

(HELD ON THURSDAY 23 rd JANUARY 2025)				
	MATHEMATICS			
	SECTIO	DN-A		
1.	The value of $\int_{e^2}^{e^4} \frac{1}{x} \left(\frac{1}{e^{(\log_e x)}} \right)^{1/2} dx$	$\frac{e^{\left((\log_{e} x)^{2}+1\right)^{-1}}}{\left((6-\log_{e} x)^{2}+1\right)^{-1}}\right) dx \text{ is }$		
	$(1) \log_{e} 2$	(2) 2		
	(3) 1	(4) e^2		
Ans.	(3)			
Sol.	Let $\ln x = t \Rightarrow \frac{dx}{x} = dt$			
	$I = \int_{2}^{4} \frac{e^{\frac{1}{1+t^{2}}}}{\frac{1}{1+t^{2}} + e^{\frac{1}{1+(6-t)^{2}}}} dt$			
	$I = \int_{2}^{4} \frac{e^{\frac{1}{1+(6-t)^{2}}}}{e^{\frac{1}{1+(6-t)^{2}}} + \frac{1}{1+t^{2}}} dt$			
	$2I = \int_{2}^{4} dt = (t)_{2}^{4} = 4 - 2 =$	= 2		
	I = 1			
2.	Let I(x) = $\int \frac{dx}{(x-11)^{\frac{11}{13}}} (x)^{\frac{11}{13}}$	$(15)^{\frac{15}{13}}$.		
	If I(37) - I(24) = $\frac{1}{4} \left(-\frac{1}{b} \right)$	$\frac{1}{\frac{1}{13}} - \frac{1}{\frac{1}{13}}$, b, c $\in \mathbb{N}$, then		
	3(b + c) is equal to			
	(1) 40	(2) 39		
	(3) 22	(4) 26		
Ans.	(2)			
Sol.	$I(x) = \int \frac{dx}{(x - 11)^{\frac{11}{13}} (x - 12)^{\frac{11}{13}}}$	$\overline{5)^{\frac{15}{13}}}$		
	$\operatorname{Put} \frac{x-11}{x+15} = t \implies \frac{26}{(x+5)}$	$\int_{0}^{2} dx = dt$		

TIME : 9:00 AM TO 12:00 NOON

TEST PAPER WITH SOLUTION

$$I(x) = \frac{1}{26} \int \frac{dt}{t^{11/13}} = \frac{1}{26} \cdot \frac{t^{2/13}}{2/13}$$

$$I(x) = \frac{1}{4} \left(\frac{x-11}{x+15}\right)^{2/13} C$$

$$I(37) - I(24) = \frac{1}{4} \left(\frac{26}{52}\right)^{2/13} - \frac{1}{4} \left(\frac{13}{39}\right)^{2/13}$$

$$= \frac{1}{4} \left(\frac{1}{2^{2/13}} - \frac{1}{2^{2/13}}\right)$$

$$\therefore b = 4, c = 9$$

$$3(b + c) = 39$$
If the function
$$f(x) \begin{cases} \frac{2}{x} \left\{ \sin(k_1 + 1)x + \sin(k_1 - 1)x \right\}, & x < 0 \\ \frac{2}{x} \log_e \left(\frac{2 + k_1 x}{2 + k_2 x}\right), & x > 0 \end{cases}$$
is continuous at x = 0, then $k_1^2 + k_2^2$ is equal to
(1) 8 (2) 20

(1)(3) 5 (4) 10

Ans. (4)

3.

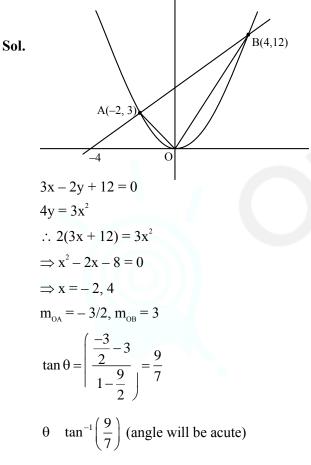
Sol.
$$\lim_{x \to 0^{-}} \frac{2}{x} \left\{ \sin(k_{1}+1)x + \sin(k_{1}-1)x \right\} = 4$$
$$\Rightarrow 2(k_{1}+1) + 2(k_{2}-1) = 4$$
$$\Rightarrow k_{1} + k_{2} = 2$$
$$\Rightarrow \lim_{x \to 0^{+}} \frac{2}{x} \ln\left(\frac{2+k_{1}x}{2+k_{2}x}\right) = 4$$
$$\Rightarrow \lim_{x \to 0^{+}} \frac{1}{x} \ln\left(1 + \frac{(k_{1}-k_{1})x}{2+k_{2}x}\right) = 2$$
$$\Rightarrow \frac{k_{1}-k_{2}}{2} = 2$$

Questpix_ \Rightarrow k₁ - k₂ = 4 $\therefore k_1 = 3, k_2 = -1$ $k_1^2 + k_2^2 = 9 + 1 = 10$

If the line 3x - 2y + 12 = 0 intersects the parabola 4. $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1)
$$\tan^{-1}\left(\frac{11}{9}\right)$$
 (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$
(3) $\tan^{-1}\left(\frac{4}{5}\right)$ (4) $\tan^{-1}\left(\frac{9}{7}\right)$

Ans. (4)



5. Let a curve y = f(x) pass through the points (0, 5) and (log₂, k). If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - (7 + e^{2x})dy = 0$, then k is equal to

(1) 16	(2) 8
(2) 22	

(3) 32 (4) 4 Ans. (2)

Sol.
$$\frac{dy}{dx} = \frac{2(3+y) \cdot e^{2x}}{7 + e^{2x}}$$

 $\frac{dy}{dx} - \frac{2y \cdot e^{2x}}{7 + e^{2x}} = \frac{6 \cdot e^{2x}}{7 - e^{2x}}$
I.F. $= e^{-\int \frac{2e^{2x}}{7 + e^{2x}}} \frac{1}{7 + e^{2x}}$
 $\therefore y \cdot \frac{1}{7 + e^{2x}} = \int \frac{6e^{2x}}{(7 + 3^{2x})^2} dx$
 $\frac{y}{7 + e^{2x}} = \frac{-3}{7 - e^{2x}} + C$
 $(0, 5) \Rightarrow \frac{5}{8} = \frac{-3}{8} + C \Rightarrow C = 1$
 $\therefore y = -3 + 7 + e^{2x}$
 $y = e^{2x} + 4$
 $\therefore k = 8$
6. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$
Then the domain of fog is
 $(1) \mathbb{R}$ $(2) (0, \infty)$

(1) 🔣

(3)
$$[0, \infty)$$
 (4) $[1, \infty)$

Ans. (1)

6.

Sol.
$$f(x) = \ln x$$

$$g(x) = \frac{x^{4} - 2x^{3} + 3x^{2} - 2x + 2}{2x^{2} - 2x - 1}$$

$$D_{g} \in \mathbb{R}$$

$$D_{f} \in (0, \infty)$$
For $D_{fog} \Rightarrow g(x) > 0$

$$\frac{x^{4} - 2x^{3} + 3x^{2} - 2x + 2}{2x^{2} - 2x + 1} > 0$$

$$\Rightarrow x^{4} - 2x^{3} + 3x^{2} - 2x + 2 > 0$$

$$(1 - 1) = 2x^{3} + 3x^{2} - 2x + 2 > 0$$

Clearly x < 0 satisfies which are included in option (1) only.

