

(Held On Saturday 27th January, 2024)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) - \tan^{-1}(2x) = \frac{\pi}{4}$

is :

- (1) More than 2
- (2) 1
- (3) 2
- (4) 0

Ans. (2)

Sol. $\tan^{-1} x - \tan^{-1} 2x = \frac{\pi}{4}; x > 0$

$$\Rightarrow \tan^{-1} 2x = \frac{\pi}{4} - \tan^{-1} x$$

Taking tan both sides

$$\Rightarrow 2x = \frac{1-x}{1+x}$$

$$\Rightarrow 2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+8}}{8} = \frac{-3 \pm \sqrt{17}}{8}$$

$$\text{Only possible } x = \frac{-3 + \sqrt{17}}{8}$$

2. Consider the function $f : (0,2) \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x}{2} + \frac{2}{x} \text{ and the function } g(x) \text{ defined by}$$

$$g(x) = \begin{cases} \min\{f(t)\}, & 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases} \text{ . Then}$$

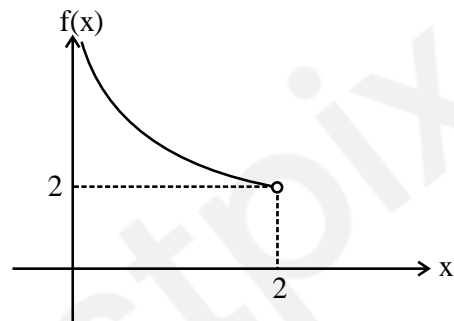
- (1) g is continuous but not differentiable at $x = 1$
- (2) g is not continuous for all $x \in (0,2)$
- (3) g is neither continuous nor differentiable at $x = 1$
- (4) g is continuous and differentiable for all $x \in (0,2)$

Ans. (1)

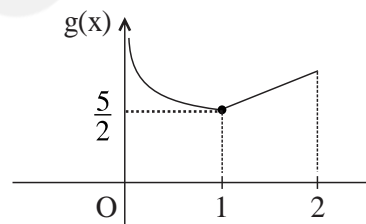
Sol. $f : (0,2) \rightarrow \mathbb{R}; f(x) = \frac{x}{2} + \frac{2}{x}$

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$\therefore f(x)$ is decreasing in domain.



$$g(x) = \begin{cases} \frac{x}{2} + \frac{2}{x} & 0 < x \leq 1 \\ \frac{3}{2} + x & 1 < x < 2 \end{cases}$$

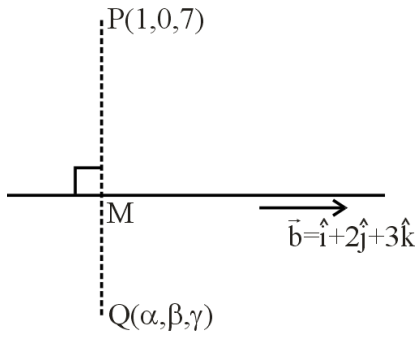


3. Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ) . Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y -axis and z -axis respectively and an acute angle with x -axis ?

- (1) $(1, -2, 1 + \sqrt{2})$
- (2) $(1, 2, 1 - \sqrt{2})$
- (3) $(3, 4, 3 - 2\sqrt{2})$
- (4) $(3, -4, 3 + 2\sqrt{2})$

Ans. (3)

Sol. $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$M(\lambda, 1+2\lambda, 2+3\lambda)$

$\vec{PM} = (\lambda - 1)\hat{i} + (1 + 2\lambda)\hat{j} + (3\lambda - 5)\hat{k}$

\vec{PM} is perpendicular to line L_1

$\vec{PM} \cdot \vec{b} = 0 \quad (\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore M = (1, 3, 5)$

$\vec{Q} = 2\vec{M} - \vec{P}$ [M is midpoint of \vec{P} & \vec{Q}]

$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$

Required line having direction cosine (l, m, n)

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l = \frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu \left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$

Option (3) satisfying for $\mu = 4$

4. Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is :

(1) $(-3, -1) \cup \left(-\frac{1}{3}, 1\right)$

(2) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

(3) $(-3, 0) \cup \left(\frac{2}{3}, 1\right)$

(4) $(-3, -1) \cup \left(\frac{1}{3}, 1\right)$

Ans. (2)

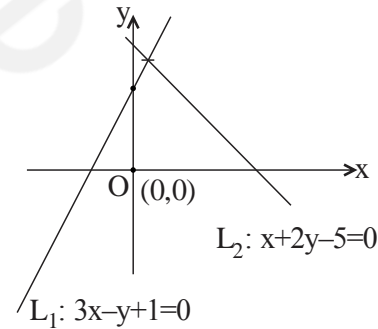
Sol. $P(a^2, a + 1)$

$L_1 = 3x - y + 1 = 0$

Origin and P lies same side w.r.t. L_1

$\Rightarrow L_1(0) \cdot L_1(P) > 0$

$\therefore 3(a^2) - (a + 1) + 1 > 0$



$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$(1)

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0) \cdot L_2(P) > 0$

$\Rightarrow a^2 + 2(a + 1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a + 3)(a - 1) < 0$

$\therefore a \in (-3, 1)$(2)

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

5. The 20th term from the end of the progression

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4} \text{ is :-}$$

- (1) -118
- (2) -110
- (3) -115
- (4) -100

Ans. (3)

Sol. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$

This is A.P. with common difference

$$d_1 = -1 + \frac{-3}{4} = -\frac{7}{4}$$

$$-129\frac{1}{4}, \dots, 19\frac{3}{4}, 20$$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

6. Let $f: \mathbb{R} - \left\{-\frac{1}{2}\right\} \rightarrow \mathbb{R}$ and $g: \mathbb{R} - \left\{-\frac{5}{2}\right\} \rightarrow \mathbb{R}$ be

defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then

the domain of the function fog is :

- (1) $\mathbb{R} - \left\{-\frac{5}{2}\right\}$
- (2) \mathbb{R}
- (3) $\mathbb{R} - \left\{-\frac{7}{4}\right\}$
- (4) $\mathbb{R} - \left\{-\frac{5}{2}, -\frac{1}{4}\right\}$

Ans. (1)

Sol. $f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$

$$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$$

Domain of $f(g(x))$

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$$x \neq -\frac{5}{2} \text{ and } \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

$$x \in \mathbb{R} - \left\{-\frac{5}{2}\right\} \text{ and } x \in \mathbb{R}$$

$$\therefore \text{Domain will be } \mathbb{R} - \left\{-\frac{5}{2}\right\}$$

7. For $0 < a < 1$, the value of the integral

$$\int_0^\pi \frac{dx}{1-2a \cos x + a^2} \text{ is :}$$

(1) $\frac{\pi^2}{\pi+a^2}$

(2) $\frac{\pi^2}{\pi-a^2}$

(3) $\frac{\pi}{1-a^2}$

(4) $\frac{\pi}{1+a^2}$

Ans. (3)

Sol. $I = \int_0^\pi \frac{dx}{1-2a \cos x + a^2}; 0 < a < 1$

$$I = \int_0^\pi \frac{dx}{1+2a \cos x + a^2}$$

$$2I = 2 \int_0^{\pi/2} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2 \cos^2 x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a) \cdot \sec^2 x}{(1+a^2)^2 \cdot \sec^2 x - 4a^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a) \cdot \sec^2 x}{(1+a^2)^2 \cdot \tan^2 x + (1-a^2)^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2 \cdot \sec^2 x}{1+a^2} \cdot dx$$

$$\tan^2 x + \left(\frac{1-a^2}{1+a^2} \right)^2$$

$$\Rightarrow I = \frac{2}{(1-a^2)} \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{1-a^2}$$

8. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for all $x \in (0, 3)$. If g is decreasing in $(0, \alpha)$ and increasing in $(\alpha, 3)$, then 8α is

(1) 24

(2) 0

(3) 18

(4) 20

Ans. (3)

Sol. $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0 \forall x \in (0, 3)$

