(Held On Monday 08th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 (1-x^k)^n dx,\ n \in \mathbb{N}$, satisfies $147\ I_{20} = 148\ I_{21}$ is :

(1) 10 (2) 8

(3) 14 (4) 7

Ans. (4)

Sol. $I_n = \int_0^1 (1-x^k)^n.1\ dx$

$I_n = (1-x^k)^n.x - nk\int_0^1 (1-x^k)^{n-1}.x^{k-1}.dx$

$I_n = nk\int_0^1 [(1-x^k)^n - (1-x^k)^{n-1}]dx$

$I_n = nkI_n - nkI_n$

$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}$

$\frac{I_{21}}{I_{20}} = \frac{21k}{1+21k}$

$= \frac{147}{148} \Rightarrow k = 7$

2. The sum of all the solutions of the equation $(8)^{2x} - 16\cdot(8)^x + 48 = 0$ is :

(1) $1 + \log_6(8)$ (2) $\log_8(6)$

(3) $1 + \log_8(6)$ (4) $\log_8(4)$

Ans. (3)

Sol. $(8)^{2x} - 16\cdot(8)^x + 48 = 0$

Put $8^x = t$

$t^2 - 16 + 48 = 0$

$\Rightarrow t = 4$ or $t = 12$

$\Rightarrow 8^x = 4 \quad 8^x = 12$

$\Rightarrow x = \log_8 x \quad x = \log_8 12$

sum of solution $= \log_8 4 + \log_8 12$

$= \log_8 48 = \log_8(6.8)$

$= 1 + \log_8 6$

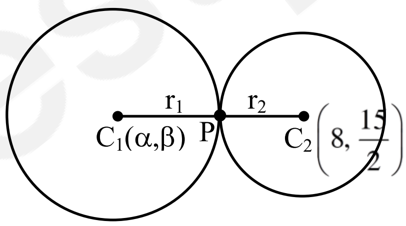
3. Let the circles $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1, then $(\alpha+\beta) + 4\left(r_1^2 + r_2^2\right)$ equals

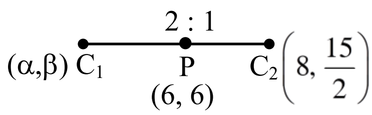
(1) 110 (2) 130

(3) 125 (4) 145

Ans. (2)

Sol.





$\because \frac{16+\alpha}{3} = 6$ and $\frac{15+\beta}{3} = 6$

$\Rightarrow (\alpha, \beta) \equiv (2, 3)$

Also, $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_2 + r$

$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$

$\therefore (\alpha+\beta) + 4(r_1 + r^2)$

$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$

4. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = γ ; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where O is the origin. Then the distance of P from the x-axis is :

(1) $\gamma\sqrt{1-\sin^2\phi\cos^2\theta}$ (2) $\gamma\sqrt{1+\cos^2\theta\sin^2\phi}$

(3) $\gamma\sqrt{1-\sin^2\theta\cos^2\phi}$ (4) $\gamma\sqrt{1+\cos^2\phi\sin^2\theta}$

Ans. (1)

Sol. P(x, y, z), Q(x, y, O) ; $x^2+y^2+z^2=\gamma^2$

$\overline{OQ}=x\hat{i}+y\hat{j}$

$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\Rightarrow \sin^2\phi = \frac{x^2+y^2}{x^2+y^2+z^2}$

distance of P from x-axis $\sqrt{y\ \ +z^2}$

$\Rightarrow \sqrt{\gamma\ \ -x^2} \ \Rightarrow \gamma\sqrt{1\ \ \frac{x^2}{\gamma^2}}$

$= \gamma\sqrt{1-\cos\ \ \theta\sin^2\phi}$

5. The number of critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ is :

(1) 2 (2) 0

(3) 1 (4) 3

Ans. (1)

Sol. $f(x) = (x-2)^{2/3}(2x+1)$

$f'(x) = \frac{2}{3}(x-2)^{-1/3}(2x+1)+(x-2)^{2/3}(2)$

$f'(x) = 2\times\frac{(2x+1)+(x-2)}{3(x-2)^{1/3}}$

$\frac{3x-1}{(x-2)^{1/3}}=0$

Critical points $x = \frac{1}{3}$ and $x = 2$

6. Let f(x) be a positive function such that the area bounded by y = f(x), y = 0 from x = 0 to x = a > 0 is $e^{-a}+4a^2+a-1$. Then the differential equation, whose general solution is $y = c_1f(x)+c_2$, where c_1 and c_2 are arbitrary constants, is :

(1) $(8e^x-1)\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

(2) $(8e^x+1)\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

(3) $(8e^x+1)\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

(4) $(8e^x-1)\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

Ans. (3)

Sol. $\int_0^a f(x)dx = e^{-a}+4a\ \ +a-1$

$f(a) = -e^{-a}+8a+1$

$f(x) = -e^{-x}+8x+1$

Now $y = C_1f(x)+C_2$

$\frac{dy}{dx}=C_1f'(x) = C_1(e^{-x}+8)$ …..(1)

$\frac{d^2y}{dx^2} = -C_1e^{-x} \Rightarrow -e^x\frac{d^2y}{dx^2}$

Put in equation (1)

$\frac{dy}{dx}=-e^x\frac{d^2y}{dx^2}(e^{-}\ \ +8)$

$(8e^x+1)\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

7. Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is:

(1) 1 (2) 2

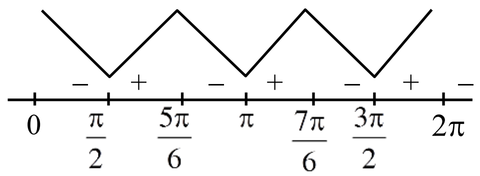
(3) 3 (4) 4

Ans. (2)

Sol. $f(x) = 4\cos^3(x) + 3\sqrt{3}\cos^2(x) - 10 \ ; x \in (0, 2\pi)$

$\Rightarrow f'(x) = 12\cos^2 x[-\sin(x)] + 3\sqrt{3}\,(2\cos(x))[-\sin(x)]$

$\Rightarrow f'(x) = -6\sin(x)\cos(x)[2\cos(x) + \sqrt{3}\,]$



local maxima at $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

8. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3×3, then $2a + 3b$ is equal to :

(1) –10 (2) –13

(3) –9 (4) –12

Ans. (2)

Sol. $A^3 - 4A^2 + A + 21\,I = 0$

$tr(A) = 4 = 5 + 6 \ \Rightarrow \ b = -1$

$|A| = -21$

$-16 + a = -21 \ \Rightarrow \ a = -5$

$2a + 3b = -13$

9. If the shortest distance between the lines

$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k},\ \lambda \in \mathbb{R}$

$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k},\ \mu \in \mathbb{R}$

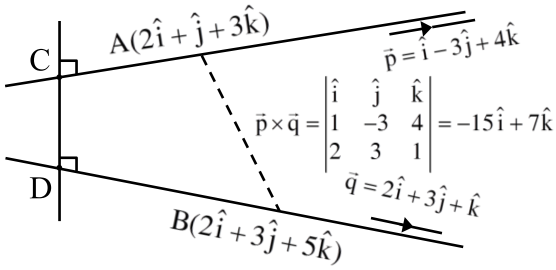
is $\frac{m}{\sqrt{n}}$, where gcd (m, n) = 1, then the value of m + n equals.

(1) 384 (2) 387

(3) 377 (4) 390

Ans. (2)

Sol.



Shortes distance (CD) = $\left|\frac{\overline{AB}\cdot\vec{p}\times\vec{q}}{|\vec{p}\times\vec{q}|}\right|$

$= \left|\frac{(0\hat{i}+2\hat{j}+2\hat{k}).(-15\hat{i}+7\hat{j}+9\hat{k})}{\sqrt{355}}\right|$

$= \frac{0+14+18}{\sqrt{355}} = \frac{32}{\sqrt{355}}$

$\therefore\ m + n = 32 + 355 = 387$

10. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m equals :

(1) 9 (2) 11

(3) 8 (4) 10

Ans. (4)

Sol. $x + y = 24,\ x, y \in N$

$AM > GM \ \Rightarrow \ xy \le 144$

$xy \ge 108$

Favorable pairs of (x, y) are

(13, 11), (12, 12), (14, 10), (15, 9), (16, 8), (17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15), (10, 14), (11, 13)

i.e. 13 cases

Total choices for $x + y = 24$ is 23

Probability $= \frac{13}{23} = \frac{m}{n}$

$n - m = 10$

11. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$,

then $80(\tan^2 x - \cos x)$ is equal to :

(1) 109 (2) 108

(3) 18 (4) 19

Ans. (1)

Sol. $\sin x = \frac{-3}{5}$, $\pi < x < \frac{3\pi}{2}$

$\tan x = \frac{3}{4}$ $\cos x = -\frac{4}{5}$

$80(\tan^2 x - \cos x)$

$= 80\left(\frac{9}{16} + \frac{4}{5}\right) = 45 + 64 = 109$

12. Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2} dx$. If $I(0) = 3$, then

$I\left(\frac{\pi}{12}\right)$ is equal to :

(1) $\sqrt{3}$ (2) $3\sqrt{3}$

(3) $6\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (2)

Sol. $I(x) = \int \frac{6dx}{\sin^2 x(1 \quad \cot x)^2} = \frac{6\,\mathrm{cosec}^2 x\, dx}{(1-\cot x)^2}$