Questpix Let the arc AC of a circle subtend a right angle at 7. the centre O. If the point B on the arc AC, divides $\frac{\text{length of arc AB}}{\text{length of arc BC}} = \frac{1}{5}$, and the arc AC such that $\overrightarrow{OC} = \alpha \overrightarrow{OA} + \beta \overrightarrow{OB}$, then $\alpha = \sqrt{2} (\sqrt{3} - 1)\beta$ is equal to (1) $2 - \sqrt{3}$ (2) $2\sqrt{3}$ (4) $2 + \sqrt{3}$ (3) $5\sqrt{3}$ Ans. (1) Sol. 75° $\vec{c} = \alpha \vec{a} + \beta \vec{b}$ (1) $\vec{a}.\vec{c} = \alpha \vec{a}.\vec{a} + \beta \vec{b}.\vec{a}$ $0 = \alpha + \beta \cos 15^{\circ} \dots (2)$ (1) $\Rightarrow \vec{b}.\vec{c} = \alpha \vec{a}.\vec{b} + \beta \vec{b}.\vec{b}$ $\Rightarrow \cos 75^\circ = \alpha \cos 15^\circ + \beta \dots (3)$ (2) & (3) $\Rightarrow \cos 75^\circ = -\beta \cos^2 15^\circ + \beta$ $\beta = \frac{\cos 75^{\circ}}{\sin^2 15^{\circ}} = \frac{1}{\sin 15^{\circ}} = \frac{2\sqrt{2}}{\sqrt{-1}}$ (2) $\Rightarrow \alpha = \frac{-\cos 15^{\circ}}{\sin 15^{\circ}} = \frac{-(\sqrt{3})}{(\sqrt{3})}$ $\therefore \vec{c} = \frac{-\left(\sqrt{3}\right)}{\left(\sqrt{3}-\right)}\vec{a} \quad \left(\frac{2\sqrt{3}}{\sqrt{3}-1}\vec{b}\right)$ Now

$$\alpha + \sqrt{2} \left(\sqrt{3} - 1 \right) \beta = \frac{-\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - \right)} + \frac{\sqrt{2} \left(\sqrt{3} - 1\right) \cdot 2\sqrt{2}}{\sqrt{3} - 1}$$

$$= \frac{-(\sqrt{3}+1)^2}{2} + 4$$
$$= \frac{-3-1-2\sqrt{3}+8}{2}$$
$$= 2-\sqrt{3}$$

8. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

$$(1) -1200 \qquad (2) -1080$$

Ans. (2)

Sol.
$$a = 3$$

 $S_4 = \frac{1}{5}(S_8 - S_4)$
 $\Rightarrow 5S_4 = S_8 - S_4$
 $\Rightarrow 6S_4 = S_8$
 $\Rightarrow 6.\frac{4}{2}[2 \times 3 + (4 - 1) \times d]$
 $= \frac{8}{2}[2 \times 3 + (8 - 1)d]$
 $\Rightarrow 12(6 + 3d) = 4(6 + 7d)$
 $\Rightarrow 18 + 9d = 6 + 7d$
 $\Rightarrow d = -6$
 $S_{20} = \frac{20}{2}[2 \times 3 + (20 - 1)(-6)]$
 $= 10[6 - 114]$
 $= -1080$
9 Let P be the foot of the perpendicular

9. Let P be the foot of the perpendicular from the point

- Q(10, -3, -1) on the line $\frac{x-3}{7} = \frac{y-2}{1} \frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point (3, -2, 1), is
 - (1) $9\sqrt{15}$ (2) $\sqrt{30}$
 - (3) $8\sqrt{15}$ (4) $3\sqrt{30}$

Ans. (4)



Sol.

$$Q(10,-3,-1)$$

$$P(\alpha,\beta,\gamma) = (10,1,-3)$$

$$R(3,-2,1)$$

$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2} = \lambda$$

$$\Rightarrow 7\lambda + 3, -\lambda + 2, -2\lambda - 1$$
dr's of QP \Rightarrow

$$7\lambda - 7, -\lambda + 5, -2\lambda$$
Now

$$(7\lambda - 7) \cdot 7 - (-\lambda + 5) + (2\lambda) \cdot 2 = 0$$

$$54\lambda - 54 = 0 \Rightarrow \lambda = 1$$

$$\therefore P = (10, 1, -3)$$

$$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$$

$$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$$
Area $= \left| \frac{1}{2} \begin{vmatrix} i & j & k \\ 0 & 4 & 2 \\ 7 & -3 & 4 \end{vmatrix} \right| = 3\sqrt{30}$

10. Let $\left|\frac{\overline{z}-i}{2\overline{z}+i}\right| = \frac{1}{3}$, $z \in \mathbb{C}$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and (α , 0) is 11 square units, then α^2 equals (1) 100 (2) 50

81

25

(3)
$$\frac{121}{25}$$
 (4)

Ans. (1)

Sol. $\left| \frac{\overline{z} - i}{2\overline{z} + i} \right| = \frac{1}{3}$ $\left| \frac{\overline{z} - i}{\overline{z} + \frac{i}{2}} \right| = \frac{2}{3}$ $3|x - iy - i| = 2|x - iy + \frac{i}{2}|$ $9(x^2 + (y+1)^2) = 4(x^2 + (y - 1/3)^2)$ $9x^2 + 9y^2 + 18y + 9 = 4x^2 + 4y^2 - 4y + 1$ $5x^2 + 5y^2 + 22y + 8 = 0$

$$x^{2} + y^{2} + \frac{22}{5}y + \frac{8}{5} = 0$$

centre $\Rightarrow (0, -\frac{11}{5})$

$$\begin{vmatrix} \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 0 & -11/5 & 1 \\ \alpha & 0 & 1 \end{vmatrix} = 11$$

 $\Rightarrow \left(-\frac{11}{5}\alpha \right)^{2} = (11 \times 2)^{2}$
 $\Rightarrow \alpha^{2} = 100$

- 11. Let R = {(1, 2), (2, 3), (3,3)} be a relation defined on the set {1, 2, 3, 4}. Then the minimum number of elements, needed to be added in R so the R becomes an equivalence relation, is :
 - (1) 10 (2) 8

Ans. (4)

Sol. $A = \{1, 2, 3, 4\}$

For relation to be reflexive

 $\mathbf{R} = \{(1,2), (2,3), (3,3)\}$

Minimum elements added will be

(1,1), (2,2), (4,4) (2,1) (3,2) (3,2) (3,1) (1,3)

 \therefore Minimum number of elements = 7

Option : (4)

- **12.** The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is
 - (1) 34000(2) 37000(3) 36000(4) 35000

Ans. (3)

Sol. DAUGHTER

Total words = 8!

Total words in which vowels are together = $6! \times 3!$

words in which all vowels are not together

 $= 8! - 6! \times 3!$ = 6! [56 - 6] = 720 × 50 = 36000 Ans.(3)



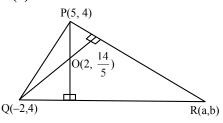
13. Let the area of a \triangle PQR with vertices P(5, 4), Q(-2, 4) and R(a, b) be 35 square units. If its orthocenter and centroid are O $\left(2, \frac{14}{5}\right)$ and C(c, d) respectively, then c + 2d is equal to

 $(4) \frac{8}{3}$

(1)
$$\frac{7}{3}$$
 (2) 3

Ans. (2)

Sol.



Equation of lines QR = 5x + 2y + 2 = 0Equation of lines PR = 10x - 3y - 38 = 0 \therefore Point R (2, -6) Centroid = $\left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right)$

$$=\left(\frac{5}{3},-\right)$$

$$c + 2d = \frac{5}{3} + \frac{4}{3} = 3$$

3

14. If
$$\frac{\pi}{-1} \le x \le \frac{3}{4}$$
, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is
equal to
(1) $x - \tan^{-1}\frac{4}{3}$ (2) $x - \tan^{-1}\frac{5}{12}$

12

(3)
$$x + \tan^{-1}\frac{4}{5}$$
 (4) $x + \tan^{-1}\frac{4}{5}$

Ans. (2)

S

ol.
$$\frac{\pi}{2} \le x \le \frac{5}{-1}$$

 $\cos^{-1} \frac{12}{13} \cos x + \frac{5}{12} \sin x$
 $\cos^{-1}(\cos x \cos \alpha + \sin x \sin \alpha)$
 $\cos^{-1}(\cos(x-\alpha))$
 $\Rightarrow x - \alpha$ because $x - \alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$
 $\Rightarrow x - \tan^{-1} \frac{5}{-12}$

12

15. The value of $(\sin 70^\circ)(\cot 10^\circ \cot 70^\circ - 1)$ is (1) 1 (2) 0

(3) 3/2 (4) 2/3

Ans. (1)

Sol. $\sin 70^{\circ} (\cot 10^{\circ} \cot 70^{\circ} - 1)$

$$\Rightarrow \frac{\cos(80^\circ)}{\sin 10} = 1$$

16. Marks obtains by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

(2) 44(4) 52

Ans. (2)

