(Held On Sunday 29th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $B \Rightarrow (A \Rightarrow B)$

(2) $A \Rightarrow (A \Leftrightarrow B)$

(3) $A \Rightarrow ((\sim A) \Rightarrow B)$

(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

Official Ans. by NTA (2)

QuestPix Ans. (1 or 3 or 4)

Sol.

A	B	~A	~A ∨ B	B ⇒ ((~A) ∨ B)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

A ⇒ B	~A ⇒ B	B ⇒ (A ⇒ B)	A ⇒ ((~A) ⇒ B)	B ⇒ ((~A) ⇒ B)
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

2. Shortest distance between the lines

$\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

(1) $2\sqrt{3}$

(2) $4\sqrt{3}$

(3) $3\sqrt{3}$

(4) $5\sqrt{3}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ $\quad \vec{a}=\hat{i}-8\hat{j}+4\hat{k}$

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ $\quad \vec{b}=\hat{i}+2\hat{j}+6\hat{k}$

$\vec{p}=2\hat{i}-7\hat{j}+5\hat{k},\ \vec{q}=2\hat{i}+\hat{j}-3\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}$$

$= \hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$

$= 16\left(\hat{i}+\hat{j}+\hat{k}\right)$

$$d=\left|\frac{(\vec{a}-\vec{b})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|=\left|\frac{\left(-10\hat{j}-2\hat{k}\right)\cdot 16\left(\hat{i}+\hat{j}+\hat{k}\right)}{16\sqrt{3}}\right|$$

$$=\left|\frac{-12}{\sqrt{3}}\right|=4\sqrt{3}$$

3. If $\vec{a}=\hat{i}+2\hat{k},\ \vec{b}=\hat{i}+\hat{j}+\hat{k},\ \vec{c}=7\hat{i}-3\hat{k}+4\hat{k}$,

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$ and $\vec{r}.\vec{a}=0$ then $\vec{r}.\vec{c}$ is equal to :

(1) 34 (2) 12

(3) 36 (4) 30

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow (\vec{r}-\vec{c})\times\vec{b}=0$

$\Rightarrow \vec{r}-\vec{c}=\lambda\vec{b}$

$\Rightarrow \vec{r}=\vec{c}+\lambda\vec{b}$

And given that $\vec{r}\cdot\vec{a}=0$

$\Rightarrow (\vec{c}+\lambda\vec{b})\cdot\vec{a}=0$

$\Rightarrow \vec{c}\cdot\vec{a}+\lambda\vec{b}\cdot\vec{a}=0$

$\Rightarrow \lambda=\frac{-\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}$

Now $\vec{r}\cdot\vec{c}=(\vec{c}+\lambda\vec{b})\cdot\vec{c}$

$$=\left(\vec{c}-\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\vec{b}\right)\cdot\vec{c}$$

$$=|\vec{c}|-\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\right)(\vec{b}\cdot\vec{c})$$

$$=74-\left[\frac{15}{3}\right]8$$

$= 74-40=34$

4. Let $S=\{w_1,w_2,....\}$ be the sample space associated to a random experiment. Let $P(w_n)=\frac{P(w_{n-1})}{2}, n\geq 2$. Let $A=\{2k+3\ell;\ k,\ell\in\mathbb{N}\}$ and $B=\{w_n;n\in A\}$. Then P(B) is equal to

(1) $\frac{3}{32}$ (2) $\frac{3}{64}$

(3) $\frac{1}{16}$ (4) $\frac{1}{32}$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Let $P(w_1)=\lambda$ then $P(w_2)=\frac{\lambda}{2}$ … $P(w_n)=\frac{\lambda}{2^{n-1}}$

As $\sum_{k=1}^{\infty}P(w_k)=1 \Rightarrow \frac{\lambda}{1-\frac{1}{2}}=1 \Rightarrow \lambda=\frac{1}{2}$

So, $P(w_n)=\frac{1}{2^n}$

$A=\{2k+3\ell;k,\ell\in\mathbb{N}\}=\{5, 7, 8, 9, 10 .....\}$

$B=\{w_n:n\in A\}$

$B=\{w_5, w_7, w_8, w_9, w_{10}, w_{11}, ...\}$

$A=\mathbb{N}-\{1,2,3,4,6\}$

$\therefore P(B)=1-[P(w_1)+P(w_2)+P(w_3)+P(w_4)+P(w_6)]$

$=1-\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{64}\right]$

$=1-\frac{32+16+8+4+1}{64}=\frac{3}{64}$

5. The value of the integral $\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$ is :

(1) $\tan^{-1}\frac{1}{2}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

(2) $\tan^{-1}2-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

(3) $\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

(4) $\tan^{-1}\frac{1}{2}-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $I=\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$

$=\int_1^2\frac{\left(t^4+1-t^2\right)+t^2}{\left(t^2+1\right)\left(t^4-t^2+1\right)}dt$

$=\int_1^2\left(\frac{1}{t^2+1}+\frac{t^2}{t^6+1}\right)dt$

$=\int_1^2\left(\frac{1}{t^2+1}+\frac{1}{3}\frac{3t^2}{(t^3)^2+1}\right)dt$

$=\tan^{-1}(t)+\frac{1}{3}\tan^{-1}(t^3)\Big|_1^2$

$=(\tan^{-1}(2)-\tan^{-1}(1))+\frac{1}{3}(\tan^{-1}(2^3)-\tan^{-1}(1^3))$

$=\tan^{-1}(2)+\frac{1}{3}\tan^{-1}(8)-\frac{\pi}{3}$

6. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99}K}{a}=\frac{2^{\ell}m}{n}$, where m and n are odd numbers, then the ordered pair (□, n) is equal to :

(1) (50, 51)

(2) (51, 99)

(3) (50, 101)

(4) (51, 101)

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. In the expansion of

$(1+x)^{99} = C_0 + C_1x + C_2x^2 + ... + C_{99}x^{99}$

$K = C_1 + C_3 + ... + C_{99} = 2^{98}$

a $\Rightarrow$ Middle in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$

$T_{\frac{200}{2}+1} = {}^{200}C_{100}\,(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100}$

$= {}^{200}C_{100} \cdot 2^{50}$

So, $\frac{{}^{200}C_{99} \times 2^{98}}{{}^{200}C_{100} \times 2^{50}} = \frac{100}{101} \times 2^{48}$

So, $\frac{25}{101} \times 2^{50} = \frac{m}{n}2^{\ell}$

$\therefore$ m, n are odd so

(□, n) become (50, 101) Ans.

7. Let f and g be twice differentiable functions on R such that

$f''(x) = g''(x) + 6x$

$f'(1) = 4g'(1) - 3 = 9$

$f(2) = 3g(2) = 12$

Then which of the following is NOT true ?

(1) $g(-2) - f(-2) = 20$

(2) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(3) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1|$

(4) There exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = g(x_0)$

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $f''(x) = g''(x) + 6x$ ...(1)

$f'(1) = 4g'(1) - 3 = 9$ ...(2)

$f(2) = 3g(2) = 12$ ...(3)

By integrating (1)

$f'(x) = g'(x) + 6\frac{x^2}{2} + C$

At $x = 1$,

$f'(1) = g'(1) + 3 + C$

$\Rightarrow 9 = 4 + 3 + C \Rightarrow C = 3$

$\therefore f'(x) = g'(x) + 3x^2 + 3$

Again by integrating,

$f(x) = g(x) + \frac{3x^3}{3} + 3x + D$

At $x = 2$,

$f(2) = g(2) + 8 + 3(2) + D$

$\Rightarrow 12 = 4 + 8 + 6 + D \Rightarrow D = -6$

So, $f(x) = g(x) + x^3 + 3x - 6$

$\Rightarrow f(x) - g(x) = x^3 + 3x - 6$

At $x = -2$,

$\Rightarrow g(-2) - f(-2) = 20$ (Option (1) is true)

Now, for $-1 < x, 2$

$h(x) = f(x) - g(x) = x^3 + 3x - 6$

$\Rightarrow h'(x) = 3x^2 + 3$

$\Rightarrow h(x)\uparrow$

So, $h(-1) < h(x) < h(2)$

$\Rightarrow -10 < h(x) < 8$

$\Rightarrow |h(x)| < 10$ (option (2) is NOT true)

Now, $h'(x) = f'(x) - g'(x) = 3x^2 + 3$

If $|h'(x)| < 6 \Rightarrow |3x^2 + 3| < 6$

$\Rightarrow 3x^2 + 3 < 6$

$\Rightarrow x^2 < 1$

$\Rightarrow -1 < x < 1$ (option (3) is True)

If $x \in (-1, 1)$ $|f'(x) - g'(x)| < 6$

option (3) is true and now to solve

$f(x) - g(x) = 0$

$\Rightarrow x^3 + 3x - 6 = 0$

$h(x) = x^3 + 3x - 6$

here, $h(1) = -ve$ and $h\left(\frac{3}{2}\right) = +ve$

So there exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = g(x_0)$

(option (4) is true)

8. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is

invertible, is

(1) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$ (2) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$

(3) $\{k\pi, k \in \mathbb{Z}\}$ (4) $\mathbb{R}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. If its invertible, then determinant value $\neq 0$

So ,

$$\begin{vmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 1 & 2\sin t + \cos t & \sin t - 2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying, $R_1 \to R_1 - R_2$ then $R_2 \to R_2 - R_3$

