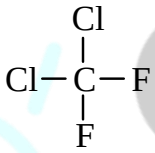
Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (1) Chloral 

(2) Gammaxene 

(3) Chloropicrin 

(4) Freon - 12 

62. Incorrect statement for the use of indicators in acid-base titration is :

(1) Methyl orange may be used for a weak acid vs weak base titration.

(2) Methyl orange is a suitable indicator for a strong acid vs weak base titration

(3) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration

(4) Phenolphthalein may be used for a strong acid vs strong base titration.

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

Indicator	pH range
Methyl orange	3.2 – 4.5
Phenolpthalein	8.3 – 10.5

Methyl orange may be used for a strong acid vs strong base and strong acid vs weak base titration.

Phenolpthalein may be used for a strong acid vs strong base and weak acid vs strong base titration.

63. Which of the following compounds are not used as disinfectants?

(A) Chloroxylenol (B) Bithional

(C) Veronal (D) Prontosil

(E) Terpineol

Choose the **correct** answer from the options given below:

(1) A, B, E (2) A, B

(3) B, D, E (4) C, D

Official Ans. by NTA (4)

QuestPix Ans. (1)

Sol. Veronal is neurological medicine, Prontosil is antibiotic, rest all are disinfectants.

64. A hydrocarbon 'X' with formula C_6H_8 uses two moles of H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is :

(1) hexa-1, 3, 5-triene

(2) 1-methylcyclopenta-1, 4-diene

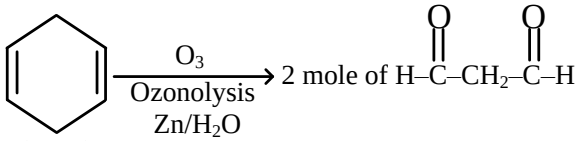
(3) cyclohexa-1, 3-diene

(4) cyclohexa-1, 4-diene

Official Ans. by NTA (4)

QuestPix Ans. (4)

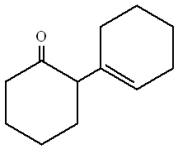
Sol.

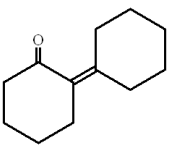


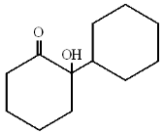
(C_6H_8)

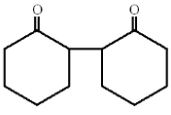
(Cyclohexa-1, 4-diene)

65. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R) The final product (R) is :

(1) 

(2) 

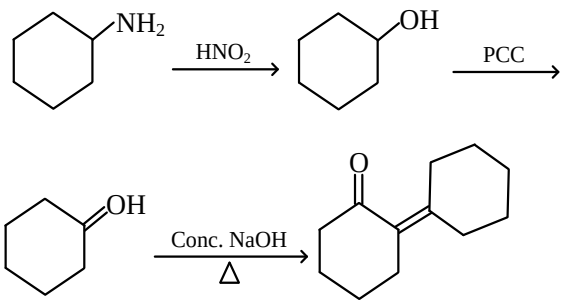
(3) 

(4) 

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



66. Given below are two statements :

Statement I : Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green.

Statement II : The green colour observed is due to the formation of copper(I) metaborate.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. (Borax Bead Test)

On treatment with metal salt, boric anhydride forms metaborate of the metal which gives different colours in oxidising and reducing flame. For example, in the case of copper sulphate, following reactions occur.

$$CuSO_4 + B_2O_3 \xrightarrow[\text{(Oxidising)}]{\text{Non -luminous flame}} \underset{\substack{\text{Cupric metaborate}\\ \text{blue-green}}}{Cu(BO_2)_2} + SO_3$$

Two reactions may take place in reducing flame (Luminous flame)

(i) The blue-green $Cu(BO_2)_2$ is reduced to colourless cuprous metaborate as :

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow[\text{flame}]{\text{Luminous}} 2CuBO_2 + Na_2B_4O_7 + CO$$

(ii) Cupric metaborate may be reduced to metallic copper and bead appears red opaque.

$$2Cu(BO_2)_2 + 4NaBO_2 + 2C \xrightarrow[\text{flame}]{\text{Luminous}} 2Cu + 2Na_2B_4O_7 + 2CO$$

67. Evaluate the following statements for their correctness.

(A) The elevation in boiling point temperature of water will be same for 0.1 M NaCl and 0.1 M urea.

