

**(HELD ON THURSDAY 23<sup>rd</sup> JANUARY 2025)**
**TIME : 9:00 AM TO 12:00 NOON**

| MATHEMATICS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | TEST PAPER WITH SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
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| <p><b>SECTION-A</b></p> <p>1. The value of <math>\int_{e^2}^{e^4} x \left( \frac{e^{((\log_e x)^2 + 1)^{-1}}}{e^{((\log_e x)^2 + 1)^{-1}} + e^{((6 - \log_e x)^2 + 1)^{-1}}} \right) dx</math> is</p> <p>(1) <math>\log_e 2</math> (2) 2<br/>(3) 1 (4) <math>e^2</math></p> <p><b>Ans. (3)</b></p> <p><b>Sol.</b> Let <math>\ln x = t \Rightarrow \frac{dx}{x} = dt</math></p> $I = \int_2^4 \frac{e^{\frac{1}{1+t^2}}}{\frac{1}{1+t^2} + e^{\frac{1}{1+(6-t)^2}}} dt$ $I = \int_2^4 \frac{e^{\frac{1}{1+(6-t)^2}}}{e^{\frac{1}{1+(6-t)^2}} + \frac{1}{1+t^2}} dt$ $2I = \int_2^4 dt = (t)_2^4 = 4 - 2 = 2$ $I = 1$ <p>2. Let <math>I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}} (x-15)^{\frac{15}{13}}}</math>.</p> <p>If <math>I(37) - I(24) = \frac{1}{4} \left( \frac{1}{b^{\frac{1}{13}}} - \frac{1}{13} \right)</math>, <math>b, c \in \mathbb{N}</math>, then</p> <p><math>3(b+c)</math> is equal to</p> <p>(1) 40 (2) 39<br/>(3) 22 (4) 26</p> <p><b>Ans. (2)</b></p> <p><b>Sol.</b> <math>I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}} (x-15)^{\frac{15}{13}}}</math></p> <p>Put <math>\frac{x-11}{x+15} = t \Rightarrow \frac{26}{(x+5)^2} dx = dt</math></p> | $I(x) = \frac{1}{26} \int \frac{dt}{t^{11/13}} = \frac{1}{26} \cdot \frac{t^{2/13}}{2/13}$ $I(x) = \frac{1}{4} \left( \frac{x-11}{x+15} \right)^{2/13} + C$ $I(37) - I(24) = \frac{1}{4} \left( \frac{26}{52} \right)^{2/13} - \frac{1}{4} \left( \frac{13}{39} \right)^{2/13}$ $= \frac{1}{4} \left( \frac{1}{2^{2/13}} - \frac{1}{2^{2/13}} \right)$ $= \frac{1}{4} \left( \frac{1}{4^{1/13}} - \frac{1}{9^{1/13}} \right)$ <p><math>\therefore b = 4, c = 9</math></p> <p><math>3(b+c) = 39</math></p> <p>3. If the function</p> $f(x) = \begin{cases} \frac{2}{x} \{ \sin(k_1 + 1)x + \sin(k_2 - 1)x \} & , x < 0 \\ 4 & , x = 0 \\ \frac{2}{x} \log_e \left( \frac{2 + k_1 x}{2 + k_2 x} \right) & , x > 0 \end{cases}$ <p>is continuous at <math>x = 0</math>, then <math>k_1^2 + k_2^2</math> is equal to</p> <p>(1) 8 (2) 20<br/>(3) 5 (4) 10</p> <p><b>Ans. (4)</b></p> <p><b>Sol.</b> <math>\lim_{x \rightarrow 0^-} \frac{2}{x} \{ \sin(k_1 + 1)x + \sin(k_2 - 1)x \} = 4</math></p> $\Rightarrow 2(k_1 + 1) + 2(k_2 - 1) = 4$ $\Rightarrow k_1 + k_2 = 2$ $\Rightarrow \lim_{x \rightarrow 0^+} \frac{2}{x} \ln \left( \frac{2 + k_1 x}{2 + k_2 x} \right) = 4$ $\Rightarrow \lim_{x \rightarrow 0^+} \frac{1}{x} \ln \left( 1 + \frac{(k_1 - k_2)x}{2 + k_2 x} \right) = 2$ $\Rightarrow \frac{k_1 - k_2}{2} = 2$ |

$$\Rightarrow k_1 - k_2 = 4$$

$$\therefore k_1 = 3, k_2 = -1$$

$$k_1^2 + k_2^2 = 9 + 1 = 10$$

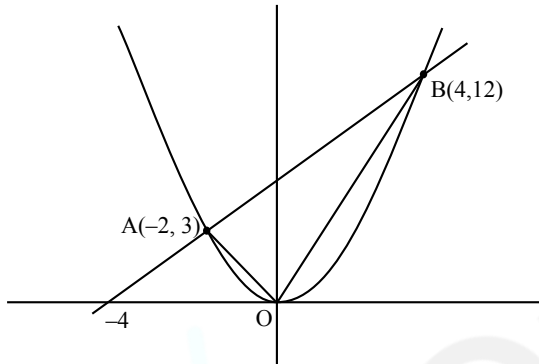
4. If the line  $3x - 2y + 12 = 0$  intersects the parabola  $4y = 3x^2$  at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1)  $\tan^{-1}\left(\frac{11}{9}\right)$                       (2)  $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$

(3)  $\tan^{-1}\left(\frac{4}{5}\right)$                       (4)  $\tan^{-1}\left(\frac{9}{7}\right)$

**Ans. (4)**

**Sol.**



$$3x - 2y + 12 = 0$$

$$4y = 3x^2$$

$$\therefore 2(3x + 12) = 3x^2$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x = -2, 4$$

$$m_{OA} = -3/2, m_{OB} = 3$$

$$\tan \theta = \left| \frac{\frac{-3}{2} - 3}{1 - \frac{9}{2}} \right| = \frac{9}{7}$$

$$\theta = \tan^{-1}\left(\frac{9}{7}\right) \text{ (angle will be acute)}$$

5. Let a curve  $y = f(x)$  pass through the points  $(0, 5)$  and  $(\log_2 k, k)$ . If the curve satisfies the differential equation  $2(3 + y)e^{2x} dx - (7 + e^{2x})dy = 0$ , then  $k$  is equal to

(1) 16                                      (2) 8

(3) 32                                      (4) 4

**Ans. (2)**

**Sol.**  $\frac{dy}{dx} = \frac{2(3 + y)e^{2x}}{7 + e^{2x}}$

$$\frac{dy}{dx} - \frac{2ye^{2x}}{7 + e^{2x}} = \frac{6e^{2x}}{7 + e^{2x}}$$

$$\text{I.F.} = e^{-\int \frac{2e^{2x}}{7 + e^{2x}} dx} = \frac{1}{7 + e^{2x}}$$

$$\therefore y \cdot \frac{1}{7 + e^{2x}} = \int \frac{6e^{2x}}{(7 + e^{2x})^2} dx$$

$$\frac{y}{7 + e^{2x}} = \frac{-3}{7 + e^{2x}} + C$$

$$(0, 5) \Rightarrow \frac{5}{8} = \frac{-3}{8} + C \Rightarrow C = 1$$

$$\therefore y = -3 + 7 + e^{2x}$$

$$y = e^{2x} + 4$$

$$\therefore k = 8$$

6. Let  $f(x) = \log_e x$  and  $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$ .

Then the domain of  $f \circ g$  is

(1)  $\mathbb{R}$                                       (2)  $(0, \infty)$

(3)  $[0, \infty)$                               (4)  $[1, \infty)$

**Ans. (1)**

**Sol.**  $f(x) = \ln x$

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$$

$$D_g \in \mathbb{R}$$

$$D_f \in (0, \infty)$$

$$\text{For } D_{f \circ g} \Rightarrow g(x) > 0$$

$$\frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$$\Rightarrow x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$$

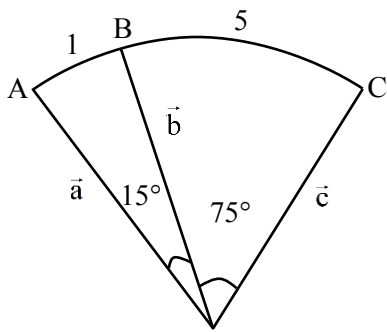
Clearly  $x < 0$  satisfies which are included in option

(1) only.

7. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC, divides the arc AC such that  $\frac{\text{length of arc AB}}{\text{length of arc BC}} = \frac{1}{5}$ , and  $\vec{OC} = \alpha\vec{OA} + \beta\vec{OB}$ , then  $\alpha = \sqrt{2}(\sqrt{3}-1)\beta$  is equal to
- (1)  $2-\sqrt{3}$                       (2)  $2\sqrt{3}$   
 (3)  $5\sqrt{3}$                       (4)  $2+\sqrt{3}$

**Ans. (1)**

**Sol.**



$$\vec{c} = \alpha\vec{a} + \beta\vec{b} \dots(1)$$

$$\vec{a} \cdot \vec{c} = \alpha\vec{a} \cdot \vec{a} + \beta\vec{b} \cdot \vec{a}$$

$$0 = \alpha + \beta \cos 15^\circ \dots(2)$$

$$(1) \Rightarrow \vec{b} \cdot \vec{c} = \alpha\vec{b} \cdot \vec{a} + \beta\vec{b} \cdot \vec{b}$$

$$\Rightarrow \cos 75^\circ = \alpha \cos 15^\circ + \beta \dots(3)$$

$$(2) \& (3) \Rightarrow \cos 75^\circ = -\beta \cos^2 15^\circ + \beta$$

$$\beta = \frac{\cos 75^\circ}{\sin^2 15^\circ} = \frac{1}{\sin 15^\circ} = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$(2) \Rightarrow \alpha = \frac{-\cos 15^\circ}{\sin 15^\circ} = \frac{-(\sqrt{3})}{(\sqrt{3}-1)}$$

$$\therefore \vec{c} = \frac{-(\sqrt{3})}{(\sqrt{3}-1)}\vec{a} + \left( \frac{2\sqrt{2}}{\sqrt{3}-1} \right)\vec{b}$$

Now

$$\alpha + \sqrt{2}(\sqrt{3}-1)\beta = \frac{-(\sqrt{3}+1)}{(\sqrt{3}-1)} + \frac{\sqrt{2}(\sqrt{3}-1) \cdot 2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{-(\sqrt{3}+1)^2}{2} + 4$$

$$= \frac{-3-1-2\sqrt{3}+8}{2}$$

$$= 2-\sqrt{3}$$

8. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

- (1) -1200                      (2) -1080  
 (3) -1020                      (4) -120

