# JEE(ADVANCED)–2024 (EXAMINATION)

**(Held On Sunday 26th MAY, 2024)**

| MATHEMATICS | TEST PAPER WITH ANSWER AND SOLUTION |
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## PAPER-1

### SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct option is chosen;
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
*Negative Marks* : −1 In all other cases.

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**1.** Let $f(x)$ be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}\quad(t)}{t\quad -x^9}=1$ for each $x > 0$. Then, for all $x > 0$, $f(x)$ is equal to :

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$ (B) $\frac{9}{11x}+\frac{13}{11}x^{10}$ (C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$ (D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

**Ans.** **(B)**

**Sol.** $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t\quad -x^9}=1$

$\lim\limits_{t\to x}\frac{10t^9f(x)-x^{10}f'(t)}{9t^8}=1$

$\Rightarrow 10xf(x) - x^2f'(x) = 9$

$\Rightarrow x^2f'(x) = 10x\ f(x) - 9$

$\Rightarrow f'(x)=\frac{10f(x)}{x}\quad\frac{9}{x^2}$

$\Rightarrow \frac{dy}{dx}-\frac{10}{x}y=-\frac{9}{x^2}$

$\Rightarrow y.\frac{1}{x^{10}}=\int-\frac{9}{x^2}.\frac{1}{x^{10}}dx$

$\Rightarrow \frac{y}{x^{10}}=\frac{\quad}{11x^{11}}+c \qquad \ldots(1)$

$\because f(1)=2\Rightarrow\frac{2}{1}=\frac{9}{11}+c\Rightarrow c=\frac{13}{11}$

$\therefore f(x)=\frac{9}{11x}+\frac{13}{11}x^{10}$

$\Rightarrow$ Option (B) is correct.

**2.** A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is :

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{5}{7}$ (D) $\frac{5}{12}$

**Ans.** **(C)**

**Sol.** $C \rightarrow$ Correct

$G \rightarrow$ Guess

$K \rightarrow$ Knows

$P\left(\frac{}{}\right)=\frac{1}{2}$ , $P\left(\frac{C}{K}\right)=$

$P\left(\frac{}{}\right)=\frac{1}{6}$

Let required probability = x

$$\therefore\ P\left(\frac{G}{C}\right)=\frac{(1-x)P\left(\frac{C}{G}\right)}{(1-x)P\left(\frac{C}{G}\right)\quad x.P\left(\frac{}{}\right)}$$

$$\frac{1}{6}=\frac{(1-x)\left(\frac{1}{2}\right)}{(1-x)\left(\frac{1}{2}\right)\quad (x)(1)}$$

$\Rightarrow x=\frac{5}{7}$ $\Rightarrow$ Option (C) is correct.

**3.** Let $\frac{\pi}{2}<x<\pi$ be such that $\cot x=\frac{-5}{\sqrt{11}}$. Then

$$\left(\sin\frac{11x}{2}\quad(\sin 6x-\cos 6x)+\left(\cos\frac{11x}{2}\quad(\sin 6x+\cos 6x)\right.\right.$$

is equal to :

(A) $\frac{\sqrt{11}-1}{2\sqrt{}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{}}$ (C) $\frac{\sqrt{11}+1}{3\sqrt{}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{}}$

**Ans.** **(B)**

**Sol.** $x \in \left(\frac{\pi}{2}\ \ \pi\right)$

$\cot x = -\frac{5}{\sqrt{11}}$

$\frac{1-\tan^2\frac{x}{2}}{2\tan\frac{x}{2}} = -\frac{5}{\sqrt{11}}$

$\tan\frac{x}{2} = \sqrt{11},\ \ \frac{}{\sqrt{11}}$

$\tan\frac{x}{2} = \sqrt{11},\ \text{As}\ \frac{}{4} < \frac{}{2} < \frac{\pi}{2}$

$\left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos\frac{11x}{2}\right)(\sin 6x + \cos 6x)$

$\Big\{\sin 6x \sin\frac{11x}{} + \cos\frac{11x}{2}\cos 6x$

$= \cos\left(6x - \frac{11x}{}\right) + \sin\left(6x - \frac{11x}{2}\right)$

$= \cos\frac{x}{2}\quad \sin\frac{}{}$

$= \frac{1}{2\sqrt{3}}\quad \frac{\sqrt{11}}{2\sqrt{3}}$

$= \frac{\sqrt{11}+1}{2\sqrt{\ }}$ $\Rightarrow$ Option (B) is correct.

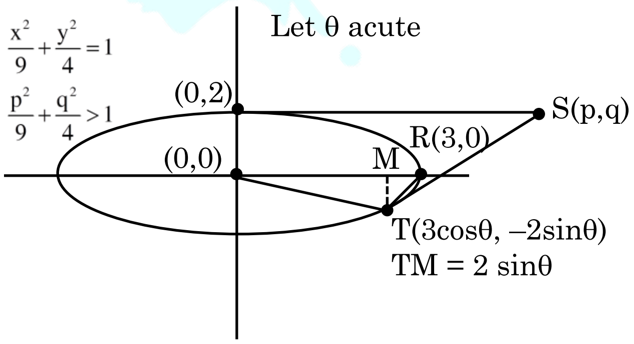
**4.** Consider the ellipse $\frac{x^2}{} + \frac{}{4} = 1$. Let S(p, q) be a point in the first quadrant such that $\frac{p^2}{9} + \frac{}{}\ \ 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle $\Delta$ORT is $\frac{3}{2}$, then which of the following options is correct ?

(A) $q = 2,\ p = 3\sqrt{3}$ (B) $q = 2,\ p = 4\sqrt{3}$

(C) $q = 1,\ p = 5\sqrt{3}$ (D) $q = 1,\ p = 6\sqrt{3}$

**Ans.** **(A)**

**Sol.**




$\text{Ar}(\ \ \text{ORT}) = \frac{3}{2}$

$$\left|\frac{1}{2}\times 3\times 2\sin\theta\right| = -$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{11\pi}{}$$

$$T\left(\frac{3\sqrt{\ }}{2}, -1\right.$$

Tangent at (0, 2) $\dfrac{x(0)}{9} + \dfrac{y(2)}{} = 1 \Rightarrow y = 2 \quad \ldots(1)$

Tangent at $\left(\dfrac{3\sqrt{\ }}{2}, -1\right)$ $\dfrac{x\left(\dfrac{3\sqrt{\ }}{2}\right.}{9} + \dfrac{y(-1)}{} \quad 1 \quad \ldots(2)$

$\therefore$ By solving (1) & (2) $\Rightarrow$ $p = 3\sqrt{3}$, $q = 2$

$\Rightarrow$ Option (A) is Correct.

**SECTION-2 : (Maximum Marks : 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
*Negative Marks* : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –2 marks.

**5.** Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \left\{\left(-1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$ and $T_2 = \left\{\left(1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$. Then which of the following statements is (are) TRUE ?

(A) $\mathbb{Z} \bigcup T_1 \bigcup T \subset S$

(B) $T_1 \bigcap \left(0, \dfrac{1}{2024}\right. = \phi$, where $\phi$ denotes the empty set.

(C) $T_2 \bigcap (2024, \infty) \neq \phi$

(D) For any given a, b $\in \mathbb{Z}$, $\cos\left(\pi(a+b\sqrt{2})\right) + i\sin\left(\pi(a+b\sqrt{2})\right) \in \mathbb{Z}$ if and only if b = 0, where $i = \sqrt{-1}$.

**Ans.** **(A,C,D)**

**Sol.** (A) $\left(-1+\sqrt{2}\right)^n = m + \sqrt{2}n, m, n \in \mathbb{Z}$

$\left(1+\sqrt{2}\right)^n = m_1 + \sqrt{2}n_1, m_1, n_1 \in \mathbb{Z}$

$\Rightarrow \mathbb{Z} \cup T_1 \cup T \subseteq S$

but $b\sqrt{2} \in S$ for negative $b \in \mathbb{Z}$.

So $\quad \mathbb{Z} \cup T_1 \cup T \subset S$

(B) $\left(\sqrt{2} - \ \right)^n = \dfrac{1}{\left(\sqrt{2} + \ \right)^n} < \dfrac{}{2024}$

$\Rightarrow 2024 < \left(\sqrt{2}+1\right)^n, \exists\, n \in \mathbb{N}$

$\Rightarrow T_1 \cap \ 0, \dfrac{1}{2024}\Big) \neq \phi$

(C) $\left(1+\sqrt{2}\right)^n > 2024, \exists n \in \mathbb{N}$

$\Rightarrow T_2 \cap (2024, \infty) \neq \phi$

(D) $\sin\left(\pi\left(a + b\sqrt{2}\right) = 0\right) \Rightarrow b = 0,\ a \in \mathbb{Z}$.

$\Rightarrow$ Options (A), (C), (D) are Correct.

**6.** Let $\mathbb{R}^2$ denote $\mathbb{R}\times\mathbb{R}$. Let

$S = \{(a, b, c) : a, b, c \in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all $(x, y) \in \mathbb{R}^2 - \{(0, 0)\}\}$.

Then which of the following statements is (are) TRUE ?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.

(C) For any given $(a, b, c) \in S$, the system of linear equations

$ax + by = 1$

$bx + cy = -1$

has a unique solution.

(D) For any given $(a, b, c) \in S$, the system of linear equations

$(a + 1)x + by = 0$

$bx + (c + 1)y = 0$

has a unique solution

**Ans.** **(B,C,D)**

**Sol.** (A) $ax^2 + 2bxy + cy^2 > 0 \ \forall (x, y) \in \mathbb{R}^2 - \{(0,0)\}$

$\Rightarrow ax^2 + 2bxy + cy^2$ must represent pair of imaginary lines and $a, c > 0$.

$\Rightarrow b^2 < ac$

(B) $b^2 < 3\times\frac{1}{12} \Rightarrow |2b| < 1$

(C) since $b^2 \neq ac$

$\Rightarrow ax + by = 1$ and $bx + cy = -1$

are not parallel lines.

(D) $ac + a + c > b^2 \Rightarrow$ lines are not parallel.

$\Rightarrow$ Options (B), (C), (D) are Correct.

**7.** Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let dist $(X, Y)$ denote the distance between two points X and Y in $\mathbb{R}^3$. Let

$S = \left\{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\right\}$ and

$T = \left\{Y \in \mathbb{R}^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\right\}$.

Then which of the following statements is (are) TRUE ?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

**Ans.** **(A,B,C,D)**

**Sol.** $S = \{X : (XP)^2 - (XQ)^2 = 50\}$

$T = \{Y : (YQ)^2 - (YP)^2 = 50\}$

for finding $S \equiv X(x, y, z)$ and for $T \equiv Y(x, y, z)$

$((x-1)^2 + (y-1)^2 + (z-1)^2) - ((x-4)^2 + (y-2)^2 + (z-7)^2) = 50$

$\Rightarrow \quad S = \{(x, y, z) : 6x + 8z = 105\}$

$T = \{(x, y, z) : 6x + 8z = 5\}$

Since S and T both are plane ;

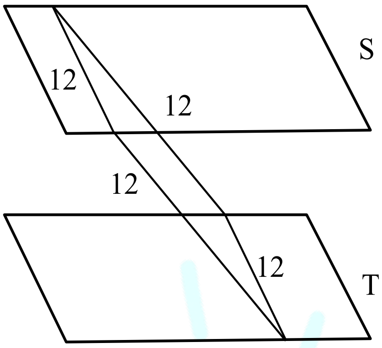
(A) There exist a triangle in plane S whose area = 1 (always)

(B) L & M lies on plane T, hence line segment joining L & M will lie on plane T.

(C) Distance between S & T

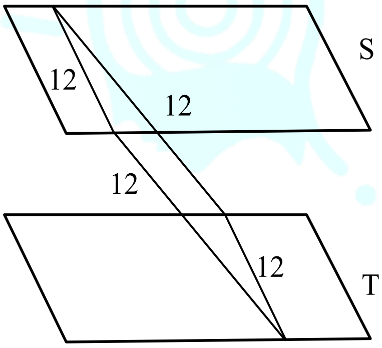
$$d = \left|\frac{105-5}{10}\right| = 10$$

Hence for rectangle of perimeter 48 can exist.




There will be infinite such rectangle possible.

(D) For square




Hence Answers A,B,C,D are correct

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** If the correct integer is entered;
*Zero Marks* : 0 In all other cases.

**8.** Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{\ }}$. If x, y $\in \mathbb{R}$ are such that

$3x + 2y = \log_a (18)^{\frac{5}{4}}$ and

$2x - y = \log_b(\sqrt{1080})$,

then 4x + 5y is equal to ……..

**Ans. (8)**

**Sol.** $3x + 2y = \log_{3\sqrt{\ }}\left(3\sqrt{2}\right)^{\frac{5}{2}} = \frac{5}{2}$

$\Rightarrow \quad 6x + 4y = 5 \quad ……(1)$

$2x - y = \log_{\frac{1}{5^{1/6}\sqrt{\ }}}(5^{\frac{1}{6}}\sqrt{6})^3 = -3$

$\Rightarrow \quad 2x - y = -3 \quad …..(2)$

equation (1) – (2)

$\Rightarrow \quad 4x + 5y = 8$

**9.** Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and $\alpha_4$ are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ………

**Ans. (20)**

**Sol.** $f(1) = 1 + a + b + c = -9 \quad \Rightarrow \quad a + b + c = -10 \quad …..(1)$

$4x^3 + 3ax^2 + 2bx = 0$ roots are $\sqrt{3}\,i, -\sqrt{3}\,i, 0$

$\Rightarrow \quad 4x^2 + 3ax + 2b = 0 \begin{cases} \sqrt{3}\,i \\ -\sqrt{3}\,i \end{cases}$

$\Rightarrow \quad a = 0 \;\&\; \frac{2b}{4} = \left(\sqrt{3}\,i\right)\left(\ \sqrt{3}\,i\right)$

$b = 6$ use a, b in (1) $\Rightarrow c = -16$

$\Rightarrow \quad f(x) = x^4 + 6x^2 - 16 = 0$

$(x^2 + 8)(x^2 - 2) = 0$

$\Rightarrow \quad x = \pm\sqrt{8}\,i, \pm\sqrt{2} \quad \Rightarrow \quad |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$

**10.** Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0,1\} \text{ and } |A| \in \{-1,1\} \right\}$, where |A| denotes the determinant of A. Then the number of elements in S is ______.

