

# PART I : PHYSICS

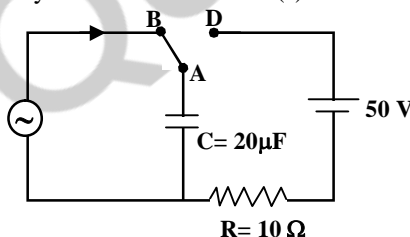
## SECTION – 1 (One or More Than One Options Correct Type)

This section contains **10 multiple choice type questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE** or **MORE THAN ONE** are correct.

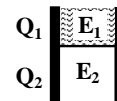
- \*1. A student is performing an experiment using a resonance column and a tuning fork of frequency  $244 \text{ s}^{-1}$ . He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is  $(0.350 \pm 0.005)\text{m}$ , the gas in the tube is (Useful information:  $\sqrt{167RT} = 640 \text{ J}^{1/2} \text{ mole}^{-1/2}$ ;  $\sqrt{140RT} = 590 \text{ J}^{1/2} \text{ mole}^{-1/2}$ . The molar masses  $M$  in grams are given in the options. Take the values of  $\sqrt{\frac{10}{M}}$  for each gas as given there.)

- (A) Neon  $\left( M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10} \right)$  (B) Nitrogen  $\left( M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5} \right)$   
 (C) Oxygen  $\left( M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16} \right)$  (D) Argon  $\left( M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32} \right)$

2. At time  $t = 0$ , terminal A in the circuit shown in the figure is connected to B by a key and an alternating current  $I(t) = I_0 \cos(\omega t)$ , with  $I_0 = 1\text{A}$  and  $\omega = 500 \text{ rad/s}$  starts flowing in it with the initial direction shown in the figure. At  $t = \frac{7\pi}{6\omega}$ , the key is switched from B to D. Now onwards only A and D are connected. A total charge  $Q$  flows from the battery to charge the capacitor fully. If  $C = 20 \mu\text{F}$ ,  $R = 10 \Omega$  and the battery is ideal with emf of  $50 \text{ V}$ , identify the correct statement(s).



- (A) Magnitude of the maximum charge on the capacitor before  $t = \frac{7\pi}{6\omega}$  is  $1 \times 10^{-3}\text{C}$ .  
 (B) The current in the left part of the circuit just before  $t = \frac{7\pi}{6\omega}$  is clockwise.  
 (C) Immediately after A is connected to D, the current in R is  $10 \text{ A}$ .  
 (D)  $Q = 2 \times 10^{-3}\text{C}$ .
3. A parallel plate capacitor has a dielectric slab of dielectric constant  $K$  between its plates that covers  $1/3$  of the area of its plates, as shown in the figure. The total capacitance of the capacitor is  $C$  while that of the portion with dielectric in between is  $C_1$ . When the capacitor is charged, the plate area covered by the dielectric gets charge  $Q_1$  and the rest of the area gets charge  $Q_2$ . The electric field in the dielectric is  $E_1$  and that in the other portion is  $E_2$ . Choose the correct option/options, ignoring edge effects.

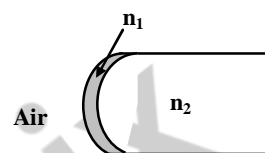


- (A)  $\frac{E_1}{E_2} = 1$  (B)  $\frac{E_1}{E_2} = \frac{1}{K}$   
 (C)  $\frac{Q_1}{Q_2} = \frac{3}{K}$  (D)  $\frac{C}{C_1} = \frac{2+K}{K}$

- \*4. One end of a taut string of length 3m along the x axis is fixed at  $x = 0$ . The speed of the waves in the string is  $100 \text{ ms}^{-1}$ . The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

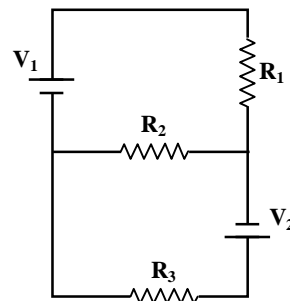
(A)  $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$  (B)  $y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$   
 (C)  $y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$  (D)  $y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$

5. A transparent thin film of uniform thickness and refractive index  $n_1 = 1.4$  is coated on the convex spherical surface of radius  $R$  at one end of a long solid glass cylinder of refractive index  $n_2 = 1.5$ , as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance  $f_1$  from the film, while rays of light traversing from glass to air get focused at distance  $f_2$  from the film. Then



- (A)  $|f_1| = 3R$  (B)  $|f_1| = 2.8 R$   
 (C)  $|f_2| = 2R$  (D)  $|f_2| = 1.4 R$
6. Heater of an electric kettle is made of a wire of length  $L$  and diameter  $d$ . It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length  $L$  and diameter  $2d$ . The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?
- (A) 4 if wires are in parallel (B) 2 if wires are in series  
 (C) 1 if wires are in series (D) 0.5 if wires are in parallel.

7. Two ideal batteries of emf  $V_1$  and  $V_2$  and three resistances  $R_1$ ,  $R_2$  and  $R_3$  are connected as shown in the figure. The current in resistance  $R_2$  would be zero if



- (A)  $V_1 = V_2$  and  $R_1 = R_2 = R_3$   
 (B)  $V_1 = V_2$  and  $R_1 = 2R_2 = R_3$   
 (C)  $V_1 = 2V_2$  and  $2R_1 = 2R_2 = R_3$   
 (D)  $2V_1 = V_2$  and  $2R_1 = R_2 = R_3$
8. Let  $E_1(r)$ ,  $E_2(r)$  and  $E_3(r)$  be the respective electric fields at a distance  $r$  from a point charge  $Q$ , an infinitely long wire with constant linear charge density  $\lambda$ , and an infinite plane with uniform surface charge density  $\sigma$ . If  $E_1(r_0) = E_2(r_0) = E_3(r_0)$  at a given distance  $r_0$ , then

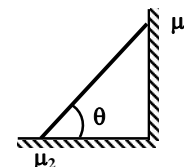
(A)  $Q = 4\sigma\pi r_0^2$  (B)  $r_0 = \frac{\lambda}{2\pi\sigma}$   
 (C)  $E_1(r_0/2) = 2E_2(r_0/2)$  (D)  $E_2(r_0/2) = 4E_3(r_0/2)$

9. A light source, which emits two wavelengths  $\lambda_1 = 400 \text{ nm}$  and  $\lambda_2 = 600 \text{ nm}$ , is used in a Young's double slit experiment. If recorded fringe widths for  $\lambda_1$  and  $\lambda_2$  are  $\beta_1$  and  $\beta_2$  and the number of fringes for them within a distance  $y$  on one side of the central maximum are  $m_1$  and  $m_2$ , respectively, then

- (A)  $\beta_2 > \beta_1$   
 (B)  $m_1 > m_2$   
 (C) From the central maximum, 3<sup>rd</sup> maximum of  $\lambda_2$  overlaps with 5<sup>th</sup> minimum of  $\lambda_1$   
 (D) The angular separation of fringes for  $\lambda_1$  is greater than  $\lambda_2$

- \*10. In the figure, a ladder of mass  $m$  is shown leaning against a wall. It is in static equilibrium making an angle  $\theta$  with the horizontal floor. The coefficient of friction between the wall and the ladder is  $\mu_1$  and that between the floor and the ladder is  $\mu_2$ . The normal reaction of the wall on the ladder is  $N_1$  and that of the floor is  $N_2$ . If the ladder is about to slip, then

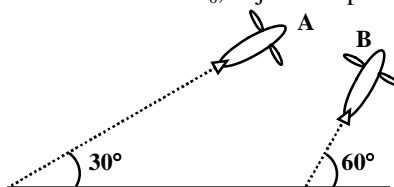
- (A)  $\mu_1 = 0$   $\mu_2 \neq 0$  and  $N_2 \tan \theta = \frac{mg}{2}$   
 (B)  $\mu_1 \neq 0$   $\mu_2 = 0$  and  $N_1 \tan \theta = \frac{mg}{2}$   
 (C)  $\mu_1 \neq 0$   $\mu_2 \neq 0$  and  $N_2 = \frac{mg}{1 + \mu_1 \mu_2}$   
 (D)  $\mu_1 = 0$   $\mu_2 \neq 0$  and  $N_1 \tan \theta = \frac{mg}{2}$



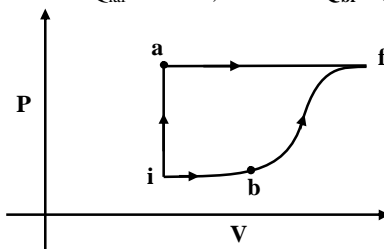
## SECTION – 2: (Only Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. During Searle's experiment, zero of the Vernier scale lies between  $3.20 \times 10^{-2}$  m and  $3.25 \times 10^{-2}$  m of the main scale. The 20<sup>th</sup> division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between  $3.20 \times 10^{-2}$  m and  $3.25 \times 10^{-2}$  m of the main scale but now the 45<sup>th</sup> division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is  $8 \times 10^{-7}$  m<sup>2</sup>. The least count of the Vernier scale is  $1.0 \times 10^{-5}$  m. The maximum percentage error in the Young's modulus of the wire is
- \*12. Airplanes A and B are flying with constant velocity in the same vertical plane at angles  $30^\circ$  and  $60^\circ$  with respect to the horizontal respectively as shown in the figure. The speed of A is  $100\sqrt{3}$  ms<sup>-1</sup>. At time  $t = 0$  s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at  $t = t_0$ , A just escapes being hit by B,  $t_0$  in seconds is