**Ans. (2)**

**Sol.**  $a = 3$

$$S_4 = \frac{1}{5}(S_8 - S_4)$$

$$\Rightarrow 5S_4 = S_8 - S_4$$

$$\Rightarrow 6S_4 = S_8$$

$$\Rightarrow 6 \cdot \frac{4}{2} [2 \times 3 + (4-1) \times d]$$

$$= \frac{8}{2} [2 \times 3 + (8-1)d]$$

$$\Rightarrow 12(6+3d) = 4(6+7d)$$

$$\Rightarrow 18+9d = 6+7d$$

$$\Rightarrow d = -6$$

$$S_{20} = \frac{20}{2} [2 \times 3 + (20-1)(-6)]$$

$$= 10 [6-114]$$

$$= -1080$$

9. Let P be the foot of the perpendicular from the point

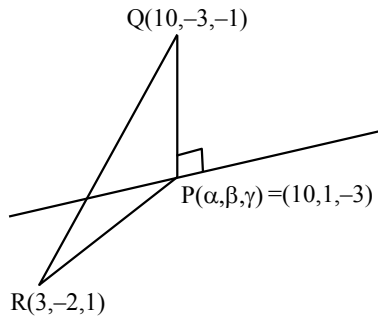
$$Q(10, -3, -1) \text{ on the line } \frac{x-3}{7} = \frac{y-2}{1} = \frac{z+1}{-2}. \text{ Then}$$

the area of the right angled triangle PQR, where R is the point (3, -2, 1), is

- (1)  $9\sqrt{15}$                       (2)  $\sqrt{30}$   
 (3)  $8\sqrt{15}$                       (4)  $3\sqrt{30}$

**Ans. (4)**

Sol.



$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2} = \lambda$$

$$\Rightarrow 7\lambda + 3, -\lambda + 2, -2\lambda - 1$$

dr's of QP  $\Rightarrow$

$$7\lambda - 7, -\lambda + 5, -2\lambda$$

Now

$$(7\lambda - 7) \cdot 7 - (-\lambda + 5) + (2\lambda) \cdot 2 = 0$$

$$54\lambda - 54 = 0 \Rightarrow \lambda = 1$$

$$\therefore P = (10, 1, -3)$$

$$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$$

$$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 2 \\ 7 & -3 & 4 \end{vmatrix} \right| = 3\sqrt{30}$$

10. Let  $\left| \frac{\bar{z}-i}{2\bar{z}+i} \right| = \frac{1}{3}$ ,  $z \in \mathbb{C}$ , be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and  $(\alpha, 0)$  is 11 square units, then  $\alpha^2$  equals

- (1) 100 (2) 50  
(3)  $\frac{121}{25}$  (4)  $\frac{81}{25}$

Ans. (1)

Sol.  $\left| \frac{\bar{z}-i}{2\bar{z}+i} \right| = \frac{1}{3}$

$$\left| \frac{\bar{z}-i}{\bar{z}+\frac{i}{2}} \right| = \frac{2}{3}$$

$$3|x-iy-i| = 2|x-iy+\frac{i}{2}|$$

$$9(x^2 + (y+1)^2) = 4(x^2 + (y-\frac{1}{2})^2)$$

$$9x^2 + 9y^2 + 18y + 9 = 4x^2 + 4y^2 - 4y + 1$$

$$5x^2 + 5y^2 + 22y + 8 = 0$$

$$x^2 + y^2 + \frac{22}{5}y + \frac{8}{5} = 0$$

$$\text{centre} \Rightarrow (0, -\frac{11}{5})$$

$$\left| \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 0 & -11/5 & 1 \\ \alpha & 0 & 1 \end{vmatrix} \right| = 11$$

$$\Rightarrow \left( -\frac{11}{5}\alpha \right)^2 = (11 \times 2)^2$$

$$\Rightarrow \alpha^2 = 100$$

11. Let  $R = \{(1, 2), (2, 3), (3, 3)\}$  be a relation defined on the set  $\{1, 2, 3, 4\}$ . Then the minimum number of elements, needed to be added in R so the R becomes an equivalence relation, is :

- (1) 10 (2) 8  
(3) 9 (4) 7

Ans. (4)

Sol.  $A = \{1, 2, 3, 4\}$

For relation to be reflexive

$$R = \{(1, 2), (2, 3), (3, 3)\}$$

Minimum elements added will be

$$(1, 1), (2, 2), (4, 4), (2, 1), (3, 2), (3, 2), (3, 1), (1, 3)$$

$$\therefore \text{Minimum number of elements} = 7$$

Option : (4)

12. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

- (1) 34000 (2) 37000  
(3) 36000 (4) 35000

Ans. (3)

Sol. DAUGHTER

Total words = 8!

Total words in which vowels are together = 6!  $\times$  3!

words in which all vowels are not together

$$= 8! - 6! \times 3!$$

$$= 6! [56 - 6]$$

$$= 720 \times 50$$

$$= 36000$$

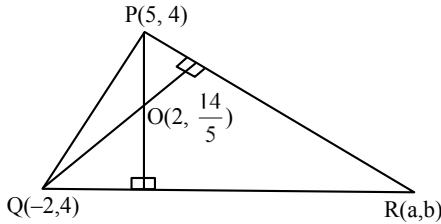
Ans.(3)

13. Let the area of a  $\Delta PQR$  with vertices  $P(5, 4)$ ,  $Q(-2, 4)$  and  $R(a, b)$  be 35 square units. If its orthocenter and centroid are  $O\left(2, \frac{14}{5}\right)$  and  $C(c, d)$  respectively, then  $c + 2d$  is equal to

- (1)  $\frac{7}{3}$  (2) 3  
(3) 2 (4)  $\frac{8}{3}$

**Ans. (2)**

**Sol.**



Equation of lines  $QR = 5x + 2y + 2 = 0$

Equation of lines  $PR = 10x - 3y - 38 = 0$

$\therefore$  Point  $R(2, -6)$

Centroid  $= \left( \frac{5-2+2}{3}, \frac{4+4-6}{3} \right)$

$= \left( \frac{5}{3}, -\frac{2}{3} \right)$

$c + 2d = \frac{5}{3} + 2\left(-\frac{2}{3}\right) = 3$

14. If  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$ , then  $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$  is equal to

- (1)  $x - \tan^{-1} \frac{4}{3}$  (2)  $x - \tan^{-1} \frac{5}{12}$   
(3)  $x + \tan^{-1} \frac{4}{5}$  (4)  $x + \tan^{-1} \frac{5}{12}$

**Ans. (2)**

**Sol.**  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$

$\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{12}\sin x\right)$

$\cos^{-1}(\cos x \cos \alpha + \sin x \sin \alpha)$

$\cos^{-1}(\cos(x-\alpha))$

$\Rightarrow x - \alpha$  because  $x - \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow x - \tan^{-1} \frac{5}{12}$

15. The value of  $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$  is

- (1) 1 (2) 0  
(3)  $3/2$  (4)  $2/3$

**Ans. (1)**

**Sol.**  $\sin 70^\circ (\cot 10^\circ \cot 70^\circ - 1)$

$\Rightarrow \frac{\cos(80^\circ)}{\sin 10^\circ} = 1$

16. Marks obtained by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

- (1) 48 (2) 44  
(3) 40 (4) 52

**Ans. (2)**

**Sol.** median  $= \ell + \left( \frac{\frac{N}{2} - F}{f} \right) \times h$

$= 12 + \left( \frac{\frac{N}{2} - 18}{12} \right) \times 6 = 14$

$\Rightarrow \left( \frac{\frac{N}{2} - 18}{12} \right) \times 6 = 2$

$\frac{N}{2} - 18 = 4 \Rightarrow N = 44$

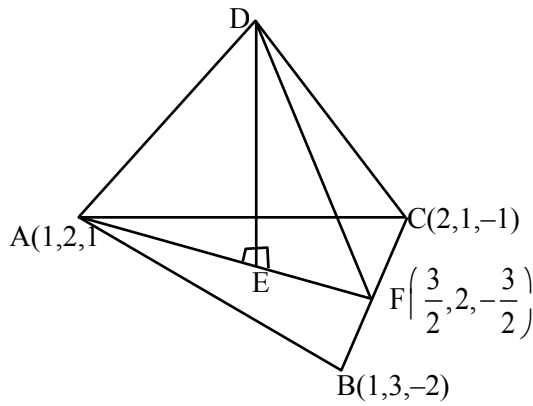
17. Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be  $\hat{i} + 2\hat{j} + \hat{k}$ ,  $\hat{i} + 3\hat{j} - 2\hat{k}$  and  $2\hat{i} + \hat{j} - \hat{k}$  respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is  $\frac{\sqrt{110}}{3}$

and the volume of the tetrahedron is  $\frac{\sqrt{805}}{6\sqrt{2}}$ , then the position vector of E is

- (1)  $\frac{1}{2}(\hat{i} + 4\hat{j} + 7\hat{k})$  (2)  $\frac{1}{12}(7\hat{i} + 4\hat{j} + 3\hat{k})$   
(3)  $\frac{1}{6}(12\hat{i} + 12\hat{j} + \hat{k})$  (4)  $\frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$

**Ans. (4)**

Sol.