Sol. medain
$$= \ell + \left(\frac{\frac{N}{2} - F}{f}\right) \times 1$$

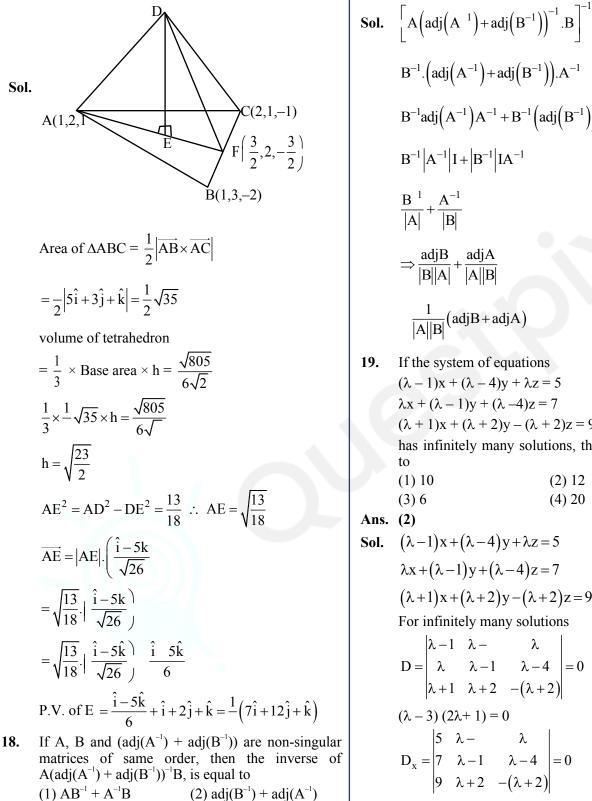
$$=12 + \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 14$$
$$\Rightarrow \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 =$$

 $\frac{N}{2} - 18 = 4 \implies N = 44$ **17.** Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then

(1)
$$\frac{1}{2}(\hat{i}+4\hat{j}+7\hat{k})$$
 (2) $\frac{1}{12}(7\hat{i}+4\hat{j}+3\hat{k})$
(3) $\frac{1}{6}(12\hat{i}+12\hat{j}+\hat{k})$ (4) $\frac{1}{6}(7\hat{i}+12\hat{j}+\hat{k})$

Ans. (4)





(3) $\frac{1}{|AB|}(adj(B) + adj(A))(4) \frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|A|}$

$$B^{-1} \cdot \left(adj (A^{-1}) + adj (B^{-1}) \right) \cdot A^{-1}$$

$$B^{-1}adj (A^{-1}) A^{-1} + B^{-1} \left(adj (B^{-1}) \right) \cdot A^{-1}$$

$$B^{-1} |A^{-1}| I + |B^{-1}| IA^{-1}$$

$$\frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$$

$$\Rightarrow \frac{adjB}{|B||A|} + \frac{adjA}{|A||B|}$$

$$\frac{1}{|A||B|} \left(adjB + adjA \right)$$
19. If the system of equations
 $(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$
 $\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$
 $(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$
has infinitely many solutions, then $\lambda^{2} + \lambda$ is equal to
(1) 10 (2) 12
(3) 6 (4) 20
Ans. (2)
Sol. $(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$
 $\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$
 $(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$
 $\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$
 $(\lambda - 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$
For infinitely many solutions
 $D = \begin{vmatrix} \lambda - 1 & \lambda - & \lambda \\ \lambda & \lambda - 1 & \lambda - 4 \\ \lambda + 1 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$
 $(\lambda - 3) (2\lambda + 1) = 0$
 $D_x = \begin{vmatrix} 5 & \lambda - & \lambda \\ 7 & \lambda - 1 & \lambda - 4 \\ 9 & \lambda + 2 & -(\lambda + 2) \end{vmatrix} = 0$
 $2(3 - \lambda) (23 - 2\lambda) = 0$
 $\lambda = 3$
 $\therefore \lambda^{2} + \lambda = 9 + 3 = 12$



20. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

(1)
$$\frac{1}{2}$$
 (2) $\frac{3}{5}$
(3) $\frac{2}{3}$ (4) $\frac{4}{9}$

Ans. (1)

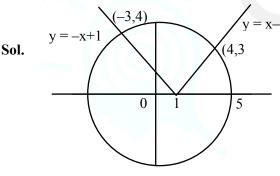
Sol. a = number or dice 1 b = number on dice 2 (a,b) = (1,3), (3,1), (2,2), (2,3), (3,2), (1,4), (4,1) Required probability $= \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{1}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{2}{6} + \frac{1}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6}$ $= \frac{18}{36} = \frac{1}{2}$

SECTION-B

21. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and y = |x - 1| is $\frac{1}{4}(b\pi + c)$,

b, $c \in \mathbb{N}$, then b + c is equal to

Ans. (77)



$$x^{2} + y^{2} = 5$$

$$x^{2} + (x - 1)^{2} = 25 \implies x = 4$$

$$x^{2} + (-x + 1)^{2} = 5 \implies x = -3$$

$$A = 25\pi - \int_{-3}^{4} \sqrt{25 - x^{2}} dx + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 3 \times 3$$

$$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2}\sqrt{25 - x^{2}} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4}$$

$$A = 25\pi + \frac{25}{2} - \left\lfloor 6 + \frac{25}{2} \sin^{-1} \frac{4}{5} + 6 + \frac{25}{2} \sin^{-1} \frac{3}{5} \right\rfloor$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2} \cdot \frac{\pi}{2} ; \quad A = \frac{75\pi}{4} + \frac{1}{2}$$

$$A = \frac{1}{4} (75\pi + 2)$$

$$b = 75, c = 2$$

$$b + c = 75 + 2 = 77$$

22. The sum of all rational terms in the expansion of $(1 + 2^{1/3} + 3^{1/2})^6$ is equal to _____

Sol.
$$\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^{6}$$

$$=\frac{\left|\frac{6}{\left|\frac{r_{1}}{\left|\frac{r_{2}}{\left|\frac{r_{3}}{2}\right|}\right|^{r_{1}}}(1)^{r_{1}}(2)^{\frac{r_{2}}{3}}(3)^{\frac{r_{3}}{2}}}{\frac{r_{3}}{2}}$$

$$\frac{\frac{r_{1}}{6} \frac{r_{2}}{1}}{\frac{r_{2}}{4}}$$

$$\frac{r_{3}}{6} \frac{r_{3}}{1}$$

$$\frac{r_{2}}{2} \frac{r_{3}}{4}$$

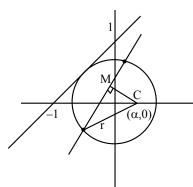
$$\frac{r_{3}}{2} \frac{r_{3}}{4}$$

$$\frac{r_{3}}{4} \frac{r_{3}$$

the centre of the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line -3x + 2y = 1. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{1}{\beta}$ 1, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to ______



Sol.



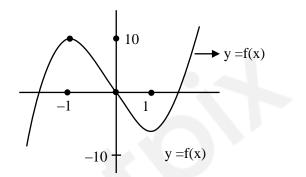
x - y + 1 = 0; p = r $\left|\frac{\alpha - 0 \quad 1}{\sqrt{2}}\right| = r \Longrightarrow (\alpha + 1)^2 = 2r^2 \dots (1)$ now $\left(\frac{-3\alpha+0-1}{\sqrt{9+4}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 r^2$ $\Rightarrow (3\alpha + 1)^2 + 4 = 13 r^2 \dots (2)$ (1) & (2) \Rightarrow $(3\alpha + 1)^2 + 4 = 13 \frac{(\alpha + 1)^2}{2}$ $\Rightarrow 18\alpha^2 + 12\alpha + 2 + 8 = 13\alpha^2 + 26\alpha + 13$ $\Rightarrow 5\alpha^2 - 14\alpha - 3 = 0$ $\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0; \Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$ $\Rightarrow \alpha = \frac{-1}{5}, 3$ \therefore r = $2\sqrt{2}$ How $\alpha e = 3$ and $2\alpha = 4\sqrt{2}$ $\alpha^2 e^2 = 9 \implies \alpha = 2\sqrt{2} \implies \alpha^2 = 8$ $\alpha^2 \left(1 + \frac{\beta^2}{\alpha^2} \right) =$ $\alpha^2 + \beta^2 = 9$ $\therefore \beta^2 = 1$ $\therefore 2\alpha^2 + 3\beta^2 = 2(8) + 3(1) = 19$