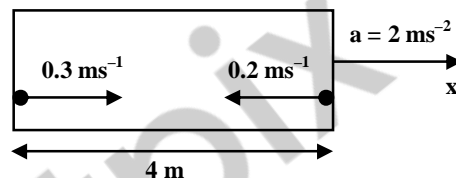
- \*13. A thermodynamic system is taken from an initial state  $i$  with internal energy  $U_i = 100$  J to the final state  $f$  along two different paths  $iaf$  and  $ibf$ , as schematically shown in the figure. The work done by the system along the paths  $af$ ,  $ib$  and  $bf$  are  $W_{af} = 200$  J,  $W_{ib} = 50$  J and  $W_{bf} = 100$  J respectively. The heat supplied to the system along the path  $iaf$ ,  $ib$  and  $bf$  are  $Q_{iaf}$ ,  $Q_{ib}$  and  $Q_{bf}$  respectively. If the internal energy of the system in the state  $b$  is  $U_b = 200$  J and  $Q_{iaf} = 500$  J, the ratio  $Q_{bf}/Q_{ib}$  is



14. Two parallel wires in the plane of the paper are distance  $X_0$  apart. A point charge is moving with speed  $u$  between the wires in the same plane at a distance  $X_1$  from one of the wires. When the wires carry current of magnitude  $I$  in the same direction, the radius of curvature of the path of the point charge is  $R_1$ . In contrast, if the currents  $I$  in the two wires have directions opposite to each other, the radius of curvature of the path is  $R_2$ . If  $\frac{X_0}{X_1} = 3$ , the value of  $\frac{R_1}{R_2}$  is

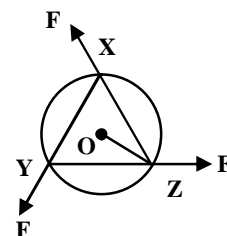
15. To find the distance  $d$  over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density  $\rho$  of the fog, intensity (power/area)  $S$  of the light from the signal and its frequency  $f$ . The engineer finds that  $d$  is proportional to  $S^{1/n}$ . The value of  $n$  is

- \*16. A rocket is moving in a gravity free space with a constant acceleration of  $2 \text{ ms}^{-2}$  along  $+x$  direction (see figure). The length of a chamber inside the rocket is  $4 \text{ m}$ . A ball is thrown from the left end of the chamber in  $+x$  direction with a speed of  $0.3 \text{ ms}^{-1}$  relative to the rocket. At the same time, another ball is thrown in  $-x$  direction with a speed of  $0.2 \text{ ms}^{-1}$  from its right end relative to the rocket. The time in seconds when the two balls hit each other is

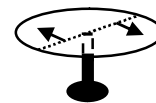


17. A galvanometer gives full scale deflection with  $0.006 \text{ A}$  current. By connecting it to a  $4990 \Omega$  resistance, it can be converted into a voltmeter of range  $0 - 30 \text{ V}$ . If connected to a  $\frac{2n}{249} \Omega$  resistance, it becomes an ammeter of range  $0 - 1.5 \text{ A}$ . The value of  $n$  is

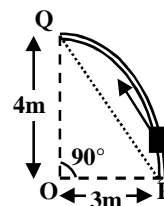
- \*18. A uniform circular disc of mass  $1.5 \text{ kg}$  and radius  $0.5 \text{ m}$  is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude  $F = 0.5 \text{ N}$  are applied simultaneously along the three sides of an equilateral triangle  $XYZ$  with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in  $\text{rad s}^{-1}$  is



- \*19. A horizontal circular platform of radius  $0.5 \text{ m}$  and mass  $0.45 \text{ kg}$  is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass  $0.05 \text{ kg}$  are attached to the platform at a distance  $0.25 \text{ m}$  from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of  $9 \text{ ms}^{-1}$  with respect to the ground. The rotational speed of the platform in  $\text{rad s}^{-1}$  after the balls leave the platform is



- \*20. Consider an elliptically shaped rail  $PQ$  in the vertical plane with  $OP = 3 \text{ m}$  and  $OQ = 4 \text{ m}$ . A block of mass  $1 \text{ kg}$  is pulled along the rail from  $P$  to  $Q$  with a force of  $18 \text{ N}$ , which is always parallel to line  $PQ$  (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches  $Q$  is  $(n \times 10)$  Joules. The value of  $n$  is (take acceleration due to gravity  $= 10 \text{ ms}^{-2}$ )

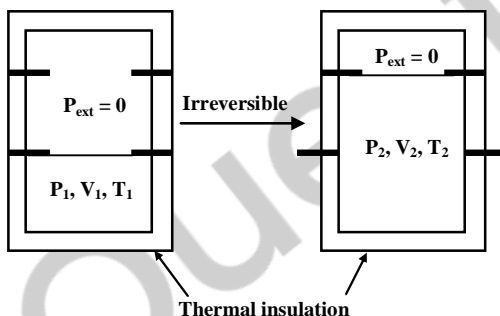


## PART - II: CHEMISTRY

### SECTION – 1 (One or More Than One Options Correct Type)

This section contains **10 multiple choice type questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

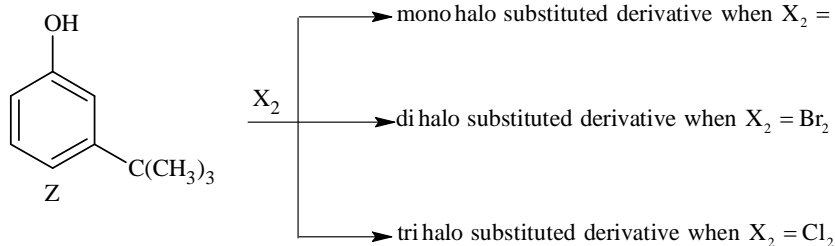
- \*21. The correct combination of names for isomeric alcohols with molecular formula  $C_4H_{10}O$  is/are  
 (A) tert-butanol and 2-methylpropan-2-ol  
 (B) tert-butanol and 1, 1-dimethylethan-1-ol  
 (C) n-butanol and butan-1-ol  
 (D) isobutyl alcohol and 2-methylpropan-1-ol
- \*22. An ideal gas in a thermally insulated vessel at internal pressure =  $P_1$ , volume =  $V_1$  and absolute temperature =  $T_1$  expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are  $P_2$ ,  $V_2$  and  $T_2$ , respectively. For this expansion,



- (A)  $q = 0$   
 (B)  $T_2 = T_1$   
 (C)  $P_2 V_2 = P_1 V_1$   
 (D)  $P_2 V_2^\gamma = P_1 V_1^\gamma$
- \*23. Hydrogen bonding plays a central role in the following phenomena:  
 (A) Ice floats in water.  
 (B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions.  
 (C) Formic acid is more acidic than acetic acid.  
 (D) Dimerisation of acetic acid in benzene.
24. In a galvanic cell, the salt bridge  
 (A) does not participate chemically in the cell reaction.  
 (B) stops the diffusion of ions from one electrode to another.  
 (C) is necessary for the occurrence of the cell reaction.  
 (D) ensures mixing of the two electrolytic solutions.
- \*25. For the reaction:  

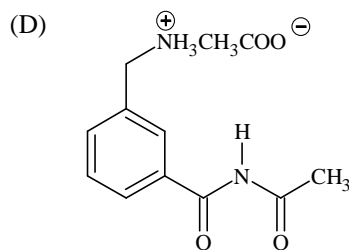
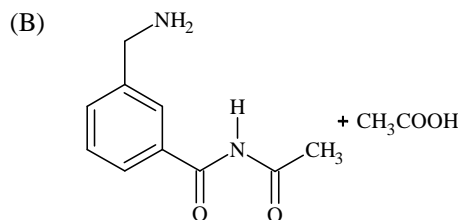
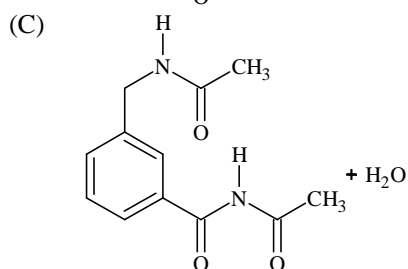
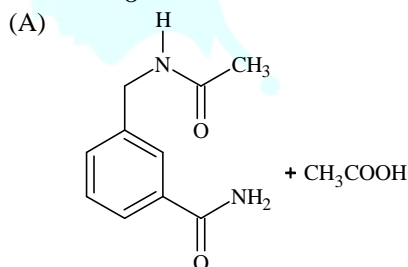
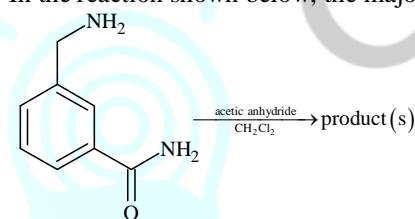
$$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$$
  
 The correct statement(s) in the balanced equation is/are:  
 (A) Stoichiometric coefficient of  $HSO_4^-$  is 6.  
 (B) Iodide is oxidized.  
 (C) Sulphur is reduced.  
 (D)  $H_2O$  is one of the products.