$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2} |5\hat{i} + 3\hat{j} + \hat{k}| = \frac{1}{2} \sqrt{35}$$

volume of tetrahedron

$$= \frac{1}{3} \times \text{Base area} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$$

$$\frac{1}{3} \times \frac{1}{2} \sqrt{35} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$$

$$h = \sqrt{\frac{23}{2}}$$

$$AE^2 = AD^2 - DE^2 = \frac{13}{18} \therefore AE = \sqrt{\frac{13}{18}}$$

$$\vec{AE} = |AE| \cdot \left( \frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left( \frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right)$$

$$= \sqrt{\frac{13}{18}} \cdot \left( \frac{\hat{i} - 5\hat{k}}{\sqrt{26}} \right) = \frac{\hat{i} - 5\hat{k}}{6}$$

$$\text{P.V. of E} = \frac{\hat{i} - 5\hat{k}}{6} + \hat{i} + 2\hat{j} + \hat{k} = \frac{1}{6} (7\hat{i} + 12\hat{j} + \hat{k})$$

18. If A, B and  $(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))$  are non-singular matrices of same order, then the inverse of  $A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B$ , is equal to

(1)  $AB^{-1} + A^{-1}B$  (2)  $\text{adj}(B^{-1}) + \text{adj}(A^{-1})$

(3)  $\frac{1}{|AB|} (\text{adj}(B) + \text{adj}(A))$  (4)  $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

Ans. (3)

Sol.  $\left[ A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B \right]^{-1}$

$$B^{-1} \cdot (\text{adj}(A^{-1}) + \text{adj}(B^{-1})) \cdot A^{-1}$$

$$B^{-1} \text{adj}(A^{-1})A^{-1} + B^{-1}(\text{adj}(B^{-1}))A^{-1}$$

$$B^{-1}|A^{-1}|I + |B^{-1}|IA^{-1}$$

$$\frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$$

$$\Rightarrow \frac{\text{adj}B}{|B||A|} + \frac{\text{adj}A}{|A||B|}$$

$$\frac{1}{|A||B|} (\text{adj}B + \text{adj}A)$$

19. If the system of equations

$$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$$

$$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$$

$$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$$

has infinitely many solutions, then  $\lambda^2 + \lambda$  is equal to

(1) 10

(2) 12

(3) 6

(4) 20

Ans. (2)

Sol.  $(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$$

$$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$$

For infinitely many solutions

$$D = \begin{vmatrix} \lambda - 1 & \lambda - 4 & \lambda \\ \lambda & \lambda - 1 & \lambda - 4 \\ \lambda + 1 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$$

$$(\lambda - 3)(2\lambda + 1) = 0$$

$$D_x = \begin{vmatrix} 5 & \lambda - 4 & \lambda \\ 7 & \lambda - 1 & \lambda - 4 \\ 9 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$$

$$2(3 - \lambda)(23 - 2\lambda) = 0$$

$$\lambda = 3$$

$$\therefore \lambda^2 + \lambda = 9 + 3 = 12$$

20. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is
- (1)  $\frac{1}{2}$

(3)  $\frac{2}{3}$

(2)  $\frac{3}{5}$

(4)  $\frac{4}{9}$

**Ans. (1)**

**Sol.** a = number on dice 1  
b = number on dice 2  
(a,b) = (1,3), (3,1), (2,2), (2,3), (3,2), (1,4), (4,1)  
Required probability  

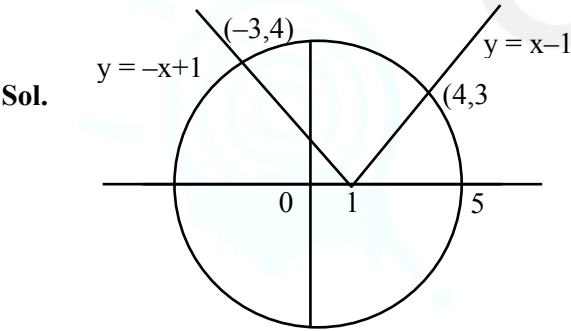
$$= \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6}$$

$$= \frac{18}{36} = \frac{1}{2}$$

### SECTION-B

21. If the area of the larger portion bounded between the curves  $x^2 + y^2 = 25$  and  $y = |x - 1|$  is  $\frac{1}{4}(b\pi + c)$ ,  $b, c \in \mathbb{N}$ , then  $b + c$  is equal to \_\_\_\_\_

**Ans. (77)**



$$x^2 + y^2 = 5$$

$$x^2 + (x - 1)^2 = 25 \Rightarrow x = 4$$

$$x^2 + (-x + 1)^2 = 5 \Rightarrow x = -3$$

$$A = 25\pi - \int_{-3}^4 \sqrt{25 - x^2} dx + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 3 \times 3$$

$$A = 25\pi + \frac{25}{2} - \left[ \frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1} \frac{x}{5} \right]_{-3}^4$$

$$A = 25\pi + \frac{25}{2} - \left[ 6 + \frac{25}{2} \sin^{-1} \frac{4}{5} + 6 + \frac{25}{2} \sin^{-1} \frac{3}{5} \right]$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2} \cdot \frac{\pi}{2} ; A = \frac{75\pi}{4} + \frac{1}{2}$$

$$A = \frac{1}{4}(75\pi + 2)$$

$$b = 75, c = 2$$

$$b + c = 75 + 2 = 77$$

22. The sum of all rational terms in the expansion of  $(1 + 2^{1/3} + 3^{1/2})^6$  is equal to \_\_\_\_\_

**Ans. (612)**

**Sol.**  $\left( 1 + 2^{\frac{1}{3}} + 3^{\frac{1}{2}} \right)^6$

$$= \frac{|6|}{|r_1| |r_2| |r_3|} (1)^{r_1} (2)^{\frac{r_2}{3}} (3)^{\frac{r_3}{2}}$$

| $r_1$ | $r_2$ | $r_3$ |
|-------|-------|-------|
| 6     |       | 0     |
| 4     |       | 2     |
| 2     |       | 4     |
| 0     |       | 6     |
| 3     |       | 0     |
| 1     |       | 2     |
| 0     |       |       |

$$\text{sum} = \frac{|6|}{|6| |0| |0|} + \frac{|6|}{|4| |0| |2|} (3) + \frac{|6|}{|2| |0| |4|} (3)^2 + \frac{|6|}{|0| |0| |6|} (3)$$

$$+ \frac{|6|}{|3| |3| |0|} (2) + \frac{|6|}{|1| |3| |2|} (2)^1 (3)^1 + \frac{|6|}{|0| |6| |0|} (2)^2$$

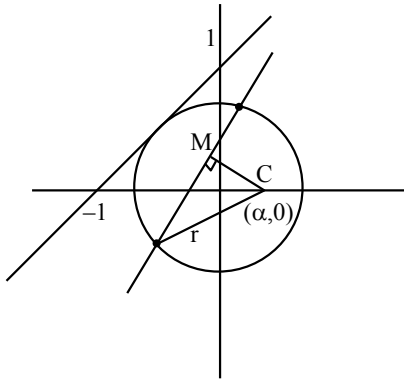
$$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$$

23. Let the circle C touch the line  $x - y + 1 = 0$ , have the centre on the positive x-axis, and cut off a chord of length  $\frac{4}{\sqrt{13}}$  along the line  $-3x + 2y = 1$ .

Let H be the hyperbola  $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ , whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then  $2\alpha^2 + 3\beta^2$  is equal to \_\_\_\_\_

**Ans. (19)**

Sol.



$$x - y + 1 = 0 ; p = r$$

$$\left| \frac{\alpha - 0}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right| = r \Rightarrow (\alpha + 1)^2 = 2r^2 \dots (1)$$

$$\text{now } \left( \frac{-3\alpha + 0 - 1}{\sqrt{9 + 4}} \right)^2 + \left( \frac{2}{\sqrt{13}} \right)^2 = r^2$$

$$\Rightarrow (3\alpha + 1)^2 + 4 = 13r^2 \dots (2)$$

$$(1) \& (2) \Rightarrow (3\alpha + 1)^2 + 4 = 13 \frac{(\alpha + 1)^2}{2}$$

$$\Rightarrow 18\alpha^2 + 12\alpha + 2 + 8 = 13\alpha^2 + 26\alpha + 13$$

$$\Rightarrow 5\alpha^2 - 14\alpha - 3 = 0$$

$$\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0 ; \Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$$

$$\Rightarrow \alpha = \frac{-1}{5}, 3$$

$$\therefore r = 2\sqrt{2}$$

$$\text{How } \alpha = 3 \text{ and } 2\alpha = 4\sqrt{2}$$

$$\alpha^2 e^2 = 9 \Rightarrow \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$$

$$\alpha^2 \left( 1 + \frac{\beta^2}{\alpha^2} \right) =$$

$$\alpha^2 + \beta^2 = 9$$

$$\therefore \beta^2 = 1$$

$$\therefore 2\alpha^2 + 3\beta^2 = 2(8) + 3(1) = 19$$

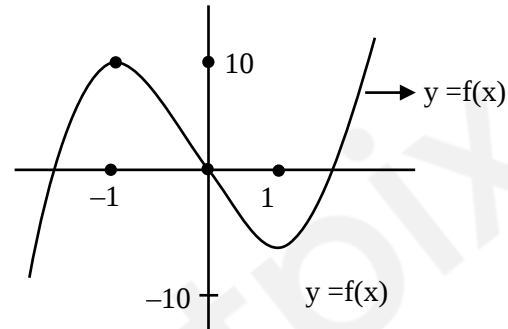
24. If the set of all values of  $a$ , for which the equation  $5x^3 - 15x - a = 0$  has three distinct real roots, is the interval  $(\alpha, \beta)$ , then  $\beta - 2\alpha$  is equal to \_\_\_\_\_

Ans. (30)

Sol.  $5x^3 - 15x - a = 0$

$$f(x) = 5x^3 - 15x$$

$$f'(x) = 15x^2 - 15 = 15(x-1)(x+1)$$



$$a \in (-10, 10)$$

$$\alpha = -10, \beta = 10$$

$$\beta - 2\alpha = 10 + 20 = 30$$

25. If the equation  $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$  has equal roots, where  $a + c = 15$  and  $b = \frac{36}{5}$ , then  $a^2 + c^2$  is equal to \_\_\_\_\_

Ans. (117)