24. If the set of all values of a, for which the equation $5x^{3} - 15x - a = 0$ has three distinct real roots, is the interval (α, β) , then $\beta - 2\alpha$ is equal to _____

Ans. (30) **Sol.** $5x^3 - 15x - a = 0$

$$f(x) = 5x^3 - 15x$$

$$f(x) = 15x^2 - 15 = 15(x-1)(x+1)$$



 $a \in (-10, 10)$

 $\alpha = -10, \beta = 10$

$$\beta - 2\alpha = 10 + 20 = 30$$

If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ 25. has equal roots, where a + c = 15 and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to _____

Ans. (117)

Sol.
$$a(b-c) x^2 + b (c-a) x + c(a-b) = 0$$

 $x = 1$ is root \therefore other root is 1
 $\alpha + \beta = -\frac{b(c-a)}{a(b-c)} = 2$
 $\Rightarrow -bc + ab = 2ab - 2ac$
 $\Rightarrow 2ac = ab + bc$
 $\Rightarrow 2ac = b(a + c)$

$$\Rightarrow 2ac = 0(a + c)$$
$$\Rightarrow 2ac = 15b \dots (1)$$
$$\Rightarrow 2ac = 15 \quad \frac{36}{5} = 108$$

$$\Rightarrow ac = 54$$

$$a + c = 15$$

$$a^{2} + c^{2} + 2ac = 225$$

$$a^{2} + c^{2} = 225 - 108 = 117$$



(HELD ON THURSDAY 23rd JANUARY 2025) TIME : 9:00 AM TO 12:00 NOON PHYSICS **TEST PAPER WITH SOLUTION SECTION-A** 28. Given below are two statements: Statement-I: The hot water flows faster than cold Regarding self-inductance : 26. A : The self-inductance of the coil depends on its water. Statement-II : Soap water has higher surface geometry. B : Self-inductance does not depend on the tension as compared to fresh water. permeability of the medium. In the light above statements, choose the correct C : Self-induced e.m.f. opposes any change in the answer from the options given below current in a circuit. (1) Statement-I is false but Statement II is true D : Self-inductance is electromagnetic analogue of (2) Statement-I is true but Statement II is false mass in mechanics. (3) Both Statement-I and Statement-II are true E : Work needs to be done against self-induced (4) Both Statement-I and Statement-II are false e.m.f. in establishing the current. Ans. (2) Choose the correct answer from the options given **Sol.** Hot water is less viscous then cold water. below: Surfactant reduces surface tension. (2) A, C, D, E only (1) A, B, C, D only A sub-atomic particle of mass 10^{-30} kg is moving 29. (3) A, B, C, E only (4) B, C, D, E only with a velocity 2.21×10^6 m/s. Under the matter wave consideration, the particle will behave Ans. (2) closely like _____. (h = 6.63×10^{-34} J.s) **Sol.** Self inductance of coil (1) Infra-red radiation (2) X-rays $L = \frac{\mu_0 N^2 A}{2\pi R}$ (4) Visible radiation (3) Gamma rays 27. A light hollow cube of side length 10 cm and mass Ans. (2) **Sol.** $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-30} \times 2.21 \ 10^6}$ 10g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, $= 3 \times 10^{-10} m$ where the value of y is Hence particle will behave as x-ray. (Acceleration due to gravity, $g = 10 \text{ m/s}^2$, density 30. A spherical surface of radius of curvature R, of water = 10^3 kg/m^3) separates air from glass (refractive index = 1.5). (1)2(2) 6The centre of curvature is in the glass medium. A (3)4(4)1point object 'O' placed in air on the optic axis of Ans. (1) the surface, so that its real image is formed at 'I' **Sol.** $a^2x \rho g = ma_{net}$ inside glass. The line OI intersects the spherical surface at P and PO = PI. The distance PO equals $\frac{L^2 \rho g}{m} x = a_{net}$ to- $T = 2\pi \sqrt{\frac{m}{L^2 \rho g}}$ (1) 5R (2) 3R(3) 2R (4) 1.5R Ans. (1) where m = 10g, L = 10 cm, $\rho = 1000 \text{ kg/m}^3$



Sol.	$\begin{array}{c c} \mu=1 & \mu=1.5\\ air & glass\\ \hline O & P & I \\ \end{array}$
	PO = u = -x $PI = v = x$ $PO = PI$
	$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ $\frac{1.5}{r} + \frac{1}{r} = \frac{1}{r}$
	$x = x = 2R$ $\frac{5}{2x} = \frac{1}{2R}$ $X = 5R$

31. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n₂ to the number of nuclei of n_1 , after one half-life of n_1 ?

(1) 1/4	(2) 1/8
(3) 4	(4) 8

Ans. (1)

- **Sol.** $N_2 = N_0 e^{-3\lambda t}$ $N_1 = N_0 e^{-\lambda t}$
 - $\frac{N_2}{N_2} = e^{-2\lambda}$ $t_{half life of N} = \frac{\ln 2}{1}$

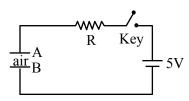
$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\lambda t = \ln 2$$
$$t = \frac{\ln 2}{\lambda}$$
$$= e^{-2\lambda \frac{\ln 2}{\lambda}}$$

$$\frac{N_2}{N_2} = \frac{1}{1}$$

$$\frac{1}{N_1} = \frac{1}{N_1}$$

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



- A. There will be no current through resistor R.
- B. There will be maximum current in the connecting wires.
- C. Potential difference between the capacitor plates A and B is minimum.
- D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below :

(1) C, D only	(2) B, C, D only	
(3) A, C only	(4) A, B, D only	

Ans. (2)

Sol. Initially capacitor behave as a short circuit.

So current will be maximum.

Charge on capacitor will be zero.

Potential difference across capacitor will be zero.

33. The position of a particle moving on x-axis is given by $x(t) = A \sin t + B \cos^2 t + Ct^2 + D$, where t

is time. The dimension of $\frac{ABC}{D}$ is-

(1) L (2)
$$L^{3} T^{-2}$$

(3) $L^{2} T^{-2}$ (4) L^{2}

Ans. (3)

Dimension [x(t)] = [L]Sol. $[\mathbf{A}] = [\mathbf{L}]$ [B] = [L] $[C] = [LT^{-2}]$ [D] = [L]

Questpix

34. Match the List-I with List-II

	List-I		List-II
А.	Pressure varies	I.	Adiabatic
	inversely with volume		process
	of an ideal gas.		
B.	Heat absorbed goes	II.	Isochoric
	partly to increase		process
	internal energy and		
	partly to do work.		
C.	Heat is neither	III	Isothermal
	absorbed nor released		process
	by a system		
D.	No work is done on or	IV	Isobaric
	by a gas	•	process

Choose the *correct* answer from the options given below :

(1) A–I, B–IV, C–II, D–III (2) A–III, B–I, C–IV, D–II

- (3) A–I, B–III, C–II, D–IV
- (4) A–III, B–IV, C–I, D–II

Ans. (4)

Sol.
$$A \rightarrow P \propto \frac{1}{V}$$

 \Rightarrow PV = constant

 \Rightarrow nRT = const. \Rightarrow T = const.

Hence Isothermal III

 $B \rightarrow IV$

 $W \neq 0, \Delta U \neq 0, \Delta Q \neq 0$ [only isobaric]

 $C \rightarrow I \Delta Q = 0$ Adiabatic

 $D \rightarrow II w = 0$ Isochoric

III IV I II

35. Consider a moving coil galvanometer (MCG) :

A : The torsional constant in moving coil galvanometer has dimensions $[ML^2T^{-2}]$

B : Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C : If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D : MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E : Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below :

(1) A, B only	(2) A, D, only	
(3) B, D, E only	(4) A, B, E only	

Ans. (1)

Sol. (A)
$$\tau = C\theta \Rightarrow [ML^2T^{-2}] = [C][1]$$

(B) C.S =
$$\frac{\theta}{I} = \frac{BNA}{I}$$

V.S. = $\frac{BNA}{RC}$ [R also depends on 'N']

(C) V.S.
$$\propto \frac{\text{NAB}}{\text{CR}}$$
 $R \rightarrow \text{NR}$

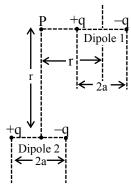
(D) False [Theory]

(E) E [False] $C.S \propto N$

 \Rightarrow : C.S. $\frac{\text{NAB}}{\text{C}}$



36. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance 2a. For the charge particle at P not to experience any net force, which of the following correctly describes the situation?



