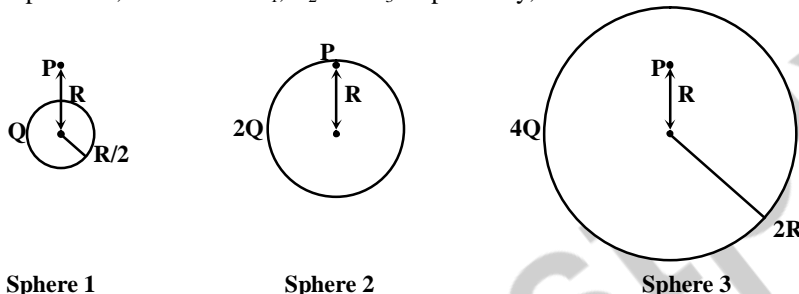


PART I : PHYSICS

SECTION – 1 : (Only One Option Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE option is correct.

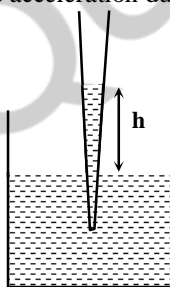
1. Charges Q , $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1 , E_2 and E_3 respectively, then



- (A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

- *2. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h , where the radius of its cross section is b . If the surface tension of water is S , its density is ρ , and its contact angle with glass is θ , the value of h will be (g is the acceleration due to gravity)

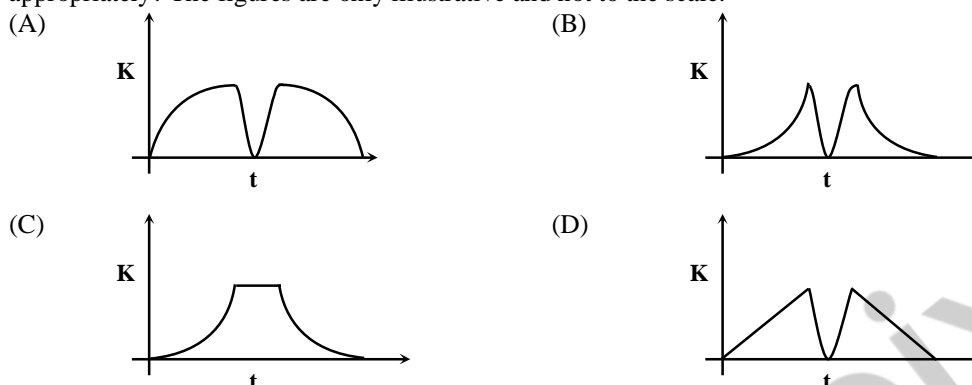
- (A) $\frac{2S}{b\rho g} \cos(\theta - \alpha)$
 (B) $\frac{2S}{b\rho g} \cos(\theta + \alpha)$
 (C) $\frac{2S}{b\rho g} \cos(\theta - \alpha / 2)$
 (D) $\frac{2S}{b\rho g} \cos(\theta + \alpha / 2)$



3. If λ_{cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{cu}/\lambda_{Mo}$ is close to
 (A) 1.99 (B) 2.14 (C) 0.50 (D) 0.48

- *4. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = $6 \times 10^6 \text{ m}$ and the acceleration due to gravity of Earth is 10 ms^{-2})
 (A) 96 N (B) 108 N (C) 120 N (D) 150 N

- *5. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.

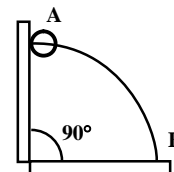


6. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2 , respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240 \text{ eV nm}$, the work function of the metal is nearly

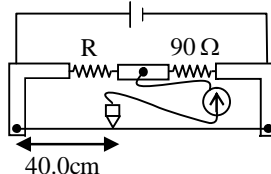
(A) 3.7 eV (B) 3.2 eV (C) 2.8 eV (D) 2.5 eV

- *7. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

(A) always radially outwards.
(B) always radially inwards.
(C) radially outwards initially and radially inwards later.
(D) radially inwards initially and radially outwards later.



8. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90Ω , as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is

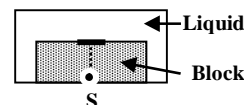


(A) $60 \pm 0.15 \Omega$ (B) $135 \pm 0.56 \Omega$ (C) $60 \pm 0.25 \Omega$ (D) $135 \pm 0.23 \Omega$

9. Parallel rays of light of intensity $I = 912 \text{ W m}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

10. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



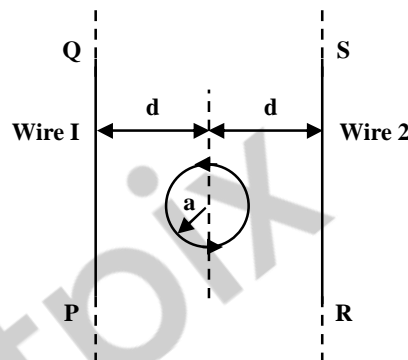
(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

SECTION – 2 : Comprehension type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

Paragraph for Questions 11 & 12

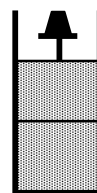
The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d . The loop and the wires are carrying the same current I . The current in the loop is in the counterclockwise direction if seen from above.



11. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case
 (A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$
 (B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$
 (C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$
 (D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$
12. Consider $d \gg a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)
 (A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$ (C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

Paragraph for Questions 13 & 14

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are

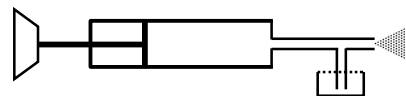


$C_v = \frac{3}{2}R$, $C_p = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$.

- *13. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be
 (A) 550 K (B) 525 K (C) 513 K (D) 490 K
- *14. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be
 (A) 250 R (B) 200 R (C) 100 R (D) -100 R

Paragraph for Questions 15 & 16

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



- *15. If the piston is pushed at a speed of 5 mms^{-1} , the air comes out of the nozzle with a speed of
 (A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}
- *16. If the density of air is ρ_a and that of the liquid ρ_ℓ , then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to
 (A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

SECTION – 3: Match List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as option (A), (B), (C) and (D) out of which one is correct.

- *17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

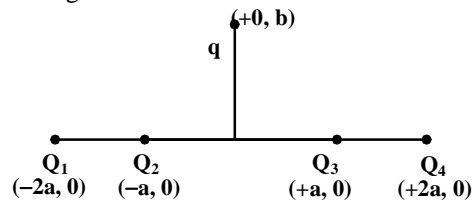
List I	List II
P. Lift is accelerating vertically up.	1. $d = 1.2 \text{ m}$
Q. Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	2. $d > 1.2 \text{ m}$
R. Lift is moving vertically up with constant speed.	3. $d < 1.2 \text{ m}$
S. Lift is falling freely.	4. No water leaks out of the jar

Code:

- (A) P-2, Q-3, R-2, S-4
 (B) P-2, Q-3, R-1, S-4
 (C) P-1, Q-1, R-1, S-4
 (D) P-2, Q-3, R-1, S-1

18. Four charges Q_1 , Q_2 , Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a$, $-a$, $+a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.

List I	List II
P. Q_1, Q_2, Q_3, Q_4 all positive	1. $+x$
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. $-x$
R. Q_1, Q_4 positive; Q_2, Q_3 negative	3. $+y$
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. $-y$



Code:

- (A) P-3, Q-1, R-4, S-2
 (B) P-4, Q-2, R-3, S-1
 (C) P-3, Q-1, R-2, S-4
 (D) P-4, Q-2, R-1, S-3

19. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.

List I

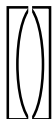
P.



Q.



R.



S.



List II

1. $2r$

2. $r/2$

3. $-r$

4. r

Code:

(A) P-1, Q-2, R-3, S-4

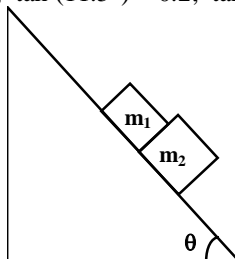
(C) P-4, Q-1, R-2, S-3

(B) P-2, Q-4, R-3, S-1

(D) P-2, Q-1, R-3, S-4

- *20. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ . Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on the block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g .

