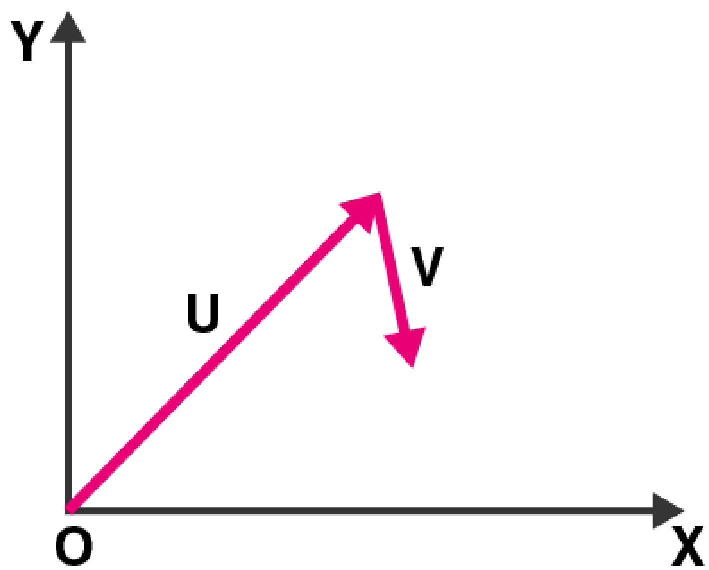
Multiple Choice Questions I

4.1. The angle between $A = \hat{i} + \hat{j}$ **and** $B = \hat{i} - \hat{j}$ **is**
a) 40º
b) 90º
c) -45º
d) 180º
Answer:
The correct answer is b) 90º

4.2. Which one of the following statements is true?
a) a scalar quantity is the one that is conserved in a process
b) a scalar quantity is the one that can never take negative values
c) a scalar quantity is the one that does not vary from one point to another in space
d) a scalar quantity has the same value for observers with different orientations of the axes
Answer:
The correct answer is d) a scalar quantity has the same value for observers with different orientations of the axes

4.3. Figure shows the orientation of two vectors u and v in the XY plane.



If $u = a\hat{i} + b\hat{j}$ **and** $v = p\hat{i} + q\hat{j}$ **which of the following is correct?**
a) a and p are positive while b and q are negative
b) a, p, and b are positive while q is negative
c) a, q, and b are positive while p is negative
d) a, b, p, and q are all positive
Answer:
The correct answer is b) a, p, and b are positive while q is negative

4.4. The component of a vector r along X-axis will have maximum value if
a) r is along positive Y-axis
b) r is along positive X-axis
c) r makes an angle of 45º with the X-axis
d) r is along negative Y-axis
Answer:
The correct answer is b) r is along positive X-axis

4.5. The horizontal range of a projectile fired at an angle of 15º is 50 m. If it is fired with the same speed at an angle of 45º, its range will be
a) 60 m
b) 71 m
c) 100 m
d) 141 m
Answer:
The correct answer is c) 100 m

4.6. Consider the quantities pressure, power, energy, impulse gravitational potential, electric charge, temperature, area. Out of these, the only vector quantities are
a) impulse, pressure, and area
b) impulse and area
c) area and gravitational potential
d) impulse and pressure
Answer:
The correct answer is b) impulse and area

4.7. In a two dimensional motion, instantaneous speed v0 is a positive constant. Then which of the following are necessarily true?
a) the average velocity is not zero at any time
b) average acceleration must always vanish
c) displacements in equal time intervals are equal
d) equal path lengths are traversed in equal intervals
Answer:
The correct answer is d) equal path lengths are traversed in equal intervals

4.8. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true?
a) the acceleration of the particle is zero
b) the acceleration of the particle is bounded
c) the acceleration of the particle is necessarily in the plane of motion
d) the particle must be undergoing a uniform circular motion
Answer:
The correct answer is d) the particle must be undergoing a uniform circular motion

4.9. Three vectors A, B, and C add up to zero. Find which is false
a) vector (A×B)C is not zero unless vectors B, C are parallel
d) vector (A×B).C is not zero unless vectors B, C are parallel
c) if vectors A, B, C define a plane, (A×B)C is in that plane

d) (A×B).C = $|A||B||C|$ **such that $C^2 = A^2 + B^2$**

Answer:

The correct answer is c) if vectors A, B, C define a plane, (A×B)C is in that plane and d) (A×B).C = $|A||B||C|$ such that $C^2 = A^2 + B^2$

4.10. It is found that $|A + B| = |A|$. This necessarily implies

a) B = 0

b) A, B are antiparallel

c) A, B are perpendicular

d) A.B ≤ 0

Answer:

The correct answer is a) B = 0

Multiple Choice Questions II

4.11. Two particles are projected in air with speed v_0, at angles θ_1 and θ_2 to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices

a) angle of project: $q_1 > q_2$

b) time of flight: $T_1 > T_2$

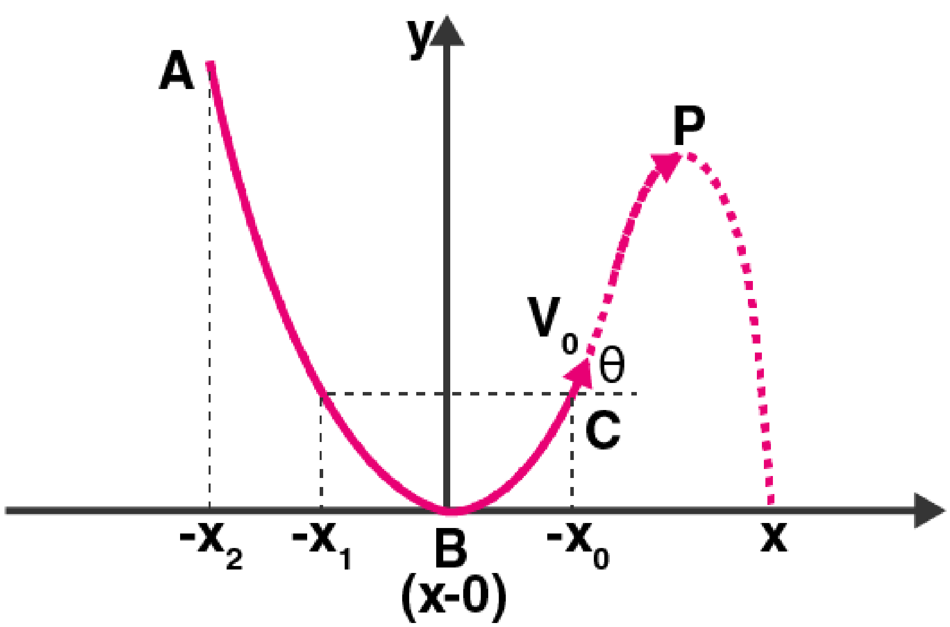
c) horizontal range: $R_1 > R_2$

d) total energy: $U_1 > U_2$

Answer:

The correct answer is a) angle of project: $q_1 > q_2$ and b) time of flight: $T_1 > T_2$

4.12. A particle slides down a frictionless parabolic track starting from rest at point A. Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then



a) KE at P = KE at B
b) height at P = height at A
c) total energy at P = total energy at A
d) time of travel from A to B = time of travel from B to P
Answer:
The correct answer is c) total energy at P = total energy at A

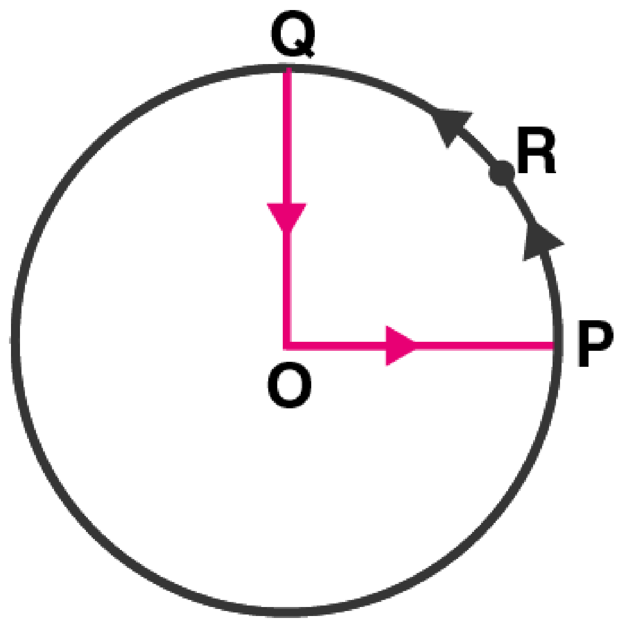
4.13. Following are four different relations about displacement, velocity, and acceleration for the motion of a particle in general. Choose the incorrect one (s):
a) $v_{av} = 1/2\ [v(t_1) + v(t_2)]$
b) $v_{av} = r(t_2)-r(t_1)/t_2-t_2$
c) $r = 1/2\ [v(t_2)-v(t_1)](t_2-t_1)$
d) $a_{av} = v(t_2)-v(t_1)/t_2-t_1$
Answer:
The correct answer is a) $v_{av} = 1/2\ [v(t_1) + v(t_2)]$ and c) $r = 1/2\ [v(t_2)-v(t_1)](t_2-t_1)$