(1)
$$\frac{a}{r} - 20$$
 (2) $\frac{a}{r} \sim 10$
(3) $\frac{a}{r} \sim 0.5$ (4) $\frac{a}{r} \sim 3$

120

Ans. (4)

Sol.
$$\frac{kq}{(r-a)^{2}} \underbrace{\xrightarrow{\theta}}_{\theta} \underbrace{\frac{kq}{r^{2}+a^{2}}}_{\frac{\theta}{r^{2}+a^{2}}} \\ \frac{kq}{(r-a)^{2}} = \frac{kq}{(r-a)} + \frac{2kq}{(r+a^{2})}\cos\theta \\ \frac{1}{(r-a)^{2}} = \frac{1}{(r-a)^{2}} \frac{2a}{(r+a^{2})^{\frac{3}{2}}} \\ \frac{1}{(r-a)^{2}} - \frac{1}{(r-a)^{2}} \frac{2a}{(r+a^{2})^{\frac{3}{2}}} \\ \frac{4ra}{(r-a^{2})^{2}} = \frac{2a}{(r+a^{2})^{\frac{3}{2}}}$$

$$\Rightarrow \frac{2r}{\left(r^{2} - a^{2}\right)^{2}} = \frac{1}{\left(r^{2} + a^{2}\right)^{\frac{3}{2}}}$$
$$\frac{4r^{2}}{\left(r^{2} - a^{2}\right)^{4}} = \frac{1}{\left(r^{2} - a^{2}\right)^{3}}$$
$$\Rightarrow 4r^{2}(r^{2} + a^{2})^{3} = (r^{2} - a^{2})^{4}$$
$$4r^{8}\left(1 + \frac{a^{2}}{2}\right)^{3} = r\left(1 - \frac{a}{r}\right)$$
$$4\left(1 + \frac{a^{2}}{r^{2}}\right)^{3} = \left(1 - \frac{a^{2}}{r^{2}}\right)^{4}$$

Exact value cannot be solved in exam for this equation to be true

$$\left|\frac{a}{r}\right| > 1 \implies a > r$$

But point charge Q lies between charges of dipole 1 hence electric field cannot be zero.

There for it should be bonus.

But by solving from mathematical software we are getting $a/r \approx 3$.

37. A gun fires a lead bullet of temperature 300K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is _____ grams.

(Latent heat of fusion of lead = $2.5 \times 10^4 \text{ JKg}^{-1}$ and specific heat capacity of lead = $125 \text{ JKg}^{-1} \text{ K}^{-1}$)

(3) 10 (4) 5

Sol.
$$625 = ms\Delta T + mL$$

 $625 = m[125 \times 300 + 2.5 \times 10^4]$

$$625 = m[37500 + 25000]$$

 $625 = m[62500]$

$$m = \frac{1}{100} kg$$
$$M = 10 grams$$

Ans. (3)



What is the lateral shift of a ray refracted through a 38. parallel-sided glass slab of thickness 'h' in terms of the angle of incidence 'i' and angle of refraction 'r', if the glass slab is placed in air medium ?

sin r

(1)
$$\frac{h \tan(i-r)}{\tan r}$$
 (2)
$$\frac{h \cos(i-r)}{\sin r}$$

(3) h (4)
$$\frac{h \sin(i-r)}{\cos r}$$

(3) h

Ans. (4)

Sol. Formula base

 $h\sin(i-r)$ cosr

39. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1 . If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2 . The ratio of $v_1^2 : v_2^2$ is

> (1) 1 : $\sqrt{2}$ (2) 1 : 3 (4) 1 : $\sqrt{3}$ (3)1:2

Ans. (1)

Sol. Lsine

using WET

$$W_{g} = k_{r} - k_{i}$$
Mg L sin $\theta = k_{r} - k_{i}$
K.E. in pure rolling $\frac{1}{2}mV_{cm}^{2} + -I_{cm}$

$$= \frac{1}{2}mV^{2} + \frac{1}{2} \times \frac{2}{5}mR \quad \frac{V^{2}}{R^{2}}$$

$$\frac{7}{10}mV^{2}$$
mgL sin $\theta = \frac{7}{10}mV_{f}^{2} - 0$

$$V_{f}^{2} \propto \sin \theta$$

$$\left(\frac{V_{1}}{V_{2}}\right)^{2} = \frac{\sin \theta}{\sin \theta} = \frac{\sin 30^{\circ}}{\sin 45^{\circ}} = \frac{1}{\sqrt{2}}$$

40. Refer to the circuit diagram given in the figure, which of the following observation are correct?

A. Total resistance of circuit is 6 Ω .

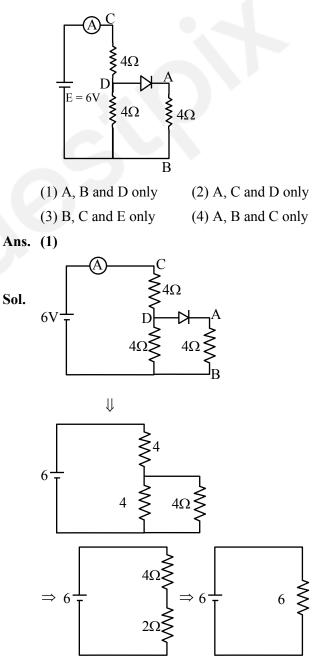
B. Current in Ammeter is 1A

C. Potential across AB is 4 Volts.

D. Potential across CD is 4 Volts.

E. Total resistance of the circuit is 8Ω .

Choose the correct answer from the options given below:





Current through ammeter = 1 A

$$R_{net} = 6\Omega$$

$$6V \qquad D \qquad A$$

$$0.5A \qquad 0.5A$$

$$V_{AB} = 0.5 \times 4 = 2 \text{ volt}$$

$$V_{CD} = 1 \times 4 = 4$$
 volt

A, B & D are correct

41. The electric flux is $\phi = \alpha \sigma + \beta \lambda$

where λ and σ are linear and surface charge

density, respectively, $\left(\frac{\alpha}{\beta}\right)$ represents

- (1) charge
- (2) electric field
- (3) displacement
- (4) area

Sol. $\phi = \alpha \sigma + \beta \lambda$

$$[\phi] = [\alpha\sigma] = [\beta\lambda]$$

$$\begin{bmatrix} \alpha \end{bmatrix} = \frac{\left[\phi \right]}{\left[\sigma \right]} \qquad \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\left[\lambda \right]}{\left[1 \right]}$$
$$\begin{bmatrix} \beta \end{bmatrix} = \frac{\left[\phi \right]}{\left[\lambda \right]} = \frac{\left[Q/L \right]}{\left[Q/Area \right]} \qquad \begin{bmatrix} Area \\ Length \end{bmatrix}$$
$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = L$$

42. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index μ_1 , $\mu_1 < \mu_2$) having radii of curvature $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place ?

$$(1) \frac{\mu_{1}|R_{1}|.|R_{2}|}{\mu_{2}(|R_{1}|+|R_{2}|)-\mu_{1}|R_{1}|}$$

$$(2) \frac{\mu_{1}|R_{1}|.|R_{2}|}{\mu_{2}(|R_{1}|+|R_{2}|)-\mu_{1}|R_{2}|}$$

$$(3) \frac{\mu_{1}|R_{1}|.|R_{2}|}{\mu_{2}(2|R_{1}|+|R_{2}|)-\mu_{1}\sqrt{|R_{1}|.|R_{2}}}$$

(4)
$$\frac{(\mu_2 + \mu_1)|R_1|}{(\mu_2 - \mu_1)}$$

Ans. (2)

Sol.