Put $1 - \cot x = t$

$\mathrm{cosec}^2 x\, dx = dt$

$I = \int \frac{6dt}{t^2} = \frac{-6}{t} \quad c$

$I(x) = \frac{-6}{1-\cot x} c$, $c = 3$

$I(x) = 3 - \frac{6}{1-\cot x}$, $I\left(\frac{\pi}{12}\right) = \quad -\frac{6}{1-(2 \quad \sqrt{3})}$

$I\left(\frac{}{12}\right) = 3 + \frac{6}{\sqrt{3}+} = 3 + \frac{6(\sqrt{3}-1)}{2} = 3\sqrt{3}\sqrt{2}$

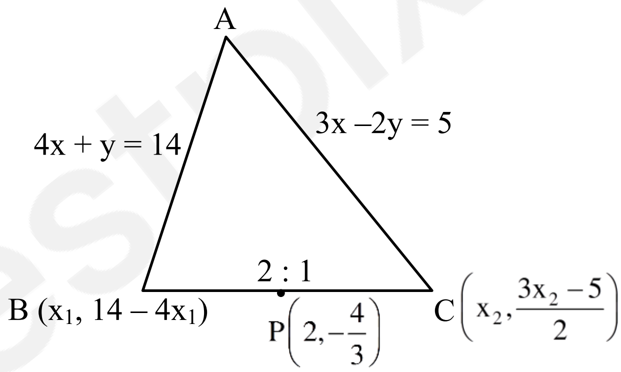
13. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2 : 1. The equation of the side BC is :

(1) $x - 6y - 10 = 0$ (2) $x - 3y - 6 = 0$

(3) $x + 3y + 2 = 0$ (4) $x + 6y + 6 = 0$

Ans. (3)

Sol.



$\frac{2x_2 + x}{3} = 2$, $\frac{2\left(\frac{3x_2-5}{2}\right) \quad (14-4x_1)}{3} = \frac{-4}{3}$

$2x_2 + x_1 = 6$, $3x_2 - 4x_1 = -13$

$x_2 = 1$, $x_1 = 4$

So, $C(1, -1)$, $B(4, -2)$

$m = \frac{-1}{3}$

Equation of BC : $y + 1 = \frac{-1}{3}(x - 1)$

$3y + 3 = -x + 1$

$x + 3y + 2 = 0$

14. Let [t] be the greatest integer less than or equal to t. Let A be the set of al prime factors of 2310 and $f : A \to \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$.

The number of one-to-one functions from A to the range of f is :

(1) 20 (2) 120

(3) 25 (4) 24

Ans. (2)

Sol. $N = 2310 = 231 \times 10$

$= 3 \times 11 \times 7 \times 2 \times 5$

$A = \{2, 3, 5, 7, 11\}$

$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$

$f(2) = [\log_2(5)] = 2$

$f(3) = [\log_2(14)] = 3$

$f(5) = [\log_2(25 + 25)] = 5$

$f(7) = [\log_2(117)] = 6$

$f(11) = [\log_2 387] = 8$

Range of f : B = {2, 3, 5, 6, 8}

No. of one-one functions = 5! = 120

15. Let z be a complex number such that |z + 2| = 1 and $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left|\text{Re}\left(\overline{z+2}\right)\right|$ is :

(1) $\frac{\sqrt{6}}{5}$ (2) $\frac{1+\sqrt{}}{5}$

(3) $\frac{24}{5}$ (4) $\frac{2\sqrt{}}{5}$

Ans. (4)

Sol. $|z + 2| = 1$, $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$

Let $z + 2 = \cos\theta + i\sin\theta$

$\frac{1}{z+2} = \cos\theta - i\sin\theta$

$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z\ \ 2} = 1 - (\cos\theta - i\sin\theta)$

$= (1 - \cos\theta) + i\sin\theta$

$\text{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta$, $\sin\theta = \frac{1}{5}$

$\cos\theta = \pm\sqrt{1 - \frac{1}{25}} = \pm\frac{2\sqrt{6}}{5}$

$\left|\text{Re}(\overline{z+2})\right| \quad \frac{2\sqrt{6}}{5}$

16. If the set R = {(a, b) ; a + 5b = 42, a, b ∈ ℕ } has m elements and $\sum_{n=1}^{m}\left(1 + i^{n!}\right) = x + iy$, where $I = \sqrt{-1}$, then the value of m + x + y is :

(1) 8 (2) 12

(3) 4 (4) 5

Ans. (2)

Sol. $a + 5b = 42$, $a, b \in N$

$a = 42 - 5b$, $b = 1$, $a = 37$

$b = 2$, $a = 32$

$b = 3$, $a = 27$

⋮

$b = 8$, $a = 2$

R has "8" elements $\Rightarrow$ m = 8

$\sum_{n=1}^{8}(1 - i^{n!}) = x + iy$

for $n \geq 4$, $i^{n!} = 1$

$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$

$= 1 - I + 2 + 1 + 1$

$= 5 - I = x + iy$

$m + x + y = 8 + 5 - 1 = 12$

17. For the function $f(x) = (\cos x) - x + 1,\ x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x is $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

(1) Both (S1) and (S2) are correct
(2) Only (S1) is correct
(3) Both (S1) and (S2) are incorrect
(4) Only (S2) is correct

Ans. (2)

Sol. $f(x) = \cos x - x + 1$

$f'(x) = -\sin x - 1$

f is decreasing $\forall\, x \in R$

$f(x) = 0$

$f(0) = 2,\ f(\pi) = -\pi$

f is strictly decreasing in $[0, \pi]$ and $f(0).f(\pi) < 0$

$\Rightarrow$ only one solution of $f(x) = 0$

S1 is correct and S2 is incorrect.

18. The set of all α, for which the vector $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$ is :

(1) $[0, 1)$
(2) $(-2, 0]$
(3) $\left(-\frac{4}{3}, 0\right]$
(4) $\left(-\frac{4}{3}, 1\right]$

Ans. (3)

Sol. $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$

$\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$

so $\vec{a}.\vec{b} < 0,\ \forall t \in R$

$\alpha t^2 - 12 + 6\alpha t < 0$

$\alpha t^2 + 6\alpha t - 12 < 0,\ \forall t \in R$

$\alpha < 0$, and $D < 0$

$36\alpha^2 + 48\alpha < 0$

$12\alpha(3\alpha + 4) < 0$

$\frac{-4}{3} < \alpha < 0$

also for $a = 0,\ \vec{a}.\vec{b} < 0$

hence a $\alpha \in \left(\frac{-4}{3}, 0\right]$

19. Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{2}{e}$
(2) $\frac{1}{e^2}$
(3) $\frac{1}{e}$
(4) $\frac{2}{e^2}$

Ans. (3)

Sol. $(1 + y^2)\, e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})\, dy = 0$

$$\int \frac{\sec^2 x\, e^{\tan x}}{1 + e^{2\tan x}}dx + \int \frac{dy}{1 \quad y^2} = C$$

$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$

for $x = 0$, $y = 1$, $\tan^{-1}(1) + \tan^{-1} 1 = C$

$$C = \frac{\pi}{2}$$

$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$

Put $x = \pi$, $\tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$

$$\tan^{-1} y = \cot^{-1} e$$

$$y = \frac{1}{e}$$

20. Let H : $\frac{-x^2}{a^2} + — \quad 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H. If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to :

(1) 170
(2) 171
(3) 169
(4) 172

Ans. (2)

Sol. $H: \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1,\ e = \sqrt{3}$

$$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \quad \Rightarrow \frac{a^2}{b^2} = 2$$

$$a^2 = 2b^2$$

$$\text{length of L.R.} = \frac{2a^2}{b} = 4\sqrt{3}$$

$$a = \sqrt{6}$$

$$P(\alpha, 6) \text{ lie on } \frac{y^2}{3} - \frac{x^2}{6} = 1$$

$$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$$

Foci $= (0, \pm be) = (0, 3)$ & $(0, -3)$

Let d_1 & d_2 be focal distances of $P(\alpha, 6)$

$$d_1 = \sqrt{\alpha \quad + (6 + be)^2},\ d_2 = \sqrt{\alpha \quad + (6 - be)^2}$$

$$d_1 = \sqrt{66 + 81},\ d_2 = \sqrt{66 + 9}$$

$$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$$

$$\alpha^2 + \beta = 66 + 105 = 171$$

SECTION-B

21. Let $A = \begin{vmatrix} 2 & - \\ 1 & \end{vmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to ______.

Ans. (7)

Sol. $A = \begin{vmatrix} 2 & - \\ 1 & \end{vmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & 9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & 9 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & 18 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & -0 \\ 0 & -27 \end{bmatrix}\begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$

$$3^7 = 3^n \Rightarrow n = 7$$

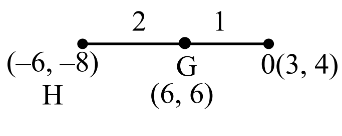
22. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumecentre and orthocentre respectively are (3, 4) and (–6, –8), then the value of |a – b| is _____.