[Useful information: $\tan(5.5^\circ) \approx 0.1$; $\tan(11.5^\circ) \approx 0.2$; $\tan(16.5^\circ) \approx 0.3$]



List I

P. $\theta = 5^\circ$

Q. $\theta = 10^\circ$

R. $\theta = 15^\circ$

S. $\theta = 20^\circ$

List II

1. $m_2 g \sin \theta$

2. $(m_1 + m_2) g \sin \theta$

3. $\mu m_2 g \cos \theta$

4. $\mu(m_1 + m_2) g \cos \theta$

Code:

(A) P-1, Q-1, R-1, S-3

(B) P-2, Q-2, R-2, S-3

(C) P-2, Q-2, R-2, S-4

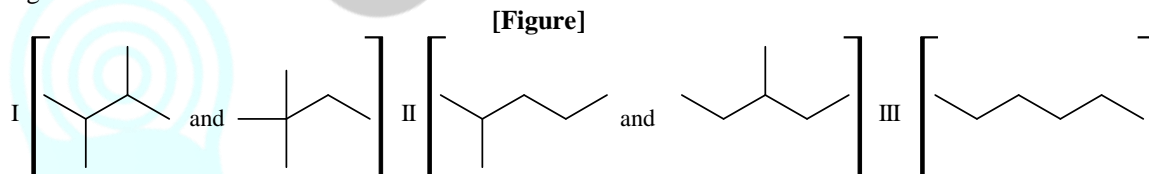
(D) P-2, Q-2, R-3, S-3

PART II : CHEMISTRY

SECTION – 1 : (Only One Option Correct Type)

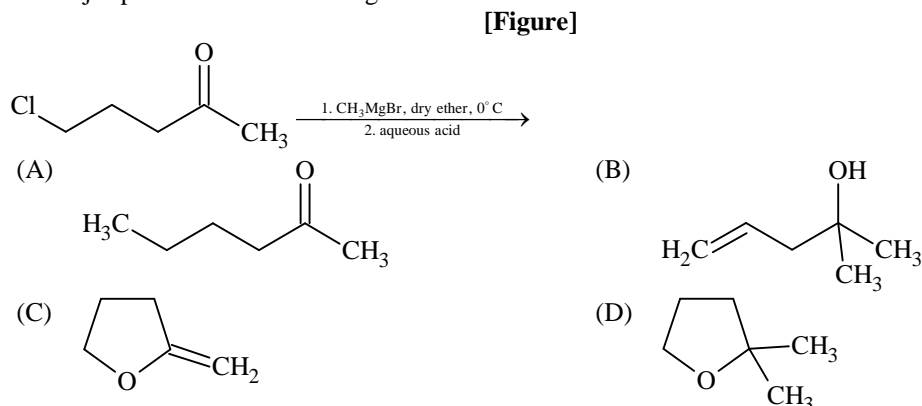
This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** option is correct.

- *21. Assuming $2s - 2p$ mixing is **NOT** operative, the paramagnetic species among the following is
 (A) Be_2 (B) B_2
 (C) C_2 (D) N_2
- *22. For the process
 $\text{H}_2\text{O}(\ell) \longrightarrow \text{H}_2\text{O}(\text{g})$
 at $T = 100^\circ\text{C}$ and 1 atmosphere pressure, the correct choice is
 (A) $\Delta S_{\text{system}} > 0$ and $\Delta S_{\text{surrounding}} > 0$ (B) $\Delta S_{\text{system}} > 0$ and $\Delta S_{\text{surrounding}} < 0$
 (C) $\Delta S_{\text{system}} < 0$ and $\Delta S_{\text{surrounding}} > 0$ (D) $\Delta S_{\text{system}} < 0$ and $\Delta S_{\text{surrounding}} < 0$
- *23. For the elementary reaction $\text{M} \rightarrow \text{N}$, the rate of disappearance of **M** increases by a factor of 8 upon doubling the concentration of **M**. The order of the reaction with respect to **M** is
 (A) 4 (B) 3
 (C) 2 (D) 1
24. For the identification of β -naphthol using dye test, it is necessary to use
 (A) dichloromethane solution of β -naphthol. (B) acidic solution of β -naphthol.
 (C) neutral solution of β -naphthol. (D) alkaline solution of β -naphthol.
- *25. Isomers of hexane, based on their branching, can be divided into three distinct classes as shown in the figure.

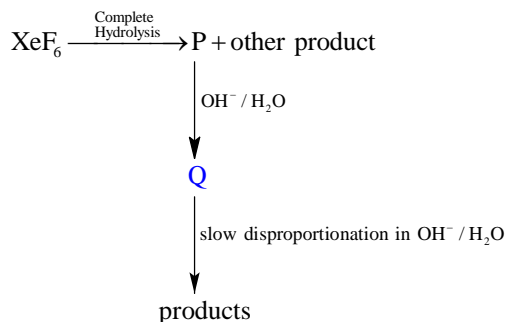


The correct order of their boiling point is

- (A) $\text{I} > \text{II} > \text{III}$ (B) $\text{III} > \text{II} > \text{I}$
 (C) $\text{II} > \text{III} > \text{I}$ (D) $\text{III} > \text{I} > \text{II}$
26. The major product in the following reaction is

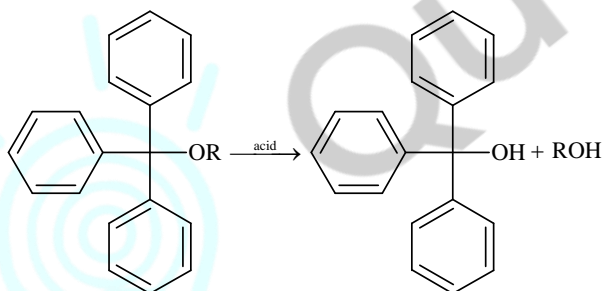


27. Under ambient conditions, the total number of gases released as products in the final step of the reaction scheme shown below is



- (A) 0 (B) 1
(C) 2 (D) 3
28. The product formed in the reaction of SOCl_2 with white phosphorous is
(A) PCl_3 (B) SO_2Cl_2
(C) SCl_2 (D) POCl_3
- *29. Hydrogen peroxide in its reaction with KIO_4 and NH_2OH respectively, is acting as a
(A) reducing agent, oxidising agent (B) reducing agent, reducing agent
(C) oxidising agent, oxidising agent (D) oxidising agent, reducing agent
30. The acidic hydrolysis of ether (X) shown below is fastest when

[Figure]



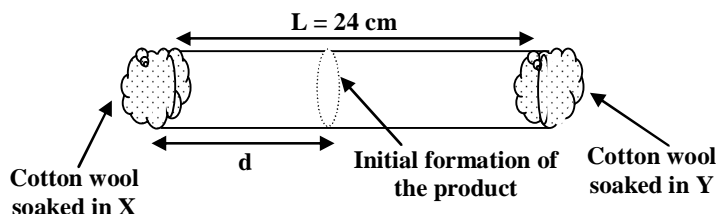
- (A) one phenyl group is replaced by a methyl group.
(B) one phenyl group is replaced by a para-methoxyphenyl group.
(C) two phenyl groups are replaced by two para-methoxyphenyl groups.
(D) no structural change is made to X.

SECTION – 2 : Comprehension type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

Paragraph for Questions 31 & 32

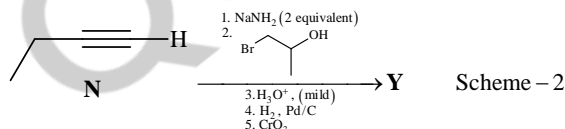
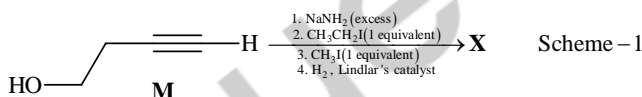
X and Y are two volatile liquids with molar weights of 10 g mol^{-1} and 40 g mol^{-1} respectively. Two cotton plugs, one soaked in X and the other soaked in Y, are simultaneously placed at the ends of a tube of length $L = 24 \text{ cm}$, as shown in the figure. The tube is filled with an inert gas at 1 atmosphere pressure and a temperature of 300 K. Vapours of X and Y react to form a product which is first observed at a distance $d \text{ cm}$ from the plug soaked in X. Take X and Y to have equal molecular diameters and assume ideal behaviour for the inert gas and the two vapours.



- *31. The value of d in cm (shown in the figure), as estimated from Graham's law, is
 (A) 8 (B) 12
 (C) 16 (D) 20
- *32. The experimental value of d is found to be smaller than the estimate obtained using Graham's law. This is due to
 (A) larger mean free path for **X** as compared to that of **Y**.
 (B) larger mean free path for **Y** as compared to that of **X**.
 (C) increased collision frequency of **Y** with the inert gas as compared to that of **X** with the inert gas.
 (D) increased collision frequency of **X** with the inert gas as compared to that of **Y** with the inert gas.

Paragraph for Questions 33 & 34

Schemes 1 and 2 describe sequential transformation of alkynes **M** and **N**. Consider only the major products formed in each step for both schemes.

