

SHORT ANSWER TYPE

1. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$. Determine

(i) $A \times B$

(ii) $B \times A$

(iii) $B \times B$

(iv) $A \times A$

Solution:

According to the question,

$A = \{-1, 2, 3\}$ and $B = \{1, 3\}$

(i) $A \times B$

$\{-1, 2, 3\} \times \{1, 3\}$

So, $A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

Hence, the Cartesian product = $\{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

(ii) $B \times A$.

$\{1, 3\} \times \{-1, 2, 3\}$

So, $B \times A = \{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$

Hence, the Cartesian product = $\{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$

(iii) $B \times B$

$\{1, 3\} \times \{1, 3\}$

So, $B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

Hence, the Cartesian product = $\{(1, 1), (1, 3), (3, 1), (3, 3)\}$

(iv) $A \times A$

$\{-1, 2, 3\} \times \{-1, 2, 3\}$

So, $A \times A = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

Hence,

the Cartesian product = $\{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

2. If $P = \{x : x < 3, x \in \mathbb{N}\}$, $Q = \{x : x \leq 2, x \in \mathbb{W}\}$. Find $(P \cup Q) \times (P \cap Q)$, where \mathbb{W} is the set of whole numbers.

Solution:

According to the question,

$P = \{x : x < 3, x \in \mathbb{N}\}$, $Q = \{x : x \leq 2, x \in \mathbb{W}\}$ where \mathbb{W} is the set of whole numbers

$P = \{1, 2\}$

$Q = \{0, 1, 2\}$

Now

$(P \cup Q) = \{1, 2\} \cup \{0, 1, 2\} = \{0, 1, 2\}$

And,

$(P \cap Q) = \{1, 2\} \cap \{0, 1, 2\} = \{1, 2\}$

We need to find the Cartesian product of $(P \cup Q) = \{0, 1, 2\}$ and $(P \cap Q) = \{1, 2\}$

So,

$(P \cup Q) \times (P \cap Q) = \{0, 1, 2\} \times \{1, 2\}$

$= \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

Hence, the Cartesian product = $\{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}$

3. If $A = \{x : x \in W, x < 2\}$, $B = \{x : x \in N, 1 < x < 5\}$, $C = \{3, 5\}$ find

(i) $A \times (B \cap C)$

(ii) $A \times (B \cup C)$

Solution:

According to the question,

$A = \{x : x \in W, x < 2\}$, $B = \{x : x \in N, 1 < x < 5\}$ $C = \{3, 5\}$; W is the set of whole numbers

$A = \{x : x \in W, x < 2\} = \{0, 1\}$

$B = \{x : x \in N, 1 < x < 5\} = \{2, 3, 4\}$

(i)

$(B \cap C) = \{2, 3, 4\} \cap \{3, 5\}$

$(B \cap C) = \{3\}$

$A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0, 3), (1, 3)\}$

Hence, the Cartesian product = $\{(0, 3), (1, 3)\}$

(ii)

$(B \cup C) = \{2, 3, 4\} \cup \{3, 5\}$

$(B \cup C) = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\} = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

Hence, the Cartesian product = $\{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

4. In each of the following cases, find a and b .

(i) $(2a + b, a - b) = (8, 3)$

(ii) $(a/4, a - 2b) = (0, 6 + b)$

Solution:

(i)

According to the question,

$(2a + b, a - b) = (8, 3)$

Given the ordered pairs are equal, so corresponding elements will be equal.

Hence,

$2a + b = 8$ and $a - b = 3$

Now $a - b = 3$

$\Rightarrow a = 3 + b$

Substituting the value of a in the equation $2a + b = 8$,

We get,

$2(3 + b) + b = 8$

$\Rightarrow 6 + 2b + b = 8$

$\Rightarrow 3b = 8 - 6 = 2$

$\Rightarrow b = 2/3$

Substituting the value of b in equation $(a - b = 3)$,

We get,

$\Rightarrow a - 2/3 = 3$

$\Rightarrow a = 3 + 2/3$

$\Rightarrow a = (9 + 2)/3$

$\Rightarrow a = 11/3$

Hence the value of $a = 11/3$ and $b = 2/3$ respectively.