4.14. For a particle performing uniform circular motion, choose the correct statement from the following:
a) magnitude of particle velocity (speed) remains constant
b) particle velocity remains directed perpendicular to radius vector
c) direction of acceleration keeps changing as particle moves
d) angular momentum is constant in magnitude but direction keep changing
Answer:
The correct answer is a) magnitude of particle velocity (speed) remains constant, b) particle velocity remains directed perpendicular to radius vector and c) direction of acceleration keeps changing as particle moves

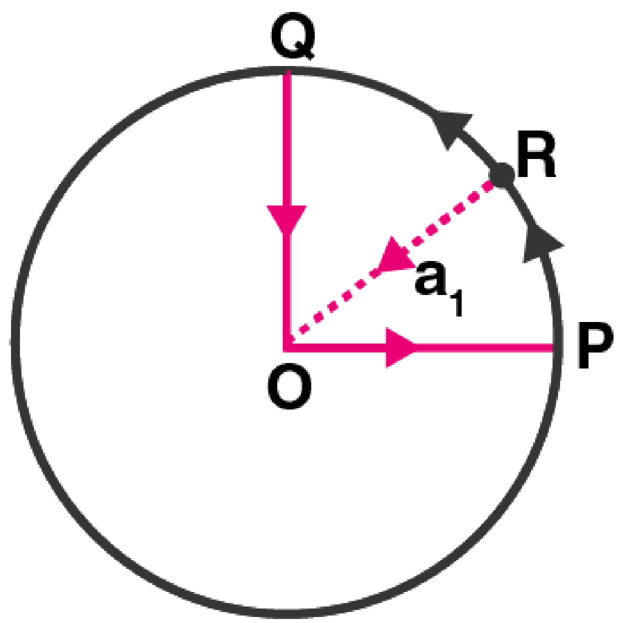
4.15. For two vectors A and B, $|A + B| = |A - B|$ **is always true when**
a) $|A| = |B| \neq 0$
b) $A \perp B$
c) $|A| = |B| \neq 0$ **and A and B are parallel or antiparallel**
d) when either $|A|\ or\ |B|$ **is zero**
Answer:
The correct answer is b) $A \perp B$ and d) when either $|A|\ or\ |B|$ is zero

Very Short Answers

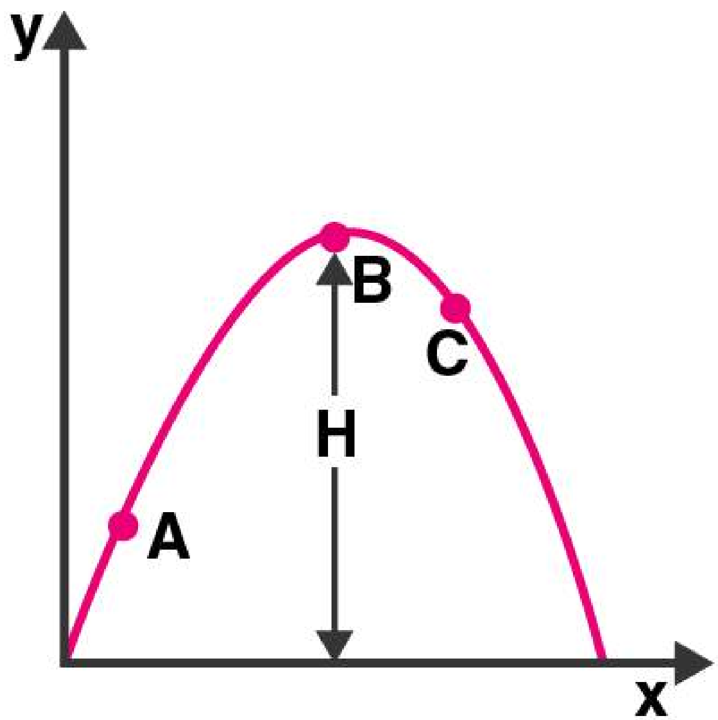
4.16. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown in the figure. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude and direction?



Answer:
The radius of the circular path is 1 km with O as the centre and at distance R.
Speed with which the cyclist is moving is 10 m/s.
Therefore, it can be said that the motion is uniform circular motion at R.
Therefore, R = 1000 m
v = 10 m/s
Therefore, acceleration, ac = $v^2/R = 0.1\ m/s^2$ along RO.

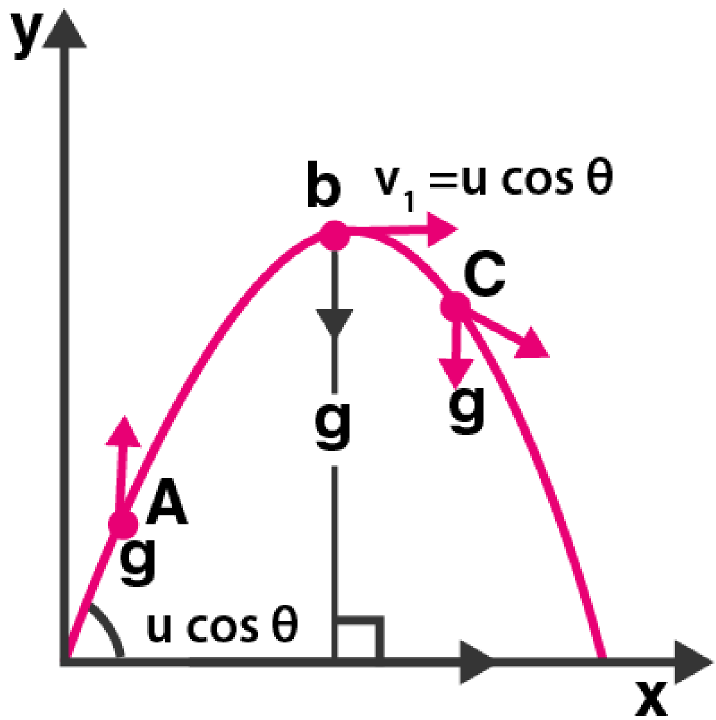


4.17. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in the figure, where x and y indicate horizontal and vertical directions respectively. Show in the diagram, direction of velocity and acceleration at points A, B, and C.

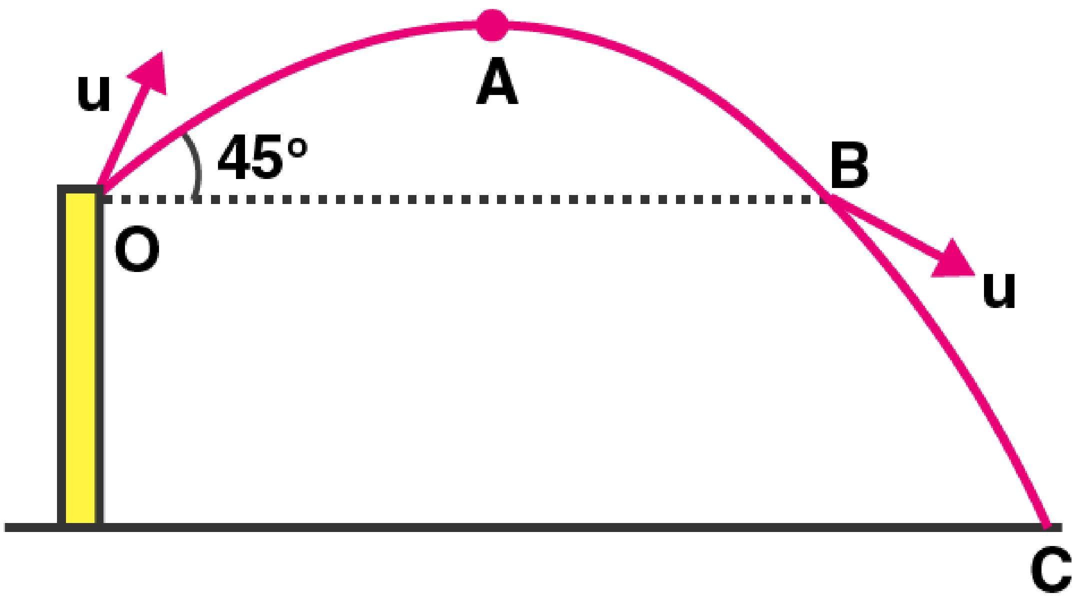


Answer:
The projectile motion is either parabolic or part of it. The velocity is always tangential to the direction of velocities of A, B and C. At point B, the maximum height is reached by the trajectory. Therefore, the vertical component $Bv_y = 0$ and horizontal component is u cos θ.

We know that the direction of acceleration is in the direction of force acting on the body. Therefore, here the direction of acceleration is in the direction of gravitational force and is always vertically downward which is equal to the acceleration due to gravity.



4.18. A ball is thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have
a) greatest speed
b) smallest speed
c) greatest acceleration?
Explain
Answer:
a) Let us assume that the ball is projected from the point O at an angle 45o. From O to A there is an increase in the height of the ball such that the KE of the ball decreases. The height from point A to B decreases making the speed increase and now it is equal to the initial speed when the ball was at point O. This is because O and B are in the same horizontal line. From point B to C, again the speed increases as the height has reduced.
Therefore, the greatest speed is at C as v_y maximum and $v_x = u/\sqrt{2}$.



b) The smallest speed will be at point A as the height at this point is the highest and $v_y = 0$ as the horizontal speed is the only constant value ie; $u/\sqrt{2}$

c) The greatest acceleration is acceleration due to gravity as only gravitational force is acting on the ball and in downward direction.