$$\begin{aligned} & \left| \begin{array}{c} R_{1} \\ \mu_{1} \\ \mu_{2} \\ \\ \hline \\ \mu_{2} \\ \\ \hline \\ \frac{1}{f_{eq}} = \frac{2}{f_{L}} - \frac{1}{f_{m}} \\ \\ f_{m} = -\frac{\left| \frac{R_{2}}{2} \right|}{2} \\ \\ \hline \\ \frac{1}{f_{L}} \quad \left(\frac{\mu_{2}}{\mu_{1}} - \right) \left(\frac{1}{R_{1}} + \frac{1}{R_{2}} \right) \\ \\ \hline \\ \frac{1}{f_{eq}} \quad 2 \left(\frac{\mu_{2} - \mu_{1}}{\mu_{1}} \right) \left(\frac{R_{1} + R_{2}}{R_{1}R_{2}} \right) + \frac{2}{R_{2}} \\ \\ = \frac{2}{R_{2}} \left| \frac{(\mu_{2} - \mu_{1})(R_{1} + R_{2}) + \mu_{1}R_{1}}{\mu_{1}} \right| \\ \\ = \frac{2}{R_{2}} \left| \frac{\mu_{2}R_{1} + \mu_{2}R_{2} - \mu_{1}R_{1} - \mu_{1}R_{2} + \mu_{1}R_{1}}{\mu_{1}} \right| \end{aligned}$$



$$\frac{1}{f_{eq}} = \frac{2[\mu_2 R_1 + \mu_2 R_2 - \mu_1 R_2]}{\mu_1 R_1 R_2}$$

For same size of image

u = 2f

$$u = \frac{\mu_1 R_1 R_2}{\mu_2 R_1 + \mu_2 R_2 - \mu_1 R_2}$$

43. The electric field of an electromagnetic wave in free space is

$$\vec{E} = 57 \cos[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y)]$$

 $(4\hat{i}-3\hat{j})N/C.$

The associated magnetic field in Tesla is-

(1)
$$\vec{B} = \frac{57}{3 \times 10^8} \cos \left[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y) \right] (5\hat{k})$$

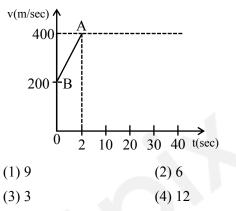
(2) $\vec{B} = \frac{57}{3 \times 10^8} \cos \left[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y) \right] (\hat{k})$
(3) $\vec{B} = -\frac{57}{3 \times 10^8} \cos \left[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y) \right] (5\hat{k})$
(4) $\vec{B} = -\frac{57}{3 \times 10^8} \cos \left[7.5 \times 10^6 t - 5 \times 10^{-3} (3x + 4y) \right] (\hat{k})$

Ans. (3)

Sol. $\vec{K} = 3\hat{i} + 4\hat{j}$

$$\hat{K} = \frac{3\hat{i} + 4\hat{j}}{5}$$
$$\hat{E} = \frac{4\hat{i} - 3\hat{j}}{5}$$
$$\hat{B} = \hat{K} \times \hat{E}$$
$$\hat{B} = -\hat{Z}$$
$$B_0 = \frac{E_0}{C} = \frac{57}{3 \times 10^8}$$

44. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is _____ km.



Ans. (4)

Sol. Total Area under curve.

45. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be-

(1) 2.0 cm	(2) 0.5 cm
------------	------------

Ans. (4)

mass of disc = m

mass of cut part =
$$\frac{m}{16}$$

$$X_{com} = \frac{m \times 0 - \frac{m}{16} \quad 15}{m - \frac{m}{16}}$$

= 1 cm.



SECTION-B

46. A positive ion A and a negative ion B has charges 6.67×10^{-19} C and 9.6×10^{-10} C, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $P \times 10^{-13}$, where the value of P is

(Take $\frac{1}{4\pi_0}$ = 9 × 10⁹ Nm²C⁻¹ and universal

gravitational constant as $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$)

Ans. (BONUS)

Sol.
$$\frac{9 \times 10^9 \times 6.67 \times 10^{-19} \times 9.6 \times 10^{-10}}{6.67 \times 10^{-11} \times 19.2 \times 10^{-27} \times 9 \times 10^{-27}}$$
$$\frac{1}{2} \times 10^{45}$$

Charge is not integral multiple of electron.

47. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n⁻¹ is _____.

Sol. A.B = 0

$$4 - 6 n + 8p = 0$$

 $|\vec{A}| = |\vec{B}|$
 $4 + 9n^2 + 4 = 4 + 4 + 16p^2$
 $9n^2 = 16p^2$
 $P = +\frac{3}{4}n$
 $4 - 6n \pm 6n = 0$
 $12n = 4$
 $n = \frac{1}{3}$

48. An ideal gas initially at 0°C temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is 3/2, the change in temperature due to the thermodynamics process is K.

Sol.
$$\gamma = \frac{3}{2}$$

 $Tv^{\gamma^{-1}} = C$
 $273 V_0^{0.5} = T\left(\frac{V_0}{4}\right)^{0.5}$
 $T = 273 \times 2 = 546$
 $\Delta T = 273$

49. A force $f = x^2y\hat{i} + y^2\hat{j}$ acts on a particle in a plane x + y = 10. The work done by this force during a displacement from (0, 0) to (4m, 2m) is ______ Joule (round off to the nearest integer)

Ans. (152)

Sol.
$$\int_{0}^{4} x^{2}(10-x)dx + y dy$$
$$= \left[\frac{10x^{3}}{3} - \frac{x^{4}}{4}\right]_{0} + \left[\frac{y^{3}}{3}\right]^{2} = \frac{640}{3} - 64 + \frac{8}{3} = 152$$

50.
$$\int_{12V}^{3H} \sqrt{2}$$

In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of 8 A/s. At an instant when R is 12 Ω , the value of the current in the circuit will be _____A.

Ans. (3)

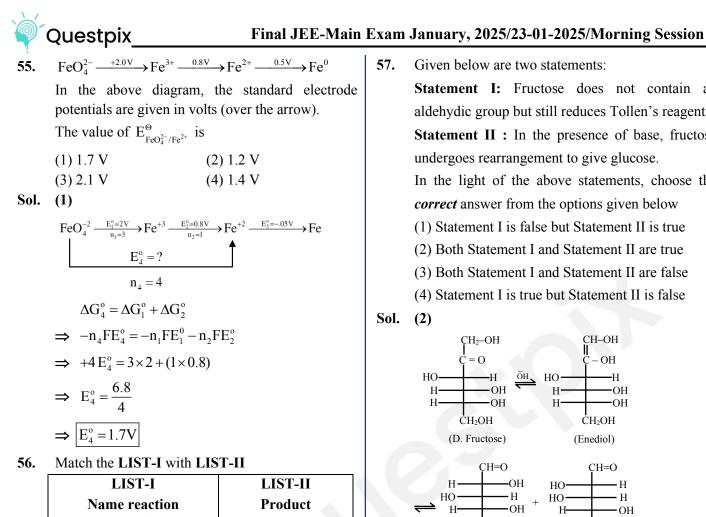
Sol.
$$\varepsilon - \frac{LdI}{dt} - IR = 0$$

 $12 - 3 \times (-8) - I \times 12 = 0$
 $I = 3$



(HELD ON THURSDAY 23rd JANUARY 2025)

TIME: 9:00 AM TO 12:00 NOON



1151-1				
	Name reaction		Product	
			obtainable	
A.	Swarts reaction	I.	Ethyl benzene	
B.	Sandmeyer's reaction	II.	Ethyl iodide	
C.	Wurtz Fittig reaction	III.	Cyanobenzene	
D.	Finkelstein reaction	IV.	Ethyl fluoride	

Choose the *correct* answer from the options given below:

(1) A-II, B-III, C-I, D-IV (2) A-IV, B-I, C-III, D-II (3) A-IV, B-III, C-I, D-II (4) A-II, B-I, C-III, D-IV

Sol. (3)

LIST-I Name reaction		LIST-II Product obtainable	
A.	Swarts reaction	I.	$Et-I \xrightarrow{KF} Et-F$
B.	Sandmeyer's reaction	II.	$PhN_{2}^{\oplus}Cl^{-} \xrightarrow{CuCN/KCN} PhCN+N_{2}$
C.	Wurtz Fittig reaction	III.	$\begin{array}{c} Ph-Cl + EtCl \xrightarrow{Na} \\ ether \end{array} $ $Ph-Et + Ph-Ph + Et-Et$
D.	Finkelstein reaction	IV.	$Et-Cl \xrightarrow{NaI} Et-I + NaCl$