- \*26. The reactivity of compound Z with different halogens under appropriate conditions is given below:



The observed pattern of electrophilic substitution can be explained by

- (A) the steric effect of the halogen  
 (B) the steric effect of the tert-butyl group  
 (C) the electronic effect of the phenolic group  
 (D) the electronic effect of the tert-butyl group
- \*27. The correct statement(s) for orthoboric acid is/are  
 (A) It behaves as a weak acid in water due to self ionization.  
 (B) Acidity of its aqueous solution increases upon addition of ethylene glycol.  
 (C) It has a three dimensional structure due to hydrogen bonding.  
 (D) It is a weak electrolyte in water.
28. Upon heating with  $Cu_2S$ , the reagent(s) that give copper metal is/are  
 (A)  $CuFeS_2$  (B)  $CuO$   
 (C)  $Cu_2O$  (D)  $CuSO_4$
- \*29. The pair(s) of reagents that yield paramagnetic species is/are  
 (A) Na and excess of  $NH_3$  (B) K and excess of  $O_2$   
 (C) Cu and dilute  $HNO_3$  (D)  $O_2$  and 2- ethylantraquinol
30. In the reaction shown below, the major product(s) formed is/are

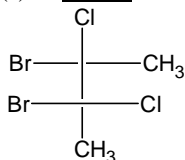


**SECTION – 2: (Only Integer Value Correct Type)**

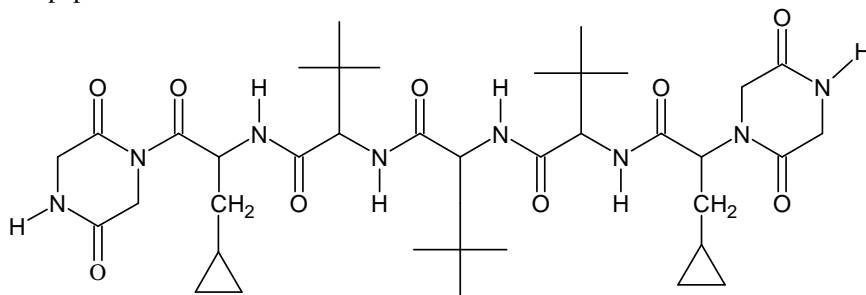
This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

31. Among PbS, CuS, HgS, MnS, Ag<sub>2</sub>S, NiS, CoS, Bi<sub>2</sub>S<sub>3</sub> and SnS<sub>2</sub>, the total number of BLACK coloured sulphides is

- \*32. The total number(s) of stable conformers with non-zero dipole moment for the following compound is(are)



33. Consider the following list of reagents:  
Acidified K<sub>2</sub>Cr<sub>2</sub>O<sub>7</sub>, alkaline KMnO<sub>4</sub>, CuSO<sub>4</sub>, H<sub>2</sub>O<sub>2</sub>, Cl<sub>2</sub>, O<sub>3</sub>, FeCl<sub>3</sub>, HNO<sub>3</sub> and Na<sub>2</sub>S<sub>2</sub>O<sub>3</sub>.  
The total number of reagents that can oxidise aqueous iodide to iodine is
34. A list of species having the formula XZ<sub>4</sub> is given below.  
XeF<sub>4</sub>, SF<sub>4</sub>, SiF<sub>4</sub>, BF<sub>4</sub><sup>-</sup>, BrF<sub>4</sub><sup>-</sup>, [Cu(NH<sub>3</sub>)<sub>4</sub>]<sup>2+</sup>, [FeCl<sub>4</sub>]<sup>2-</sup>, [CoCl<sub>4</sub>]<sup>2-</sup> and [PtCl<sub>4</sub>]<sup>2-</sup>.  
Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is
35. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with NaBH<sub>4</sub> (**NOTE:** stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are
- \*36. In an atom, the total number of electrons having quantum numbers  $n = 4$ ,  $|m_l| = 1$  and  $m_s = -1/2$  is
- \*37. If the value of Avogadro number is  $6.023 \times 10^{23} \text{ mol}^{-1}$  and the value of Boltzmann constant is  $1.380 \times 10^{-23} \text{ JK}^{-1}$ , then the number of significant digits in the calculated value of the universal gas constant is
38. MX<sub>2</sub> dissociates in M<sup>2+</sup> and X<sup>-</sup> ions in an aqueous solution, with a degree of dissociation ( $\alpha$ ) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is
39. The total number of distinct naturally occurring amino acids obtained by complete acidic hydrolysis of the peptide shown below is



- \*40. A compound H<sub>2</sub>X with molar weight of 80g is dissolved in a solvent having density of 0.4 gml<sup>-1</sup>. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is



## PART - III: MATHEMATICS

### SECTION – 1 : (One or More than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE are correct**.

41. Let  $f: (0, \infty) \rightarrow \mathbb{R}$  be given by  $f(x) = \int_{1/x}^x e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$ , then  
 (A)  $f(x)$  is monotonically increasing on  $[1, \infty)$  (B)  $f(x)$  is monotonically decreasing on  $(0, 1)$   
 (C)  $f(x) + f\left(\frac{1}{x}\right) = 0$ , for all  $x \in (0, \infty)$  (D)  $f(2^x)$  is an odd function of  $x$  on  $\mathbb{R}$
42. Let  $a \in \mathbb{R}$  and let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^5 - 5x + a$ , then  
 (A)  $f(x)$  has three real roots if  $a > 4$  (B)  $f(x)$  has only one real roots if  $a > 4$   
 (C)  $f(x)$  has three real roots if  $a < -4$  (D)  $f(x)$  has three real roots if  $-4 < a < 4$
43. For every pair of continuous functions  $f, g: [0, 1] \rightarrow \mathbb{R}$  such that  $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$ , the correct statement(s) is(are)  
 (A)  $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0, 1]$   
 (B)  $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0, 1]$   
 (C)  $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$  for some  $c \in [0, 1]$   
 (D)  $(f(c))^2 = (g(c))^2$  for some  $c \in [0, 1]$
- \*44. A circle  $S$  passes through the point  $(0, 1)$  and is orthogonal to the circles  $(x-1)^2 + y^2 = 16$  and  $x^2 + y^2 = 1$ . Then  
 (A) radius of  $S$  is 8 (B) radius of  $S$  is 7  
 (C) centre of  $S$  is  $(-7, 1)$  (D) centre of  $S$  is  $(-8, 1)$
45. Let  $\vec{x}, \vec{y}$  and  $\vec{z}$  be three vectors each of magnitude  $\sqrt{2}$  and the angle between each pair of them is  $\frac{\pi}{3}$ . If  $\vec{a}$  is a non-zero vector perpendicular to  $\vec{x}$  and  $\vec{y} \times \vec{z}$  and  $\vec{b}$  is a non-zero vector perpendicular to  $\vec{y}$  and  $\vec{z} \times \vec{x}$ , then  
 (A)  $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$  (B)  $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$   
 (C)  $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$  (D)  $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{z} - \vec{y})$
46. From a point  $P(\lambda, \lambda, \lambda)$ , perpendiculars PQ and PR are drawn respectively on the lines  $y = x, z = 1$  and  $y = -x, z = -1$ . If P is such that  $\angle QPR$  is a right angle, then the possible value(s) of  $\lambda$  is(are)  
 (A)  $\sqrt{2}$  (B) 1  
 (C)  $-1$  (D)  $-\sqrt{2}$
47. Let  $M$  be a  $2 \times 2$  symmetric matrix with integer entries. Then  $M$  is invertible if  
 (A) the first column of  $M$  is the transpose of the second row of  $M$   
 (B) the second row of  $M$  is the transpose of the first column of  $M$   
 (C)  $M$  is a diagonal matrix with non-zero entries in the main diagonal  
 (D) the product of entries in the main diagonal of  $M$  is not the square of an integer



48. Let  $M$  and  $N$  be two  $3 \times 3$  matrices such that  $MN = NM$ . Further, if  $M \neq N^2$  and  $M^2 = N^4$ , then  
 (A) determinant of  $(M^2 + MN^2)$  is 0  
 (B) there is a  $3 \times 3$  non-zero matrix  $U$  such that  $(M^2 + MN^2)U$  is the zero matrix  
 (C) determinant of  $(M^2 + MN^2) \geq 1$   
 (D) for a  $3 \times 3$  matrix  $U$ , if  $(M^2 + MN^2)U$  equals the zero matrix then  $U$  is the zero matrix

49. Let  $f: [a, b] \rightarrow [1, \infty)$  be a continuous function and let  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t) dt & \text{if } a \leq x \leq b, \\ \int_a^b f(t) dt & \text{if } x > b \end{cases}$$