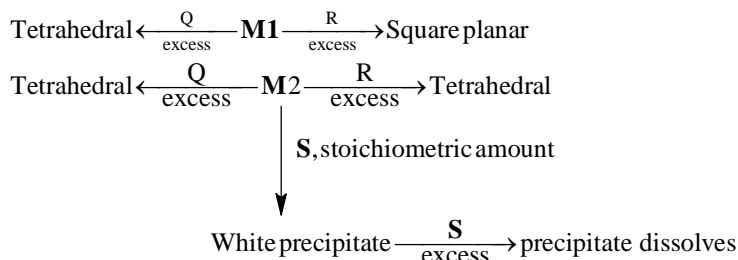


33. The product **X** is
 (A) CH3COCH2CH=CHCH3
 (B) CH3COCH2CH=CHCH2CH3
 (C) CH3CH2OCH2CH=CHCH3
 (D) CH3CH2OCH2CH=CHCH2CH3
34. The correct statement with respect to product **Y** is
 (A) It gives a positive Tollens test and is a functional isomer of **X**.
 (B) It gives a positive Tollens test and is a geometrical isomer of **X**.
 (C) It gives a positive iodoform test and is a functional isomer of **X**.
 (D) It gives a positive iodoform test and is a geometrical isomer of **X**.

Paragraph for Questions 35 & 36

An aqueous solution of metal ion **M1** reacts separately with reagents **Q** and **R** in excess to give tetrahedral and square planar complexes, respectively. An aqueous solution of another metal ion **M2** always forms tetrahedral complexes with these reagents. Aqueous solution of **M2** on reaction with reagent **S** gives white precipitate which dissolves in excess of **S**. The reactions are summarized in the scheme given below.

SCHEME:



35. **M1**, **Q** and **R**, respectively are
- (A) Zn^{2+} , KCN and HCl (B) Ni^{2+} , HCl and KCN
- (C) Cd^{2+} , KCN and HCl (D) Co^{2+} , HCl and KCN
36. Reagent **S** is
- (A) $\text{K}_4[\text{Fe}(\text{CN})_6]$ (B) Na_2HPO_4
- (C) K_2CrO_4 (D) KOH

SECTION – 3: Match List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as option (A), (B), (C) and (D) out of which one is correct.

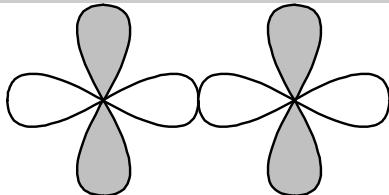
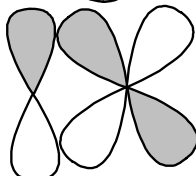
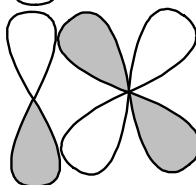
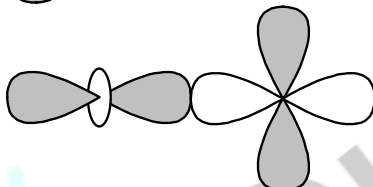
37. Match each coordination compound in **List-I** with an appropriate pair of characteristics from **List-II** and select the correct answer using the code given below the lists
 {en = $\text{H}_2\text{NCH}_2\text{CH}_2\text{NH}_2$; atomic numbers: Ti = 22, Cr = 24; Co = 27; Pt = 78}

List – I		List - II	
P.	$[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}$	1.	Paramagnetic and exhibits ionization isomerism
Q.	$[\text{Ti}(\text{H}_2\text{O})_5\text{Cl}](\text{NO}_3)_2$	2.	Diamagnetic and exhibits <i>cis-trans</i> isomerism
R.	$[\text{Pt}(\text{en})(\text{NH}_3)\text{Cl}]\text{NO}_3$	3.	Paramagnetic and exhibits <i>cis-trans</i> isomerism
S.	$[\text{Co}(\text{NH}_3)_4(\text{NO}_3)_2]\text{NO}_3$	4.	Diamagnetic and exhibits ionization isomerism

Code:

	P	Q	R	S
(A)	4	2	3	1
(B)	3	1	4	2
(C)	2	1	3	4
(D)	1	3	4	2

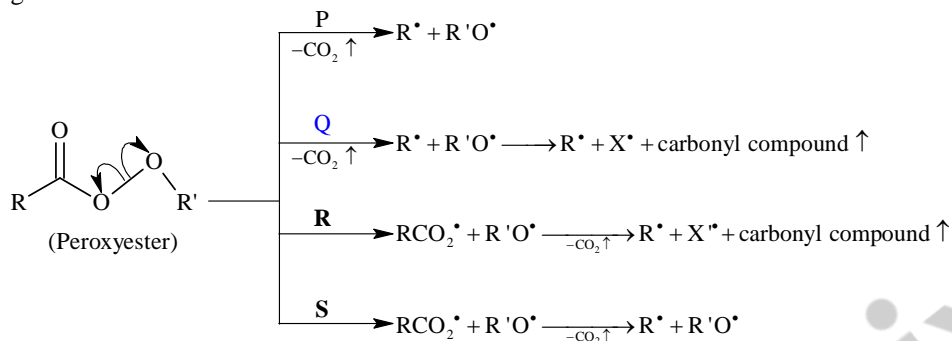
- *38. Match the orbital overlap figures shown in **List-I** with the description given in **List-II** and select the correct answer using the code given below the lists.

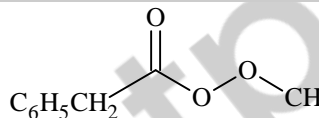
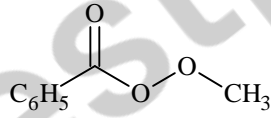
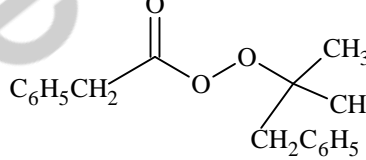
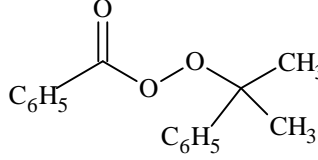
	List - I	List - II
P.		1. $p - d \pi$ antibonding
Q.		2. $d - d \sigma$ bonding
R.		3. $p - d \pi$ bonding
S.		4. $d - d \sigma$ antibonding

Code:

	P	Q	R	S
(A)	2	1	3	4
(B)	4	3	1	2
(C)	2	3	1	4
(D)	4	1	3	2

39. Different possible **thermal** decomposition pathways for peroxyesters are shown below. Match each pathway from **List I** with an appropriate structure from **List II** and select the correct answer using the code given below the lists.



List – I		List – II	
P.	Pathway P	1.	
Q.	Pathway Q	2.	
R.	Pathway R	3.	
S.	Pathway S	4.	

Code:

	P	Q	R	S
(A)	1	3	4	2
(B)	2	4	3	1
(C)	4	1	2	3
(D)	3	2	1	4



40. Match the four starting materials (**P**, **Q**, **R**, **S**) given in **List I** with the corresponding reaction schemes (**I**, **II**, **III**, **IV**) provided in **List II** and select the correct answer using the code given below the lists.

List - I		List - II	
P.		1. Scheme I	$\xrightarrow[\text{(iii) SOCl}_2 \text{ (iv) NH}_3]{\text{(i) KMnO}_4, \text{HO}^-, \text{heat (ii) H}^+, \text{H}_2\text{O}} \text{C}_7\text{H}_6\text{N}_2\text{O}_3$
Q.		2. Scheme II	$\xrightarrow[\text{(iv) HNO}_3 \text{ (v) dil. H}_2\text{SO}_4, \text{heat (vi) HO}^-]{\text{(i) Sn/HCl (ii) CH}_3\text{COCl (iii) conc. H}_2\text{SO}_4} \text{C}_6\text{H}_6\text{N}_2\text{O}_2$
R.		3. Scheme III	$\xrightarrow[\text{(iii) H}_2\text{S.NH}_3 \text{ (iv) NaNO}_2, \text{H}_2\text{SO}_4 \text{ (v) hydrolysis}]{\text{(i) red hot iron, 873 K (ii) fu ming HNO}_3, \text{H}_2\text{SO}_4, \text{heat}} \text{C}_6\text{H}_5\text{NO}_3$
S.		4. Scheme IV	$\xrightarrow[\text{(ii) conc. HNO}_3, \text{conc. H}_2\text{SO}_4 \text{ (iii) dil. H}_2\text{SO}_4, \text{heat}]{\text{(i) conc. H}_2\text{SO}_4, 60^\circ \text{C}} \text{C}_6\text{H}_5\text{NO}_4$

Code:

	P	Q	R	S
(A)	1	4	2	3
(B)	3	1	4	2
(C)	3	4	2	1
(D)	4	1	3	2

PART III : MATHEMATICS

SECTION - 1 : (Only One Option Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE option is correct.

41. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is
- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$ (D) $\frac{3}{4}$
- *42. In a triangle the sum of two sides is x and the product of the same two sides is y . If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is
- (A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$
 (C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$

- *43. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is
 (A) 264 (B) 265
 (C) 53 (D) 67
- *44. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is
 (A) 3 (B) 6
 (C) 9 (D) 15
- *45. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has
 (A) only purely imaginary roots (B) all real roots
 (C) two real and two purely imaginary roots (D) neither real nor purely imaginary roots
46. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17} dx$ is equal to
 (A) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$ (B) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$
 (C) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$ (D) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$
47. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in $(-1, 1)$ satisfying $f(0) = 0$. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x) dx$ is
 (A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
 (C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$
48. Let $f : [0, 2] \rightarrow \mathbb{R}$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$. Let $F(x) = \int_0^{x^2} f(\sqrt{t}) dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then $F(2)$ equals
 (A) $e^2 - 1$ (B) $e^4 - 1$
 (C) $e - 1$ (D) e^4
- *49. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is
 (A) 1051 (B) 1106
 (C) 1113 (D) 1120

- *50. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has
 (A) infinitely many solutions (B) three solutions
 (C) one solution (D) no solution

SECTION – 2 : Comprehension Type (Only One Option Correct)

This section contains 3 paragraph, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

Paragraph For Questions 51 and 52

Box 1 contains three cards bearing numbers 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.

51. The probability that $x_1 + x_2 + x_3$ is odd, is
 (A) $\frac{29}{105}$ (B) $\frac{53}{105}$
 (C) $\frac{57}{105}$ (D) $\frac{1}{2}$
52. The probability that x_1, x_2, x_3 are in an arithmetic progression, is
 (A) $\frac{9}{105}$ (B) $\frac{10}{105}$
 (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

Paragraph For Questions 53 and 54

Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at)$, $Q, R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$.

- *53. The value of r is
 (A) $-\frac{1}{t}$ (B) $\frac{t^2 + 1}{t}$
 (C) $\frac{1}{t}$ (D) $\frac{t^2 - 1}{t}$
- *54. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is
 (A) $\frac{(t^2 + 1)^2}{2t^3}$ (B) $\frac{a(t^2 + 1)^2}{2t^3}$
 (C) $\frac{a(t^2 + 1)^2}{t^3}$ (D) $\frac{a(t^2 + 2)^2}{t^3}$

Paragraph For Questions 55 and 56

Given that for each $a \in (0, 1)$, $\lim_{h \rightarrow 0^+} \int_h^{1-h} t^{-a} (1-t)^{a-1} dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$.

55. The value of $g\left(\frac{1}{2}\right)$ is

(A) π

(B) 2π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

56. The value of $g'\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$

(B) π

(C) $-\frac{\pi}{2}$

(D) 0

SECTION – 3 : Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching list. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D), out of which ONE is correct.

57. Match the following:

List – I		List – II	
(P)	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2 , satisfying $f(0) = 0$ and $\int_0^1 f(x) dx = 1$, is	(1)	8
(Q)	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	(2)	2
(R)	$\int_{-2}^2 \frac{3x^2}{(1+e^x)} dx$ equals	(3)	4
(S)	$\frac{\left(\int_{-1/2}^{1/2} \cos 2x \cdot \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{1/2} \cos 2x \cdot \log\left(\frac{1+x}{1-x}\right) dx\right)}$ equals	(4)	0

Codes:

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	1	4

58. Match the following:

List – I	List – II
(P) Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \neq \pm \frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)} \left\{ (x^2 - 1) \frac{d^2 y(x)}{dx^2} + x \frac{dy(x)}{dx} \right\}$ equals	(1) 1
(Q) Let $A_1, A_2, \dots, A_n (n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let \vec{a}_k be the position vector of the point A_k , $k = 1, 2, \dots, n$. If $\left \sum_{k=1}^{n-1} (\vec{a}_k \times \vec{a}_{k+1}) \right = \left \sum_{k=1}^{n-1} (\vec{a}_k \cdot \vec{a}_{k+1}) \right $, then the minimum value of n is	(2) 2
*(R) If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	(3) 8
(S) Number of positive solutions satisfying the equation $\tan^{-1} \left(\frac{1}{2x+1} \right) + \tan^{-1} \left(\frac{1}{4x+1} \right) = \tan^{-1} \left(\frac{2}{x^2} \right)$ is	(4) 9

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

59. Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$, $f_2 : [0, \infty) \rightarrow \mathbb{R}$, $f_3 : \mathbb{R} \rightarrow \mathbb{R}$ and $f_4 : \mathbb{R} \rightarrow [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases}; f_2(x) = x^2; f_3(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \text{ and } f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$$

List – I	List – II
(P) f_4 is	(1) onto but not one-one
(Q) f_3 is	(2) neither continuous nor one-one
(R) $f_2 \circ f_1$ is	(3) differentiable but not one-one
(S) f_2 is	(4) continuous and one-one

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

*60. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i \sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \dots, 9$.

List – I		List – II	
(P)	For each z_k there exists a z_j such $z_k \cdot z_j = 1$	(1)	True
(Q)	There exists a $k \in \{1, 2, \dots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers	(2)	False
(R)	$\frac{ 1 - z_1 1 - z_2 \dots 1 - z_9 }{10}$ equals	(3)	1
(S)	$1 - \sum_{k=1}^9 \cos\left(\frac{2k\pi}{10}\right)$ equals	(4)	2

Codes:

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

ANSWERS

PAPER-2 [Code – 8] JEE(ADVANCED) 2014 ANSWERS

PART I : PHYSICS

- | | | | |
|-------|-------|-------|-------|
| 1. C | 2. D | 3. B | 4. B |
| 5. B | 6. A | 7. D | 8. C |
| 9. A | 10. C | 11. C | 12. B |
| 13. D | 14. D | 15. C | 16. A |
| 17. C | 18. A | 19. B | 20. D |

PART II : CHEMISTRY

- | | | | |
|-------|-------|-------|-------|
| 21. C | 22. B | 23. B | 24. D |
| 25. B | 26. D | 27. C | 28. A |
| 29. A | 30. C | 31. C | 32. D |
| 33. A | 34. C | 35. B | 36. D |
| 37. B | 38. C | 39. A | 40. C |

PART III : MATHEMATICS

- | | | | |
|-------|-------|-------|-------|
| 41. A | 42. B | 43. C | 44. D |
| 45. D | 46. A | 47. B | 48. B |
| 49. C | 50. D | 51. B | 52. C |
| 53. D | 54. B | 55. A | 56. D |
| 57. D | 58. A | 59. D | 60. C |

PART I : PHYSICS

1. For point outside dielectric sphere $E = \frac{Q}{4\pi\epsilon_0 r^2}$

For point inside dielectric sphere $E = E_s \frac{r}{R}$

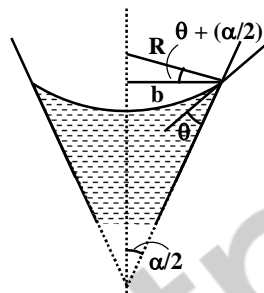
Exact Ratio $E_1 : E_2 : E_3 = 2 : 4 : 1$

2. If R be the meniscus radius
 $R \cos(\theta + \alpha/2) = b$

Excess pressure on concave side of meniscus $= \frac{2S}{R}$

$$h\rho g = \frac{2S}{R} = \frac{2S}{b} \cos\left(\theta + \frac{\alpha}{2}\right)$$

$$\Rightarrow h = \frac{2S}{b\rho g} \cos\left(\theta + \frac{\alpha}{2}\right)$$



3. $\frac{\lambda_{Cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo} - 1}{Z_{Cu} - 1}\right)^2$

4. Inside planet

$$g_i = g_s \frac{r}{R} = \frac{4}{3} G\pi r\rho$$

Force to keep the wire at rest (F)
 = weight of wire

$$= \int_{4R/5}^R (\lambda dr) \left(\frac{4}{3} G\pi r\rho\right) = \left(\frac{4}{3} G\pi\rho\right) \left(\frac{9\lambda}{50}\right) R^2$$

Here, ρ = density of earth $= \frac{M_e}{\frac{4}{3}\pi R_e^3}$

Also, $R = \frac{R_e}{10}$; putting all values, $F = 108 \text{ N}$

5. $\frac{d(KE)}{dt} = mv \frac{dv}{dt}$

6. $\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{u_1^2}{u_2^2}$

$\phi = 3.7 \text{ eV}$

7. Initially bead is applying radially inward normal force.
 During motion at an instant, $N = 0$, after that N will act radially outward.