We get

$$e^{-t} \begin{vmatrix} 0 & -\sin t - \cos t & -3\sin t + \cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$$e^{-t} \times 1\left(2\sin t\cos t + 6\cos^2 t + 6\sin^2 t - 2\sin t\cos t\right) \neq 0$$

$$\Rightarrow e^{-t} \times 6 \neq 0$$

for $\forall\, t \in \mathbb{R}$

9. The area of the region

$$A = \left\{(x,y) : |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\right\}$$

(1) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$ (2) $\sqrt{5} + 2\sqrt{2} - 4.5$

(3) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$ (4) $\sqrt{5} - 2\sqrt{2} + 1$

Official Ans. by NTA (4)

QuestPix Ans. (4)

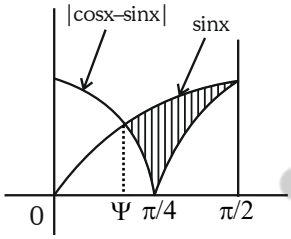
Sol. $|\cos x - \sin x| \le y \le \sin x$

Intersection point of $\cos x - \sin x = \sin x$

$$\Rightarrow \tan x = \frac{1}{2}$$

Let $\psi = \tan^{-1}\frac{1}{2}$

So, $\tan\psi = \frac{1}{2}$, $\sin\psi = \frac{1}{\sqrt{5}}$, $\cos\psi = \frac{2}{\sqrt{5}}$



$$\text{Area} = \int_{\psi}^{\pi/2} (\sin x - |\cos x - \sin x|)dx$$

$$= \int_{\psi}^{\pi/4} (\sin x - (\cos x - \sin x))dx + \int_{\pi/4}^{\pi/2} (\sin x - (\sin x - \cos x))dx$$

$$= \int_{\psi}^{\pi/4} (2\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2} \cos x\, dx$$

$$= \left[-2\cos x - \sin x\right]_{\psi}^{\pi/4} + \left[\sin x\right]_{\pi/4}^{\pi/2}$$

$$= -\sqrt{2} - \frac{1}{\sqrt{2}} + 2\cos\psi + \sin\psi + \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= -\sqrt{2} - \frac{1}{\sqrt{2}} + 2\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}}\right) + 1 - \frac{1}{\sqrt{2}}$$

$$= \sqrt{5} - 2\sqrt{2} + 1$$

10. The set of all values of λ for which the equation

$\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$

(1) $[-2, -1]$ (2) $\left[-2, -\frac{3}{2}\right]$

(3) $\left[-1, -\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2}, -1\right]$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $\lambda = \cos^2 2x - 2\sin^4 x - 2\cos^2 x$

convert all in to cos x.

$\lambda = (2\cos^2 x - 1)^2 - 2(1 - \cos^2 x)^2 - 2\cos^2 x$

$= 4\cos^4 x - 4\cos^2 x + 1 - 2(1 - 2\cos^2 x + \cos^4 x) - 2\cos^2 x$

$= 2\cos^4 x - 2\cos^2 x + 1 - 2$

$= 2\cos^4 x - 2\cos^2 x - 1$

$= 2\left[\cos^4 x - \cos^2 x - \frac{1}{2}\right]$

$= 2\left[\left(\cos^2 x - \frac{1}{2}\right)^2 - \frac{3}{4}\right]$

$\lambda_{max} = 2\left[\frac{1}{4} - \frac{3}{4}\right] = 2 \times \left(-\frac{2}{4}\right) = -1$ (max Value)

$\lambda_{min} = 2\left[0 - \frac{3}{4}\right] = -\frac{3}{2}$ (Minimum Value)

So, Range $= \left[-\frac{3}{2}, -1\right]$

11. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is :

(1) 89 (2) 84

(3) 86 (4) 79

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Lets arrange the letters of OUGHT in alphabetical order.

G, H, O, T, U

Words starting with

G $\dashrightarrow$ 4!

H $\dashrightarrow$ 4!

O $\dashrightarrow$ 4!

T G $\dashrightarrow$ 3!

T H $\dashrightarrow$ 3!

T O G $\dashrightarrow$ 2!

T O H $\dashrightarrow$ 2!

T O U G H $\rightarrow$ 1!

Total = 89

12. The plane $2x - y + z = 4$ intersects the line segment joining the points A(a, –2, 4) and B(2, b, –3) at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point (a – b, b, 2b –a) then CP^2 is equal to :

(1) $\frac{17}{3}$ (2) $\frac{16}{3}$

(3) $\frac{73}{3}$ (4) $\frac{97}{3}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. A(a, –2, 4), B(2, b, –3)

AC : CB = 2 : 1

$\Rightarrow C \equiv \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{-2}{3}\right)$

C lies on $2x - y + 2 = 4$

$\Rightarrow \frac{2a+8}{3} - \frac{2b-2}{3} - \frac{2}{3} = 4$

$\Rightarrow a - b = 2 \ldots (1)$

Also $OC = \sqrt{5}$

$\Rightarrow \left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5 \quad \ldots (2)$

Solving, (1) and (2)

$(b + 6)^2 + (2b - 2)^2 = 41$

$\Rightarrow 5b^2 + 4b - 1 = 0$

$\Rightarrow b = -1$ or $\frac{1}{5}$

$\Rightarrow a = 1$ or $\frac{11}{5}$

But $ab < 0 \Rightarrow (a, b) = (1, -1)$

$C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$, $P \equiv (2, -1, -3)$

$CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$

13. Let $\vec{a}=4\hat{i}+3\hat{j}$ and $\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}$ is a vector such that $\vec{c}.(\vec{a}\times\vec{b})+25=0$, $\vec{c}.(\hat{i}+\hat{j}+\hat{k})=4$ and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals :

(1) $\frac{5}{\sqrt{2}}$

(2) $\frac{1}{5}$

(3) $\frac{1}{\sqrt{2}}$

(4) $\frac{3}{\sqrt{2}}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\vec{a}\times\vec{b}=15\hat{i}-20\hat{j}-25\hat{k}$

Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$

$\Rightarrow$ $15x-20y-25z+25=0$

$\Rightarrow$ $3x-4y-5z=-5$

Also $x+y+z=4$

and $\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}=1 \Rightarrow 4x+3y=5$

$\Rightarrow$ $\vec{c}=2\hat{i}-\hat{j}+3\hat{k}$

Projection of $\vec{c}$ or $\vec{b}$ $=\frac{25}{5\sqrt{2}}=\frac{5}{\sqrt{2}}$

14. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersects at the point P, then the distance of the point P from the plane z = a is :

(1) 16 (2) 28

(3) 10 (4) 22

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Point on $L_1 \equiv (\lambda+1, 2\lambda+2, \lambda-3)$

Point on $L_2 \equiv (2\mu+a, 3\mu-2, \mu+3)$

$\lambda-3=\mu+3 \quad \Rightarrow \lambda=\mu+6 \quad \ldots(1)$

$2\lambda+2=3\mu-2 \quad \Rightarrow 2\lambda=3\mu-4 \quad \ldots(2)$

Solving, (1) and (2)

$\Rightarrow$ $\lambda=22$ & $\mu=16$

$\Rightarrow$ $P \equiv (23, 46, 19)$

$\Rightarrow$ $a=-9$

Distance of P from z = –9 is 28

15. The value of the integral $\int_{1/2}^{2}\frac{\tan^{-1}x}{x}dx$ is equal to

(1) $\pi \log_e 2$ (2) $\frac{1}{2}\log_e 2$

(3) $\frac{\pi}{4}\log_e 2$ (4) $\frac{\pi}{2}\log_e 2$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $I=\int_{1/2}^{2}\frac{\tan^{-1}x}{x}dx \quad \ldots\ldots (i)$

Put $x=\frac{1}{t} \quad dx=-\frac{1}{t^2}dt$

$$I=-\int_{2}^{1/2}\frac{\tan^{-1}\frac{1}{t}}{\frac{1}{t}}\cdot\frac{1}{t^2}dt=-\int_{2}^{1/2}\frac{\tan^{-1}\frac{1}{t}}{t}dt$$

$$I=\int_{1/2}^{2}\frac{\cot^{-1}t}{t}dt=\int_{1/2}^{2}\frac{\cot^{-1}x}{x}dx \quad \ldots\ldots (ii)$$

Add both equation

$$2I=\int_{1/2}^{2}\frac{\tan^{-1}x+\cot^{-1}x}{x}dx=\frac{\pi}{2}\int_{1/2}^{2}\frac{dx}{x}=\frac{\pi}{2}(\ell n2)_{1/2}^{2}$$

$$=\frac{\pi}{2}\left(\ell n2-\ell n\frac{1}{2}\right)=\pi\ell n2$$

$$I=\frac{\pi}{2}\ell n2$$

16. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line x + 2y = 1 and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line x – y = 2, then the area of the triangle PQR is:

(1) $\frac{9}{\sqrt{5}}$ (2) $5\sqrt{3}$

(3) $\frac{3}{2}\sqrt{5}$ (4) $3\sqrt{5}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $y^2 = 3x$

