(Held On Friday 05th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point (7, 8, 9). Then $d^2 + 6$ is equal to :

(1) 72 (2) 69

(3) 75 (4) 78

Ans. (3)

Sol. $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$ …(1)

$x = 3\lambda - 6, y = 2\lambda, z = \lambda - 1$

$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$ …(2)

$x = 4\mu + 7, y = 3\mu + 9, z = 2\mu + 4$

$3\lambda - 6 = 4\mu + 7 \Rightarrow 3\lambda - 4\mu = 13$ …(3) × 2

$2\lambda = 3\mu + 9 \Rightarrow 2\lambda - 3\mu = 9$ …(4) × 3

$6\lambda - 8\mu = 26$

$6\lambda - 9\mu = 27$

$-\quad +\quad -$

$\mu = -1$

$\Rightarrow 3\lambda - 4(-1) = 13$

$3\lambda = 9$

$\lambda = 3$

int. point (3, 6, 2) ; (7, 8, 9)

$d^2 = 16 + 4 + 49 = 69$

Ans. $d^2 + 6 = 69 + 6 = 75$

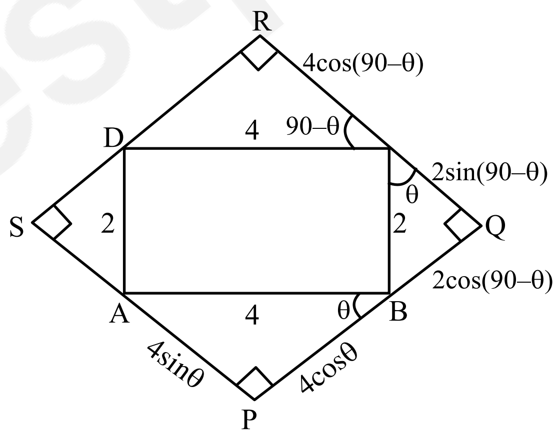
2. Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a + b)^2$ is equal to :

(1) 72 (2) 60

(3) 80 (4) 64

Ans. (1)

Sol.



Area = $(4\cos\theta + 2\sin\theta)(2\cos\theta + 4\sin\theta)$

$= 8\cos^2\theta + 16\sin\theta\cos\theta + 4\sin\theta\cos\theta + 8\sin^2\theta$

$= 8 + 20\sin\theta\cos\theta$

$= 8 + 10\sin2\theta$

Max Area = 8 + 10 = 18 ($\sin2\theta = 1$) $\theta = 45°$

$(a + b)^2 = (4\cos\theta + 2\sin\theta + 2\cos\theta + 4\sin\theta)^2$

$= (6\cos\theta + 6\sin\theta)^2$

$= 36(\sin\theta + \cos\theta)^2$

$= 36(\sqrt{2})^2$

$= 72$

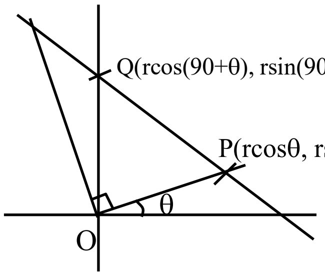
3. Let two straight lines drawn from the origin O intersect the line 3x + 4y = 12 at the points P and Q such that ΔOPQ is an isosceles triangle and ∠POQ = 90°. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

(1) 44 (2) 48

(3) 46 (4) 42

Ans. (3)

Sol.



$3x + 4y = 12$

$3(r\cos\theta) + 4(r\sin\theta) = 12$

$r(3\cos\theta + 4\sin\theta) = 12 \quad ...(1)$

$3(-r\sin\theta) + 4(r\cos\theta) = 12$

$r(-3\sin\theta + 4\cos\theta) = 12 \quad ...(2)$

$$\left(\frac{12}{}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta) \ + (-3\sin\theta + 4\cos\theta)^2$$

$$2\left(\frac{12}{r}\right)^2 = 9 + 16$$

$$\frac{2\times 144}{r^2} = 25 \Rightarrow 288 = 25r^2$$

$$\Rightarrow \frac{288}{25} = r^2$$

$$\Rightarrow \sqrt{2}\left(\frac{12}{5}\right)$$

$\ell = OP^2 + PQ^2 + QO^2$

$\ell = r^2 + r^2 \quad + r^2(\cos\theta + \sin\theta)^2 + r^2(\sin\theta + \cos\theta)^2$

$= 2r^2 + r^2(1 + \sin2\theta + 1 - 2\sin2\theta)$

$= 2r^2 + 2r^2$

$= 4r^2$

$$= 4 \ \frac{288}{25}\Big) = \frac{1152}{25} = 46.08$$

$[\ell] = 46$

4. If y = y(x) is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(1) $e^{-\pi/8}$ (2) $e^{-\pi/4}$

(3) $e^{\pi/4}$ (4) $e^{\pi/8}$

Ans. (2)

Sol. $\frac{dy}{dx} + 2y = \sin 2x$, $y(0) = \frac{3}{4}$

$I.F = e^{\int 2\,dx} = e^{2x}$

$y.e^{2x} = \int e^{2x}\sin 2x\,dx$

$$y.e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4+4} + C$$

$$x = 0,\ y = \frac{3}{4} \Rightarrow \frac{3}{4}.1 = \frac{1(0-2)}{8} + C$$

$$\frac{3}{4} = -\frac{1}{4} + C$$

$1 = C$

$$y = \frac{2\sin 2x - 2\cos 2x}{8} \quad 1.e^{-2x}$$

$$x = \frac{\pi}{8},\ y = \frac{1}{8}\left(2\sin\frac{\pi}{4} - 2\cos\frac{}{4} \quad + e^{-2\left(\frac{\pi}{8}\right)}\right.$$

$$y = 0 + e^{-\frac{\pi}{4}}$$

5. For the function

$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$,

consider the following two statements :

(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$.

(II) f′ is decreasing in $\left(0, \frac{\pi}{2}\right)$.

Between the above two statements,

(1) only (I) is true.

(2) only (II) is true.

(3) neither (I) nor (II) is true.

(4) both (I) and (II) are true.

Ans. (4)

Sol. $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x) \quad x \in \left[0, \frac{\pi}{2}\right]$

$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x + 1) > 0 \quad f(x) \uparrow$

$f'(x) = -\sin x + 0 - \frac{\pi}{2}(2)$

$= -\sin x - \frac{4}{\pi} < 0 \quad f'(x) \downarrow$

$0 < x < \frac{\pi}{2}$

$\Rightarrow -\frac{2}{\pi}\left(0 < 2x < \pi\right)$ (+, +1, 1)

$-\frac{2}{\pi} > \frac{-2}{\pi}(2x+1) > -\frac{2}{\pi}(\pi+1)$ (+3, +3, +3)

$3 - \frac{2}{\pi} > 3 - \frac{2}{\pi}(2x+1) > 3 - \frac{2}{\pi}(\pi+1)$ (+ve) (ve)

6. If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(1) 49 (2) 45

(3) 47 (4) 51

Ans. (3)

Sol. $11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

for infinite sol.

$$D = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & 19 & -39 \end{vmatrix} \quad 0$$

$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$

$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$

$\Rightarrow 62\lambda = 118 - 242$

$\Rightarrow \lambda = \frac{-124}{62} = -2$

$$D_1 = \begin{vmatrix} -5 & 1 & -2 \\ 3 & 3 & 5 \\ \mu & -19 & -39 \end{vmatrix} = 0$$

$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$

$\Rightarrow -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$

$\Rightarrow 11\mu = -110 - 231 = -341$

$\Rightarrow \mu = -31$

$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$

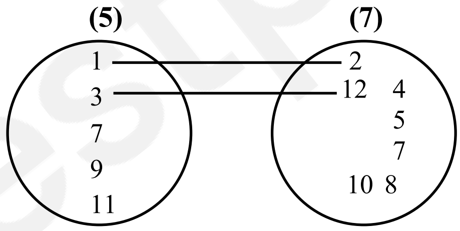
7. Let A = {1, 3, 7, 9, 11} and B = {2, 4, 5, 7, 8, 10, 12}. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$, is :

(1) 180 (2) 120

(3) 480 (4) 240

Ans. (4)

Sol.