4.19. A football is kicked into the air vertically upwards. What is its
a) acceleration
b) velocity at the highest point
Answer:
a) The acceleration of the football will be downward when it is kicked upwards as only gravitational force is acting on it.

b) The football will have its highest velocity when $v_y = 0$.

4.20. A, B, and C are three non-collinear, non co-planar vectors. What can you say about direction of A(B×C)?
Answer:
The direction of product of vectors B and C is perpendicular to the plane that contains vector B and C which is based on the Right hand grip rule.
The direction of vector A(B×C) is perpendicular to vector A and is in the plane which contains vectors B and C and is again based on the Right hand grip rule.

Short Answers

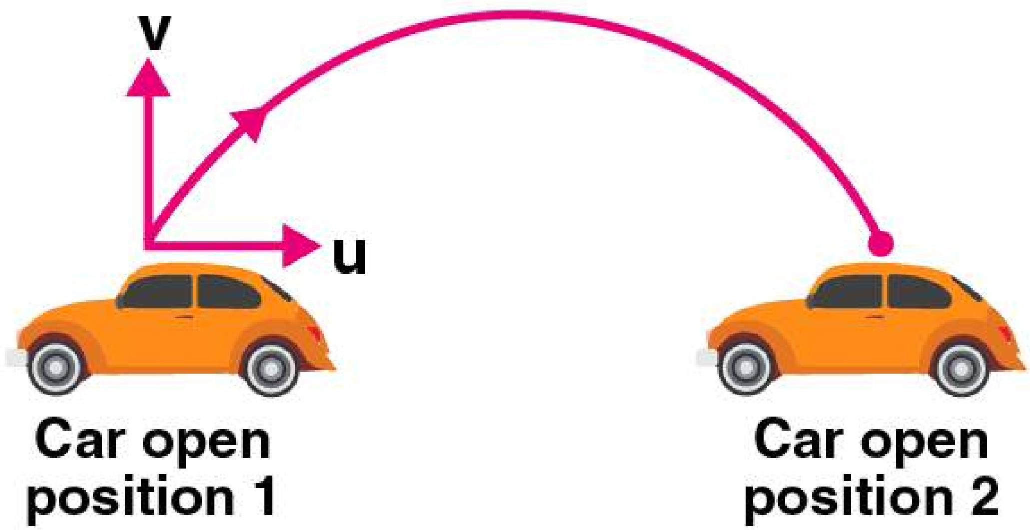
4.21. A boy travelling in an open car moving on a labelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.

Answer:

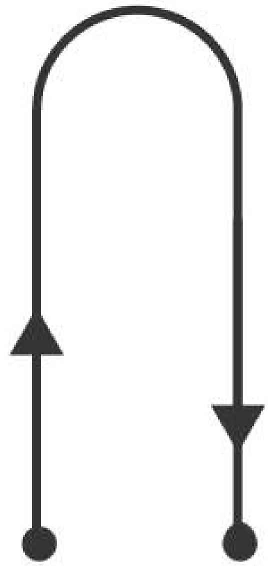
Given,

v = vertical velocity of the ball that the boy has

u = velocity of the car which is equal to the horizontal velocity of the ball



Above is the diagram which is viewed by the observer on the footpath. The diagram is parabolic as the vertical and horizontal components of the velocities are viewed by the observer.



Above is the diagram which is viewed by the boy sitting in the same car which is moving vertically up-down and is under the action of gravity. For car and boy to catch up, the car should move with constant velocity.

4.22. A boy throws a ball in air at 60º to the horizontal along a road with a speed of 10 m/s. Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the bot in the car, if car has a speed of 18 km/h. Give explanation to support your diagram.

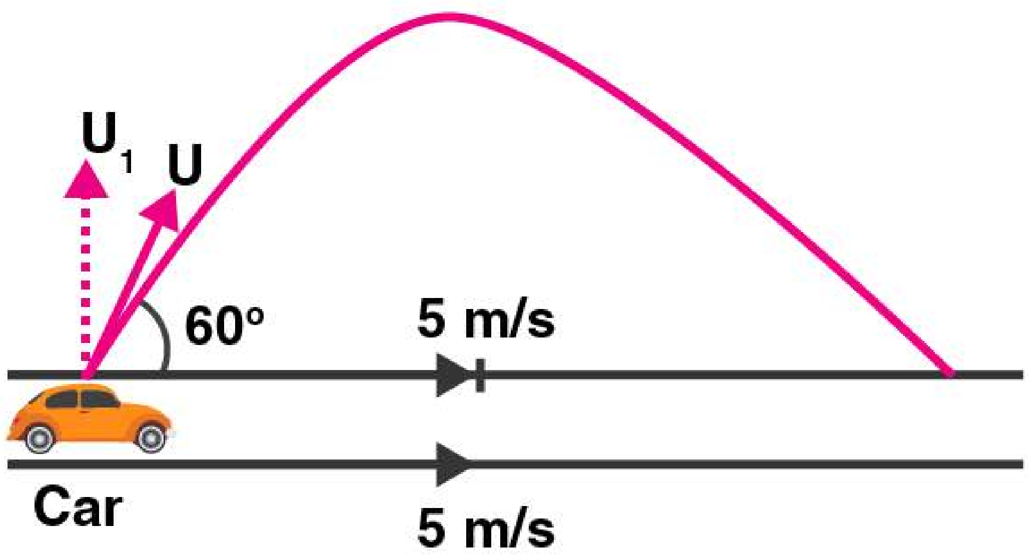
Answer:

Given,

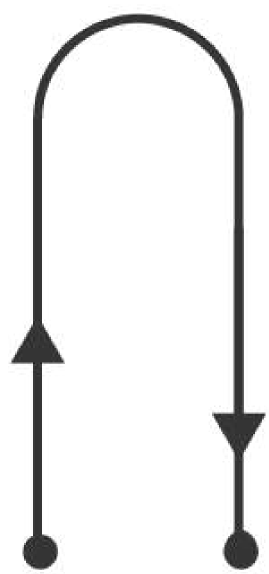
u = 36 km/h = 10 m/s

u_x = u cos 60º = 5 m/s

Speed of the car in the direction of motion of ball = (18)(5/18) = 5 m/s



Following is the diagram of the ball thrown by the boy when the car passes him. Both, car as well as ball are in the horizontal distances and speeds that are equal to each other. But the vertical component of velocity is
$u_y = u \cos 30^o = 5\sqrt{3}$ m/s.
The motion of the ball according to the boy sitting in the car is in vertically up-down motion.



4.23. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives

trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.

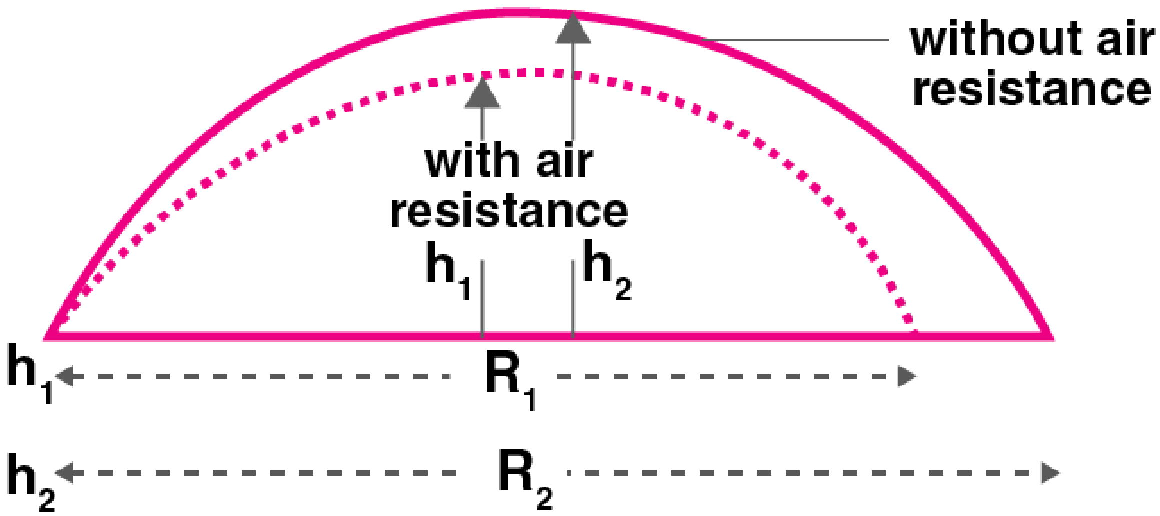
Answer:

When the air resistance acting on the projectile motion is considered, the vertical and horizontal velocity decreases due to air resistance. Following is the formula which is responsible for reducing the height of the motion and this is smaller than the height which exists when there is no force of friction:

$R = (u^2/g) \sin 2\theta$

$H_{max} = u^2 \sin^2 \theta/2g$

Following is the diagram which explains that the time of flight will remain constant with and without air resistance.



Therefore, $h_1 < h_2$ and $R_1 < R_2$.

4.24. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight when the target is seen, should the pilot drop the bomb in order to attack the target?

Answer:

Given,

u = 720 km/h = 200 m/s

Let t be the time at which pilot drops the bomb, then Q will be the point vertically above the target T.

We also know that the horizontal velocity of the bomb is equal to the velocity of the fighter plane. But the vertical component is zero. When the bomb reaches TQ, it is a free falling object with initial velocity equal to zero.

u = 0

$H = 1.5 \text{ km} = 10 \text{ m/s}^2$

$H = ut + 1/2\, gt^2$

Substituting the values, we get

$t = 10\sqrt{3}$ second.