Tangent $P(x_1, y_1)$ is parallel to $x + 2y = 1$

Then slope at $P = -\frac{1}{2}$

$$2y\frac{dy}{dx} = 3$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2y} = -\frac{1}{2}$$

$$\Rightarrow y_1 = -3$$

Coordinates of $P(3, -3)$

Similarly $Q\left(\frac{4}{\sqrt{3}}, \frac{1}{\sqrt{5}}\right)$, $R\left(-\frac{4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3\left(\frac{2}{\sqrt{5}}\right) + 3\left(\frac{8}{\sqrt{5}}\right) + 0\right] = \frac{30}{2\sqrt{5}} = 3\sqrt{5}$$

17. Let $y = y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx} + y = x^2\log_e x, (x > 1)$. If $y(2) = 2$, then $y(e)$ is equal to

(1) $\frac{4 + e^2}{4}$

(2) $\frac{1 + e^2}{4}$

(3) $\frac{2 + e^2}{2}$

(4) $\frac{1 + e^2}{2}$

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $x\log_e x\frac{dy}{dx} + y = x^2\log_e x, (x > 1).$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\ln x} = x$$

Linear differential equation

I.F. $= e^{\int \frac{1}{x\ln x}dx} = |\ln x|$

$\therefore$ Solution of differential equation

$$y|\ln x| = \int x|\ln x|dx$$

$$= |\ln x|\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$\Rightarrow y|\ln x| = |\ln x|\left(\frac{x^2}{2}\right) - \frac{x^2}{4} + c$$

For constant

$y(2) = 2 \Rightarrow c = 1$

So, $y(x) = \frac{x^2}{2} - \frac{x^2}{4|\ln x|} + \frac{1}{|\ln x|}$

Hence, $y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = 1 + \frac{e^2}{4}$

18. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 472 (2) 432

(3) 507 (4) 400

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Total 3 digit number = 900

Divisible by 3 = 300 (Using $\frac{900}{3} = 300$)

Divisible by 4 = 225 (Using $\frac{900}{4} = 225$)

Divisible by 3 & 4 = 108, ….

(Using $\frac{900}{12} = 75$)

Number divisible by either 3 or 4

$= 300 + 2250 - 75 = 450$

We have to remove divisible by 48,

144, 192, ….., 18 terms

Required number of numbers $= 450 - 18 = 432$

19. Let R be a relation defined on $\mathbb{N}$ as a R b is $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

(1) not reflexive

(2) transitive but not symmetric

(3) symmetric but not transitive

(4) an equivalence relation

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. a R a $\Rightarrow$ 5a is multiple it 5

So reflexive

a R b $\Rightarrow 2a + 3b = 5\alpha$,

Now b R a

$$2b + 3a = 2b + \left(\frac{5\alpha - 3b}{2}\right)\cdot 3$$

$$= \frac{15}{2}\alpha - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$$

$$= \frac{5}{2}(2a + 2b - 2\alpha)$$

$$= 5(a + b - \alpha)$$

Hence symmetric

a R b $\Rightarrow$ $2a + 3b = 5\alpha$.

b R c $\Rightarrow$ $2b + 3c = 5\beta$

Now $2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 3c = 5(\alpha + \beta - b)$

$\Rightarrow$ a R c

Hence relation is equivalence relation.

20. Consider a function $f: \mathbb{N} \to \mathbb{R}$, satisfying

$f(1) + 2f(2) + 3f(3) + \ldots + xf(x) = x(x + 1) f(x); x \geq 2$

with $f(1)=1$. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

(1) 8200

(2) 8000

(3) 8400

(4) 8100

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Given for $x \geq 2$

$f(1) + 2f(2) + \ldots + xf(x) = x(x + 1) f(x)$

replace x by x + 1

$\Rightarrow$ $x(x + 1) f(x) + (x + 1) f(x + 1)$

$= (x + 1)(x + 2) f(x + 1)$

$$\Rightarrow \frac{x}{f(x+1)} + \frac{1}{f(x)} = \frac{(x+2)}{f(x)}$$

$$\Rightarrow x f(x) = (x + 1) f(x + 1) = \frac{1}{2}, x \geq 2$$

$$f(2) = \frac{1}{4},\ f(3) = \frac{1}{6}$$

Now $f(2022) = \frac{1}{4044}$

$f(2028) = \frac{1}{4056}$

So, $\frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

SECTION-B

21. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

Official Ans. by NTA (3000)

QuestPix Ans. (3000)

Sol. N should be divisible by 2 but not by 3

N = (Numbers divisible by 2) – (Numbers divisible by 6)

$$N = \frac{9000}{2} - \frac{9000}{6} = 4500 - 1500 = 3000$$

22. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _____.

Official Ans. by NTA (10)

QuestPix Ans. (10)

Sol. $S_1 : y^2 = 2x$ $\quad S_2 : x^2 + y^2 = 4x$

P(2,2) is common point on S_1 & S_2

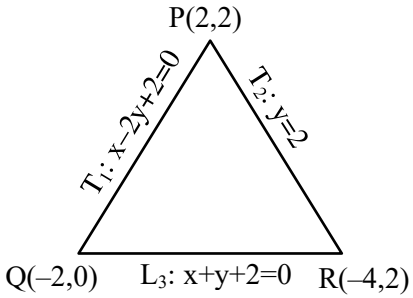
T_1 is tangent to S_1 at P $\Rightarrow T_1 : y.2 = x + 2$

$\Rightarrow T_1 : x - 2y + 2 = 0$

T_2 is tangent to S_2 at P $\Rightarrow T_2 : x.2 + y.2 = 2(x+2)$

$\Rightarrow T_2 : y = 2$

& $L_3 : x + y + 2 = 0$ is third line



$PQ = a = \sqrt{20}$

$QR = b = \sqrt{8}$

$RP = c = 6$

Area $(\Delta PQR) = \Delta = \frac{1}{2} \times 6 \times 2 = 6$

$\therefore\ r = \frac{abc}{4\Delta} = \frac{\sqrt{160}}{4} = \sqrt{10} \Rightarrow r^2 = 10$

23. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _____.

Official Ans. by NTA (11)

QuestPix Ans. (11)

Sol. The given line is polar or $P(2, \beta)$ w.r.t. given circle

$x^2 + y^2 - 4x - 6y - 3 = 0$

Chord or contact

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$\Rightarrow (\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$ ..... (i)

☐ But the equation of chord of contact is given

as : $x + y - 3 = 0$ $\quad$ ..... (ii)

comparing the coefficients

$$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = -\left(\frac{2\alpha + 3\beta + 3}{-3}\right)$$

On solving $\alpha = -6$

$\beta = -5$

Now $4\alpha - 7\beta = 11$

24. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n - 1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall\ n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to _____.

Official Ans. by NTA (461)

QuestPix Ans. (461)

Sol. As, $S = \frac{b_1}{2} + \frac{b_2}{2^2} + ...+ \frac{b_9}{2^9} + \frac{b_{10}}{2^{10}}$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + ...+ \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}}$

subtracting

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} .....+ \frac{a_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$

$\Rightarrow S = b_1 - \frac{b_{10}}{2^{10}} + \left(\frac{a_1}{2} + \frac{a_2}{2^2} .....+ \frac{a_9}{2^9}\right)$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} .....+ \frac{a_9}{2^{10}}\right)$

subtracting

$$\Rightarrow \frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2}-\frac{a_9}{2^{10}}\right)+\left(\frac{1}{2^2}+\frac{2}{2^3}+...+\frac{8}{2^9}\right)$$

$$\Rightarrow \frac{S}{2}=\frac{a_1+b_1}{2}-\frac{(b_{10}+2a_9)}{2^{11}}+\frac{T}{4}$$

$$\Rightarrow 2S=2(a_1+b_1)-\frac{(b_{10}+2a_9)}{2^9}+T$$

$$\Rightarrow 2^7(2S-T)=2^8(a_1+b_1)-\frac{(b_{10}+2a_9)}{4}$$

Given $a_n - a_{n-1} = n - 1$,

$\therefore$ $a_2 - a_1 = 1$

$a_3 - a_2 = 2$

$\vdots$

$a_9 - a_8 = 8$

$a_9 - a_1 = 1 + 2 + \ldots + 8 = 36$

$\Rightarrow$ $a_9 = 37$ $(a_1 = 1)$

Also, $b_n - b_{n-1} = a_{n-1}$

$\therefore$ $b_{10} - b_1 = a_1 + a_2 + \ldots. + a_9$

$= 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$\Rightarrow$ $b_{10} = 130$ (As $b_1 = 1$)

$\therefore$ $2^7(2S - T) = 2^8(1 + 1) - (130 + 2 \times 37)$

$$2^9-\frac{204}{4}=461$$

25. If the equation of the normal to the curve $y=\frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to _____.

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $$y=\frac{x-a}{(x+b)(x-2)}$$

At point $(1, -3)$,

$$-3=\frac{1-9}{(1+b)(1-2)}$$

$\Rightarrow$ $1 - a = 3(1 + b)$ ..... (1)

Now, $y=\frac{x-a}{(x+b)(x-2)}$

$$\Rightarrow \frac{dy}{dx}=\frac{(x+b)(x-2)\times(1)-(x-a)(2x+b-2)}{(x+b)^2(x-2)^2}$$