8. $R = \frac{x}{100-x} 90$
 $\therefore R = 60 \Omega$
 $\frac{dR}{R} = \frac{100}{(x)(100-x)} dx$
 $\therefore dR = \frac{100}{(40)(60)} 0.1 \times 60$
 $= 0.25 \Omega$

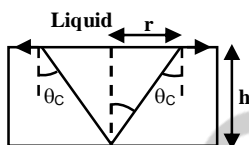
Alternatively

$$\frac{dR}{R} = \frac{0.1}{40} + \frac{0.1}{60}$$

$$\therefore dR = 0.25 \Omega$$

9. Rate of radiation energy lost by the sphere
 = Rate of radiation energy incident on it
 $\Rightarrow \sigma \times 4\pi r^2 [T^4 - (300)^4] = 912 \times \pi r^2$
 $\Rightarrow T = \sqrt[4]{11} \times 10^2 \approx 330 \text{ K}$

10. $\tan \theta_c = \frac{r}{h} = \frac{5.77}{10} \approx \sqrt{3}$
 $\Rightarrow \theta_c = 30^\circ$
 $\Rightarrow \sin \theta_c = \frac{\mu_\ell}{\mu_b}$
 $\Rightarrow \mu_\ell = 2.72 \times \frac{1}{2} = 1.36$



11. The net magnetic field at the given point will be zero if.

$$|\vec{B}_{\text{wires}}| = |\vec{B}_{\text{loop}}|$$

$$\Rightarrow 2 \frac{\mu_0 I}{2\pi \sqrt{a^2 + h^2}} \times \frac{a}{\sqrt{a^2 + h^2}} = \frac{\mu_0 I a^2}{2(a^2 + h^2)^{3/2}}$$

$$\Rightarrow h \approx 1.2 a$$

The direction of magnetic field at the given point due to the loop is normally out of the plane. Therefore, the net magnetic field due the both wires should be into the plane. For this current in wire I should be along PQ and that in wire RS should be along SR.

12. $\tau = MB \sin \theta = I \pi a^2 \times 2 \times \frac{\mu_0 I}{2\pi d} \sin 30^\circ = \frac{\mu_0 I^2 a^2}{2d}$

13. Heat given by lower compartment = $2 \times \frac{3}{2} R \times (700 - T) \dots (i)$

Heat obtained by upper compartment = $2 \times \frac{7}{2} R \times (T - 400) \dots (ii)$

equating (i) and (ii)

$$3(700 - T) = 7(T - 400)$$

$$2100 - 3T = 7T - 2800$$

$$4900 = 10T \Rightarrow T = 490 \text{ K}$$

at eq. TK

400 K

700 K

14. Heat given by lower compartment = $2 \times \frac{5}{2} R \times (700 - T)$

...(i)

$$\text{Heat obtained by upper compartment} = 2 \times \frac{7}{2} R \times (T - 400) \quad \dots(ii)$$

By equating (i) and (ii)

$$5(700 - T) = 7(T - 400)$$

$$3000 - 5T = 7T - 2800$$

$$6300 = 12T$$

$$T = 525K$$

$$\therefore \text{Work done by lower gas} = nR\Delta T = -350 R$$

$$\text{Work done by upper gas} = nR\Delta T = +250 R$$

$$\text{Net work done} = 100 R$$

15. By $A_1 V_1 = A_2 V_2$

$$\Rightarrow \pi(20)^2 \times 5 = \pi(1)^2 V_2 \Rightarrow V_2 = 2 \text{ m/s}^2$$

16. $\frac{1}{2} \rho_a V_a^2 = \frac{1}{2} \rho_\ell V_\ell^2$

For given V_a

$$V_\ell \propto \sqrt{\frac{\rho_a}{\rho_\ell}}$$

17. In P, Q, R no horizontal velocity is imparted to falling water, so d remains same.

In S, since its free fall, $a_{\text{eff}} = 0$

\therefore Liquid won't fall with respect to lift.

18. **P:** By Q_1 and Q_4 , Q_3 and Q_2 F is in +y

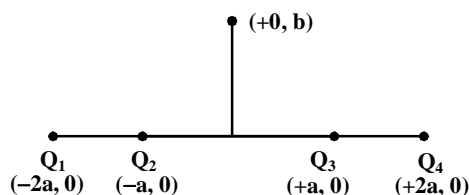
Q: By Q_1 and Q_4 , Q_2 and Q_3 F is in +ve x.

R: By Q_1 and Q_4 , F is in +ve y

By Q_2 and Q_3 , F is in -ve y

But later has more magnitude, since its closer to (0, b). Therefore net force is in -y

S: By Q_1 and Q_4 , F is in +ve x and by Q_2 and Q_3 , F is in -x, but later is more in magnitude, since its closer to (0, b). Therefore net force is in -ve x.



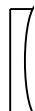
19. $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$



$$f = R$$



$$f = 2R$$



$$f = -2R$$

$$\text{Use } \frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

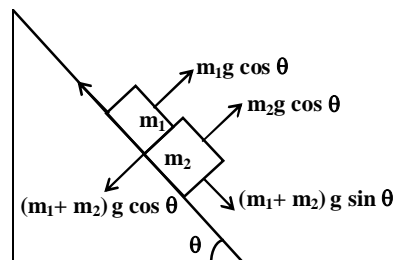
$$(P) \frac{1}{f_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}; f_{\text{eq}} = \frac{R}{2}$$

$$(Q) \frac{1}{f_{\text{eq}}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R}; f_{\text{eq}} = R$$

$$(R) \frac{1}{f_{\text{eq}}} = -\frac{1}{2R} - \frac{1}{2R} = -\frac{1}{R}; f_{\text{eq}} = -R$$

$$(S) \frac{1}{f_{\text{eq}}} = \frac{1}{R} - \frac{1}{2R} = \frac{1}{2R}; f_{\text{eq}} = 2R$$

20. Condition for not sliding,
 $f_{\max} > (m_1 + m_2) g \sin \theta$
 $\mu N > (m_1 + m_2) g \sin \theta$
 $0.3 m_2 g \cos \theta \geq 30 \sin \theta$
 $6 \geq 30 \tan \theta$
 $1/5 \geq \tan \theta$
 $0.2 \geq \tan \theta$
 \therefore for P, Q
 $f = (m_1 + m_2) g \sin \theta$
 For R and S
 $F = f_{\max} = \mu m_2 g \sin \theta$

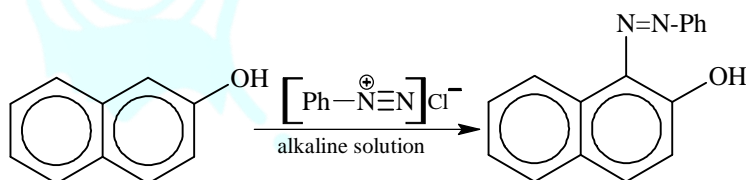


PART II : CHEMISTRY

21. Assuming that no 2s-2p mixing takes place
 (A) $\text{Be}_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$ (diamagnetic)
 (B) $\text{B}_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^0, \pi 2p_y^0$ (diamagnetic)
 (C) $\text{C}_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^1, \pi 2p_y^1, \pi^* 2p_x^0, \pi^* 2p_y^0, \sigma^* 2p_z^0$ (paramagnetic)
 (D) $\text{N}_2 \rightarrow \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^0, \pi^* 2p_y^0, \sigma^* 2p_z^0$ (diamagnetic)
22. At 100°C and 1 atmosphere pressure $\text{H}_2\text{O}(\ell) \rightleftharpoons \text{H}_2\text{O}(\text{g})$ is at equilibrium. For equilibrium
 $\Delta S_{\text{total}} = 0$ and $\Delta S_{\text{system}} + \Delta S_{\text{surrounding}} = 0$
 $\therefore \Delta S_{\text{system}} > 0$ and $\Delta S_{\text{surrounding}} < 0$

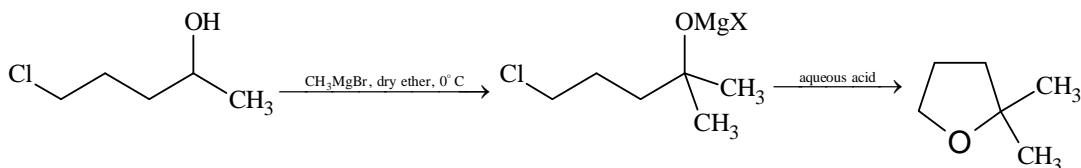
23. $\frac{r_1}{r_2} = \frac{1}{8} = \frac{[M]^n}{[2M]^n} \Rightarrow n = 3$

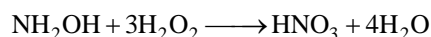
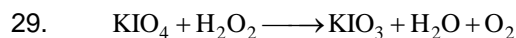
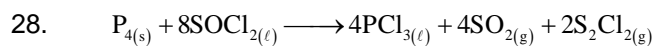
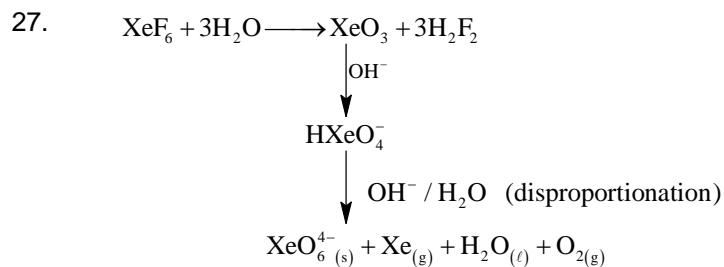
24.