Ans. (16)

Sol. $2x + 3y - 1 = 0$

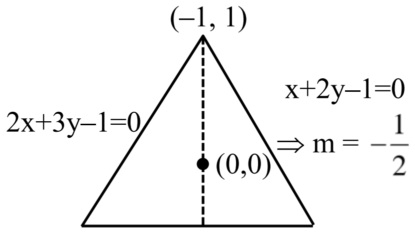
$x + 2y - 1 = 0$

$ax + by - 1 = 0$



$$\left(\frac{6-6}{3}, \frac{8 \quad 8}{}\right.$$

$= (0, 0)$



ax + by – 1 = 0

$$\left(\frac{1-0}{-1-0}\right)\left(\frac{-a}{b}\right)=-1$$

$\Rightarrow -a = b$

$\Rightarrow \quad ax - ay - 1 = 0$

$$ax - a\left(1-\frac{2x}{3}\right.$$

$$x\left(a+\frac{2a}{3}\right) \quad \frac{a}{3}$$

$$x=\frac{a+3}{5a}$$

$$2\left(\frac{a+3}{5a}\right)+3y-1=0$$

$$y=\frac{1-\frac{2a+6}{5a}}{3} \quad \frac{3a-6}{3\times 5a}$$

$$y=\frac{a-2}{5a}$$

$$\frac{\left(\frac{a-2}{5a}\right)}{\left(\frac{a+3}{5a}\right)}=2 \Rightarrow a-2=2a+6$$

a = –8

b = 8

–8x + 8y – 1 = 0

|a – b| = 16

23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to ______.

Ans. (17)

Sol.

Blue balls	0	1	2	3	4	5
Prob.	$\frac{^5C_0 \cdot {}^4C_1}{^9C_3}$	$\frac{^5C_1 \cdot {}^4C_2}{^9C_3}$	$\frac{^5C_2\ {}^4C_1}{^9C_3}$	$\frac{^5C_3 \cdot {}^4C_0}{^9C_3}$	0	0

$$7\bar{x}=\frac{^5C_1{}^4C_2+{}^5C_2\cdot{}^4C_1\times 2+{}^5C_3\cdot{}^4C_0\times 3}{^9C_3}\times 7$$

$$\frac{30+80+30}{84}\times 7$$

$$=\frac{140}{12}=\frac{70}{6}=\frac{35}{3}$$

yellow	0	1	2	3	4
		$^5C_2{}^4C_1$	$^5C_1{}^4C_2$	$^5C_0{}^4C_3$	0

$$4\bar{y}=\frac{40+60\quad 12}{84}\times 4=\frac{112}{21}=\frac{16}{3}$$

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to ______.

Ans. (36)

Sol. **2, 3, 4, 5, 7**

total number of three digit numbers not divisible by 3 will be formed by using the digits

(4, 5, 7)
(3, 4, 7)
(2, 5, 7)
(2, 4, 7)
(2, 4, 5)
(2, 3, 5)

number of ways = 6 × 3! = 36

25. Let the positive integers be written in the form :

1

2 3

4 5 6

7 8 9 10

. . . . . .

If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is ______.

Ans. (103)

Sol. $S = 1 + 2 + 4 + 7 + \ldots.. + T_n$

$S = 1 + 2 + 4 + \ldots\ldots$

$Tn = 1 + 1 + 2 + 3 + \ldots\ldots + (T_n - T_{n-1})$

$T_n = 1 + \left(\frac{n-1}{2}\right)[2 + (n-2)\times 1]$

$T_n = 1 + 1 + \frac{n(n-1)}{2}$

$n = 100 \quad T_n = 1 + \frac{100\times 99}{2} = 4950 + 1$

$n = 101 \quad T_n = 1 + \frac{101\times 100}{2} = 5050 + 1 = 5051$

$n = 102 \quad T_n = 1 + \frac{102\times 101}{2} = 5151 + 1 = 5152$

$n = 103 \quad T_n = 1 + \frac{103\times 102}{2} = 5254$

$n = 104 \quad T_n = 1 + \frac{104\times 103}{2} = 5357$

26. If the range of $f(\theta) = \frac{\sin^4\theta + 3\cos\ \theta}{\sin^4\theta + \cos\ \theta}$, $\theta \quad \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to ______.

Ans. (96)

Sol. $f(\theta) \quad \frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$

$f(\theta) = 1 + \frac{2\cos^2\theta}{\sin^4\theta + \cos\ \theta}$

$f(\theta) = \frac{2\cos^2\theta}{\cos^4\theta - \cos^2\theta + 1} + 1$

$f(\theta) = \frac{2}{\cos^2\theta + \sec^2\theta - 1} + 1$

$f(\ )|_{min.} = 1$

$f(\theta)_{max.} = 3$

$S = \frac{64}{1 - 1/3} = 96$

27. Let $\alpha = \sum_{r=0}^{n}(4r^2 + 2r + 1)\,{}^nC_r$ and $\beta = \left(\sum_{r=0}^{n}\frac{{}^nC_r}{r+1}\right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is ______.

Ans. (5)

Sol. $\alpha = \sum_{r=0}^{n}(4r^2 + 2r + 1).\ C_r$

$\alpha = 4\sum_{r=0}^{n} r^2.-.{}^{n-1}C_{r-1} + 2\sum_{r=0}^{n} r.\frac{n}{r}.{}^{n\ 1}C_r \ + \sum_{r=0}^{n}{}^nC_r$

$+4n\sum_{r=0}^{n}{}^{n-1}C_{r-1} \ +2n\sum_{r=0}^{n}{}^{n-1}C_{r-1} \quad {}^nC_r \quad r\ 0$

$\alpha = 4n(n-1)\ .\ 2^{n-2} + 4n.2^{n-1} + 2n.2^{n-1} + 2^n$

$\alpha = 2^{n-2}[4n(n-1) + 8n + 4n + 4]$

$\alpha = 2^{n-2}\ [4n^2 + 8n + 4]$

$\alpha = 2n(n+1)^2$

$\beta = \sum_{r=0}^{n}\frac{{}^nC_r}{r+1} + \frac{1}{n+1}$

$= \sum_{r=0}^{n}\frac{{}^{n+1}C_{r+1}}{n+1} \quad \frac{1}{n \quad 1}$

$= \frac{1}{n+1}\left(1 + {}^{n+1}C_1 + \ldots. + {}^{n\ 1}C_{n+1}\right)$

$= \frac{2^{n+1}}{n+1}$

$\frac{2\alpha}{\beta} = \frac{2^{n+1}(n \quad 1)^2}{2^{n+1}}.(n+1) = (n+1)^3$

$140 < (n+1)^3 < 281$

$n = 4 \Rightarrow (n+1)^3 = 125$

$n = 5 \Rightarrow (n+1)^3 = 216$

$n = 6 \Rightarrow (n+1)^3 = 343$

$\therefore\ n = 5$

28. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectros. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r}.(\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to ______ .

Ans. (569)

Sol. $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$

$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$

$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} + \vec{c} = 20\hat{i} + 5\hat{j} - 12\hat{k}$

$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$

$(\vec{r} - (\vec{b} + \vec{c})) \times \vec{a} = 0$

$r - (\vec{b} + \vec{c}) = \lambda\vec{a}$

$\vec{r} = \lambda\vec{a} + \vec{b} + \vec{c}$

But $\vec{r}.(\vec{b} - \vec{c}) = 0$

$\Rightarrow (\lambda\vec{a} + \vec{b} + \vec{c}).(\vec{b} - \vec{c}) = 0$

$\Rightarrow \lambda\vec{a}.\vec{b} + \vec{b}.\vec{b} + \vec{c}.\vec{b} - \lambda\vec{a}.\vec{c} - \vec{b}.\vec{c} - \vec{c}.\vec{c} = 0$

$\lambda = \frac{\vec{c}.\vec{c} - \vec{b}.\vec{b}}{\vec{a}.\vec{b} - \vec{a}.\vec{c}} = \frac{294 - 227}{-389 \quad 204} = \frac{-67}{593}$

$\therefore \vec{r} = \vec{b} + \vec{c} - \frac{67}{593}\vec{a}$

$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b} + \vec{c})$

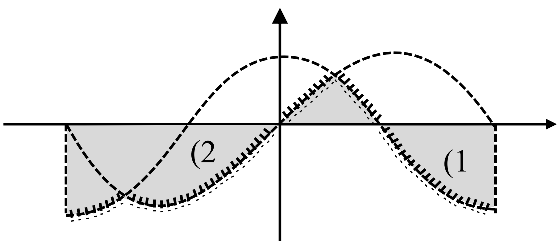
$\Rightarrow |\vec{b} + \vec{c}|^2 = 569$

29. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x-axis between $x = -\pi$ to $x = \pi$ be A. Then A^2 is equal to ______.

Ans. (16)

Sol. $y = \min\{\sin x, \cos x\}$

x–axis x–π x=π



$\int_0^{\pi/4} \sin x = (\cos x)_{\pi/4}^{0} = 1 - \frac{1}{\sqrt{2}}$

$\int_{-\pi}^{-3\pi/4} (\sin x - \cos x) = (-\cos x - \sin x)_{-\pi}^{3\pi/4}$

$= (\cos x + \sin x)_{3\pi/4}^{-\pi}$

$= (-1 + 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

$= -1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$\int_{\pi/4}^{\pi/2} \cos x dx = (\sin x)_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}}$

A = 4

$A^2 = 16$

30. The value of

$\lim_{x \to 0} 2\left(\frac{1 - \cos x\sqrt{\cos 2x}\sqrt[3]{\cos 3x}.....\sqrt[10]{\cos 10x}}{x^2}\right)$ is ______.