**Ans.** **(16)**

**10.** $|A| = 0(ae - bd) - 1(e - d) + c(b - a)$

$= c(b - a) + (d - e)$

$|A| \in \{-1, 1\}$ and $a, b, c, d, e \in \{0, 1\}$

Case-I

$c = 0 \quad d = 1, e = 0, a, b \in (0, 1)$

$d = 0, e = 1$

a b c d e

$\downarrow$

2 2 1 2 $\rightarrow$ 8 cases

Case-II

$c = 1 \quad b = 1, a = 0, \quad d = 0, e = 0, d = 1, e = 1$

$b = 0, a = 1, \quad d = 0, e = 0, d = 1, e = 1$

$b = 0, a = 0, \quad d = 1, e = 0$

$d = 0, e = 1$

$b = 1, a = 1, \quad d = 1, e = 0$

$d = 0, e = 1$

$\rightarrow$8 cases

$\Rightarrow$ Total 16 cases

**11.** A group of 9 students, $s_1, s_2, \ldots\ldots, s_9$, is to be divided to from three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that $s_1$ cannot be selected for the team X, and $s_2$ cannot be selected for the team Y. Then the number of ways to from such teams, is ______.

**Ans.** **(665)**

**Sol.**

| x | y | z |
|---|---|---|
| 2 | 3 | 4 |
| $\bar{S}_1$ | $\bar{S}_2$ | |

C-i) when x does not contain $S_1$, but contains $S_2$

$$\underset{\text{for x}}{{}^7C_1} \times \underset{\text{for y,z}}{\frac{7!}{3!4!}} = 245$$

C-ii) When x does not contain $S_1$, $S_2$ and y does not contain $S_2$

$$\text{i.e. } \underset{\text{for x}}{{}^7C_2} \times \underset{\text{for y,z}}{\frac{6!}{3!3!}} = 420$$

so total No. of ways 665

**12.** Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i}+\hat{j}+\hat{k}$, $\overrightarrow{OQ} = \hat{i}+\frac{\beta-1}{\beta}\hat{j}+\hat{k}$ and $\overrightarrow{OR} = \hat{i}+\hat{j}+\frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R}-\{0\}$ and O denotes the origin. If $\left(\overrightarrow{OP}\times\overrightarrow{OQ}\right).\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of $l$ is ………..

**Ans.** **(5)**

**Sol.** $\left(\overrightarrow{OP}\times\overrightarrow{OQ}\right).\overrightarrow{OR} = 0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & & \frac{1}{2} \end{vmatrix} = 0$$

$\alpha + \beta + 1 = 0$ …(1)

Also $(\alpha, \beta, 2)$ lies on $3x + 3y - z + l = 0$

$\Rightarrow 3\alpha + 3\beta - 2 + l = 0 \quad \Rightarrow \quad l = 2 - 3(\alpha + \beta)$

use (1) in it $\Rightarrow l = 5$

**13.** Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy-plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is $\alpha$, then the value of $24\alpha$ is ……..

**Ans.** **(42)**

**Sol.** Let equation of line is $y = mx + c$

| x | 0 | 1 | 2 | 3 | 4 | R−{0,1,2,3,4} |
|---|---|---|---|---|---|---|
| P(x) | c | m + c | 2m + c | 3m + c | 4m + c | 0 |

$\sum_{x=0}^{4}(mx+c) = 1 \Rightarrow 10m + 5c = 1 \Rightarrow 2m + c = \frac{1}{5}$ …(1)

mean $= \Sigma x_i P_i = \sum_{i=0}^{4}(mx \;+c).x_i \;= 30m + 10c = \frac{5}{2}$

$\therefore 3m + c = \frac{1}{4}$ …(2)

from (1) and (2) $m = \frac{1}{20}$, $c = \frac{1}{10}$

$\Sigma P_i x_i \;= \sum_{i=0}^{4}\left(mx_i + c\right)x_1^2$

$= \sum_{i=0}^{4}\left(mx_i^3 + cx \;\right) \Rightarrow 100m + 30c$ (Now putting m and c)

$\Rightarrow \Sigma P_i x_i^2 = 5 + 3 = 8$

Variance $= \Sigma P_i x_i^2 - \left(\Sigma P_i x_i\right)^2 \;= 8 - \left(\frac{5}{2}\right)^2 = -$

$\therefore 24\alpha = 42$

## SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  *Negative Marks* : –1 In all other cases.

**14.** Let $\alpha$ and $\beta$ be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a $3\times3$ matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i = 1, 2, 3 and j = 1, 2, 3.

Match each entry in **List-I** to the correct entry in **List-II**.

| | **List-I** | | **List-II** |
|---|---|---|---|
| (P) | The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is | (1) | 1 |
| (Q) | The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is | (2) | 12 |
| (R) | Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is | (3) | infinite |
| (S) | Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is | (4) | 6 |
| | | (5) | 0 |

The correct options is

(A) (P) $\rightarrow$ (4) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (5) (S) $\rightarrow$ (1)

(B) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (2) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (3) (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (1) (Q) $\rightarrow$ (5) (R) $\rightarrow$ (3) (S) $\rightarrow$ (4)

**Ans.** **(C)**

**Sol.** $\alpha, \beta$ are roots of $x^2 + x - 1 = 0$

$\therefore \alpha + \beta = -1 \Rightarrow 1 + \alpha + \beta = 0$

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(P) $$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & \\ & 1 & \alpha \end{bmatrix} \Rightarrow 3! \times 2 = 12$$

For one arrangement of row 1 we can arrange other two rows exactly in two ways and row 1 can be arranged in 3! ways

$\therefore 3! \times 2 = 12$ ways

(Q) $$M = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} \Rightarrow \text{Consider one such arrangement with } a = \alpha, b = \beta, c = 1$$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ & 1 & \alpha \end{bmatrix}$$

a, b, c can be arranged in 3! ways and corresponding entries can be arranged in 1 way.

(R) $$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} a \\ 0 \\ -c \end{bmatrix}$$

$ay + bz = a$

$-ax + cz = 0$

$-bx - cy = -c$

It is observed that $D = D_x = D_y = D_z = 0$

$\therefore$ infinite solution

(S) $$\begin{bmatrix} 1 & \alpha & \beta \\ \beta & \alpha & 1 \\ \alpha & 1 & \end{bmatrix}$$

$\Rightarrow \alpha\beta - 1 - \alpha\beta^2 + \alpha^2 + \beta^2 - \alpha^2\beta = 0$ (since $\alpha\beta = \alpha + \beta = -1$)

**15.** Let the straight line y = 2x touch a circle with center (0, α), α > 0, and radius r at a point $A_1$. Let $B_1$ be the point on the circle such that the line segment $A_1B_1$ is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

| List-I | | List-II | |
|---|---|---|---|
| (P) | α equals | (1) | (–2, 4) |
| (Q) | r equals | (2) | $\sqrt{5}$ |
| (R) | $A_1$ equals | (3) | (–2, 6) |
| (S) | $B_1$ equals | (4) | 5 |
| | | (5) | (2, 4) |

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

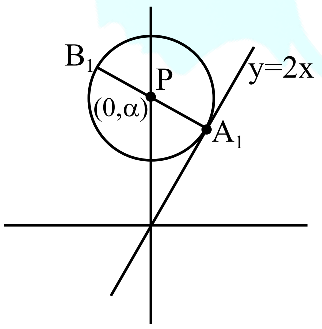
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)

(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)

(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

**Ans.** **(C)**

**Sol.**




Consider centre as P(0, α), α > 0

$$\left|\frac{2(0)-\alpha}{\sqrt{5}}\right| = r$$

$|-\alpha| = \sqrt{5}\,r$

$\alpha = \sqrt{5}\,r$

$\therefore\ \alpha + r = 5 + \sqrt{5}$

$\sqrt{5}\,r + r = \sqrt{5}\left(\sqrt{5}+1\right)$

$r = \sqrt{5},\ \ \alpha = 5$

$\therefore\ P(0, 5)$

Foot of perpendicular from P to line $2x - y = 0$

$\dfrac{x-0}{2} = \dfrac{y-5}{-1} = \dfrac{-(2(0)-5)}{5} = 1$

$x = 2,\ \ y = 4 \quad A_1(2, 4)$

Let $B(p, q)$ $\therefore\ \dfrac{p+2}{2} = 0,\ \dfrac{q+4}{2} = 5$

$\therefore\ p = -2,\ q = 6 \quad B(-2, 6)$

**16.** Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \dfrac{x+11}{1} = \dfrac{y+21}{2} = \dfrac{z+29}{3}$ and $L_2 : \dfrac{x+16}{3} = \dfrac{y+11}{2} = \dfrac{z+4}{\gamma}$ intersect. Let $R_1$ be the point of intersection of $L_1$ and $L_2$. Let O = (0, 0, 0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines $L_1$ and $L_2$.

Match each entry in **List-I** to the correct entry in **List-II**.

| List-I | | List-II | |
|---|---|---|---|
| (P) | $\gamma$ equals | (1) | $-\hat{i} - \hat{j} + \hat{k}$ |
| (Q) | A possible choice for $\hat{n}$ is | (2) | $\sqrt{\dfrac{3}{2}}$ |
| (R) | $\overrightarrow{OR_1}$ equals | (3) | 1 |
| (S) | A possible value of $\overrightarrow{OR_1}.\hat{n}$ is | (4) | $\dfrac{1}{\sqrt{6}}\hat{i} - \dfrac{2}{\sqrt{6}}\hat{j} + \dfrac{1}{\sqrt{6}}\hat{k}$ |
| | | (5) | $\sqrt{\dfrac{2}{3}}$ |

The correct option is

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (2)

(B) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (2)

(C) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

**Ans.** **(C)**

**Sol.** $L_1 : \frac{x+11}{1} = \frac{y+21}{2} \quad \frac{z+29}{3} \quad a$

$L_2 : \frac{x+16}{} = \frac{y+11}{2} = \frac{z+4}{\gamma} = b$

$x = a - 11 = 3b - 16 \quad \Rightarrow a - 3b = -5 \qquad \ldots(1)$

$y = 2a - 21 = 2b - 11 \quad \Rightarrow 2a - 2b = 10 \qquad \ldots(2)$

$z = 3a - 29 = br - 4 \Rightarrow 3a - b\gamma = 25 \qquad \ldots(3)$

from (1) & (2)

$a = 10\ ,\ b = 5$

Now from (3)

$3(10) - 5\gamma = 25 \qquad \therefore \gamma = 1$

$R_1 \equiv (-1, -1, 1)$

$OR_1 = -\hat{i} - \hat{j} + \hat{k}$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} - (-8)\hat{j} - 4\hat{k}$$

$\vec{n} = -4\hat{i} + 8\hat{j} + 4\hat{k} = -4(\hat{i} - 2\hat{j} + \hat{k})$

$$\hat{n} = \pm\frac{4(\hat{i} - 2\hat{j} + \hat{k})}{4\sqrt{6}} = \pm\frac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$$

$$\overrightarrow{OR} \cdot \hat{n} = \pm\left(-\hat{i} - \hat{j} + \hat{k}\right)\left(\frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}\right) = \pm\frac{2}{\sqrt{}} = \pm\sqrt{\frac{4}{6}} = \pm\sqrt{\frac{2}{}}$$

**17.** Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be functions defined by

$$f(x)=\begin{cases} x\,|x|\sin\left(\dfrac{1}{x}\right), & x\neq 0,\\ 0, & x=0,\end{cases} \quad \text{and} \quad g(x)=\begin{cases} 1-2x, & 0\le x\le \dfrac{1}{2},\\ 0, & \text{otherwise.}\end{cases}$$

Let a, b, c, d $\in \mathbb{R}$. Define the function $h:\mathbb{R}\to\mathbb{R}$ by

$$h(x)=a\,f(x)+b\left(g(x)+g\left(\frac{1}{2}-x\right)\right)+c(x-g(x))+d\,g(x),\ x\in\mathbb{R}$$

Match each entry in **List-I** to the correct entry in **List-II**.

| | **List-I** | | **List-II** |
|---|---|---|---|
| (P) | If a = 0, b = 1, c = 0 and d = 0, then | (1) | $h$ is one-one. |
| (Q) | If a = 1, b = 0, c = 0 and d = 0, then | (2) | $h$ is onto. |
| (R) | If a = 0, b = 0, c = 1 and d = 0, then | (3) | $h$ is differentiable on $\mathbb{R}$. |
| (S) | If a = 0, b = 0, c = 0 and d = 1, then | (4) | the range of $h$ is [0, 1]. |
| | | (5) | the range of $h$ is {0, 1}. |

The correct option is :