Then

- (A)  $g(x)$  is continuous but not differentiable at  $a$   
 (B)  $g(x)$  is differentiable on  $\mathbb{R}$   
 (C)  $g(x)$  is continuous but not differentiable at  $b$   
 (D)  $g(x)$  is continuous and differentiable at either  $a$  or  $b$  but not both
50. Let  $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$  be given by  $f(x) = (\log(\sec x + \tan x))^3$ . Then  
 (A)  $f(x)$  is an odd function  
 (B)  $f(x)$  is a one-one function  
 (C)  $f(x)$  is an onto function  
 (D)  $f(x)$  is an even function

### SECTION – 2 : (One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

- \*51. Let  $n_1 < n_2 < n_3 < n_4 < n_5$  be positive integers such that  $n_1 + n_2 + n_3 + n_4 + n_5 = 20$ . Then the number of such distinct arrangements  $(n_1, n_2, n_3, n_4, n_5)$  is \_\_\_\_\_
- \*52. Let  $n \geq 2$  be an integer. Take  $n$  distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of  $n$  is \_\_\_\_\_
53. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be respectively given by  $f(x) = |x| + 1$  and  $g(x) = x^2 + 1$ . Define  $h: \mathbb{R} \rightarrow \mathbb{R}$  by  

$$h(x) = \begin{cases} \max \{f(x), g(x)\} & \text{if } x \leq 0 \\ \min \{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$
  
 Then number of points at which  $h(x)$  is not differentiable is \_\_\_\_\_
- \*54. Let  $a, b, c$  be positive integers such that  $\frac{b}{a}$  is an integer. If  $a, b, c$  are in geometric progression and the arithmetic mean of  $a, b, c$  is  $b + 2$ , then the value of  $\frac{a^2 + a - 14}{a + 1}$  is \_\_\_\_\_
55. Let  $\vec{a}, \vec{b}$ , and  $\vec{c}$  be three non-coplanar unit vectors such that the angle between every pair of them is  $\frac{\pi}{3}$ .  
 If  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ , where  $p, q$  and  $r$  are scalars, then the value of  $\frac{p^2 + 2q^2 + r^2}{q^2}$  is \_\_\_\_\_



56. The slope of the tangent to the curve  $(y - x^5)^2 = x(1 + x^2)^2$  at the point  $(1, 3)$  is \_\_\_\_\_
57. The value of  $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1 - x^2)^5 \right\} dx$  is \_\_\_\_\_
58. The largest value of the non-negative integer  $a$  for which  $\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$  is \_\_\_\_\_
- \*59. Let  $f : [0, 4\pi] \rightarrow [0, \pi]$  be defined by  $f(x) = \cos^{-1}(\cos x)$ . The number of points  $x \in [0, 4\pi]$  satisfying the equation  $f(x) = \frac{10-x}{10}$  is \_\_\_\_\_
- \*60. For a point  $P$  in the plane, let  $d_1(P)$  and  $d_2(P)$  be the distances of the point  $P$  from the lines  $x - y = 0$  and  $x + y = 0$  respectively. The area of the region  $R$  consisting of all points  $P$  lying in the first quadrant of the plane and satisfying  $2 \leq d_1(P) + d_2(P) \leq 4$ , is \_\_\_\_\_

\*\*\*\*\*

# JEE(ADVANCED)-2014

## PAPER 1 CODE 5

### ANSWERS

#### PART - I: PHYSICS

- |                   |                 |                   |                   |
|-------------------|-----------------|-------------------|-------------------|
| 1. <b>D</b>       | 2. <b>C, D</b>  | 3. <b>A, D</b>    | 4. <b>A, C, D</b> |
| 5. <b>A, C</b>    | 6. <b>B, D</b>  | 7. <b>A, B, D</b> | 8. <b>C</b>       |
| 9. <b>A, B, C</b> | 10. <b>C, D</b> | 11. <b>4</b>      | 12. <b>5</b>      |
| 13. <b>2</b>      | 14. <b>3</b>    | 15. <b>3</b>      | 16. <b>2</b>      |
| 17. <b>5</b>      | 18. <b>2</b>    | 19. <b>4</b>      | 20. <b>5</b>      |

#### PART - II: CHEMISTRY

- |                    |                    |                    |                    |
|--------------------|--------------------|--------------------|--------------------|
| 21. <b>A, C, D</b> | 22. <b>A, B, C</b> | 23. <b>A, B, D</b> | 24. <b>A</b>       |
| 25. <b>A, B, D</b> | 26. <b>A, B, C</b> | 27. <b>B, D</b>    | 28. <b>B, C, D</b> |
| 29. <b>A, B, C</b> | 30. <b>A</b>       | 31. <b>6</b>       | 32. <b>3</b>       |
| 33. <b>7</b>       | 34. <b>4</b>       | 35. <b>5</b>       | 36. <b>6</b>       |
| 37. <b>4</b>       | 38. <b>2</b>       | 39. <b>1</b>       | 40. <b>8</b>       |

#### PART - III: MATHEMATICS

- |                    |                    |                 |                 |
|--------------------|--------------------|-----------------|-----------------|
| 41. <b>A, C, D</b> | 42. <b>B, D</b>    | 43. <b>A, D</b> | 44. <b>B, C</b> |
| 45. <b>A, B, C</b> | 46. <b>C</b>       | 47. <b>C, D</b> | 48. <b>A, B</b> |
| 49. <b>A, C</b>    | 50. <b>A, B, C</b> | 51. <b>7</b>    | 52. <b>5</b>    |
| 53. <b>3</b>       | 54. <b>4</b>       | 55. <b>4</b>    | 56. <b>8</b>    |
| 57. <b>2</b>       | 58. <b>2</b>       | 59. <b>3</b>    | 60. <b>6</b>    |

# HINTS AND SOLUTIONS

## PART - I: PHYSICS

1.  $\ell = \frac{1}{4\nu} \sqrt{\frac{\gamma RT}{M}}$

Calculations for  $\frac{1}{4\nu} \sqrt{\frac{\gamma RT}{M}}$  for gases mentioned in options A, B, C and D, work out to be 0.459 m, 0.363 m, 0.340 m & 0.348 m respectively. As  $\ell = (0.350 \pm 0.005)\text{m}$ ; Hence correct option is D.

2. As current leads voltage by  $\pi/2$  in the given circuit initially, then ac voltage can be represent as  $V = V_0 \sin \omega t$   
 $\therefore q = CV_0 \sin \omega t = Q \sin \omega t$   
 where,  $Q = 2 \times 10^{-3} \text{ C}$

- At  $t = 7\pi/6\omega$ ;  $I = -\frac{\sqrt{3}}{2} I_0$  and hence current is anticlockwise.

- Current 'i' immediately after  $t = \frac{7\pi}{6\omega}$  is

$$i = \frac{V_c + 50}{R} = 10 \text{ A}$$

- Charge flow =  $Q_{\text{final}} - Q_{(7\pi/6\omega)} = 2 \times 10^{-6} \text{ C}$   
 Hence C & D are correct options.

3. As  $E = V/d$   
 $E_1/E_2 = 1$  (both parts have common potential difference)  
 Assume  $C_0$  be the capacitance without dielectric for whole capacitor.

$$k \frac{C_0}{3} + \frac{2C_0}{3} = C$$

$$\frac{C}{C_1} = \frac{2+k}{k}$$

$$\frac{Q_1}{Q_2} = \frac{k}{2}$$

4. Taking  $y(t) = A f(x) g(t)$  & Applying the conditions:  
 1; here  $x = 3\text{m}$  is antinode &  $x = 0$  is node  
 2; possible frequencies are odd multiple of fundamental frequency.

$$\text{where, } v_{\text{fundamental}} = \frac{v}{4\ell} = \frac{25}{3} \text{ Hz}$$

The correct options are A, C, D.

5. For air to glass

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

$$\therefore f_1 = 3R$$

For glass to air.