Ans. (55)

Sol.

$$\lim_{x \to 0} 2\left(\frac{1 - \left(1 - \frac{x^2}{2!}\right)\left(1 - \frac{4x^2}{2!}\right)\left(1 - \frac{9x^2}{2!}\right).....\left(1 - \frac{100x^2}{2!}\right)}{x^2}\right)$$

By expansion

$$\lim_{x \to 0} \frac{2 \; 1 - \left(1 - \frac{x^2}{2}\right)\left(1 - \frac{1}{2}.\frac{4x^2}{2}\right)\left(1 - \frac{1}{3}.\frac{9x^2}{2}\right).....\left(1 - \frac{1}{10}.\frac{100x^2}{2}\right)}{x^2}$$

$$\lim_{x \to 0} 2\left(\frac{1 - \left(1 - \frac{x^2}{2}\right)\left(1 - \frac{2x^2}{2}\right)\left(1 - \frac{3x^2}{2}\right).....\left(1 - \frac{10x^2}{2}\right)}{x^2}\right)$$

$$\lim_{x \to 0} \frac{2 \; 1 - 1 + x^2\left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + ... + \frac{10}{2}\right)}{x^2}$$

$$2\left(\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + ... + \frac{10}{2}\right)$$

$1 + 2 + ... + 10 = \frac{10 \times 11}{2} = 55$

(Held On Monday 08th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

31. Three bodies A, B and C have equal kinetic energies and their masses are 400 g, 1.2 kg and 1.6 kg respectively. The ratio of their linear momenta is :

(1) $1:\sqrt{3}:2$ (2) $1:\sqrt{3}:\sqrt{2}$

(3) $\sqrt{2}:\sqrt{3}:1$ (4) $\sqrt{3}:\sqrt{2}:1$

Ans. (1)

Sol. $KE = \frac{P^2}{2m}$

$P \propto \sqrt{m}$

Hence, $P_A : P_B : P_C$

$= \sqrt{400}:\sqrt{1200}:\sqrt{1600} = 1:\sqrt{3}:2$

32. Average force exerted on a non-reflecting surface at normal incidence is 2.4×10^{-4}N. If 360 W/cm^2 is the light energy flux during span of 1 hour 30 minutes. Then the area of the surface is:

(1) 0.2 m^2 (2) 0.02 m^2

(3) 20 m^2 (4) 0.1 m^2

Ans. (2)

Sol. Pressure $= \frac{I}{C} = \frac{F}{A}$

$\Rightarrow \frac{360}{10^{-4} \times 3 \times 10^8} = \frac{2.4 \times 10^{-4}}{A}$

$\Rightarrow A = 2 \times 10^{-2}\,m^2 = 0.02\,m^2$

33. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.0 \times 10^{-31}$ kg and m_p = 1836 times m_e)

(1) 1 : 1836 (2) $1 : \frac{1}{1836}$

(3) $1 : \frac{1}{\sqrt{1836}}$ (4) $1 : \sqrt{1836}$

Ans. (1)

Sol. λ is same for both

$P = \frac{h}{\lambda}$ same for both

$P = \sqrt{2mK}$

Hence,

$K \propto \frac{1}{m}$

$\Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{1}{1836}$

34. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (27°C). The ratio of specific heat of gases at constant volume respectively is:

(1) $\frac{7}{5}$ (2) $\frac{3}{2}$

(3) $\frac{3}{5}$ (4) $\frac{5}{3}$

Ans. (3)

Sol. $\frac{(C_v)_{mono}}{(C_v)_{dia}} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$

35. In an expression $a \times 10^b$:

(1) a is order of magnitude for $b \le 5$

(2) b is order of magnitude for $a \le 5$

(3) b is order of magnitude for $5 < a \le 10$

(4) b is order of magnitude for $a \ge 5$

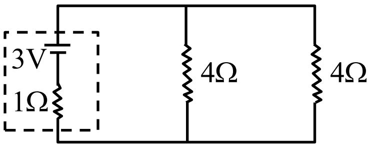
Ans. (2)

Sol. $a \times 10^b$

if $a \le 5$ order is b

$a > 5$ order is b + 1

36. In the given circuit, the terminal potential difference of the cell is :

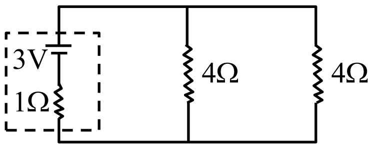


(1) 2 V (2) 4 V

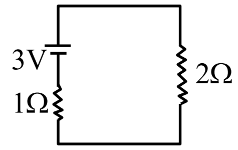
(3) 1.5 V (4) 3 V

Ans. (1)

Sol.



⇒



$i = \frac{3}{1+2} = 1A$

$v = E - ir$

$= 3 - 1 \times 1 = 2V$

37. Binding energy of a certain nucleus is 18×10^8 J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

(1) 0.2 μg (2) 20 μg

(3) 2 μg (4) 10 μg

Ans. (2)

Sol. $\Delta mc^2 = 18 \times 10^8$

$\Delta m \times 9 \times 10^{16} = 18 \times 10^8$

$\Delta m = 2 \times 10^{-8}$ kg $= 20$ μg

38. Paramagnetic substances:

A. align themselves along the directions of external magnetic field.

B. attract strongly towards external magnetic field.

C. has susceptibility little more than zero.

D. move from a region of strong magnetic field to weak magnetic field.

Choose the **most appropriate** answer from the options given below:

(1) A, B, C, D (2) B, D Only

(3) A, B, C Only (4) A, C Only

Ans. (4)

Sol. A, C only

39. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take π = 3.14) :

(1) 139.4 (2) 140.5

(3) 220.0 (4) 118.9

Ans. (1)

Sol. $x_{min} = \pi \times r_{min}$

$= \pi \times \frac{60}{100}$ m.

$x_{second} = 30 \times 2\pi \times r_{second}$

$= 30 \times 2\pi \times \frac{75}{100}$

$x = x_{second} - x_{min}$

$= 139.4$ m

40. Young's modulus is determined by the equation given by $Y = 49000 \frac{m}{\ell} \frac{dyne}{cm^2}$ where M is the mass and ℓ is the extension of wire used in the experiment. Now error in Young modules(Y) is estimated by taking data from M-ℓ plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of M and ℓ are 500 g and 2 cm respectively then percentage error of Y is :

(1) 0.2 % (2) 0.02 %

(3) 2 % (4) 0.5 %

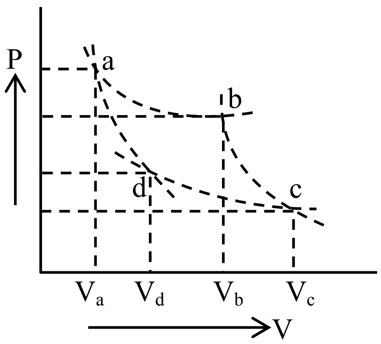
Ans. (3)

Sol. $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell}$

$= \frac{5}{500} + \frac{0.02}{2} = 0.01 + 0.01$

$\frac{\Delta Y}{Y} = 0.02 \Rightarrow \% \frac{\Delta Y}{Y} = 2\%$

41. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is:



(1) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$ (2) $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$

(3) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$ (4) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^2$

Ans. **(3)**

Sol. For adiabatic process

$TV^{\gamma-1} = \text{constant}$

$T_a \cdot V_a^{\gamma-1} = T_d \cdot V_d^{\gamma-1}$

$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_d}{T_a}$

$T_b \cdot V_b^{\gamma-1} = T_c \cdot V_c^{\gamma-1}$

$\left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_c}{T_b}$

$\frac{V_a}{V_d} = \frac{V_b}{V_c} \quad \left(\begin{matrix} \because T_d = T_c \\ T_a = T_b \end{matrix}\right)$

42. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is 3L, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is:

(1) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$ (2) $\left(\frac{r_1}{r_2}\right)^3$

(3) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$ (4) $27\left(\frac{m_1}{m_2}\right)^3$

Ans. **(3)**

Sol. $\frac{\pi r_1^2}{T_A} = \frac{L}{2m_1}$ .... (1)

$\frac{\pi r_2^2}{T_B} = \frac{3L}{2m_2}$ .... (2)

$\Rightarrow \frac{T_A}{T_B} = 3.\frac{m_1}{m_2}.\left(\frac{r_1}{r_2}\right)^2$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{T_A}{T_B}\right)^{\frac{4}{3}}$

$\Rightarrow \frac{1}{27}.\left(\frac{m_2}{m_1}\right)^3 = \left(\frac{T_A}{T_B}\right)$

43. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:

(1) Zero (2) double

(3) same (4) halved

Ans. **(2)**

Sol. In resonance $Z = R$

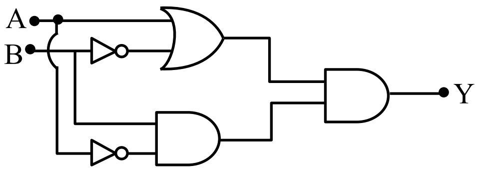
$I = \frac{V}{R}$

$R \rightarrow$ halved

$\Rightarrow I \rightarrow 2I$

I becomes doubled.