(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

**Ans.** **(C)**

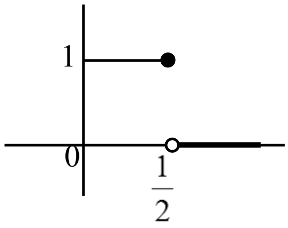
**Sol.** $$f(x)=\begin{cases} x|x|\sin\dfrac{1}{x} & ;\ x\neq 0\\ 0 & ;\ x=0\end{cases} \qquad g(x)=\begin{cases} 1-2x & ;\ 0\le x\le\dfrac{1}{2}\\ 0 & ;\ \text{otherwise}\end{cases}$$

$$g\left(\frac{1}{2}-\quad\right|=\begin{cases} 2x & ;\ 0\le --x\le\dfrac{1}{2}\\ 0 & ;\ \text{otherwise}\end{cases}=\begin{cases} 2x & ;\ 0\le x\quad \dfrac{1}{2}\\ 0 & ;\ \text{otherwise}\end{cases}$$

$$g(x)+g\left(\frac{1}{2}-x\right)=\begin{cases} 1 & ;\ 0\le x\le\dfrac{1}{2}\\ 0 & ;\ \text{otherwise}\end{cases}$$

(P) Now a = 0, b = 1, c = 0, d = 0

$$\because\ h(x) = g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1 & ; \quad 0 \le x \le \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$




Hence Range of h(x) is {0, 1}

(Q) a = 1, b = 0, c = 0, d = 0

$$h(x) = f(x) = \begin{cases} x|x|\sin\frac{1}{x} & ; \quad x \ne 0 \\ 0 & ; \quad x = 0 \end{cases}$$

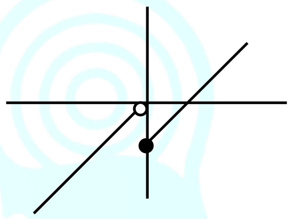
$$\text{RHD} = \lim_{x \to 0} \frac{x^2 \sin\frac{1}{x} - 0}{x} = 0$$

$$\text{LHD} = \lim_{x \to 0} \frac{-x^2 \sin\frac{1}{x} \quad 0}{x} = 0$$

Hence h(x) is differentiable on R

(R) a = 0, b = 0, c = 1, d = 0

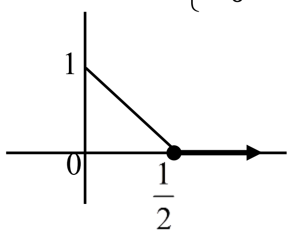
$$h(x) = x - g(x) = \begin{cases} 3x - 1 & ; \quad 0 \le x \le \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$



$\therefore$ h(x) is ONTO

(S) a = 0, b = 0, c = 0, d = 1

$$h(x) = g(x) = \begin{cases} 1 - 2x & ; \quad 0 \le x \le \frac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$




Range of h(x) is [0, 1]

# JEE(ADVANCED)–2024 (EXAMINATION)
**(Held On Sunday 26th MAY, 2024)**

| PHYSICS | TEST PAPER WITH ANSWER AND SOLUTION |
|---|---|

## PAPER-1

### SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct option is chosen;
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
*Negative Marks* : −1 In all other cases.

**1.** A dimensionless quantity is constructed in terms of electronic charge *e*, permittivity of free space $\varepsilon_0$, Planck's constant h, and speed of light *c*. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and *n* is a non-zero integer, then (α, β, γ, δ)is given by

(A) $(2n, -n, -n, -n)$
(B) $(n, -n, -2n, -n)$
(C) $(n, -n, -n, -2n)$
(D) $(2n, -n, -2n, -2n)$

**Ans.** **(A)**

**Sol.** For the quantity to be dimensionless

$$e^{\alpha}\,{}_{0}^{\beta}h^{\gamma}c^{d} = M^0L^0T^0A^0$$

$$(AT)^{\alpha}\left(M^{-1}L^{-3}T^{4}A^{2}\right)^{\beta}\left(ML^{2}T^{-1}\right)^{\gamma}\left(LT^{-1}\right)^{\delta} = A^0M^0L^0T^0$$

$\therefore\ \alpha + 2\beta = 0,\ \alpha + 4\beta - \gamma - \delta = 0,\ -\beta + \gamma = 0\ \&\ -3\beta + 2\gamma + \delta = 0$

$\therefore\ \alpha = -2\beta,\ \beta = \gamma\ \&\ \gamma = \delta$

$\therefore$ Option (A) satisfies the given condition

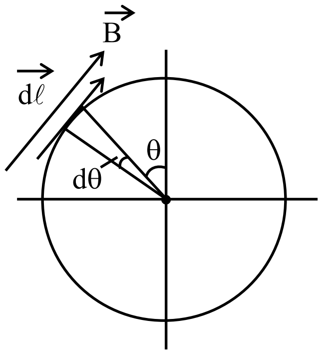
**2.** An infinitely long wire, located on the *z*-axis, carries a current *I* along the +z-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B}.\vec{dl}$ along a straight line from the point $(-\sqrt{3}a, a, 0)$ to $(a, a, 0)$ is given by

[$\mu_0$ is the magnetic permeability of free space.]

(A) $7\mu_0 I/24$
(B) $7\mu_0 I/12$
(C) $\mu_0 I/8$
(D) $\mu_0 I/6$

**Ans.** **(A)**

**Sol.**




$\Rightarrow \left|\vec{d\ell}\right| = rd\theta$

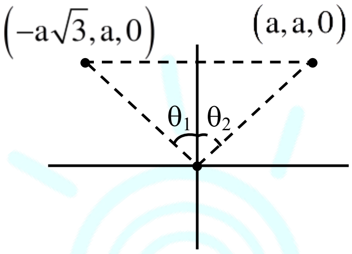
$\Rightarrow \left|\vec{B}\right| \quad \frac{\mu_0 I}{2\pi r}$

$\Rightarrow \int \vec{B}\cdot\vec{d\ell} = \int \left|\vec{B}\right|\left|\vec{d\ell}\right| \cos 0°$

$= \int \left(\frac{\mu_0 I}{2\pi r}\right) \times (rd\theta)$

$= \int_{\theta_1}^{\theta_2} \frac{\ _0}{\pi} d\theta = \frac{\mu\ I}{2}\left[\theta_2 - (-\theta\ )\right]$

[$\theta_1$ is anticlockwise hence taken negative]




$\Rightarrow \tan\theta_1 = \frac{a\sqrt{3}}{a} = \sqrt{3}$

$\theta_1 \quad \frac{\pi}{3}$

$\Rightarrow \tan\theta_2 = \frac{a}{a} = 1$

$\theta_2 \quad \frac{\pi}{4}$

$\Rightarrow \int B\cdot d\ell = \frac{\mu_0 I}{2\pi}\left[\frac{\pi}{3} + \frac{\pi}{4}\right]$

$= \frac{7\mu_0 I}{24}$

$\Rightarrow$ Ans. Option (A)

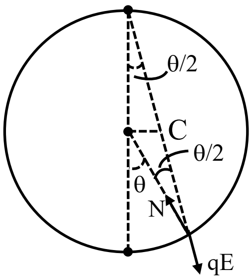
**3.** Two beads, each with charge $q$ and mass $m$, are on a horizontal, frictionless, non-conducting, circular hoop of radius $R$. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [$\varepsilon_0$ is the permittivity of free space]

(A) $q^2 / (4\pi\varepsilon_0 R^3 m)$ (B) $q^2 / (32\pi\varepsilon_0 R^3 m)$

(C) $q^2 / (8\pi\varepsilon_0 R^3 m)$ (D) $q^2 / (16\pi\varepsilon_0 R^3 m)$

**Ans.** **(B)**

**Sol.**




Restoring force $= qE\sin\left(\frac{\theta}{2}\right)$

$$\therefore \tau = qE\sin\left(\frac{\theta}{2}\right)R = I\alpha$$

$$E = \frac{Kq}{\left(2R\cos\frac{\theta}{2}\right)^2} \quad \frac{1}{4\pi{}_0} \frac{q}{4R \ \cos^2\left(\frac{\theta}{2}\right)}$$

$$\therefore \frac{1}{4\pi{}_0} \frac{qR}{4R \ \cos^2\left(\frac{\theta}{2}\right)} \sin\left(\frac{\theta}{}\right) q = mR^2\alpha$$

For $\theta$ very small,

$$\frac{-q^2}{32\pi{}_0 R^3 m}\theta = \alpha$$

$$\therefore \omega^2 = \frac{q^2}{32\pi{}_0 mR^3}$$

Hence option (B)

**4.** A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of $x$ in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are

(A) –0.5 m, 5 kg m/s (B) 0.5 m, 0 kg m/s (C) 0.5 m, –5 kg m/s (D) –1 m, 5 kg m/s

**Ans.** **(C)**

**Sol.** $F = -20\left(x - \frac{1}{2}\right) = -20X \qquad \left(X = x - \frac{1}{2}\right)$

$\therefore$ Particle will perform SHM about $x = \frac{1}{2}$ with

$\omega = 2$ rad/sec $\Rightarrow T = \pi$ sec.

$\therefore$ Phase covered in $t = \frac{\pi}{4}$ second $= 90°$.

Given particle is at rest at x = 1m $\Rightarrow$ x = 1 is extreme position.

$\therefore$ In $\frac{\pi}{4}$ sec, it will be at equilibrium

$\therefore$ x = 0.5 m and momentum = $m\omega A = 5 \times 2 \times 0.5 = 5$ kg m/s

Direction will be towards –ve x.

Hence option (C)

**SECTION-2 : (Maximum Marks : 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

**5.** A particle of mass $m$ is moving in a circular orbit under the influence of the central force F($r$) = –$kr$, corresponding to the potential energy $V(r) = kr^2 / 2$, where $k$ is a positive force constant and $r$ is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, $h$ is the Planck's constant, and $n$ a positive integer. If $v$ and $E$ are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

(A) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$

(B) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(C) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$

(D) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

**Ans.** **(A,B,C)**

**Sol.** The central force will provide necessary centripetal force

$$kr = \frac{mv^2}{r}$$

or, $kr^2 = mv^2$ …. (1)

By quantisation rule

$n\hbar = mvr$

or, $\frac{n\hbar}{r} = mv$ … (2)

$$\frac{(1)}{(2)^2} \Rightarrow \frac{kr^2}{\frac{n^2\hbar^2}{r^2}} \quad \frac{mv}{m\ v^2}$$

$$\Rightarrow \frac{k}{n^2\hbar} r^4 \quad \frac{}{m}$$

$$\Rightarrow r = \left(\frac{n^2\hbar}{km}\right)^{\frac{1}{4}} \Rightarrow \ ^2 = \frac{\hbar}{\sqrt{mk}}$$

(B) Using (1), $K \cdot \frac{n\hbar}{\sqrt{mk}} \quad mv^2$

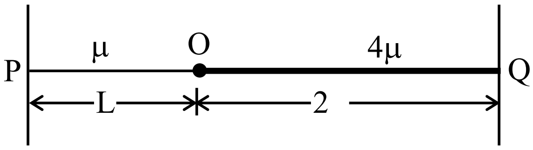
$$\Rightarrow \ ^2 = n\hbar\sqrt{\frac{k}{m^3}}$$

(C) $\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ from (1)

(D) $E = \frac{1}{2}mv^2 + \frac{1}{2}kr \;= \frac{n\hbar}{2}\sqrt{\frac{k}{m}} + \frac{1}{2}k\frac{n\hbar}{\sqrt{mk}}$

$$E = n\hbar\sqrt{\frac{k}{m}}$$

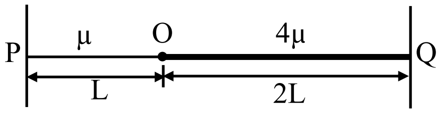
**6.** Two uniform string of mass per unit length μ and 4μ, and length *L* and 2*L*, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct ?




(A) With a node at O, the minimum frequency of vibration of the composite string is $v_0$.
(B) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.
(C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.
(D) No vibrational mode with an antinode at O is possible for the composite string.

**Ans.** **(A,C,D)**

**Sol.**




$$C_1 = \sqrt{\frac{T}{\mu}}, C \quad = \sqrt{\frac{}{4\mu}} = \frac{C_1}{}$$

For node at O :

$$L = \frac{1}{2}, 2L \quad \frac{m\lambda}{} \quad \text{(n, m are integers)}$$

$$\lambda_1 = \frac{2L}{n}, \lambda \quad = \frac{4L}{}$$

$$\frac{C_1}{\lambda_1} = \frac{}{}$$

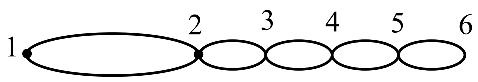
$$\Rightarrow \frac{\frac{C_1}{2L}}{n} \quad \frac{\frac{\frac{C_1}{2}}{4L}}{m}$$

$\Rightarrow 4n = m$

For minimum frequency, n = 1, m = 4

$$\therefore \nu_{min} = \frac{C_1 \times 1}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \nu_0$$

The string will look like



Total no. of nodes = 6 including the end nodes

<u>For antinode at O :</u>

$$L = (2n \quad 1)\frac{\lambda_1}{4};\ 2L = (2n \quad 1)\frac{\lambda_2}{4} \qquad \text{(n, m are integers)}$$

$$\lambda_1 \quad \frac{4L}{(2n+1)};\ \lambda_2 \quad \frac{8L}{(2m+1)}$$

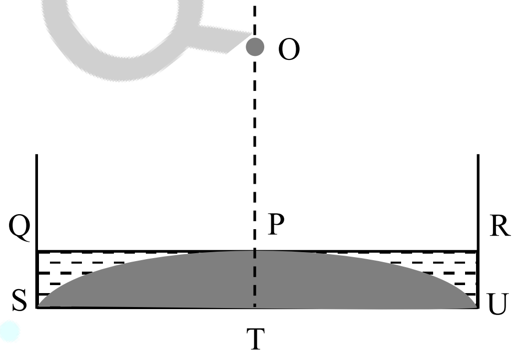
$$\frac{C_1}{\lambda_1} = \frac{}{_2}$$

$$\frac{C_1}{C_2} = \frac{\lambda}{\lambda_2}$$

$$2 = \frac{\frac{4L}{(2n+1)}}{\frac{8L}{(2m+1)}}$$

$$4 = \frac{(2m+1)}{(2n+1)} \Rightarrow \text{even} = \frac{\text{odd}}{\text{odd}} \Rightarrow \text{This node is not possible}$$

**7.** A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index $n$ up to the level QPR. If the image of a point object O at a height of $h$ (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct?