25. $\text{III} > \text{II} > \text{I}$
 More the branching in an alkane, lesser will be the surface area, lesser will be the boiling point.

26.





30. When two phenyl groups are replaced by two para methoxy group, carbocation formed will be more stable

31.
$$\frac{r_X}{r_Y} = \frac{d}{24-d} = \sqrt{\frac{40}{10}} = 2$$

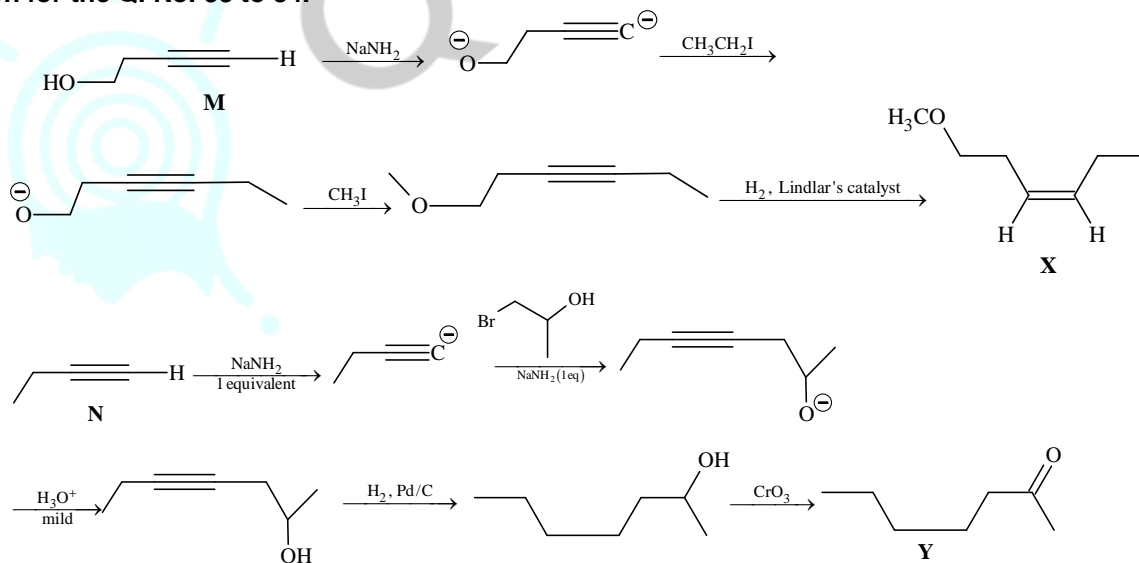
$$d = 48 - 2d$$

$$3d = 48$$

$$d = 16\text{cm}$$

32. As the collision frequency increases then molecular speed decreases than the expected.

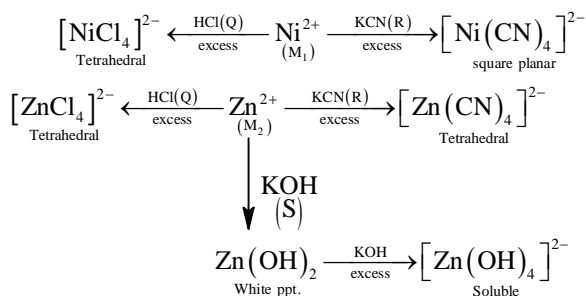
Solution for the Q. No. 33 to 34.



X and Y are functional isomers of each other and Y gives iodoform test.

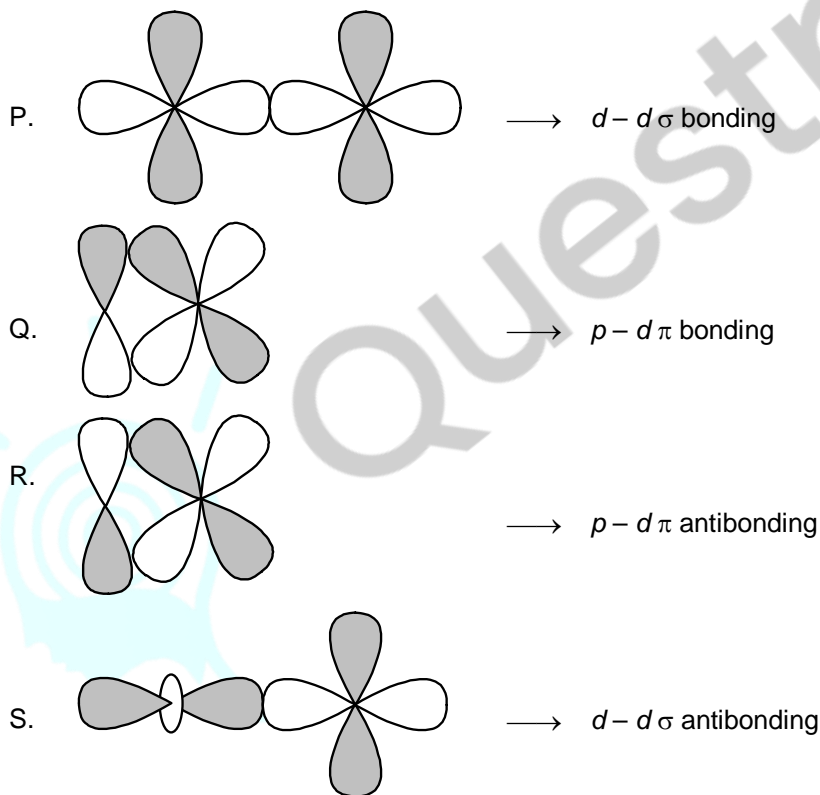


Solution for the Q. No. 35 to 36.



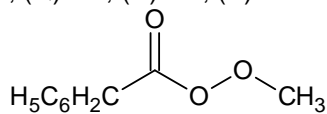
37. (P) $[\text{Cr}(\text{NH}_3)_4\text{Cl}_2]\text{Cl} \longrightarrow$ Paramagnetic and exhibits *cis-trans* isomerism
 (Q) $[\text{Ti}(\text{H}_2\text{O})_5\text{Cl}](\text{NO}_3)_2 \longrightarrow$ Paramagnetic and exhibits ionization isomerism
 (R) $[\text{Pt}(\text{en})(\text{NH}_3)\text{Cl}]\text{NO}_3 \longrightarrow$ Diamagnetic and exhibits ionization isomerism
 (S) $[\text{Co}(\text{NH}_3)_4(\text{NO}_3)_2]\text{NO}_3 \longrightarrow$ Diamagnetic and exhibits *cis-trans* isomerism

38.

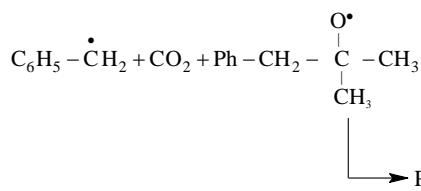
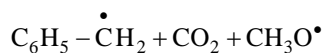
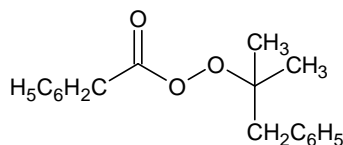


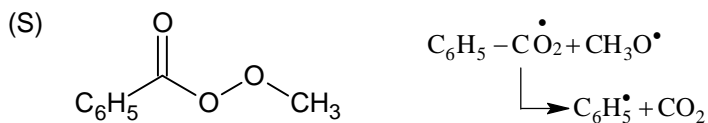
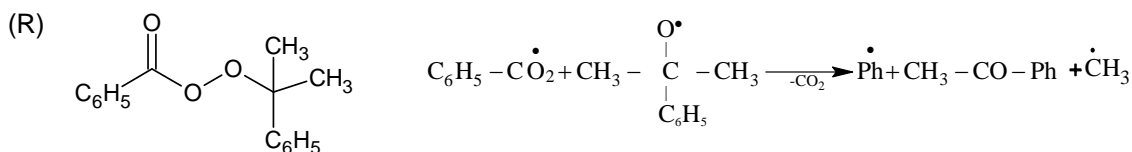
39. (P) - 1; (Q) - 3; (R) - 4; (S) - 2

(P)