44. The output Y of following circuit for given inputs is :



(1) $A \cdot B(A + B)$ (2) $A \cdot B$

(3) 0 (4) $\overline{A} \cdot B$

Ans. (3)

Sol. By truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

45. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is:

(1) $\sqrt{ab}$ (2) ab

(3) $\frac{a}{b}$ (4) $\frac{b}{a}$

Ans. (3)

Sol. Potential at surface will be same

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}$$

$$\frac{q_1}{q_2} = \frac{a}{b}$$

46. Correct Bernoulli's equation is (symbols have their usual meaning) :

(1) $P + mgh + \frac{1}{2}mv^2 = \text{constant}$

(2) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

(3) $P + \rho gh + \rho v^2 = \text{constant}$

(4) $P + \frac{1}{2}\rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Ans. (2)

Sol. $P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$

47. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is:

(1) 150 N (2) 3 N

(3) 30 N (4) 300 N

Ans. (3)

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{mv - 0}{0.1}$

$$= \frac{150 \times 10^{-3} \times 20}{0.1} = 30\,\text{N}$$

48. A stationary particle breaks into two parts of masses m_A and m_B which move with velocities v_A and v_B respectively. The ratio of their kinetic energies $(K_B : K_A)$ is :

(1) $v_B : v_A$ (2) $m_B : m_A$

(3) $m_B v_B : m_A v_A$ (4) 1 : 1

Ans. (1)

Sol. Initial momentum is zero.

Hence $|P_A| = |P_B|$

$\Rightarrow m_A v_B = m_B V_B$

$$\frac{(KE)_A}{(KE)_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{v_A}{v_B}$$

$$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$$

49. Critical angle of incidence for a pair of optical media is 45°. The refractive indices of first and second media are in the ratio:

(1) $\sqrt{2}:1$ (2) 1 : 2

(3) $1:\sqrt{2}$ (4) 2 : 1

Ans. (1)

Sol. $\sin\theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$\sin 45° = \frac{\mu_2}{\mu_1}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_2}{\mu_1}$

$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{1}$

50. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is:

(1) 2.5 g/cm^3 (2) 1.7 g/cm^3

(3) 2.2 g/cm^3 (4) 2.0 g/cm^3

Ans. **(4)**

Sol. Given 9MSD = 10VSD

mass = 8.635 g

LC = 1 MSD – 1 VSD

$LC = 1\ MSD - \frac{9}{10} MSD$

$LC = \frac{1}{10} MSD$

LC = 0.01 cm

Reading of diameter = MSR + LC × VSR

= 2 cm + (0.01) × (2)

= 2.02 cm

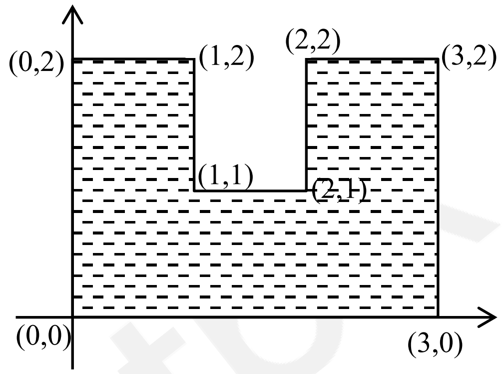
$\text{Volume of sphere} = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{2.02}{2}\right)^3$

$= 4.32\ cm^3$

$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{8.635}{4.32} = 1.998 \sim 2.00\,g$

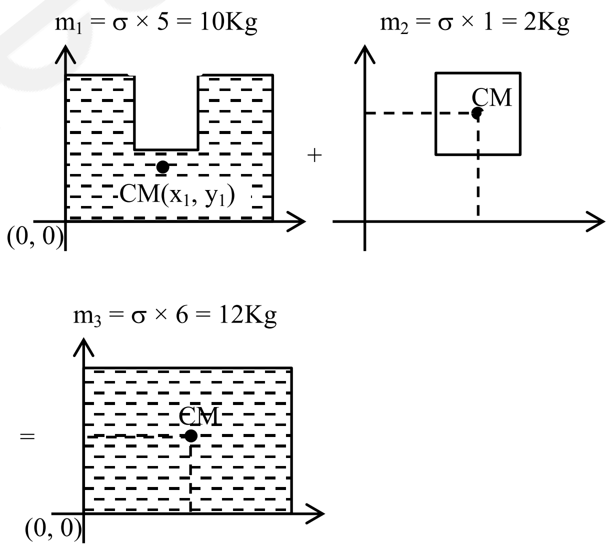
SECTION-B

51. A uniform thin metal plate of mass 10 kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{9}$. The value of n is_______.



Ans. **(15)**

Sol. $m_1 = \sigma \times 5 = 10$ Kg



$\Rightarrow m_1 x_1 + m_2 x_2 = m_3 x_3$

$10x_1 + 2(1.5) = 12(1.5) \Rightarrow x_1 = 1.5$ cm

$\Rightarrow m_1 y_1 + m_2 y_2 = m_3 y_3$

$10y_1 + 2(1.5) = 12 \times 1 \Rightarrow y_1 = 0.9$ cm

$\frac{x_1}{y_1} = \frac{1.5}{0.9} = \frac{15}{9}$

n = 15

52. An electron with kinetic energy 5 eV enters a region of uniform magnetic field of 3 μT perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E, so that electron moves along the same path, is_______ NC^{-1}.

(Given, mass of electron = 9×10^{-31} kg, electric charge = 1.6×10^{-19}C)

Ans. **(4)**

Sol. For the given condition of moving undeflected, net force should be zero.

$qE = qVB$

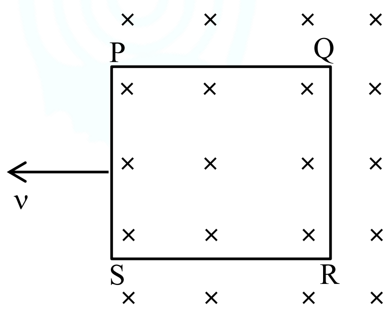
$E = VB$

$$= \sqrt{\frac{2 \times KE}{m}} \times B$$

$$= \sqrt{\frac{2 \times 5 \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}} \times 3 \times 10^{-6}$$

$= 4$ N/C

53. A square loop PQRS having 10 turns, area 3.6×10^{-3} m^2 and resistance 100 Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude B = 0.5 T as shown. Work done in pulling the loop out of the field in 1.0 s is _________$\times 10^{-6}$ J.



Ans. **(3)**

Sol. $\in = NB\ell v$

$$i = \frac{\in}{R} = \frac{NB\ell v}{R}$$

$$F = N(i\ell B) = \frac{N^2B^2\ell^2 v}{R}$$

$$W = F \times \ell = \frac{N^2B^2\ell^3}{R}\left(\frac{\ell}{t}\right)$$

$A = \ell^2$

$$W = \frac{(10 \times 10)(0.5)^2 \times (3.6 \times 10^{-3})^2}{100 \times 1}$$

$W = 3.24 \times 10^{-6}$ J

54. Resistance of a wire at 0 °C, 100 °C and t °C is found to be 10 Ω, 10.2 Ω and 10.95 Ω respectively. The temperature t in Kelvin scale is______.

Ans. **(748)**

Sol. $R = R_0(1 + \alpha\Delta T)$

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

Case-I

0 °C → 100 °C

$$\frac{10.2 - 10}{10} = \alpha(100 - 0) \quad \text{..... (1)}$$

Case-II

0 °C → t °C

$$\frac{10.95 - 10}{10} = \alpha(t - 0) \quad \text{...... (2)}$$

$$\Rightarrow \frac{t}{100} = \frac{0.95}{0.2} = 475°C$$

$t = 475 + 273 = 748$ K

55. An electric field, $\vec{E} = \frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}}$ passes through the surface of 4 m^2 area having unit vector $\hat{n} = \left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}\right)$. The electric flux for that surface is _________ V m.

Ans. **(12)**

Sol. $\phi = \vec{E} \cdot \vec{A}$

$$= \left(\frac{2\hat{i} + 6\hat{j} + 8\hat{k}}{\sqrt{6}}\right) \cdot 4\left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}\right)$$

$$= \frac{4}{6} \times (4 + 6 + 8) = 12\,\text{Vm}$$

56. A liquid column of height 0.04 cm balances excess pressure of soap bubble of certain radius. If density of liquid is 8×10^3 kg m^{-3} and surface tension of soap solution is 0.28 Nm^{-1}, then diameter of the soap bubble is________cm.

(if $g = 10$ ms^{-2})

Ans. (7)

Sol. $\rho g h = \frac{4S}{R}$

$$\Rightarrow R = \frac{4 \times 0.28}{8 \times 10^3 \times 10 \times 4 \times 10^{-4}}$$

$$\Rightarrow \frac{0.28}{8} m = \frac{28}{8} cm$$

$\Rightarrow R = 3.5$ cm

Diameter = 7 cm

57. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is _____.

Ans. (16)

Sol. For closed organ pipe

$$f_c = (2n+1)\frac{v}{4\ell} = \frac{15v}{4\ell}$$

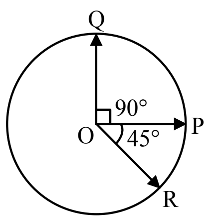
For open organ pipe

$$f_o = (n+1)\frac{v}{2\ell} = \frac{8v}{2\ell}$$

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a-1}{a}$$

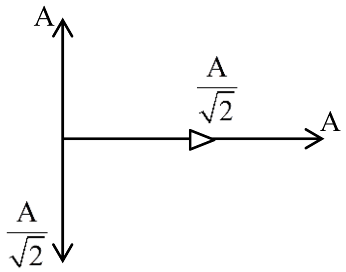
$\Rightarrow a = 16$

58. Three vectors $\overrightarrow{OP}, \overrightarrow{OQ}$ and $\overrightarrow{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is _____.