(ii)

According to the question,

$$\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$$

Given the ordered pairs are equal, so corresponding elements will be equal.

$$a/4 = 0 \text{ and } a - 2b = 6 + b$$

$$\text{Now } a/4 = 0$$

$$\Rightarrow a = 0$$

Substituting the value of a in the equation $(a - 2b = 6 + b)$,

We get,

$$0 - 2b = 6 + b$$

$$\Rightarrow -2b - b = 6$$

$$\Rightarrow -3b = 6$$

$$\Rightarrow b = -6/3$$

$$\Rightarrow b = -2$$

Hence, the value of $a = 0$ and $b = -2$ respectively

5. Given $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$. Find the ordered pairs which satisfy the conditions given below:

(i) $x + y = 5$

(ii) $x + y < 5$

(iii) $x + y > 8$

Solution:

According to the question, $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$

(i) $x + y = 5$

So, we find the ordered pair such that $x + y = 5$, where x and y belongs to set $A = \{1, 2, 3, 4, 5\}$,

$$1 + 1 = 2 \neq 5$$

$$1 + 2 = 3 \neq 5$$

$$1 + 3 = 4 \neq 5$$

$$1 + 4 = 5 \Rightarrow \text{the ordered pair is } (1, 4)$$

$$1 + 5 = 6 \neq 5$$

$$2 + 1 = 3 \neq 5$$

$$2 + 2 = 4 \neq 5$$

$$2 + 3 = 5 \Rightarrow \text{the ordered pair is } (2, 3)$$

$$2 + 4 = 6 \neq 5$$

$$2 + 5 = 7 \neq 5$$

$$3 + 1 = 4 \neq 5$$

$$3 + 2 = 5 \Rightarrow \text{the ordered pair is } (3, 2)$$

$$3 + 3 = 6 \neq 5$$

$$3 + 4 = 7 \neq 5$$

$$3 + 5 = 8 \neq 5$$

$$4 + 1 = 5 \Rightarrow \text{the ordered pair is } (4, 1)$$

$$4 + 2 = 6 \neq 5$$

$$4 + 3 = 7 \neq 5$$

$$4 + 4 = 8 \neq 5$$

$$4 + 5 = 9 \neq 5$$

$$5 + 1 = 6 \neq 5$$

$$5 + 2 = 7 \neq 5$$

$$5 + 3 = 8 \neq 5$$

$$5 + 4 = 9 \neq 5$$

$$5 + 5 = 10 \neq 5$$

Therefore, the set of ordered pairs satisfying $x + y = 5 = \{(1,4), (2,3), (3,2), (4,1)\}$.

(ii) $x + y < 5$

So, we find the ordered pair such that $x + y < 5$, where x and y belongs to set $A = \{1, 2, 3, 4, 5\}$

$$1 + 1 = 2 < 5 \Rightarrow \text{the ordered pairs is } (1, 1)$$

$$1 + 2 = 3 < 5 \Rightarrow \text{the ordered pairs is } (1, 2)$$

$$1 + 3 = 4 < 5 \Rightarrow \text{the ordered pairs is } (1, 3)$$

$$1 + 4 = 5$$

$$1 + 5 = 6 > 5$$

$$2 + 1 = 3 < 5 \Rightarrow \text{the ordered pairs is } (2, 1)$$

$$2 + 2 = 4 < 5 \Rightarrow \text{the ordered pairs is } (2, 2)$$

$$2 + 3 = 5$$

$$2 + 4 = 6 > 5$$

$$2 + 5 = 7 > 5$$

$$3 + 1 = 4 < 5 \Rightarrow \text{the ordered pairs is } (3, 1) \quad 3 + 2 = 5$$

$$3 + 3 = 6 > 5$$

$$3 + 4 = 7 > 5$$

$$3 + 5 = 8 > 5$$

$$4 + 1 = 5$$

$$4 + 2 = 6 > 5$$

$$4 + 3 = 7 > 5$$

$$4 + 4 = 8 > 5$$

$$4 + 5 = 9 > 5$$

$$5 + 1 = 6 > 5$$

$$5 + 2 = 7 > 5$$

$$5 + 3 = 8 > 5$$

$$5 + 4 = 9 > 5$$

$$5 + 5 = 10 > 5$$

Therefore, the set of ordered pairs satisfying $x + y < 5 = \{(1,1), (1,2), (1,3), (2, 1), (2,2), (3,1)\}$.