Sol.  $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

$$x = 1 \text{ is root } \therefore \text{ other root is } 1$$

$$\alpha + \beta = -\frac{b(c - a)}{a(b - c)} = 2$$

$$\Rightarrow -bc + ab = 2ab - 2ac$$

$$\Rightarrow 2ac = ab + bc$$

$$\Rightarrow 2ac = b(a + c)$$

$$\Rightarrow 2ac = 15b \dots (1)$$

$$\Rightarrow 2ac = 15 \left( \frac{36}{5} \right) = 108$$

$$\Rightarrow ac = 54$$

$$a + c = 15$$

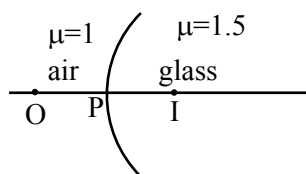
$$a^2 + c^2 + 2ac = 225$$

$$a^2 + c^2 = 225 - 108 = 117$$

**(HELD ON THURSDAY 23<sup>rd</sup> JANUARY 2025)**
**TIME : 9:00 AM TO 12:00 NOON**

| PHYSICS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | TEST PAPER WITH SOLUTION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p align="center"><b>SECTION-A</b></p> <p><b>26.</b> Regarding self-inductance :</p> <p>A : The self-inductance of the coil depends on its geometry.</p> <p>B : Self-inductance does not depend on the permeability of the medium.</p> <p>C : Self-induced e.m.f. opposes any change in the current in a circuit.</p> <p>D : Self-inductance is electromagnetic analogue of mass in mechanics.</p> <p>E : Work needs to be done against self-induced e.m.f. in establishing the current.</p> <p>Choose the correct answer from the options given below:</p> <p>(1) A, B, C, D only                      (2) A, C, D, E only</p> <p>(3) A, B, C, E only                      (4) B, C, D, E only</p> <p><b>Ans. (2)</b></p> <p><b>Sol.</b> Self inductance of coil</p> $L = \frac{\mu_0 N^2 A}{2\pi R}$ <p><b>27.</b> A light hollow cube of side length 10 cm and mass 10g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is <math>y\pi \times 10^{-2}</math> s, where the value of y is</p> <p>(Acceleration due to gravity, <math>g = 10 \text{ m/s}^2</math>, density of water = <math>10^3 \text{ kg/m}^3</math>)</p> <p>(1) 2                                              (2) 6</p> <p>(3) 4                                              (4) 1</p> <p><b>Ans. (1)</b></p> <p><b>Sol.</b> <math>a^2 \times \rho g = m a_{\text{net}}</math></p> $\frac{L^2 \rho g}{m} x = a_{\text{net}}$ $T = 2\pi \sqrt{\frac{m}{L^2 \rho g}}$ <p>where <math>m = 10\text{g}</math>, <math>L = 10 \text{ cm}</math>, <math>\rho = 1000 \text{ kg/m}^3</math></p> | <p><b>28.</b> Given below are two statements:</p> <p><b>Statement-I :</b> The hot water flows faster than cold water.</p> <p><b>Statement-II :</b> Soap water has higher surface tension as compared to fresh water.</p> <p>In the light above statements, choose the <b>correct</b> answer from the options given below</p> <p>(1) Statement-I is false but Statement II is true</p> <p>(2) Statement-I is true but Statement II is false</p> <p>(3) Both Statement-I and Statement-II are true</p> <p>(4) Both Statement-I and Statement-II are false</p> <p><b>Ans. (2)</b></p> <p><b>Sol.</b> Hot water is less viscous than cold water. Surfactant reduces surface tension.</p> <p><b>29.</b> A sub-atomic particle of mass <math>10^{-30} \text{ kg}</math> is moving with a velocity <math>2.21 \times 10^6 \text{ m/s}</math>. Under the matter wave consideration, the particle will behave closely like _____. (<math>h = 6.63 \times 10^{-34} \text{ J.s}</math>)</p> <p>(1) Infra-red radiation                      (2) X-rays</p> <p>(3) Gamma rays                                      (4) Visible radiation</p> <p><b>Ans. (2)</b></p> <p><b>Sol.</b> <math>\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-30} \times 2.21 \times 10^6}</math></p> $= 3 \times 10^{-10} \text{ m}$ <p>Hence particle will behave as x-ray.</p> <p><b>30.</b> A spherical surface of radius of curvature R, separates air from glass (refractive index = 1.5). The centre of curvature is in the glass medium. A point object 'O' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and <math>PO = PI</math>. The distance PO equals to-</p> <p>(1) 5R                                              (2) 3R</p> <p>(3) 2R                                              (4) 1.5R</p> <p><b>Ans. (1)</b></p> |

Sol.



$$PO = u = -x$$

$$PI = v = x$$

$$PO = PI$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{x} + \frac{1}{x} = \frac{1}{2R}$$

$$\frac{5}{2x} = \frac{1}{2R}$$

$$X = 5R$$

31. A radioactive nucleus  $n_2$  has 3 times the decay constant as compared to the decay constant of another radioactive nucleus  $n_1$ . If initial number of both nuclei are the same, what is the ratio of number of nuclei of  $n_2$  to the number of nuclei of  $n_1$ , after one half-life of  $n_1$ ?

- (1)  $1/4$  (2)  $1/8$   
(3) 4 (4) 8

Ans. (1)

Sol.  $N_2 = N_0 e^{-3\lambda t}$

$$N_1 = N_0 e^{-\lambda t}$$

$$\frac{N_2}{N_1} = e^{-2\lambda t}$$

$$t_{\text{half life of } N_1} = \frac{\ln 2}{\lambda} \cdot 1$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

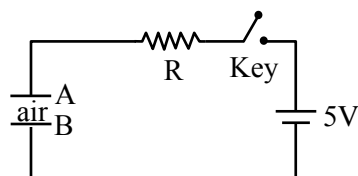
$$\lambda t = \ln 2$$

$$t = \frac{\ln 2}{\lambda}$$

$$= e^{-2\lambda \frac{\ln 2}{\lambda}}$$

$$\frac{N_2}{N_1} = \frac{1}{4}$$

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



- A. There will be no current through resistor R.  
B. There will be maximum current in the connecting wires.  
C. Potential difference between the capacitor plates A and B is minimum.  
D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below :

- (1) C, D only (2) B, C, D only  
(3) A, C only (4) A, B, D only

Ans. (2)

Sol. Initially capacitor behave as a short circuit.

So current will be maximum.

Charge on capacitor will be zero.

Potential difference across capacitor will be zero.

33. The position of a particle moving on x-axis is given by  $x(t) = A \sin t + B \cos^2 t + Ct^2 + D$ , where t is time. The dimension of  $\frac{ABC}{D}$  is-

- (1) L (2)  $L^3 T^{-2}$   
(3)  $L^2 T^{-2}$  (4)  $L^2$

Ans. (3)

Sol. Dimension  $[x(t)] = [L]$

$$[A] = [L]$$

$$[B] = [L]$$

$$[C] = [LT^{-2}]$$

$$[D] = [L]$$

$$\left[ \frac{ABC}{D} = \frac{L \times L \times LT^{-2}}{L} \right] = [L^2 T^{-2}]$$

34. Match the List-I with List-II

| List-I |                                                                              | List-II |                    |
|--------|------------------------------------------------------------------------------|---------|--------------------|
| A.     | Pressure varies inversely with volume of an ideal gas.                       | I.      | Adiabatic process  |
| B.     | Heat absorbed goes partly to increase internal energy and partly to do work. | II.     | Isochoric process  |
| C.     | Heat is neither absorbed nor released by a system                            | III.    | Isothermal process |
| D.     | No work is done on or by a gas                                               | IV.     | Isobaric process   |

Choose the **correct** answer from the options given below :

- (1) A-I, B-IV, C-II, D-III  
 (2) A-III, B-I, C-IV, D-II  
 (3) A-I, B-III, C-II, D-IV  
 (4) A-III, B-IV, C-I, D-II

**Ans. (4)**

**Sol.**  $A \rightarrow P \propto \frac{1}{V}$

$\Rightarrow PV = \text{constant}$

$\Rightarrow nRT = \text{const.} \Rightarrow T = \text{const.}$

Hence Isothermal III

$B \rightarrow IV$

$W \neq 0, \Delta U \neq 0, \Delta Q \neq 0$  [only isobaric]

$C \rightarrow I \Delta Q = 0$  Adiabatic

$D \rightarrow II w = 0$  Isochoric

III IV I II

35. Consider a moving coil galvanometer (MCG) :

A : The torsional constant in moving coil galvanometer has dimensions  $[ML^2T^{-2}]$

B : Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C : If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D : MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E : Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below :

- (1) A, B only (2) A, D, only  
 (3) B, D, E only (4) A, B, E only

**Ans. (1)**

**Sol.** (A)  $\tau = C\theta \Rightarrow [ML^2T^{-2}] = [C][1]$

(B)  $C.S = \frac{\theta}{I} = \frac{BNA}{I}$ ;

$V.S. = \frac{BNA}{RC}$  [R also depends on 'N']

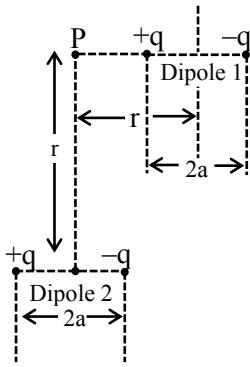
(C)  $V.S. \propto \frac{NAB}{CR}$   $R \rightarrow NR$

(D) False [Theory]

(E) E [False]  $C.S \propto N$

$\Rightarrow \therefore C.S. \propto \frac{NAB}{C}$

36. A point particle of charge  $Q$  is located at  $P$  along the axis of an electric dipole 1 at a distance  $r$  as shown in the figure. The point  $P$  is also on the equatorial plane of a second electric dipole 2 at a distance  $r$ . The dipoles are made of opposite charge  $q$  separated by a distance  $2a$ . For the charge particle at  $P$  not to experience any net force, which of the following correctly describes the situation?



- (1)  $\frac{a}{r} \sim 20$                       (2)  $\frac{a}{r} \sim 10$   
 (3)  $\frac{a}{r} \sim 0.5$                       (4)  $\frac{a}{r} \sim 3$

Ans. (4)

**Sol.**

$$\frac{kq}{(r-a)^2} = \frac{kq}{(r+a)^2} + \frac{2kq}{(r+a^2)^2} \cos \theta$$

$$\frac{1}{(r-a)^2} = \frac{1}{(r+a)^2} + \frac{2a}{(r+a^2)^{\frac{3}{2}}}$$

$$\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} = \frac{2a}{(r+a^2)^{\frac{3}{2}}}$$

$$\frac{4ra}{(r-a^2)^2} = \frac{2a}{(r+a^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{2r}{(r^2 - a^2)^2} = \frac{1}{(r^2 + a^2)^{\frac{3}{2}}}$$

$$\frac{4r^2}{(r^2 - a^2)^4} = \frac{1}{(r^2 + a^2)^3}$$

$$\Rightarrow 4r^2(r^2 + a^2)^3 = (r^2 - a^2)^4$$

$$4r^8 \left(1 + \frac{a^2}{r^2}\right)^3 = r^8 \left(1 - \frac{a^2}{r^2}\right)^4$$

$$4 \left(1 + \frac{a^2}{r^2}\right)^3 = \left(1 - \frac{a^2}{r^2}\right)^4$$

Exact value cannot be solved in exam for this equation to be true

$$\left|\frac{a}{r}\right| > 1 \Rightarrow a > r$$

But point charge  $Q$  lies between charges of dipole 1 hence electric field cannot be zero.

There for it should be bonus.

But by solving from mathematical software we are getting  $a/r \approx 3$ .