$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$$\therefore f_2 = 2R$$

6.  $H = \frac{V^2}{R} 4 = \frac{V^2}{R/2} t_1 = \frac{V^2}{R/8} t_2$   
 $t_1 = 2 \text{ min.}$   
 $t_2 = 0.5 \text{ min.}$

7.  $V_1 = \frac{R_1(V_1 + V_2)}{R_1 + R_3} \Rightarrow V_1 R_3 = V_2 R_1$   
 $V_2 = \frac{R_3(V_1 + V_2)}{R_1 + R_3} \Rightarrow V_2 R_1 = V_2 R_3$

8.  $\frac{Q}{4\pi\epsilon_0 r_0^2} = \frac{\lambda}{2\pi\epsilon_0 r_0} = \frac{\sigma}{2\epsilon_0}$   
 $E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\epsilon_0 r_0^2}, E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\epsilon_0 r_0}, E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\epsilon_0}$   
 $\therefore E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$

9.  $\beta = \frac{D\lambda}{d}$   
 $\therefore \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$   
 Also  $m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$   
 Also  $3\left(\frac{D}{d}\right)(600 \text{ nm}) = (2 \times 5 - 1)\left(\frac{D}{2d}\right)400 \text{ nm}$

Angular width  $\theta = \frac{\lambda}{d}$

10. Condition of translational equilibrium

$$N_1 = \mu_2 N_2$$

$$N_2 + \mu_1 N_1 = Mg$$

Solving  $N_2 = \frac{mg}{1 + \mu_1 \mu_2}$

$$N_1 = \frac{\mu_2 mg}{1 + \mu_1 \mu_2}$$

Applying torque equation about corner (left) point on the floor

$$mg \frac{\ell}{2} \cos \theta = N_1 \ell \sin \theta + \mu_1 N_1 \ell \cos \theta$$

Solving  $\tan \theta = \frac{1 - \mu_1 \mu_2}{2\mu_2}$

11.  $Y = \frac{FL}{\ell A}$  since the experiment measures only change in the length of wire

$$\therefore \frac{\Delta Y}{Y} \times 100 = \frac{\Delta \ell}{\ell} \times 100$$

From the observation  $\ell_1 = \text{MSR} + 20 \text{ (LC)}$

$$\ell_2 = \text{MSR} + 45 \text{ (LC)}$$

$$\Rightarrow \text{change in lengths} = 25 \text{ (LC)}$$

and the maximum permissible error in elongation is one LC

$$\therefore \frac{\Delta Y}{Y} \times 100 = \frac{(\text{LC})}{25(\text{LC})} \times 100 = 4$$

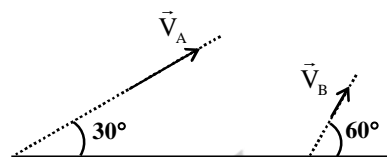
12. The relative velocity of B with respect to A is perpendicular to line of motion of A.

$$\therefore V_B \cos 30^\circ = V_A$$

$$\Rightarrow V_B = 200 \text{ m/s}$$

And time  $t_0 = (\text{Relative distance}) / (\text{Relative velocity})$

$$= \frac{500}{V_B \sin 30^\circ} = 5 \text{ sec}$$



13.  $U_b = 200 \text{ J}$ ,  $U_i = 100 \text{ J}$

Process iaf

Process	W(in Joule)	$\Delta U$ (in Joule)	Q(in Joule)
ia		0	
af		200	
Net	300	200	500

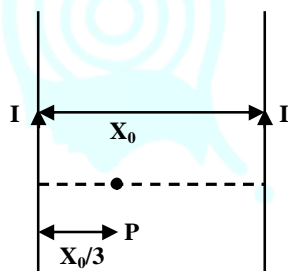
$$\Rightarrow U_f = 400 \text{ Joule}$$

Process ibf

Process	W(in Joule)	$\Delta U$ (in Joule)	Q(in Joule)
ib	100	50	150
bf	200	100	300
Net	300	150	450

$$\Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

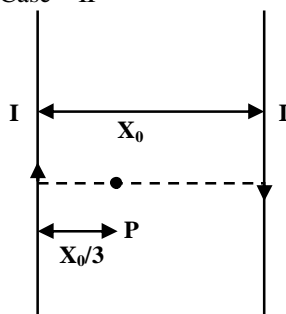
14. Case – I



$$B_1 = \frac{1}{2} \left( \frac{\mu_0}{2\pi} \right) \left( \frac{3I}{x_0} \right)$$

$$R_1 = \frac{mv}{qB_1}$$

Case – II



$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{1/3}{1/9} = 3$$

15.  $d \propto \rho^x S^y F^z$

$$\Rightarrow [L] = [ML^{-3}]^x [MT^{-3}]^y [T^{-1}]^z$$

$$\Rightarrow x + y = 0, -3x = 1, -3y - z = 0$$

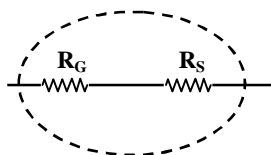
$$\Rightarrow x = \frac{-1}{3}, y = \frac{1}{3}, z = -1$$

$$\Rightarrow y = \frac{1}{n}$$

$$\Rightarrow n = 3$$

16. Maximum displacement of the left ball from the left wall of the chamber is 2.25 cm, so the right ball has to travel almost the whole length of the chamber (4m) to hit the left ball. So the time taken by the right ball is 1.9 sec (approximately 2 sec)

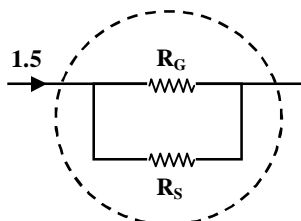
17.



$$i = \frac{V}{R}$$

$$0.006 = \frac{30}{4990 + R}$$

$$R = 10$$



$$i_{RG} = 0.006$$

$$i_{RS} = 1.494$$

Since  $R_G$  and  $R_S$  are in parallel,  $i_G R_G = i_S R_S$

$$0.006R = 1.494 \left( \frac{2n}{249} \right)$$

$$\therefore n = 5$$

18.

$$\tau = I\alpha$$

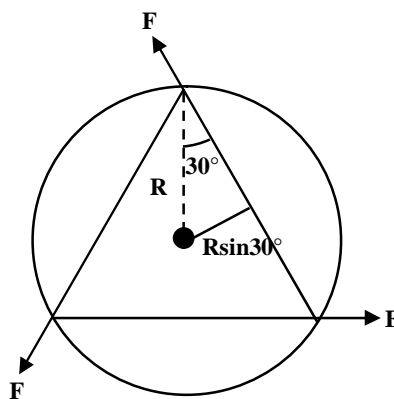
$$3FR\sin 30^\circ = I\alpha$$

$$I = \frac{MR^2}{2}$$

$$\alpha = 2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 2 \text{ rad/s}$$



19. Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$$L_i = L_f$$

$$0 = I\omega + 2mv(r/2); \text{ comparing magnitude}$$

$$\therefore \left( \frac{0.45 \times 0.5 \times 0.5}{2} \right) \omega = 0.05 \times 9 \times \frac{0.5}{2} \times 2$$

$$\therefore \omega = 4$$

20.

Using work energy theorem

$$W_{mg} + W_F = \Delta KE$$

$$-mgh + Fd = \Delta KE$$

$$-1 \times 10 \times 4 + 18(5) = \Delta KE$$

$$\Delta KE = 50$$

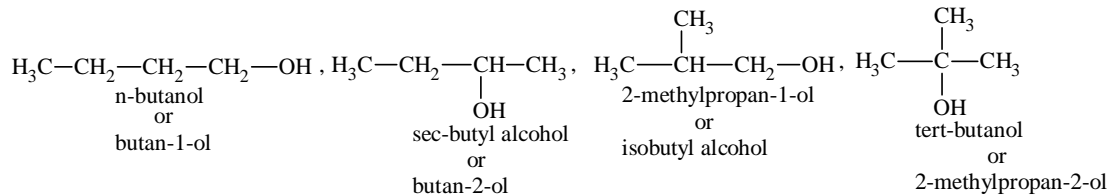
$$\therefore n = 5$$



## PART - II: CHEMISTRY

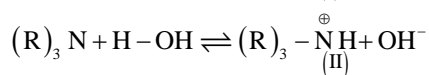
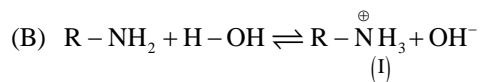
### SECTION – 1:

21. Isomeric alcohols of  $C_4H_{10}O$  are



22. Since container is thermally insulated. So,  $q = 0$ , and it is a case of free expansion therefore  $W = 0$  and  $\Delta E = 0$   
 So,  $T_1 = T_2$   
 Also,  $P_1 V_1 = P_2 V_2$

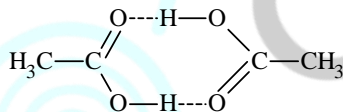
23. (A) Ice has cage-like structure in which each water molecule is surrounded by four other water molecules tetrahedrally through hydrogen bonding, due to this density of ice is less than water and it floats in water.



The cation (I) more stabilized through hydrogen bonding than cation (II). So,  $\text{R}-\text{NH}_2$  is better base than  $(\text{R})_3\text{N}$  in aqueous solution.

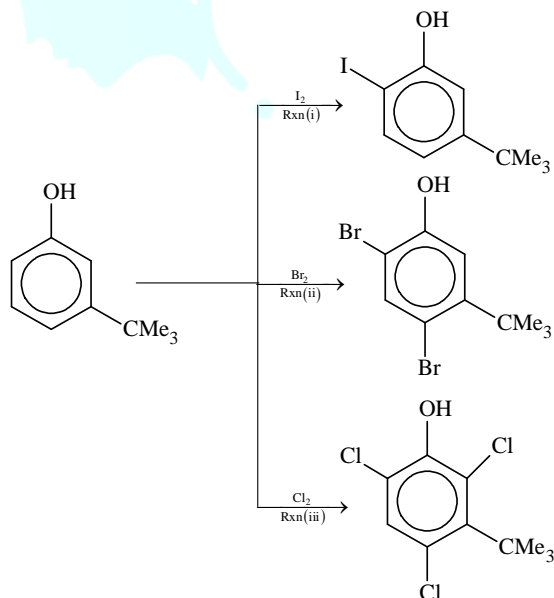
(C)  $\text{HCOOH}$  is stronger acid than  $\text{CH}_3\text{COOH}$  due to inductive effect and not due to hydrogen bonding.