(iii) $x + y > 8$

So, we find the ordered pair such that $x + y > 8$, where x and y belongs to set $A = \{1, 2, 3, 4, 5\}$

$$1 + 1 = 2 < 8$$

$$1 + 2 = 3 < 8$$

$$1 + 3 = 4 < 8$$

$$1 + 4 = 5 < 8$$

$$1 + 5 = 6 < 8$$

$$2 + 1 = 3 < 8$$

$$2 + 2 = 4 < 8$$

$$2 + 3 = 5 < 8$$

$$2 + 4 = 6 < 8$$

$$2 + 5 = 7 < 8$$

$$3 + 1 = 4 < 8$$

$$3 + 2 = 5 < 8$$

$$3 + 3 = 6 < 8$$

$$3 + 4 = 7 < 8$$

$$3 + 5 = 8$$

$$4 + 1 = 5 < 8$$

$$4 + 2 = 6 < 8$$

$$4 + 3 = 7 < 8$$

$$4 + 4 = 8$$

$$4 + 5 = 9 > 8, \text{ so one of the ordered pairs is } (4, 5)$$

$$5 + 1 = 6 < 8$$

$$5 + 2 = 7 < 8$$

$$5 + 3 = 8$$

$$5 + 4 = 9 > 8, \text{ so one of the ordered pairs is } (5, 4)$$

$$5 + 5 = 10 > 8, \text{ so one of the ordered pairs is } (5, 5)$$

Therefore, the set of ordered pairs satisfying $x + y > 8 = \{(4, 5), (5, 4), (5, 5)\}$.

6. Given $R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$. Find the domain and Range of R.

Solution:

According to the question,

$$R = \{(x, y) : x, y \in W, x^2 + y^2 = 25\}$$

$$R = \{(0, 5), (3, 4), (4, 3), (5, 0)\}$$

The domain of R consists of all the first elements of all the ordered pairs of R.

$$\text{Domain of } R = \{0, 3, 4, 5\}$$

The range of R contains all the second elements of all the ordered pairs of R.

$$\text{Range of } R = \{5, 4, 3, 0\}$$

7. If $R_1 = \{(x, y) \mid y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of R_1 .

Solution:

According to the question,

$$R_1 = \{(x, y) \mid y = 2x + 7, \text{ where } x \in R \text{ and } -5 \leq x \leq 5\} \text{ is a relation}$$

The domain of R_1 consists of all the first elements of all the ordered pairs of R_1 , i.e., x,

It is also given $-5 \leq x \leq 5$.

Therefore,

$$\text{Domain of } R_1 = [-5, 5]$$

The range of R contains all the second elements of all the ordered pairs of R_1 , i.e., y

$$\text{It is also given } y = 2x + 7$$

$$\text{Now } x \in [-5, 5]$$

Multiply LHS and RHS by 2,

We get,

$$2x \in [-10, 10]$$

Adding LHS and RHS with 7,

We get,

$$2x + 7 \in [-3, 17]$$

$$\text{Or, } y \in [-3, 17]$$

So,

$$\text{Range of } R_1 = [-3, 17]$$

8. If $R_2 = \{(x, y) \mid x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$ is a relation. Then find R_2 .

Solution:

We have,

$$R_2 = \{(x, y) \mid x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$$

So, we get,

$$x^2 = 0 \text{ and } y^2 = 64 \text{ or } x^2 = 64 \text{ and } y^2 = 0$$

$$x = 0 \text{ and } y = \pm 8 \text{ or } x = \pm 8 \text{ and } y = 0$$

$$\text{Therefore, } R_2 = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$$

9. If $R_3 = \{(x, |x|) \mid x \text{ is a real number}\}$ is a relation. Then find domain and range of R_3 .

Solution:

According to the question,

$$R_3 = \{(x, |x|) \mid x \text{ is a real number}\} \text{ is a relation}$$

Domain of R_3 consists of all the first elements of all the ordered pairs of R_3 , i.e., x ,

It is also given that x is a real number,

So, Domain of $R_3 = \mathbb{R}$

Range of R contains all the second elements of all the ordered pairs of R_3 , i.e., $|x|$