(A) For $n = 1.42$, $h = 50$ cm.

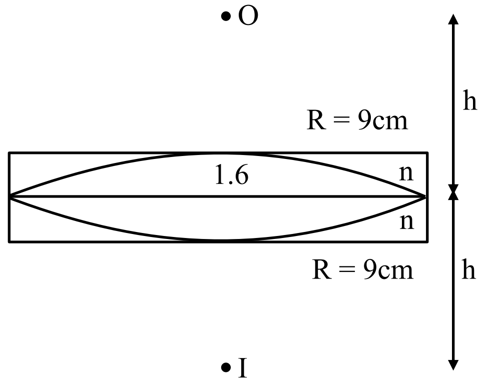
(B) For $n = 1.35$, $h = 36$ cm.

(C) For $n = 1.45$, $h = 65$ cm.

(D) For $n = 1.48$, $h = 85$ cm.

**Ans.** **(A,B)**

**Sol.** Since STU is a plane mirror, we can take mirror image of the whole situation about it and final image can be assumed to be at a distance h below the base.




Since object and image are at same distance from equivalent lens, hence h = $2F_{eq}$

$$\frac{1}{F_{eq}} \quad \left(\frac{1.6-1}{1}\right)\left(\frac{2}{9}\right)+\frac{(n-1)}{1}\left(-\frac{2}{9}\right)$$

$$\frac{1}{\frac{h}{2}}=\frac{1.2}{9}+\frac{2(1-n)}{}$$

$$\frac{2}{h}=\frac{3.2-2n}{9}$$

$$h=\frac{9}{1.6-n}\text{cm}$$

(A) for n = 1.42, h = 50 cm
(B) for n = 1.35, h = 36 cm
(C) for n = 1.45, h = 60 cm
(D) for n = 1.48, h = 75 cm

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** If the correct integer is entered;
*Zero Marks* : 0 In all other cases.

---

**8.** The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where $k$ is a constant of suitable dimensions in SI units, and $T$ is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from – 73°C to 27°C is n$k$, the value of n is _______.
[Given : 0 K = –273 °C.]

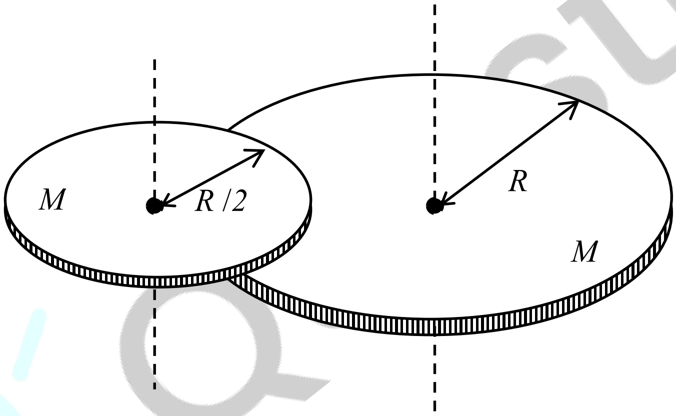
**Ans.** **(25000)**

**Sol.** $T_i$ = – 73°C = 200 K
$T_f$ = 27°C = 300 K

$$Q=\int msdT$$
$$=\int 1\cdot kT\ dT$$
$$=\int kT\ dT\ =K\int_{200}^{300} T\ dT$$
$$=\frac{K}{2}\left[T^2\right]_{200}^{300}\quad \frac{K}{2}\ \ 300^2-200\ \ ]$$
$Q = 25000\ K$

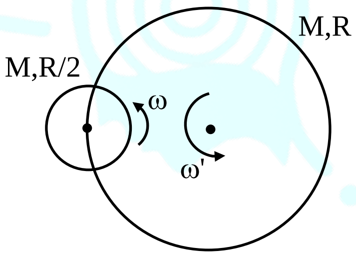
Hence $\eta = 25000$

**9.** A disc of mass $M$ and radius $R$ is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass $M$ and radius $R/2$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed $\omega$. If the angular speed at which the large disc rotates is $\omega/n$, then the value of $n$ is ______.




**Ans.** **(12)**

**Sol.**




On applying conservation of angular momentum about axis of larger disc.

$$\frac{1}{2}\cdot M\left(\frac{R}{2}\right)^2\cdot\omega-M(\omega' R)\cdot R-\frac{MR^2}{2}\cdot\omega'=0$$
$$\Rightarrow \frac{\quad}{8}=\frac{3\omega'}{2}$$
$$\Rightarrow \omega'=\frac{\omega}{12}\qquad \text{Hence, } n=12$$

**10.** A point source S emits unpolarized light uniformly in all directions. At two points A and B, the ratio $r = I_A / I_B$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of $r$ will be _____.

**Ans.** **(8)**

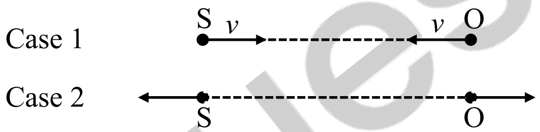
**Sol.** New intensity at B

$$I'_B \quad \left(\frac{I_B}{2}\right)\cos^2 45° \ = \frac{I_B}{4}$$

New value of $\alpha = \dfrac{I_A}{\frac{I_B}{4}} = \dfrac{4I}{I_B}$

$= 4 \times 2;\ \alpha = 8$

**11.** A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed $v$ with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed $v$ with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be $n$ Hz. The value of $n$ is _____.




**Ans.** **(200)**

**Sol.** Frequency received by observer

$$f_0 = \left(\frac{C \pm V_0}{C \pm V_S}\right) \quad \text{, C is speed of sound}$$

**Case-1:**

$$f_1 = \left(\frac{C+V}{C-V}\right)$$

$$288 = \left(\frac{C+V}{C-V}\right)240$$

**Case-2:**

$$f_2 = \left(\frac{C-V}{C+V}\right)$$

$$n = \left(\frac{C-V}{C+V}\right)240$$

multiply the two equations, we get.

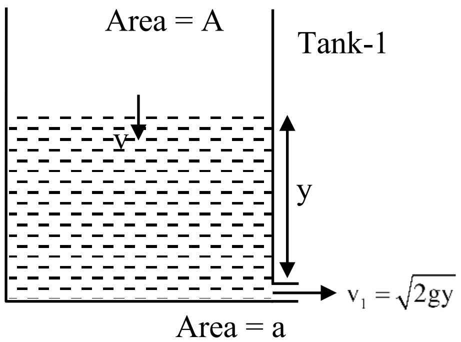
(288) (n) = (240) (240)

N = 200

**12.** Two large, identical water tanks, 1 and 2, kept on the top of a building of height $H$, are filled with water up to height $h$ in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are $t_1$ and $t_2$, respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio $t_1/t_2$ is _____.

**Ans.** **(3)**

**Sol.**

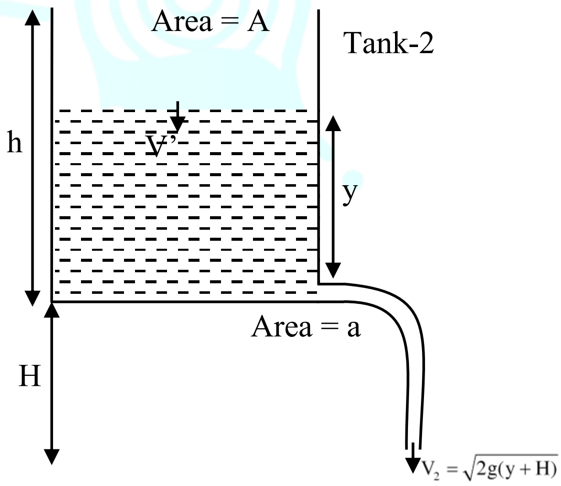



$Av = av_1$

$$A\left(-\frac{dy}{dt}\right) \quad a\sqrt{2gy}\ ;\ dt = \frac{A}{a\sqrt{2g}} \cdot \frac{-dy}{\sqrt{y}}$$

$$\int_0^{t_1} dt = \frac{A}{a\sqrt{2g}} \int_h^0 \frac{dy}{\sqrt{y}}$$

$$t_1 = \frac{A}{a\sqrt{2g}} 2\sqrt{h}\ ;\ t_1 = \frac{A}{a}\sqrt{\frac{2h}{g}}$$




$Av' = av_2$

$$A\left(-\frac{dy}{dt}\right)=a\sqrt{2g(H+y)}$$

$$dt=-\frac{A}{a\sqrt{2g}}\frac{dy}{\sqrt{H+y}}$$

$$\int_0^{t_2} dt=-\frac{A}{a\sqrt{2g}}\int_H^0\frac{dy}{\sqrt{H+y}}$$

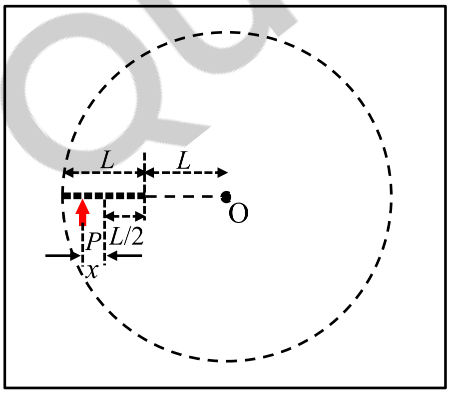
$$t_2=\frac{A}{a\sqrt{2g}}(2)(\sqrt{H+h}-\sqrt{H})\ \&\ H=\frac{16h}{9}$$

$$=\frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{5}{3}\quad\frac{4}{3}\right)$$

$$t_2=\frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{1}{3}\right)$$

ratio $\frac{t_1}{t_2}=3$

**13.** A thin uniform rod of length *L* and certain mass is kept on a frictionless horizontal table with a massless string of length *L* fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse *P* is imparted to the rod at a distance $x = L/n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of *n* is _____.




**Ans.** **(18)**

**Sol.** Linear impulse $\int F dt = \Delta$ momentum

$$= m\,(V_{cm}-0)$$

$$P = m\,(\omega\, r_{cm})$$

$$= m\omega\left(L+\frac{L}{2}\right)$$

$$P = m\left(\frac{3L}{2}\right) \qquad \text{....(i)}$$

Angular impulse $\int \tau dt = \Delta$ angular momentum

$$\int r\times F dt = \Delta L$$

$r\times\int F dt = I(\omega - 0)$ , and I is moment of inertia about axis of rotation.

$$\left(L+\frac{L}{2}+x\right)\times P = (I_{cm}+md^2)\omega$$

$$= \left(\frac{mL^2}{12}+m\left(L+\frac{L}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2}+x\right)P = mL^2\left(\frac{1}{12}+\left(\frac{3}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2}+x\right)P = mL^2\left(\frac{7}{3}\right)\omega \qquad \ldots.(ii)$$

Divide eq.-(i) & (ii)

$$\left(\frac{3L}{2}+x\right) = \frac{L\left(\frac{7}{3}\right)}{\left(\frac{3}{2}\right)}$$

$$\frac{3L}{2}+x = L\left(\frac{14}{9}\right)$$

$$x = \frac{L}{18}$$

## SECTION-4 : (Maximum Marks : 12)

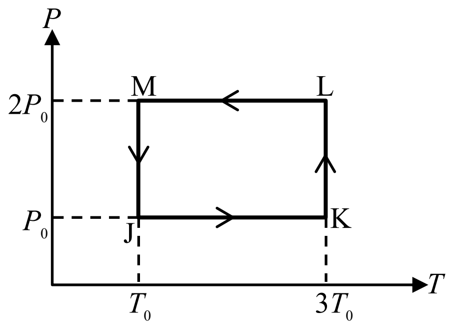
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : –1 In all other cases.

**14.** One mole of a monatomic ideal gas undergoes the cyclic process J → K → L → M → J, as shown in the P-T diagram.




Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [$R$ is the gas constant.]

| List-I | | List-II |
|---|---|---|
| (P) | Work done in the complete cyclic process | (1) $RT_0 - 4RT_0 \ln 2$ |
| (Q) | Change in the internal energy of the gas in the process JK | (2) 0 |
| (R) | Heat given to the gas in the process KL | (3) $3RT_0$ |
| (S) | Change in the internal energy of the gas in the process MJ | (4) $-2RT_0 \ln 2$ |
| | | (5) $-3RT_0 \ln 2$ |

(A) P → 1; Q → 3; R → 5; S → 4
(B) P → 4; Q → 3; R → 5; S → 2
(C) P → 4; Q → 1; R → 2; S → 2
(D) P → 2; Q → 5; R → 3; S → 4

**Ans.** **(B)**

**Sol.** $J\ (P_0, V_0, T_0)$

$K\ (P_0, 3V_0, 3T_0)$

$M\ \left(2P_0, \dfrac{V_0}{2}, T_0\right)$

$L\ \left(2P_0, \dfrac{3V_0}{2}, 3T_0\right)$

$P_0V_0 = nRT_0$

$JK \rightarrow \text{isobaric} \Rightarrow W = P_0\,(2V_0) = 2nRT_0$

$\Delta U = \dfrac{3}{2}nR(2T_0) = 3nRT$

$KL \rightarrow \text{isothermal} \rightarrow W = nR(3T)\ \ell n\left(\dfrac{1}{2}\right) = -\,3nRT_0\ell n2$

$\Delta U = 0 \Rightarrow Q = -3nRT_0\ell n2$

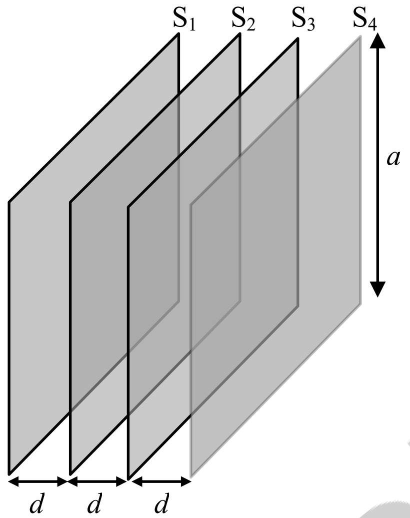
$LM \rightarrow \text{isobaric} = 2P_0\,(-V_0) = -2nRT_0$

$MJ \rightarrow \text{isothermal} \Rightarrow nRT_0\ell n2;\ \Delta U = 0$

$WD_{net} = -2nRT_0\ \ell n2$

P → 4, Q → 3, R → 5, S → 2

**15.** Four identical thin, square metal sheets, $S_1$, $S_2$, $S_3$ and $S_4$, each of side $a$ are kept parallel to each other with equal distance $d$ ($<< a$) between them, as shown in the figure. Let $C_0 = \varepsilon_0 a^2/d$, where $\varepsilon_0$ is the permittivity of free space.




Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

| List-I | | List-II |
|---|---|---|
| (P) | The capacitance between $S_1$ and $S_4$, with $S_2$ and $S_3$ not connected, is | (1) $3C_0$ |
| (Q) | The capacitance between $S_1$ and $S_4$, with $S_2$ shorted to $S_3$, is | (2) $C_0/2$ |
| (R) | The capacitance between $S_1$ and $S_3$, with $S_2$ shorted to $S_4$, is | (3) $C_0/3$ |
| (S) | The capacitance between $S_1$ and $S_2$, with $S_3$ shorted to $S_1$, and $S_2$ shorted to $S_4$, is | (4) $2C_0/3$ |
| | | (5) $2C_0$ |

(A) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 5

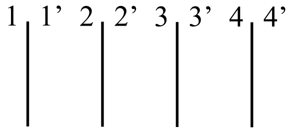
(B) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 1

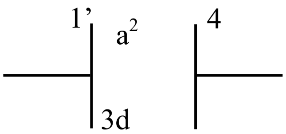
(C) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1

(D) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 2; S $\rightarrow$ 5

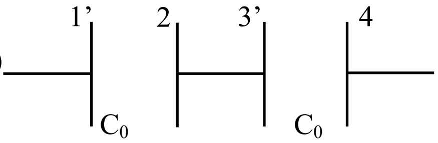
**Ans.** **(C)**

**Sol.**

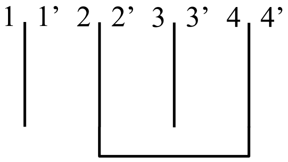


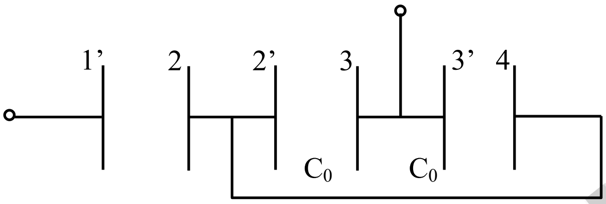

(P) 


$\Rightarrow C = \frac{\varepsilon_0 q^2}{3d} \quad \frac{}{3}$

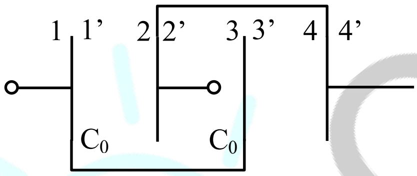
(Q) 

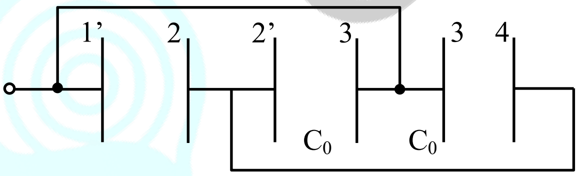

$\Rightarrow \left(\frac{C_0}{2}\right|$

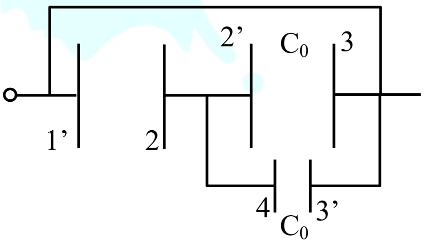
(R) 

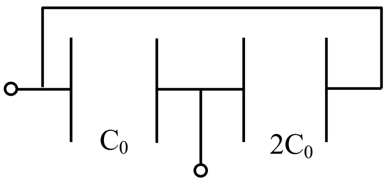




$\Rightarrow \frac{(2C_0)(C_0)}{3C_0} \quad \frac{2C_0}{3}$

(S) 


$\Rightarrow$ 


$\Rightarrow 3C_0$

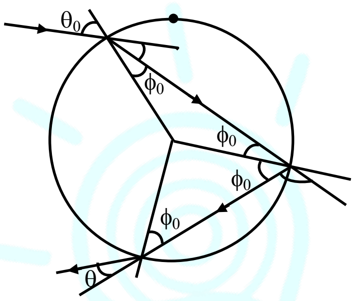
**16.** A light ray is incident on the surface of a sphere of refractive index $n$ at an angle of incidence $\theta_0$. The ray partially refracts into the sphere with angle of refraction $\phi_0$ and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is $\alpha$. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

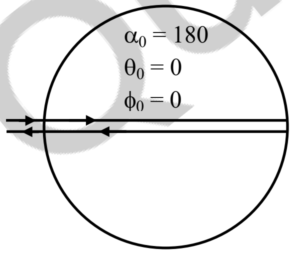
| | **List-I** | **List-II** |
|---|---|---|
| (P) | If $n = 2$ and $\alpha = 180°$, then all the possible values of $\theta_0$ will be | (1) 30° and 0° |
| (Q) | If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of $\theta_0$ will be | (2) 60° and 0° |
| (R) | If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of $\phi_0$ will be | (3) 45° and 0° |
| (S) | If $n = \sqrt{2}$ and $\theta_0 = 45°$, then all the possible values of $\alpha$ will be | (4) 150° |
| | | (5) 0° |

(A) $P \to 5;\ Q \to 2;\ R \to 1;\ S \to 4$
(B) $P \to 5;\ Q \to 1;\ R \to 2;\ S \to 4$
(C) $P \to 3;\ Q \to 2;\ R \to 1;\ S \to 4$
(D) $P \to 3;\ Q \to 1;\ R \to 2;\ S \to 5$

**Ans.** **(A)**

**Sol.**




$\alpha = (\theta_0 - \phi_0) + (180 - 2\phi_0) + (\theta_0 - 2\phi_0)$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

(P) $\alpha = 180 + 2\theta_0 - 4\phi_0$

$180 = 180 + 2\theta_0 - 4\phi_0 \Rightarrow \theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = 2\sin\phi_0$ ....(ii)

From (i) & (ii)

$\sin\theta_0 = 2\sin(\theta_0/2) \Rightarrow \cos\left(\frac{\theta_0}{2}\right)$ 1

$\frac{\theta_0}{2} = 0$

$\Rightarrow \theta_0 = 0$

(Q) $\theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = \sqrt{3}\sin\phi_0$ ....(ii)

From (i) & (ii)

$\sin\theta_0 \quad \sqrt{3}\sin\left(\frac{\theta_0}{2}\right)$

$\Rightarrow \cos\left(\frac{\theta_0}{2}\right) \quad \frac{\sqrt{3}}{}$

$\frac{\theta_0}{2} = 30, 150$

$\theta_0 = 60, 300$ (Rejected)

$\theta_0 = 60$ & $0$

(R) $\theta_0 = 2\phi_0$

$\sin\theta_0 = \sqrt{3}\sin\phi$

$\sin 2\theta_0 = \sqrt{3}\sin\phi$

$\cos\phi_0 = \frac{\sqrt{3}}{2}$

$\phi_0 = 30, 150$ (Rejected)

$\phi_0 = 30$ & $0$ ....(i)

(S) $\sin 45 = \sqrt{2}\cos\phi_0$

$\cos\phi_0 = 1/2$

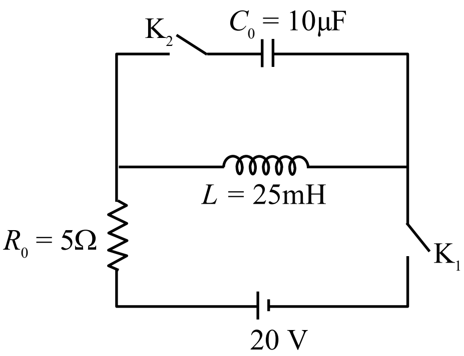
$\phi_0 = 60$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

$\alpha = 180 + 90 - 120$ ....(iv)

$= 180 - 30; \alpha = 150°$

**17.** The circuit shown in the figure contains an inductor $L$, a capacitor $C_0$, a resistor $R_0$ and an ideal battery. The circuit also contains two keys $K_1$ and $K_2$. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key $K_1$ is closed and immediately after this the current in $R_0$ is found to be $I_1$. After a long time, the current attains a steady state value $I_2$. Thereafter, $K_2$ is closed and simultaneously $K_1$ is opened and the voltage across $C_0$ oscillates with amplitude $V_0$ and angular frequency $\omega_0$.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

| **List-I** | **List-II** |
|---|---|
| (P) The value of $I_1$ in Ampere is | (1) 0 |
| (Q) The value of $I_2$ in Ampere is | (2) 2 |
| (R) The value of $\omega_0$ in kilo-radians/s is | (3) 4 |
| (S) The value of $V_0$ in Volt is | (4) 20 |
| | (5) 200 |

(A) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 5

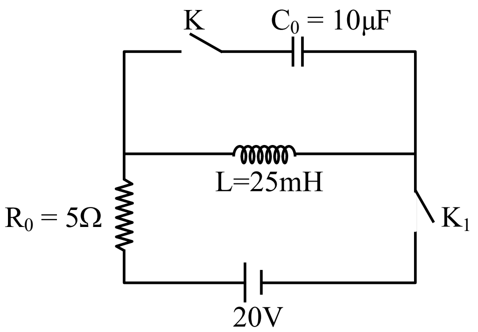
(B) P $\rightarrow$ 1; Q $\rightarrow$ 2; R $\rightarrow$ 3; S $\rightarrow$ 5

(C) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

(D) P $\rightarrow$ 2; Q $\rightarrow$ 5; R $\rightarrow$ 3; S $\rightarrow$ 4

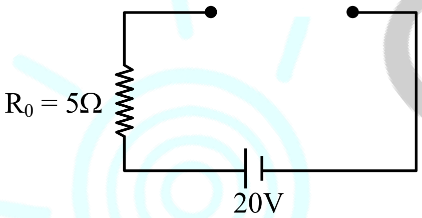
**Ans.** **(A)**

**Sol.**




(P) When $K_1$ is closed current in $R_0$ is $I_1$

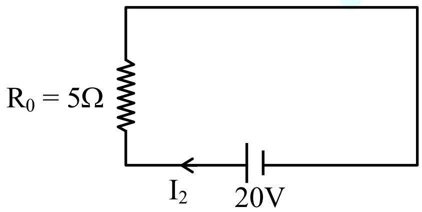
At t = 0; circuit will be




$I_1 = 0$

P $\rightarrow$ (1)

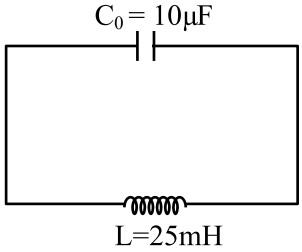
(Q) After long time inductor behave as a wire so $I_2$




$I_2 = \frac{20}{5}$ 4A

Q $\rightarrow$ (3)

(R) When $K_2$ is closed and $K_1$ open




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

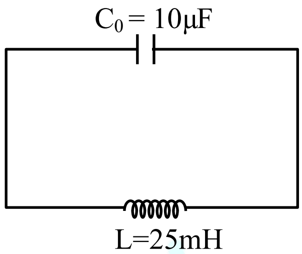
$$\omega_0 = \frac{1}{\sqrt{25\times10^{-3}\times10\times10}} = \frac{\ }{5\times10^{-4}}$$

$\omega_0 = 2 \times 10^3$ rad/s

$\omega_0 = 2$ kilo-radian/s

$R \rightarrow (2)$

(S) Now $K_2$ is closed and $K_1$ open




$$\frac{1}{2}LI_2^2 = \frac{\ }{2}CV$$

$25 \times 10^{-3} \times (4)^2 = 10 \times 10^{-6} \times V_0^2$

$V_0^2 = 2500\times16$

$V_0 = 50 \times 4 = 200$ V

$S \rightarrow (5)$

# JEE(ADVANCED)–2024 (EXAMINATION)

**(Held On Sunday 26th MAY, 2024)**

| CHEMISTRY | TEST PAPER WITH ANSWER AND SOLUTION |
|---|---|

## PAPER-1

### SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  *Full Marks* : +3 If **ONLY** the correct option is chosen;

  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

  *Negative Marks* : −1 In all other cases.

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**1.** A closed vessel contains 10 g of an ideal gas **X** at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas **Y** is added to it and the pressure becomes 6 atm. The ratio of root mean square velocities of **X** and **Y** at 300 K is

(A) $2\sqrt{2}:\sqrt{3}$ (B) $2\sqrt{2}:1$ (C) 1 : 2 (D) 2 : 1

**Ans. (D)**

**Sol.** For Ideal Gas

$PV = nRT$

$\therefore$ $n \propto P$ at constant T & V.