37. A gun fires a lead bullet of temperature 300K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is \_\_\_\_ grams.

(Latent heat of fusion of lead =  $2.5 \times 10^4 \text{ J Kg}^{-1}$  and specific heat capacity of lead =  $125 \text{ J Kg}^{-1} \text{ K}^{-1}$ )

- (1) 20                                      (2) 15  
 (3) 10                                      (4) 5

Ans. (3)

**Sol.**  $625 = ms\Delta T + mL$   
 $625 = m[125 \times 300 + 2.5 \times 10^4]$   
 $625 = m[37500 + 25000]$   
 $625 = m[62500]$   
 $m = \frac{1}{100} \text{ kg}$   
 $M = 10 \text{ grams}$



38. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness 'h' in terms of the angle of incidence 'i' and angle of refraction 'r', if the glass slab is placed in air medium ?

- (1)  $\frac{h \tan(i-r)}{\tan r}$  (2)  $\frac{h \cos(i-r)}{\sin r}$   
 (3) h (4)  $\frac{h \sin(i-r)}{\cos r}$

Ans. (4)

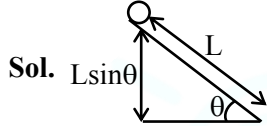
Sol. Formula base

$$\frac{h \sin(i-r)}{\cos r}$$

39. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle  $30^\circ$  with the horizontal. The speed of the particle at the bottom of the plane is  $v_1$ . If the angle of inclination is increased to  $45^\circ$  while keeping L constant. Then the new speed of the sphere at the bottom of the plane is  $v_2$ . The ratio of  $v_1^2 : v_2^2$  is

- (1)  $1 : \sqrt{2}$  (2)  $1 : 3$   
 (3)  $1 : 2$  (4)  $1 : \sqrt{3}$

Ans. (1)



using WET

$$W_g = k_f - k_i$$

$$Mg L \sin \theta = k_f - k_i$$

$$\text{K.E. in pure rolling } \frac{1}{2} m V_{cm}^2 + I_{cm} \omega^2$$

$$= \frac{1}{2} m V^2 + \frac{1}{2} \times \frac{2}{5} m R \frac{V^2}{R^2}$$

$$\frac{7}{10} m V^2$$

$$mgL \sin \theta = \frac{7}{10} m V_f^2 - 0$$

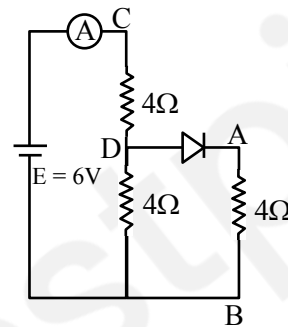
$$V_f^2 \propto \sin \theta$$

$$\left( \frac{V_1}{V_2} \right)^2 = \frac{\sin \theta}{\sin \theta} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{\sqrt{2}}$$

40. Refer to the circuit diagram given in the figure, which of the following observation are correct?

- A. Total resistance of circuit is  $6 \Omega$ .  
 B. Current in Ammeter is 1A  
 C. Potential across AB is 4 Volts.  
 D. Potential across CD is 4 Volts.  
 E. Total resistance of the circuit is  $8 \Omega$ .

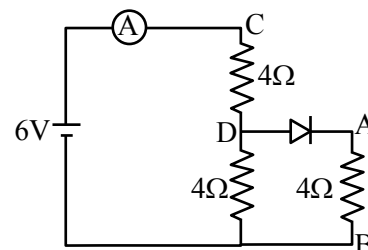
Choose the correct answer from the options given below:



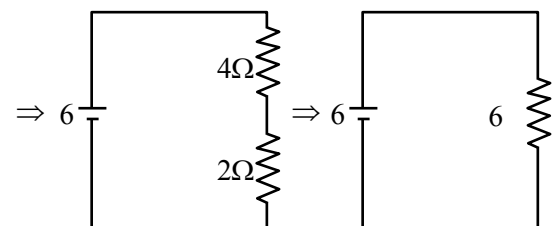
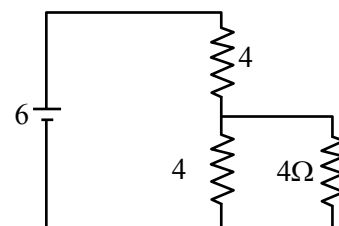
- (1) A, B and D only (2) A, C and D only  
 (3) B, C and E only (4) A, B and C only

Ans. (1)

Sol.

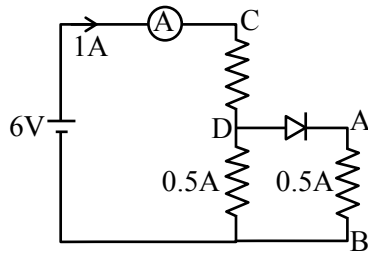


⇓



Current through ammeter = 1 A

$$R_{\text{net}} = 6\Omega$$



$$V_{AB} = 0.5 \times 4 = 2 \text{ volt}$$

$$V_{CD} = 1 \times 4 = 4 \text{ volt}$$

A, B & D are correct

41. The electric flux is  $\phi = \alpha\sigma + \beta\lambda$

where  $\lambda$  and  $\sigma$  are linear and surface charge

density, respectively,  $\left(\frac{\alpha}{\beta}\right)$  represents

- (1) charge
- (2) electric field
- (3) displacement
- (4) area

Ans. (3)

Sol.  $\phi = \alpha\sigma + \beta\lambda$

$$[\phi] = [\alpha\sigma] = [\beta\lambda]$$

$$[\alpha] = \frac{[\phi]}{[\sigma]} \quad \left[ \frac{\alpha}{\beta} \right] = \frac{[\lambda]}{[\sigma]}$$

$$[\beta] = \frac{[\phi]}{[\lambda]} = \frac{[Q/L]}{[Q/\text{Area}]} \left[ \frac{\text{Area}}{\text{Length}} \right]$$

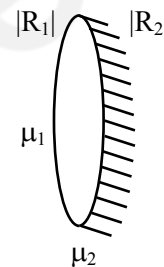
$$\left[ \frac{\alpha}{\beta} \right] = L$$

42. Given a thin convex lens (refractive index  $\mu_2$ ), kept in a liquid (refractive index  $\mu_1$ ,  $\mu_1 < \mu_2$ ) having radii of curvature  $|R_1|$  and  $|R_2|$ . Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place ?

- (1)  $\frac{\mu_1 |R_1| \cdot |R_2|}{\mu_2 (|R_1| + |R_2|) - \mu_1 |R_1|}$
- (2)  $\frac{\mu_1 |R_1| \cdot |R_2|}{\mu_2 (|R_1| + |R_2|) - \mu_1 |R_2|}$
- (3)  $\frac{\mu_1 |R_1| \cdot |R_2|}{\mu_2 (2|R_1| + |R_2|) - \mu_1 \sqrt{|R_1| \cdot |R_2|}}$
- (4)  $\frac{(\mu_2 + \mu_1) |R_1|}{(\mu_2 - \mu_1)}$

Ans. (2)

Sol.



$$\frac{1}{f_{\text{eq}}} = \frac{2}{f_L} - \frac{1}{f_m}$$

$$f_m = -\frac{|R_2|}{2}$$

$$\frac{1}{f_L} = \left( \frac{\mu_2}{\mu_1} - 1 \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$= \frac{2}{f_{\text{eq}}} \left( \frac{\mu_2 - \mu_1}{\mu_1} \right) \left( \frac{R_1 + R_2}{R_1 R_2} \right) + \frac{2}{R_2}$$

$$= \frac{2}{R_2} \left[ \frac{(\mu_2 - \mu_1)(R_1 + R_2) + \mu_1 R_1}{\mu_1} \right]$$

$$= \frac{2}{R_2} \left[ \frac{\mu_2 R_1 + \mu_2 R_2 - \mu_1 R_1 - \mu_1 R_2 + \mu_1 R_1}{\mu_1} \right]$$



$$\frac{1}{f_{eq}} = \frac{2[\mu_2 R_1 + \mu_2 R_2 - \mu_1 R_2]}{\mu_1 R_1 R_2}$$

For same size of image

$$u = 2f$$

$$u = \frac{\mu_1 R_1 R_2}{\mu_2 R_1 + \mu_2 R_2 - \mu_1 R_2}$$

43. The electric field of an electromagnetic wave in free space is

$$\vec{E} = 57 \cos[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)]$$

$$(4\hat{i} - 3\hat{j}) \text{ N/C.}$$

The associated magnetic field in Tesla is-

$$(1) \vec{B} = \frac{57}{3 \times 10^8} \cos [7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)] (5\hat{k})$$

$$(2) \vec{B} = \frac{57}{3 \times 10^8} \cos [7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)] (\hat{k})$$

$$(3) \vec{B} = -\frac{57}{3 \times 10^8} \cos [7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)] (5\hat{k})$$

$$(4) \vec{B} = -\frac{57}{3 \times 10^8} \cos [7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)] (\hat{k})$$

Ans. (3)

Sol.  $\vec{K} = 3\hat{i} + 4\hat{j}$

$$\hat{K} = \frac{3\hat{i} + 4\hat{j}}{5}$$

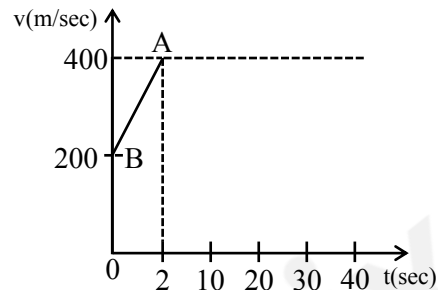
$$\hat{E} = \frac{4\hat{i} - 3\hat{j}}{5}$$

$$\hat{B} = \hat{K} \times \hat{E}$$

$$\hat{B} = -\hat{Z}$$

$$B_0 = \frac{E_0}{C} = \frac{57}{3 \times 10^8}$$

44. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is \_\_\_\_ km.



- (1) 9 (2) 6  
(3) 3 (4) 12

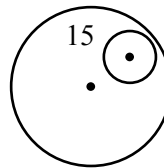
Ans. (4)

Sol. Total Area under curve.

45. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be-

- (1) 2.0 cm (2) 0.5 cm  
(3) 1.5 cm (4) 1.0 cm

Ans. (4)



Sol.

mass of disc = m

mass of cut part =  $\frac{m}{16}$

$$X_{com} = \frac{m \times 0 - \frac{m}{16} \times 15}{m - \frac{m}{16}}$$

$$= 1 \text{ cm.}$$

SECTION-B

46. A positive ion A and a negative ion B has charges  $6.67 \times 10^{-19} \text{ C}$  and  $9.6 \times 10^{-10} \text{ C}$ , and masses  $19.2 \times 10^{-27} \text{ kg}$  and  $9 \times 10^{-27} \text{ kg}$  respectively. At an instant, the ions are separated by a certain distance  $r$ . At that instant the ratio of the magnitudes of electrostatic force to gravitational force is  $P \times 10^{-13}$ , where the value of  $P$  is \_\_\_\_.

