

## EXERCISE

### SHORT ANSWER TYPE

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**1. Prove that**

$$\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$$

**Solution:**

According to the question,

$$\begin{aligned} \text{LHS} &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} \\ &= \frac{\frac{\sin A}{\cos A} + \frac{1}{\cos A} - 1}{\frac{\sin A}{\cos A} - \frac{1}{\cos A} + 1} \\ &= \frac{\frac{\sin A + 1 - \cos A}{\cos A}}{\frac{\sin A - 1 + \cos A}{\cos A}} \\ &= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A} \end{aligned}$$

Using the identity,

$\sin^2 A + \cos^2 A = 1$ , we get,

$\sin A + (1 - \cos A)$ .

$$\begin{aligned} \therefore \text{LHS} &= \frac{\sin A + (1 - \cos A)}{\sin A - (1 - \cos A)} \times \frac{\sin A + (1 - \cos A)}{\sin A + (1 - \cos A)} \\ &= \frac{\{\sin A + (1 - \cos A)\}^2}{\sin^2 A - (1 - \cos A)^2} \\ &= \frac{\sin^2 A + (1 - \cos A)^2 + 2 \sin A (1 - \cos A)}{\sin^2 A - (1 - \cos A)^2} \\ &= \frac{(\sin^2 A + \cos^2 A) + 1 - 2 \cos A + 2 \sin A (1 - \cos A)}{\sin^2 A - \{1 + \cos^2 A - 2 \cos A\}} \\ &= \frac{(1) + 1 - 2 \cos A + 2 \sin A (1 - \cos A)}{(\sin^2 A - 1) - \cos^2 A + 2 \cos A} \\ &= \frac{2(1 - \cos A) + 2 \sin A (1 - \cos A)}{(-\cos^2 A) - \cos^2 A + 2 \cos A} \\ &= \frac{2(1 + \sin A)(1 - \cos A)}{2(1 + \sin A)(1 - \cos A)} \\ &= \frac{2 \cos A (1 - \cos A)}{(1 + \sin A)} \\ &= \frac{2 \cos A (1 - \cos A)}{\cos A} = \text{RHS} \end{aligned}$$

Hence, L.H.S = R.H.S

2. If  $[2\sin\alpha / (1+\cos\alpha+\sin\alpha)] = y$ , then prove that  $[(1-\cos\alpha+\sin\alpha) / (1+\sin\alpha)]$  is also equal to  $y$ .

[Hint : Express  $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$ ]

**Solution:**

According to the question,

$$y = 2\sin\alpha / (1+\cos\alpha+\sin\alpha)$$

Multiplying numerator and denominator by  $(1-\cos\alpha+\sin\alpha)$ ,

We get,

$$\begin{aligned} \Rightarrow y &= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} \times \frac{1-\cos\alpha+\sin\alpha}{1-\cos\alpha+\sin\alpha} \\ &= \frac{2\sin\alpha}{(1+\sin\alpha)+\cos\alpha} \times \frac{(1+\sin\alpha)-\cos\alpha}{(1+\sin\alpha)-\cos\alpha} \end{aligned}$$

Using  $(a+b)(a-b) = a^2 - b^2$ , we get:

$$\begin{aligned} &= \frac{2\sin\alpha \{(1+\sin\alpha) - \cos\alpha\}}{(1+\sin\alpha)^2 - \cos^2\alpha} \\ &= \frac{2\sin\alpha(1+\sin\alpha) - 2\sin\alpha\cos\alpha}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha} \end{aligned}$$

Since,  $1-\cos^2\alpha = \sin^2\alpha$

$$\begin{aligned} \therefore y &= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{\sin^2\alpha+2\sin\alpha+\sin^2\alpha} \\ &= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{2\sin\alpha(1+\sin\alpha)} \\ \Rightarrow y &= \frac{(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)} \\ \Rightarrow y &= \frac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)} \end{aligned}$$

Hence Proved

3. If  $m \sin \theta = n \sin (\theta + 2\alpha)$ , then prove that

$$\tan (\theta + \alpha) \cot \alpha = (m + n)/(m - n)$$

[Hints: Express  $\sin(\theta + 2\alpha) / \sin\theta = m/n$  and apply componendo and dividendo]

**Solution:**

According to the question,

$$m \sin \theta = n \sin (\theta + 2\alpha)$$

To prove:

$$\tan (\theta + \alpha) \cot \alpha = (m + n)/(m - n)$$

Proof:

$$m \sin \theta = n \sin (\theta + 2\alpha)$$

$$\Rightarrow \sin(\theta + 2\alpha) / \sin \theta = m/n$$

Applying componendo-dividendo rule, we have,

$$\frac{\sin(\theta+2\alpha)+\sin \theta}{\sin(\theta+2\alpha)-\sin \theta} = \frac{m+n}{m-n}$$