$\Rightarrow f'(x)$ is increasing function

$$g'(x) = 3 \times \frac{1}{3} \cdot f'\left(\frac{x}{3}\right) - f'(3-x)$$

$$= f'\left(\frac{x}{3}\right) - f'(3-x)$$

If g is decreasing in $(0, \alpha)$

$$g'(x) < 0$$

$$f'\left(\frac{x}{3}\right) - f'(3-x) < 0$$

$$f'\left(\frac{x}{3}\right) < f'(3-x)$$

$$\Rightarrow \frac{x}{3} < 3-x$$

$$\Rightarrow x < \frac{9}{4}$$

Therefore $\alpha = \frac{9}{4}$

Then $8\alpha = 8 \times \frac{9}{4} = 18$

9. If $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$, then

$2\alpha - \beta$ is equal to :

(1) 2

(2) 7

(3) 5

(4) 1

Ans. (3)

Sol. $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{3 + \alpha \left[x - \frac{x^3}{3!} + \dots \right] + \beta \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right] + \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \dots \right)}{3 \tan^2 x} = \frac{1}{3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(3 + \beta) + (\alpha - 1)x + \left(-\frac{1}{2} - \frac{\beta}{2} \right) x^2 + \dots}{3x^2} \times \frac{x^2}{\tan^2 x} = \frac{1}{3}$$

$$\Rightarrow \beta + 3 = 0, \alpha - 1 = 0 \text{ and } \frac{-\frac{1}{2} - \frac{\beta}{2}}{3} = \frac{1}{3}$$

$$\Rightarrow \beta = -3, \alpha = 1$$

$$\Rightarrow 2\alpha - \beta = 2 + 3 = 5$$

10. If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

(1) $2S_{12} = S_{11} + S_{10}$

(2) $S_{12} = S_{11} + S_{10}$

(3) $2S_{11} = S_{12} + S_{10}$

(4) $S_{11} = S_{10} + S_{12}$

Ans. (2)

Sol. $x^2 - x - 1 = 0$

$$S_n = 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = 2023\alpha^{n-1} + 2024\beta^{n-1} + 2023\alpha^{n-2} + 2024\beta^{n-2}$$

$$= 2023\alpha^{n-2}[1 + \alpha] + 2024\beta^{n-2}[1 + \beta]$$

$$= 2023\alpha^{n-2}[\alpha^2] + 2024\beta^{n-2}[\beta^2]$$

$$= 2023\alpha^n + 2024\beta^n$$

$$S_{n-1} + S_{n-2} = S_n$$

Put $n = 12$

$$S_{11} + S_{10} = S_{12}$$

11. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(-2, -3) is

- (1) 10
- (2) 6
- (3) 4
- (4) 8

Ans. (1)

Sol. $2^m - 2^n = 56$

$$2^n(2^{m-n} - 1) = 2^3 \times 7$$

$$2^n = 2^3 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow n = 3 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow n = 3 \text{ and } m - n = 3$$

$$\Rightarrow n = 3 \text{ and } m = 6$$

$$P(6,3) \text{ and } Q(-2, -3)$$

$$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Hence option (1) is correct

12. The values of α , for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0, \text{ lie in the interval}$$

- (1) (-2, 1)
- (2) (-3, 0)
- (3) $(-\frac{3}{2}, -\frac{1}{2})$
- (4) (0, 3)

Ans. (2)

Sol. $\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$

$$\Rightarrow (2\alpha + 3) \left\{ \frac{7}{6} \right\} - (3\alpha + 1) \left\{ \frac{-1}{6} \right\} = 0$$

$$\Rightarrow (2\alpha + 3) \cdot \frac{7\alpha}{6} + (3\alpha + 1) \cdot \frac{1}{6} = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

Hence option (2) is correct.

13. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

- (1) $\frac{5}{256}$
- (2) $\frac{5}{715}$
- (3) $\frac{3}{715}$
- (4) $\frac{3}{256}$

Ans. (3)

Sol. $\frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$

Hence option (3) is correct.

14. The integral $\int \frac{(x^8 - x) dx}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is

equal to :

- (1) $\log_e \left(\left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right|^{1/3} \right) + C$
- (2) $\log_e \left(\left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right|^{1/2} \right) + C$
- (3) $\log_e \left(\left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right| \right) + C$
- (4) $\log_e \left(\left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right|^3 \right) + C$

Ans. (1)

Sol.
$$I = \int \frac{x - x^2}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$$

Let $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$

$$\Rightarrow \frac{1}{1 + \left(x^3 + \frac{1}{x^3}\right)^2} \cdot \left(3x^2 - \frac{1}{x^4}\right) dx = dt$$

$$\Rightarrow \frac{x^6}{x^{12} + 3x^6 + 1} \cdot \frac{3x^2 - 1}{x^4} dx = dt$$

$$I = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln |t| + C$$

$$I = \frac{1}{3} \ln \left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right| + C$$

$$I = \ln \left| \tan^{-1}\left(x + \frac{1}{x^3}\right) \right|^{1/3} + C$$

Hence option (1) is correct

15. If $2 \tan^2 \theta - 5 \sec \theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in \mathbb{N}$

then $\sum_{k=1}^n \frac{k}{2^k}$ is equal to :

(1) $\frac{1}{2^{15}}(2^{14} - 14)$

(2) $\frac{1}{2^{14}}(2^{15} - 15)$

(3) $1 - \frac{15}{2^{13}}$

(4) $\frac{1}{2^{13}}(2^{14} - 15)$

Ans. (4)

Sol. $2 \tan^2 \theta - 5 \sec \theta - 1 = 0$
 $\Rightarrow 2 \sec^2 \theta - 5 \sec \theta - 3 = 0$
 $\Rightarrow (2 \sec \theta + 1)(\sec \theta - 3) = 0$
 $\Rightarrow \sec \theta = -\frac{1}{2}, 3$
 $\Rightarrow \cos \theta = -2, \frac{1}{3}$
 $\Rightarrow \cos \theta = \frac{1}{3}$

For 7 solutions $n = 13$

So, $\sum_{k=1}^{13} \frac{k}{2^k} = S$ (say)

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{13}{2^{13}}$$

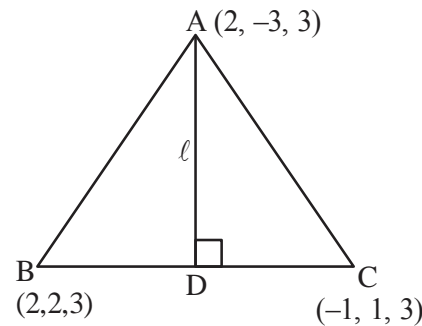
$$\frac{1}{2} S = \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{13}}}{1 - \frac{1}{2}} - \frac{13}{2^{14}} \Rightarrow S = 2 \cdot \left(\frac{2^{13} - 1}{2^{13}} - \frac{13}{2^{14}} \right)$$

16. The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :
- (1) 49
 (2) 42
 (3) 50
 (4) 45

Ans. (4)

Sol. $AB = 5$
 $AC = 5$



\therefore D is midpoint of BC

$$D\left(\frac{2+(-1)}{2}, \frac{-3+1}{2}, 3\right)$$

$$\therefore l = \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-3 - \frac{1}{2}\right)^2 + (3-3)^2}$$

$$l = \sqrt{\frac{45}{2}}$$

$\therefore 2l^2 = 45$

17. If $y = y(x)$ is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, $x > 2, y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of $y(10)$ equals :

- (1) $\frac{3}{1+(8)^{1/4}}$
- (2) $\frac{3}{1+2\sqrt{2}}$
- (3) $\frac{3}{1-2\sqrt{2}}$
- (4) $\frac{3}{1-(8)^{1/4}}$

Ans. (1)

Sol. $(x - 4)dy - (y^2 - 3y)dx = 0$

$$\Rightarrow \frac{dy}{y - 3y} \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3} \frac{y - (y - 3)}{y(y - 3)} dy \int \frac{dx}{x^2 - 4}$$

$$\Rightarrow \frac{1}{3} (\ln |y - 3| - \ln |y|) = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + C$$

$$\Rightarrow \frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + C$$

At $x = 4, y = \frac{3}{2}$

$$\therefore C = \frac{1}{4} \ln 3$$

$$\therefore \frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{x - 2}{x + 2} \right| + \frac{1}{4} \ln 3$$