(Take  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-1}$  and universal

gravitational constant as  $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ )

Ans. (BONUS)

Sol. 
$$\frac{9 \times 10^9 \times 6.67 \times 10^{-19} \times 9.6 \times 10^{-10}}{6.67 \times 10^{-11} \times 19.2 \times 10^{-27} \times 9 \times 10^{-27}}$$
  

$$\frac{1}{2} \times 10^{45}$$

Charge is not integral multiple of electron.

47. Two particles are located at equal distance from origin. The position vectors of those are represented by  $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$  and  $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$ , respectively. If both the vectors are at right angle to each other, the value of  $n^{-1}$  is \_\_\_\_.

Ans. (3)

Sol.  $\vec{A} \cdot \vec{B} = 0$

$$4 - 6n + 8p = 0$$

$$|\vec{A}| = |\vec{B}|$$

$$4 + 9n^2 + 4 = 4 + 4 + 16p^2$$

$$9n^2 = 16p^2$$

$$P = +\frac{3}{4}n$$

$$4 - 6n \pm 6n = 0$$

$$12n = 4$$

$$n = \frac{1}{3}$$

48. An ideal gas initially at  $0^\circ\text{C}$  temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is  $3/2$ , the change in temperature due to the thermodynamics process is \_\_\_\_ K.

Ans. (273)

Sol.  $\gamma = \frac{3}{2}$

$$TV^{\gamma-1} = C$$

$$273 V_0^{0.5} = T \left( \frac{V_0}{4} \right)^{0.5}$$

$$T = 273 \times 2 = 546$$

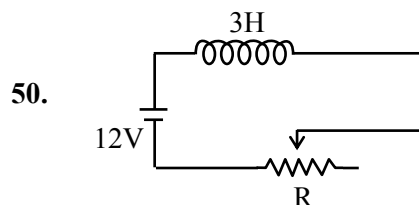
$$\Delta T = 273$$

49. A force  $\vec{f} = x^2y\hat{i} + y^2\hat{j}$  acts on a particle in a plane  $x + y = 10$ . The work done by this force during a displacement from  $(0, 0)$  to  $(4\text{m}, 2\text{m})$  is \_\_\_\_ Joule (round off to the nearest integer)

Ans. (152)

Sol. 
$$\int_0^4 x^2(10-x)dx + \int_0^2 y^2 dy$$
  

$$= \left[ \frac{10x^3}{3} - \frac{x^4}{4} \right]_0^4 + \left[ \frac{y^3}{3} \right]_0^2 = \frac{640}{3} - 64 + \frac{8}{3} = 152$$



In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of  $8 \text{ A/s}$ . At an instant when  $R$  is  $12 \Omega$ , the value of the current in the circuit will be \_\_\_\_ A.

Ans. (3)

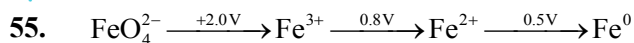
Sol.  $\epsilon - \frac{LdI}{dt} - IR = 0$

$$12 - 3 \times (-8) - I \times 12 = 0$$

$$I = 3$$

**(HELD ON THURSDAY 23<sup>rd</sup> JANUARY 2025)**
**TIME : 9 : 00 AM TO 12 : 00 NOON**

| CHEMISTRY                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | TEST PAPER WITH SOLUTIONS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p align="center"><b>SECTION-A</b></p> <p><b>51.</b> The element that does not belong to the same period of the remaining elements (modern periodic table) is:</p> <p>(1) Palladium<br/>(2) Iridium<br/>(3) Osmium<br/>(4) Platinum</p> <p><b>Sol. (1)</b></p> <p>Palladium <math>\Rightarrow</math> 5<sup>th</sup> period<br/>Iridium, Osmium, Platinum <math>\Rightarrow</math> 6<sup>th</sup> Period</p> <p><b>52.</b> Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this ?</p> <p>Given: Rydberg constant<br/><math>R_H = 10^5 \text{ cm}^{-1}</math>, <math>h = 6.6 \times 10^{-34} \text{ J s}</math>, <math>c = 3 \times 10^8 \text{ m/s}</math></p> <p>(1) Paschen series, <math>\infty \rightarrow 3</math><br/>(2) Lyman series, <math>\infty \rightarrow 1</math><br/>(3) Balmer series, <math>\infty \rightarrow 2</math><br/>(4) Paschen series, <math>5 \rightarrow 3</math></p> <p><b>Sol. (1)</b></p> <p><math>\lambda = 900 \text{ nm}</math>                      H-atom (<math>Z = 1</math>)<br/> <math>= 9 \times 10^{-5} \text{ cm}</math><br/> <math>R_H = 10^5 \text{ cm}^{-1}</math></p> <p>Rydberg eq. <math>= \frac{1}{\lambda} = R_H Z^2 \times \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)</math></p> <p><math>\Rightarrow \frac{1}{\lambda R_H} = \frac{1}{n_1^2} - \frac{1}{n_2^2}</math></p> <p><math>\Rightarrow \frac{1}{9 \times 10^5 \text{ cm} \times 10^5 \text{ cm}^{-1}} = \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)</math></p> <p><math>\Rightarrow \frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{9}</math></p> <p>It is possible when <math>n_1 = 3</math>, <math>n_2 = \infty</math><br/>Possible series : <math>\infty \rightarrow 3</math></p> | <p><b>53.</b> The <b>incorrect</b> statements among the following is</p> <p>(1) <math>\text{PH}_3</math> shows lower proton affinity than <math>\text{NH}_3</math>.<br/>(2) <math>\text{PF}_3</math> exists but <math>\text{NF}_5</math> does not.<br/>(3) <math>\text{NO}_2</math> can dimerise easily.<br/>(4) <math>\text{SO}_2</math> can act as an oxidizing agent, but not as a reducing agent.</p> <p><b>Sol. (4)</b></p> <p><math>\text{SO}_2</math> can oxidise as well as reduce.<br/>Hence it can act as both oxidising and reducing agent.</p> <p><b>54.</b> <math>\text{CrCl}_3 \cdot x\text{NH}_3</math> can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of <math>0.558^\circ\text{C}</math>. Assuming 100% ionisation of this complex and coordination number of Cr is 6, the complex will be</p> <p>(Given <math>K_f = 1.86 \text{ K kg mol}^{-1}</math>)</p> <p>(1) <math>[\text{Cr}(\text{NH}_3)_6] \text{Cl}_3</math><br/>(2) <math>[\text{Cr}(\text{NH}_3)_4 \text{Cl}_2] \text{Cl}</math><br/>(3) <math>[\text{Cr}(\text{NH}_3)_5 \text{Cl}] \text{Cl}_2</math><br/>(4) <math>[\text{Cr}(\text{NH}_3)_3 \text{Cl}_3]</math></p> <p><b>Sol. (3)</b></p> <p>Given : <math>\Delta T_f = 0.558^\circ\text{C}</math></p> <p><math>k_f = 1.86 \frac{\text{K} \times \text{kg}}{\text{mol}}</math></p> <p>0.1 m aq. sol.</p> <p><math>\Rightarrow \Delta T_f = i \times k_f \times m</math></p> <p><math>\Rightarrow 0.558 = i \times 1.86 \times 0.1</math></p> <p><math>\Rightarrow i = 3</math></p> |

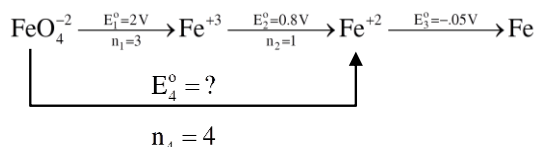


In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of  $E^\ominus_{\text{FeO}_4^{2-}/\text{Fe}^{2+}}$  is

- (1) 1.7 V (2) 1.2 V  
(3) 2.1 V (4) 1.4 V

Sol. (1)



$$\Delta G_4^\ominus = \Delta G_1^\ominus + \Delta G_2^\ominus$$

$$\Rightarrow -n_4 E_4^\ominus = -n_1 E_1^\ominus - n_2 E_2^\ominus$$

$$\Rightarrow +4 E_4^\ominus = 3 \times 2 + (1 \times 0.8)$$

$$\Rightarrow E_4^\ominus = \frac{6.8}{4}$$

$$\Rightarrow E_4^\ominus = 1.7\text{V}$$

56. Match the LIST-I with LIST-II

| LIST-I        |                       | LIST-II            |                |
|---------------|-----------------------|--------------------|----------------|
| Name reaction |                       | Product obtainable |                |
| A.            | Swarts reaction       | I.                 | Ethyl benzene  |
| B.            | Sandmeyer's reaction  | II.                | Ethyl iodide   |
| C.            | Wurtz Fittig reaction | III.               | Cyanobenzene   |
| D.            | Finkelstein reaction  | IV.                | Ethyl fluoride |

Choose the **correct** answer from the options given below:

- (1) A-II, B-III, C-I, D-IV  
(2) A-IV, B-I, C-III, D-II  
(3) A-IV, B-III, C-I, D-II  
(4) A-II, B-I, C-III, D-IV

Sol. (3)

| LIST-I        |                       | LIST-II            |                                                                                                                |
|---------------|-----------------------|--------------------|----------------------------------------------------------------------------------------------------------------|
| Name reaction |                       | Product obtainable |                                                                                                                |
| A.            | Swarts reaction       | I.                 | $\text{Et-I} \xrightarrow[\text{DMF}]{\text{KF}} \text{Et-F}$                                                  |
| B.            | Sandmeyer's reaction  | II.                | $\text{PhN}_2^+\text{Cl}^- \xrightarrow{\text{CuCN/KCN}} \text{PhCN} + \text{N}_2$                             |
| C.            | Wurtz Fittig reaction | III.               | $\text{Ph-Cl} + \text{Et-Cl} \xrightarrow[\text{ether}]{\text{Na}} \text{Ph-Et} + \text{Ph-Ph} + \text{Et-Et}$ |
| D.            | Finkelstein reaction  | IV.                | $\text{Et-Cl} \xrightarrow[\text{acetone}]{\text{NaI}} \text{Et-I} + \text{NaCl}$                              |