At $(1, -3)$ slope of normal is $\frac{1}{4}$ hence $\frac{dy}{dx} = -4$,

So, $-4=\frac{(1+b)(-1)-(1-a)b}{(1+b)^2(-1)^2}$

Using equation (1)

$$\Rightarrow -4=\frac{(1+b)(-1)-3(b+1)b}{(1+b)^2}$$

$$\Rightarrow -4=\frac{(-1)-3b}{(1+b)}(b\neq -1)$$

$\Rightarrow$ $b = -3$

So, $a = 7$

Hence, $a + b = 7 - 3 = 4$

26. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}A=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $$\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}\begin{bmatrix}a & b\\ b & c\end{bmatrix}=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$$

Now $ac - b^2 = 2$ and $2a + b = 1$

and $2b + c = 2$

solving all these above equations we get

$$\frac{1-b}{2}\times\left(\frac{2-2b}{1}\right)-b^2=2$$

$\Rightarrow$ $(1 - b)^2 - b^2 = 2$

$\Rightarrow$ $1 - 2b = 2$

$$\Rightarrow b=-\frac{1}{2} \text{ and } a=\frac{3}{4} \text{ and } c=3$$

$$\text{Hence } \alpha=3a+\frac{3b}{2}=\frac{9}{4}-\frac{3}{4}=\frac{3}{2}$$

$$\text{and } \beta=3b+\frac{3c}{2}=-\frac{3}{2}+\frac{9}{2}=3$$

$$\text{also } s=a+c=\frac{15}{4}$$

$$\therefore \frac{\beta s}{\alpha^2}=\frac{3\times 15}{4\times\frac{9}{4}}=5$$

87. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratio r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to _____.

Official Ans. by NTA (9)

QuestPix Ans. (9)

Sol. Given that

$c_k = a_k + b_k$ and $a_1 = b_1 = 4$

also $a_2 = 4r_1$ $a_3 = 4r_1^2$

$b_2 = 4r_2$ $b_3 = 4r_2^2$

Now $c_2 = a_2 + b_2 = 5$ and $c_3 = a_3 + b_3 = \frac{13}{4}$

$\Rightarrow r_1 + r_2 = \frac{5}{4}$ and $r_1^2 + r_2^2 = \frac{13}{16}$

Hence $r_1 r_2 = \frac{3}{8}$ which gives $r_1 = \frac{1}{2}$ & $r_2 = \frac{3}{4}$

$$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$$

$$= \frac{4}{1-r_1} + \frac{4}{1-r_2} - \left(\frac{48}{32} + \frac{27}{2}\right)$$

$$= 24 - 15 = 9$$

88. Let X = {11, 12, 13, ….., 40, 41} and Y = {61, 62, 63, ….., 90, 91} be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in X ∪ Y, then $\left|\bar{x} + \bar{y} - \sigma^2\right|$ is equal to _____.

Official Ans. by NTA (603)

QuestPix Ans. (603)

Sol. $\bar{x} = \frac{\sum_{i=11}^{41} i}{31} = \frac{11+41}{2} = 26$ (31 elements)

$\bar{y} = \frac{\sum_{j=61}^{91} j}{31} = \frac{61+91}{2} = 76$ (31 elements)

Combined mean, $\mu = \frac{31\times 26 + 31\times 76}{31+31}$

$= \frac{26+76}{2} = 51$

$\sigma^2 = \frac{1}{62} \times \left(\sum_{i=1}^{31} (x_i - \mu)^2 + \sum_{i=1}^{31} (y_i - \mu)^2 \right) = 705$

Since, $x_i \in X$ are in A.P. with 31 elements & common difference 1, same is $y_i \in y$, when written in increasing order.

$\therefore \sum_{i=1}^{31} (x_i - \mu)^2 = \sum_{i=1}^{31} (y_i - \mu)^2$

$= 10^2 + 11^2 + \ldots\ldots + 40^2$

$= \frac{40\times 41\times 81}{6} - \frac{9\times 10\times 19}{6} = 21855$

$\therefore \left|\bar{x} + \bar{y} - \sigma^2\right| = \left|26 + 76 - 705\right| = 603$

89. Let $\alpha = 8 - 14i$, $A = \left\{ z \in \mathbb{C} : \frac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1 \right\}$

and $B = \left\{ z \in \mathbb{C} : \left| z + 3i \right| = 4 \right\}$.

Then $\sum_{z \in A \cap B} (\mathrm{Re}\, z - \mathrm{Im}\, z)$ is equal to _____.

Official Ans. by NTA (14)

QuestPix Ans. (14)

Sol. $\alpha = 8 - 14i$

$z = x + iy$

$az = (8x + 14y) + i(-14x + 8y)$

$z+\bar{z}=2x \quad z-\bar{z}=2iy$

Set A: $\frac{2i(-14x+8y)}{i(4xy-112)}=1$

$(x-4)(y+7)=0$

$x=4$ or $y=-7$

Set B: $x^2+(y+3)^2=16$

when $x=4$ $y=-3$

when $y=-7$ $x=0$

$\therefore A\cap B=\{4-3i, 0-7i\}$

So, $\sum_{z\in A\cap B}(\text{Re}\,z-\text{Im}\,z)=4-(-3)+(0-(-7))=14$

90. Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7+3x^5-13x^3-15x=0$ and $|\alpha_1|\geq|\alpha_2|\geq\ldots.\geq|\alpha_7|$.

Then $\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$ is equal to_____.

Official Ans. by NTA (9)

QuestPix Ans. (9)

Sol. Given equation can be rearranged as

$x(x^6+3x^4-13x^2-15)=0$

clearly $x=0$ is one of the root and other part can be observed by replacing $x^2=t$ from which we have $t^3+3t^2-13t-15=0$

$\Rightarrow (t-3)(t^2+6t+5)=0$

So, $t=3,\ t=-1,\ t=-5$

Now we are getting $x^2=3, x^2=-1, x^2=-5$

$\Rightarrow x=\pm\sqrt{3}, \ x=\pm i, \ x=\pm\sqrt{5}i$

From the given condition $|\alpha_1|\geq|\alpha_2|\geq\ldots.\geq|\alpha_7|$

We can clearly say that $|\alpha_7|=0$ and

and $|\alpha_6|=\sqrt{5}=|\alpha_5|$

and $|\alpha_4|=\sqrt{3}=|\alpha_3|$ and $|\alpha_2|=1=|\alpha_1|$

So we can have, $\alpha_1=\sqrt{5}i$, $\alpha_2=-\sqrt{5}i$, $\alpha_3=\sqrt{3}i$, $\alpha_4=-\sqrt{3}$, $\alpha_5=i$, $\alpha_6=-i$

Hence

$\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$

$=1-(-3)+5=9$

(Held On Sunday 29th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

31. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(1) 3.38×10^{24} (2) 6.76×10^{24}

(3) 6.76×10^{23} (4) 1.69×10^{24}

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $(N_0)A = \frac{320}{16} = 20$ moles

$(N_0)B = \frac{320}{32} = 10$ moles

$N_A = \frac{(N_0)_A}{(2)^{2/1}} = \frac{20}{4} = 5$

$N_B = \frac{(N_0)_B}{(2)^{2/.5}} = \frac{10}{2^4} = 0.625$

Total $N = 5.625$

No. of atoms $= 5.625 \times 6.023 \times 10^{23}$

$= 3.38 \times 10^{24}$

Ans. (1)

32. At 300 K, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be (used $\pi = \frac{22}{7}$)

(1) 32 (2) 28

(3) 24 (4) 27

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$

$3 = \frac{\alpha+5}{\alpha}\frac{8}{\pi}$

$\alpha = 28$

Ans. 2

33. The ratio of de-Broglie wavelength of an α-particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is

(1) 4 (2) 16

(3) 8 (4) 2

Official Ans. by NTA (3)

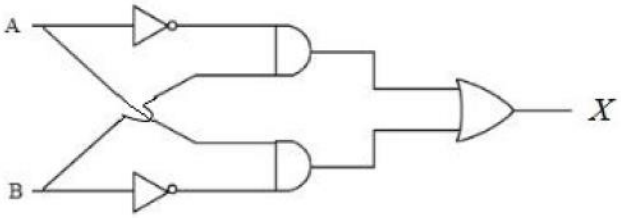
QuestPix Ans. (3)

Sol. $\frac{\lambda_\alpha}{\lambda_p} = \frac{h}{\sqrt{2m_\alpha q_\alpha V}} \Big/ \frac{h}{\sqrt{2m_p q_p V}}$

$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{8}} \quad m = 8$

Ans. 3

34. For the given logic gates combination, the correct truth table will be



(1)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

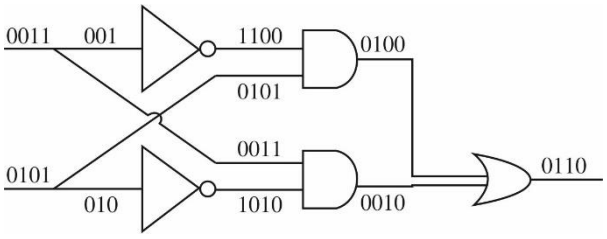
(4)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



35. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is

(1) $\sqrt{\frac{1}{1-n^2}}$ (2) $\sqrt{1-\frac{1}{n^2}}$

(3) $1+\frac{1}{n^2}$ (4) $1-\frac{1}{n^2}$

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $a_1 = g\sin\theta = g/\sqrt{2}$

$$a_2 = g\sin\theta - Kg\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}$$

$$t_2 = nt_1 \quad \& \quad a_1t_1^2 = a_2t_2^2$$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{kg}{\sqrt{2}}\right)n^2t_1^2$$

$$K = 1 - \frac{1}{n^2}$$ **Ans. 4**

36. Force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be :

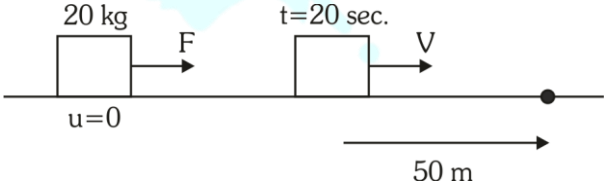
(1) 40 N (2) 5 N

(3) 20 N (4) 10 N

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$50 = V \times 10$

$V = 5$ m/s

$V = 0 + a \times 20$

$5 = a \times 20$

$a = \frac{1}{4}$ m/s^2

$F = ma = 20 \times \frac{1}{4} = 5N$

37. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be (g = 10 m/s^2)

(1) 16 (2) 6

(3) 8 (4) 10

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $P_2A - P_1A = 5.4 \times 10^5 \times g$

$$P_2 - P_1 = \frac{5.4\times10^6}{500} = 5.4\times2\times10^2\times10$$

$$= 10.8 \times 10^3$$

$$P_2 + 0 + \frac{1}{2}\rho V_2^2 = P_1 + 0 + \frac{1}{2}\rho V_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \frac{1}{2}\rho(V_1 - V_2)(V_1 + V_2)$$

$$10.8\times10^3 = \frac{1}{2}\times1.2(V_1 - V_2)\times2\times3\times10^2$$

$$10.8\times10 = 3.6(V_1 - V_2)$$

$$V_1 - V_2 = 30$$

$$\left(\frac{V_1 - V_2}{V}\right)\times100 = \frac{30}{300}\times100 = 10\%$$

38. Identify the correct statements from the following:

(A) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

(B) Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

(C) Work done by friction on a body sliding down an inclined plane is positive.

(D) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in zero.

(E) Work done by the air resistance on an oscillating pendulum in negative.

Choose the correct answer from the options given below:

(1) B and E only (2) A and C only

(3) B, D and E only (4) B and D only

Official Ans. by NTA (1)

QuestPix Ans. (1)

39. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at x = +2m, its velocity is $-4\hat{j}$ m / s. The object's velocity (v) and acceleration (a) at x = –2 m will be

(1) $v = 4\hat{i}$ m / s, $a = 8\hat{j}$ m / s^2

(2) $v = 4\hat{j}$ m / s, $a = 8\hat{i}$ m / s^2

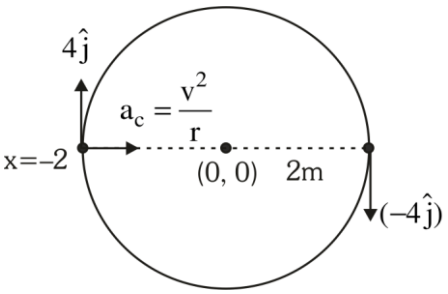
(3) $v = -4\hat{j}$ m / s, $a = 8\hat{i}$ m / s^2

(4) $v = -4\hat{i}$ m / s, $a = -8\hat{j}$ m / s^2

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



$$a_c = \frac{V^2}{r} = \frac{4^2}{2} = \frac{16}{2} = 8\,m/s^2$$

$$\vec{V} = 4\hat{j}$$

$$\vec{a}_c = 8\hat{i}$$

40. A point charge 2 × 10^{-2} C is moved from P to S in a uniform electric field of 30 NC^{-1} directed along positive x-axis. If coordinates of P and S are (1, 2, 0) m and (0, 0, 0) m respectively, the work done by electric field will be

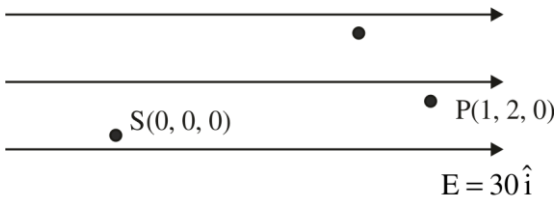
(1) 1200 mJ (2) 600 mJ

(3) –600 mJ (4) –1200 mJ

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol.



$$\omega_E = q\vec{E}\cdot\vec{S}$$

$$= 2\times10^{-2}\left[30\hat{i}\cdot(-\hat{i})\right]$$

$$= 2\times10^{-2}(-30)$$

$$= -60\times10^{-2}$$

$$= -\frac{60}{100} = -0.6J$$

$$= -600\,mJ$$

41. The modulation index for an A.M. wave having maximum and minimum peak to peak voltages of 14 mV and 6 mV respectively is :

(1) 1.4 (2) 0.4

(3) 0.2 (4) 0.6

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. μ = modulation index = $\frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$$= \frac{14-6}{14+6} = 0.4$$

42. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be :

(1) 8T (2) 4T

(3) 2T (4) 16T

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $B = \frac{\mu_0 i}{2R} \times 4$

$$B' = \frac{\mu_0 i}{2R'}$$

R' = 4R

$$B' = \frac{\mu_0 i}{8R}$$

$$\frac{B'}{B} = \frac{1}{16}$$

B' = 2T

43. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is

(A) directly proportional to the length of the potentiometer wire

(B) directly proportional to the potential gradient of the wire

(C) inversely proportional to the potential gradient of the wire

(D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements:

(1) B and D only (2) A and C only

(3) A only (4) C only

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Sensitivity of potentiometer wire is inversely proportional to potential gradient.

34. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

(1) Increase the wave length of the light

(2) Increase the refractive index of the medium between the object and objective lens

(3) Decrease the focal length of the eye piece

(4) Decrease the diameter of the objective lens

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. $P = \frac{2\mu \sin\theta}{1.22\lambda}$

45. Given below are two statements:

Statement I : Electromagnetic waves are not deflected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\, B_0$

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but statement II is true

(4) Both Statement I and Statement II are false

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Statement-I is correct as EMW are neutral.

Statement-II is wrong.

$$E_0 = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} B_0$$

46. Heat energy of 184 kJ is given to ice of mass 600 g at -12°C, Specific heat of ice is 2222.3 J kg^{-1° C^{-1} and latent heat of ice in 336 kJ/kg^{-1}

(A) Final temperature of system will be 0°C.

(B) Final temperature of the system will be greater than 0°C.

(C) The final system will have a mixture of ice and water in the ratio of 5 : 1.

(D) The final system will have a mixture of ice and water in the ratio of 1 : 5.

(E) The final system will have water only.

Choose the correct answer from the options given below:

(1) A and D only (2) B and D only

(2) A and E only (4) A and C only

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $\Delta Q = 184\times10^3$

$m = 0.600$ kg at –12°C

$S = 222.3$ J/kg/°C

$L = 336 \times 10^3$ J/kg

$Q_1 = 0.600 \times 2222.3 \times 12 = 16000.56$ J

Remaining heat $\Delta Q_1 = 184000 - 16000.56$

$= 167999.44$ J

For meeting at 0°C

$\Delta Q_2 = 0.600 \times 336000 = 201600$ J needed

$\therefore$ 100% ice is not melted

Amount of ice melted

$167999.44 = m \times 336000 = 0.4999$ kg

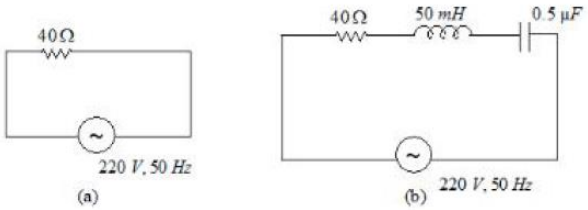
$\therefore$ mass of water = 0.4999 kg

Mass of ice = 0.1001

$\therefore$ Ratio $= \frac{0.1001}{0.4999} \approx 1:5$

Correct Ans (1) : A and D only

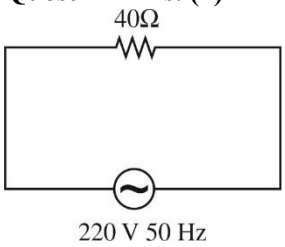
47. For the given figures, choose the correct options:



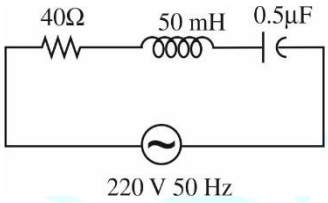
(1) The rms current in circuit (b) can never be larger than that in (a)
(2) The rms current in figure (a) is always equal to that in figure (b)
(3) The rms current in circuit (b) can be larger than that in (a)
(4) At resonance, current in (b) is less than that in (a)

Official Ans. by NTA (1)
QuestPix Ans. (1)

Sol.



$$I_{rms} = \frac{220}{40} = 5.5A$$



X_L is not equal to X_C. So rms current in (b) can never be larger than (a).

48. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.

(1) 4 hours (2) 6 hours
(3) 12 hours (4) 3 hours

Official Ans. by NTA (4)
QuestPix Ans. (4)

Sol. $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = 64$$

$$\therefore \quad T_2^2 = \frac{T_1^2}{64}$$

$$\therefore \quad T_2 = \frac{24}{8} = 3$$

49. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of A and B in a new equation : $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$.