$\because$ $\text{mole} = \dfrac{\text{Mass}}{\text{Molar mass}}$

For gas X : $\dfrac{10}{M_X} \propto 2\text{ atm}$ ......... (1)

For gas X & Y : $\dfrac{10}{M_X} + \dfrac{80}{} \propto 6\text{ atm}$ ......... (2)

From (2) – (1)

$\dfrac{80}{M_y} \propto 4$ ......... (3)

On dividing (1) by (3)

$\dfrac{M_Y}{8M_X} = \dfrac{1}{2}$

$\therefore$ $\dfrac{M_Y}{M_X} = 4$ ......... (4)

$\because$ $V_{rms} = \sqrt{\dfrac{3RT}{M}} \Rightarrow V_{rms} \propto \dfrac{1}{\sqrt{M}}$

$\therefore$ $\dfrac{(V_{rms})}{(V_{rms})_Y} = \sqrt{\dfrac{M_Y}{M_X}} = \sqrt{\dfrac{4}{1}} = \dfrac{\ }{1}$

**2.** At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid ($HNO_2$) gives the species

(A) $H_3O^+$, $NO_3^-$ and NO (B) $H_3O^+$, $NO_3^-$ and $NO_2$

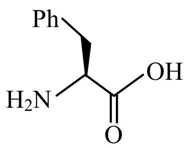
(C) $H_3O^+$, $NO^-$ and $NO_2$ (D) $H_3O^+$, $NO_3^-$ and $N_2O$

**Ans. (A)**

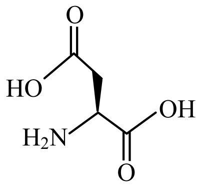
**Sol.** $3HNO_2(aq) \rightleftharpoons H_3O^+ + NO_3^- + 2NO$

**3.** Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is

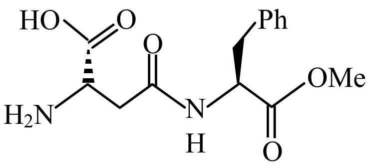
Structures of phenylalanine and aspartic acid are given below.

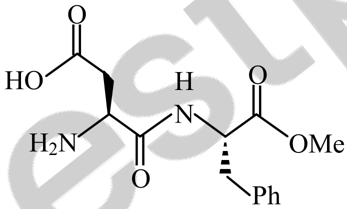


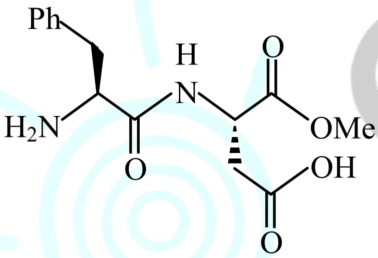
Phenylalanine

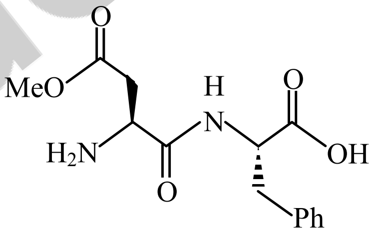


Aspartic acid

(A)


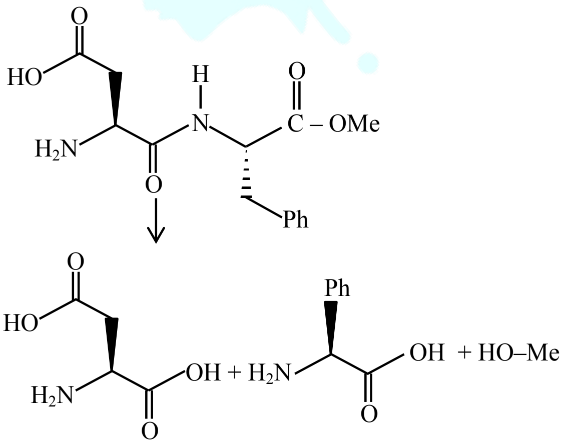

(B)



(C)



(D)



**Ans. (B)**

**Sol.** Aspartame structure is a dipeptide consisting aspartic acid and methyl ester of phenylalanine



Aspartic acid Phenylalanine

**4.** Among the following options, select the option in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.

[en = $H_2NCH_2CH_2NH_2$]

(A) **Set-I** : $[Ni(CO)_4]$ and $[PdCl_2(PPh_3)_2]$

**Set-II** : $[Co(NH_3)_5Cl]\ SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(B) **Set-I** : $[Co(en)(NH_3)_2Cl_2]$ and $[PdCl_2(PPh_3)_2]$

**Set-II** : $[Co(NH_3)_6]\ [Cr(CN)_6]$ and $[Cr(NH_3)_6]\ [Co(CN)_6]$

(C) **Set-I** : $[Co(NH_3)_3(NO_2)_3]$ and $[Co(en)_2Cl_2]$

**Set-II** : $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(D) **Set-I** : $[Cr(NH_3)_5Cl]Cl_2$ and $[Co(en)(NH_3)_2Cl_2]$

**Set-II** : $[Cr(H_2O)_6]Cl_3$ and $[Cr(H_2O)_5Cl]Cl_2 \cdot H_2O$

**Ans. (C)**

**Sol. Set-I :** $[Co(NH_3)_3(NO_2)_3]$ shows two geometrical isomers :- facial and meridional

$[Co(en)_2Cl_2]$ shows two geometrical isomers :- cis and trans

**Set-II :** $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5SO_4]Cl$ are ionization isomers of each other.

## SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if (all) the correct option(s) is(are) chosen; |
| *Partial Marks* | : +3 | If all the four options are correct but **ONLY** three options are chosen; |
| *Partial Marks* | : +2 | If three or more options are correct but **ONLY** two options are chosen, both of which are correct; |
| *Partial Marks* | : +1 | If two or more options are correct but **ONLY** one option is chosen and it is a correct option; |
| *Zero Marks* | : 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : −2 | In all other cases. |

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –2 marks.

**5.** Among the following, the correct statement(s) for electrons in an atom is(are)

(A) Uncertainty principle rules out the existence of definite paths for electrons.

(B) The energy of an electron in 2*s* orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.

(C) According to Bohr's model, the most negative energy value for an electron is given by n = 1, which corresponds to the most stable orbit.

(D) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n.

**Ans. (A,B,C)**

**Sol.** (A) Uncertainity principle talks about probability of finding electrons in different regions around the nucleus rather than definite paths.

(B) With increase in distance of electron from the nucleus, its energy increases.

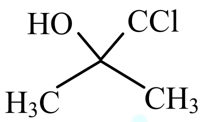
(C) Energy of electron $E_n = -13.6 \times \frac{Z^2}{n^2}$ eV/atom.

(D) Velocity of electron $V_n = 2.19 \times 10^6 \times \frac{Z}{n}$ m/sec.

**6.** Reaction of *iso*-propylbenzene with $O_2$ followed by the treatment with $H_3O^+$ forms phenol and a by-product **P**. Reaction of **P** with 3 equivalents of $Cl_2$ gives compound **Q**. Treatment of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.

The correct statement(s) regarding **P**, **Q**, **R** and **S** is(are)

(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives



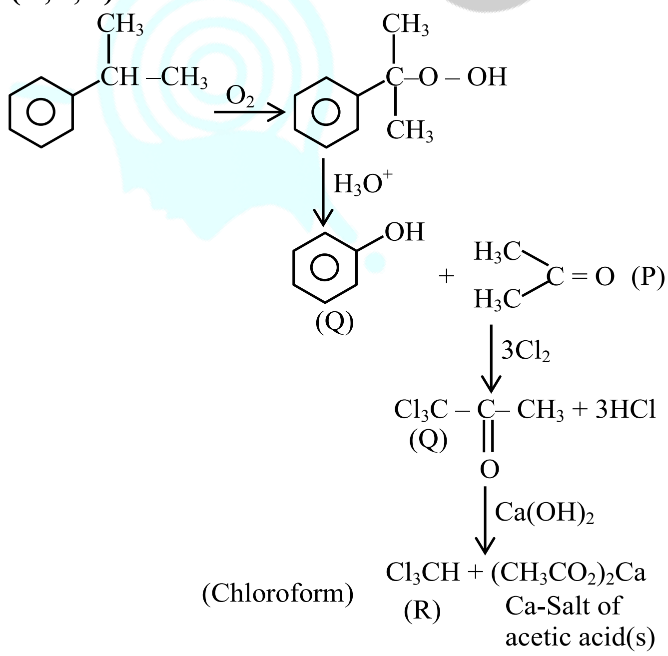

(B) Reaction of **R** with $O_2$ in the presence of light gives phosgene gas

(C) **Q** reacts with aqueous NaOH to produce $Cl_3CCH_2OH$ and $Cl_3CCOONa$

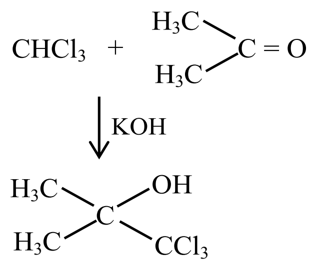
(D) **S** on heating gives **P**

**Ans. (A,B,D)**

**Sol.**



**(A)**

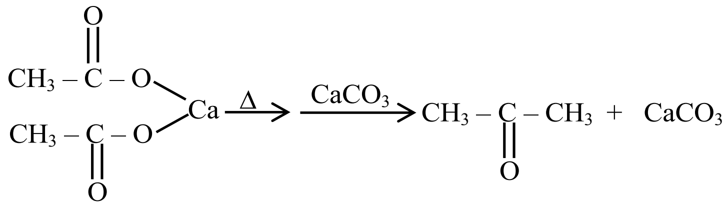



(Chloritone)

**(B)** $\underset{(R)}{CHCl_3} \xrightarrow{O_2} \underset{\text{Phosgene gas}}{COCl} + HCl$

**(C)** Q does not undergo Cannizaro reaction

**(D)**




**7.** The option(s) in which at least three molecules follow Octet Rule is(are)

(A) $CO_2$, $C_2H_4$, NO and HCl

(B) $NO_2$, $O_3$, HCl and $H_2SO_4$

(C) $BCl_3$, NO, $NO_2$ and $H_2SO_4$

(D) $CO_2$, $BCl_3$, $O_3$ and $C_2H_4$

**Ans. (A,D)**

**Sol.** NO, $NO_2$ , $BCl_3$ and $H_2SO_4$ do not follow octet rule.

## SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 **ONLY** If the correct integer is entered;
*Zero Marks* : 0 In all other cases.

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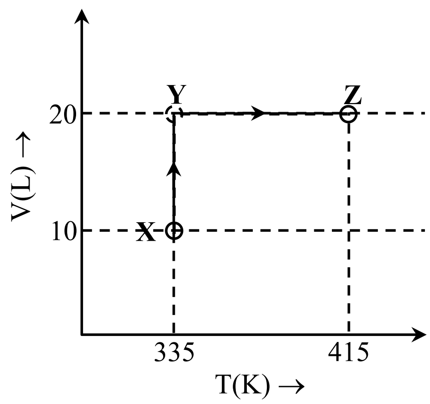
**8.** Consider the following volume-temperature (V – T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Considering only P-V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence $\mathbf{X} \rightarrow \mathbf{Y} \rightarrow \mathbf{Z}$ is ______ .

[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{V,m} = 12\ J\ K^{-1}\ mol^{-1}$ and gas constant, $R = 8.3\ J\ K^{-1}\ mol^{-1}$]

**Ans. (8120)**

**Sol.** For ideal gas

$\Delta H = nC_P\Delta T$

$\because\ C_P = C_V + R = 12 + 8.3 = 20.3$ J/K-mole

$\therefore\ \Delta H = 5 \times 20.3 \times (415 - 335)$

$\boxed{\Delta H = 8120 \text{ Joule}}$

**9.** Consider the following reaction,

$$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$$

which follows the mechanism given below:

$$2NO(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} N_2O\ \ (g) \qquad \text{(fast equilibrium)}$$

$$N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g) \qquad \text{(slow reaction)}$$

$$N_2O(g) + H_2(g) \xrightarrow{k_3} N_2(g) + H_2O(g) \qquad \text{(fast reaction)}$$

The order of the reaction is________.