A = {1, 3, 7, 9, 11}

B = {2, 4, 5, 7, 8, 10, 12}

f(1) + f(3) = 14

(i) 2 + 12

(ii) 4 + 10

$2 \times (2 \times 5 \times 4 \times 3) = 240$

8. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$, is continuous at x = 0, then f(0) is equal to :

(1) 2 (2) –2

(3) 4 (4) –4

Ans. (4)

Sol. $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$

is continuous at x = 0

$$\lim_{x\to 0} = \frac{3x - \frac{(3x)^3}{\lfloor 3} + \ldots + \alpha\left(x - \frac{x^3}{\lfloor 3}\ldots\right) - \beta\left(1 - \frac{(3x)^2}{\lfloor 2}\ldots\right)}{x^3} = f(0)$$

$$\lim_{x\to 0} = \frac{-\beta + x(3+\alpha) + \frac{9\beta x^2}{\lfloor 2} + \left(\frac{-27}{\lfloor 3} - \frac{\alpha}{\lfloor 3}\right)x^3 \ldots}{x^3} = f(0)$$

for exist

$$\beta = 0,\ 3+\alpha = 0,\ -\frac{27}{\lfloor 3} - \frac{\alpha}{\lfloor 3} = f(0)$$

$$\alpha = -3,\ -\frac{27}{6} - \frac{(-3)}{6} = f(0)$$

$$f(0) = \frac{-27+3}{6} = -4$$

9. The integral $\int_0^{\frac{\pi}{4}} \frac{136\sin x}{3\sin x + 5\cos x} dx$ is equal to :

(1) $3\pi - 50 \log_e 2 + 20 \log_e 5$

(2) $3\pi - 25 \log_e 2 + 10 \log_e 5$

(3) $3\pi - 10 \log_e\left(2\sqrt{2}\right) + 10 \log_e 5$

(4) $3\pi - 30 \log_e 2 + 20 \log_e 5$

Ans. (1)

Sol. $I = \int_0^{\pi/4} \frac{136\sin x}{3\sin x + 5\cos x} dx$

136sinx = A(3sinx + 5cosx) + B(3cosx – 5sinx)

$136 = 3A - 5B \quad \ldots(1)$

$0 = 5A + 3B \quad \ldots(2)$

$$3B = -5A \Rightarrow B = -\frac{5}{3}$$

$$136 = 3A - 5\left(-\frac{5}{3}A\right)$$

$$136 = 3A \quad \frac{25}{3}A$$

$$136 = \frac{34A}{3}$$

$$\Rightarrow A = \frac{136\times 3}{34} \quad 12$$

$$B = \frac{-5}{3}(12) \quad -20$$

$$I = \int_0^{\pi/4} \frac{A(3\sin x \quad 5\cos x)}{3\sin x + 5\cos x} + \int_0^{\pi/4} \frac{B(3\cos x - 5\sin x)}{3\sin x + 5\cos x}$$

$$= A\left(x\right)_0^{\pi/4} + B\left[\ell n\left(3\sin x + 5\cos x\right)\right]_0^{\pi/4}$$

$$= 12\left(\frac{\pi}{4}\right) - 20\ell n\left(\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}}\right) - \ell n\left(0+5\right)$$

$$= 3\pi - 20\ell n 4\sqrt{2} + 20\ell n 5$$

$$= 3\pi - 20 \times \frac{5}{2}\ell n 2 + 20\ell n 5$$

$$= 3\pi - 50\ell n 2 + 20\ell n 5$$

10. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8}. The probability of this equation having repeated roots is :

(1) $\frac{3}{256}$ (2) $\frac{1}{128}$

(3) $\frac{1}{64}$ (4) $\frac{3}{128}$

Ans. (3)

Sol. $ax^2 + bx + c = 0$

a, b, c ∈ {1, 2, 3, 4, 5, 6,7, 8}

Repeated roots D = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

$$\text{Prob} = \frac{8}{8\times 8\times 8} = \frac{\quad}{64}$$

⇒ (a, b, c)

(1, 2, 1) ; (2, 4, 2) ; (1, 4, 4) ; (4, 4, 1) ; (3, 6, 3) ; (2, 8, 8) ; (8, 8, 2) ; (4, 8, 4)

8 case

11. Let A and B be two square matrices of order 3 such that |A| = 3 and |B| = 2.

Then $|A^T A(\text{adj}(2A))^{-1} (\text{adj}(4B))(\text{adj}(AB))^{-1}AA^T|$ is equal to :

(1) 64 (2) 81

(3) 32 (4) 108

Ans. (1)

Sol. |A| = 3, |B| = 2

$|A^TA(\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1}AA^T|$

$= 3\times3\times|(\text{adj}(2A))^{-1}| \times |\text{adj}(4B)| \times |(\text{adj}(AB))^{-1}|\times3\times3$

$\downarrow \quad \downarrow \quad \downarrow$

$\dfrac{1}{|\text{adj}(2A)|} \qquad 2^{12}\times 2^2 \qquad \dfrac{1}{|\text{adj}(AB)|}$

$= \dfrac{1}{2^6\,|\text{adj}A|} \qquad = \dfrac{1}{|\text{adj}B \cdot \text{adj}A|}$

$= \dfrac{1}{2^6 \cdot 3^2} \qquad = \dfrac{1}{2^2 \cdot 3^2}$

$= 3^4 \cdot \dfrac{1}{2^6 \cdot 3^2} \cdot 2^{12} \cdot 2^2 \cdot \dfrac{}{2^2 \cdot 3^2} \quad 64$

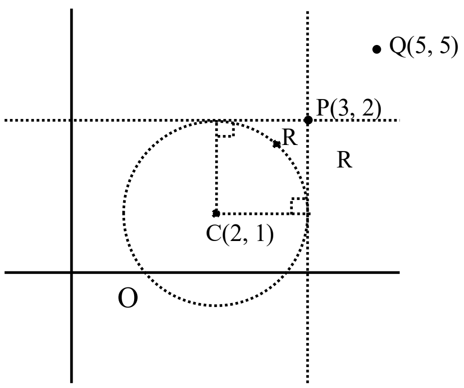
12. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point (3, 2) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (5, 5) is :

(1) $2\sqrt{2}$ (2) 5

(3) $4\sqrt{2}$ (4) 4

Ans. (4)

Sol.



Coordinates of the centre will be (2, 1)

Equation of circle will be

$(x - 2)^2 + (y - 1)^2 = 1$

$QC = \sqrt{(5-2)^2 + (5-1)^2}$

QC = 5

shortest distance

= RQ = CQ – CR

= 5 – 1

= 4

13. Let the line 2x + 3y – k = 0, k > 0, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + y^2 - 3x - 2y = 0$ and the length of the latus rectum of the ellipse $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then 2m + n is equal to

(1) 10 (2) 11

(3) 13 (4) 12

Ans. (2)

Sol. Centre of the circle = $\left(\frac{3}{2}, 1\right)$

Equation of diameter = 2x + 3y – k = 0

$2\left(\frac{3}{2}\right) \quad 3(1) - k = 0$

$\Rightarrow k = 6$

Now, Equation of ellipse becomes

$x^2 + 9y^2 = 36$

$\dfrac{x^2}{6^2} + \dfrac{}{} \quad 1$

length of LR $= \frac{2b^2}{a} = \frac{2.2^2}{6} = \frac{8}{6} = \frac{4}{3} = \frac{m}{n}$

$\therefore$ 2m + n = 2(4) + 3 = 11

14. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2

$$\left(|z_1|+|z_2|\right)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2\left(|z_1|+|z_2|\right) \text{ and}$$

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} \quad \frac{c}{|x-y|}$, then

$$\frac{a^2}{y-z}+\frac{b^2}{z-x}+\frac{c^2}{x-y}=1.$$

Between the above two statements,

(1) both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) both Statement I and Statement II are correct.

Ans. (3)

Sol. **Statement I :**

$$\left(|z_1|+|\quad|\right)\left|\frac{z_1}{|z_1|} \quad \frac{}{|\quad|}\right|$$

Since $\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le \left|\frac{z_1}{|z_1|}\right| + \left|\frac{z_2}{|z_2|}\right|$

$$\left|\frac{z_1}{|z_1|}+\frac{}{|z_2|}\right| \le \frac{|z_1|}{|z_1|}+\frac{|\quad|}{|z_2|}$$

$$\left|\frac{z_1}{|z_1|}+\frac{}{|\quad|}\right| \quad 2$$

$$\left(|z_1|+|z_2|\right)\left(\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right|\right) \le 2\left(|z_1|+|z_2|\right)$$

$\therefore$ statement I is correct

For **Statement II :**

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} \quad \frac{c}{|x-y|}$$

$$\frac{a^2}{|y-z|^2} = \frac{b^2}{|z-x|^2} = \frac{c^2}{|x-y|^2} = \lambda$$

$$a^2 = \lambda(|y-z|^2) = \lambda(y-z)\left(\bar{y}-\bar{z}\right)$$

$$b^2 = \lambda(z-x)\left(\bar{z}-\bar{x}\right) \text{ and } c^2 = \lambda(x-y)\left(\bar{x}-\bar{y}\right)$$

$$\frac{a^2}{y-z}+\frac{b^2}{z-x}+\frac{c^2}{x-y} = \lambda\left(\bar{y}-\bar{z}+\bar{z}-\bar{x}+\bar{x}-\bar{y}\right) = 0$$

Statement II is false

15. Suppose θ $\left[0,\frac{\pi}{4}\right]$ is a solution of 4 cosθ – 3 sinθ = 1.

Then cosθ is equal to :

(1) $\frac{4}{\left(3\sqrt{6}-2\right)}$

(2) $\frac{6-\sqrt{6}}{\left(3\sqrt{6}-2\right)}$

(3) $\frac{6+\sqrt{\ }}{\left(3\sqrt{6}+2\right)}$

(4) $\frac{4}{\left(3\sqrt{6}+2\right)}$

Ans. (1)