At $x = 10$

$$\frac{1}{3} \ln \left| \frac{y - 3}{y} \right| = \frac{1}{4} \ln \left| \frac{2}{3} \right| + \frac{1}{4} \ln 3$$

$$\ln \left| \frac{y - 3}{y} \right| = \ln 2^{3/4}, \forall x > 2, \frac{dy}{dx} < 0$$

as $y(4) = \frac{3}{2} \Rightarrow y \in (0, 3)$

$$-y + 3 = 8^{1/4} \cdot y$$

$$y = \frac{3}{1 + 8^{1/4}}$$

18. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, which passes through the foci of the hyperbola. If $e_1 e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through $(0, 2)$ is :

- (1) $4\sqrt{5}$
- (2) $\frac{8\sqrt{5}}{3}$
- (3) $\frac{10\sqrt{5}}{3}$
- (4) $3\sqrt{5}$

Ans. (3)

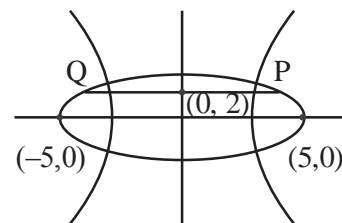
Sol. H: $\frac{x^2}{16} - \frac{y^2}{9} = 1$ $e_1 = \frac{5}{4}$

$$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$$

Also, ellipse is passing through $(\pm 5, 0)$

$$\therefore a = 5 \text{ and } b = 3$$

$$E: \frac{x^2}{25} + \frac{y^2}{9} = 1$$



End point of chord are $\left(\pm \frac{5\sqrt{5}}{3}, 2 \right)$

$$\therefore L_{PQ} = \frac{10\sqrt{5}}{3}$$

19. Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

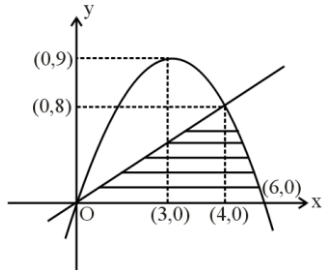
- (1) $\alpha \in \mathbb{N}$ and $\beta \notin \mathbb{N}$
- (2) $\alpha \notin \mathbb{N}$ and $\beta \in \mathbb{N}$
- (3) $\alpha \in \mathbb{N}$ and $\beta \in \mathbb{N}$
- (4) $\alpha \notin \mathbb{N}$ and $\beta \notin \mathbb{N}$

Ans. (3)

22. If the area of the region $\{(x, y) : 0 \leq y \leq \min\{2x, 6x - x^2\}\}$ is A, then $12A$ is equal to.....

Ans. (304)

Sol. We have



$$A = \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2) dx$$

$$A = \frac{76}{3}$$

$$12A = 304$$

23. Let A be a 2×2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A - xI| = 0$ be -1 and 3 , then the sum of the diagonal elements of the matrix A^2 is.....

Ans. (10)

Sol. $|A - xI| = 0$

Roots are -1 and 3

Sum of roots = $\text{tr}(A) = 2$

Product of roots = $|A| = -3$

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

We have $a + d = 2$

$ad - bc = -3$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

We need $a^2 + bc + bc + d$

$$\begin{aligned} &= a^2 + 2bc + d^2 \\ &= (a + d)^2 - 2ad + 2bc \\ &= 4 - 2(ad - bc) \\ &= 4 - 2(-3) \\ &= 4 + 6 \\ &= 10 \end{aligned}$$

24. If the sum of squares of all real values of α , for which the lines $2x - y + 3 = 0$, $6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is p, then the greatest integer less than or equal to p is

Ans. (32)

Sol. $2x - y + 3 = 0$

$$6x + 3y + 1 = 0$$

$$\alpha x + 2y - 2 = 0$$

Will not form a Δ if $\alpha x + 2y - 2 = 0$ is concurrent with $2x - y + 3 = 0$ and $6x + 3y + 1 = 0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2} = \frac{-6}{3} \text{ or } -\frac{\alpha}{2} = 2$$

$$\Rightarrow \alpha = 4 \text{ or } \alpha = -4$$

$$P = 16 + 16 + \frac{16}{25}$$

$$[P] = \left\lfloor 32 + \frac{16}{25} \right\rfloor = 32$$

25. The coefficient of x^{2012} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to

Ans. (0)

Sol. $(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$
 $(1-x)(1-x^3)^{2007}$
 $(1-x)({}^{2007}C_0 - {}^{2007}C_1(x^3) + \dots)$

General term

$$(1-x)((-1)^r {}^{2007}C_r x^{3r})$$

$$(-1)^r {}^{2007}C_r x^{3r} - (-1)^r {}^{2007}C_r x^{3r+1}$$

$$3r = 2012$$

$$r \neq \frac{2012}{3}$$

$$3r + 1 = 2012$$

$$3r = 2011$$

$$r \neq \frac{2011}{3}$$

Hence there is no term containing x^{2012} .

So coefficient of $x^{2012} = 0$

26. If the solution curve, of the differential equation

$$\frac{dy}{dx} = \frac{x+y-2}{x-y} \text{ passing through the point } (2, 1) \text{ is}$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta} \log_e \left(\alpha + \left(\frac{y-1}{x-1}\right)^2 \right) = \log |x-1|,$$

then $5\beta + \alpha$ is equal to

Ans. (11)

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$$x = X+h, y = Y+k$$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$\left. \begin{aligned} h+k-2=0 \\ h-k=0 \end{aligned} \right\} h=k=1$$

$$Y = vX$$

$$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow -\frac{dv}{dX} = \frac{1-v^2}{1-v}$$

$$\frac{1-v}{1+v^2} dv = \frac{dX}{X}$$

$$\tan^{-1} v - \frac{1}{2} \ln(1+v^2) = \ln |X| + C$$

As curve is passing through (2, 1)

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{y-1}{x-1}\right)^2\right) = \ln |x-1|$$

$$\therefore \alpha = 1 \text{ and } \beta = 2$$

$$\Rightarrow 5\beta + \alpha = 11$$

27. Let $f(x) = \int_0^x g(t) \log_e \left(\frac{1-t}{1+t} \right) dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x} \right) dx = \left(\frac{\pi}{\alpha} \right)^2 - \alpha$, then α is equal to.....

Ans. (2)

Sol. $f(x) = \int_0^x g(t) \ln \left(\frac{1-t}{1+t} \right) dt$

$$f(-x) = \int_0^{-x} g(t) \ln \left(\frac{1-t}{1+t} \right) dt$$

$$f(-x) = -\int_0^x g(-y) \ln \left(\frac{1+y}{1-y} \right) dy$$

$$= -\int_0^x g(y) \ln \left(\frac{1-y}{1+y} \right) dy \text{ (g is odd)}$$

$$f(-x) = -f(x) \Rightarrow f \text{ is also odd}$$

Now,

$$I = \int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x} \right) dx \quad \dots(1)$$

$$I = \int_{-\pi/2}^{\pi/2} \left(f(-x) + \frac{x^2 e^x \cos x}{1+e^x} \right) dx \quad \dots(2)$$

$$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x dx = 2 \int_0^{\pi/2} x \cos x dx$$

$$I = (x^2 \sin x)_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x dx$$

$$= \frac{\pi^2}{4} - 2(-x \cos x + \int \cos x dx)_0^{\pi/2}$$

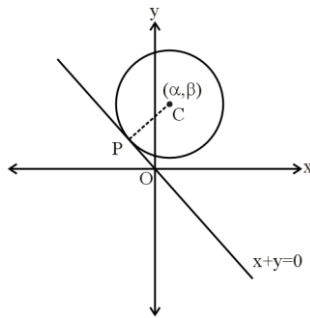
$$= \frac{\pi^2}{4} - 2(0+1) = \frac{\pi}{4} - 2 \Rightarrow \left(\frac{\pi}{2}\right)^2 - 2$$

$\therefore \alpha = 2$

28. Consider a circle $(x-\alpha)^2 + (y-\beta)^2 = 50$, where $\alpha, \beta > 0$. If the circle touches the line $y + x = 0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha + \beta)^2$ is equal to.....