(B) Azeotropic mixtures boil without change in their composition

(C) Osmosis always takes place from hypertonic to hypotonic solution

(D) The density of 32% H_2SO_4 solution having molarity 4.09 M is approximately 1.26 g mL^{-1}

(E) A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the **correct** answer from the options given below :

(1) B, D, and E only (2) A, B, and D only

(3) A and C only (4) B and D only

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. **(A)** $\Delta T_b \propto i \times c$

(B) Azeotropic mixtures have same composition in both liquid and vapour phase.

(C) Osmosis always takes place from hypotonic to hypertonic solution.

(D) $M = \dfrac{30 \times 10 \times 1.26}{98} \approx 4.09\,M$

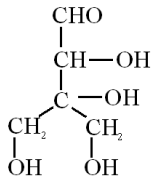
(E) When KI solutions is added to $AgNO_3$ solution, positively charged solution results due to adsorption of Ag^+ ions from dispersion medium

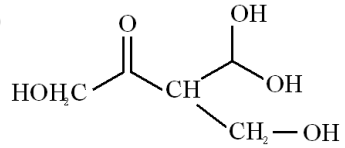
$$AgI / Ag^+$$

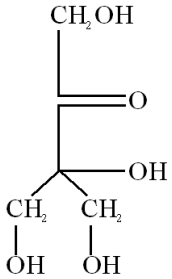
Positively charged

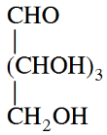
68. Compound A, $C_5H_{10}O_5$, given a tetraacetate with Ac_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is :

(1)



(2) 

(3) 

(4) 

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol. **(i)** Formation of tetraacetete with Ac_2O means compound A has four –OH linkage.
Reduction of A with HI gives Isopentane i.e. molecule contains five carbon atom.

69. Arrange the following orbitals in decreasing order of energy ?
(A) n = 3, l = 0, m = 0
(B) n = 4, l = 0, m = 0
(C) n = 3, l = 1, m = 0
(D) n = 3, l = 2, m = 1
The correct option for the order is :
(1) B > D > C > A
(2) D > B > C > A
(3) A > C > B > D
(4) D > B > A > C

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. (A) n = 3; l = 0; m = 0 ; 3s orbital
(B) n = 4; l = 0; m = 0 ; 4s orbital
(C) n = 3; l = 1; m = 0 ; 3p orbital
(D) n = 3; l = 2; m = 0 ; 3d orbital
As per Hund's rule energy is given by (n+*l*) value. If value of (n+*l*) remains same then energy is given by n only.

70. The Lewis acid character of boron tri halides follows the order :
(1) $BBr_3 > BI_3 > BCl_3 > BF_3$
(2) $BCl_3 > BF_3 > BBr_3 > BI_3$
(3) $BF_3 > BCl_3 > BBr_3 > BI_3$
(4) $BI_3 > BBr_3 > BCl_3 > BF_3$

Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. Extent of back bonding, reduces down the group leading to more Lewis acidic strength

$$\underset{(2p-2p)}{BF_3} > \underset{(2p-3p)}{BCl_3} > \underset{(2p-4p)}{BBr_3} > \underset{(2p-5p)}{BI_3}\ (\text{extent of back bonding})$$

$$BF_3 < BCl_3 < BBr_3 < BI_3\ (\text{lewis acidic nature})$$

71. Match **List-I** with **List-II**

	List-I		List-II
(A)	Physisorption	I	Single layer adsorption
(B)	Chemisorption	II	20-40 kJ mol^{-1}
(C)	$N_2(g)+3H_2(g) \xrightarrow{Fe(s)} 2NH_3(g)$	III	Chromatography
(D)	Analytical Application or Adsorption	IV	Heterogeneous catalysis

Choose the correct answer from the options given below :
(1) A – II, B – III, C – I, D – IV
(2) A – III, B – IV, C – I, D - II
(3) A – IV, B – II, C – III, D - I
(4) A – II, B – I, C – IV, D – III

Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. (A) Physisorption = 20 – 40 kJ/mol and Chemisorption = 80 – 240 kJ/mol
(B) Physisorption is multi-layered and chemisorption is unimolecular layered.
(C) In heterogeneous catalysis, medium and catalyst are in different phases.
(D) Chromatography uses adsorption to purify/separate mixtures.

72. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**
Assertion (A) : The first ionization enthalpy of 3d series elements is more than that of group 2 metals
Reason (R) : In 3d series of elements successive filling of d-orbitals takes place.
In the light of the above statements, choose the **correct** answer from the options given below :
(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(2) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of (A)
(3) **(A)** is false but **(R)** is true
(4) **(A)** is true but **(R)** is false

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol. From Sc to Mn ionization energy is less than that of Mg.

For 3d series :

	Sc	Ti	V	Cr	Mn
IE (KJ/mol)	631	656	650	653	717
	Fe	Co	Ni	Cu	Zn
IE (KJ/mol)	762	758	736	745	906

For 2nd Group

	Be	Mg	Ca	Sr	Ba	Ra
IE (KJ/mol)	631	656	650	653	717	762

73. The element playing significant role in neuromuscular function and interneuronal transmission is :

(1) Be (2) Ca (3) Li (4) Mg

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Calcium plays important role in neuromuscular function, interneuronal transmission, cell membrane etc.

74. Given below are two statements :

Statement I : H_2O_2 is used in the synthesis of Cephalosporin

Statement II : H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are correct

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are incorrect

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. It is used in the synthesis of hydroquinone, tartaric acid and certain food products and pharmaceuticals (cephalosporin) etc. Restoration of aerobic conditions to sewage wastes etc.

75. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following ?

(1) $CO_2 + H_2O \rightarrow H_2CO_3$

(2) $4NO_2 + O_2 + 2H_2O \rightarrow 4HNO_3$

(3) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

(4) $N_2O_5 + H_2O \rightarrow 2HNO_3$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. We are aware that normally rain water has a pH of 5.6 due to the presence of H^+ ions formed by the reactions of rain water with carbon dioxide present in the atmosphere.

$$H_2O(l) + CO_2(g) \rightleftharpoons H_2CO_3(aq)$$
$$H_2CO_3(aq) \rightleftharpoons H^+(aq) + HCO_3^-(aq)$$

76. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A ?

(1) C_9H_8

(2) $C_{11}H_4$

(3) C_5H_8

(4) $C_{11}H_8$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

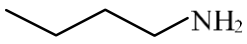
$\frac{y}{2} = 4 \ \therefore \ y = 8$

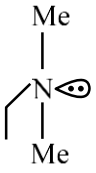
$x + \frac{8}{4} = 11$

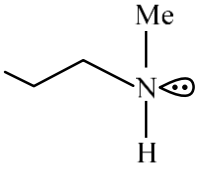
$\therefore \ x = 9$

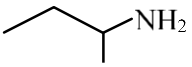
$\therefore$ Hydrocarbon will be $= C_9H_8$

77. An organic compound $[A](C_4H_{11}N)$, shows optical activity and gives N_2 gas on treatment with HNO_2. The compound $[A]$ reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH. The structure of A is:

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $C_4H_{11}N$ releases N_2 with HNO_2 i.e. it is primary amine.

After reacting with Hinsberg reagent it forms a compound which is soluble in KOH,

Hence, the amine is primary.

78. Which one of the following statements is incorrect ?

(1) Boron and Indium can be purified by zone refining method.

(2) Van- Arkel method is used to purify tungsten.

(3) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.

(4) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Van – Arkel process is used for purification of Ti, Zr, Hf and B.