Sol. $4\left(\frac{1 \quad \tan^2\theta/2}{1 \quad \tan^2\theta/2}\right) - 3\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) \quad 1$

let $\tan\frac{\theta}{2} = t$

$$\frac{4-4t^2 \quad 6t}{1+t^2} = 1$$

$4 - 4t^2 - 6t = 1 + t^2$

$\Rightarrow 5t^2 + 6t - 3 = 0$

$\Rightarrow t = \frac{-6 \pm \sqrt{36 - 4(5)(-3)}}{2(5)}$

$= \frac{-6 \pm \sqrt{96}}{10}$

$= \frac{-6 \pm 4\sqrt{6}}{10}$

$t = \frac{-3 + 2\sqrt{6}}{5}$

$\cos\theta = \frac{1-t^2}{1+t^2} = \frac{\left(\frac{2\sqrt{6}-3}{5}\right)^2}{1+\left(\frac{2\sqrt{6}-3}{5}\right)^2} = \frac{1-\left(\frac{24+9-12\sqrt{6}}{25}\right)}{1+\left(\frac{24+9-12\sqrt{6}}{25}\right)}$

$= \frac{25-33+12\sqrt{6}}{25+33-12\sqrt{6}} = \frac{12\sqrt{6}-8}{58-12\sqrt{6}} = \frac{6\sqrt{6}-4}{29-6\sqrt{6}} \times \frac{29+6\sqrt{6}}{29+6\sqrt{6}}$

$= \frac{100+150\sqrt{6}}{625} = \frac{4+6\sqrt{6}}{25} \times \frac{4-6\sqrt{6}}{4-6\sqrt{6}}$

$= \frac{-200}{25\left(4-6\sqrt{6}\right)} = \frac{8}{4-6\sqrt{6}} = \frac{4}{3\sqrt{6}} \quad 2$

16. If $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}} = m$ and $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{99\cdot 100} = n$, then the point (m, n) lies on the line

(1) $11(x-1) - 100(y-2) = 0$

(2) $11(x-2) - 100(y-1) = 0$

(3) $11(x-1) - 100y = 0$

(4) $11x - 100y = 0$

Ans. (4)

Sol. $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}} = m$

$\frac{\sqrt{1}-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} \ldots \frac{\sqrt{99}-\sqrt{100}}{-1} \quad m$

$\sqrt{100} - 1 = m \Rightarrow m = 9$

$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots \frac{1}{99\cdot 100} = n$

$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \ldots \frac{1}{99} - \frac{1}{100} = n$

$1 - \frac{1}{100} = n$

$\frac{99}{100} = n$

$(m, n) = \left(9, \frac{99}{100}\right)$

$\Rightarrow 11(9) - 100\left(\frac{99}{100}\right)$

$= 99 - 99 = 0$

Ans. option (4) $11x - 100y = 0$

17. Let $f(x) = x^5 + 2x^3 + 3x + 1$, $x \in R$, and g(x) be a function such that $g(f(x)) = x$ for all $x \in R$. Then $\frac{g(7)}{g'(7)}$ is equal to :

(1) 7 (2) 42

(3) 1 (4) 14

Ans. (4)

Sol. $f(x) = x^5 + 2x^3 + 3x + 1$

$f'(x) = 5x^4 + 6x^2 + 3$

$f'(1) = 5 + 6 + 3 = 14$

$g(f(x)) = x$

$g'(f(x))f'(x) = 1$

for $f(x) = 7$

$\Rightarrow x^5 + 2x^3 + 3x + 1 = 7$

$\Rightarrow x = 1$

$g'(7)\, f'(1) = 1 \Rightarrow g'(7) = \frac{1}{f'(1)} \quad \overline{14}$

$x = 1, f(x) = 7 \Rightarrow g(7) = 1$

$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14$

18. If A(l, –1, 2), B(5, 7, –6), C(3, 4, –10) and D(–l, –4, –2) are the vertices of a quadrilateral ABCD, then its area is :

(1) $12\sqrt{29}$ (2) $24\sqrt{29}$

(3) $24\sqrt{7}$ (4) $48\sqrt{7}$

Ans. (1)

Sol. A(1, –1, 2)
B(5, 7, –6)
C(3, 4, –10)
D(–1, –4, –2)

Area = $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right| = \frac{1}{2}\left|\left(2\hat{i}+5\hat{j}-12\hat{k}\right)\times\left(6\hat{i}+11\hat{j}-4\hat{k}\right)\right|$

$= \frac{1}{2}\left|112\hat{i}-64\text{j}-8\hat{k}\right|$

$= 4\left|14\hat{i}-8\hat{j}-\hat{k}\right|$

$= 4\sqrt{196+64+1}$

$= 4\sqrt{261}$

$= 12\sqrt{29}$

19. The value of $\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y}dy$ is :

(1) π^2 (2) $\frac{\pi^2}{2}$

(3) $\frac{\pi}{2}$ (4) $2\pi^2$

Ans. (1)

Sol. $\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y}dy$

$= \int_{-\pi}^{\pi} \frac{2y}{1 \quad \cos^2 y}dy + \int_{-\pi}^{\pi} \frac{2y\sin y}{1+\cos^2 y}dy$

(Odd) (Even)

$= 0 + 2.\int_{0}^{\pi} y\left(\frac{\sin y}{1+\cos^2 y}\right)dy$

$I = 4\int_{0}^{\pi} \frac{y\sin y}{1+\cos^2 y}dy$

$I = 4\int_{0}^{\pi} \frac{(\pi \quad y)\sin y}{1+\cos^2 y}dy$

$2I = 4\int_{0}^{\pi} \frac{\pi\sin y}{1+\cos^2 y}dy$

$I = 2\int_{0}^{\pi} \frac{\sin y}{1+\cos^2 y}dy$

$= 2\pi\left(-\tan^{-1}(\cos y)\right)_0^{\pi}$

$= -2\pi\left|\left(-\frac{\pi}{4}\right)-\left(\frac{\pi}{4}\right)\right]$

$= -2\pi\left| -\frac{2\pi}{4} \right| = \pi^2$

20. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda \quad 1} = 4 -$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} \quad \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

(1) 13 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. $\frac{2-x}{3} = \frac{3y-2}{4\lambda \quad 1} = 4 -$ ...(1)

$\frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda +}{3}\right)} \quad \frac{z \quad 4}{(\quad 1)}$

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7} \qquad \ldots(2)$$

$$\frac{x+3}{3\mu} \quad \frac{y-\frac{1}{2}}{(\ 3)} = \frac{z-5}{(-7)}$$

Right angle $\Rightarrow (-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$

$-9\mu - 4\lambda - 1 + 7 = 0$

$4\lambda + 9\mu = 6$

SECTION-B

21. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to__________.

Ans. (56)

Sol. X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x_1^2	0	1	4	9
$P_i x_1^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$p_i x_i$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$$\mu = \Sigma p_i x_i = \frac{18}{12}$$

$$\Sigma p_i x_1^2 \quad \frac{34}{12}$$

$$\sigma^2 = \Sigma p_i x_1^2 - (\mu)^2$$

$$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$$

$$\frac{34-27}{12} = \frac{7}{12}$$

$$96\sigma^2 = 96 \times \frac{7}{12} = 56$$

22. If the constant term in the expansion of $(1 + 2x - 3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is p, then 108p is equal to

Ans. (54)

Sol. $\left(1+2x-3x^3\right)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

General term m $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$$= {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}}(-1)^r \cdot x^{18-3r}$$

Put r = 6 to get coeff. of $x^0 = {}^9C_6 \cdot \frac{1}{6^3} \cdot x^0 = \frac{7}{18}x^0$

Put r = 7 to get coeff. of $x^{-3} = {}^9C_r \cdot \frac{3^{-5}}{2^2}(-1)^7 \cdot x^{-3}$

$$= -{}^9C_7 \cdot \frac{1}{3^5 \cdot 2^2} \cdot x^{-3} = \frac{-1}{27}x^{-3}$$

$$\left(1+2x-3x^3\right)\left(\frac{7}{18}x^0 - \frac{}{27}x^{-3}\right)$$

$$\frac{7}{18} + \frac{3}{27} = \frac{7}{18} + \frac{1}{9} = \frac{7+2}{18} = \frac{9}{18} = \frac{1}{2}$$

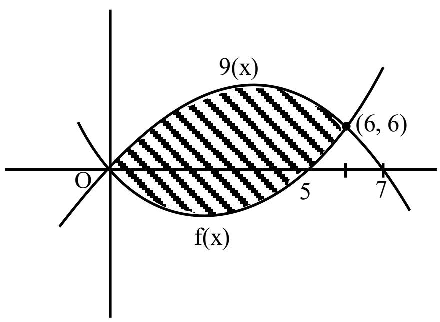
$$\therefore\ 108 \cdot \frac{1}{2} = 54$$

23. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is __________.

Ans. (72)

NTA Ans. (198)

Sol. $y = x^2 - 5x$ and $y = 7x - x^2$



$\int_0^6 (g(x) - f(x))dx$

$\int_0^6 \left((7x - x^2) - (x^2 - 5x)\right)dx$

$\int_0^6 \left(12x - 2x^2\right)dx = \left[12\frac{x^2}{2} - \frac{2x^3}{3}\right]_0^6$

$\Rightarrow 6(6) \ \ -\frac{2}{3}(6)^3$

$= 216 - 144 = 72$ unit^2

24. The number of ways of getting a sum 16 on throwing a dice four times is ___________.

Ans. (125)

Sol. $(x^1 + x^2 \ldots. + x^6)^4$

$x^4 \cdot \left(\frac{1 - x^6}{1 - x}\right)^4$

$x^4 \cdot (1 - x^6)^4 \cdot (1 - x)^{-4}$

$x^4[1 - 4x^6 + 6x^{12} \ldots.]\ [(1 - x)^{-4}]$

$(x^4 - 4x^{10} + 6x^{16} \ldots.)\ (1 - x)^{-4}$

$(x^4 - 4x^{10} + 6x^{16})\ (1 + {}^{15}C_{12}\, x^{12} + {}^{9}C_6 x^6 \ldots.)$

$({}^{15}C_{12} - 4 \cdot {}^{9}C_6 + 6)x^{16}$

$({}^{15}C_3 - 4 \cdot {}^{9}C_6 + 6)$

$= 35 \times 13 - 6 \times 8 \times 7 + 6$

$= 455 - 336 + 6$

$= 125$

25. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$, where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t, then $72\sum_{a \in S} a$ is equal to ___________.