Ans. (100)

Sol.



$S: (x-\alpha)^2 + (y-\beta)^2 = 50$

$CP = r$

$$\frac{|\alpha + \beta|}{\sqrt{2}} = 5\sqrt{2}$$

$$\Rightarrow (\alpha + \beta)^2 = 100$$

29. The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$$

intersect at the point P. If the

distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l ,

then $14l^2$ is equal to.....

Ans. (108)

Sol. $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda$

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k$$

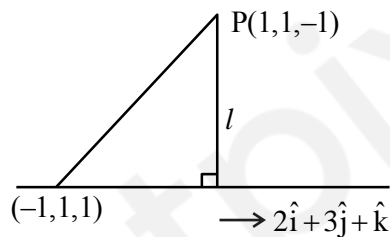
$$\Rightarrow \lambda + 2 = 4k - 3$$

$$-\lambda = 3k - 2$$

$$\Rightarrow k = 1, \lambda = -1$$

$$8\lambda + 7 = k - 2$$

$$\therefore P = (1, 1, -1)$$



Projection of $2\hat{i} - 2\hat{k}$ on $2\hat{i} + 3\hat{j} + \hat{k}$ is

$$= \frac{4 - 2}{\sqrt{4 + 9 + 1}} = \frac{2}{\sqrt{14}}$$

$$\therefore l^2 = 8 - \frac{4}{14} = \frac{108}{14}$$

$$\Rightarrow 14l^2 = 108$$

30. Let the complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100|\alpha|^2$ is.....

Ans. (20)

Sol. $|z - z_0|^2 = 4$

$$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$$

$$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$$

$$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} = 2 \dots\dots\dots(1)$$

$$|z - z_0|^2 = 16$$



$$\Rightarrow \left(\frac{1}{\alpha} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$$

$$\Rightarrow (1 - \alpha z_0)(1 - \alpha \bar{z}_0) = 16 |\alpha|^2$$

$$\Rightarrow 1 - \alpha z_0 - \alpha \bar{z}_0 + |\alpha|^2 |z_0|^2 = 16 |\alpha|^2$$

$$\Rightarrow 1 - \alpha z_0 - \alpha \bar{z}_0 = 14 |\alpha|^2 \dots\dots\dots(2)$$

From (1) and (2)

$$\Rightarrow 5 |\alpha|^2 = 1$$

$$\Rightarrow 100 |\alpha|^2 = 20$$



(Held On Saturday 27th January, 2024)
TIME : 3 : 00 PM to 6 : 00 PM
PHYSICS
TEST PAPER WITH SOLUTION
SECTION-A

31. The equation of state of a real gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT, \text{ where } P, V \text{ and } T \text{ are}$$

pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of

$\frac{a}{b^2}$ is similar to that of :

- (1) PV
- (2) P
- (3) RT
- (4) R

Ans. (2)

Sol. $[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2]$

And $[V] = [b]$

$$\frac{[a]}{[b^2]} = \frac{[PV^2]}{[V^2]} [P]$$

32. Wheatstone bridge principle is used to measure the specific resistance (S_1) of given wire, having length L, radius r. If X is the resistance of wire,

then specific resistance is : $S_1 = X \left(\frac{\pi r^2}{L}\right)$. If the

length of the wire gets doubled then the value of specific resistance will be :

- (1) $\frac{S_1}{4}$
- (2) $2S_1$
- (3) $\frac{S_1}{2}$
- (4) S_1

Ans. (4)

Sol. As specific resistance does not depend on dimension of wire so, it will not change.

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.

Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) (A) is correct but (R) is not correct
- (2) Both (A) and (R) are correct and (R) is the correct explanation of (A)
- (3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (4) (A) is not correct but (R) is correct

Ans. (2)

Sol. $\omega = \frac{2\pi}{T} \Rightarrow \omega \propto \frac{1}{T}$

$$T_{\text{moon}} = 27 \text{ days}$$

$$T_{\text{earth}} = 365 \text{ days } 4 \text{ hour}$$

$$\Rightarrow \omega_{\text{moon}} > \omega_{\text{earth}}$$

34. Given below are two statements :

Statement (I) : The limiting force of static friction depends on the area of contact and independent of materials.

Statement (II) : The limiting force of kinetic friction is independent of the area of contact and depends on materials.

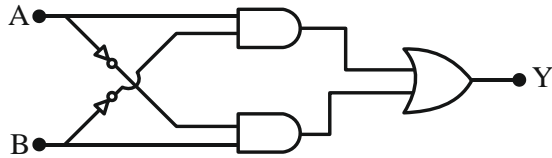
In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Statement I is correct but Statement II is incorrect
- (2) Statement I is incorrect but Statement II is correct
- (3) Both Statement I and Statement II are incorrect
- (4) Both Statement I and Statement II are correct

Ans. (2)

Sol. Co-efficient of friction depends on surface in contact So, depends on material of object.

35. The truth table of the given circuit diagram is :



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

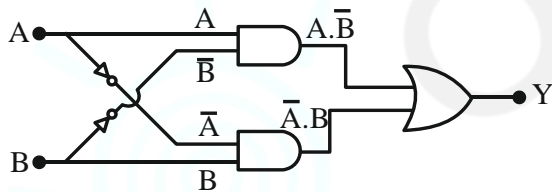
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Ans. (2)

Sol.



$$Y = A \cdot \bar{B} + \bar{A} \cdot B$$

This is XOR GATE

36. A current of $200 \mu\text{A}$ deflects the coil of a moving coil galvanometer through 60° . The current to

cause deflection through $\frac{\pi}{10}$ radian is :

- (1) $30 \mu\text{A}$ (2) $120 \mu\text{A}$
 (3) $60 \mu\text{A}$ (4) $180 \mu\text{A}$

Ans. (3)

Sol. $i \propto \theta$ (angle of deflection)

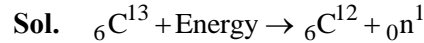
$$\therefore \frac{i_1}{i_2} = \frac{\theta_1}{\theta_2} \Rightarrow \frac{200 \mu\text{A}}{i_2} = \frac{\pi/10}{\pi/3} = \frac{3}{10}$$

$$\Rightarrow i_2 = 60 \mu\text{A}$$

37. The atomic mass of ${}^6\text{C}^{12}$ is 12.000000 u and that of ${}^6\text{C}^{13}$ is 13.003354 u . The required energy to remove a neutron from ${}^6\text{C}^{13}$, if mass of neutron is 1.008665 u , will be :

- (1) 62.5 MeV (2) 6.25 MeV
 (3) 4.95 MeV (4) 49.5 MeV

Ans. (3)



$$\Delta m = (12.000000 + 1.008665) - 13.003354$$

$$= -0.00531 \text{ u}$$

$$\therefore \text{Energy required} = 0.00531 \times 931.5 \text{ MeV}$$

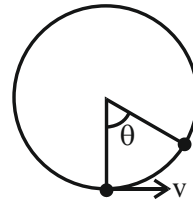
$$= 4.95 \text{ MeV}$$

38. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be :

- (1) $\tan^{-1}(\sqrt{2})$ (2) $2 \tan^{-1}\left(\frac{1}{2}\right)$
 (3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $2 \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Ans. (2)

Sol.



Loss in kinetic energy = Gain in potential energy

$$\Rightarrow \frac{1}{2}mv^2 = mg\ell(1 - \cos\theta)$$

$$\Rightarrow \frac{v^2}{\ell} = 2g(1 - \cos\theta)$$

$$\text{Acceleration at lowest point} = \frac{v^2}{\ell}$$

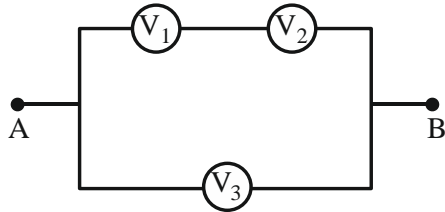
$$\text{Acceleration at extreme point} = g\sin\theta$$

$$\text{Hence, } \frac{v^2}{\ell} = g\sin\theta$$

$$\therefore \sin\theta = 2(1 - \cos\theta)$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2 \tan^{-1}\left(\frac{1}{2}\right)$$

39. Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B, their readings are V_1 , V_2 and V_3 . Choose the correct option.