(D) Acetic acid dimerizes in benzene through intermolecular hydrogen bonding.

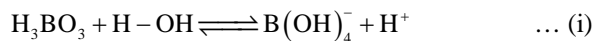


25. The balanced equation is,  
 $\text{ClO}_3^- + 6\text{I}^- + 6\text{H}_2\text{SO}_4 \longrightarrow 3\text{I}_2 + \text{Cl}^- + 6\text{HSO}_4^- + 3\text{H}_2\text{O}$

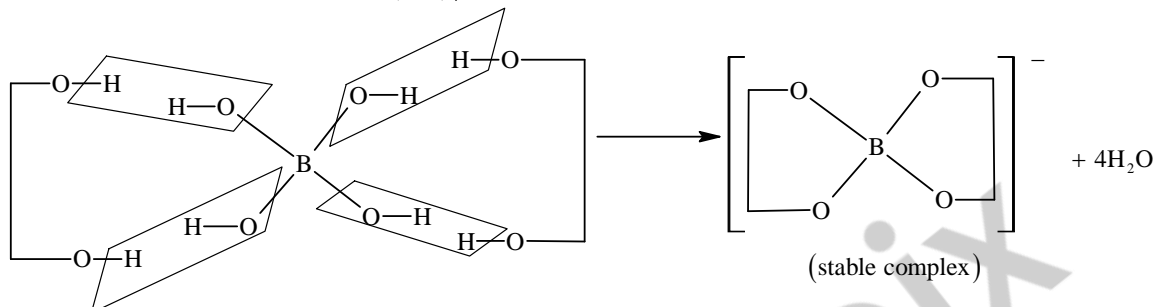
- 26.



27. (A)  $\text{H}_3\text{BO}_3$  is a weak monobasic Lewis acid.



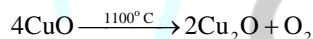
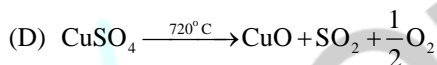
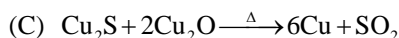
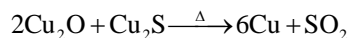
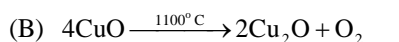
- (B) Equilibrium (i) is shifted in forward direction by the addition of syn-diols like ethylene glycol which forms a stable complex with  $\text{B}(\text{OH})_4^-$ .



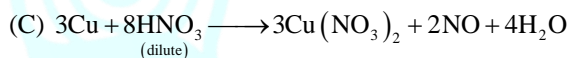
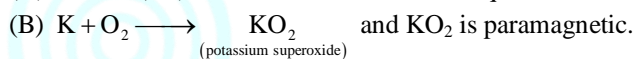
- (C) It has a planar sheet like structure due to hydrogen bonding.

- (D)  $\text{H}_3\text{BO}_3$  is a weak electrolyte in water.

28. (A)  $2\text{CuFeS}_2 + \text{O}_2 \xrightarrow{\Delta} \text{Cu}_2\text{S} + 2\text{FeS} + \text{SO}_2$

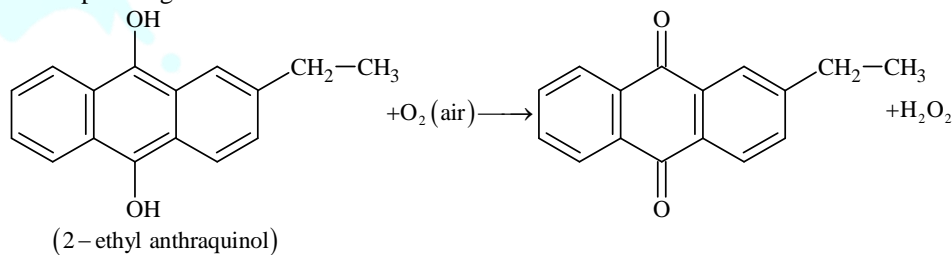


29. (A) sodium (Na) when dissolved in excess liquid ammonia, forms a blue coloured paramagnetic solution.



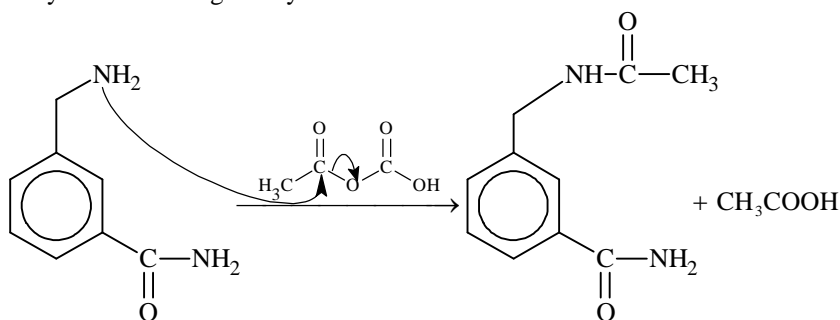
Where "NO" is paramagnetic.

- (D)



Where " $\text{H}_2\text{O}_2$ " is diamagnetic.

30. Only amines undergo acetylation and not acid amides.

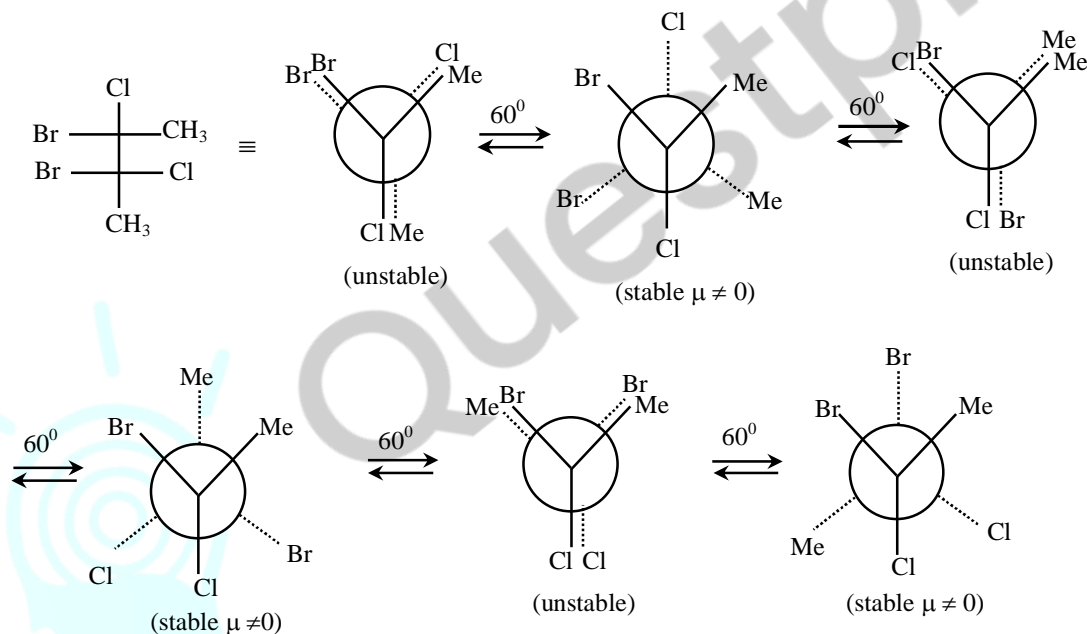


## SECTION – 2:

31. Black coloured sulphides {PbS, CuS, HgS, Ag<sub>2</sub>S, NiS, CoS}

\* Bi<sub>2</sub>S<sub>3</sub> in its crystalline form is dark brown but Bi<sub>2</sub>S<sub>3</sub> precipitate obtained is black in colour.

- 32.



33.  $\text{K}_2\text{Cr}_2\text{O}_7, \text{CuSO}_4, \text{H}_2\text{O}_2, \text{Cl}_2, \text{O}_3, \text{FeCl}_3, \text{HNO}_3$   
 $\text{K}_2\text{Cr}_2\text{O}_7 + 7\text{H}_2\text{SO}_4 + 6\text{KI} \longrightarrow 4\text{K}_2\text{SO}_4 + \text{Cr}_2(\text{SO}_4)_3 + 3\text{I}_2 + 7\text{H}_2\text{O}$   
 $2\text{CuSO}_4 + 4\text{KI} \longrightarrow \text{Cu}_2\text{I}_2 + \text{I}_2 + 2\text{K}_2\text{SO}_4$   
 $\text{H}_2\text{O}_2 + 2\text{KI} \longrightarrow \text{I}_2 + 2\text{KOH}$   
 $\text{Cl}_2 + 2\text{KI} \longrightarrow 2\text{KCl} + \text{I}_2$   
 $\text{O}_3 + 2\text{KI} + \text{H}_2\text{O} \longrightarrow 2\text{KOH} + \text{I}_2 + \text{O}_2$   
 $2\text{FeCl}_3 + 2\text{KI} \longrightarrow 2\text{FeCl}_2 + \text{I}_2 + 2\text{KCl}$   
 $8\text{HNO}_3 + 6\text{KI} \longrightarrow 6\text{KNO}_3 + 2\text{NO} + 4\text{H}_2\text{O} + 3\text{I}_2$   
 $2\text{KMnO}_4 + \text{KI} + \text{H}_2\text{O} \longrightarrow \text{KIO}_3 + 2\text{MnO}_2 + 2\text{KOH}$