Ans. (18)

Sol. $|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case-1 : $a > \frac{1}{2}$

$2a - 1 = [a] + 2a$

$[a] = -1 \quad \therefore a \in [-1, 0)$ Reject

Case-2 : $a < \frac{1}{2}$

$-2a + 1 = [a] + 2a$

$a = I + f$

$-2(I + f) + 1 = I + 2I + 2f$

$I = 0,\ f = \frac{1}{4} \quad \therefore a = \frac{1}{4}$

Hence $a = \frac{1}{4}$

$72\sum_{a \in S} a = 72 \times \frac{1}{4} = 18$

26. Let f be a differentiable function in the interval $(0, \infty)$ such that f(l) = 1 and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2\ f(2) + 3\ f(3)$ is equal to ___________.

Ans. (24)

Sol. $\lim_{t \to x} \frac{t\ f(x) - x^2 f(t)}{t - x} = 1$

$\lim_{t \to x} \frac{2t.f(x) - x^2 f'(x)}{1} = 1$

$2x.f(x) - x2f'(x) = 1$

$\frac{dy}{dx} - \frac{2}{x} \cdot y = \frac{-1}{x^2}$

$I.f. = e^{\int -\frac{2}{x} dx} \quad \frac{1}{x^2}$

$\therefore \frac{y}{x^2} = \int \frac{-}{} dx + C$

$\frac{}{x^2} = \frac{1}{3x} \quad C$

Put $f(1) = 1$

$C = \frac{2}{3}$

$y = \frac{1}{3x} \quad \frac{2x^2}{3}$

$y = \frac{2x^3 + 1}{3x}$

$f(2) = \frac{17}{6}$

$f(3) = \frac{55}{9}$

$2f(2) + 3f(3) = \frac{17}{3} + \frac{55}{3} = \frac{72}{3} = 24$

27. Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to __________.

Ans. (910)

Sol. $d \rightarrow$ common diff.

$A_k = -kd[2a + (2k - 1)d]$

$A_3 = -153$

$\Rightarrow 153 = 13d[2a + 5d]$

$51 = d[2a + 5d] \qquad \ldots(1)$

$A_5 = -435$

$435 = 5d[2a + 9d]$

$87 = d[2a + 9d]$

$(2) - (1)$

$36 = 4d^2$

$d = 3, a = 1$

$a_{17} - A_7 = 49 - [-7.3[2 + 39]] = 910$

28. Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$. If $\vec{a} \cdot \vec{c} = 130$, then $\vec{b} \cdot \vec{c}$ is equal to __________.

Ans. (30)

Sol. $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$

$(2\vec{b} + 4\vec{a}) \times \vec{c} = 0$

$\vec{c} = \lambda(4\vec{a} + 2\vec{b}) = \lambda(8\hat{i} - 14\hat{j} + 30\hat{k})$

$\vec{a} \cdot \vec{c} = 130$

$8\lambda + 42\lambda + 210\lambda = 130$

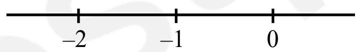
$\lambda = \frac{1}{2}$

$\vec{c} = 4\hat{i} - 7\hat{j} + 15\hat{k}$

$\vec{b} \cdot \vec{c} = 8 + 7 + 15 = 30$

29. The number of distinct real roots of the equation $|x|\,|x + 2| - 5|x + 1| - 1 = 0$ is __________.

Ans. (3)



Sol.

Case-1

$x \geq 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

$x = \frac{3 \pm \sqrt{9 + 24}}{2} \quad \overline{3 \pm \sqrt{33}}$

One positive root

Case-2

$-1 \leq x < 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$

$x = -1$

one root in range

Case-3

$-2 \leq x < -1$

$x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

No root in range

Case-4

$x < -2$

$x^2 + 7x + 4 = 0$

$$x = \frac{-7 \pm \sqrt{49-16}}{2} = \frac{7 \pm \sqrt{33}}{2}$$

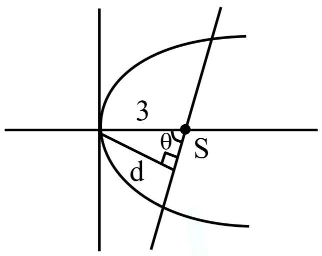
one root in range

Total number of distinct roots are 3

30. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to ___________.

Ans. (108)

Sol.



$\ell = 4a \operatorname{cosec}^2\theta$

$$\ell = 12 \times \frac{9}{d^2}$$

$\ell d^2 = 108$

(Held On Friday 05th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

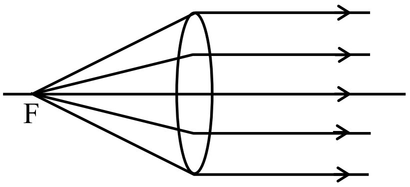
31. Light emerges out of a convex lens when a source of light kept at its focus. The shape of wavefront of the light is:

(1) Both spherical and cylindrical
(2) Cylindrical
(3) Spherical
(4) Plane

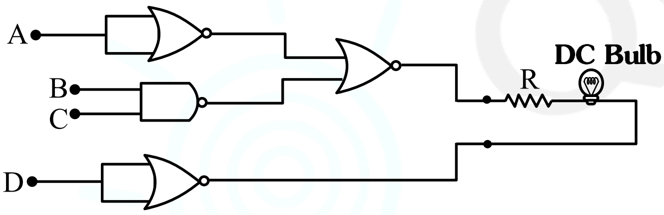
Ans. (4)

Sol. Light emerges parallel

$\therefore$ planor wavefront



32. Following gates section is connected in a complete suitable circuit.



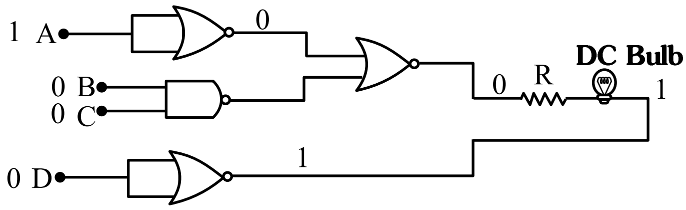
For which of the following combination, bulb will glow (ON):

(1) A = 0, B = 1, C = 1, D = 1
(2) A = 1, B = 0, C = 0, D = 0
(3) A = 0, B = 0, C = 0, D = 1
(4) A = 1, B = 1, C = 1, D = 0

Ans. (2)

Sol. Bulb will glow if bulb have potential drop on it. One end of bulb must be at high (1) and other must be at low (0).

Option (2) satisfy this condition



33. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimension as that the $\sqrt{uG}$:

(1) Pressure gradient per unit mass
(2) Force per unit mass
(3) Gravitational potential
(4) Energy per unit mass

Ans. (2)

Sol. $[uG] = [(M^1L^{-1}T^{-2})\ (M^{-1}L^3T^{-2})]$

$[uG] = [M^0L^2T^{-4}]$

$[\sqrt{uG}] = [L^1T^{-2}]$

Option (2) is correct

34. Given below are two statements :

Statement-I: When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be 0°.

Statement-II: The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well :

In the light of above statement, choose the **correct** answer from the options given below.

(1) **Statement-I** is false but **Statement-II** is true.
(2) Both **Statement-I** and **Statement-II** are true.
(3) Both **Statement-I** and **Statement-II** are false.
(4) **Statement-I** is true and **Statement-II** is false.

Ans. (1)

Sol. Capillary rise

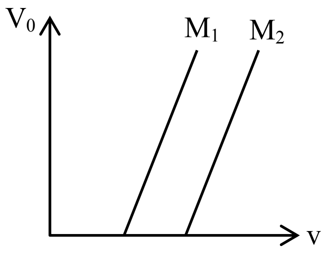
$$h = \frac{2T\cos\theta}{\rho g r};$$

If $\theta = 0°$ then rise is non-zero

$\therefore$ Statement-1 is incorrect.

Option(1) is correct

35. Given below are two statements:



Statement-I: Figure shows the variation of stopping potential with frequency (ν) for the two photosensitive materials M_1 and M_2. The slope gives value of $\frac{h}{e}$, where h is Planck's constant, e is the charge of electron.

Statement-II: M_2 will emit photoelectrons of greater kinetic energy for the incident radiation having same frequency.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) **Statement-I** is correct and **Statement-II** is incorrect.

(2) **Statement-I** is incorrect but **Statement-II** is correct.

(3) Both **Statement-I** and **Statement-II** are incorrect.

(4) Both **Statement-I** and **Statement-II** are correct.

Ans. (1)

Sol. $eV_0 = h\nu - \phi$

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

M_2 material has higher work function, so statement-(II) is incorrect.

Option (1) is correct.