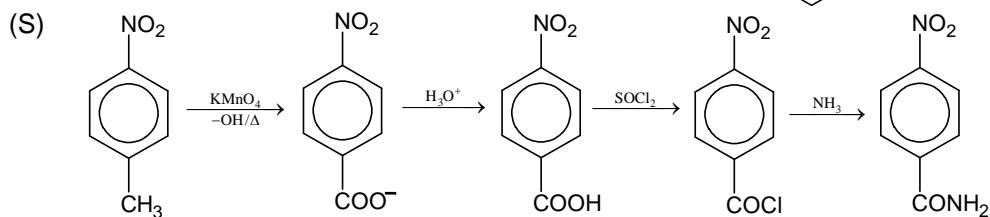
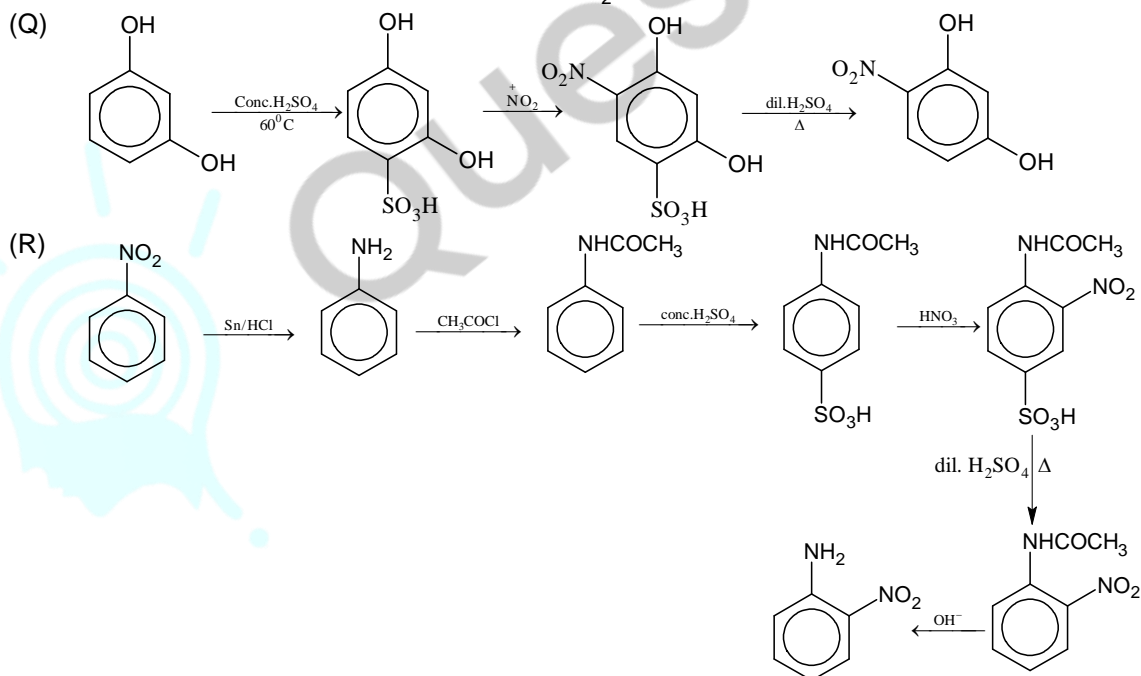
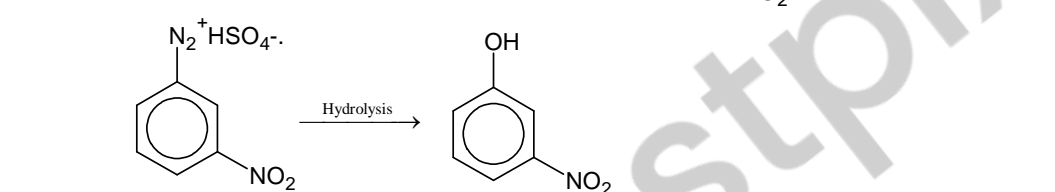
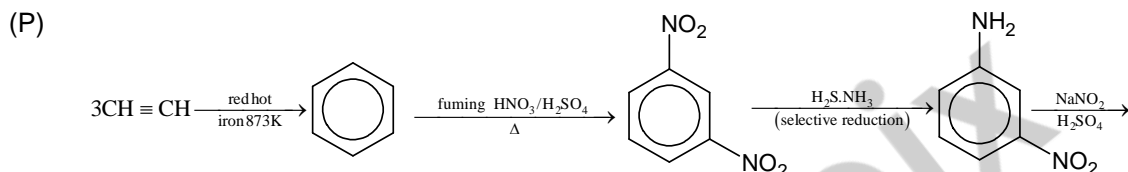


(Q)





40.



PART III : MATHEMATICS

41. Either a girl will start the sequence or will be at second position and will not acquire the last position as well.

$$\text{Required probability} = \frac{{}^3C_1 + {}^2C_1}{{}^5C_2} = \frac{1}{2}.$$

42. $x = a + b$

$$y = ab$$

$$x^2 - c^2 = y$$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2} = \cos(120^\circ)$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

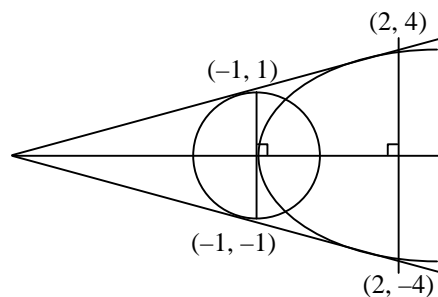
$$\Rightarrow R = \frac{abc}{4\Delta}, r = \frac{\Delta}{s}$$

$$\Rightarrow \frac{r}{R} = \frac{4\Delta^2}{s(abc)} = \frac{4\left[\frac{1}{2}ab\sin\left(\frac{2\pi}{3}\right)\right]^2}{\frac{x+c}{2} \cdot y \cdot c}$$

$$\frac{r}{R} = \frac{3y}{2c(x+c)}.$$

43. Number of required ways = $5! - \{4 \cdot 4! - {}^4C_2 \cdot 3! + {}^4C_3 \cdot 2! - 1\} = 53$.

44. Area = $\left(\frac{1}{2}(1+4)3\right) \times 2 = 15$



45. $P(x) = ax^2 + b$ with a, b of same sign.

$$P(P(x)) = a(ax^2 + b)^2 + b$$

$$\text{If } x \in \mathbb{R} \text{ or } ix \in \mathbb{R}$$

$$\Rightarrow x^2 \in \mathbb{R}$$

$$\Rightarrow P(x) \in \mathbb{R}$$

$$\Rightarrow P(P(x)) \neq 0$$

Hence real or purely imaginary number can not satisfy $P(P(x)) = 0$.

46. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \operatorname{cosec} x)^{17} dx$

$$\text{Let } e^u + e^{-u} = 2 \operatorname{cosec} x, x = \frac{\pi}{4} \Rightarrow u = \ln(1 + \sqrt{2}), x = \frac{\pi}{2} \Rightarrow u = 0$$

$$\Rightarrow \operatorname{cosec} x + \cot x = e^u \text{ and } \operatorname{cosec} x - \cot x = e^{-u} \Rightarrow \cot x = \frac{e^u - e^{-u}}{2}$$

$$(e^u - e^{-u}) dx = -2 \operatorname{cosec} x \cot x dx$$

$$\Rightarrow -\int (e^u + e^{-u})^{17} \frac{(e^u - e^{-u})}{2 \operatorname{cosec} x \cot x} du$$

$$= -2 \int_{\ln(1+\sqrt{2})}^0 (e^u + e^{-u})^{16} du$$

$$= \int_0^{\ln(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$$

47. $\frac{dy}{dx} + \frac{x}{x^2-1} y = \frac{x^4+2x}{\sqrt{1-x^2}}$

This is a linear differential equation

$$\text{I.F.} = e^{\int \frac{x}{x^2-1} dx} = e^{\frac{1}{2} \ln|x^2-1|} = \sqrt{1-x^2}$$

\Rightarrow solution is

$$y\sqrt{1-x^2} = \int \frac{x(x^3+2)}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} dx$$

$$\text{or } y\sqrt{1-x^2} = \int (x^4+2x) dx = \frac{x^5}{5} + x^2 + c$$

$$f(0) = 0 \Rightarrow c = 0$$

$$\Rightarrow f(x)\sqrt{1-x^2} = \frac{x^5}{5} + x^2$$

$$\text{Now, } \int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x) dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}} dx \quad (\text{Using property})$$

$$= 2 \int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}} dx = 2 \int_0^{\pi/3} \frac{\sin^2 \theta}{\cos \theta} \cos \theta d\theta \quad (\text{Taking } x = \sin \theta)$$

$$= 2 \int_0^{\pi/3} \sin^2 \theta d\theta = 2 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/3} = 2 \left(\frac{\pi}{6} \right) - 2 \left(\frac{\sqrt{3}}{8} \right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}.$$

48. $F(0) = 0$

$$F'(x) = 2x f(x) = f'(x)$$

$$f(x) = e^{x^2+c}$$

$$f(x) = e^{x^2} \quad (\because f(0) = 1)$$

$$F(x) = \int_0^{x^2} e^x dx$$

$$F(x) = e^{x^2} - 1 \quad (\because F(0) = 0)$$

$$\Rightarrow F(2) = e^4 - 1$$

49. $2x_1 + 3x_2 + 4x_3 = 11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).