The dimensions of t is given as $[T^{-1}]$.

(1) $A = [L^{-1}\,T]$, $B = [LT^{-1}]$
(2) $A = (LT]$, $B = [L^{-1}T^{-1}]$
(3) $A = [L^{-1}T^{-1}]$, $B = [LT^{-1}]$
(4) $A = [L^{-1}T^{-1}]$, $B = [LT]$

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. $\left(x - At\right)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$

$$[At] = A \times \frac{1}{T} = L$$

$$\therefore \quad [A] = T^1L^1$$

$\frac{t}{B}$ is in meters

$$\therefore \quad \frac{1}{T[B]} = L$$

$$\therefore \quad [B] = T^{-1}L^{-1}$$

$\therefore$ Correct Ans. (2)

50. A square loop of area $25cm^2$ has a resistance of 10Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be

(1) 2.5×10^{-3} J (2) 1.0×10^{-3} J
(3) 1.0×10^{-4} J (4) 5×10^{-3} J

Official Ans. by NTA (2)
QuestPix Ans. (2)

Sol. $\ell = 50$ cm

t = 1 sec

$$\therefore \quad V = \frac{0.05}{1} = 0.05 \text{ m/s.}$$

$$i = \frac{40 \times 0.05 \times 0.05}{10} = 0.01A$$

$$\therefore \quad F = B_i\ell = 40 \times 0.01 \times 0.05$$

$$F = 0.02 \text{ N}$$

$$\therefore \quad W = 0.02 \times \ell = 0.02 \times .05$$

$$\therefore \quad W = 1 \times 10^{-3}J$$

SECTION-B

51. When two resistance R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3 Ω is introduced into the right-gap to get null point at 40 cm from left end. The product of R_1 R_2 is _____ Ω^2

Official Ans. by NTA (30)

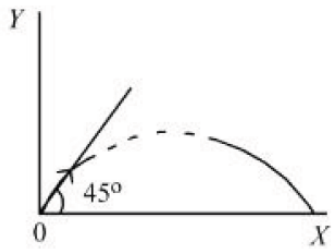
QuestPix Ans. (30)

Sol. $\frac{R_1+R_2}{10}=\frac{60}{40}=\frac{3}{2}\Rightarrow R_1+R_2=15$

Now $\frac{R_1R_2}{(R_1+R_2)\times 3}=\frac{40}{60}=\frac{2}{3}\Rightarrow R_1R_2=30$

52. A particle of mass 100 g is projected at time t = 0 with a speed 20 ms^{-1} at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time t = 2s is found to be $\sqrt{K}kg\,m^2/s$. The value of K is _____.

(Take g = 10 ms^{-2})



Official Ans. by NTA (800)

QuestPix Ans. (800)

Sol. Use $\Delta L=\int_0^t \tau\,dt$

$L_0=\int_0^2 mg(v_x t)\,dt$

$=mgv_x\frac{t^2}{2}=(0.1)(10)(10\sqrt{2})\frac{2^2}{2}$

$=20\sqrt{2}$

$=\sqrt{800}\,kg\,m^2/s$

53. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab at 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on Vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10}\times 10^{-3}$, where x is _____

Official Ans. by NTA (41)

QuestPix Ans. (41)

Sol. $u=\frac{h}{h'}=\frac{5.25}{5.00}$

Least count = $\frac{1}{20}cm-\frac{49}{50}\cdot\frac{1}{20}cm$

$=\frac{1}{50}\times\frac{1}{20}cm=0.01\ mm$

$\ln u=\ln h-\ln h'$

$\frac{du}{u}=\frac{dh}{h}-\frac{dh'}{h'}$

$du=\left[\frac{0.01}{5.25}+\frac{0.01}{5.00}\right]\frac{5.25}{5.00}$

$=\frac{41}{10}\times 10^{-3}$

Ans. = 41

54. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V=2ar^2+b$.

Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _____.

ε = permittivity of medium.

Official Ans. by NTA (12)

QuestPix Ans. (12)

Sol. $E=-\frac{dV}{dr}=-4ar\equiv\frac{\rho r}{3\varepsilon_0}$ (compare)

Result inside uniformly charged solid sphere.

$\rho=-12a\varepsilon_0$

$\lambda=12$

55. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1-e^{-\pi/2})$s. The value of t is _____.

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. $$v\frac{dv}{dx}=\frac{v^2}{R}\Rightarrow\int_{15}^{v}\frac{dv}{v}=\frac{1}{R}\int_{0}^{x}dx$$

$$v=15e^{x/R}$$

$$\frac{dx}{dt}=15e^{x/R}$$

$$\int_{0}^{\frac{\pi R}{2}}e^{-x/R}dx=15\int_{0}^{t_0}dt$$

$$t_0=40\left(1-e^{-\pi/2}\right)$$

56. An inductor of inductance 2 μH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}$F, where the value of x is_____.

$\left(\text{Take } \pi=\frac{22}{7}\right)$

Official Ans. by NTA (3872)

QuestPix Ans. (3872)

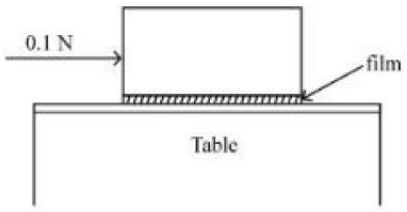
Sol. $$\frac{1}{2\pi fC}=2\pi fL$$

$$C=\frac{1}{4\pi^2f^2L}=\frac{1}{4\times\pi^2\times49\times10^6\times2\times10^{-6}}$$

$$C=\frac{1}{3872}F$$

x = 3872 **Ans.**

57. A metal block of base area 0.20 m^2 is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is _____$\times10^{-3}$ m/s.



Official Ans. by NTA (25)

QuestPix Ans. (25)

Sol. $$|F|=\eta A\frac{\Delta v}{\Delta h}:0.1=5\times10^{-3}\times0.2\times\frac{v}{.25\times10^{-3}}$$

$$v=0.025m/s \text{ or } v=25\times10^{-3}m/s$$

58. A particle of mass 250 g executes a simple harmonic motion under a periodic force F = (–25 x) N. The particle attains a maximum speed of 4m/s during its oscillation. The amplitude of the motion is _____cm.

Official Ans. by NTA (40)

QuestPix Ans. (40)

Sol. $$\frac{1}{4}a=-25x \quad ; \quad a=-100x$$

$$\omega^2=100 \quad \omega=10,$$

$$\omega A=4 \qquad A=\frac{4}{10}=0.4m$$

A = 40 cm

59. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium –1) and 6.8 (medium – 2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is _____.

(Given for dielectric media, $\mu_r = 1$)

Official Ans. by NTA (7)

QuestPix Ans. (7)

Sol. $\mu_1 = \sqrt{2.8 \times 1} = \sqrt{2.8}$

$\mu_2 = \sqrt{6.8 \times 1} = \sqrt{6.8}$

$\mu_1 \sin i = \mu_2 \cos i$ $\qquad \tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}}$

$\tan i = \left(\frac{2.8+4}{2.8}\right)^{1/2}$ $\qquad i = \tan^{-1}\left(1+\frac{10}{7}\right)^{1/2}$

$\theta = 7$ Ans.

60. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15 Ω is used for shunting null point moves to 300 cm. The internal resistance of the cell is _____Ω.

Official Ans. by NTA (5)

QuestPix Ans. (5)

Sol. $\frac{\varepsilon}{r+5} \times 5 = 200x$ $\qquad$ ….(1)

$\frac{\varepsilon \times 15}{r+15} = 300x$ $\qquad$ …(2)

$\Rightarrow r = 5\ \Omega$

Ans. 5

(Held On Sunday 29th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

61. Given below are two statements:

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct.

(2) Both the statements I and II are correct

(3) Statement I is correct but statement II is incorrect

(4) Both the statements I and II are incorrect

Official Ans. by NTA (2)

QuestPix Ans. (1)

Sol. The first ionization energies (as in NCERT) are as follows:

B : 801 kJ/mol

Al : 577 kJ/mol

Ga : 579 kJ/mol

Ga : $[Ar]3d^{10}4s^24p^1$

62. Correct order of spin only magnetic moment of the following complex ions is:

(Given At. No. Fe: 26, Co:27)

(1) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(2) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$

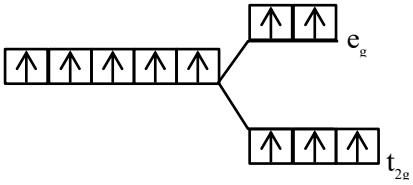
(3) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

(4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Official Ans. by NTA (1)

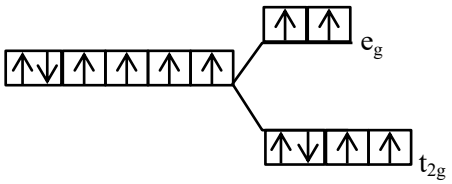
QuestPix Ans. (1)

Sol. $[FeF_6]^{3-}$: $Fe^{3+} = 3d^5$ $\Delta_O < P$



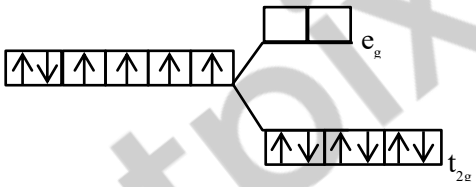
Number of unpaired $e^- = 5$ $\therefore$ $\mu = \sqrt{35}$ BM

$[CoF_6]^{3-}$: $Co^{3+} = 3d^6$ $(\Delta_O < P)$



Number of unpaired $e^- = 4$ $\therefore$ $\mu = \sqrt{24}$ BM

$[Co(C_2O_4)_3]^{3-}$: $Co^{3+} = 3d^6$ $(\Delta_O > P)$



Number of unpaired $e^- = 0$ $\therefore$ $\mu = 0$ BM

63. Match **List-I** and **List-II**.

List-I	List-II
A. Osmosis	I. Solvent molecules pass through semi permeable membrane towards solvent side.
B. Reverse osmosis	II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C. Electro osmosis	III. Solvent molecules pass through semi permeable membrane towards solution side.
D. Electrophoresis	IV. Dispersion medium moves in an electric field.