36. The angle between vector $\vec{Q}$ and the resultant of $(2\vec{Q}+2\vec{P})$ and $(2\vec{Q}-2\vec{P})$ is:

(1) $0°$

(2) $\tan^{-1}\frac{(2\vec{Q}-2\vec{P})}{2\vec{Q}+2\vec{P}}$

(3) $\tan^{-1}\left(\frac{P}{Q}\right)$

(4) $\tan^{-1}\left(\frac{2Q}{P}\right.$

Ans. (1)

Sol. $\vec{R} = (2\vec{Q}+2\vec{P}) + (2\vec{Q}-2\vec{P})$

$\vec{R} = 4\vec{Q}$

Angle between $\vec{Q}$ and $\vec{R}$ is zero

Option (1) is correct

37. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of:

(1) 10^{39} (2) 10^{19}

(3) 10^{29} (4) 10^{36}

Ans. (1)

Sol. $F_e = \frac{kQ_1Q}{r^2}$ $\frac{9\times10^{9}\times1.6\times10^{-19}\times1.6\times10^{19}}{}$

$F_g = \frac{Gm_1m}{r^2}$ $\frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{}$

$$\frac{F_e}{F_g} \cong 0.23\times10^{40} \cong 2.3\times10^{39}$$

Option (1)

38. In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.

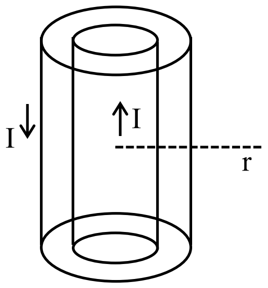
(1) inside the outer conductor

(2) in between the two conductors

(3) outside the cable

(4) inside the inner conductor

Ans. (3)



Sol.

$\oint \vec{B}.d\vec{\ell} = \mu_0 i_{enc} = 0$

$\therefore$ B = 0 outside the cable

39. An electron rotates in a circle around a nucleus having positive charge Ze. Correct relation between total energy (E) of electron to its potential energy (U) is:

(1) E = 2U (2) 2E = 3U
(3) E = U (4) 2E = U

Ans. (4)

Sol. $F = \frac{k(Ze)(e)}{r^2} \quad \frac{mv^2}{}$

$KE = \frac{1}{2}mv^2 \quad \frac{1}{2}\frac{K(Ze)(e)}{r}$

$PE = -\frac{K(Ze)(e)}{r}$

$TE = \frac{K(Ze)(e)}{2r} \quad \frac{K(Ze)(e)}{r} \quad \frac{-K(Ze)(e)}{2r}$

$TE = \frac{PE}{2}$

2TE = PE

Option (4)

40. If the collision frequency of hydrogen molecules in a closed chamber at 27°C is Z, then the collision frequency of the same system at 127° C is :

(1) $\frac{\sqrt{3}}{2}Z$ (2) $\frac{4}{3}Z$
(3) $\frac{2}{\sqrt{3}}Z$ (4) $\frac{3}{4}Z$

Ans. (3)

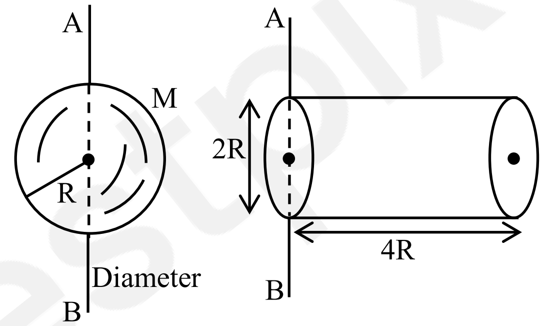
Sol. Assuming mean free path constant.

$f \propto v \propto \sqrt{T}$

$\frac{1}{f_2} = \sqrt{\frac{T}{T}} \quad \sqrt{\frac{300}{400}}$

$f_2 = \sqrt{\frac{4}{3}} = f = \frac{}{\sqrt{3}}Z$

41. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{\frac{8}{x}}$. The value of x is:



(1) 34 (2) 17
(3) 67 (4) 51

Ans. (3)

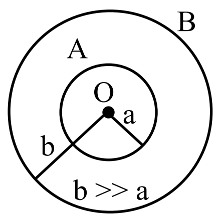
Sol. $I_{sphere} = \frac{2}{3}MR^2 \quad Mk_1$

$I_{cylinder} = \frac{1}{12}M(4R^2) + \frac{}{4}MR^2 + M(2R)^2$

$= \frac{67}{12}MR^2 \quad Mk_2$

$\frac{k_1}{k_2} = \sqrt{\frac{2}{3}.\frac{12}{67}} \quad \sqrt{\frac{8}{67}}$

42. Two conducting circular loops A and B are placed in the same plane with their centres coinciding as shown in figure. The mutual inductance between them is:



(1) $\frac{\mu_0 \pi a^2}{2b}$ (2) $\frac{\mu_0}{2\pi}.\frac{b^2}{a}$

(3) $\frac{\mu_0 \pi b^2}{2a}$ (4) $\frac{\mu_0}{2\pi}.\frac{a^2}{b}$

Ans. (1)

Sol. $\phi = Mi = BA$

$$\Rightarrow Mi = \frac{\mu_0 i}{2b}\pi a^2$$

$$\therefore M = \frac{\mu_0 \pi a^2}{2b}$$

43. Match **list-I** with **list-II**:

	List-I		List-II
(A)	Kinetic energy of planet	(I)	$-\frac{GMm}{a}$
(B)	Gravitation Potential energy of Sun-planet system.	(II)	$\frac{GMm}{2a}$
(C)	Total mechanical energy of planet	(III)	$\frac{Gm}{r}$
(D)	Escape energy at the surface of planet for unit mass object	(IV)	$-\frac{GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet)

Choose the correct answer from the options given below:

(1) (A) – II, (B) – I, (C) – IV, (D) – III

(2) (A) – III, (B) – IV, (C) – I, (D) – II

(3) (A) – I, (B) – IV, (C) – II, (D) – III

(4) (A) – I, (B) – II, (C) – III, (D) – IV

Ans. (1)

Sol. $KE = \frac{1}{2}mv^2 \quad \frac{GMm}{2a}$

$PE = -2KE$

$TE = -KE$

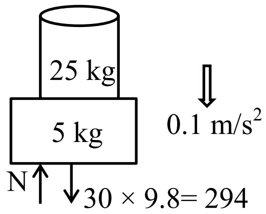
44. A wooden block of mass 5kg rests on soft horizontal floor. When an iron cylinder of mass 25 kg is placed on the top of the block, the floor yields and the block and the cylinder together go down with an acceleration of 0.1 ms^{-2}. The action force of the system on the floor is equal to:

(1) 297 N (2) 294 N

(3) 291 N (4) 196 N

Ans. (3)

Sol. Taking $g = 9.8\ m/s^2$



$294 - N = 30 \times 0.1$

$N = 291$

45. A simple pendulum doing small oscillations at a place R height above earth surface has time period of $T_1 = 4$ s. T_2 would be it's time period if it is brought to a point which is at a height 2R from earth surface. Choose the correct relation [R = radius of Earth]:

(1) $T_1 = T_2$ (2) $2T_1 = 3T_2$

(3) $3T_1 = 2T_2$ (4) $2T_1 = T_2$

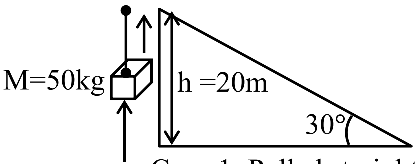
Ans. (3)

Sol. $T_1 = 2\sqrt{\frac{\ell}{GM}(2R)^2}$

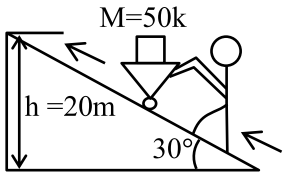
$$T_2 = 2\sqrt{\frac{\ell}{GM}(3R)^2}$$

$$\therefore \frac{T_1}{T_2} \quad \frac{2}{3}$$

46. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be:



Case-1: Pulled straight up



Case-2: Along the ramp

(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{3} : 2$ (4) 1 : 2

Ans. (1)

Sol. Work done by gravity is independent of path. It depends only on vertical displacement so work done in both cases will be same.

Option (1) is correct

47. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632 s, 4.6 s and 4.64 s. The arithmetic mean of these reading in correct significant figure is.

(1) 4.623 s (2) 4.62 s

(3) 4.6 s (4) 5 s

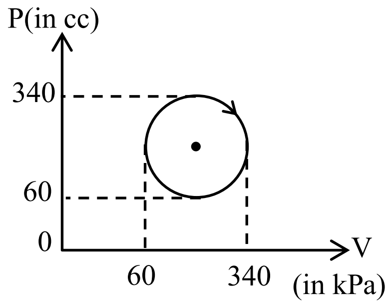
Ans. (3)

Sol. Sum of number by considering significant digit

sum = 4.6 + 4.6 + 4.6 + 4.6 = 18.4

$$\text{Arithmetic Mean} = \frac{\text{sum}}{4} = \frac{18.4}{} = 4.6$$

48. The heat absorbed by a system in going through the given cyclic process is :



(1) 61.6 J (2) 431.2 J

(3) 616 J (4) 19.6 J

Ans. (1)

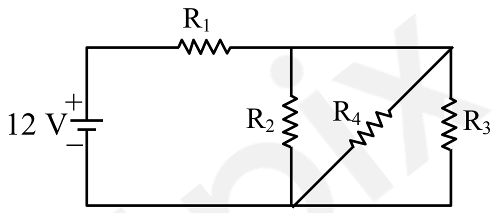
Sol. $\Delta U = 0$ (Cyclic process)

$\Delta Q = W$ = area of P-V curve.