Given below are two statements: Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent Statement II : In the presence of base, fructose undergoes rearrangement to give glucose. In the light of the above statements, choose the

correct answer from the options given below

- (1) Statement I is false but Statement II is true
- (2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

CH-OH CH2-OH – OH = 0 Ōн, HO· **-**H -H -OH **<** H -OH -OH -OH H CH₂OH ĊH₂OH (D. Fructose) (Enediol) CH=O CH=O -OH **-** H HO-·H • H HO + -OH H -OH -OH H •OH H-CH₂OH ĊH₂OH D-Glucose D-Mannose

 2.8×10^{-3} mol of CO, is left after removing 10^{21} 58. molecules from its 'x' mg sample. The mass of CO, taken initially is

> Given : $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$ (1) 196.2 mg (2) 98.3 mg

> (3) 150.4 mg (4) 48.2 mg

Sol. (1)

$$(\text{moles})_{\text{initial}} = \frac{x \times 10^{-3}}{44}$$
$$(\text{moles})_{\text{removal}} = \frac{10^{21}}{6.02 \times 10^{23}}$$
$$(\text{moles})_{\text{left}} \quad (\text{moles})_{\text{initial}} - (\text{moles})_{\text{removed}}$$

$$2.8 \times 10^{-10} = \frac{1}{44} = \frac{1}{6.02 \times 10^{23}}$$

⇒ x = 196.2 mg



59. Ice at -5°C is heated to become vapor with temperature of 110°C at atmospheric pressure. The entropy change associated with this process can be obtained from :

$$(1) \int_{268K}^{383K} C_{p} dT + \frac{H_{melting}}{273} + \frac{\Delta H_{boiling}}{373}$$

$$(2) \int_{268K}^{273K} \frac{C_{p,m}}{T} dT + \frac{\Delta H_{m}, fusion}{T_{f}} + \frac{\Delta}{T_{b}} \frac{M_{m}, aporisation}{T_{b}}$$

$$+ \int_{273K}^{373K} \frac{C_{p,m}}{T} dT + \int_{373K}^{383K} \frac{C_{p,m}}{T} dT$$

$$(3) \int_{268K}^{383K} C_{p} dT + \frac{q_{rev}}{T}$$

$$(4) \int_{268K}^{273K} C_{p,m} dT + \frac{\Delta H_{m}, fusion}{T_{f}} + \frac{\Delta H_{m,vaporisation}}{T_{b}}$$

$$+ \int_{273K}^{373K} C_{p,m} dT + \int_{373K}^{383K} C_{p,m} dT$$

Ice
$$\rightarrow$$
 Ice \rightleftharpoons Water \rightarrow Water \rightleftharpoons Water \rightarrow Water
 $268 \text{ K} 273 \text{ K} 273 \text{ K} 273 \text{ K} 373 \text{ K} 383 \text{ K}$
(1) (2) (3) (4) (5)
 $\Delta S_{\text{overall}} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$
 $\Delta S_2 = \frac{\Delta H_{\text{m fusion}}}{273}$ $T_f = 273 \text{ 'K'}$
 $\Delta S_3 = \int_{273}^{373} \frac{C_{p,m} dT}{T}$
 $\Delta S_4 = \frac{\Delta H_{\text{m vaporisation}}}{373}$ $T_b = 373 \text{ 'K'}$
 $\Delta S_5 = \int_{373}^{383} \frac{C_{p,m} dT}{T}$

Answer = (2)

60. The d-electronic configuration of an octahedral Co(II) complex having magnetic moment of 3.95 BM is :

(1)
$$t_{2g}^{6} e_{g}^{1}$$
 (2) $t_{2g}^{3} e_{g}^{0}$
(3) $t_{2g}^{5} e_{g}^{2}$ (4) $e^{4} t_{2}^{3}$
Sol. (3)
 $Co^{+2} = (Ar)_{18} 3d^{7} 4s^{\circ}$
 $1 \quad 1 \quad e_{g}^{2}$
 $1 \quad 1 \quad e_{g}^{2}$
(1) $L \quad 1t_{2g}^{5}$
61. The complex that shows Facial – Meridional isomerism is
(1) $[Co(NH_{*}), CL_{*}]$ (2) $[Co(NH_{*}), CL_{*}]^{+}$

(1) $[Co(NH_3)_3Cl_3]$ (2) $[Co(NH_3)_4Cl_2]^+$ (3) $[Co(en)_3]^{3+}$ (4) $[Co(en)_2Cl_2]^+$ Sol. (1) Ma_3b_3 type complexes show Facial - Meridional isomerism (i) $[Co(NH_3)_3Cl_3] \Rightarrow Ma_3b_3$ (ii) $[Co(NH_3)_4Cl_2]^+ \Rightarrow Ma_4b_2$ (iii) $[Co(en)_3]^{3+} \Rightarrow M(AA)_3$ (iv) $[Co(en)_2Cl_2]^+ \Rightarrow M(AA)_2b_2$ a, b, = NH_3, Cl⁻ AA = en 62. The major product of the following reaction is :

$$CH_{3}CH_{2}CH=O \xrightarrow{excess HCHO}{alkali}?$$

$$(1) CH_{3}-CH_{2}-CH_{2}-OH$$

$$(2) CH_{3}-CH-CH=O$$

$$(2) CH_{3}-CH-CH=O$$

$$(3) CH_{3}-C-CH_{2}-OH$$

$$(3) CH_{3}-C-CH_{2}-OH$$

$$(4) CH_{3}-C-CH=O$$

$$(4) CH_{3}-C-CH=O$$

$$(4) CH_{3}-C-CH=O$$

$$(4) CH_{3}-C-CH=O$$

Sol. (3)

This is an example of Tollen's reaction i.e. multiple cross aldol followed by cross Cannizaro reaction

$$CH_{2}-OH$$

$$CH_{3}CH_{2}CH=O \xrightarrow{2HCHO} CH_{3}-C-CHO$$

$$I$$

$$CH_{2}-OH$$

$$CH_{2}-OH$$

$$CH_{2}-OH$$

$$HCHO$$

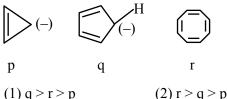
$$CH_{3}-C-CH_{2}-OH$$

$$HCOO^{\Theta}$$

$$CH_{2}-OH$$



The correct stability order of the following 63. species/molecules is :



(3)
$$q > p > r$$
 (4) $p > q > r$

Sol. (1)

q is aromatic r is nonaromatic p is antiaromatic

- chlorination 64. Propane molecule on under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x" when it is further treated with chlorine under the photochemical condition? (1)4(2) 2
 - (3) 5 (4) 3
- Sol. (4)

"X" is
$$CH_3$$
- CH_2
 Cl Cl_2
 Cl_2
 CH_3 - CH_2 - CH_2 - CH_2 - Cl_2 -

- 65. What amount of bromine will be required to convert 2 g of phenol into 2, 4, 6-tribromophenol? (Given molar mass in g mol^{-1} of C, H, O, Br are 12, 1, 16, 80 respectively)
 - (1) 10.22 g (2) 6.0 g (4) 20.44 g (3) 4.0 g
- Sol. (1)

$$\overset{OH}{\longleftarrow} + 3Br_2 \xrightarrow{Br} \overset{OH}{\longleftarrow} Br_{+} 3HBr$$
Moles of phenol = $\frac{2}{94} = 0.021$

∴ Moles of bromine = $0.021 \times 3 = 0.064$

∴ Mass of bromine = $0.064 \times 160 = 10.22$ g

The correct set of ions (aqueous solution) with **66**. same colour from the following is :

(1)
$$V^{2+}$$
, Cr^{3+} , Mn^{3+}
(2) Zn^{2+} , V^{3+} , Fe^{3+}
(3) Ti^{4+} , V^{4+} , Mn^{2+}
(4) Sc^{3+} , Ti^{3+} , Cr^{2+}

Sol. (1)

- (1) V^{2+} (Violet), Cr^{3+} (Violet), Mn^{3+} (Violet)
- (2) Zn^{2+} (Colourless), V^{3+} (Green), Fe^{3+} (Yellow)
- (3) Ti^{4+} (Colourless), V^{4+} (Blue), Mn^{2+} (Pink)
- (4) Sc^{3+} (Colourless), Ti^{3+} (Purple), Cr^{2+} (Blue)

67. Given below are two statements :

> Statement I : In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

> Statement II: The sodium fusion extract of an organic compound having N and S gives prussian blue colour with FeSO, and Na, [Fe(CN),]

> In the light of the above statements, choose the **correct** answer from the options given below

- (1) Both Statement I and Statement II are true
- (2) Both Statement I and Statement II are false
- (3) Statement I is false but Statement II is true
- (4) Statement I is true but Statement II is false

Sol. (4)

The sodium fusion extract of organic compound having N & S gives blood red colour with FeSO, and $Na_{4}[Fe(CN)_{6}]$

Which of the following happens when NH₄OH is **68**. added gradually to the solution containing $1M A^{2+}$ and 1M B^{3+} ions ?