∴ Required coefficients

$$\begin{aligned}
 &= ({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) + ({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1) \\
 &= (1 \times 7 \times 66) + (4 \times 35 \times 1) + (6 \times 7 \times 12) + (1 \times 7) \\
 &= 462 + 140 + 504 + 7 = 1113.
 \end{aligned}$$

50. $\sin x + 2 \sin 2x - \sin 3x = 3$
 $\sin x + 4 \sin x \cos x - 3 \sin x + 4 \sin^3 x = 3$
 $\sin x [-2 + 4 \cos x + 4(1 - \cos^2 x)] = 3$
 $\sin x [2 - (4 \cos^2 x - 4 \cos x + 1) + 1] = 3$
 $\sin x [3 - (2 \cos x - 1)^2] = 3$
 $\Rightarrow \sin x = 1 \text{ and } 2 \cos x - 1 = 0$

$$\Rightarrow x = \frac{\pi}{2} \text{ and } x = \frac{\pi}{3}$$

which is not possible at same time

Hence, no solution

51. Case I : One odd, 2 even

Total number of ways = $2 \times 2 \times 3 + 1 \times 3 \times 3 + 1 \times 2 \times 4 = 29$.

Case II: All 3 odd

Number of ways = $2 \times 3 \times 4 = 24$

Favourable ways = 53

$$\text{Required probability} = \frac{53}{3 \times 5 \times 7} = \frac{53}{105}.$$

52. Here $2x_2 = x_1 + x_3$

$\Rightarrow x_1 + x_3 = \text{even}$

Hence number of favourable ways = ${}^2C_1 \cdot {}^4C_2 + {}^1C_1 \cdot {}^3C_1 = 11$.

53. Slope (QR) = Slope (PK)

$$\frac{2at - 0}{at^2 - 2a} = \frac{-\frac{2a}{t} - 2ar}{\frac{a}{t^2} - ar^2}$$

$$\Rightarrow \frac{t}{t^2 - 2} = - \left(\frac{\frac{1}{t} + r}{\frac{1}{t^2} - r^2} \right) \Rightarrow r = \frac{t^2 - 1}{t}$$

54. Tangent at P: $ty = x + at^2$ or $y = \frac{x}{t} + at$

$$\text{Normal at S: } y + \frac{x}{t} = \frac{2a}{t} + \frac{a}{t^3}$$

$$\text{Solving, } 2y = at + \frac{2a}{t} + \frac{a}{t^3}$$

$$y = \frac{a(t^2 + 1)^2}{2t^3}$$

55. $g\left(\frac{1}{2}\right) = \lim_{h \rightarrow 0^+} \int_h^{1-h} t^{-1/2} (1-t)^{-1/2} dt$

$$= \int_0^1 \frac{dt}{\sqrt{t-t^2}} = \int_0^1 \frac{dt}{\sqrt{\frac{1}{4} - \left(t - \frac{1}{2}\right)^2}} = \sin^{-1} \left(\frac{t - \frac{1}{2}}{\frac{1}{2}} \right) \Bigg|_0^1$$

$$= \sin^{-1} 1 - \sin^{-1}(-1) = \pi.$$

56. We have $g(a) = g(1-a)$ and g is differentiable

Hence $g'\left(\frac{1}{2}\right) = 0$.

57. (P) $f(x) = ax^2 + bx$, $\int_0^1 f(x) dx = 1$

$$\Rightarrow 2a + 3b = 6$$

$$\Rightarrow (a, b) \equiv (0, 2) \text{ and } (3, 0).$$

(Q) $f(x) = \sqrt{2} \cos\left(x^2 - \frac{\pi}{4}\right)$

For maximum value, $x^2 - \frac{\pi}{4} = 2n\pi$

$$\Rightarrow x^2 = 2n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = \pm \sqrt{\frac{\pi}{4}}, \pm \sqrt{\frac{9\pi}{4}} \text{ as } x \in [-\sqrt{3}, \sqrt{13}].$$

(R) $\int_0^2 \left(\frac{3x^2}{1+e^x} + \frac{3x^2}{1+e^{-x}} \right) dx = \int_0^2 3x^2 dx = 8.$

(S) $\int_{-1/2}^{1/2} \cos 2x \ln\left(\frac{1+x}{1-x}\right) dx = 0$ as it is an odd function.

58. (P) $y = \cos(3 \cos^{-1} x)$

$$y' = \frac{3 \sin(3 \cos^{-1} x)}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} y' = 3 \sin(3 \cos^{-1} x)$$

$$\Rightarrow \frac{-x}{\sqrt{1-x^2}} y' + \sqrt{1-x^2} y'' = 3 \cos(3 \cos^{-1} x) \cdot \frac{-3}{\sqrt{1-x^2}}$$

$$\Rightarrow -xy' + (1-x^2)y'' = -9y$$

$$\Rightarrow \frac{1}{y} \left[(x^2-1)y'' + xy' \right] = 9.$$

(Q) $(a_k \times a_{k+1}) = r^2 \sin \frac{2\pi}{n}$

$$a_k \cdot a_{k+1} = r^2 \cos \frac{2\pi}{n}$$

$$\Rightarrow \left| \sum_{k=1}^{n-1} \bar{a}_k \times \bar{a}_{k+1} \right| = \left| \sum_{k=1}^{n-1} a_k \cdot a_{k+1} \right|$$

$$\Rightarrow r^2 (n-1) \sin \frac{2\pi}{n} = r^2 (n-1) \cos \frac{2\pi}{n}$$

$$\tan \frac{2\pi}{n} = 1 \Rightarrow n = \frac{8}{4k+1}$$

$$\Rightarrow n = 8.$$

$$(R) \frac{h^2}{6} + \frac{l^2}{3} = 1, h = \pm 2$$

$$\text{Tangent at } (2, 1) \text{ is } \frac{2x}{6} + \frac{y}{3} = 1 \Rightarrow x + y = 3.$$

$$(S) \tan^{-1} \left(\frac{1}{2x+1} \right) + \tan^{-1} \frac{1}{4x+1} = \tan^{-1} \frac{2}{x^2}$$

$$\tan^{-1} \left(\frac{3x+1}{4x^2+3x} \right) = \tan^{-1} \frac{2}{x^2}$$

$$\Rightarrow 3x^2 - 7x - 6 = 0$$

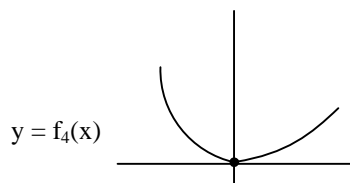
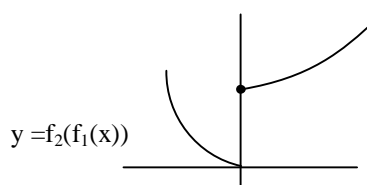
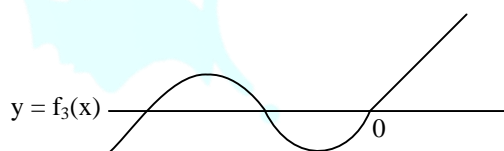
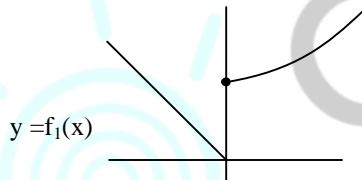
$$x = -\frac{2}{3}, 3.$$

$$59. f_2(f_1) = \begin{cases} x^2, & x < 0 \\ e^{2x}, & x \geq 0 \end{cases}$$

$$f_4: \mathbb{R} \rightarrow [0, \infty)$$

$$f_4(x) = \begin{cases} f_2(f_1(x)) & , x < 0 \\ f_2(f_1(x)) - 1 & , x \geq 0 \end{cases}$$

$$= \begin{cases} x^2 & , x < 0 \\ e^{2x} - 1 & , x \geq 0 \end{cases}$$



$$60. (P) z_k \text{ is } 10^{\text{th}} \text{ root of unity} \Rightarrow \bar{z}_k \text{ will also be } 10^{\text{th}} \text{ root of unity. Take } z_j \text{ as } \bar{z}_k.$$

$$(Q) z_1 \neq 0 \text{ take } z = \frac{z_k}{z_1}, \text{ we can always find } z.$$

$$(R) z^{10} - 1 = (z - 1)(z - z_1) \dots (z - z_9)$$

$$\Rightarrow (z - z_1)(z - z_2) \dots (z - z_9) = 1 + z + z^2 + \dots + z^9 \quad \forall z \in \text{complex number.}$$

Put $z = 1$

$$(1 - z_1)(1 - z_2) \dots (1 - z_9) = 10.$$

(S) $1 + z_1 + z_2 + \dots + z_9 = 0$

$$\Rightarrow \operatorname{Re}(1) + \operatorname{Re}(z_1) + \dots + \operatorname{Re}(z_9) = 0$$

$$\Rightarrow \operatorname{Re}(z_1) + \operatorname{Re}(z_2) + \dots + \operatorname{Re}(z_9) = -1.$$

$$\Rightarrow 1 - \sum_{k=1}^9 \cos \frac{2k\pi}{10} = 2.$$