$= \pi \times (140 \times 10^3 \text{ Pa}) \times (140 \times 10^{-6} \text{ m}^3)$

$\Delta Q = 61.6$ J

49. In the given figure $R_1 = 10\Omega$, $R_2 = 8\Omega$, $R_3 = 4\Omega$ and $R_4 = 8\Omega$. Battery is ideal with emf 12V. Equivalent resistant of the circuit and current supplied by battery are respectively.



(1) 12 Ω and 11.4 A (2) 10.5 Ω and 1.14 A

(3) 10.5 Ω and 1 A (4) 12 Ω and 1 A

Ans. (4)

Sol. Here R_2, R_3, R_4 are in parallel

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$R_{234} = 2\Omega$

R_{234} is in series with R_1 so

$R_{eq} = R_{234} + R_1 = 2 + 10 = 12\Omega$

$$i = \frac{12}{12} = 1\text{Amp}$$

50. An alternating voltage of amplitude 40 V and frequency 4 kHz is applied directly across the capacitor of 12 μF. The maximum displacement current between the plates of the capacitor is nearly:

(1) 13 A (2) 8 A

(3) 10 A (4) 12 A

Ans. (4)

Sol. Displacement current is same as conduction current in capacitor.

$$X_C = \frac{1}{\omega C} \quad \frac{}{2\ \ fC}$$

$$= \frac{1}{2\pi \times 4 \times 10^3 \times 12 \times 10^{-}} = 3.317\Omega$$

$$I = \frac{V}{X_C} = \frac{40}{3.317} = 12A$$

SECTION-B

51. In Young's double slit experiment, carried out with light of wavelength 5000Å, the distance between the slits is 0.3 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0 cm. The value of x for third maxima is .... mm.

Ans. (10)

Sol. $\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{3\times10^{-4}} = \frac{10\times10^{-3}}{3}$ m

For 3rd maxima $y_3 = 3\beta = 10\times10^{-3}$ m = 10 mm

52. A 2A current carrying straight metal wire of resistance 1 Ω, resistivity 2 × 10^{-6} Ωm, area of cross-section 10 mm^2 and mass 500 g is suspended horizontally in mid air by applying a uniform magnetic field $\vec{B}$. The magnitude of B is .... × 10^{-1} T (given, g = 10 m/s^2)

Ans. (5)

Sol. $R = \frac{\rho\ell}{A} \Rightarrow \frac{2\times10^{-6}\times\ell}{10^{-5}} = 1 \Rightarrow \ell = 5$

$mg = Bi\ell$

$B = \frac{mg}{i\ell} = \frac{5}{2\times5} = 0.5 = 5\times10^{-1}$ Tesla

53. The electric field between the two parallel plates of a capacitor of 1.5 μF capacitance drops to one third of its initial value in 6.6 μs when the plates are connected by a thin wire. The resistance of this wire is .... Ω. (Given, log 3 = 1.1)

Ans. (4)

Sol. $E = \frac{E_0}{3} \Rightarrow V = \frac{V_0}{3}$

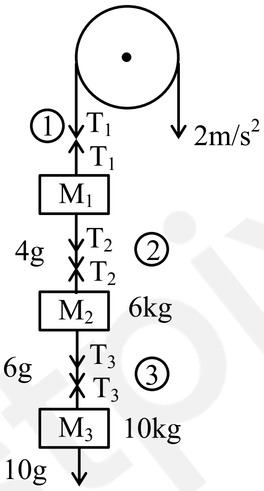
$\frac{V_0}{3} = V_0 e^{-\frac{t}{\tau}}$

$t = \tau \ell n 3$

$6.6\times10^{-6} = R(1.5\times10^{-6})(1.1)$

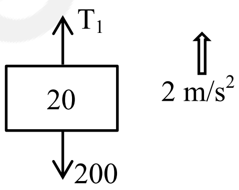
$R = \frac{6}{1.5} = 4\Omega$

54. Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg and 10 kg respectively are hanging from a smooth pully using rope 1, 2 and 3 as shown in figure. The tension in the rope 1, T_1 when they are moving upward with acceleration of $2ms^{-2}$ is .... N (if g = 10 m/s^2)



Ans. (240)

Sol. FBD of M_1 :



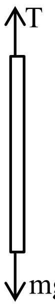
$T_1 - 200 = (4 + 6 + 10) \times 2$

$\therefore T_1 = 240$

55. The density and breaking stress of a wire are 6 × 10^4 kg /m^3 and 1.2 × 10^8 N/m^2 respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{1^{rd}}{3}$ of the value on the surface of earth. The maximum length of the wire with breaking is .... m (take, g = 10 m/s^2)

Ans. (600)

Sol.



$T = mg$

$\sigma = \frac{T}{A} = \frac{mg}{A}$

$\frac{(\sigma A\ell)g}{A}$

$\Rightarrow \ell = \frac{\sigma}{\rho g} = \frac{1.2\times10^8\times3}{6\times10^4\times10} = 600$

56. A body moves on a frictionless plane starting from rest. If S_n is distance moved between t = n – 1 and t = n and S_{n-1} is distance moved between t = n –2 and t = n – 1, then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1-\frac{2}{x}\right)$ for n = 10. The value of x is ....

Ans. (19)

Sol. $S_n = \frac{1}{2}a(2n-1) = \frac{19a}{2}$

$S_{n-1} = \frac{1}{2}a(2n-3) = \frac{17a}{2}$

$\frac{S_{n-1}}{S_n} = \frac{17}{19} = 1-\frac{2}{x} \Rightarrow x = 19$

57. If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is .... × 10^{-2} MeV. (Given 1 u = 931 MeV/c^2, atomic mass of helium = 4.002603 u)

Ans. (727)

Sol. Reaction :

$3\,{}_2^4He \longrightarrow {}_6^{12}C + \gamma \text{ rays}$

Mass defect = $\Delta m = (3m_{He} - m_C)$

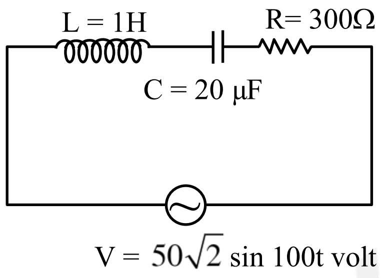
= (3 × 4.002603 – 12) = 0.007809 u

Energy released

= 931 Δm MeV

= 7.27 MeV = 727 × 10^{-2} MeV

58. An ac source is connected in given series LCR circuit. The rms potential difference across the capacitor of 20 μF is .... V.



Ans. (50)

Sol. $X_L = \omega L = 100 \times 1 = 100\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{100\times20\times10^{-6}} = 500\Omega$

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\sqrt{(100-500)^2+300^2}$

$Z = 500\Omega$

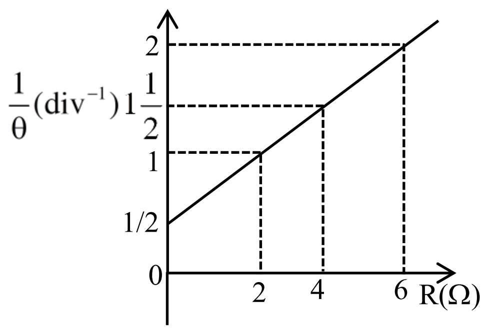
$i_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = 0.1A$

rms voltage across capacitor

$V_{rms} = X_C\, i_{rms}$

=500 × 0.1 = 50V

59. In the experiment to determine the galvanometer resistance by half-deflection method, the plot of $\frac{1}{\theta}$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is .... $\times10^{-1}$ A/division. [The source has emf 2V]



Ans. (5)

Sol. $i = K\theta$

$$\frac{2}{G+R} = K\theta$$

$$\Rightarrow \frac{1}{\theta} = \frac{(G+R)K}{2} = R\left(\frac{K}{2}\right) + \frac{KG}{2}$$

$$\text{Slope} = \frac{K}{2} = \frac{1}{4} \Rightarrow K = 0.5 = 5\times10^{-1}\,A$$

60. Three capacitors of capacitances 25 μF, 30 μF and 45 μF are connected in parallel to a supply of 100 V. Energy stored in the above combination is E. When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}E$. The value of x is ....

Ans. (86)

Sol. In parallel combination : Potential difference is same across all

$$\text{Energy} = \frac{1}{2}(C_1 + C_2 + C_3)V^2$$

$$= \frac{1}{2}(25+30+45)\times(100)^2\times10^{-6} = 0.5 = E$$

In series combination: Charge is same on all.

$$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45}$$

$$\frac{1}{C_{equ}} = \frac{(18+15+10)}{450} = \frac{43}{450} \Rightarrow C_{equ} = \frac{450}{43}$$

$$\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$= \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{(V\times C_{equ})^2}{2}\times\frac{1}{C_{equ}} = \frac{V^2 C_{equ}}{2}$$

$$\frac{(100)^2}{2}\times\frac{450}{43}\times10^{-6}$$

$$\Rightarrow \frac{4.5}{86} = \frac{9}{x}E = \frac{9}{x}\times0.5 \Rightarrow x = 86$$

(Held On Friday 05th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61. The **incorrect** postulates of the Dalton's atomic theory are :

(A) Atoms of different elements differ in mass.