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

(4) A-I, B-III, C-II, D-IV

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. A. Osmosis III

B. Reverse osmosis I

C. Electro osmosis IV

D. Electrophoresis II

64. The set of correct statements is:

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidising nature in +6 oxidation state.

(1) (ii) and (iii) (2) (i), (ii) and (iv)

(3) (i) and (iii) (4) (ii), (iii) and (iv)

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. (i), (ii) and (iv) correct.

Manganese exhibits +7 oxidation state in its oxide. (Mn_2O_7)

Ru & Os from RuO_4 & OsO_4 oxide in +8 oxidation state

Cr in +6 oxidation act is oxidizing.

Sc does not show +4 oxidation state.

65. Match **List-I** and **List-II**.

List-I	List-II
A. Elastomeric polymer	I. Urea formaldehyde resin
B. Fibre polymer	II. Polystyrene
C. Thermosetting polymer	III. Polyester
D. Thermoplastic polymer	IV. Neoprene

Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV

(2) A-II, B-I, C-IV, D-III

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-I, C-III, D-II

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Neoprene : Elastomer

Polyester : Fibre

Polystyrene : Thermoplastic

Urea–Formaldhyde Resin: Thermosetting polymer

66. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are

(1) Starch and iodine

(2) Methyl orange and H_2O_2

(3) Starch and H_2O_2

(4) Methyl orange and iodine

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. $I^- + H_2O_2 \longrightarrow \underset{(A)}{I_2} + H_2O$

$I_2 + \underset{(Indicator)}{Starch} \longrightarrow Blue$

67. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Stomach ulcers

(2) Hyperacidity

(3) Anxiety and stress

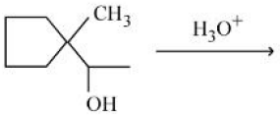
(4) Depression and hypertension

Official Ans. by NTA (4)

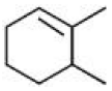
QuestPix Ans. (4)

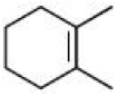
Sol. Theory based.

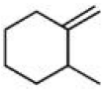
68. Find out the major product for the following reaction.



Major Product

(1) 

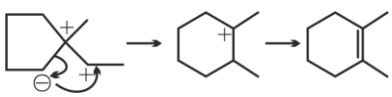
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. 

69. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

(1) 1-Bromo-2-methylbutane

(2) 2-Bromopropane

(3) 2-Bromopentane

(4) 2-Bromo-3,3-dimethylpentane

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $CH_3-CH_2-CH(CH_3)-CH_2-Br \longrightarrow$ $C-C-C(C)=C$ (1)

$CH_3-CH(Br)-CH_3 \longrightarrow CH_3CH=CH_2$ (1)

$CH_3-CH_2-CH_2-CH(Br)-CH_3 \longrightarrow$

C-C-C-C=C + C-C-C=C-C, cis & trans (3)

$C-C-C(CH_3)_2-C(Br)-C \longrightarrow C-C-C(C)(C)-C=C$ (1)

70. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

(1) C_8H_6 (2) C_9H_9

(3) C_6H_6 (4) C_9H_6

Official Ans. by NTA (1)

QuestPix Ans. (1)

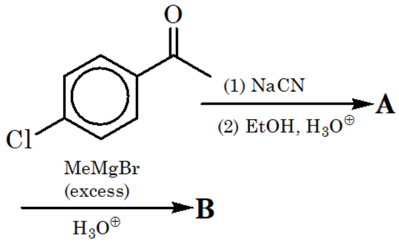
Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

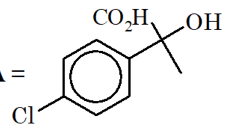
$x+\frac{y}{4}=9.5$

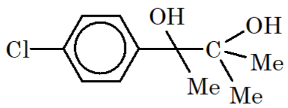
$\frac{y}{2}=3$

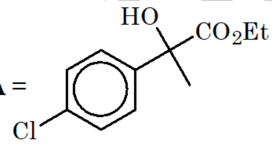
$\Rightarrow x=8, y=6$

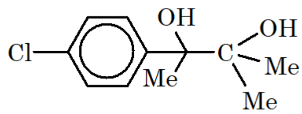
71. Find out the major products from the following reaction sequence.

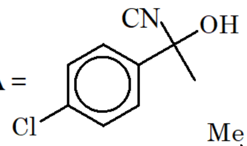


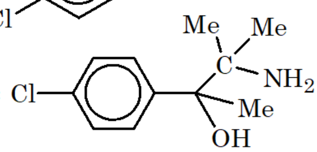
(1) A = 

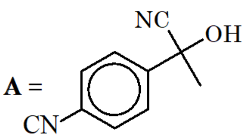
B = 

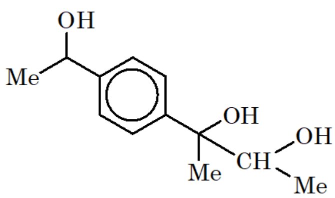
(2) A = 

B = 

(3) A = 

B = 

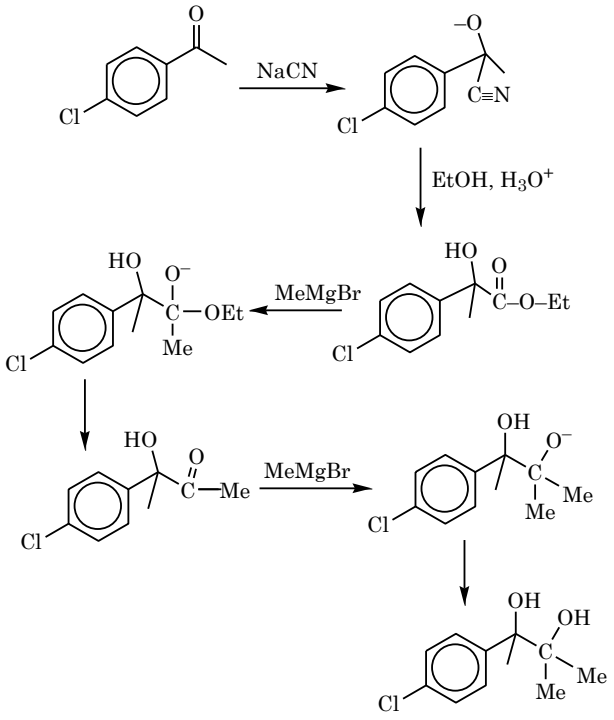
(4) A = 

B = 

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol.



72. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are

(1) 1, 3 and 3 (2) 1, 3 and 2

(3) 1, 2 and 3 (4) 2, 3 and 3

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. Theory based.

73. A solution of CrO_5 in amyl alcohol has a….colour

(1) Green (2) Orange–Red

(3) Yellow (4) Blue

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. A solution of CrO_5 in amyl alcohol has a blue colour. So, option (4) is correct.

74. The concentration of dissolved Oxygen in water for growth of fish should be more than X ppm and Biochemical Oxygen Demand in clean water should be less than Y ppm. X and Y in ppm are, respectively.

(1) X Y 6 5 (2) X Y 4 8

(3) X Y 4 15 (4) X Y 6 12

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. The growth of fish gets inhibited if the concentration of dissolved Oxygen in water is less than 6 ppm and Biochemical Oxygen demand in clean water should be less than 5 ppm.