By transformation formula of T-ratios,

We know that,

$$\sin A + \sin B = 2 \sin ((A+B)/2) \cos ((A - B)/2)$$

And,

$$\sin A - \sin B = 2 \cos ((A+B)/2) \sin ((A - B)/2)$$

On applying the formula, we get,

$$\frac{2 \sin \left( \frac{2\theta + 2\alpha}{2} \right) \cos \left( \frac{\theta + 2\alpha - \theta}{2} \right)}{2 \cos \left( \frac{2\theta + 2\alpha}{2} \right) \sin \left( \frac{\theta + 2\alpha - \theta}{2} \right)} = \frac{m+n}{m-n}$$

$$\frac{\sin(\theta+\alpha) \cos(\alpha)}{\cos(\theta+\alpha) \sin(\alpha)} = \frac{m+n}{m-n}$$

$$\{\because \tan x = (\sin x)/(\cos x)\}$$

$$\Rightarrow \tan(\theta + \alpha) \cot \alpha = \frac{m+n}{m-n}$$

$$\text{Therefore, } \tan (\theta + \alpha) \cot \alpha = (m + n)/(m - n)$$

Hence Proved

$$\cos(\alpha + \beta) = \frac{4}{5} \text{ and } \sin(\alpha - \beta) = \frac{5}{13},$$

4. If  $2\alpha$  where  $\alpha$  lie between 0 and  $\pi/4$ , find value of  $\tan 2\alpha$

[Hint: Express  $\tan 2\alpha$  as  $\tan (\alpha + \beta + \alpha - \beta)$ ]

**Solution:**

According to the question,

$$\cos(\alpha + \beta) = 4/5 \dots(i)$$

We know that,

$$\sin x = \sqrt{1 - \cos^2 x}$$

Therefore,

$$\sin (\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\Rightarrow \sin (\alpha + \beta) = \sqrt{1 - (4/5)^2} = 3/5 \dots(ii)$$

Also,

$$\sin(\alpha - \beta) = 5/13 \text{ \{given\} } \dots(iii)$$

we know that,

$$\cos x = \sqrt{1 - \sin^2 x}$$

Therefore,

$$\cos (\alpha - \beta) = \sqrt{1 - \sin^2(\alpha - \beta)}$$

$$\Rightarrow \cos (\alpha - \beta) = \sqrt{1 - (5/13)^2} = 12/13 \dots(iv)$$

Therefore,

$$\tan 2\alpha = \tan (\alpha + \beta + \alpha - \beta)$$

We know that,

$$\begin{aligned}\tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\ \tan(\alpha+\beta) + \tan(\alpha-\beta) \\ \therefore \tan 2\alpha &= \frac{1 - \tan(\alpha+\beta) \tan(\alpha-\beta)}{\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} + \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}} \\ &= \frac{1 - \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \times \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}{\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} + \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}\end{aligned}$$

From equation i, ii, iii and iv we have,

$$\begin{aligned}\Rightarrow \tan 2\alpha &= \frac{\frac{3}{5} + \frac{5}{13}}{1 - \frac{3}{5} \times \frac{5}{13}} \\ &= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} \\ &= \frac{\frac{9+5}{12}}{1 - \frac{15}{48}} \\ \Rightarrow \tan 2\alpha &= \frac{14}{12 \left( \frac{33}{48} \right)} \\ &= \frac{56}{33}\end{aligned}$$

Hence,  $\tan 2\alpha = 56/33$

**5. If  $\tan x = b/a$  then find the value of**

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

**Solution:**

According to the question,

$$\tan x = b/a$$

Let,

$$\begin{aligned}
 y &= \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \\
 \therefore y &= \sqrt{\frac{a(1+\frac{b}{a})}{a(1-\frac{b}{a})}} + \sqrt{\frac{a(1-\frac{b}{a})}{a(1+\frac{b}{a})}} \\
 &= \sqrt{\frac{(1+\tan x)}{(1-\tan x)}} + \sqrt{\frac{(1-\tan x)}{(1+\tan x)}} \\
 &= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}} \\
 &= \frac{(\sqrt{1+\tan x})^2 + (\sqrt{1-\tan x})^2}{(\sqrt{1-\tan x})(\sqrt{1+\tan x})} \\
 &= \frac{1+\tan x + 1-\tan x}{\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\tan^2 x}} \\
 \therefore y &= \frac{\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}}{2} = \frac{2}{\sqrt{1-\tan^2 x}} \\
 &= \frac{2}{\sqrt{1-\frac{\sin^2 \theta}{\cos^2 \theta}}} \\
 &= \frac{2 \cos \theta}{\sqrt{\cos^2 \theta - \sin^2 \theta}} \\
 \therefore \cos^2 \theta - \sin^2 \theta &= \cos 2\theta \\
 &= \frac{2 \cos \theta}{\sqrt{\cos 2\theta}}
 \end{aligned}$$

**6. Prove that  $\cos \theta \cos \theta/2 - \cos 3\theta \cos 9\theta/2 = \sin 7\theta \sin 4\theta$**

**[Hint: Express L.H.S. =  $\frac{1}{2} [2\cos \theta \cos \theta/2 - 2\cos 3\theta \cos 9\theta / 2]$**

**Solution:**

Using transformation formula, we get,

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$-2 \sin A \sin B = \cos(A + B) - \cos(A - B)$$

Multiplying and dividing the expression by 2.