(B) Matter consists of divisible atoms.

(C) Compounds are formed when atoms of different element combine in a fixed ratio.

(D) All the atoms of given element have different properties including mass.

(E) Chemical reactions involve reorganisation of atoms.

Choose the **correct** answer from the options given below :

(1) (B), (D), (E) only

(2) (A), (B), (D) only

(3) (C), (D), (E) only

(4) (B), (D) only

Ans. **(4)**

Sol. B, D

62. The following reaction occurs in the Blast furnance where iron ore is reduced to iron metal

$$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons Fe_{(l)} + 3CO_{2(g)}$$

Using the Le-chatelier's principle, predict which one of the following will not disturb the equilibrium.

(1) Addition of Fe_2O_3

(2) Addition of CO_2

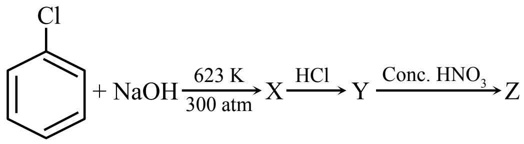
(3) Removal of CO

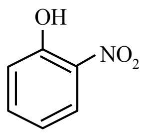
(4) Removal of CO_2

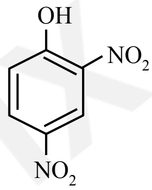
Ans. **(1)**

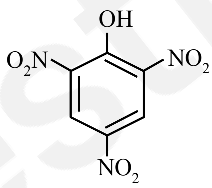
Sol. When solid added no effect on equilibrium.

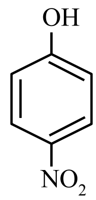
63. Identify compound (Z) in the following reaction sequence.



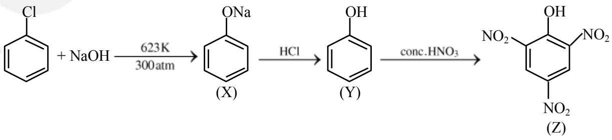
(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. 

64. Given below are two statements : One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A): Enthalpy of neutralisation of strong monobasic acid with strong monoacidic base is always –57 kJ mol^{-1}

Reason (R): Enthalpy of neutralisation is the amount of heat liberated when one mole of H^+ ions furnished by acid combine with one mole of ^-OH ions furnished by base to form one mole of water.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Ans. **(2)**

Sol. Enthalpy of neutralization of SA & SB is always –57 kJ / mol because strong monoacid gives one mole of H^+ and strong mono base gives one mole of OH^- which form one mole of water.

65. The statement(s) that are **correct** about the species O^{2-}, F^-, Na^+ and Mg^{2+}.

(A) All are isoelectronic

(B) All have the same nuclear charge

(C) O^{2-} has the largest ionic radii

(D) Mg^{2+} has the smallest ionic radii

Choose the **most appropriate** answer from the options given below :

(1) (B), (C) and (D) only

(2) (A), (B), (C) and (D)

(3) (C) and (D) only

(4) (A), (C) and (D) only

Ans. **(4)**

Sol.

	O^{-2}	F^-	Na^+	Mg^{+2}
(No. of e^-)	10	10	10	10

(Ionic radius) $O^{-2} > F^- > Na^+ > Mg^{+2}$

Zeff $O^{-2} < F^- < Na^+ < Mg^{+2}$

66. For the compounds:

(A) $H_3C–CH_2–O–CH_2–CH_2–CH_3$

(B) $H_3C–CH_2–CH_2–CH_2–CH_3$

(C) $CH_3–CH_2–\underset{\overset{||}{O}}{C}–CH_2–CH_3$

(D) $H_3C–\underset{\overset{|}{OH}}{CH}–CH_2–CH_2–CH_3$

The increasing order of boiling point is :

Choose the **correct** answer from the options given below :

(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (C) < (D)

(3) (D) < (C) < (A) < (B)

(4) (B) < (A) < (D) < (C)

Ans. **(2)**

Sol. Compounds having same number of carbon atoms follow the boiling point order as:

$(\text{Boiling point})_{\text{Hydrogen bonding}} > (\text{Boiling point})_{\text{high polarity}} > (\text{Boiling point})_{\text{low polarity}} > (\text{Boiling point})_{\text{non polar}}$

67. Given below are two statements :

Statement I: In group 13, the stability of +1 oxidation state increases down the group.

Statement II: The atomic size of gallium is greater than that of aluminium.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are correct

(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is correct but **Statement II** is incorrect

Ans. **(4)**

Sol. **Statement I :** Number of d & f electrons, increases down the group and due to poor shielding of d & f e^-, stability of lower oxidation states increases down the group

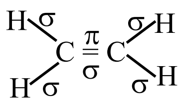
Statement II : The atomic size of aluminium is greater than that of gallium.

68. Number of σ and π bonds present in ethylene molecule is respectively :

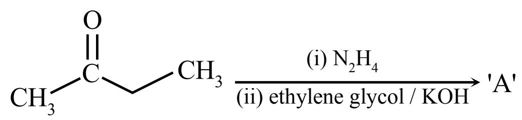
(1) 3 and 1 (2) 5 and 2

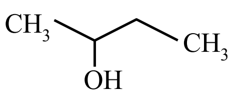
(3) 4 and 1 (4) 5 and 1

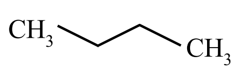
Ans. **(4)**

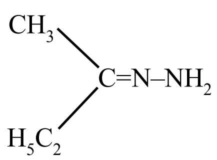
Sol. ethylene is , it has 5σ bonds and 1π bond.

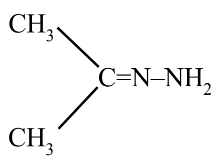
69. Identify 'A' in the following reaction :



(1) 

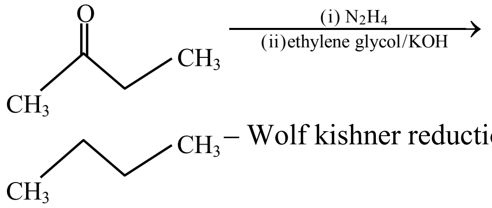
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



– Wolf kishner reduction.

70. The reaction at cathode in the cells commonly used in clocks involves.

(1) reduction of Mn from +4 to +3

(2) oxidation of Mn from +3 to +4

(3) reduction of Mn from + 7 to +2

(4) oxidation of Mn from + 2 to +7

Ans. **(1)**

Sol. In the cathode reaction manganese (Mn) is reduced from the +4 oxidation state to the +3 state.

71. Which one of the following complexes will exhibit the least paramagnetic behaviour ?

[Atomic number, Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Co(H_2O)_6]^{2+}$ (2) $[Fe(H_2O)_6]^{2+}$

(3) $[Mn(H_2O)_6]^{2+}$ (4) $[Cr(H_2O)_6]^{2+}$

Ans. **(1)**

Sol.

	Number of unpaired e^-	$\mu = \sqrt{n(n+2)}$ B.M.
$[Co(H_2O)_6]^{2+}$	3	3.87
$[Fe(H_2O)_6]^{2+}$	4	4.89
$[Mn(H_2O)_6]^{2+}$	5	5.92
$[Cr(H_2O)_6]^{2+}$	4	4.89

Least paramagnetic behaviour = $[Co(H_2O)_6]^{2+}$

72. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): Cis form of alkene is found to be more polar than the trans form

Reason (R): Dipole moment of trans isomer of 2-butene is zero.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

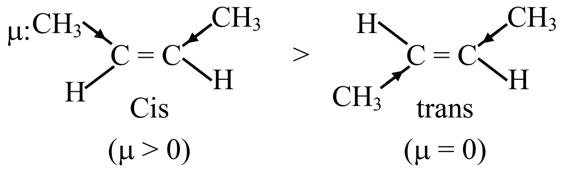
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(4) **(A)** is false but **(R)** is true

Ans. **(3)**

Sol. Dipole moment is a vector quantity and for compound net dipole moment is the vector sum of all dipoles hence dipole moment of cis form is greater than trans form.



73. Given below are two statements :

Statement I: Nitration of benzene involves the following step –

$$H-\overset{\oplus}{\underset{\cdot\cdot}{O}}(H)-NO_2 \rightleftharpoons H_2O + \overset{\oplus}{N}O_2$$

Statement II: Use of Lewis base promotes the electrophilic substitution of benzene.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is correct but **Statement II** is incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is incorrect but **Statement II** is correct

Ans. **(2)**

Sol. In nitration of benzene concentrated H_2SO_4 and HNO_3 is used as reagent which generates electrophile $\overset{\oplus}{N}O_2$ in following steps:

$$H_2SO_4 + HNO_3 \rightleftharpoons HSO_4^{\ominus} + H-\overset{H}{\underset{\oplus}{O}}-NO_2$$

$$\rightleftharpoons HSO_4^{\ominus} + H_2O + NO_2^{\oplus}$$

Lewis acids can promote the formation of electrophiles not Lewis base

74. The correct order of ligands arranged in increasing field strength.

(1) $Cl^- < {}^-OH < Br^- < CN^-$

(2) $F^- < Br^- < I^- < NH_3$

(3) $Br^- < F^- < H_2O < NH_3$

(4) $H_2O < {}^-OH < CN^- < NH_3$

Ans. **(3)**

Sol. Experimental order $Br^- < F^- < H_2O < NH_3$

75. Which of the following gives a positive test with ninhydrin ?

(1) Cellulose (2) Starch

(3) Polyvinyl chloride (4) Egg albumin

Ans. **(4)**

Sol. Ninhydrin test is a test of amino acids. Egg albumin contains protein which is a natural polymer of amino acids which will show positive ninhydrin test

76. The metal that shows highest and maximum number of oxidation state is:

(1) Fe (2) Mn

(3) Ti (4) Co

Ans. **(2)**

Sol. Mn shows highest oxidation state (Mn^{+7}) in 3d series metals.