34.  $\text{XeF}_4 \rightarrow$  Square planar  
 $\text{BrF}_4^- \rightarrow$  Square planar  
 $[\text{Cu}(\text{NH}_3)_4]^{2+} \rightarrow$  Square planar  
 $[\text{PtCl}_4]^{2-} \rightarrow$  Square planar  
 $\text{SF}_4 \rightarrow$  See-saw  
 $\text{SiF}_4 \rightarrow$  Tetrahedral  
 $\text{BF}_4^- \rightarrow$  Tetrahedral  
 $[\text{FeCl}_4]^{2-} \rightarrow$  Tetrahedral  
 $[\text{CoCl}_4]^{2-} \rightarrow$  Tetrahedral

35. (1)  $\text{CH}_3 - \text{CH}_2 - \overset{*}{\underset{\text{CH}_3}{\text{CH}}} - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_3$  Will not give a racemic mixture on reduction with  $\text{NaBH}_4$   
(2)  $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{CH}_2 - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_3$  Will give a racemic mixture on reduction with  $\text{NaBH}_4$   
(3)  $\text{CH}_3 - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_2 - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_3$  Will give a racemic mixture on reduction with  $\text{NaBH}_4$   
(4)  $\begin{array}{c} \text{CH}_3 \text{ O} \\ | \parallel \\ \text{H}_3\text{C} - \text{C} - \text{C} - \text{CH}_3 \\ | \\ \text{CH}_3 \end{array}$  Will give a racemic mixture on reduction with  $\text{NaBH}_4$   
(5)  $\text{CH}_3 - \text{CH}_2 - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_2 - \text{CH}_2 - \text{CH}_3$  Will give a racemic mixture on reduction with  $\text{NaBH}_4$   
(6)  $\text{CH}_3 - \text{CH}_2 - \overset{\text{O}}{\parallel} \text{C} - \underset{\text{CH}_3}{\text{CH}} - \text{CH}_3$  Will give a racemic mixture on reduction with  $\text{NaBH}_4$

36.  $n = 4$   
 $\ell = 0, 1, 2, 3$   
 $|m_\ell| = 1 \Rightarrow \pm 1$   
 $m_s = -\frac{1}{2}$   
**For**  $\ell = 0, m_\ell = 0$   
 $\ell = 1, m_\ell = -1, 0, +1$   
 $\ell = 2, m_\ell = -2, -1, 0, +1, +2$   
 $\ell = 3, m_\ell = -3, -2, -1, 0, +1, +2, +3$

So, six electrons can have  $|m_\ell| = 1$  &  $m_s = -\frac{1}{2}$  37.  $k = \frac{R}{N_A}$

$$R = kN_A$$

$$= 1.380 \times 10^{-23} \times 6.023 \times 10^{23}$$

$$= 8.31174$$

$$\approx 8.312$$

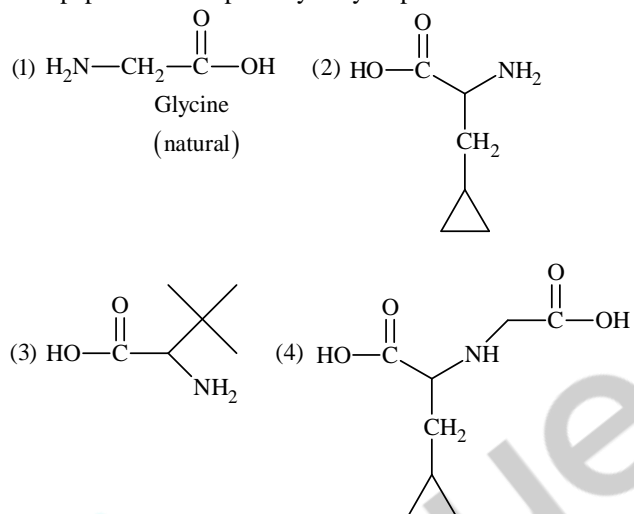
38.  $MX_2 \rightleftharpoons M^{2+} + 2X^-$

$$1 - \alpha \quad \alpha \quad 2\alpha$$

$$i = 1 + 2\alpha \quad \{\alpha = 0.5\}$$

$$i = 2$$

39. This peptide on complete hydrolysis produced 4 distinct amino acids which are given below:



Only glycine is naturally occurring amino acid.

40. Here,  $V_{\text{solution}} \approx V_{\text{solvent}}$

Since, in 1ℓ solution, 3.2 moles of solute are present,

So, 1ℓ solution  $\approx$  1ℓ solvent ( $d = 0.4\text{g/ml}$ )  $\approx$  0.4 kg

$$\text{So, molality (m)} = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}} = \frac{3.2}{0.4} = 8$$

## PART - III: MATHEMATICS

41.  $f'(x) = \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}$

Which is increasing in  $[1, \infty)$

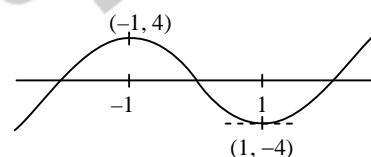
Also,  $f(x) + f\left(\frac{1}{x}\right) = 0$

$$g(x) = f(2^x) = \int_{2^{-x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$g(-x) = \int_{2^x}^{2^{-x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt = -g(x)$$

Hence, an odd function

42. Let  $y = x^5 - 5x$



43. Let  $f(x)$  and  $g(x)$  achieve their maximum value at  $x_1$  and  $x_2$  respectively

$$h(x) = f(x) - g(x)$$

$$h(x_1) = f(x_1) - g(x_1) \geq 0$$

$$h(x_2) = f(x_2) - g(x_2) \leq 0$$

$$\Rightarrow h(c) = 0 \text{ where } c \in [0, 1] \Rightarrow f(c) = g(c).$$

44. Given circles

$$x^2 + y^2 - 2x - 15 = 0$$

$$x^2 + y^2 - 1 = 0$$

$$\text{Radical axis } x + 7 = 0 \quad \dots (1)$$

Centre of circle lies on (1)

Let the centre be  $(-7, k)$

$$\text{Let equation be } x^2 + y^2 + 14x - 2ky + c = 0$$

Orthogonality gives

$$-14 = c - 15 \Rightarrow c = 1 \quad \dots (2)$$

$$(0, 1) \rightarrow 1 - 2k + 1 = 0 \Rightarrow k = 1$$

$$\text{Hence radius} = \sqrt{7^2 + k^2 - c} = \sqrt{49 + 1 - 1} = 7$$

**Alternate solution**

$$\text{Given circles } x^2 + y^2 - 2x - 15 = 0$$

$$x^2 + y^2 - 1 = 0$$

$$\text{Let equation of circle } x^2 + y^2 + 2gx + 2fy + c = 0$$

Circle passes through  $(0, 1)$

$$\Rightarrow 1 + 2f + c = 0$$

Applying condition of orthogonality

$$-2g = c - 15, 0 = c - 1$$

$$\Rightarrow c = 1, g = 7, f = -1$$

$$r = \sqrt{49 + 1 - 1} = 7; \text{ centre } (-7, 1)$$

45.  $\vec{a}$  is in direction of  $\vec{x} \times (\vec{y} \times \vec{z})$

i.e.  $(\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z}$

$$\Rightarrow \vec{a} = \lambda_1 \left[ 2 \times \frac{1}{2} (\vec{y} - \vec{z}) \right]$$

$$\vec{a} = \lambda_1 (\vec{y} - \vec{z}) \quad \dots (1)$$

Now  $\vec{a} \cdot \vec{y} = \lambda_1 (\vec{y} \cdot \vec{y} - \vec{y} \cdot \vec{z})$

$$= \lambda_1 (2 - 1) \Rightarrow \lambda_1 = \vec{a} \cdot \vec{y} \quad \dots (2)$$

From (1) and (2),  $\vec{a} = \vec{a} \cdot \vec{y} (\vec{y} - \vec{z})$

Similarly,  $\vec{b} = (\vec{b} \cdot \vec{z}) (\vec{z} - \vec{x})$

Now,  $\vec{a} \cdot \vec{b} = (\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z}) [(\vec{y} - \vec{z}) \cdot (\vec{z} - \vec{x})]$

$$= (\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z}) [1 - 1 - 2 + 1]$$

$$= - (\vec{a} \cdot \vec{y}) (\vec{b} \cdot \vec{z})$$

46. Line 1:  $\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r, Q(r, r, 1)$

Line 2:  $\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k, R(k, -k, -1)$

$$\overrightarrow{PQ} = (\lambda - r)\hat{i} + (\lambda - r)\hat{j} + (\lambda - 1)\hat{k}$$

and  $\lambda - r + \lambda - r = 0$  as  $\overrightarrow{PQ}$  is  $\perp$  to  $L_1$

$$\Rightarrow 2\lambda = 2r \Rightarrow \lambda = r$$

$$\overrightarrow{PR} = (\lambda - k)\hat{i} + (\lambda + k)\hat{j} + (\lambda + 1)\hat{k}$$

and  $\lambda - k - \lambda - k = 0$  as  $\overrightarrow{PR}$  is  $\perp$  to  $L_2$

$$\Rightarrow k = 0$$

so  $PQ \perp PR$

$$(\lambda - r)(\lambda - k) + (\lambda - r)(\lambda + k) + (\lambda - 1)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 1, -1$$

For  $\lambda = 1$  as points P and Q coincide

$$\Rightarrow \lambda = -1.$$

47. Let  $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$  (where  $a, b, c \in \mathbb{I}$ )

then  $\text{Det } M = ab - c^2$

if  $a = b = c$ ,  $\text{Det}(M) = 0$

if  $c = 0$ ,  $a, b \neq 0$ ,  $\text{Det}(M) \neq 0$

if  $ab \neq \text{square of integer}$ ,  $\text{Det}(M) \neq 0$

48.  $M^2 = N^4 \Rightarrow M^2 - N^4 = O \Rightarrow (M - N^2)(M + N^2) = O$  (As  $M, N$  commute.)