Ans. (3)

Sol.



$$\vec{R} = \left(A + \frac{A}{\sqrt{2}}\right)\hat{i} + \left(A - \frac{A}{\sqrt{2}}\right)\hat{j}$$

$$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{2}}\right)^2 + \left(A - \frac{A}{\sqrt{2}}\right)^2} = \sqrt{3}A$$

59. A parallel beam of monochromatic light of wavelength 600 nm passes through single slit of 0.4 mm width. Angular divergence corresponding to second order minima would be _____$\times 10^{-3}$ rad.

Ans. (6)

Sol. $\sin\theta \simeq \theta \simeq \frac{2\lambda}{b}$

$$= \frac{2 \times 600 \times 10^{-9}}{4 \times 10^{-4}} = 3 \times 10^{-3} \text{ rad}$$

Total divergence = $(3 + 3) \times 10^{-3} = 6 \times 10^{-3}$ rad

60. In an alpha particle scattering experiment distance of closest approach for the α particle is 4.5×10^{-14} m. If target nucleus has atomic number 80, then maximum velocity of α-particle is _______$\times 10^5$ m/s approximately.

($\frac{1}{4\pi \epsilon_0} = 9 \times 10^9$ SI unit, mass of α particle = 6.72×10^{-27} kg)

Ans. (156)

Sol. $v = \sqrt{\frac{4KZe^2}{m r_{min}}}$

$$= \sqrt{\frac{4 \times 9 \times 10^9 \times 80}{6.72 \times 10^{-27} \times 4.5 \times 10^{-14}}} \times 1.6 \times 10^{-19}$$

$= 9.759 \times 10^{25} \times 1.6 \times 10^{-19}$

$= 156 \times 10^5$ m/s

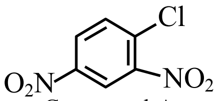
(Held On Monday 08th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61. Given below are two statements:

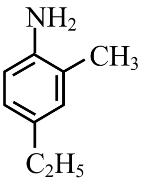
Statement I :



Compound-A

IUPAC name of Compound A is 4-chloro-1, 3-dinitrobenzene:

Statement II:



Compound-B

IUPAC name of Compound B is 4-ethyl-2-methylaniline.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both Statement I and Statement II are correct

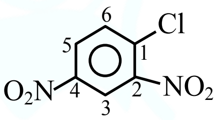
(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are incorrect

Ans. (2)

Sol. Statement I :

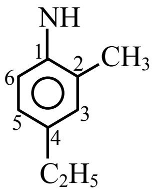


IUPAC name

⇒ 1-chloro-2, 4-dinitrobenzene

⇒ statement-I is incorrect

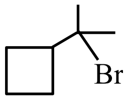
Statement-II :

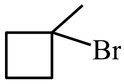


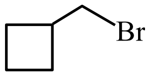
⇒ 4-ethyl-2-methylaniline

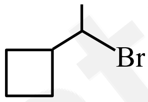
⇒ statement-II is correct

62. Which among the following compounds will undergo fastest S_N2 reaction.

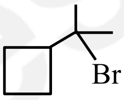
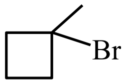
(1) 

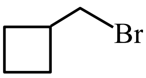
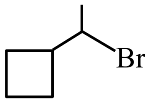
(2) 

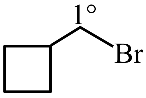
(3) 

(4) 

Ans. (3)

Sol. 1  2 

3  4 

fastest SN^2 reaction give 

Rate of SN^2 is Me – x > 1° – x > 2° – x > 3° – x

63. Combustion of glucose ($C_6H_{12}O_6$) produces CO_2 and water. The amount of oxygen (in g) required for the complete combustion of 900 g of glucose is:

[Molar mass of glucose in g mol^{-1} = 180]

(1) 480 (2) 960

(3) 800 (4) 32

Ans. (2)

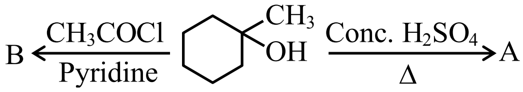
Sol. $C_6H_{12}O_{6(s)} + 6O_{2(g)} \longrightarrow 6CO_{2(g)} + 6H_2O_{(\ell)}$

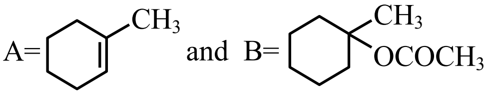
$\frac{900}{180}$

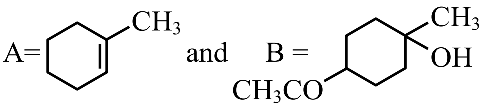
= 5 mol 30 mol

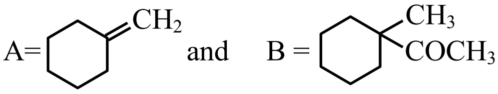
Mass of O_2 required = 30 × 32 = 960 gm

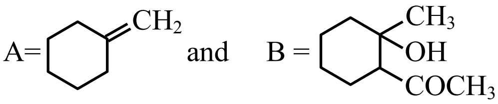
64. Identify the major products A and B respectively in the following set of reactions.



(1) 

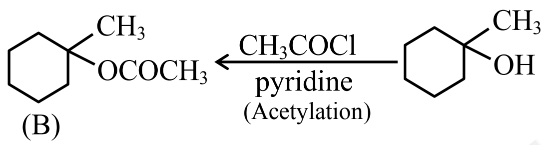
(2) 

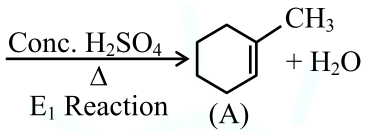
(3) 

(4) 

Ans. (1)

Sol.





65. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A : The stability order of +1 oxidation state of Ga, In and Tl is Ga < In < Tl.

Reason R : The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the *correct* answer from the options given below :

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(2) **A** is true but **R** is false.

(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(4) **A** is false but **R** is true.

Ans. (1)

Sol. The relative stability of +1 oxidation state progressively increases for heavier elements due to inert pair effect.

$\therefore$ Stability of $A\ell^{+1} < Ga^{+1} < In^{+1} < T\ell^{+1}$

66. Match **List I** with **List-II**

	List-I (Name of the test)		**List-II (Reaction sequence involved) [M is metal]**
A	Borax bead test	I.	$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)} CoO.\ MO$
B.	Charcoal cavity test	II.	$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$
C.	Cobalt nitrate test	III	$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$
D.	Flame test	IV	$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

Choose the **correct** answer from the option below :

(1) A-III, B-I, C-IV, D-II

(2) A-III, B-II, C-IV, D-I

(3) A-III, B-I, C-II, D-IV

(4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol. Cobalt nitrate test

$$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)} CoO.\ MO$$

Flame test

$$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$$

Borax Bead test

$$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$$

Charcoal cavity test

$$MSO_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$$

67. Match **List I** and with **List II**

List-I (Molecule)		List-II(Shape)	
A	NH_3	I.	Square pyramid
B.	BrF_5	II.	Tetrahedral
C.	PCl_5	III	Trigonal pyramidal
D.	CH_4	IV	Trigonal bipyramidal

Choose the **correct** answer from the option below :

(1) A-IV, B-III, C-I, D-II

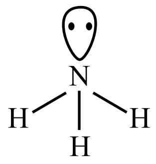
(2) A-II, B-IV, C-I, D-III

(3) A-III, B-I, C-IV, D-II

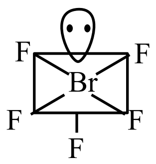
(4) A-III, B-IV, C-I, D-II

Ans. (3)

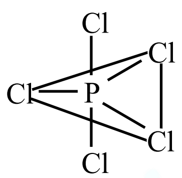
Sol.



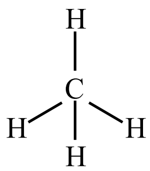
Trigonal pyramidal



Square pyramidal



Trigonal bipyramidal



Tetrahedral

68. For the given hypothetical reactions, the equilibrium constants are as follows:

$X \rightleftharpoons Y ; K_1 = 1.0$

$Y \rightleftharpoons Z ; K_2 = 2.0$

$Z \rightleftharpoons W ; K_3 = 4.0$

The equilibrium constant for the reaction

$X \rightleftharpoons W$ is

(1) 6.0 (2) 12.0

(3) 8.0 (4) 7.0

Ans. (3)

Sol.

$\rightleftharpoons Y$ $\quad k_1 = 1$

$\rightleftharpoons Z$ $\quad k_2 = 2$

$Z \rightleftharpoons \omega$ $\quad k_3 = 4$

$X \rightleftharpoons \omega$ $\quad k_1 \cdot k_2 \cdot k_3$

$k = 1 \times 2 \times 4$

$k = 8$

69. Thiosulphate reacts differently with iodine and bromine in the reaction given below :

$$2S_2O_3^{2-} + I_2 \rightarrow S_4O_6^{2-} + 2I^-$$

$$S_2O_3^{2-} + 5Br_2 + 5H_2O \rightarrow 2SO_4^{2-} + 4Br^- + 10H^+$$

Which of the following statement justifies the above dual behaviour of thiosulphate?