79. Which of the following elements have half-filled f-orbitals in their ground state ?

(Given : atomic number

Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61)

A. Sm

B. Eu

C. Tb

D. Gd

E. Pm

Choose the **correct** answer from the options given below:

(1) B and D only (2) A and E only

(3) A and B only (4) C and D only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol.

1. $_{62}Sm : 4f^6 6s^2$
2. $_{64}Gd : 4f^7 5d^1 6s^2$
3. $_{63}Eu : 4f^7 6s^2$
4. $_{65}Tb: 4f^9 6s^2$
5. $_{61}Pm: 4f^5 6s^2$

80. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over :

(1) Ni (2) Copper gauze

(3) Pd (4) Copper oxide

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Duma's method.

The nitrogen containing organic compound, when heated with CuO in a atmosphere of CO_2, yields free N_2 in addition to CO_2 and H_2O.

$$C_xH_yN_z + (2x + \frac{y}{2})\,CuO \rightarrow$$

$$x\,CO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + (2x + \frac{y}{2})\,Cu$$

Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over heated copper gauze.

SECTION-B

81. If the CFSE of $\left[Ti(H_2O)_6\right]^{3+}$ is -96.0 kJ / mol , this complex will absorb maximum at wavelength ___ nm. (nearest integer)

Assume Planck's constant $(h) = 6.4 \times 10^{-34}$ Js Speed of light $(c) = 3.0 \times 10^8$ m / s and Avogadro's constant $(N_A) = 6 \times 10^{23}$ / mol .

Official Ans. by NTA (480)

QuestPix Ans. (480)

Sol. $\left(Ti^{+3}(H_2O)_6\right)^{3+}$

$Ti^{+3} : 3d'$

C.F.S.E. $= -0.4 \times \Delta_0$

$$= -\frac{96 \times 10^3}{N_0} J$$

$$\Delta_0 = \frac{96 \times 10^3}{0.4 \times 6 \times 10^{23}}$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{96 \times 10^3}{0.4 \times 6 \times 10^{23}}$$

$$\lambda = \frac{0.4 \times 6 \times 10^{23} \times 6.4 \times 10^{-34} \times 3 \times 10^8}{96 \times 10^3}$$

$= 0.48 \times 10^{-6}$m

$= 480 \times 10^{-9}$m

$= 480$ nm

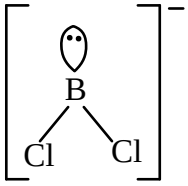
82. Amongst the following, the number of species having the linear shape is _____.

$XeF_2, I_3^+, C_3O_2, I_3^-, CO_2, SO_2$, BeCl and $BCl_2^{\ominus}$

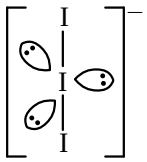
Official Ans. by NTA (5)

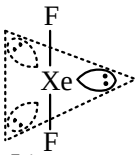
QuestPix Ans. (5)

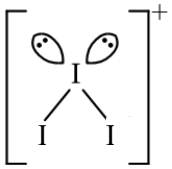
Sol.

(i) 

(ii) Cl—Be—Cl

(iii) I_3^-  Linear

(iv) XeF_2  Linear

(v)  V - Shape

(vi) CO_2 O = C = O Linear

(vii C_3O_2 (O=C=C=C=O) Linear

(viii)  V-

83. The resistivity of a 0.8 M solution of an electrolyte is $5\times10^{-3}\ \Omega cm$. Its molar conductivity is _____ $\times10^4\ \Omega^{-1}\ cm^2\ mol^{-1}$. (Nearest integer)

Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol. $\Lambda_m=\frac{\kappa\times1000}{M}$

$\Lambda_m=\frac{1}{\rho}\times\frac{1000}{M}$

$\frac{1}{5\times10^{-3}}\times\frac{1000}{0.8}$

Ans. $25\times10^4\ \Omega^{-1}\ cm^{-2}\ mol^{-1}$

84. At 298 K, the solubility of silver chloride in water is $1.434\times10^{-3}\ g\ L^{-1}$. The value of $-\log K_{sp}$ for silver chloride is _____.