Let the distance between PQ be ut.

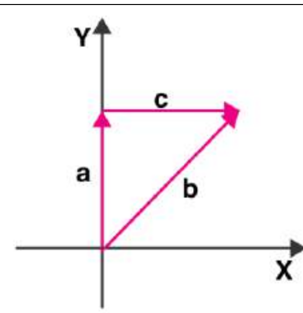
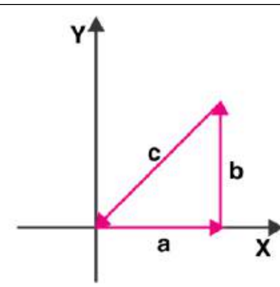
Therefore, PQ = $2000\sqrt{3}$ m
$\tan\theta = \sqrt{3}/4$
$\tan\theta = \tan^{-1}23^\circ42'$
$\theta = 23^\circ42'$

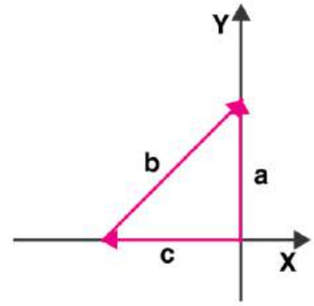
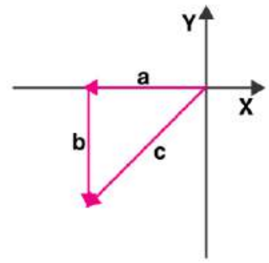
4.25. a) Earth can be thought of as a sphere of radius 6400 km. Any object is performing circular motion around the axis of earth due to earth's rotation. What is acceleration of object on the surface of the earth towards its centre? What is it at latitude θ? How does these accelerations compare with g = 9.8 m/s²?

b) Earth also moves in circular orbit around sun once every year with an orbital radius of 1.5 × 10¹¹m. What is the acceleration of earth towards the centre of the sun? How does this acceleration compare with g = 9.8 m/s²?
Answer:
a) From the problem,
Radius of the earth, R = 6400 km = 6.4×10^6 m
Time period, T = 1 day = 24 × 60 × 60 = 86400 s
Centripetal acceleration, $a_c = \omega^2 R = (4\pi^2 R)/T$
Substituting the vales, we get
$a_c = 0.034\ m/s^2$
At equator, latitude $\theta = 0^\circ$
We know that,
$a_c/g = 0.034/9.8 = 1/288$

b) Orbital radius of the earth around the sun, R = 1.5×10^{11} m
Time period = 1 year = 365 days = 3.15×10^7 s
Centripetal acceleration, $a_c = \omega^2 R = (4\pi^2 R)/T^2 = 5.97 \times 10^{-3}\ m/s^2$
$a_c/g = 5.97 \times 10^{-3}/9.8 = 1/1642$

4.26 Given below in column I are the relations between vectors a, b, and c and in column II are the orientations of a, b, and c in the XY plane. Match the relation in column I to correct orientations in column II.

Column I	**Column II**
a) a + b = c	 **i)**
b) a – c = b	 **ii)**

c) b – a = c	 **iii)**
d) a + b + c = 0	 **iv)**

Answer:
a) matches with iv)
b) matches with iii)
c) matches with i)
d) matches with ii)

4.27. If $|A| = 2$ **and** $|B| = 4$ **, then match the relations in column I with the angle between θ between A and B in column II.**

Column I	**Column II**
a) A.B = 0	**i) θ = 0**
b) A.B = +8	**ii) θ = 90°**
c) A.B = 4	**iii) θ = 180°**
d) A.B = -8	**iv) θ = 0°**

Answer:
a) matches with ii)
b) matches with i)
c) matches with iv)
d) matches with iii)

4.28. If $|A| = 2$ **and** $|B| = 4$ **, then match the relations in column I with the angle θ between A and B in column II.**

Column I	**Column II**
a) [latex]\left | A \times B\right |=0[/latex]	**i) θ = 30°**
b) [latex]\left | A \times B\right |=8[/latex]	**ii) θ = 45°**
c) [latex]\left | A \times B\right |=4[/latex]	**iii) θ = 90°**
d) [latex]\left | A \times B\right |=4\sqrt{2}[/latex]	**iv) θ = 0°**

Answer:
a) matches with iv)
b) matches with iii)
c) matches with i)
d) matches with ii)

Long Answers

4.29. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take g = 10 m/s^2.

Answer:
Given,
Speed of packets = 125 m/s
Height of the hill = 500 m
Distance between the canon and the foot of the hill, d = 800 m
The vertical component of the velocity should be minimum so that the time taken to cross the hill will be the shortest.
$u_y = \sqrt{2gh} \geq \sqrt{(2)(10)(500)} \geq 100$ m/s
But,

$$u^2 = u_x^2 + u_y^2$$

Therefore, horizontal component of the initial velocity,

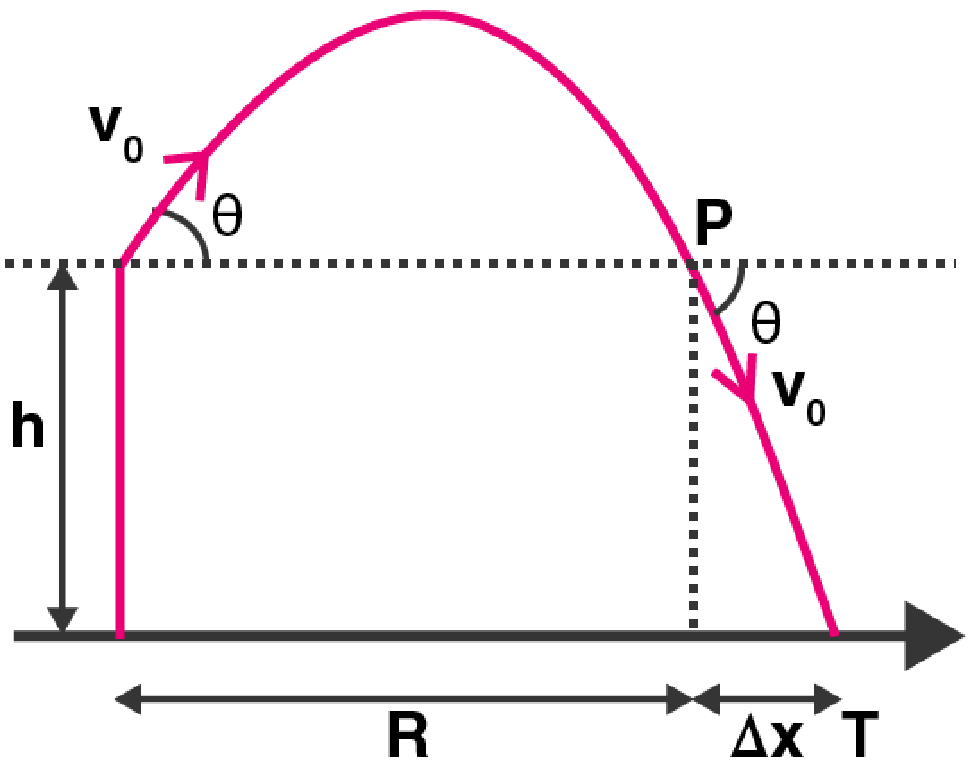
$$u_x = \sqrt{u^2 - u_y^2}$$

u_x = 75 m/s
Time taken by the packet to reach the top of the hill,

$$t = \sqrt{\frac{2h}{g}}$$

t = 10s
Time taken to reach the ground from the top of the hill = t' = t = 10s
Horizontal distance covered in 10s = (75)(10) = 750 m
Therefore, time taken by canon = 50/2 = 25 s
Therefore, total time taken by a packet to reach the ground = 25 + 10 + 10 = 45 s.

4.30. A gun can fire shells with maximum speed v0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$ **. If a target farther away by distance Δx has to be hit with the same gun, show that it could be achieved by raising the gun to a height at least h = Δx[1+Δx/R]**