It is also given that x is a real number,

$$\text{So, } |x| = |R|$$

$$\Rightarrow |x| \geq 0,$$

i.e., $|x|$ has all positive real numbers including 0

Hence,

$$\text{Range of } R_3 = [0, \infty)$$

10. Is the given relation a function? Give reasons for your answer.

(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

(ii) $f = \{(x, x) \mid x \text{ is a real number}\}$

(iii) $g = n, (1/n) \mid n \text{ is a positive integer}$

(iv) $s = \{(n, n^2) \mid n \text{ is a positive integer}\}$

(v) $t = \{(x, 3) \mid x \text{ is a real number}\}$

Solution:

(i) According to the question,

$$h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$$

Therefore, element 3 has two images, namely, 9 and 11.

A relation is said to be function if every element of one set has one and only one image in other set.

Hence, h is not a function.

(ii) According to the question,

$$f = \{(x, x) \mid x \text{ is a real number}\}$$

This means the relation f has elements which are real number.

Therefore, for every $x \in \mathbb{R}$ there will be unique image.

A relation is said to be function if every element of one set has one and only one image in other set.

Hence, f is a function.

(iii) According to the question,

$g = n, (1/n) \mid n$ is a positive integer

Therefore, the element n is a positive integer and the corresponding $1/n$ will be a unique and distinct number.

Therefore, every element in the domain has unique image.

A relation is said to be function if every element of one set has one and only one image in other set.

Hence, g is a function.

(iv) According to the question,

$s = \{(n, n^2) \mid n \text{ is a positive integer}\}$

Therefore, element n is a positive integer and the corresponding n^2 will be a unique and distinct number, as square of any positive integer is unique.

Therefore, every element in the domain has unique image.

A relation is said to be function if every element of one set has one and only one image in other set.

Hence, s is a function.

(v) According to the question,

$t = \{(x, 3) \mid x \text{ is a real number}\}$

Therefore, the domain element x is a real number.

Also, range has one number i.e., 3 in it.

Therefore, for every element in the domain has the image 3, it is a constant function.

A relation is said to be function if every element of one set has one and only one image in other set.

Hence, t is a function.

11. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following

(a) $f(3) + g(-5)$

(b) $f(1/2) \times g(14)$

(c) $f(-2) + g(-1)$

(d) $f(t) - f(-2)$

(e) $(f(t) - f(5))/(t - 5)$, if $t \neq 5$

Solution:

According to the question,

f and g are real functions such that $f(x) = x^2 + 7$ and $g(x) = 3x + 5$

(a) $f(3) + g(-5)$

$f(x) = x^2 + 7$

Substituting $x = 3$ in $f(x)$, we get

$$f(3) = 3^2 + 7 = 9 + 7 = 16 \dots(i)$$

And,

$$g(x) = 3x + 5$$

Substituting $x = -5$ in $g(x)$, we get

$$g(-5) = 3(-5) + 5 = -15 + 5 = -10 \dots(ii)$$

Adding equations (i) and (ii),

We get,

$$f(3) + g(-5) = 16 - 10 = 6$$

$$(b) f(1/2) \times g(14)$$

$$f(x) = x^2 + 7$$

Substituting $x = 1/2$ in $f(x)$, we get

$$f(1/2) = (1/2)^2 + 7 = 1/4 + 7 = 29/4 \dots(i)$$

And,

$$g(x) = 3x + 5$$

Substituting $x = 14$ in $g(x)$, we get

$$g(14) = 3(14) + 5 = 42 + 5 = 47 \dots(ii)$$

Multiplying equation (i) and (ii),

We get,

$$f(1/2) \times g(14) = (29/4) \times 47 = 1363/4$$

$$(c) f(-2) + g(-1)$$

$$f(x) = x^2 + 7$$

Substituting $x = -2$ in $f(x)$, we get

$$f(-2) = (-2)^2 + 7 = 4 + 7 = 11 \dots(i)$$

And,

$$g(x) = 3x + 5$$

Substituting $x = -1$ in $g(x)$, we get

$$g(-1) = 3(-1) + 5 \\ = -3 + 5 = 2 \dots(ii)$$

Adding equation (i) and (ii),

We get,

$$f(-2) + g(-1) = 11 + 2 = 13$$

$$(d) f(t) - f(-2)$$

$$f(x) = x^2 + 7$$

Substituting $x = t$ in $f(x)$, we get

$$f(t) = t^2 + 7 \dots(i)$$

Considering the same function,

$$f(x) = x^2 + 7$$

Substituting $x = -2$ in $f(x)$, we get

$$f(-2) = (-2)^2 + 7 = 4 + 7 = 11 \dots(ii)$$

Subtracting equation (i) with (ii),

We get,

$$f(t) - f(-2) = t^2 + 7 - 11 = t^2 - 4$$

(e) $(f(t) - f(5)) / (t - 5)$, if $t \neq 5$

$$f(x) = x^2 + 7$$

Substituting $x = t$ in $f(x)$, we get

$$f(t) = t^2 + 7 \dots\dots\dots(i)$$

Considering the same function,

$$f(x) = x^2 + 7$$

Substituting $x = 5$ in $f(x)$, we get

$$f(5) = (5)^2 + 7 = 25 + 7 = 32 \dots\dots\dots(ii)$$

From equation (i) and (ii), we get

$$\begin{aligned} \frac{f(t) - f(5)}{t - 5} &= \frac{t^2 + 7 - 32}{t - 5} \\ &= \frac{t^2 - 25}{t - 5} \\ &= \frac{t^2 - 5^2}{t - 5} \end{aligned}$$

But we know $a^2 - b^2 = (a + b)(a - b)$, so above equation becomes,

$$\frac{f(t) - f(5)}{t - 5} = \frac{(t + 5)(t - 5)}{t - 5}$$

Cancelling the like terms, we get

$$\frac{f(t) - f(5)}{t - 5} = t + 5$$

12. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.

(a) For what real numbers x , $f(x) = g(x)$?

(b) For what real numbers x , $f(x) < g(x)$?

Solution:

According to the question,

f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$

(a) For what real numbers x , $f(x) = g(x)$

To satisfy the condition $f(x) = g(x)$,

Should also satisfy,

$$2x + 1 = 4x - 7$$

$$\Rightarrow 7 + 1 = 4x - 2x$$

$$\Rightarrow 8 = 2x$$

$$\text{Or, } 2x = 8$$

$$\Rightarrow x = 4$$

Hence, we get,

For $x = 4$, $f(x) = g(x)$

(b) For what real numbers x , $f(x) < g(x)$

To satisfy the condition $f(x) < g(x)$,

Should also satisfy,

$$2x + 1 < 4x - 7$$

$$\Rightarrow 7 + 1 < 4x - 2x$$

$$\Rightarrow 8 < 2x$$

$$\text{Or, } 2x > 8$$

$$\Rightarrow x > 4$$

Hence, we get,

$$\text{For } x > 4, f(x) > g(x)$$

13. If f and g are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$, then find.

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) f/g

Solution:

According to the question,

f and g be real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$,

(i) $f + g$

$$\begin{aligned}\Rightarrow f + g &= f(x) + g(x) \\ &= 2x + 1 + x^2 + 1 \\ &= x^2 + 2x + 2\end{aligned}$$

(ii) $f - g$

$$\begin{aligned}\Rightarrow f - g &= f(x) - g(x) \\ &= 2x + 1 - (x^2 + 1) \\ &= 2x - x^2\end{aligned}$$

(iii) fg

$$\begin{aligned}\Rightarrow fg &= f(x) g(x) \\ &= (2x + 1)(x^2 + 1) \\ &= 2x(x^2) + 2x(1) + 1(x^2) + 1(1) \\ &= 2x^3 + 2x + x^2 + 1 \\ &= 2x^3 + x^2 + 2x + 1\end{aligned}$$

(iv) f/g

$$\begin{aligned}f/g &= f(x)/g(x) \\ \Rightarrow \frac{f}{g} &= \frac{(2x + 1)}{x^2 + 1}\end{aligned}$$

14. Express the following functions as set of ordered pairs and determine their range.

$f: X \rightarrow R, f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$

Solution:

According to the question,

A function $f: X \rightarrow R, f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$

Domain = f is a function such that the first elements of all the ordered pair belong to the set $X = \{-1, 0, 3, 9, 7\}$.