**Ans. (3)**

**Sol.** Rate law = $k_2\,[N_2O_2]\,[H_2]$ [$\because$ slowest step of reaction is RDS]

$\because\ \dfrac{k_1}{k_{-1}} = \dfrac{[N_2O_2]}{[NO]^2}$

$\therefore\ [N_2O_2] = \dfrac{k_1}{k_{-1}}\,[NO]^2$

$\therefore$ Rate $= k_2 \times \dfrac{k_1}{k_{-1}}\,[NO]^2\,[H_2]$

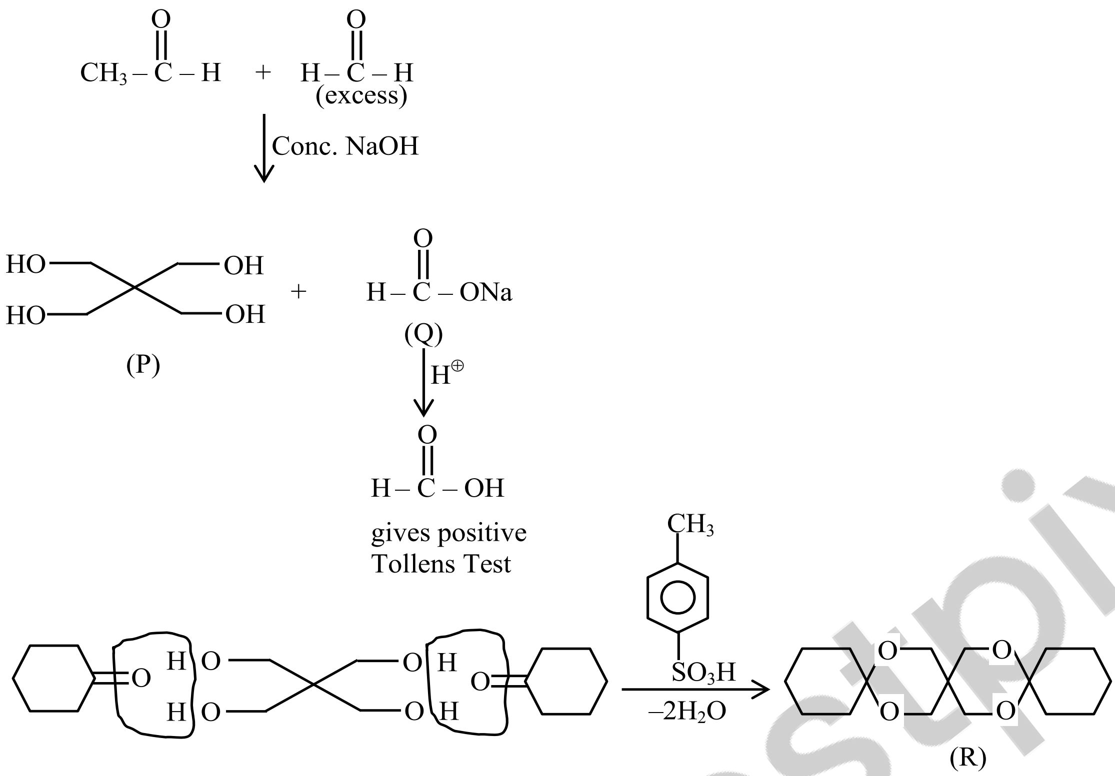
$\therefore$ Order of reaction is (3)

**10.** Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollens' test, whereas **Q** on acidification gives positive Tollens' test. Treatment of **P** with excess cyclohexanone in the presence of catalytic amount of *p*-toluenesulfonic acid (PTSA) gives product **R**.

Sum of the number of methylene groups ($-CH_2-$) and oxygen atoms in **R** is ______.

**Ans. (18)**

**Sol.**




Total $CH_2$ in R = 14

and oxygen in R= 4

So 14 + 4 = 18

**11.** Among $V(CO)_6$, $Cr(CO)_5$, $Cu(CO)_3$, $Mn(CO)_5$, $Fe(CO)_5$, $[Co(CO)_3]^{3-}$, $[Cr(CO)_4]^{4-}$, and $Ir(CO)_3$, the total number of species isoelectronic with $Ni(CO)_4$ is_______.

[Given, atomic number : V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

**Ans. (3)**

**Sol.** In case of complexes, isoelectronic species should be those having same effective atomic number (EAN)

$Ni(CO)_4 \Rightarrow 28 + 4 \times 2 = 36$

(i) $V(CO)_6 \Rightarrow 23 + 2 \times 6 = 35$

(ii) $Cr(CO)_5 \Rightarrow 24 + 2 \times 5 = 34$

(iii) $Cu(CO)_3 \Rightarrow 29 + 2 \times 3 = 35$

(iv) $Mn(CO)_5 \Rightarrow 25 + 2 \times 5 = 35$

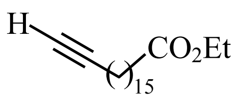
(v) $Fe(CO)_5 \Rightarrow 26 + 2 \times 5 = 36$

(vi) $[Co(CO)_3]^{3-} \Rightarrow 27 + 3 + 2 \times 3 = 36$

(vii) $[Cr(CO)_4]^{4-} \Rightarrow 24 + 4 + 2 \times 4 = 36$

(viii) $[Ir(CO)_3] \Rightarrow 77 + 2 \times 3 = 83$

**12.** In the following reaction sequence, the major product **P** is formed.



$$\xrightarrow[\text{iii) } H_3O^+,\ \Delta]{\text{i) } Hg^{2}\ ,\ H_3O^+ \quad \text{ii) Zn–Hg/HCl}} \mathbf{P}$$

Glycerol reacts completely with excess **P** in the presence of an acid catalyst to form **Q**. Reaction of **Q** with excess NaOH followed by the treatment with $CaCl_2$ yields Ca-soap **R**, quantitatively.

Starting with one mole of **Q**, the amount of **R** produced in gram is_______.

[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

**Ans. (909)**

**Sol.**

$$H-C\equiv C-(CH_2)_{15}-CO_2Et$$

$$\Big\downarrow Hg^{2+} / H_3O^+$$

$$H_3C-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_{15}-CO_2Et$$

$$\Big\downarrow \text{Zn(Hg) + Conc. HCl}$$

$$H_3C-CH_2-(CH_2)_{16}-CO_2Et$$

$$\Big\downarrow H_3O^+$$

$$H_3C-(CH_2)_{16}-CO_2H$$

$$\Big\downarrow \text{Glycerol} / H^+$$

Glyceryl stearate

$$\begin{array}{l} CH_2-O-\overset{\overset{\displaystyle O}{\|}}{C}-C_{17}H_{35} \\ \ | \\ CH-O-\underset{\underset{\displaystyle O}{\|}}{C}-C_{17}H_{35} \\ \ | \\ CH_2-O-\underset{\underset{\displaystyle O}{\|}}{C}-C_{17}H_{35} \end{array} \quad \text{(Q)} \quad \text{(1 mole)}$$

$$\Big\downarrow \text{NaOH (excess)}$$

$$\begin{array}{l} CH_2OH \\ \ | \\ CH_2OH \\ \ | \\ CH_2OH \end{array} + \underset{\text{(3 mole)}}{3C_{17}H_{35}CO_2Na}$$

$$\Big\downarrow CaCl_2$$

$$\underset{3/2 \text{ mole}}{(C_{17}H_{35}CO_2)_2Ca} \quad \text{(Soap)}$$

$$\frac{3}{2} \text{ mole soap.} = \frac{3}{2} \times 606 \text{ gm} = 909 \text{ gm}$$

**13.** Among the following complexes, the total number of diamagnetic species is________.

$[Mn(NH_3)_6]^{3+}$, $[MnCl_6]^{3-}$, $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Fe(NH_3)_6]^{3+}$, and $[Co(en)_3]^{3+}$

[Given, atomic number : Mn = 25, Fe = 26, Co = 27;

en = $H_2NCH_2CH_2NH_2$]

**Ans. (1)**

**Sol.** $Mn^{3+} \Rightarrow [Ar]3d^4$

$d^4$ configuration in $t_{2g}$ and $e_g$ orbitals will always have unpaired electrons irrespective of SFL and WFL.

$Fe^{3+} \Rightarrow [Ar]3d^5$

$d^5$ configuration will also have unpaired electron irrespective of SFL and WFL.

$Co^{3+} \Rightarrow [Ar]3d^6$

$d^6 \Rightarrow$ it can be both paramagnetic or diamagnetic based on field of ligands.

In case of $F^- \Rightarrow$ weak field ligand, configuration will be $t_{2g}e_g^2$ hence it is paramagnetic but in case of en $\Rightarrow$ strong filed ligand, configuration will be $t_{2g}^6e_g$ hence it will be diamagnetic.

## SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 **ONLY** if the option corresponding to the correct combination is chosen;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : –1 In all other cases.

**14.** In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.

The limiting ionic conductivity $(\Lambda_0)$ values (in mS $m^2$ $mol^{-1}$) for different ions in aqueous solutions are given below :

| Ions | $Ag^+$ | $K^+$ | $Na^+$ | $H^+$ | $NO_3^-$ | $Cl^-$ | $SO_4^{2-}$ | $OH^-$ | $CH_3COO^-$ |
|---|---|---|---|---|---|---|---|---|---|
| $\Lambda_0$ | 6.2 | 7.4 | 5.0 | 35.0 | 7.2 | 7.6 | 16.0 | 19.9 | 4.1 |

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.

Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

**List-I**

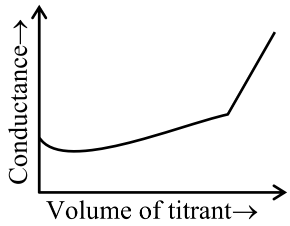
(P) Titrate : KCl
Titrant : $AgNO_3$

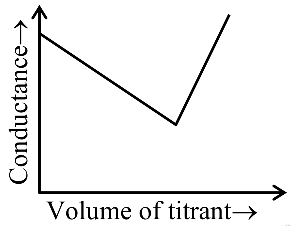
(Q) Titrate : $AgNO_3$
Titrant : KCl

(R) Titrate : NaOH
Titrant : HCl

(S) Titrate : NaOH
Titrant : $CH_3COOH$

**List-II**

(1)



(2)
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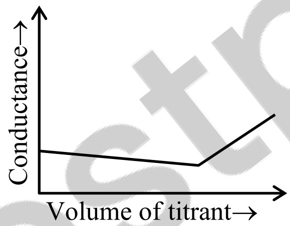
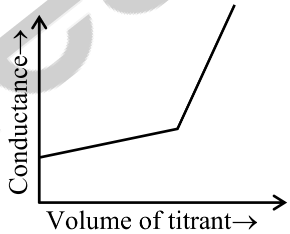
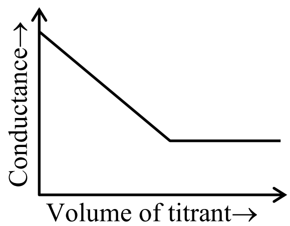



(4)



(5)



(A) P → 4, Q → 3, R → 2, S → 5
(B) P → 2, Q → 4, R → 3, S → 1
(C) P → 3, Q → 4, R → 2, S → 5
(D) P → 4, Q → 3, R → 2, S → 1

**Ans. (C)**

**Sol. Option (P) :**

On adding $AgNO_3$ solution to KCl solution precipitation of AgCl will occur due to which $Cl^-$ already present will be replaced by $NO_3^-$ ions. So conductance of solution will decrease till equivalence point. After complete precipitation of AgCl, further added $AgNO_3$ will increase the number of ions in resulting solution so conductance will increase.

**Option (Q) :**

On adding KCl solution to $AgNO_3$ solution precipitation of AgCl will occur due to which already present $Ag^+$ ions will be replaced by $K^+$ ions in solution. So conductance of solution will increase. After complete precipitation of AgCl further added KCl will increase the number of ions in resulting solution so conductance will increase further.

**Option (R) :**

On adding HCl solution to NaOH solution, $OH^-$ will be replaced by $Cl^-$ ions so conductance of solution decreases. After complete neutralisation further added HCl will increase number of ions in the solution. So conductance will increase futher.

**Option (S) :**

On adding $CH_3COOH$ solution to NaOH solution $OH^-$ will be replaced by $CH_3COO^-$ ions, so conductance of solution decreases. After complete neutralisation further added $CH_3COOH$ will remain undissociated because it is a weak acid and there is also common ion effect on acetate ions. So number of ions in solution will remain almost constant therefore conductance of solution will remain constant.

**15.** Based on **VSEPR** model, match the xenon compounds given in **List-I** with the corresponding geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

| | **List-I** | | **List-II** |
|---|---|---|---|
| (P) | $XeF_2$ | (1) | Trigonal bipyramidal and two lone pair of electrons |
| (Q) | $XeF_4$ | (2) | Tetrahedral and one lone pair of electrons |
| (R) | $XeO_3$ | (3) | Octahedral and two lone pair of electrons |
| (S) | $XeO_3F_2$ | (4) | Trigonal bipyramidal and no lone pair of electrons |
| | | (5) | Trigonal bipyramidal and three lone pair of electrons |

(A) $P \rightarrow 5, Q \rightarrow 2, R \rightarrow 3, S \rightarrow 1$

(B) $P \rightarrow 5, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 4$

(C) $P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$

(D) $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 3$

**Ans. (B)**

**Sol.** $XeF_2 \Rightarrow$ 2 sigma bonds and 3 lone pairs on Xe, number of hybrid orbitals = 5, $sp^3d$ hybridisation , geometry will be trigonal bipyramidal.

**P–5**

$XeF_4 \Rightarrow$ 4 sigma bonds and 2 lone pairs on Xe, number of hybrid orbitals = 6, $sp^3d^2$ hybridisation , geometry will be octahedral.

**Q–3**

$XeO_3 \Rightarrow$ 3 sigma bonds and 1 lone pairs on Xe, number of hybrid orbitals = 4, $sp^3$ hybridisation , geometry will be tetrahedral.

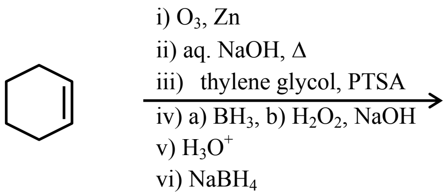
**R–2**

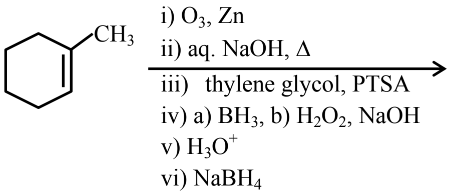
$XeO_3F_2 \Rightarrow$ 5 sigma bonds and 0 lone pairs on Xe, number of hybrid orbitals = 5, $sp^3d$ hybridisation , geometry will be trigonal bipyramidal.