- (1) $V_1 = V_2$ (2) $V_1 \neq V_3 - V_2$
 (3) $V_1 + V_2 > V_3$ (4) $V_1 + V_2 = V_3$

Ans. (4)

Sol. From KVL,

$$V_1 + V_2 - V_3 = 0 \Rightarrow V_1 + V_2 = V_3$$

40. The total kinetic energy of 1 mole of oxygen at 27°C is :

[Use universal gas constant (R) = 8.31 J/mole K]

- (1) 6845.5 J (2) 5942.0 J
 (3) 6232.5 J (4) 5670.5 J

Ans. (3)

Sol. Kinetic energy = $\frac{f}{2} nRT$

$$= \frac{5}{2} \times 1 \times 8.31 \times 300 \text{ J}$$

$$= 6232.5 \text{ J}$$

41. Given below are two statements : one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion (A) : In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R) : The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both (A) and (R) are correct and (R) is the correct explanation of (A)
 (2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
 (3) (A) is true but (R) is false
 (4) (A) is false but (R) is true

Ans. (2)

Sol. Assertion & Reason both are correct Theory

42. Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46Ω . The power consumed in it is :

- (1) 12.5 W (2) 10.0 W
 (3) 11.5 W (4) 12.0 W

Ans. (3)

Sol. $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\frac{230}{V_2} = \frac{10}{1}$$

$$V_2 = 23 \text{ V}$$

$$\text{Power consumed} = \frac{V_2^2}{R}$$

$$= \frac{23 \times 23}{46} = 11.5 \text{ W}$$

43. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute

temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is :

- (1) $\frac{5}{3}$ (2) $\frac{3}{2}$
 (3) $\frac{7}{5}$ (4) $\frac{9}{7}$

Ans. (2)

Sol. $P \propto T^3 \Rightarrow PT^{-3} = \text{constant}$

$$PV^\gamma = \text{constant}$$

$$P \left(\frac{nRT}{P} \right)^\gamma = \text{constant}$$

$$P^{1-\gamma} T^\gamma = \text{constant}$$

$$PT^{\frac{\gamma}{1-\gamma}} = \text{constant}$$

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3 + 3\gamma$$

$$3 = 2\gamma$$

$$\gamma = \frac{3}{2}$$

44. The threshold frequency of a metal with work function 6.63 eV is :

- (1) 16×10^{15} Hz
- (2) 16×10^{12} Hz
- (3) 1.6×10^{12} Hz
- (4) 1.6×10^{15} Hz

Ans. (4)

Sol. $\phi_0 = h\nu_0$

$$6.63 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \nu_0$$

$$\nu_0 = \frac{1.6 \times 10^{-19}}{10^{-34}}$$

$$\nu_0 = 1.6 \times 10^{15} \text{ Hz}$$

45. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.

Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.

In the light of the above statements, choose the correct answer from the options given below :

- (1) (A) is false but (R) is true
- (2) (A) is true but (R) is false
- (3) Both (A) and (R) are true and (R) is the correct explanation (A)
- (4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. (3 or 4)

Sol. Theory

46. When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :

- (1) 60°
- (2) 30°
- (3) 90°
- (4) 45°

Ans. (4)

Sol. Let I_0 be intensity of unpolarised light incident on first polaroid.

$$I_1 = \text{Intensity of light transmitted from 1}^{\text{st}} \text{ polaroid} \\ = \frac{I_0}{2}$$

θ be the angle between 1st and 2nd polaroid

ϕ be the angle between 2nd and 3rd polaroid

$\theta + \phi = 90^\circ$ (as 1st and 3rd polaroid are crossed)

$$\phi = 90^\circ - \theta$$

$I_2 =$ Intensity from 2nd polaroid

$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$

$I_3 =$ Intensity from 3rd polaroid

$$I_3 = I_2 \cos^2 \phi$$

$$I_3 = I_1 \cos^2 \theta \cos^2 \phi$$

$$I_3 = \frac{I_0}{2} \cos^2 \theta \cos^2 \phi$$

$$\phi = 90^\circ - \theta$$

$$I_3 = \frac{I_0}{2} \cos^2 \theta \sin^2 \theta$$

$$I_3 = \frac{I_0}{2} \left[\frac{2 \sin \theta \cos \theta}{2} \right]^2$$

$$I_3 = \frac{I_0}{8} \sin^2 2\theta$$

I_3 will be maximum when $\sin 2\theta = 1$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

47. An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity $6 \times 10^8 \text{ W/m}^2$ falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $= 3 \times 10^8 \text{ m/s}$) :

- (1) 36 Nm^{-2}
- (2) 18 Nm^{-2}
- (3) 6 Nm^{-2}
- (4) 2 Nm^{-2}

Ans. (3)

Sol. Radiation pressure $= \frac{I}{v}$

$$= \frac{I \cdot \mu}{c}$$

$$= \frac{6 \times 10^8 \times 3}{3 \times 10^8}$$

$$= 6 \text{ N/m}^2$$

48. Given below are two statements : one is labelled a Assertion (A) and the other is labelled as Reason(R)

Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.

Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (2) ((A) is correct but (R) is not correct
- (3) (A) is not correct but (R) is correct
- (4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (4)

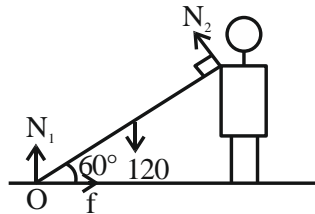
Sol. Electric line of force are always perpendicular to equipotential surface so angle between force and displacement will always be 90° . So work done equal to 0.

49. A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is :

- (1) 6 kg
- (2) 12 kg
- (3) 3 kg
- (4) $6\sqrt{3}$ kg

Ans. (3)

Sol.



Torque about O = 0

$$120 \left(\frac{L}{2} \cos 60^\circ \right) - N_2 L = 0$$

$$N_2 = 30 \text{ N}$$

50. A bullet is fired into a fixed target loses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is :

- (1) 2
- (2) 5
- (3) 3
- (4) 4

Ans. (Bonus)

Sol. $v^2 - u^2 = 2aS$

$$\left(\frac{2u}{3} \right)^2 = u^2 + 2(-a)(4 \times 10^{-2})$$

$$\frac{4u^2}{9} = u^2 - 2a(4 \times 10^{-2})$$

$$-\frac{5u^2}{9} = -2a(4 \times 10^{-2}) \dots(1)$$

$$0 = \left(\frac{2u}{3} \right)^2 + 2(-a)(x)$$

$$-\frac{4u^2}{9} = -2ax \dots(2)$$

(1) / (2)

$$\frac{5}{4} = \frac{4 \times 10^{-2}}{x}$$

$$x = \frac{16}{5} \times 10^{-2}$$

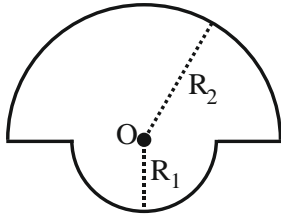
$$x = 3.2 \times 10^{-2} \text{ m}$$

$$x = 32 \times 10^{-3} \text{ m}$$

Note : Since no option is matching, Question should be bonus.

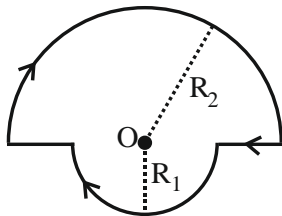
SECTION-B

51. The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi$ m and $R_2 = 4\pi$ m carrying current $I = 4$ A as per figure given below is $\alpha \times 10^{-7}$ T. The value of α is _____. (Centre O is common for all segments)



Ans. (3.00)

Sol.



$$\frac{\mu_0 i}{2R_2} \left(\frac{\pi}{2\pi} \right) \otimes + \frac{\mu_0 i}{2R_1} \left(\frac{\pi}{2\pi} \right) \otimes$$

$$\left(\frac{\mu_0 i}{4R_2} + \frac{\mu_0 i}{4R_1} \right)$$

$$\frac{4\pi \times 10^{-7} \times 4}{4 \times 4\pi} + \frac{4\pi \times 10^{-7} \times 4}{4 \times 2\pi}$$

$$= 3 \times 10^{-7} = \alpha \times 10^{-7}$$

$$\alpha = 3$$

52. Two charges of $-4 \mu\text{C}$ and $+4 \mu\text{C}$ are placed at the points $A(1, 0, 4)$ m and $B(2, -1, 5)$ m located in an electric field $\vec{E} = 0.20 \hat{i}$ V/cm. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha} \times 10^{-5}$ Nm, Where $\alpha =$ _____.