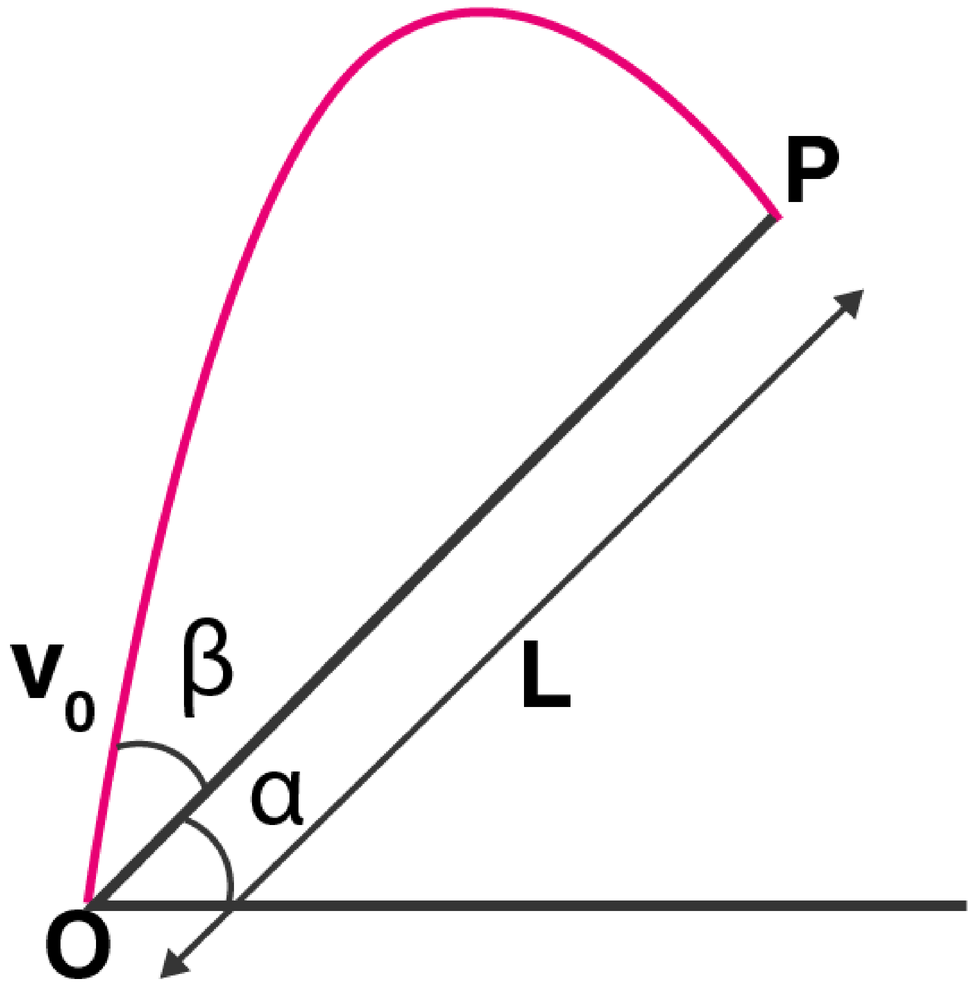


Answer:

$$R = \frac{v_0^2}{g}$$

Which is the maximum range of projectile
Therefore, the angle of projection, $\theta = 45^o$
The gun is placed at a height h so that it can hit the target
The vertical downward direction is taken as negative
Horizontal component of initial velocity = $v_o \cos \theta$
Vertical component of initial velocity = $v_0 \sin \theta$
Displacement along y-axis, $-h = (v_o \sin \theta)t + 1/2\ (-g)t^2$
Displacement along x-axis, $t = (R + \Delta x)/v_o \cos \theta$
Substituting all the equations, we get
$h = \Delta x\ (1 + \Delta x/R)$

4.31. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal.
a) find an expression of range on the plane surface
b) time of flight
c) β at which range will be maximum
Answer:



a) Time of flight, T =

$$\frac{2u\ sin(\alpha+\beta)}{g\ cos\ \beta}$$

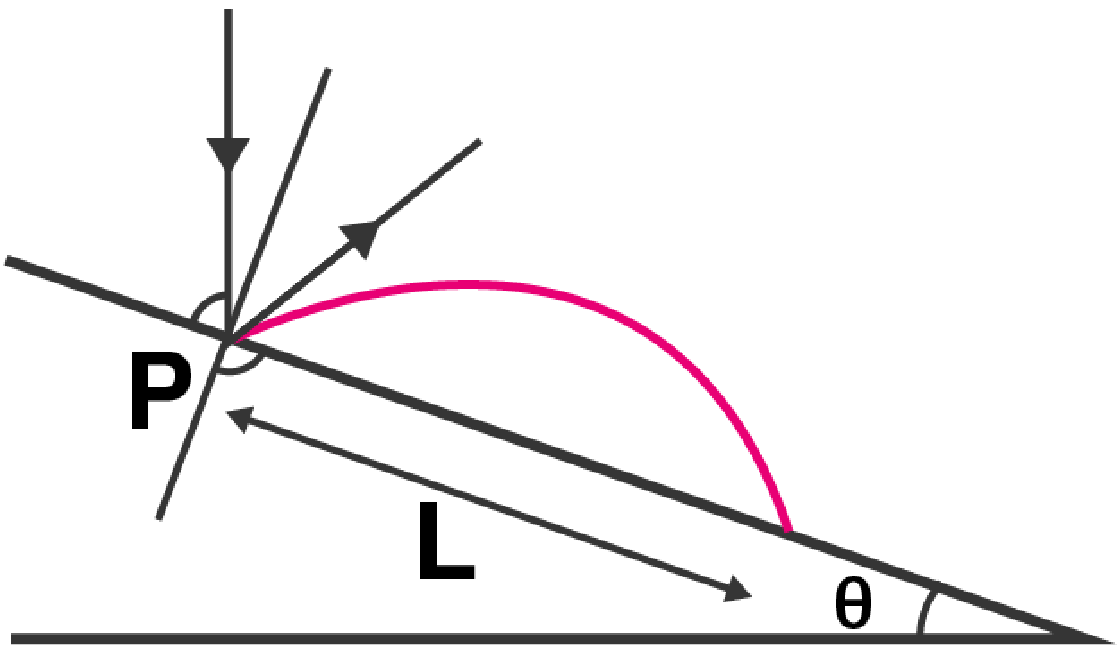
b) Range down an inclined plane, R =

$$\frac{2u^2 sin(\alpha+\beta) cos\ \alpha}{g\ cos^2\beta}$$

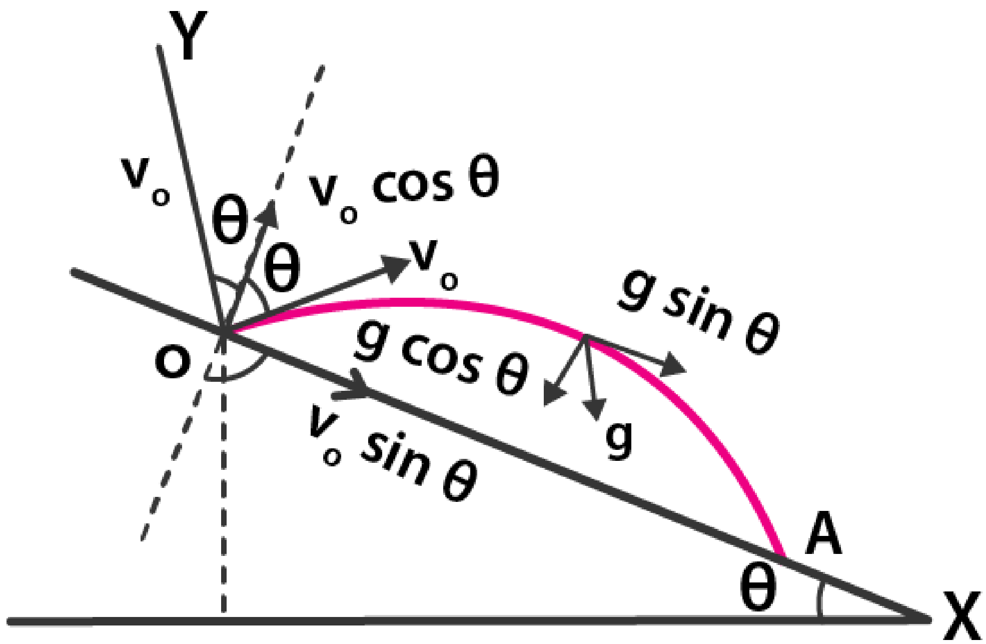
c) Maximum grange down an inclined plane, Rmax =

$$\frac{u^2}{g(1 - sin\ \beta)}$$

4.32. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v0 and rebounds elastically. Find the distance along the plane where it will hit second time.



Answer:



When x and y axes are selected as shown in the diagram, the motion of projectile from O to A is
$y = 0$
$u_y = v_o \cos\theta$
$a_y = -g \cos\theta$
$t = T$
With the help of kinematics for y-axis
$y = u_y t + 1/2\, a_y t^2$
$T = 2v_o \cos\theta / g \cos\theta$
With the help of kinematics for x-axis
$x = u_x t + 1/2\, a_x t^2$

$$L = \frac{4v_0^2}{g} sin\theta$$

4.33. A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?
Answer:
Let V_{rg} be the velocity of the rain drop that appears to the girl.
All the vectors are drawn in reference to the frame from the ground.