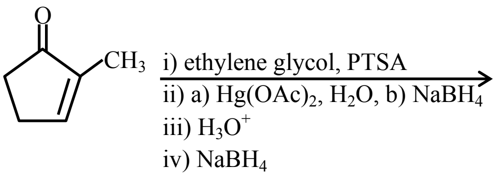
**S–4**

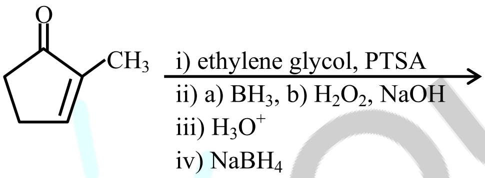
**16.** **List-I** contains various reaction sequences and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

**List-I**

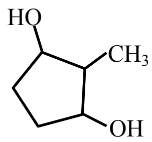
(P) 

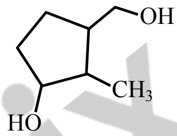

(Q) 

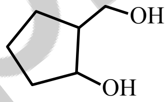

(R) 

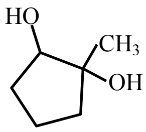

(S) 

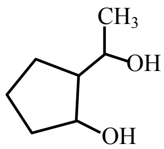

**List-II**

(1) 


(2) 


(3) 


(4) 


(5) 


(A) P → 3, Q → 5, R → 4, S → 1

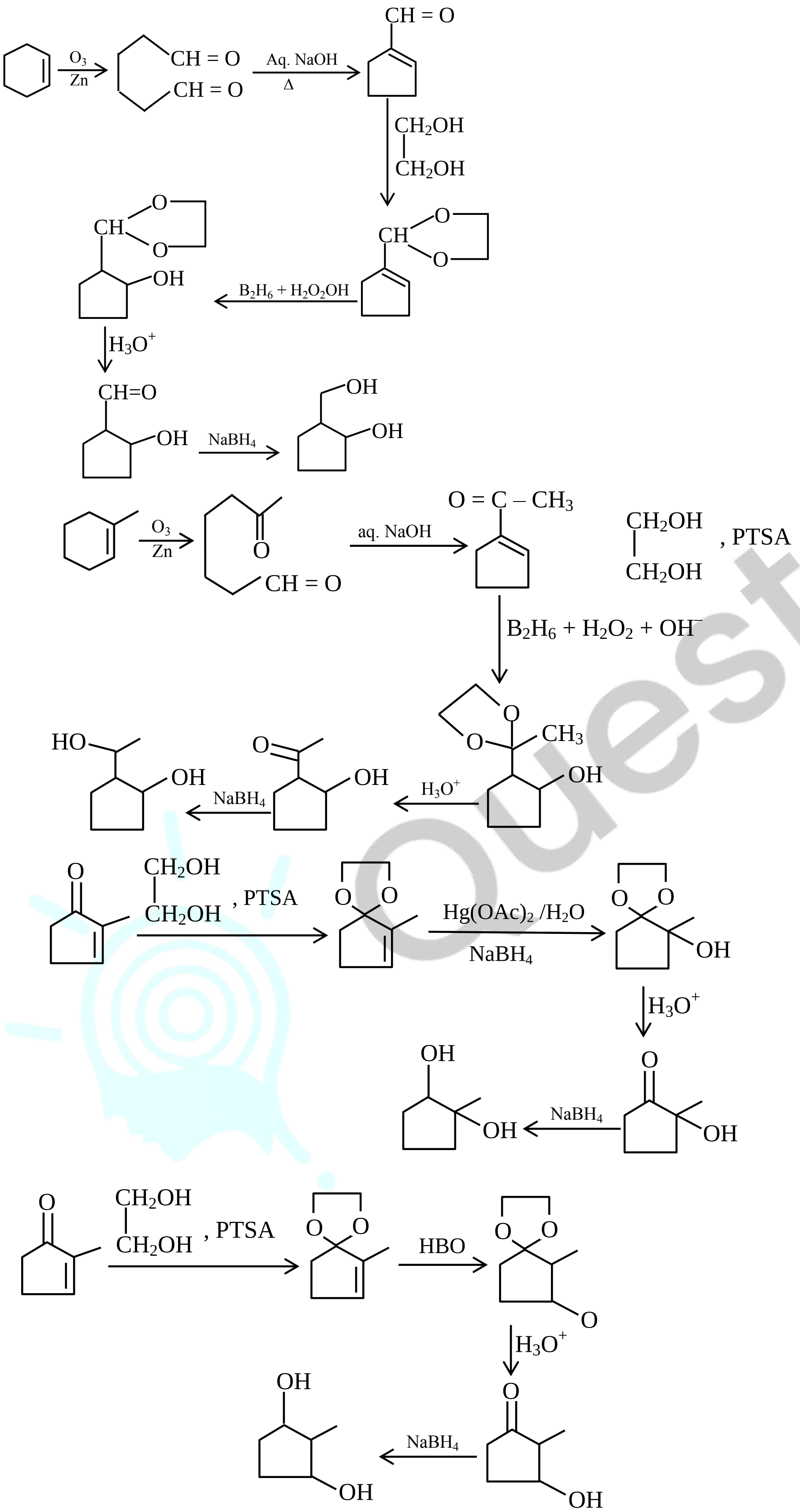
(B) P → 3, Q → 2, R → 4, S → 1

(C) P → 3, Q → 5, R → 1, S → 4

(D) P → 5, Q → 2, R → 4, S → 1

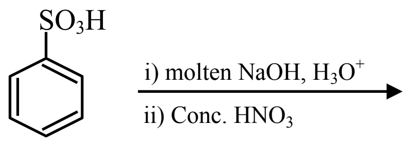
**Ans. (A)**

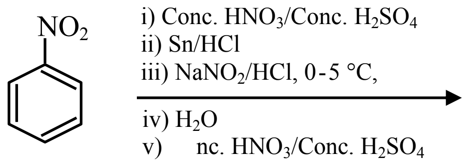
**Sol.**

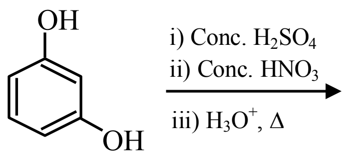

O3 / Zn
CH = O
CH = O
Aq. NaOH / Δ
CH = O
CH2OH
CH2OH
CH
O
O
B2H6 + H2O2OH
OH
H3O+
CH=O
NaBH4
OH
O3 / Zn
O
CH = O
aq. NaOH
O = C – CH3
CH2OH
CH2OH
, PTSA
B2H6 + H2O2 + OH–
CH3
HO
Hg(OAc)2 /H2O
NaBH4
HBO

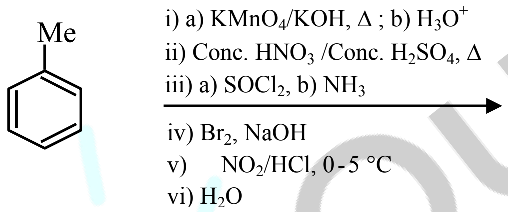
**17.** **List-I** contains various reaction sequences and **List-II** contains different phenolic compounds. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

**List-I**

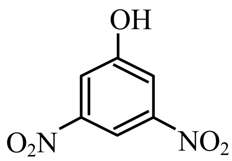
(P) 

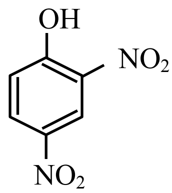

(Q) 

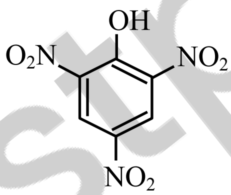

(R) 

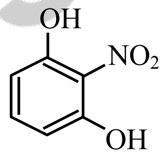

(S) 

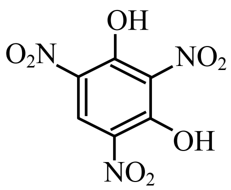

**List-II**

(1) 


(2) 

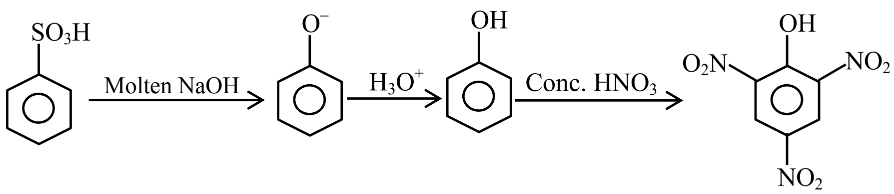

(3) 

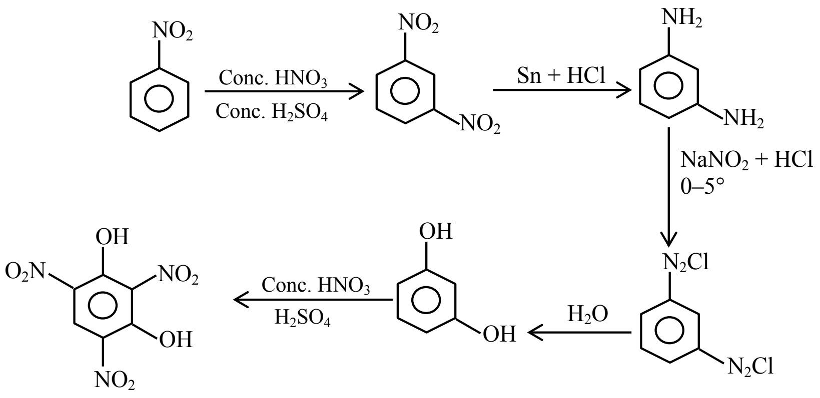

(4) 

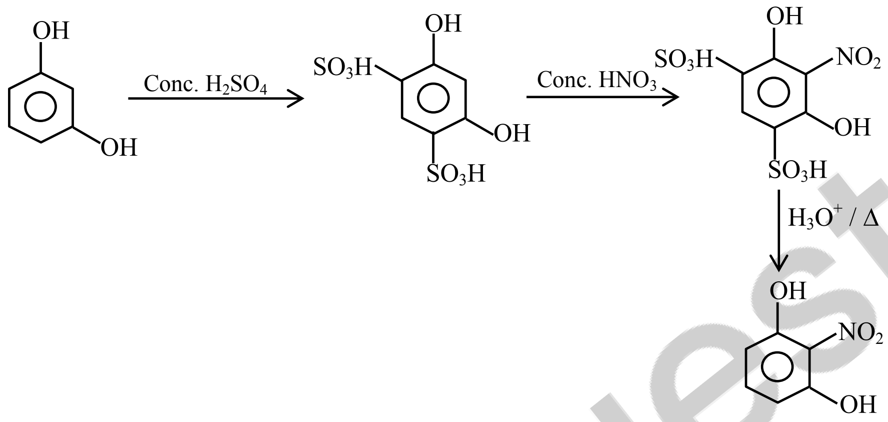

(5) 

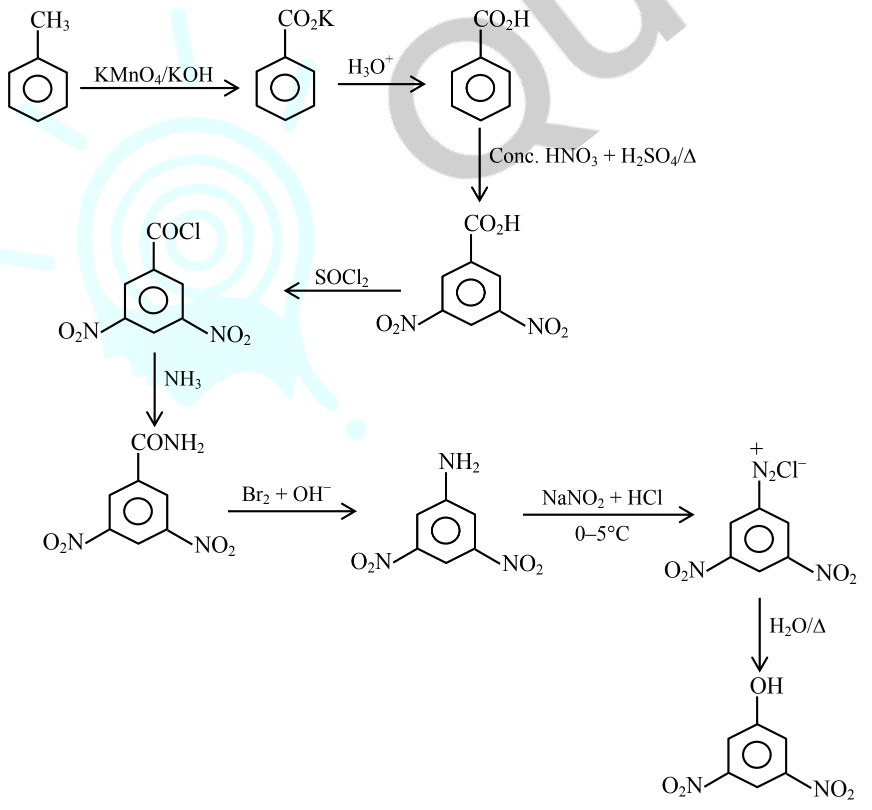

(A) P-2, Q-3, R-4, S-5
(B) P-2, Q-3, R-5, S-1
(C) P-3, Q-5, R-4, S-1
(D) P-3, Q-2, R-5, S-4

**Ans.** **(C)**

**Sol.** 


NO2
Conc. HNO3
Conc. H2SO4
NO2
NO2
Sn + HCl
NH2
NH2
NaNO2 + HCl
0–5°
N2Cl
N2Cl
H2O
OH
OH
Conc. HNO3
H2SO4
OH
O2N
NO2
OH
NO2


OH
OH
Conc. H2SO4
OH
SO3H
OH
SO3H
Conc. HNO3
OH
SO3H
NO2
OH
SO3H
H3O+ / Δ
OH
NO2
OH


CH3
KMnO4/KOH
CO2K
H3O+
CO2H
Conc. HNO3 + H2SO4/Δ
CO2H
O2N
NO2
SOCl2
COCl
O2N
NO2
NH3
CONH2
O2N
NO2
Br2 + OH−
NH2
O2N
NO2
NaNO2 + HCl
0–5°C
N2Cl−
O2N
NO2
H2O/Δ
OH
O2N
NO2