Ans. (2.00)

Sol.

$$\begin{matrix} (1, 0, 4) & & (2, -1, 5) \\ \bullet & \cdots & \bullet \\ A & & B \\ -4\mu\text{C} & & 4\mu\text{C} \end{matrix}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\vec{p} = q\vec{\ell}$$

$$\vec{E} = 0.2 \frac{\text{V}}{\text{cm}} = 20 \frac{\text{V}}{\text{m}}$$

$$\vec{p} = 4 \times (\hat{i} - \hat{j} + \hat{k})$$

$$= (4\hat{i} - 4\hat{j} + 4\hat{k}) \mu\text{C} \cdot \text{m}$$

$$\vec{\tau} = (4\hat{i} - 4\hat{j} + 4\hat{k}) \times (20\hat{i}) \times 10^{-6} \text{ Nm}$$

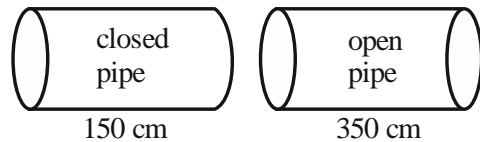
$$= (8\hat{k} + 8\hat{j}) \times 10^{-5} = 8\sqrt{2} \times 10^{-5}$$

$$\alpha = 2$$

53. A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is _____ m/s.

Ans. (294.00)

Sol.



$$f_c = \frac{v}{4l_1} \quad f_o = \frac{v}{2l_2}$$

$$|f_c - f_o| = 7$$

$$\frac{v}{4 \times 150} - \frac{v}{2 \times 350} = 7$$

$$\frac{v}{600 \text{ cm}} - \frac{v}{700 \text{ cm}} = 7$$

$$\frac{v}{6\text{m}} - \frac{v}{7\text{m}} = 7$$

$$v \left(\frac{1}{42} \right) =$$

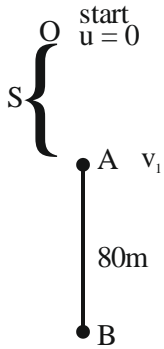
$$v = 42 \times 7$$

$$= 294 \text{ m/s}$$

54. A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is _____ m (use $g = 10 \text{ ms}^{-2}$)

Ans. (45.00)

Sol.



From A \rightarrow B

$$-80 = -v_1 t - \frac{1}{2} \times 10 t^2$$

$$-80 = -2v_1 - \frac{1}{2} \times 10 \times 2^2$$

$$-80 = -2v_1 - 20$$

$$-60 = -2v_1$$

$$v_1 = 30 \text{ m/s}$$

From O to A

$$v^2 = u^2 + 2gS$$

$$30^2 = 0 + 2 \times (-10)(-S)$$

$$900 = 20 S$$

$$S = 45 \text{ m}$$

55. The reading of pressure metre attached with a closed pipe is $4.5 \times 10^4 \text{ N/m}^2$. On opening the valve, water starts flowing and the reading of pressure metre falls to $2.0 \times 10^4 \text{ N/m}^2$. The velocity of water is found to be $\sqrt{V} \text{ m/s}$. The value of V is _____

Ans. (50)

Sol. Change in pressure $= \frac{1}{2} \rho v^2$

$$4.5 \times 10^4 - 2.0 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$$2.5 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$$v^2 = 50$$

$$v = \sqrt{50}$$

$$\text{Velocity of water} = \sqrt{V} = \sqrt{50}$$

$$= V = 50$$

56. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is _____.

Ans. (7.00)

Sol. In pure rolling work done by friction is zero. Hence potential energy is converted into kinetic energy. Since initially the ring and the sphere have same potential energy, finally they will have same kinetic energy too.

$$\therefore \text{Ratio of kinetic energies} = 1$$

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

57. A parallel beam of monochromatic light of wavelength 5000 \AA is incident normally on a single narrow slit of width 0.001 mm . The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of _____ (degree).

Ans. (30.00)

Sol. For first minima

$$a \sin \theta = \lambda$$

$$\Rightarrow \sin \theta = \frac{\lambda}{a} = \frac{5000 \times 10^{-10}}{1 \times 10^{-6}} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$



58. The electric potential at the surface of an atomic nucleus ($z = 50$) of radius 9×10^{-13} cm is $\times 10^6$ V.

Ans. (8.00)

Sol. Potential = $\frac{kQ}{R} = \frac{kZe}{R}$
 $= \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-13} \times 10^2}$
 $= 8 \times 10^6$ V

59. If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where

$$\alpha = \underline{\hspace{2cm}}$$

Ans. (144.00)

Sol. Longest wavelength corresponds to transition between $n = 3$ and $n = 4$

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{3^2} - \frac{1}{4} \right) = RZ \left(\frac{1}{9} - \frac{1}{16} \right)$$

$$= \frac{7RZ^2}{9 \times 16}$$

$$\Rightarrow \lambda = \frac{144}{7R} \text{ for } Z = 1 \quad \therefore \alpha = 144$$

60. A series LCR circuit with $L = \frac{100}{\pi}$ mH, $C = \frac{10^{-3}}{\pi}$ F and $R = 10 \Omega$, is connected across an ac source of 220 V, 50 Hz supply. The power factor of the circuit would be _____.

Ans. (1.00)

Sol. $X_C = \frac{1}{\omega C} = \frac{\pi}{2\pi \times 50 \times 10^{-3}} = 10 \Omega$

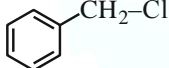
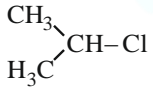
$$X_L = \omega L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3}$$