57. Given below are two statements:

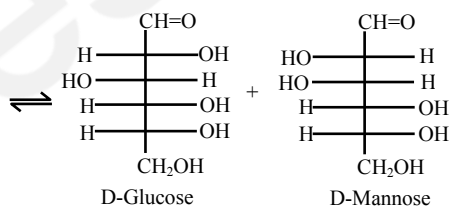
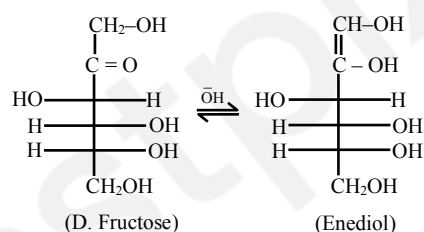
**Statement I:** Fructose does not contain an aldehydic group but still reduces Tollen's reagent

**Statement II :** In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the **correct** answer from the options given below

- (1) Statement I is false but Statement II is true  
(2) Both Statement I and Statement II are true  
(3) Both Statement I and Statement II are false  
(4) Statement I is true but Statement II is false

Sol. (2)



58.  $2.8 \times 10^{-3}$  mol of  $\text{CO}_2$  is left after removing  $10^{21}$  molecules from its 'x' mg sample. The mass of  $\text{CO}_2$  taken initially is

$$\text{Given : } N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

- (1) 196.2 mg (2) 98.3 mg  
(3) 150.4 mg (4) 48.2 mg

Sol. (1)

$$(\text{moles})_{\text{initial}} = \frac{x \times 10^{-3}}{44}$$

$$(\text{moles})_{\text{removal}} = \frac{10^{21}}{6.02 \times 10^{23}}$$

$$(\text{moles})_{\text{left}} = (\text{moles})_{\text{initial}} - (\text{moles})_{\text{removed}}$$

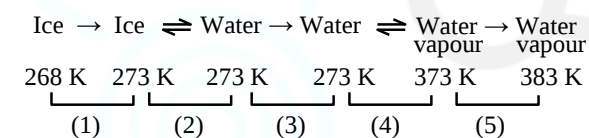
$$2.8 \times 10^{-3} = \frac{x \times 10^{-3}}{44} - \frac{10^{21}}{6.02 \times 10^{23}}$$

$$\Rightarrow x = 196.2 \text{ mg}$$

59. Ice at  $-5^{\circ}\text{C}$  is heated to become vapor with temperature of  $110^{\circ}\text{C}$  at atmospheric pressure. The entropy change associated with this process can be obtained from :

$$\begin{aligned}
 (1) \quad & \int_{268\text{K}}^{383\text{K}} C_p dT + \frac{H_{\text{melting}}}{273} + \frac{\Delta H_{\text{boiling}}}{373} \\
 (2) \quad & \int_{268\text{K}}^{273\text{K}} \frac{C_{p,m}}{T} dT + \frac{\Delta H_{m,\text{fusion}}}{T_f} + \frac{\Delta H_{m,\text{vaporisation}}}{T_b} \\
 & + \int_{273\text{K}}^{373\text{K}} \frac{C_{p,m}}{T} dT + \int_{373\text{K}}^{383\text{K}} \frac{C_{p,m}}{T} dT \\
 (3) \quad & \int_{268\text{K}}^{383\text{K}} C_p dT + \frac{q_{\text{rev}}}{T} \\
 (4) \quad & \int_{268\text{K}}^{273\text{K}} C_{p,m} dT + \frac{\Delta H_{m,\text{fusion}}}{T_f} + \frac{\Delta H_{m,\text{vaporisation}}}{T_b} \\
 & + \int_{273\text{K}}^{373\text{K}} C_{p,m} dT + \int_{373\text{K}}^{383\text{K}} C_{p,m} dT
 \end{aligned}$$

**Sol. (2)**



$$\Delta S_{\text{overall}} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$$

$$\Delta S_2 = \frac{\Delta H_{m,\text{fusion}}}{273} \quad T_f = 273\text{ 'K'}$$

$$\Delta S_3 = \int_{273}^{373} \frac{C_{p,m}}{T} dT$$

$$\Delta S_4 = \frac{\Delta H_{m,\text{vaporisation}}}{373} \quad T_b = 373\text{ 'K'}$$

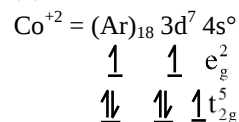
$$\Delta S_5 = \int_{373}^{383} \frac{C_{p,m}}{T} dT$$

Answer = (2)

60. The d-electronic configuration of an octahedral  $\text{Co(II)}$  complex having magnetic moment of 3.95 BM is :

- (1)  $t_{2g}^6 e_g^1$  (2)  $t_{2g}^3 e_g^0$   
 (3)  $t_{2g}^5 e_g^2$  (4)  $e^4 t_{2g}^3$

**Sol. (3)**



61. The complex that shows Facial – Meridional isomerism is

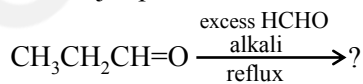
- (1)  $[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$  (2)  $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$   
 (3)  $[\text{Co}(\text{en})_3]^{3+}$  (4)  $[\text{Co}(\text{en})_2\text{Cl}_2]^+$

**Sol. (1)**

$\text{Ma}_3\text{b}_3$  type complexes show Facial - Meridional isomerism

- (i)  $[\text{Co}(\text{NH}_3)_3\text{Cl}_3] \Rightarrow \text{Ma}_3\text{b}_3$   
 (ii)  $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+ \Rightarrow \text{Ma}_4\text{b}_2$   
 (iii)  $[\text{Co}(\text{en})_3]^{3+} \Rightarrow \text{M}(\text{AA})_3$   
 (iv)  $[\text{Co}(\text{en})_2\text{Cl}_2]^+ \Rightarrow \text{M}(\text{AA})_2\text{b}_2$   
 $a, b = \text{NH}_3, \text{Cl}^-$   
 $\text{AA} = \text{en}$

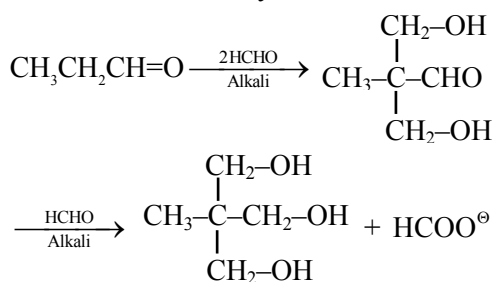
62. The major product of the following reaction is :



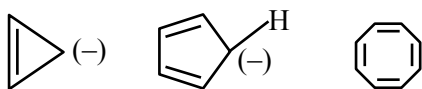
- (1)  $\text{CH}_3-\text{CH}_2-\text{CH}_2-\text{OH}$   
 (2)  $\text{CH}_3-\text{CH}-\text{CH}=\text{O}$   
 $\quad \quad \quad |$   
 $\quad \quad \quad \text{CH}_2-\text{OH}$   
 (3)  $\text{CH}_3-\text{C}-\text{CH}_2-\text{OH}$   
 $\quad \quad \quad \diagup \quad \diagdown$   
 $\quad \quad \quad \text{CH}_2-\text{OH} \quad \text{CH}_2-\text{OH}$   
 (4)  $\text{CH}_3-\text{C}-\text{CH}=\text{O}$   
 $\quad \quad \quad ||$   
 $\quad \quad \quad \text{CH}_2$

**Sol. (3)**

This is an example of Tollen's reaction i.e. multiple cross aldol followed by cross Cannizzaro reaction



63. The correct stability order of the following species/molecules is :



p

q

r

- (1)  $q > r > p$  (2)  $r > q > p$   
(3)  $q > p > r$  (4)  $p > q > r$

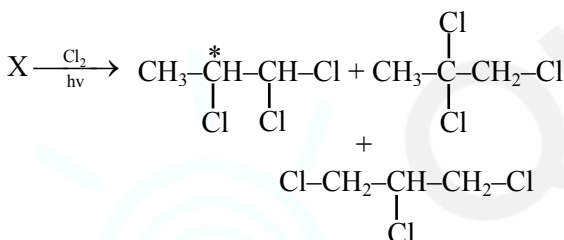
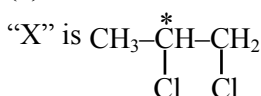
Sol. (1)

q is aromatic r is nonaromatic p is antiaromatic

64. Propane molecule on chlorination under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x" when it is further treated with chlorine under the photochemical condition?

- (1) 4 (2) 2  
(3) 5 (4) 3

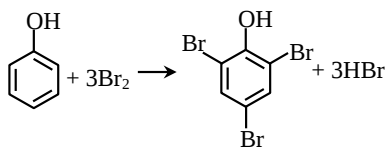
Sol. (4)



65. What amount of bromine will be required to convert 2 g of phenol into 2, 4, 6-tribromophenol ? (Given molar mass in  $\text{g mol}^{-1}$  of C, H, O, Br are 12, 1, 16, 80 respectively)

- (1) 10.22 g (2) 6.0 g  
(3) 4.0 g (4) 20.44 g

Sol. (1)



$$\text{Moles of phenol} = \frac{2}{94} = 0.021$$

$$\therefore \text{Moles of bromine} = 0.021 \times 3 = 0.064$$

$$\therefore \text{Mass of bromine} = 0.064 \times 160 = 10.22 \text{ g}$$

66. The correct set of ions (aqueous solution) with same colour from the following is :

- (1)  $\text{V}^{2+}$ ,  $\text{Cr}^{3+}$ ,  $\text{Mn}^{3+}$  (2)  $\text{Zn}^{2+}$ ,  $\text{V}^{3+}$ ,  $\text{Fe}^{3+}$   
(3)  $\text{Ti}^{4+}$ ,  $\text{V}^{4+}$ ,  $\text{Mn}^{2+}$  (4)  $\text{Sc}^{3+}$ ,  $\text{Ti}^{3+}$ ,  $\text{Cr}^{2+}$

Sol. (1)

- (1)  $\text{V}^{2+}$  (Violet),  $\text{Cr}^{3+}$  (Violet),  $\text{Mn}^{3+}$  (Violet)  
(2)  $\text{Zn}^{2+}$  (Colourless),  $\text{V}^{3+}$  (Green),  $\text{Fe}^{3+}$  (Yellow)  
(3)  $\text{Ti}^{4+}$  (Colourless),  $\text{V}^{4+}$  (Blue),  $\text{Mn}^{2+}$  (Pink)  
(4)  $\text{Sc}^{3+}$  (Colourless),  $\text{Ti}^{3+}$  (Purple),  $\text{Cr}^{2+}$  (Blue)