Let the velocity of rain be:

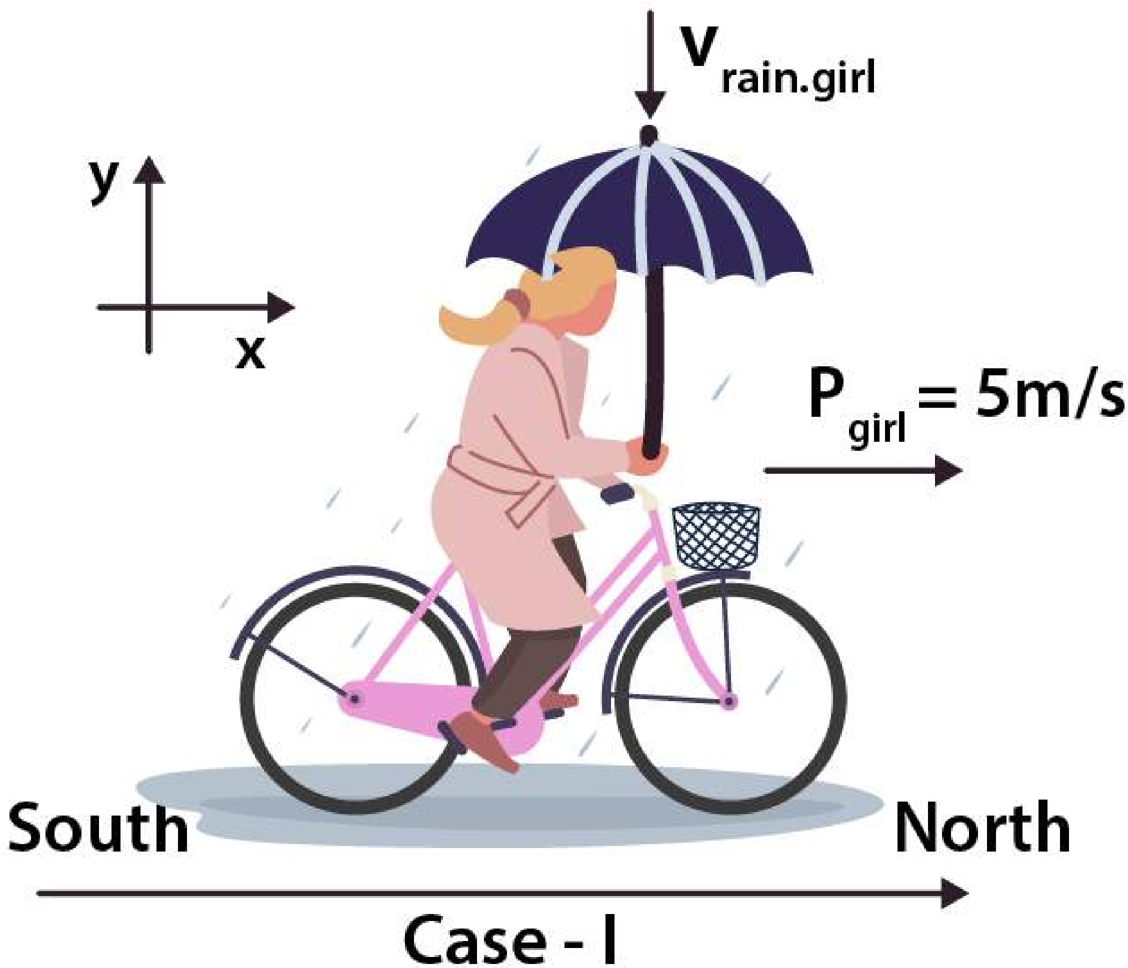
$$v_r = a\hat{i} + b\hat{j}$$

Case I

According to the problem, the velocity of girl = v_g
Let v_{rg} be the velocity of rain with respect to the girl
The equation which is used for determining a is:

$$v_r - v_g = (a\hat{i} + b\hat{j}) - 5\hat{i} = (a - 5)\hat{i} + b\hat{j}$$

Therefore, a = 5



Case II

Now the velocity of the girl has increased = v_g
Following is the equation that is used for determining the value of b at tan 45°.

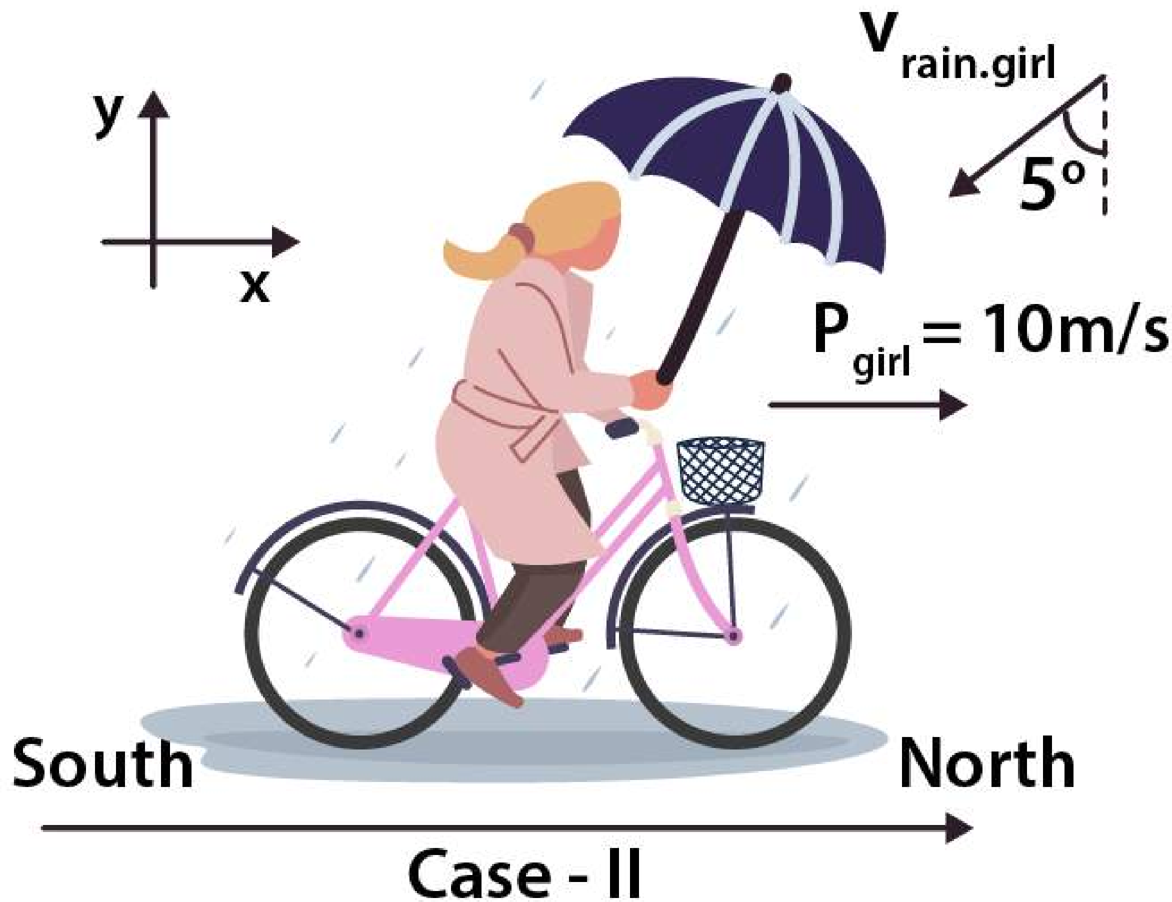
$$v_r - v_g = (a\hat{i} + b\hat{j}) - 10\hat{i} = (a - 10)\hat{i} + b\hat{j}$$

Therefore, the velocity of rain is:

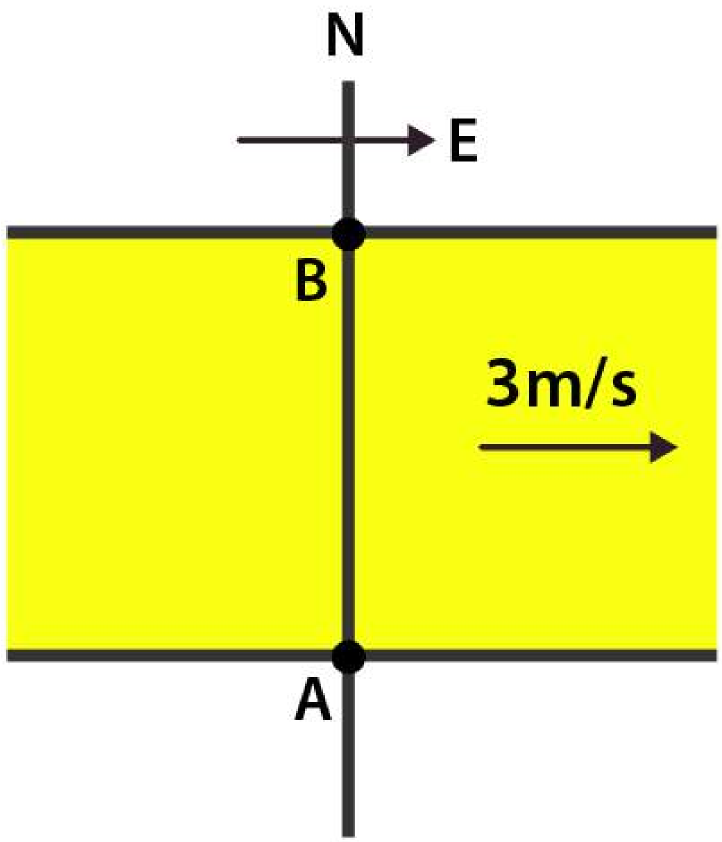
$$v_r = 5\hat{i} - 5\hat{j}$$

Speed of rain is:

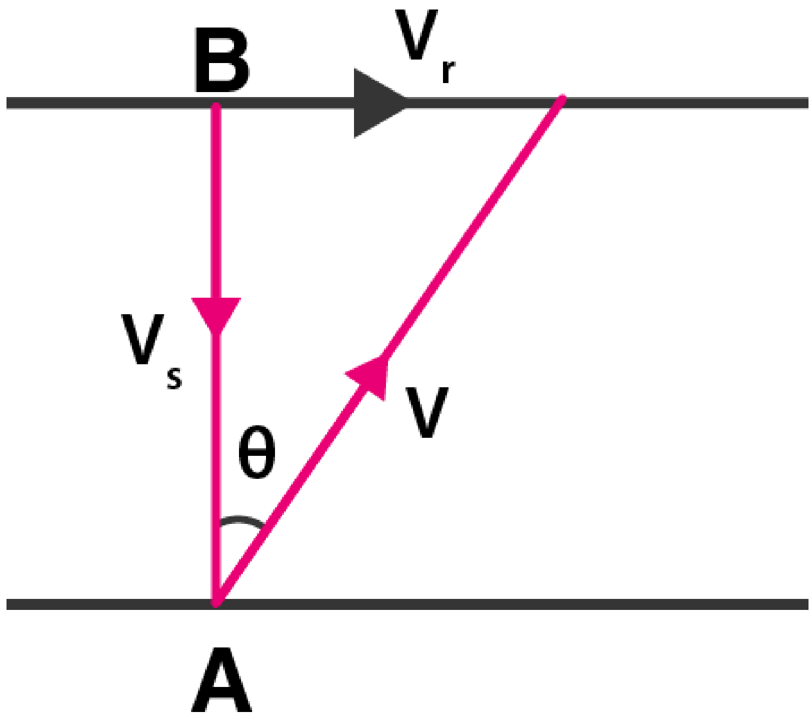
$$|v_r| = \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}\ m/s$$



4.34. A river is flowing due east with a speed 3 m/s. A swimmer can swim in still water at a speed of 4 m/s.
a) if swimmer starts swimming due north, what will be his resultant velocity?
b) if he wants to start from point A on south bank and reach opposite point B on north bank,
i) which direction should he swim?
ii) what will be his resultant speed?
c) from two different cases as mentioned in a) and b) above, in which case will he reach opposite bank in shorter time?



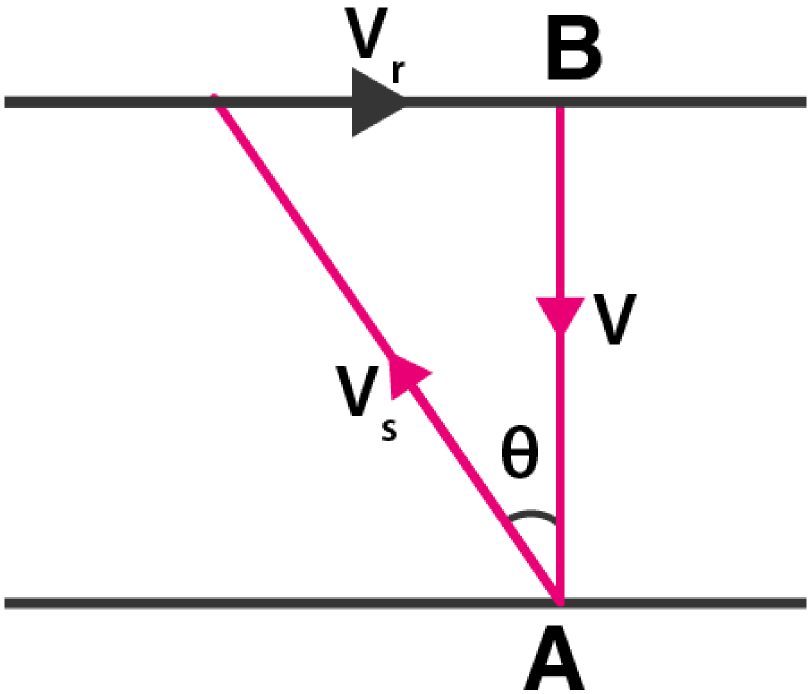
Answer:
Given,
Speed of the river, $v_r = 3$ m/s
Velocity of swimmer, $v_s = 4$ m/s
a) When the swimmer swims due north, the y-component has the velocity 4 m/s and x-component will have 3 m/s.
following is the resultant velocity with respect to tan θ



$$v = \sqrt{v_r^2 + v_s^2} = \sqrt{(3)^2 + (4)^2} = 5\ m/s$$

θ = 37ºN

b) Following is the resultant speed of the swimmer when he wants to reach the point which is directly opposite ti him and which is with respect to tan θ

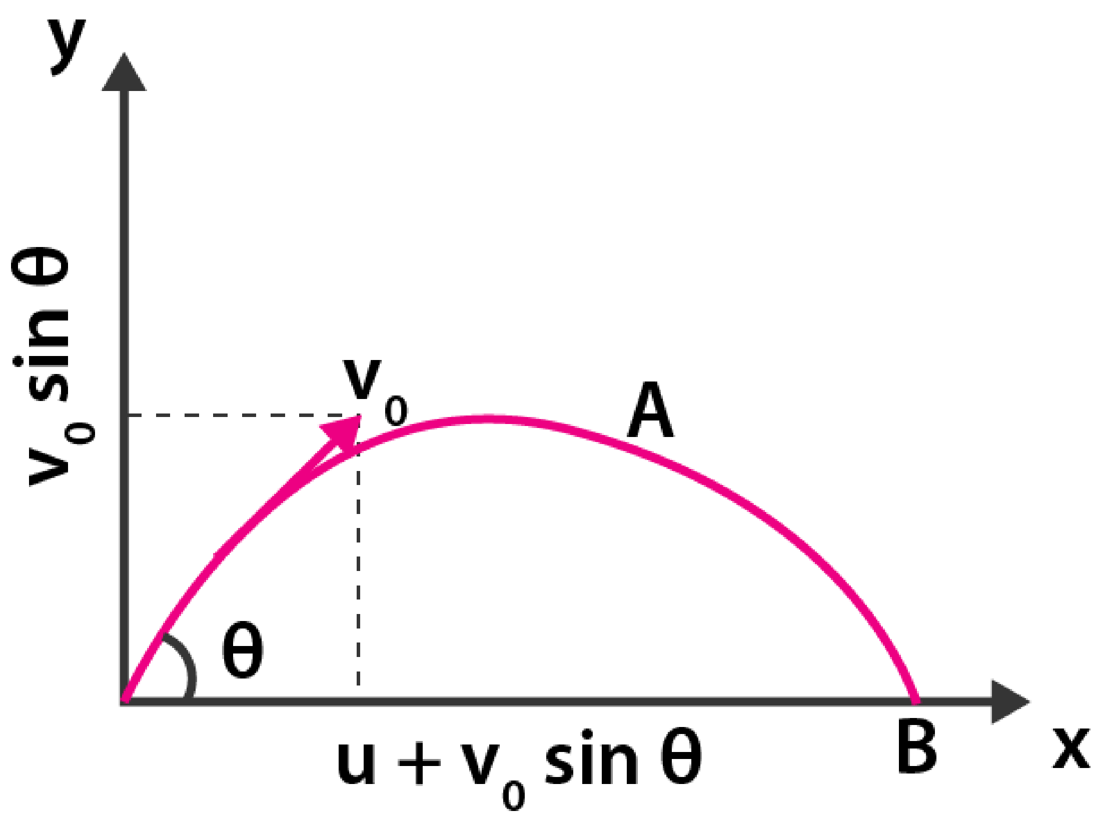


$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{(4)^2 - (3)^2} = \sqrt{7}\ m/s$$

$\theta = \tan^{-1}(3/\sqrt{7})$ of north

c) Time taken in case a) is d/4 sec
Time taken in case b) is $d/\sqrt{7}$
As $d/4 < d/\sqrt{7}$, time taken in case a) is shorter than case b).

4.35. A cricket fielder can throw the cricket ball with a speed vo. If he throws the ball while running with speed u at an angle θ to the horizontal, find
a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator
b) what will be time of flight?
c) what is the distance from the point of projection at which the ball will land?
d) find θ at which he should throw the ball that would maximise the horizontal range as found c)
e) how does θ for maximum range change if u > vo, u = vo and u < vo
f) how does θ in e) compare with that for u = 0?
Answer:
Initial velocity in x-direction,
$u_x = u + v_o \cos\theta$
Initial velocity in y-direction,
$u_y = v_o \sin\theta$
where θ is the angle of projection



a) The angle of projection with the horizontal with respect to spectator is:
$\tan\theta = u_y/u_x$

$$\theta = tan^{-1}(\frac{v_0 sin\theta}{u + v_0 cos\theta})$$

b) When the displacement along the y-axis is zero over the time period T
$y = 0$
$u_y = v_o \sin\theta$
$a_y = -g$
$t = T$
We know that,
$y = u_yT + 1/2\ a_yt^2$
Solving for T, we get
$T = 2\ u_o \sin\theta/g$

c) Horizontal range is

$$R = \frac{v_0}{g}[2u\ sin\theta + v_0\ sin2\theta]$$

d) For horizontal range, $dR/d\theta = 0$
Then θmax is:

$$\theta_{max} = cos^{-1}(\frac{-u + \sqrt{u^2} + 8v_0^2}{4v_0})$$

e) If $u = v_o$,
$\theta max = \pi/2$

f) If $u = 0$, $\theta max = 45^o$
Then, $u = 0$ and $\theta \geq 45^o$

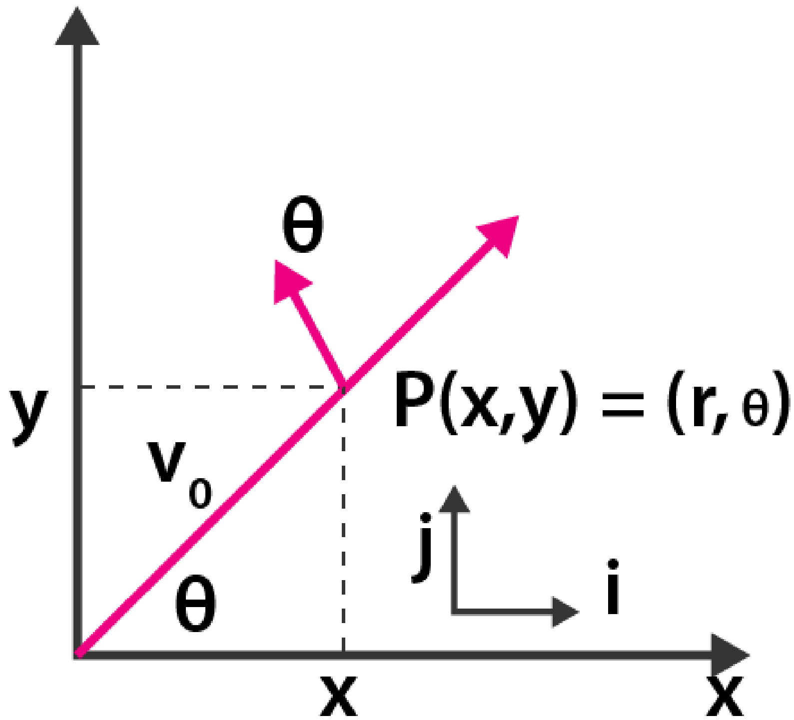
4.36. Motion in two dimensions, in a plane can be studied by expressing position, velocity, and acceleration as vector in Cartesian coordinates $A = A_x\hat{i} + A_y\hat{j}$ **where** $\hat{i}$ **and** $\hat{j}$ **are unit vectors along x and y directions, respectively and Ax and Ay are corresponding components of A. Motion can also be studied by expressing vectors in circular polar coordinates as** $A = A_r\hat{r} + A_\theta\hat{\theta}$ **where** $\hat{r} = \frac{r}{r} = cos\theta\hat{i} + sin\theta\hat{j}\ and\ \hat{\theta} = -sin\theta\hat{i} + cos\theta\hat{j}$ **are unit vectors along direction in which r and θ are increasing.**

a) express $\hat{i}$ **and** $\hat{j}$ **in terms of** $\hat{r}$ **and** $\hat{\theta}$

b) show that both $\hat{r}$ **and** $\hat{\theta}$ **are unit vectors and are perpendicular to each other**

c) show that $\frac{d}{dt}(\hat{r}) = \omega\hat{\theta}\ where\ \omega = \frac{d\theta}{dt}\ and\ \frac{d}{dt}(\hat{\theta}) = -\omega\hat{r}$

d) for a particle moving along a spiral given by $r = a\theta\hat{r}$ **where a = 1 find dimensions of 'a'**
e) find velocity and acceleration in polar vector representation for particle moving along spiral described in d) above



Answer:
a) The unit vector is given as:

$$\hat{r} = cos\theta\hat{i} + sin\theta\hat{j} \ and \ \hat{\theta} = -sin\theta\hat{i} + cos\theta\hat{j}$$

Solving the above equations we get unit vector as:

$$\hat{i} = n(\hat{r}cos\theta - \hat{\theta}sin\theta)$$

b)

$$\hat{r}.\hat{i} = (cos\theta\hat{i} + sin\theta\hat{j}).(-sin\theta\hat{i} + cos\theta\hat{j})$$

$$\theta = 90^\circ, angle \ between \ \hat{r} \ and \ \hat{\theta}$$

c) Given,

$$\hat{r} = cos\theta\hat{i} + sin\theta\hat{j}$$

Solving the above gives:

$$\hat{r} = \omega[-sin\theta\hat{i} + cos\theta\hat{j}]$$

d) Given,

$$r = a\theta\hat{r}$$

[a] = L = $[M^0L^1T^0]$

e) Given,
a = 1 unit

$$r = \theta\hat{r} = \theta[cos\theta\hat{i} + sin\theta\hat{j}]$$

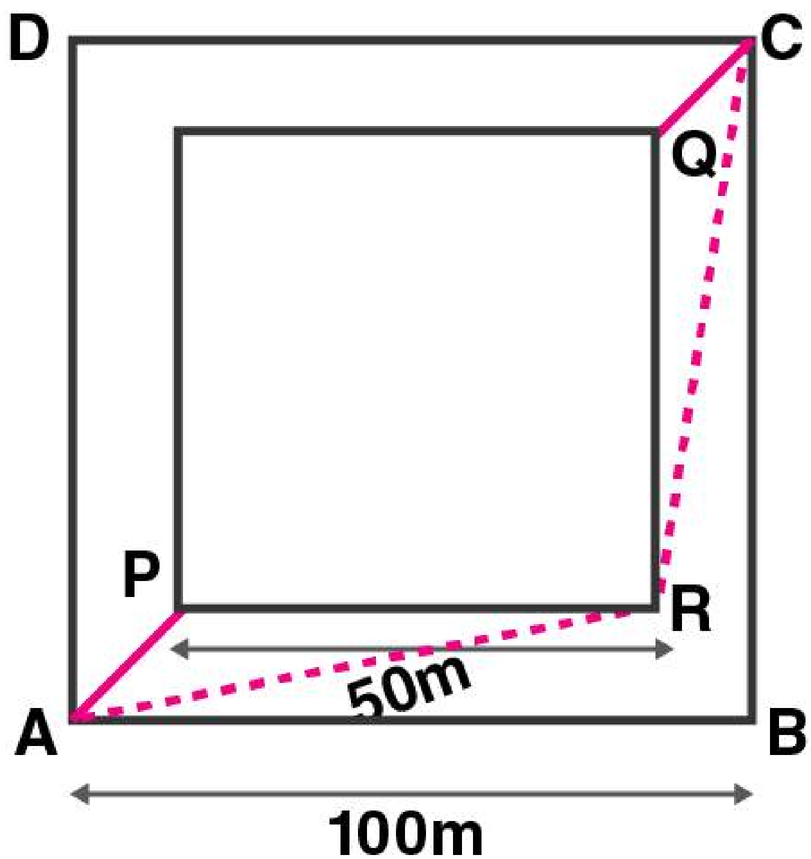
Differentiating the above equation, we get velocity as:

$$v = \frac{d\theta}{dt}\hat{r} + \theta\hat{\theta} = \omega\hat{r} + \omega\theta\hat{r}$$

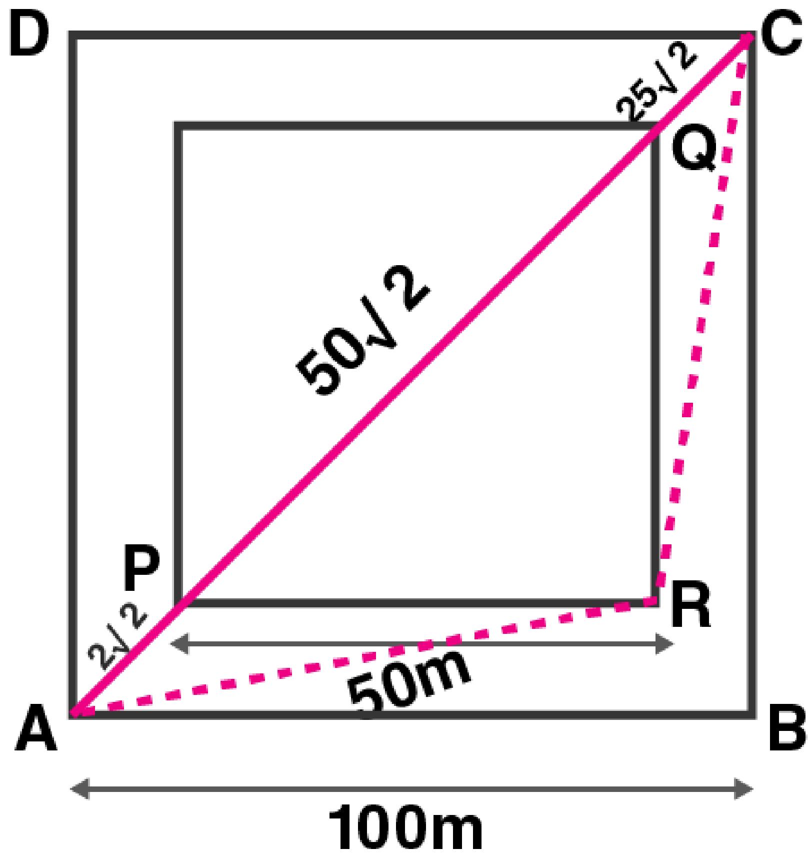
By differentiating the above equation we can find acceleration as:

$$a = (\frac{d^2\theta}{dtv^2} - \omega^2)\hat{r} + (2\omega^2 + \frac{d^2\theta}{dt^2}\theta)\theta$$

4.37. A man wants to reach from A to the opposite comer of the square C. The sides of the square are 100 m. A central square of 50 m × 50 m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he walk only at a speed of v m/s. What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?



Answer:



As shown in the diagram, APQC is the path taken by the man through the sand, time taken by him to go from A to C,

$$T_{sand} = \frac{AP + QC}{1} + \frac{PQ}{v}$$

= 50√2(1/v + 1)

The shortest path will be ARC from the diagram.

$T_{outside}$ = AR+RC/1 s

$$AR = \sqrt{75^2 + 25^2}$$

RC = AR = 25√10 s

$T_{sand} < T_{outside}$

V > 0.81 m/s