$$= 10 \Omega$$

$\therefore X_C = X_L$, Hence, circuit is in resonance

$$\therefore \text{power factor} = \frac{R}{Z} = \frac{R}{R} = 1$$

(Held On Saturday 27th January, 2024)
TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY	TEST PAPER WITH SOLUTION
SECTION-A	
<p>61. The order of relative stability of the contributing structure is:</p> $ \begin{array}{ccc} \begin{array}{c} \text{:O:} \\ \parallel \\ \text{CH}_2=\text{CH}-\text{C}-\text{H} \\ \text{I} \end{array} & \longleftrightarrow & \begin{array}{c} \oplus \\ \text{CH}_2-\text{CH}=\text{C}-\text{H} \\ \text{II} \end{array} \\ & & \begin{array}{c} \text{:O:} \ominus \\ \\ \text{CH}_2-\text{CH}=\text{C}-\text{H} \\ \text{III} \end{array} \\ & & \begin{array}{c} \ominus \\ \\ \text{CH}_2-\text{CH}=\text{C}-\text{H} \\ \text{III} \end{array} \end{array} $ <p>Choose the correct answer from the options given below:</p> <p>(1) I > II > III (2) II > I > III (3) I = II = III (4) III > II > I</p> <p>Ans. (1)</p> <p>Sol. I > II > III, since neutral resonating structures are more stable than charged resonating structure. II > III, since stability of structure with -ve charge on more electronegative atom is higher.</p> <p>62. Which among the following halide/s will not show S_N1 reaction:</p> <p>(A) H₂C = CH - CH₂Cl (B) CH₃ - CH = CH - Cl (C)  (D) </p> <p>Choose the most appropriate answer from the options given below:</p> <p>(1) (A), (B) and (D) only (2) (A) and (B) only (3) (B) and (C) only (4) (B) only</p> <p>Ans. (4)</p> <p>Sol. Since CH₃ - CH = CH⁺ is very unstable, CH₃ - CH = CH - Cl cannot give S_N1 reaction.</p>	<p>63. Which of the following statements is not correct about rusting of iron?</p> <p>(1) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off. (2) When pH lies above 9 or 10, rusting of iron does not take place. (3) Dissolved acidic oxides SO₂, NO₂ in water act as catalyst in the process of rusting. (4) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.</p> <p>Ans. (1)</p> <p>Sol. As tin coating is peeled off, then iron is exposed to atmosphere.</p> <p>64. Given below are two statements: Statement (I) : In the Lanthanoids, the formation of Ce⁺⁴ is favoured by its noble gas configuration. Statement (II) : Ce⁺⁴ is a strong oxidant reverting to the common +3 state.</p> <p>In the light of the above statements, choose the most appropriate answer from the options given below:</p> <p>(1) Statement I is false but Statement II is true (2) Both Statement I and Statement II are true (3) Statement I is true but Statement II is false (4) Both Statement I and Statement II are false</p> <p>Ans. (2)</p> <p>Sol. Statement (1) is true, Ce⁺⁴ has noble gas electronic configuration. Statement (2) is also true due to high reduction potential for Ce⁴⁺/Ce³⁺ (+1.74V), and stability of Ce³⁺, Ce⁴⁺ acts as strong oxidizing agent.</p> <p>65. Choose the correct option having all the elements with d¹⁰ electronic configuration from the following:</p> <p>(1) ²⁷Co, ²⁸Ni, ²⁶Fe, ²⁴Cr (2) ²⁹Cu, ³⁰Zn, ⁴⁸Cd, ⁴⁷Ag (3) ⁴⁶Pd, ²⁸Ni, ²⁶Fe, ²⁴Cr (4) ²⁸Ni, ²⁴Cr, ²⁶Fe, ²⁹Cu</p> <p>Ans. (2)</p> <p>Sol. [Cr] = [Ar]4s¹3d⁵ [Cd] = [Kr]5s²4d¹⁰ [Cu] = [Ar]4s¹3d¹⁰ [Ag] = [Kr]5s¹4d¹⁰ [Zn] = [Ar]4s²3d¹⁰</p>

66. Phenolic group can be identified by a positive:
- (1) Phthalein dye test
 - (2) Lucas test
 - (3) Tollen's test
 - (4) Carbylamine test

Ans. (1)

Sol. Carbylamine Test-Identification of primary amines
Lucas Test - Differentiation between 1°, 2° and 3° alcohols

Tollen's Test - Identification of Aldehydes

Phthalein Dye Test - Identification of phenols

67. The molecular formula of second homologue in the homologous series of mono carboxylic acids is _____.

- (1) C₃H₆O₂
- (2) C₂H₄O₂
- (3) CH₂O
- (4) C₂H₂O₂

Ans. (2)

Sol. HCOOH, CH₃COOH



Second homologue

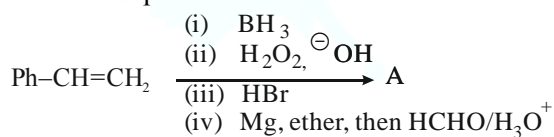
68. The technique used for purification of steam volatile water immiscible substance is:

- (1) Fractional distillation
- (2) Fractional distillation under reduced pressure
- (3) Distillation
- (4) Steam distillation

Ans. (4)

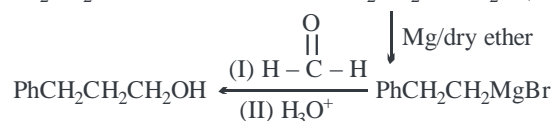
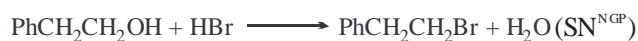
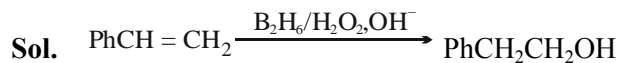
Sol. Steam distillation is used for those liquids which are insoluble in water, containing non-volatile impurities and are steam volatile.

69. The final product A, formed in the following reaction sequence is:



- (1) Ph-CH₂-CH₂-CH₃
- (2) Ph-CH(CH₃)-CH₃
- (3) Ph-CH(CH₃)-CH₂OH
- (4) Ph-CH₂-CH₂-CH₂-OH

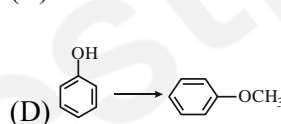
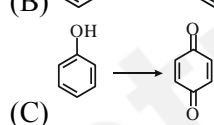
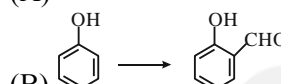
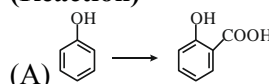
Ans. (4)



70. Match List-I with List-II.

List - I

(Reaction)



List - II

(Reagent(s))

(I) Na₂Cr₂O₇, H₂SO₄

(II) (i) NaOH (ii) CH₃Cl

(III) (i) NaOH, CHCl₃

(ii) NaOH (iii) HCl

(IV) (i) NaOH (ii) CO₂

(iii) HCl

Choose the correct answer from the options given below:

- (1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
- (2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
- (3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
- (4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

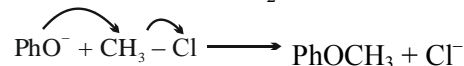
Ans. (4)

Sol. (A) → Kolbe Schmidt Reaction

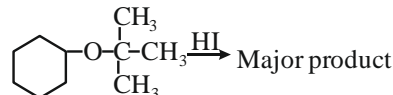
(B) → Reimer Tiemann Reaction

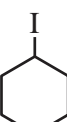
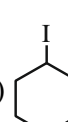
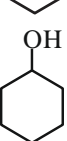
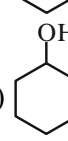
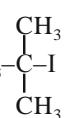
(C) → Oxidation of phenol to p-benzoquinone

(D) → PhOH + NaOH → H₂O + PhO⁻



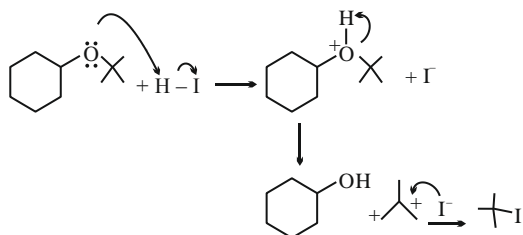
71. Major product formed in the following reaction is a mixture of:



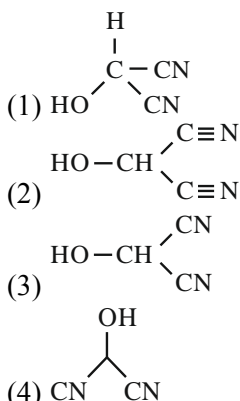
- (1)  and (CH₃)₃CI
- (2)  and (CH₃)₃COH
- (3)  and (CH₃)₃COH
- (4)  and 

Ans. (4)

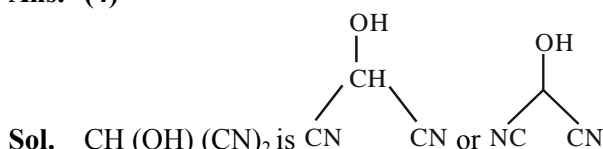
Sol.



72. Bond line formula of $\text{HOCH}(\text{CN})_2$ is:



Ans. (4)



73. Given below are two statements:

Statement (I) : Oxygen being the first member of group 16 exhibits only -2 oxidation state.

Statement (II) : Down the group 16 stability of $+4$ oxidation state decreases and $+6$ oxidation state increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) Statement I is correct but Statement II is incorrect
- (2) Both Statement I and Statement II are correct
- (3) Both Statement I and Statement II are incorrect
- (4) Statement I is incorrect but Statement II is correct

Ans. (3)

Sol. Statement-I: Oxygen can have oxidation state from -2 to $+2$, so statement I is incorrect

Statement- II: On moving down the group stability of $+4$ oxidation state increases whereas stability of $+6$ oxidation state decreases down the group, according to inert pair effect.

So both statements are wrong.