Also,  $M \neq N^2$ ,  $\text{Det}((M - N^2)(M + N^2)) = 0$

As  $M - N^2$  is not null  $\Rightarrow \text{Det}(M + N^2) = 0$

Also  $\text{Det}(M^2 + MN^2) = (\text{Det } M)(\text{Det}(M + N^2)) = 0$

$\Rightarrow$  There exist non-null  $U$  such that  $(M^2 + MN^2)U = O$



49. Since  $f(x) \geq 1 \forall x \in [a, b]$   
for  $g(x)$   
LHD at  $x = a$  is zero

$$\int_a^x f(t) dt - 0$$

and RHD at  $(x = a) = \lim_{x \rightarrow a^+} \frac{a}{x - a} = \lim_{x \rightarrow a^+} f(x) \geq 1$

Hence  $g(x)$  is not differentiable at  $x = a$   
Similarly LHD at  $x = b$  is greater than 1  
 $g(x)$  is not differentiable at  $x = b$

50.  $f(x) = (\log(\sec x + \tan x))^3 \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
 $f(-x) = -f(x)$ , hence  $f(x)$  is odd function  
Let  $g(x) = \sec x + \tan x \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
 $\Rightarrow g'(x) = \sec x (\sec x + \tan x) > 0 \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
 $\Rightarrow g(x)$  is one-one function  
Hence  $(\log_e(g(x)))^3$  is one-one function.  
and  $g(x) \in (0, \infty) \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
 $\Rightarrow \log(g(x)) \in \mathbb{R}$ . Hence  $f(x)$  is an onto function.

51. When  $n_5$  takes value from 10 to 6 the carry forward moves from 0 to 4 which can be arranged in

$${}^4C_0 + \frac{{}^4C_1}{4} + \frac{{}^4C_2}{3} + \frac{{}^4C_3}{2} + \frac{{}^4C_4}{1} = 7$$

**Alternate solution**

Possible solutions are

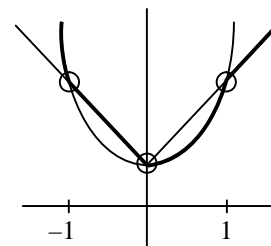
1, 2, 3, 4, 10  
1, 2, 3, 5, 9  
1, 2, 3, 6, 8  
1, 2, 4, 5, 8  
1, 2, 4, 6, 7  
1, 3, 4, 5, 7  
2, 3, 4, 5, 6

Hence 7 solutions are there.

52. Number of red lines =  ${}^nC_2 - n$   
Number of blue lines =  $n$   
Hence,  ${}^nC_2 - n = n$   
 ${}^nC_2 = 2n$   
 $\frac{n(n-1)}{2} = 2n$   
 $n - 1 = 4 \Rightarrow n = 5$ .

$$53. \quad h(x) = \begin{cases} x^2 + 1 & , \quad x \in (-\infty, -1] \\ -x + 1 & , \quad x \in [-1, 0] \\ x^2 + 1 & , \quad x \in [0, 1] \\ x + 1 & , \quad x \in [1, \infty) \end{cases}$$

Hence, not differentiable at  $x = -1, 0, 1$



$$54. \quad \frac{b}{a} = \frac{c}{b} = (\text{integer})$$

$$b^2 = ac \Rightarrow c = \frac{b^2}{a}$$

$$\frac{a+b+c}{3} = b+2$$

$$a+b+c = 3b+6 \Rightarrow a-2b+c = 6$$

$$a-2b+\frac{b^2}{a} = 6 \Rightarrow 1 - \frac{2b}{a} + \frac{b^2}{a^2} = \frac{6}{a}$$

$$\left(\frac{b}{a} - 1\right)^2 = \frac{6}{a} \Rightarrow a = 6 \text{ only}$$

$$55. \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = p + q(\vec{a} \cdot \vec{b}) + r(\vec{a} \cdot \vec{c})$$

$$\text{And } [\vec{a} \quad \vec{b} \quad \vec{c}] = \frac{1}{\sqrt{2}}$$

$$p + \frac{q}{2} + \frac{r}{2} = [\vec{a} \quad \vec{b} \quad \vec{c}] \quad \dots (1)$$

$$\frac{p}{2} + q + \frac{r}{2} = 0 \quad \dots (2)$$

$$\frac{p}{2} + \frac{q}{2} + r = [\vec{a} \quad \vec{b} \quad \vec{c}] \quad \dots (3)$$

$$\Rightarrow p = r = -q$$

$$\frac{p^2 + 2q^2 + r^2}{q^2} = 4$$

$$56. \quad 2(y - x^5) \left( \frac{dy}{dx} - 5x^4 \right) = 1(1 + x^2)^2 + (x)(2(1 + x^2)(2x))$$

$$\text{Now put } x = 1, y = 3 \text{ and } \frac{dy}{dx} = m.$$

$$2(3 - 1)(m - 5) = 1(4) + (1)(4)(2)$$

$$m - 5 = \frac{12}{4}$$

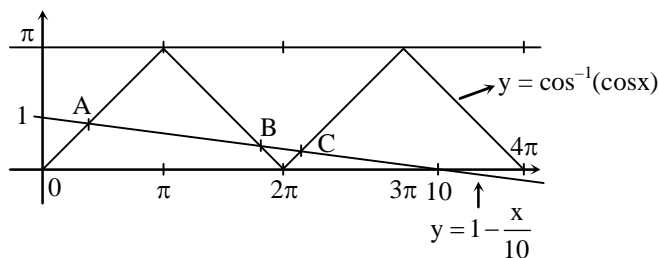
$$m = 5 + 3 = 8$$

$$\frac{dy}{dx} = m = 8.$$

$$\begin{aligned}
 57. \quad & \int_0^1 4x^3 \frac{d^2}{dx^2} (1-x^2)^5 dx \\
 &= \left[ 4x^3 \frac{d}{dx} (1-x^2)^5 \right]_0^1 - \int_0^1 12x^2 \frac{d}{dx} (1-x^2)^5 dx \\
 &= \left[ 4x^3 \times 5(1-x^2)^4 (-2x) \right]_0^1 - 12 \left[ \left[ x^2 (1-x^2)^5 \right]_0^1 - \int_0^1 2x (1-x^2)^5 dx \right] \\
 &= 0 - 0 - 12[0 - 0] + 12 \int_0^1 2x (1-x^2)^5 dx \\
 &= 12 \times \left[ -\frac{(1-x^2)^6}{6} \right]_0^1 \\
 &= 12 \left[ 0 + \frac{1}{6} \right] = 2
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \lim_{x \rightarrow 1} \left( \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right)^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4} \\
 & \lim_{x \rightarrow 1} \left( \frac{\frac{\sin(x-1)}{(x-1)} - a}{\frac{\sin(x-1)}{(x-1)} + 1} \right)^{(1+\sqrt{x})} = \frac{1}{4} \Rightarrow \left( \frac{1-a}{2} \right)^2 = \frac{1}{4} \\
 & \Rightarrow a = 0, a = 2 \\
 & \Rightarrow a = 2
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & f: [0, 4\pi] \rightarrow [0, \pi], f(x) = \cos^{-1}(\cos x) \\
 & \Rightarrow \text{point A, B, C satisfy } f(x) = \frac{10-x}{10} \\
 & \text{Hence, 3 points}
 \end{aligned}$$



$$\begin{aligned}
 60. \quad & 2 \leq d_1(p) + d_2(p) \leq 4 \\
 & \text{For } P(\alpha, \beta), \alpha > \beta \\
 & \Rightarrow 2\sqrt{2} \leq 2\alpha \leq 4\sqrt{2} \\
 & \sqrt{2} \leq \alpha \leq 2\sqrt{2} \\
 & \Rightarrow \text{Area of region} = \left( (2\sqrt{2})^2 - (\sqrt{2})^2 \right) \\
 & \quad = 8 - 2 = 6 \text{ sq. units}
 \end{aligned}$$