(Given mass of Ag is $107.9\ g\ mol^{-1}$ and mass of Cl is $35.5\ g\ mol^{-1}$)

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. $AgCl(s) \rightarrow \underset{S}{Ag^+(aq.)} + \underset{S}{Cl^-(aq.)}$

$K_{sp}=S^2=\left(\frac{1.43}{143.4}\times10^{-3}\right)^2=10^{-10}$

$-\log K_{sp}=10$

85. A sample of a metal oxide has formula $M_{0.83}O_{1.00}$. The metal M can exist in two oxidation states +2 and +3. In the sample of $M_{0.83}O_{1.00}$, the percentage of metal ions existing in +2 oxidation state is _____ % (nearest integer)

Official Ans. by NTA (59)

QuestPix Ans. (59)

Sol.

M $\begin{cases}+2\longrightarrow x\\ +3\longrightarrow (0.83-x)\end{cases}$

$2x+3(0.83-x)=2$

$x=0.49$

$\%\ M^{2+}=\frac{0.49}{0.83}\times100$

$=59\%$

86. Assume carbon burns according to following equation :

$2C_{(s)}+O_{2(g)}\rightarrow2CO(g)$

When 12 g carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is _____ $\times10^{-1}$ L at STP [nearest integer]

[Given : Assume CO as ideal gas, Mass of C is $12\ g\ mol^{-1}$, Mass of O is $16\ g\ mol^{-1}$ and molar volume of an ideal gas at STP is $22.7\ L\ mol^{-1}$]

Official Ans. by NTA (227)

QuestPix Ans. (227)

Sol. $2C(s) + O_2(g) \rightarrow 2CO(g)$

1mol 1.5 mol

Limiting reagent is carbon. One mole carbon produces one mole CO. Hence, volume at STP is 227×10^{-1} litre

87. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxide is _____.

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. K, Rb and Cs form stable super oxides but Cs has ionisation enthalpy less than 400 kJ.

88. Enthalpies of formation of $CCl_4(g), H_2O(g), CO_2(g)$ and $HCl(g)$ are $-105, -242, -394$ and -92 kJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is _____ kJ mol^{-1} (nearest integer)

$CCl_4(g)+2H_2O(g)\rightarrow CO_2(g)+4HCl(g)$

Official Ans. by NTA (173)

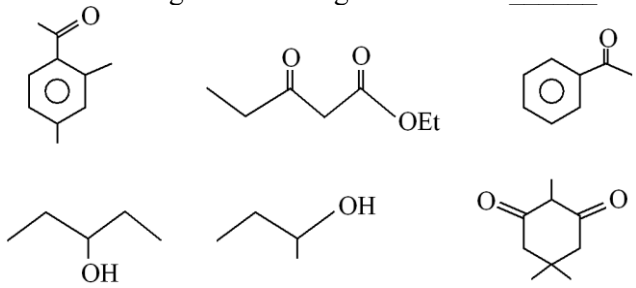
QuestPix Ans. (173)

Sol. $\Delta_rH=\sum H_p-\sum H_R$

$=(-394+4\times-92)-(-105+(2\times-242))$

$=-173\ kJ/mol$

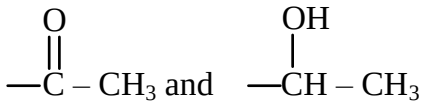
89. The number of molecules which gives haloform test among the following molecules is _____ .



Official Ans. by NTA (3)
QuestPix Ans. (3)

Sol.

Molecules having



gives positive haloform test.

90.. The rate constant for a first order reaction is $20\ \text{min}^{-1}$. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is _____ $\times 10^{-2}$ min . (Nearest integer)
(Given : $\ln 10 = 2.303$
$\log 2 = 0.3010$)

Official Ans. by NTA (17)
QuestPix Ans. (17)

Sol.

$$C = \frac{C_o}{2^n} = \frac{C_o}{32}$$

$$n = 5$$

$$t = 5t_{1/2}$$

$$= \frac{5 \times 0.693}{20} = \frac{0.693}{4}$$

$$= 0.17325 \text{ min} = 17.325 \times 10^{-2} \text{ min.}$$