(1) Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions

(2) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reaction

(3) Bromine is a stronger oxidant than iodine

(4) Bromine is a weaker oxidant than iodine

Ans. (3)

Sol. In the reaction of $S_2O_3^{2-}$ with I_2, oxidation state of sulphur changes to +2 to +2.5

In the reaction of $S_2O_3^{2-}$ with Br_2, oxidation state of sulphur changes from +2 to +6.

$\therefore$ Both I_2 and Br_2 are oxidant (oxidising agent) and Br_2 is stronger oxidant than I_2.

70. An octahedral complex with the formula $CoCl_3nNH_3$ upon reaction with excess of $AgNO_3$ solution given 2 moles of AgCl. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is ______.

(1) 3 (2) 6

(3) 8 (4) 5

Ans. (3)

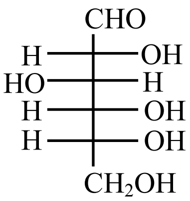
Sol. $[\overset{+3}{Co}(NH_3)_5Cl]Cl_2 + \text{excess } AgNO_3 \longrightarrow 2AgCl$ (2 moles)

$x + 0 - 1 - 2 = 0$

$x = +3$

$n = 5$

$\therefore\ x + n = 8$

71. 

The **incorrect** statement regarding the given structure is

(1) Can be oxidized to a dicarboxylic acid with Br_2 water

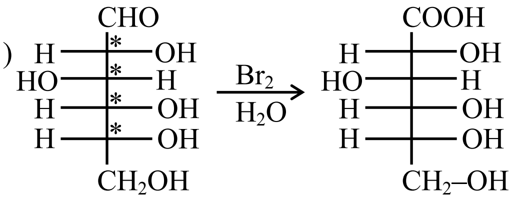
(2) despite the presence of – CHO does not give Schiff's test

(3) has 4-asymmetric carbon atom

(4) will coexist in equilibrium with 2 other cyclic structure

Ans. (1)

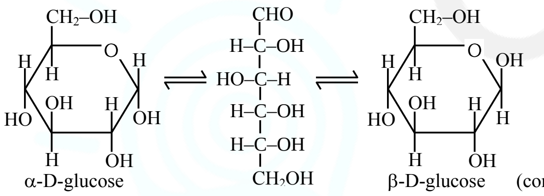
Sol.

(1) 

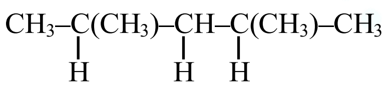
statement 1 is incorrect (monocarboxylic acid)

(2) correct

(3) c.c. is 4 (correct)

(4)  (correct)

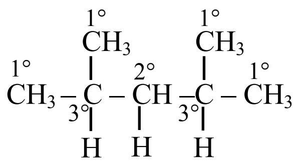
72. In the given compound, the number of 2° carbon atom/s is ______.



(1) Three (2) One

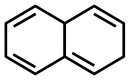
(3) Two (4) Four

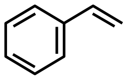
Ans. (2)

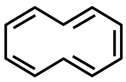
Sol. 

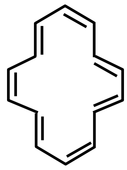
only one 2° carbon is present in this compound.

73. Which of the following are aromatic?

A. 

B. 

C. 

D. 

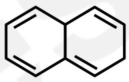
(1) B and D only

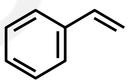
(2) A and C only

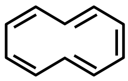
(3) A and B only

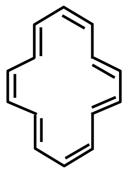
(4) C and D only

Ans. (1)

Sol. A.  ⟶ Non aromatic

B.  ⟶ Aromatic

C.  ⟶ Non aromatic

D.  ⟶ Aromatic

74. Among the following halogens

F_2, Cl_2, Br_2 and I_2

Which can undergo disproportionation reaction?

(1) Only I_2

(2) Cl_2, Br_2 and I_2

(3) F_2, Cl_2 and Br_2

(4) F_2 and Cl_2

Ans. (2)

Sol. F_2 do not disproportionate because fluorine do not exist in positive oxidation state however Cl_2, Br_2 & I_2 undergoes disproportionation.

75. Given below are two statements:

Statement I : $N(CH_3)_3$ and $P(CH_3)_3$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from same group, the nature of bonding of $N(CH_3)_3$ and $P(CH_3)_3$ is always same with transition metals.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is incorrect but Statement II is correct
(2) Both Statement I and Statement II are correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are incorrect

Ans. **(3)**

Sol. $N(CH_3)_3$ and $P(CH_3)_3$ both are Lewis base and acts as ligand, However, $P(CH_3)_3$ has a π-acceptor character.

76. Match **List I** with **List II**

List-I (Elements)		**List-II(Properties in their respective groups)**	
A	Cl,S	I.	Elements with highest electronegativity
B.	Ge, As	II.	Elements with largest atomic size
C.	Fr, Ra	III	Elements which show properties of both metals and non metal
D.	F, O	IV	Elements with highest negative electron gain enthalpy

Choose the **correct** answer from the options given below :

(1) A-II, B-III, C-IV, D-I
(2) A-III, B-II, C-I, D-IV
(3) A-IV, B-III, C-II, D-I
(4) A-II, B-I, C-IV, D-III

Ans. **(3)**

Sol. Elements with highest electronegativity $\rightarrow$ F, O

Elements with largest atomic size $\rightarrow$ Fr, Ra

Elements which shows properties of both metal and non-metals i.e. metalloids $\rightarrow$ Ge, As

Elements with highest negative electron gain enthalpy $\rightarrow$ Cl, S

77. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

A. Fe^{3+} oxidises the iodide ion
B. Fe^{3+} oxidises the persulphate ion
C. Fe^{2+} reduces the iodide ion
D. Fe^{2+} reduces the persulphate ion

Choose the **most appropriate** answer from the options given below:

(1) B and C only
(2) B only
(3) A only
(4) A and D only

Ans. **(4)**

Sol. $2Fe^{3+} + 2I^{-} \longrightarrow 2Fe^{2+} + I_2$

$2Fe^{2+} + S_2O_8^{2-} \longrightarrow 2Fe^{3+} + 2SO_4^{2-}$

Fe^{+3} oxidises I^- to I_2 and convert itself into Fe^{+2}. This Fe^{+2} reduces $S_2O_8^{2-}$ to SO_4^{2-} and converts itself into Fe^{+3}.

78. Match **List I** with **List II**

List-I (Compound)		**List-II (Colour)**	
A	$Fe_4[Fe(CN)_6]_3.xH_2O$	I.	Violet
B.	$[Fe(CN)_5NOS]^{4-}$	II.	Blood Red
C.	$[Fe(SCN)]^{2+}$	III.	Prussian Blue
D.	$(NH_4)_3PO_4.12MoO_3$	IV.	Yellow

Choose the **correct** answer from the options given below :

(1) A-III, B-I, C-II, D-IV
(2) A-IV, B-I, C-II, D-III
(3) A-II, B-III, C-IV, D-I
(4) A-I, B-II, C-III, D-IV

Ans. **(1)**

Sol. $Fe_4[Fe(CN)_6]_3 .xH_2O \rightarrow$ Prussian Blue

$[Fe(CN)_5NOS]^{4-} \rightarrow$ Violet

$[Fe(SCN)]^{2+} \rightarrow$ Blood Red

$(NH_4)_3PO_4.12MoO_3 \rightarrow$ Yellow

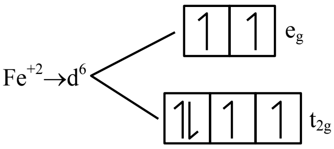
79. Number of complexes with even number of electrons in t_{2g} orbitals is -

$[Fe(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{2+}$, $[Co(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$, $[Cr(H_2O)_6]^{2+}$

(1) 1
(2) 3
(3) 2
(4) 5

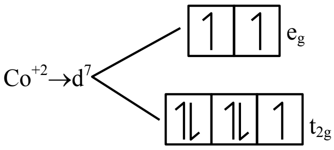
Ans. **(2)**

Sol. $[Fe(H_2O)_6]^{2+}$



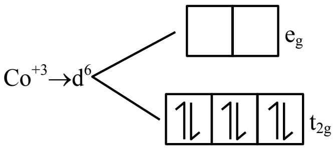
Electron in t_{2g} = 4(even)

$[Co(H_2O)_6]^{2+}$



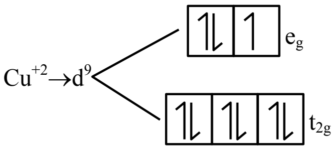
Electron in t_{2g} = 5(odd)

$[Co(H_2O)_6]^{3+}$



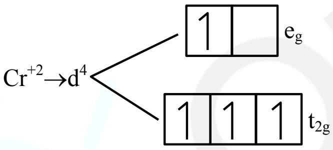
Electron in t_{2g} = 6(even)

$[Cu(H_2O)_6]^{2+}$



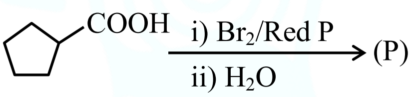
Electron in t_{2g} = 6(even)