Given : $K_{-}[A(OH)_{2}] = 9 \times 10^{-10}$ and

 $K_{av}[B(OH)_{2}] = 27 \times 10^{-18}$ at 298 K.

- (1) B(OH), will precipitate before A(OH),
- (2) A(OH), and B(OH), will precipitate together
- (3) A(OH), will precipitate before $B(OH)_{3}$
- (4) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH₄OH



Sol.	(1)	
501.	(\mathbf{I})	

Condition for precipitation $Q_{ip} > K_{sp}$ For $[A(OH)_2]$ $[A^{2+}] [OH^{-}]^2 > 9 \times 10^{-10}$ $[A^{+2}] = 1 M$ $\Rightarrow [OH^{-}] > 3 \times 10^{-5} M$ For $[B(OH)_3]$ $[B^{3+}][OH^{-}]^3 > 27 \times 10^{-18}$ $[B^{3+}] = 1M$ $\Rightarrow [OH^{-}] > 3 \times 10^{-6} M$ So, B(OH), will precipitate before A(OH),

69. Match the LIST-I with LIST-II

LIST-I (Classification of molecules based on octet rule)		LIST-II (Example)	
А.	Molecules obeying octet rule	I.	NO, NO ₂
B.	Molecules with incomplete octet	II.	BCl ₃ , AlCl ₃
C.	Molecules with incomplete octet with odd electron	III.	H ₂ SO ₄ , PCl ₅
D.	Molecules with expanded octet	IV.	CCl ₄ , CO ₂

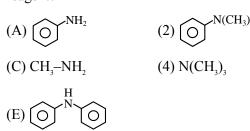
Choose the **correct** answer from the options given below :

- (1) A-IV, B-II, C-I, D-III
- (2) A-III, B-II, C-I, D-IV
- (3) A-IV, B-I, C-III, D-II
- (4) A-II, B-IV, C-III, D-I

Sol. (1)

- $(A) A \to IV$
- $(B) B \rightarrow II$
- $(C) C \rightarrow I$
- $(D) D \rightarrow III$

70. Which among the following react with Hinsberg's reagent?



Choose the correct answer from the options given below :

- (1) B and D only (2) C and D only
- (3) A, B and E only (4) A, C and E only

Sol. (4)

B and D are 3° amine which does not have replaceable H on N, So does not react.

SECTION-B

- 71. If 1 mM solution of ethylamine produces pH = 9, then the ionization constant (K_b) of ethylamine is 10^{-x}. The value of x is ______ (nearest integer). [The degree of ionization of ethylamine can be neglected with respect to unity.]
- Sol. (7)
- Sol. $C_2H_5NH_2(aq)$ $H_2O \Longrightarrow C_2H_2NH_3^+ + \overset{\odot}{O}H$ $C = 10^{-3} M$ - - - $C(1 - \alpha)$ $C\alpha$ $C\alpha$ $\Rightarrow C = 10^{-3}$ $= 10^{-5}$ $= 10^{-5}$ $\boxed{1 - \alpha \approx 1}$ Given, $P^H = 9 \Rightarrow P^{OH} = 5 \Rightarrow \begin{bmatrix} \overset{\odot}{O}H \end{bmatrix} = 10^{-5}M$ $Now, K_b = \frac{[C_2H_5NH_3^+][\overset{\odot}{O}H]}{[C_2H_5NH_2]}$ $\Rightarrow K_b = \frac{10^{-5} \times 10}{10^{-3}} = 10^{-7}$



72. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is _____%.

(Given molar mass in g mol⁻¹ of Ba : 137, S : 32, O : 16)

Sol. (40)

Millimoles of $BaSO_4 = \frac{466}{233} = 2m \text{ mol}$

$$%S = \frac{\frac{466}{233} \times 32}{160} \times 100 = 40\%$$

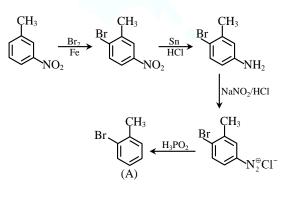
 Consider the following sequence of reactions to produce major product (A)

Molar mass of product (A) is $____ g mol^{-1}$.

(Given molar mass in g mol $^{\mathchar`-1}$ of C : 12, H : 1,

O: 16, Br: 80, N: 14, P: 31)

Sol. (171)



Molar mass of product (C_7H_7Br) (A) is 171 g mol⁻¹

74. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below :

 $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

S.No.	Time/s	Total pressure / (atm)
1.	0	0.6
2.	100	'x'
x =	× 10	⁻³ atm [nearest integer]

Given : Rate constant for the reaction is $4.606 \times 10^{-2} \text{ s}^{-1}$.

Р

 $\overline{2}$

Sol. (900)

NTA. (897)

P.

$$K_{N_2O_5} = 2 \times 4.606 \times 10^{-2} \,\mathrm{S}^{-1}$$
$$2N_2O_5(g) \longrightarrow 2N_2O_4(g) + O_2(g)$$
$$0.6 \qquad 0 \qquad 0$$

$$P_{f} = 0.6 - P = P$$

$$2 \times 4.606 \times 10^{-2} = \frac{2.303}{100} \log \frac{0.6}{0.6 - P}$$

$$4\log_{10}\frac{0.6}{0.6-P}$$

Z

$$10^4 = \frac{0.6}{0.6 - P}$$

$$\Rightarrow 0.6 \times 10^{4} - 10^{4} P = 0.6$$

$$\Rightarrow 10^{4} P = 0.6(10^{4} - 1)$$

$$P = (6000 - 0.6) \times 10^{-4}$$

$$= 5999. \times 10^{-4}$$

$$= 0.59994$$

$$P_{Total} = 0.6 \quad \frac{P}{2}$$

$$= 0.6 + 0.29997$$
$$= 0.89997$$
$$899.97 \times 10^{-3}$$

Ans. 900



Given by NTA

Given : $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$						
t = 0	0.6	0	0			
t = 100s	0.6 - x	х	x/2			

$$P_{Total} = 0.6 + \frac{x}{2}$$

As given in equation

$$K_r = 4.606 \times 10^{-2} \text{ sec}^{-1}$$

(Here language conflict in question)

$$(K_r = \frac{KA}{2} \text{ not considered})$$

 $K_r t = \ln \frac{0.6}{0.6 - x}$

 $4.606 \times 10^{-2} \times 100 = 2.303 \ log \ \frac{0.6}{0.6-x}$

$$P_{Total} = 0.6 + \frac{0.594}{2} = 0.897 \text{ atm}$$

= 897 × 10⁻³ atm

75. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are 55.0 kJ mol⁻¹ and 175.0 J/K/mol respectively. The standard free energy change for this reaction at 25°C in J mol⁻¹ is ______(Nearest integer)

Sol. (2850)

$$\Delta H^{\circ}_{rxn} = 55 \text{ kJ/mol}, \qquad T = 298 \text{ K}$$

$$\Delta S^{\circ}_{rxn} = 175 \text{ J/mol}$$

$$\Delta G^{\circ}_{rxn} = \Delta H^{\circ}_{rxn} - T\Delta S^{\circ}_{rxn}$$

$$\Rightarrow \Delta G^{\circ}_{rxn} = 55000 \text{ J/mol} - 298 \times 175 \text{ J/mol}$$

$$\Rightarrow \Delta G^{\circ}_{rxn} = 55000 - 52150$$

$$\Rightarrow \Delta G^{\circ}_{rxn} = 2850 \text{ J/mol}$$