The second element of all the ordered pair are such that they satisfy the condition $f(x) = x^3 + 1$

When $x = -1$,

$$f(x) = x^3 + 1$$

$$f(-1) = (-1)^3 + 1 = -1 + 1 = 0 \Rightarrow \text{ordered pair} = (-1, 0)$$

When $x = 0$,
 $f(x) = x^3 + 1$
 $f(0) = (0)^3 + 1 = 0 + 1 = 1 \Rightarrow$ ordered pair $= (0, 1)$
When $x = 3$,
 $f(x) = x^3 + 1$
 $f(3) = (3)^3 + 1 = 27 + 1 = 28 \Rightarrow$ ordered pair $= (3, 28)$
When $x = 9$,
 $f(x) = x^3 + 1$
 $f(9) = (9)^3 + 1 = 729 + 1 = 730 \Rightarrow$ ordered pair $= (9, 730)$
When $x = 7$,
 $f(x) = x^3 + 1$
 $f(7) = (7)^3 + 1 = 343 + 1 = 344 \Rightarrow$ ordered pair $= (7, 344)$
Therefore, the given function as a set of ordered pairs is
 $f = \{(-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)\}$
And,
Range of $f = \{0, 1, 28, 730, 344\}$

15. Find the values of x for which the functions **$f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal****Solution:**

According to the question,
 f and g functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$
For what real numbers x , $f(x) = g(x)$
To satisfy the condition $f(x) = g(x)$,
Should also satisfy,
 $3x^2 - 1 = 3 + x$
 $\Rightarrow 3x^2 - x - 3 - 1 = 0$
 $\Rightarrow 3x^2 - x - 4 = 0$
Splitting the middle term,
We get,
 $\Rightarrow 3x^2 + 3x - 4x - 4 = 0$
 $\Rightarrow 3x(x + 1) - 4(x + 1) = 0$
 $\Rightarrow (3x - 4)(x + 1) = 0$
 $\Rightarrow 3x - 4 = 0$ or $x + 1 = 0$
 $\Rightarrow 3x = 4$ or $x = -1$
 $\Rightarrow x = 4/3, -1$
Hence, for $x = 4/3, -1$, $f(x) = g(x)$,
i.e., given functions are equal.

LONG ANSWER TYPE**16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β ?****Solution:**

According to the question,
 $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$, and is described by relation $g(x) = \alpha x + \beta$

Now, given the relation,

$$g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$$

$$g(x) = \alpha x + \beta$$

For ordered pair (1,1), $g(x) = \alpha x + \beta$, becomes

$$g(1) = \alpha(1) + \beta = 1$$

$$\Rightarrow \alpha + \beta = 1$$

$$\Rightarrow \alpha = 1 - \beta \dots(i)$$

Considering ordered pair (2, 3), $g(x) = \alpha x + \beta$, becomes

$$g(2) = \alpha(2) + \beta = 3$$

$$\Rightarrow 2\alpha + \beta = 3$$

Substituting value of α from equation (i), we get

$$\Rightarrow 2(2) + \beta = 3$$

$$\Rightarrow \beta = 3 - 4 = -1$$

Substituting value of β in equation (i), we get

$$\alpha = 1 - \beta = 1 - (-1) = 2$$

Now, the given equation becomes,

$$\text{i.e., } g(x) = 2x - 1$$

17. Find the domain of each of the following functions given by

$$(i) \quad f(x) = \frac{1}{\sqrt{1 - \cos x}}$$

$$(ii) \quad f(x) = \frac{1}{\sqrt{x + |x|}}$$

$$(iii) \quad f(x) = x |x|$$

$$(iv) \quad f(x) = \frac{x^3 - x + 3}{x^2 - 1}$$

$$(v) \quad f(x) = \frac{3x}{2x - 8}$$

Solution:

(i)

$$f(x) = \frac{1}{\sqrt{1 - \cos x}}$$

According to the question,

We know the value of $\cos x$ lies between $-1, 1$,

$$-1 \leq \cos x \leq 1$$

Multiplying by negative sign, we get

$$\text{Or } 1 \geq -\cos x \geq -1$$

Adding 1, we get

$$2 \geq 1 - \cos x \geq 0 \dots(i)$$

Now,

$$f(x) = \frac{1}{\sqrt{1-\cos x}},$$

$$1 - \cos x \neq 0$$

$$\Rightarrow \cos x \neq 1$$

$$\text{Or, } x \neq 2n\pi \forall n \in \mathbb{Z}$$

Therefore, the domain of $f = \mathbb{R} - \{2n\pi : n \in \mathbb{Z}\}$

(ii)

$$f(x) = \frac{1}{\sqrt{x + |x|}}$$

According to the question,

For real value of f ,

$$x + |x| > 0$$

When $x > 0$,

$$x + |x| > 0 \Rightarrow x + x > 0 \Rightarrow 2x > 0 \Rightarrow x > 0$$

When $x < 0$,

$$x + |x| > 0 \Rightarrow x - x > 0 \Rightarrow 2x > 0 \Rightarrow x > 0$$

So, $x > 0$, to satisfy the given equation.

Therefore, the domain of $f = \mathbb{R}^+$

(iii)

$$f(x) = x|x|$$

According to the question,

We know x and $|x|$ are defined for all real values.

Therefore, the domain of $f = \mathbb{R}$

(iv)

$$f(x) = \frac{(x^3 - x + 3)}{x^2 - 1}$$

According to the question,

For real value of

$$x^2 - 1 \neq 0$$

$$\Rightarrow (x-1)(x+1) \neq 0$$

$$\Rightarrow x-1 \neq 0 \text{ or } x+1 \neq 0$$

$$\Rightarrow x \neq 1 \text{ or } x \neq -1$$

Therefore, the domain of $f = \mathbb{R} - \{-1, 1\}$

(v)

$$f(x) = \frac{3x}{2x-8}$$

According to the question,

For real value of

$$2x - 8 \neq 0$$

$$\Rightarrow x \neq 4$$

Therefore, the domain of $f = \mathbb{R} - \{28\}$

18. Find the range of the following functions given by

(i) $f(x) = \frac{3}{2-x^2}$

(ii) $f(x) = 1 - |x-2|$

(iii) $f(x) = |x-3|$

(iv) $f(x) = 1 + 3 \cos 2x$

Solution:

(i)

$$f(x) = \frac{3}{2-x^2}$$

According to the question,

Let $f(x) = y$,

$$y = \frac{3}{2-x^2}$$

$$\Rightarrow 2-x^2 = \frac{3}{y}$$

$$\Rightarrow x^2 = 2 - \frac{3}{y}$$

But, we know that, $x^2 \geq 0$

$$2 - \frac{3}{y} \geq 0$$

$$\Rightarrow \frac{2y-3}{y} \geq 0$$

$$\Rightarrow y > 0 \text{ and } 2y-3 \geq 0$$

$$\Rightarrow y > 0 \text{ and } 2y \geq 3$$

$$\Rightarrow y > 0 \text{ and } y \geq \frac{3}{2}$$

Or $f(x) > 0$ and $f(x) \geq \frac{3}{2}$

$$f(x) \in (-\infty, 0) \cup [\frac{3}{2}, \infty)$$

$$\Rightarrow f(x) \in (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$$

Therefore, the range of $f = (-\infty, 0) \cup [\frac{3}{2}, \infty)$

(ii) $f(x) = 1 - |x-2|$

According to the question,

For real value of f ,

$$|x-2| \geq 0$$

Adding negative sign, we get

$$\text{Or } -|x-2| \leq 0$$

Adding 1 we get

$$\Rightarrow 1 - |x-2| \leq 1$$

$$\text{Or } f(x) \leq 1$$

$$\Rightarrow f(x) \in (-\infty, 1]$$

Therefore, the range of $f = (-\infty, 1]$

(iii) $f(x) = |x-3|$

According to the question,

We know $|x|$ are defined for all real values.