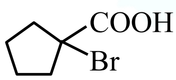
$[Cr(H_2O)_6]^{2+}$

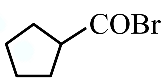


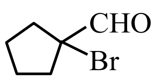
Electron in t_{2g} = 3(odd)

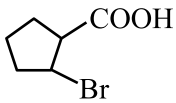
80. Identify the product (P) in the following reaction:



(1) 

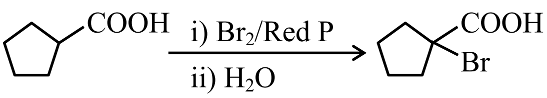
(2) 

(3) 

(4) 

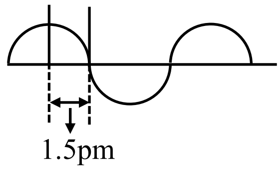
Ans. **(1)**

Sol. HVZ Reaction



SECTION-B

81. A hypothetical electromagnetic wave is show below.



The frequency of the wave is $x \times 10^{19}$ Hz.

x = ______ (nearest integer)

Ans. **(5)**

Sol. $\lambda = 1.5 \times 4$ pm

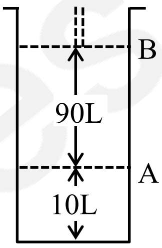
$= 6 \times 10^{-12}$ meter

$\lambda\nu = C$

$6 \times 10^{-12} \times \nu = 3 \times 10^8$

$\nu = 5 \times 10^{19}$ Hz

82.



Consider the figure provided.

1 mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at 18°C. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

x = ______ L atm. (nearest integer)

[Given : Absolute temperature = °C + 273.15, $R = 0.08206$ L atm mol^{-1} K^{-1}]

Ans. **(55)**

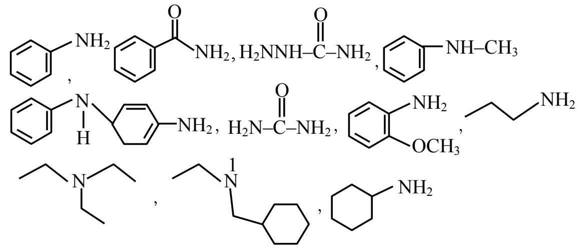
Sol. $\omega = -nRT \ln\left(\frac{V_2}{V_1}\right)$

$= -1 \times \cdot 08206 \times 291\cdot 15 \ln\left(\frac{100}{10}\right)$

$= -55.0128$

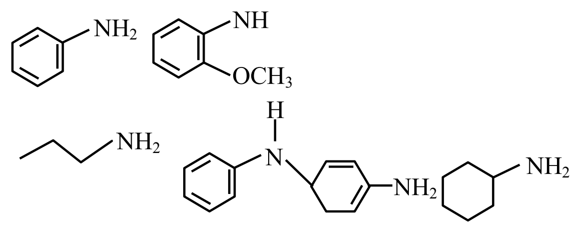
Work done by system ≈ 55 atm lit.

83. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is _____.

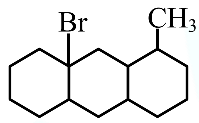


Ans. (5)

Sol. Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.

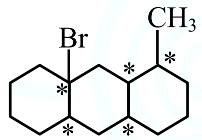


84. The number of optical isomers in following compound is : _____.



Ans. (32)

Sol.



Total chiral centre = 5

No. of optical isomers = $2^5 = 32$.

85. The 'spin only' magnetic moment value of MO_4^{2-} is ____ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn).
(Given atomic number : Sc = 21, Ti = 22, V = 23, Cr = 24, Mn = 25 and Zn = 30)

Ans. (0)

Sol. Metal having least metallic radii among Sc, Ti, V, Cr, Mn & Zn is Cr.

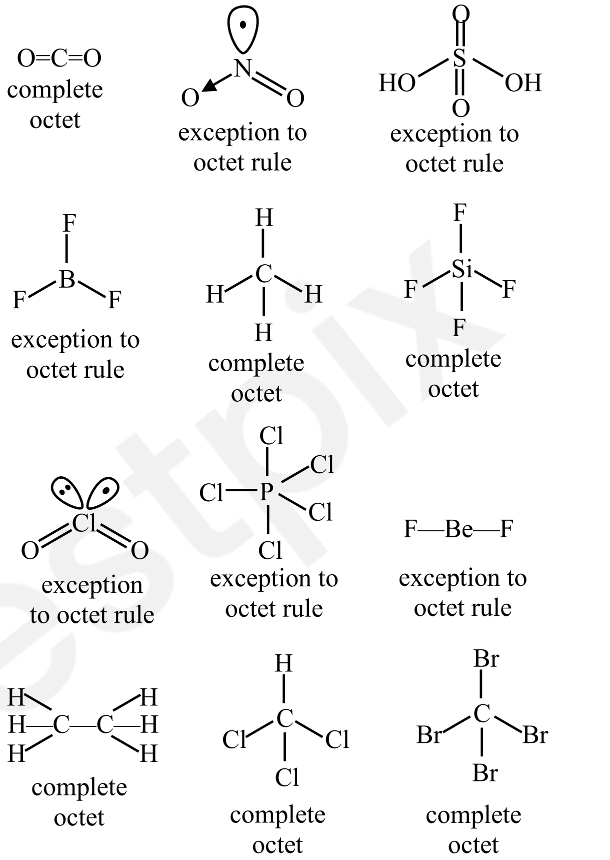
Spin only magnetic moment of CrO_4^{2-}.

Here Cr^{+6} is in d^0 configuration (diamagnetic).

86. Number of molecules from the following which are exceptions to octet rule is _____.
CO_2, NO_2, H_2SO_4, BF_3, CH_4, SiF_4, ClO_2, PCl_5, BeF_2, C_2H_6, $CHCl_3$, CBr_4

Ans. (6)

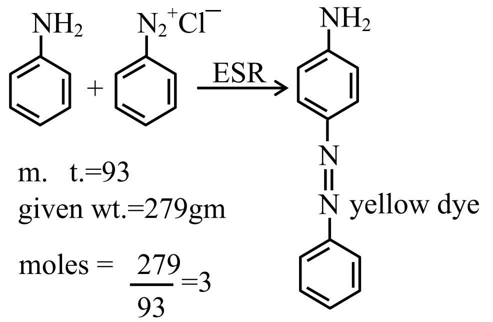
Sol.



87. If 279 g of aniline is reacted with one equivalent of benzenediazonium chloride, the mximum amount of aniline yellow formed will be _____ g. (nearest integer)
(consider complete conversion)

Ans. (591)

Sol.



m. t.=93
given wt.=279gm

moles = $\frac{279}{93} = 3$

moles formed =3
m. t. = 197
amount formed
$= 197 \times 3 = 591$ gm

88. Consider the following reaction

$A + B \rightarrow C$

The time taken for A to become $1/4^{th}$ of its initial concentration is twice the time taken to become 1/2 of the same. Also, when the change of concentration of B is plotted against time, the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

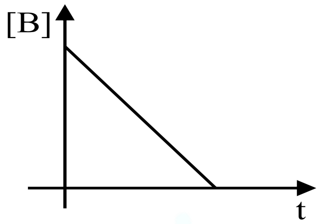
The overall order of the reaction is ____.

Ans. **(1)**

Sol. For 1^{st} order reaction

75% life = 2 × 50% life

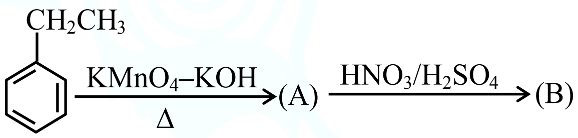
So order with respect to A will be first order.



So order with respect to B will be zero.

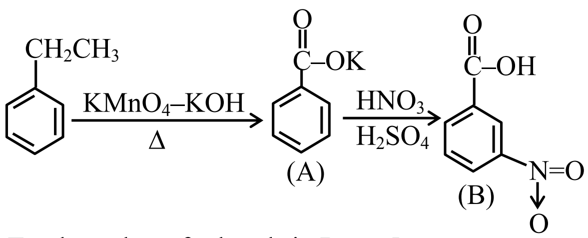
Overall order of reaction = 1 + 0 = 1

89. Major product B of the following reaction has ____ π-bond.



Ans. **(5)**

Sol. Major product B is →



Total number of π bonds in B are 5

90. A solution containing 10g of an electrolyte AB_2 in 100g of water boils at 100.52°C. The degree of ionization of the electrolyte (α) is _____ × 10^{-1}. (nearest integer)

[Given : Molar mass of AB_2 = 200g mol^{-1}. K_b (molal boiling point elevation const. of water) = 0.52 K kg mol^{-1}, boiling point of water = 100°C ; AB_2 ionises as $AB_2 \rightarrow A^{2+} + 2B^{-}$]

Ans. **(5)**

Sol. $AB_2 \rightarrow A^{+2} + 2B^{\ominus}$

$i = 1 + (3 - 1)\alpha$

$i = 1 + 2\alpha$

$\Delta T_b = k_b\, im$

$$0.52 = 0.52\,(1 + 2\alpha)\,\frac{\frac{10}{200}}{\frac{100}{1000}}$$

$$1 = (1 + 2\alpha)\,\frac{10}{20}$$

$2 = 1 + 2\alpha$

$\alpha = 0.5$

Ans. $\alpha = 5 \times 10^{-1}$