74. Identify from the following species in which d^2sp^3 hybridization is shown by central atom:

- (1) $[\text{Co}(\text{NH}_3)_6]^{3+}$
- (2) BrF_5
- (3) $[\text{Pt}(\text{Cl})_4]^{2-}$
- (4) SF_6

Ans. (1)

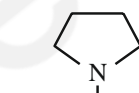
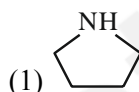
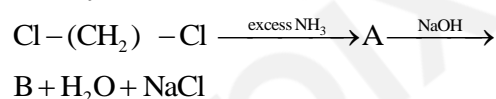
Sol. $[\text{Co}(\text{NH}_3)_6]^{3+}$ – d^2sp^3 hybridization

BrF_5 – sp^3d^2 hybridization

$[\text{PtCl}_4]^{2-}$ – dsp^2 hybridization

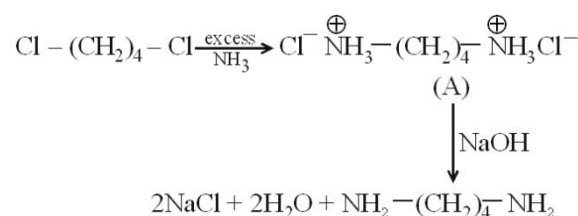
SF_6 – sp^3d^2 hybridization

75. Identify B formed in the reaction.



Ans. (2)

Sol.



76. The quantity which changes with temperature is:

- (1) Molarity
- (2) Mass percentage
- (3) Molality
- (4) Mole fraction

Ans. (1)

Sol. $\text{Molarity} = \frac{\text{Moles of solute}}{\text{Volume of solution}}$

Since volume depends on temperature, molarity will change upon change in temperature.

77. Which structure of protein remains intact after coagulation of egg white on boiling?

- (1) Primary
- (2) Tertiary
- (3) Secondary
- (4) Quaternary

Ans. (1)

Sol. Boiling an egg causes denaturation of its protein resulting in loss of its quaternary, tertiary and secondary structures.

78. Which of the following cannot function as an oxidising agent?

- (1) N^{3-}
- (2) SO_4^{2-}
- (3) BrO_3^-
- (4) MnO_4^-

Ans. (1)

Sol. In N^{3-} ion 'N' is present in its lowest possible oxidation state, hence it cannot be reduced further because of which it cannot act as an oxidizing agent.

79. The incorrect statement regarding conformations of ethane is:

- (1) Ethane has infinite number of conformations
- (2) The dihedral angle in staggered conformation is 60°
- (3) Eclipsed conformation is the most stable conformation.
- (4) The conformations of ethane are inter-convertible to one-another.

Ans. (3)

Sol. Eclipsed conformation is the least stable conformation of ethane.

80. Identify the incorrect pair from the following:

- (1) Photography - AgBr
- (2) Polythene preparation - $TiCl_4$, $Al(CH_3)_3$
- (3) Haber process - Iron
- (4) Wacker process - Pt Cl_2

Ans. (4)

Sol. The catalyst used in Wacker's process is $PdCl_2$

SECTION-B

81. Total number of ions from the following with noble gas configuration is _____.

Sr^{2+} ($Z = 38$), Cs^+ ($Z = 55$), La^{2+} ($Z = 57$) Pb^{2+} ($Z = 82$), Yb^{2+} ($Z = 70$) and Fe^{2+} ($Z = 26$)

Ans. (2)

Sol. Noble gas configuration = $ns^2 np^6$

$[Sr^{2+}] = [Kr]$

$[Cs^+] = [Xe]$

$[Yb^{2+}] = [Xe] 4f^{14}$

$[La^{2+}] = [Xe] 5d^1$

$[Pb^{2+}] = [Xe] 4f^{14} 5d^{10} 6s^2$

$[Fe^{2+}] = [Ar] 3d^6$

82. The number of non-polar molecules from the following is _____

HF, H_2O , SO_2 , H_2 , CO_2 , CH_4 , NH_3 , HCl, $CHCl_3$, BF_3

Ans. (4)

Sol. The non-polar molecules are CO_2 , H_2 , CH_4 and BF_3

83. Time required for completion of 99.9% of a First order reaction is _____ times of half life ($t_{1/2}$) of the reaction.

Ans. (10)

Sol.

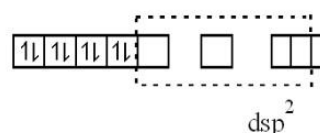
$$\frac{t_{99.9\%}}{t_{1/2}} = \frac{\frac{2.303 \left(\frac{a}{a-x} \right)}{k} \log \left(\frac{100}{100-99.9} \right)}{\frac{2.303}{k} \log 2} = \frac{\log \left(\frac{100}{100-99.9} \right)}{\log 2} = \frac{\log 10^3}{\log 2} = \frac{3}{0.3} = 10$$

84. The Spin only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is _____ B.M. (Nearest integer)

(Given atomic number for Pt = 78)

Ans. (0)

Sol. Pt^{2+} (d^8)



$Pt^{2+} \rightarrow dsp$ hybridization and have no unpaired e^- s.

\therefore Magnetic moment = 0

85. For a certain thermochemical reaction $M \rightarrow N$ at $T = 400$ K, $\Delta H^\circ = 77.2$ kJ mol^{-1} , $\Delta S = 122$ JK $^{-1}$, log equilibrium constant (logK) is _____ $\times 10^{-1}$.

Ans. (37)

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$$= 77.2 \times 10^3 - 400 \times 122 = 28400 \text{ J}$$

$$\Delta G^\circ = -2.303 RT \log K$$

$$\Rightarrow 28400 = -2.303 \times 8.314 \times 400 \log K$$

$$\Rightarrow \log K = -3.708 = -37.08 \times 10^{-1}$$

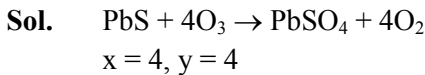
86. Volume of 3 M NaOH (formula weight 40 g mol⁻¹) which can be prepared from 84 g of NaOH is _____ × 10⁻¹ dm³.

Ans. (7)

$$\text{Sol. } M = \frac{n_{\text{NaOH}}}{V_{\text{sol}} (\text{in L})} \Rightarrow 3 = \frac{(84/40)}{V} \Rightarrow V = 0.7\text{L} = 7 \times 10^{-1}\text{L}$$

87. 1 mole of PbS is oxidised by "X" moles of O₃ to get "Y" moles of O₂. X + Y = _____

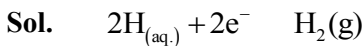
Ans. (8)



88. The hydrogen electrode is dipped in a solution of pH = 3 at 25°C. The potential of the electrode will be - _____ × 10⁻² V.

$$\left(\frac{2.303RT}{F} = 0.059\text{V} \right)$$

Ans. (18)

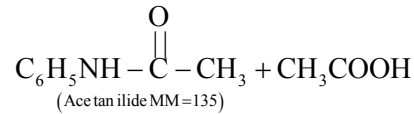
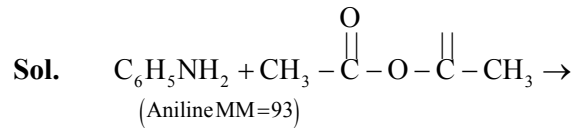


$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.059}{2} \log \frac{P_{\text{H}_2}}{[\text{H}^+]^2}$$

$$= 0 - 0.059 \times 3 = -0.177 \text{ volts.} = -17.7 \times 10^{-2} \text{ V.}$$

89. 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _____ × 10⁻¹ g. (Given molar mass in g mol⁻¹ N : 14, O : 16, C : 12, H : 1)

Ans. (135)

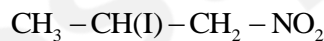
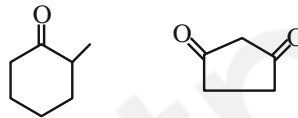


$$n_{\text{Acetanilide}} = n_{\text{Aniline}}$$

$$\Rightarrow \frac{m}{135} = \frac{9.3}{93}$$

$$\Rightarrow m = 13.5\text{g}$$

90. Total number of compounds with Chiral carbon atoms from following is _____.



Ans. (5)

Sol. Chiral carbons are marked by.