And $|x-3|$ will always be greater than or equal to 0.

i.e., $f(x) \geq 0$

Therefore, the range of $f = [0, \infty)$

(iv) $f(x) = 1 + 3 \cos 2x$

According to the question,

We know the value of $\cos 2x$ lies between $-1, 1$, so

$$-1 \leq \cos 2x \leq 1$$

Multiplying by 3, we get

$$-3 \leq 3 \cos 2x \leq 3$$

Adding 1, we get

$$-2 \leq 1 + 3 \cos 2x \leq 4$$

Or, $-2 \leq f(x) \leq 4$

Hence $f(x) \in [-2, 4]$

Therefore, the range of $f = [-2, 4]$

19. Redefine the function $f(x) = |x-2| + |2+x|, -3 \leq x \leq 3$

Solution:

According to the question,

function $f(x) = |x-2| + |2+x|, -3 \leq x \leq 3$

We know that,

when $x > 0$,

$|x-2|$ is $(x-2), x \geq 2$

$|2+x|$ is $(2+x), x \geq -2$

when $x < 0$

$|x-2|$ is $-(x-2), x < 2$

$|2+x|$ is $-(2+x), x < -2$

Given that, $f(x) = |x-2| + |2+x|, -3 \leq x \leq 3$

It can be rewritten as,

$$f(x) = \begin{cases} -(x-2) - (2+x), & -3 \leq x < -2 \\ -(x-2) + (2+x), & -2 \leq x < 2 \\ (x-2) + (2+x), & 2 \leq x \leq 3 \end{cases}$$

Or

$$f(x) = \begin{cases} -x + 2 - 2 - x, & -3 \leq x < -2 \\ -x + 2 + 2 + x, & -2 \leq x < 2 \\ x - 2 + 2 + x, & 2 \leq x \leq 3 \end{cases}$$

Or,

$$f(x) = \begin{cases} -2x, & -3 \leq x < -2 \\ 4, & -2 \leq x < 2 \\ 2x, & 2 \leq x \leq 3 \end{cases}$$

20. If

$$f(x) = \frac{x-1}{x+1}, \text{ then show that:}$$

$$(i) \quad f\left(\frac{1}{x}\right) = -f(x)$$

$$(ii) \quad f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Solution:

(i)

$$f(x) = \frac{x-1}{x+1}$$

Substituting x by $1/x$, we get

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1}$$

$$= \frac{\frac{1-x}{x}}{\frac{1+x}{x}}$$

$$= \frac{1-x}{1+x}$$

$$= \frac{-(x-1)}{1+x}$$

$$= -\frac{x-1}{x+1}$$

Therefore,

We get,

$$f\left(\frac{1}{x}\right) = -f(x)$$

Hence proved

(ii)

$$f(x) = \frac{x-1}{x+1}$$

Substituting x by $-1/x$, we get

$$\begin{aligned}
 f\left(-\frac{1}{x}\right) &= \frac{\left(-\frac{1}{x}\right) - 1}{\left(-\frac{1}{x}\right) + 1} \\
 &= \frac{\frac{-1 - x}{x}}{\frac{-1 + x}{x}} \\
 &= \frac{-1 - x}{-1 + x} \\
 &= \frac{-(x + 1)}{x - 1} \\
 &= \frac{-1}{\frac{x - 1}{x + 1}}
 \end{aligned}$$

Therefore,

$$f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$$

Hence proved

21. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $\mathbb{R}^+ \cup \{0\}$. Find

(i) $(f + g)(x)$

(ii) $(f - g)(x)$

(iii) $(fg)(x)$

(iv) $(f/g)(x)$

Solution:

(i)

$$(f + g)(x)$$

$$\Rightarrow (f + g)(x) = f(x) + g(x)$$

$$\Rightarrow f(x) + g(x) = \sqrt{x} + x$$

(ii)

$$(f - g)(x)$$

$$\Rightarrow (f - g)(x) = f(x) - g(x)$$

$$\Rightarrow f(x) - g(x) = \sqrt{x} - x$$

(iii)

$$(fg)(x)$$

$$\Rightarrow (fg)(x) = f(x) \cdot g(x)$$

$$\Rightarrow (fg)(x) = (\sqrt{x})(x)$$

$$\Rightarrow f(x)g(x) = x\sqrt{x}$$

(iv)

$$(f/g)(x) = f(x)/g(x)$$

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x}$$

Multiplying and dividing by \sqrt{x} ,

We get

$$= \frac{\sqrt{x}}{x} \times \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{x}{x\sqrt{x}}$$

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$$