77. Ail organic compound has 42.1% carbon, 6.4% hydrogen and remainder is oxygen. If its molecular weight is 342, then its molecular formula is :

(1) $C_{11}H_{18}O_{12}$ (2) $C_{12}H_{20}O_{12}$

(3) $C_{14}H_{20}O_{10}$ (4) $C_{12}H_{22}O_{11}$

Ans. **(4)**

Sol. only $C_{12}H_{22}O_{11}$ has 42.1% carbon, 6.4% hydrogen & 51.5 percent oxygen.

78. Given below are two statement :

Statement I : Bromination of phenol in solvent with low polarity such as $CHCl_3$ or CS_2 requires Lewis acid catalyst.

Statement II : The lewis acid catalyst polarises the bromine to generate Br^+.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Statement I is true but Statement II is false.

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Ans. **(4)**

Sol. Phenol is a highly activated compound which can undergo bromination directly with Bromine without any lewis acid.

79. Molar ionic conductivities of divalent cation and anion are 57 S cm^2 mol^{-1} and 73 S cm^2 mol^{-1} respectively. The molar conductivity of solution of an electrolyte with the above cation and anion will be :

(1) 65 S cm^2 mol^{-1} (2) 130 S cm^2 mol^{-1}

(3) 187 S cm^2 mol^{-1} (4) 260 S cm^2 mol^{-1}

Ans. **(2)**

Sol. $\Lambda_C^{+2} = 57\,S\,cm^2 mol^{-1}$

$\Lambda_A^{+2} = 73\,S\,cm^2 mol^{-1}$

$\Lambda_{Solution} = \lambda_C^{2} + \Lambda_A^{-2}$

$= 57 + 73 = 130$

80. The number of neutrons present in the more abundant isotope of boron is 'x'. Amorphous boron upon heating with air forms a product, in which the oxidation state of boron is 'y'. The value of x + y is …

(1) 4 (2) 6

(3) 3 (4) 9

Ans. **(4)**

Sol. More abundant isotope = B^{11}

[Number of neutrons = 6]

x = 6

$B + O_2 \rightarrow B_2O_3$

Oxidation state of B in B_2O_3 = +3

So, y = 3

Hence x + y = 9

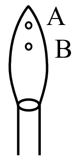
SECTION-B

81. The value of Rydberg constant (R_H) is 2.18×10^{-18} J. The velocity of electron having mass 9.1×10^{-31} kg in Bohr's first orbit of hydrogen atom = ……… $\times10^5$ ms^{-1} (nearest integer)

Ans. **(22)**

Sol. $V = 2.18 \times 10^6 \times \dfrac{Z}{n}$

$= 21.8 \times 10^5 \times \dfrac{1}{1} \approx 22 \times 10^5$ (nearest)

82. 

In a borax bead test under hot condition, a metal salt (one from the given) is heated at point B of the flame, resulted in green colour salt bead. The spin-only magnetic moment value of the salt is ………… BM (Nearest integer)

[Given atomic number of Cu = 29, Ni = 28, Mn = 25, Fe = 26]

Ans. **(6)**

Sol. Fe^{+3} will give green coloured bead when heated at point B.

Number of unpaired e^- in $Fe^{+3} = 5$

$\mu = 5.92$

Nearest integer = 6

83. The heat of combustion of solid benzoic acid at constant volume is –321.30 kJ at 27°C. The heat of combustion at constant pressure is (–321.30 – xR) kJ, the value of x is ……………

Ans. **(150)**

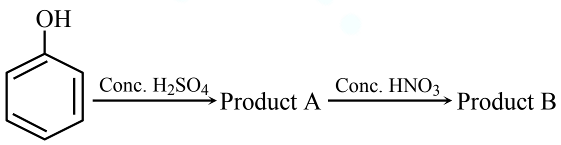
Sol. $C_6H_5COOH(S) + \frac{15}{2}O_2(g) \rightarrow 7CO_2(g) + 3H_2O(\ell)$

$\Delta H = \Delta U + \Delta n_g RT$

$= -321.30 - \frac{1}{2}\frac{R}{100}\times 300$

$= (-321.30 - 150R)$ kJ

84. Consider the given chemical reaction sequence :



Total sum of oxygen atoms in Product A and Product B are ………

Ans. **(14)**

Sol. Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid and then with concentrated nitric acid to get 2, 4, 6 trinitrophenol.

85. The spin only magnetic moment value of the ion among Ti^{2+}, V^{2+}, Co^{3+} and Cr^{2+}, that acts as strong oxidising agent in aqueous solution is ……….. BM (Near integer).

(Given atomic numbers : Ti : 22, V : 23, Cr : 24, Co : 27)

Ans. **(5)**

Sol. Strong oxidising agent = Co^{+3}

No. of unpaired e^- in $Co^{+3}[3d^6] = 4$

Hence $\mu = \sqrt{n(n+2)} = \sqrt{24}$ BM

Nearest integer = 5

86. During Kinetic study of reaction $2A + B \rightarrow C + D$, the following results were obtained :

	A[M]	B[M]	initial rate of formation of D
I	0.1	0.1	6.0×10^{-3}
II	0.3	0.2	7.2×10^{-2}
III	0.3	0.4	2.88×10^{-1}
IV	0.4	0.1	2.40×10^{-2}

Based on above data, overall order of the reaction is ……….

Ans. **(3)**

Sol. $r = K[A]^x[B]^y$

(I) $6 \times 10^{-3} = K[0.1]^x[0.1]^y$

(IV) $2.4 \times 10^{-2} = K[0.4]^x[0.1]^y$

(IV)/(I)

$4 = (4)^x$

$x = 1$

$r = K[A]^x[B]^y$

(III) $2.88 \times 10^{-1} = K[0.3]^x[0.4]^y$

(II) $7.2 \times 10^{-2} = K[0.3]^x[0.2]^y$

(III)/(II)

$4 = 2^y$

$y = 2$

Overall order = x + y = 1 + 2 = 3

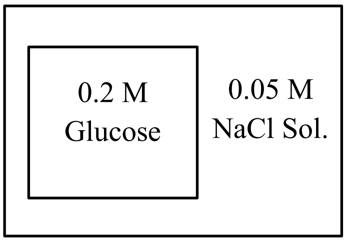
87. An artificial cell is made by encapsulating 0.2 M glucose solution within a semipermeable membrane. The osmotic pressure developed when the artificial cell is placed within a 0.05 M solution of NaCl at 300 K is _______ × 10^{-1} bar. (Nearest Integer)

[Given : R = 0.083 L bar mol^{-1} K^{-1}]

Assume complete dissociation of NaCl

Ans. **(25)**

Sol.



$NaCl \longrightarrow Na^+ + Cl^-$

0.05M 0.05M 0.05M

Total $C_1 = 0.05 + 0.05 = 0.1$ M (NaCl)

$C_2 = 0.2$ M (glucose)

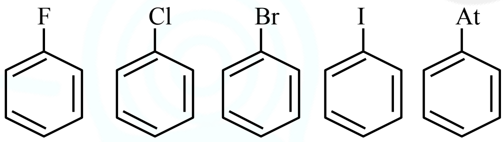
$\pi = (C_2 - C_1)$ RT

$= (0.2 - 0.1) \times 0.083 \times 300$

$= 2.49$ bar

$= 24.9 \times 10^{-1}$ bar

88. The number of halobenzenes from the following that can be prepared by Sandmeyer's reaction is ....



I **II** **III** **IV** **V**

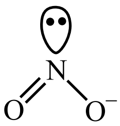
Ans. **(2)**

Sol. In Sandmayer reaction only bromobenzene & chlorobenzene are prepared

89. In the lewis dot structure for NO_2^-, total number of valence electrons around nitrogen is …….

Ans. **(8)**

Sol.

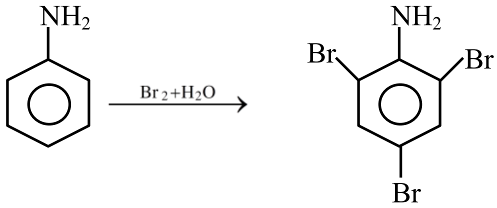


Number of valence e^- around N-atom = 8

90. 9.3 g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtained is 26.4 g. The percentage yield is ……… %.

Ans. **(80)**

Sol.



(white ppt)

93 g of aniline produces 330 g of 2, 4, 6-tribromoaniline. Hence 9.3 g of aniline should produce 33g of 2, 4, 6-tribromoaniline. Hence percentage yield $\frac{26.4 \times 100}{33} = 80\%$