75. Reaction of propanamide with Br_2 / KOH (aq) produces :

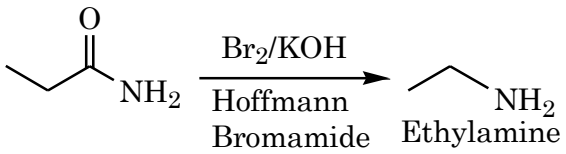
(1) Ethylnitrile (2) Propylamine

(3) Propanenitrile (4) Ethylamine

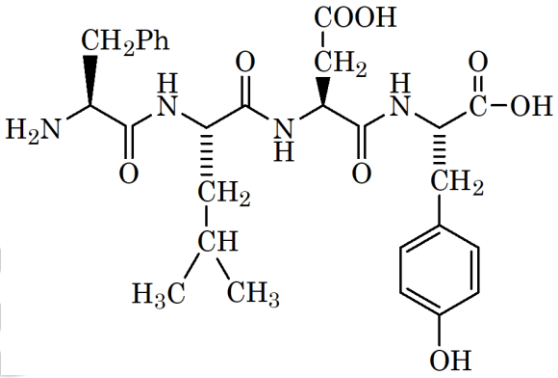
Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



76. Following tetrapeptide can be represented as



(F, L, D, Y, I, Q, P are one letter codes for amino acids)

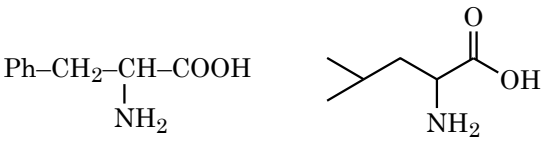
(1) FIQY (2) FLDY

(3) YQLF (4) PLDY

Official Ans. by NTA (2)

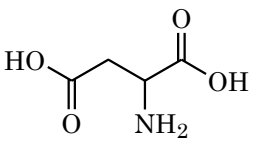
QuestPix Ans. (2)

Sol. Hydrolysis of the given tetrapeptide will give the following:

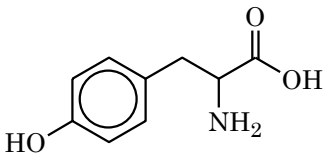


(F)
(Phenylalanine)

(L)
(Leucine)



(D)
(Aspartic acid)



(Y)
(Tyrosine)

77. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below :

(1) C and D only (2) B and C only

(3) A and B only (4) B and D only

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. Only (B) and (C) are correct.

(B) $G = H - TS$

At constant T

$\Delta G = \Delta H - T\Delta S$

(A) First law is given by

$\Delta U = Q + W$

If we apply constant P and reversible work.

$\Delta U = Q - P\Delta V$

(C) By definition of entropy change

$dS = \frac{dq_{rev}}{T}$

At constant T

$\Delta S = \frac{q_{rev}}{T}$

(D) $H = U + PV$

For ideal gas

$H = U + nRT$

At constant T

$\Delta H = \Delta U + \Delta nRT$

78. The major component of which of the following ore is sulphide based mineral?

(1) Calamine (2) Siderite

(3) Sphalerite (4) Malachite

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. Calamine : $ZnCO_3$

Siderite : $FeCO_3$

Sphalerite : ZnS

Malachite : $CuCO_3.Cu(OH)_2$

79. Given below are two statements:

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the statements I and II are correct

(2) Statement I is incorrect but statement II is correct

(3) Both the statements I and II are incorrect

(4) Statement I is correct but statement II is incorrect

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Statement–I is correct.

Ni is used in Hydrogenation of unsaturated fat to make edible fats.

Statements–II is false as hydride of Silicon is electron precise & neither electron deficient nor electron rich.

80. Match **List I** with **List II**.

List I		List II	
A.	van't Hoff factor, i	I.	Cryoscopic constant
B.	k_f	II.	Isotonic solutions
C.	Solutions with same osmotic pressure	III.	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D.	Azeotropes	IV.	Solutions with same composition of vapour above it

Choose the correct answer from the options given below :

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-II, C-I, D-IV

(C) A-III, B-I, C-IV, D-II

(D) A-I, B-III, C-II, D-IV

Official Ans. by NTA (1)

QuestPix Ans. (1)

Sol. (A) van't Hoff factor, i

$i = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$

(B) k_f = Cryoscopic constant

(C) Solutions with same osmotic pressure are known as isotonic solutions.

(D) Solutions with same composition of vapour over them are called Azeotrope.

SECTION-B

81. On heating, $LiNO_3$ gives how many compounds among the following?

Li_2O, N_2, O_2, $LiNO_2$, NO_2

Official Ans. by NTA (3)

QuestPix Ans. (3)

Sol. $2\,LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$

Hence three products Li_2O, NO_2 and O_2

82. At 298 K

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), K_1 = 4 \times 10^5$

$N_2(g) + O_2(g) \rightleftharpoons 2NO\,(g), K_2 = 1.6 \times 10^{12}$

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O\,(g), K_3 = 1.0 \times 10^{-13}$

Based on above equilibria, the equilibrium constant of the reaction,

$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO\,(g) + 3H_2O\,(g)$

is ___ $\times 10^{-33}$ (Nearest integer)

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), K_1 = 4 \times 10^5$ ...(i)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g), K_2 = 1.6 \times 10^{12}$ ...(ii)

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3 = 1.0 \times 10^{-13}$ ...(iii)

(ii) + 3×(iii) – (i)

$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$

$k_{eq} = \frac{k_2 \times k_3^3}{k_1} = \frac{1.6 \times 10^{12} \times (10^{-13})^3}{4 \times 10^5}$

$= \frac{1.6}{4} \times 10^{-32} = 4 \times 10^{-33}$

83. For conversion of compound A $\rightarrow$B, the rate constant of the reaction was found to be 4.6×10^{-5} L mol^{-1} s^{-1}. The order of the reaction is _____ .

Official Ans. by NTA (2)

QuestPix Ans. (2)

Sol. As unit of rate constant is $(conc.)^{1-n}$ $time^{-1}$

$\Rightarrow$ (L mol^{-1}) $\Rightarrow$ 1–n = –1 n = 2

84. Total number of acidic oxides among N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O and NO is _____ .

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol. Acidic oxides are N_2O_3, NO_2, Cl_2O_7, SO_2

85. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is _____ g mol^{-1} (Nearest integer)

Official Ans. by NTA (200)

QuestPix Ans. (200)

Sol. Let M is the molar mass of the compound (g/mol)

mass of compound = 0.01 M gm

mass of carbon = $0.01\,M \times \frac{60}{100}$

moles of carbon = $\frac{0.01M}{12} \times \frac{60}{100}$

moles of CO_2 from combustion = $\frac{4.4}{44}$ = moles of carbon

$\frac{0.01M}{12} \times \frac{60}{100} = \frac{4.4}{44}$

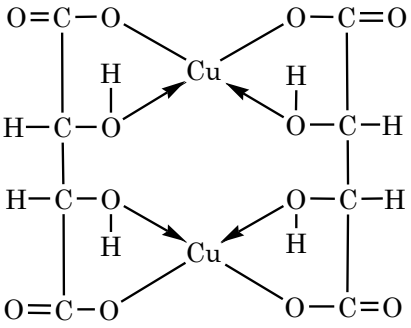
$M = \frac{4.4}{44} \times \frac{100}{60} \times \frac{12}{0.01} = 200 gm/mol$

86. The denticity of the ligand present in the Fehling's reagent is _____ .

Official Ans. by NTA (4)

QuestPix Ans. (4)

Sol.



Copper tartarate complex

Denticity = 2

87. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is _____ $\times 10^{21}$ (Nearest integer)

(Given $N_A = 6.02 \times 10^{23}$)

Official Ans. by NTA (36)

QuestPix Ans. (36)

Sol. One unit cell of hcp contains = 18 voids

No. of voids in 0.02 mol of hcp

$= \frac{18}{6} \times 6.02 \times 10^{23} \times 0.02$

$\approx 3.6 \times 10^{22}$

$\approx 36 \times 10^{21}$

88. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 Å. The radius of the third Bohr orbit of He^+ is _____ picometer. (Nearest Integer)

Official Ans. by NTA (270)

QuestPix Ans. (270)

Sol. $r \propto \frac{n^2}{Z}$

$r_{He^+} = r_H \times \frac{n^2}{Z}$

$r_{He^+} = 0.6 \times \frac{(3)^2}{2}$

$= 2.7$ Å

$r_{He^+} = 270\text{pm}$

89. The equilibrium constant for the reaction $Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{\circ}_{Zn^{2+}/Zn} = -0.76$ V is _____ $\times 10^{-2}$ V. (Nearest integer)

Given : $\frac{2.303RT}{F} = 0.059V$

Official Ans. by NTA (17)

QuestPix Ans. (17)

Sol. $Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$

$\Delta G^\circ = -2.303RT \log_{10} Keq$

$-nF\left(E^0_{cell}\right) = -2.303RT \log_{10} Keq$

$E^0_{Zn/Zn^{2+}} + E^0_{Sn^{2+}/Sn} = \frac{0.059}{2} \log_{10} Keq$

$0.76 + E^0_{Sn^{2+}/Sn} = \frac{0.059}{2} \log_{10} 10^{20}$

$0.76 + E^0_{Sn^{2+}/Sn} = \frac{0.059 \times 20}{2}$

$E^0_{Sn^{2+}/Sn} = 0.59 - 0.76 = -0.17$

$E^0_{Sn/Sn^{2+}} = 17 \times 10^{-2}$ V

Ans. = 17

90. The volume of HCl, containing 73 g L^{-1}, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is _____ mL. (Nearest Integer)

(Given : molar Masses of Na, Cl, O, H are 23, 35.5, 16 and 1 g mol^{-1} respectively)

Official Ans. by NTA (15)

QuestPix Ans. (15)

Sol. Mole of Na $= \frac{0.69}{23} = 3 \times 10^{-2}$

$Na + H_2O \longrightarrow NaOH + \frac{1}{2} H_2$

By using POAC

Moles of NaOH = 3×10^{-2}

NaOH reacts with HCl

No. of equivalent of NaOH = No. of equivalent of HCl

$3 \times 10^{-2} \times 1 = \frac{73}{36.5} \times V(\text{in L}) \times 1$

V = 1.5×10^{-2} L

Volume of HCl = 15 ml.