67. Given below are two statements :

**Statement I :** In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

**Statement II :** The sodium fusion extract of an organic compound having N and S gives prussian blue colour with  $\text{FeSO}_4$  and  $\text{Na}_4[\text{Fe}(\text{CN})_6]$

In the light of the above statements, choose the **correct** answer from the options given below

- (1) Both **Statement I** and **Statement II** are true  
(2) Both **Statement I** and **Statement II** are false  
(3) **Statement I** is false but **Statement II** is true  
(4) **Statement I** is true but **Statement II** is false

Sol. (4)

The sodium fusion extract of organic compound having N & S gives blood red colour with  $\text{FeSO}_4$  and  $\text{Na}_4[\text{Fe}(\text{CN})_6]$

68. Which of the following happens when  $\text{NH}_4\text{OH}$  is added gradually to the solution containing  $1\text{M A}^{2+}$  and  $1\text{M B}^{3+}$  ions ?

$$\text{Given : } K_{\text{sp}}[\text{A}(\text{OH})_2] = 9 \times 10^{-10} \text{ and}$$

$$K_{\text{sp}}[\text{B}(\text{OH})_3] = 27 \times 10^{-18} \text{ at } 298 \text{ K.}$$

- (1)  $\text{B}(\text{OH})_3$  will precipitate before  $\text{A}(\text{OH})_2$   
(2)  $\text{A}(\text{OH})_2$  and  $\text{B}(\text{OH})_3$  will precipitate together  
(3)  $\text{A}(\text{OH})_2$  will precipitate before  $\text{B}(\text{OH})_3$   
(4) Both  $\text{A}(\text{OH})_2$  and  $\text{B}(\text{OH})_3$  do not show precipitation with  $\text{NH}_4\text{OH}$

**Sol. (1)**

 Condition for precipitation  $Q_{ip} > K_{sp}$ 

 For  $[A(OH)_2]$ 

$$[A^{2+}][OH^-]^2 > 9 \times 10^{-10}$$

$$[A^{2+}] = 1 \text{ M}$$

$$\Rightarrow [OH^-] > 3 \times 10^{-5} \text{ M}$$

 For  $[B(OH)_3]$ 

$$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$$

$$[B^{3+}] = 1 \text{ M}$$

$$\Rightarrow [OH^-] > 3 \times 10^{-6} \text{ M}$$

 So,  $B(OH)_3$  will precipitate before  $A(OH)_2$ 
**69. Match the LIST-I with LIST-II**

| LIST-I<br>(Classification of molecules<br>based on octet rule) |                                                   | LIST-II<br>(Example) |                                                   |
|----------------------------------------------------------------|---------------------------------------------------|----------------------|---------------------------------------------------|
| A.                                                             | Molecules obeying octet rule                      | I.                   | NO, NO <sub>2</sub>                               |
| B.                                                             | Molecules with incomplete octet                   | II.                  | BCl <sub>3</sub> , AlCl <sub>3</sub>              |
| C.                                                             | Molecules with incomplete octet with odd electron | III.                 | H <sub>2</sub> SO <sub>4</sub> , PCl <sub>5</sub> |
| D.                                                             | Molecules with expanded octet                     | IV.                  | CCl <sub>4</sub> , CO <sub>2</sub>                |

 Choose the **correct** answer from the options given below :

(1) A-IV, B-II, C-I, D-III

(2) A-III, B-II, C-I, D-IV

(3) A-IV, B-I, C-III, D-II

(4) A-II, B-IV, C-III, D-I

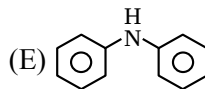
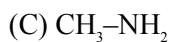
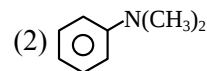
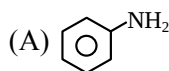
**Sol. (1)**

(A) A → IV

(B) B → II

(C) C → I

(D) D → III

**70.** Which among the following react with Hinsberg's reagent?


Choose the correct answer from the options given below :

(1) B and D only

(2) C and D only

(3) A, B and E only

(4) A, C and E only

**Sol. (4)**

B and D are 3° amine which does not have replaceable H on N, So does not react.

### SECTION-B

**71.** If 1 mM solution of ethylamine produces pH = 9, then the ionization constant ( $K_b$ ) of ethylamine is  $10^{-x}$ . The value of x is \_\_\_\_\_ (nearest integer).  
 [The degree of ionization of ethylamine can be neglected with respect to unity.]

**Sol. (7)**


$$C = 10^{-3} \text{ M}$$

$$C(1 - \alpha)$$

$$\Rightarrow C = 10^{-3}$$

$$-$$

$$C\alpha$$

$$= 10^{-5}$$

$$1 - \alpha \approx 1$$

$$\text{Given, } P^H = 9 \Rightarrow P^{OH} = 5 \Rightarrow [OH^-] = 10^{-5} \text{ M}$$

$$\text{Now, } K_b = \frac{[C_2H_5NH_3^+][OH^-]}{[C_2H_5NH_2]}$$

$$\Rightarrow K_b = \frac{10^{-5} \times 10^{-5}}{10^{-3}} = 10^{-7}$$

72. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is \_\_\_\_\_ %.

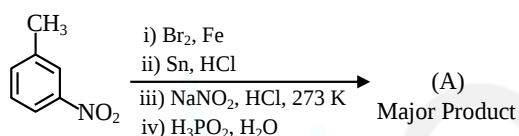
(Given molar mass in  $\text{g mol}^{-1}$  of Ba : 137, S : 32, O : 16)

**Sol. (40)**

$$\text{Millimoles of BaSO}_4 = \frac{466}{233} = 2 \text{ m mol}$$

$$\%S = \frac{\frac{466}{233} \times 32}{160} \times 100 = 40\%$$

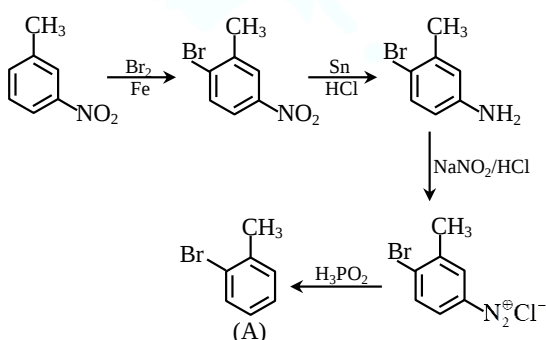
73. Consider the following sequence of reactions to produce major product (A)



Molar mass of product (A) is \_\_\_\_\_  $\text{g mol}^{-1}$ .

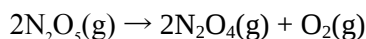
(Given molar mass in  $\text{g mol}^{-1}$  of C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31)

**Sol. (171)**



Molar mass of product ( $\text{C}_7\text{H}_7\text{Br}$ ) (A) is  $171 \text{ g mol}^{-1}$

74. For the thermal decomposition of  $\text{N}_2\text{O}_5(\text{g})$  at constant volume, the following table can be formed, for the reaction mentioned below :



| S.No. | Time/s | Total pressure / (atm) |
|-------|--------|------------------------|
| 1.    | 0      | 0.6                    |
| 2.    | 100    | 'x'                    |

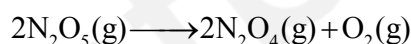
$$x = \text{_____} \times 10^{-3} \text{ atm [nearest integer]}$$

Given : Rate constant for the reaction is  $4.606 \times 10^{-2} \text{ s}^{-1}$ .

**Sol. (900)**

**NTA. (897)**

$$K_{\text{N}_2\text{O}_5} = 2 \times 4.606 \times 10^{-2} \text{ s}^{-1}$$



$$P_i \quad 0.6 \quad 0 \quad 0$$

$$P_f \quad 0.6 - P \quad P \quad \frac{P}{2}$$

$$2 \times 4.606 \times 10^{-2} = \frac{2.303}{100} \log \frac{0.6}{0.6 - P}$$

$$4 \log_{10} \frac{0.6}{0.6 - P}$$

$$10^4 = \frac{0.6}{0.6 - P}$$

$$\Rightarrow 0.6 \times 10^4 - 10^4 P = 0.6$$

$$\Rightarrow 10^4 P = 0.6(10^4 - 1)$$

$$P = (6000 - 0.6) \times 10^{-4}$$

$$= 5999. \times 10^{-4}$$

$$= 0.59994$$

$$P_{\text{Total}} = 0.6 + \frac{P}{2}$$

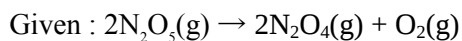
$$= 0.6 + 0.29997$$

$$= 0.89997$$

$$= 899.97 \times 10^{-3}$$

Ans. 900

Given by NTA



|          |         |   |     |
|----------|---------|---|-----|
| t = 0    | 0.6     | 0 | 0   |
| t = 100s | 0.6 - x | x | x/2 |

$$P_{\text{Total}} = 0.6 + \frac{x}{2}$$

As given in equation

$$K_r = 4.606 \times 10^{-2} \text{ sec}^{-1}$$

(Here language conflict in question)

$$(K_r = \frac{KA}{2} \text{ not considered})$$

$$K_r t = \ln \frac{0.6}{0.6 - x}$$

$$4.606 \times 10^{-2} \times 100 = 2.303 \log \frac{0.6}{0.6 - x}$$

$$P_{\text{Total}} = 0.6 + \frac{0.594}{2} = 0.897 \text{ atm}$$

$$= 897 \times 10^{-3} \text{ atm}$$

75. The standard enthalpy and standard entropy of decomposition of  $\text{N}_2\text{O}_4$  to  $\text{NO}_2$  are  $55.0 \text{ kJ mol}^{-1}$  and  $175.0 \text{ J/K/mol}$  respectively. The standard free energy change for this reaction at  $25^\circ\text{C}$  in  $\text{J mol}^{-1}$  is \_\_\_\_\_ (Nearest integer)

**Sol. (2850)**

$$\Delta H_{\text{rxn}}^\circ = 55 \text{ kJ/mol}, \quad T = 298 \text{ K}$$

$$\Delta S_{\text{rxn}}^\circ = 175 \text{ J/mol}$$

$$\Delta G_{\text{rxn}}^\circ = \Delta H_{\text{rxn}}^\circ - T\Delta S_{\text{rxn}}^\circ$$

$$\Rightarrow \Delta G_{\text{rxn}}^\circ = 55000 \text{ J/mol} - 298 \times 175 \text{ J/mol}$$

$$\Rightarrow \Delta G_{\text{rxn}}^\circ = 55000 - 52150$$

$$\Rightarrow \Delta G_{\text{rxn}}^\circ = 2850 \text{ J/mol}$$