$$\therefore \text{LHS} = \frac{1}{2} \left( 2 \cos \theta \cos \frac{\theta}{2} - 2 \cos 3\theta \cos \frac{9\theta}{2} \right)$$

Applying transformation formula, we get,

$$\text{LHS} = \frac{1}{2} \left( \cos \left( \theta + \frac{\theta}{2} \right) + \cos \left( \theta - \frac{\theta}{2} \right) - \left\{ \cos \left( 3\theta + \frac{9\theta}{2} \right) + \cos \left( 3\theta - \frac{9\theta}{2} \right) \right\} \right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \left( \cos \frac{3\theta}{2} + \cos \frac{\theta}{2} - \cos \left( \frac{15\theta}{2} \right) - \cos \left( -\frac{3\theta}{2} \right) \right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \left( \cos \frac{3\theta}{2} + \cos \frac{\theta}{2} - \cos \frac{15\theta}{2} - \cos \frac{3\theta}{2} \right) \{ \because \cos(-x) = \cos x \}$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \left( \cos \frac{\theta}{2} - \cos \frac{15\theta}{2} \right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \left( 2 \sin \left( \frac{\frac{\theta}{2} + \frac{15\theta}{2}}{2} \right) \sin \left( \frac{\frac{15\theta}{2} - \frac{\theta}{2}}{2} \right) \right)$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \left( 2 \sin \left( \frac{8\theta}{2} \right) \sin \left( \frac{7\theta}{2} \right) \right)$$

$$\therefore \text{LHS} = \sin 4\theta \sin \left( \frac{7\theta}{2} \right) = \text{RHS}$$

Hence,

$$\cos \theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 4\theta \sin \left( \frac{7\theta}{2} \right)$$

**7. If  $a \cos \theta + b \sin \theta = m$  and  $a \sin \theta - b \cos \theta = n$ , then show that  $a^2 + b^2 = m^2 + n^2$ .**

**Solution:**

According to the question,

$$a \cos \theta + b \sin \theta = m \dots (i)$$

$$a \sin \theta - b \cos \theta = n \dots (ii)$$

Squaring and adding equation 1 and 2, we get,

$$(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = m^2 + n^2$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = m^2 + n^2$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta = m^2 + n^2$$

$$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$$

$$\text{Using, } \sin^2 \theta + \cos^2 \theta = 1,$$

We get,

$$\Rightarrow a^2 + b^2 = m^2 + n^2$$

**8. Find the value of  $\tan 22^\circ 30'$ .**

$$\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \frac{\sin \theta}{1 + \cos \theta}$$

[Hint: Let  $\theta = 45^\circ$ , use

**Solution:**

$$\text{Let, } \theta = 45^\circ$$

$$\text{As we need to find: } \tan 22^\circ 30' = \tan (\theta/2)$$

We know that,

$$\sin \theta = \cos \theta = 1/\sqrt{2} \text{ (for } \theta = 45^\circ)$$

Since,

$$\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

Multiplying  $2\cos \frac{\theta}{2}$  in numerator and denominator, we get,

$$\Rightarrow \tan \frac{\theta}{2} = \frac{2 \cos \frac{\theta}{2} \sin \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$$

By applying formula of T-ratios of multiple angles-

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2\cos^2 x - 1 \text{ or } 1 + \cos 2x = 2\cos^2 x$$

$$\therefore \tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$

$$\Rightarrow \tan 22^\circ 30' = \frac{\sin 45^\circ}{1 + \cos 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}$$

Therefore,  $\tan 22^\circ 30' = \sqrt{2} - 1$

**9. Prove that  $\sin 4A = 4\sin A \cos^3 A - 4 \cos A \sin^3 A$ .**

**Solution:**

$$\sin 4A = \sin (2A + 2A)$$

We know that,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\text{Therefore, } \sin 4A = \sin 2A \cos 2A + \cos 2A \sin 2A$$

$$\Rightarrow \sin 4A = 2 \sin 2A \cos 2A$$

From T-ratios of multiple angle,

We get,

$$\sin 2A = 2 \sin A \cos A \text{ and } \cos 2A = \cos^2 A - \sin^2 A$$

$$\Rightarrow \sin 4A = 2(2 \sin A \cos A)(\cos^2 A - \sin^2 A)$$

$$\Rightarrow \sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$$

$$\text{Hence, } \sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$$

**10. If  $\tan \theta + \sin \theta = m$  and  $\tan \theta - \sin \theta = n$ , then prove that  $m^2 - n^2 = 4 \sin \theta \tan \theta$**

**[Hint:  $m + n = 2 \tan \theta$ ,  $m - n = 2 \sin \theta$ , then use  $m^2 - n^2 = (m + n)(m - n)$ ]**

**Solution:**

According to the question,

$$\tan \theta + \sin \theta = m \dots (i)$$

$$\tan \theta - \sin \theta = n \dots (ii)$$

Adding equation i and ii,

$$2 \tan \theta = m + n \dots (iii)$$

Subtracting equation ii from i,

We get,

$$2\sin \theta = m - n \dots(\text{iv})$$

Multiplying equations (iii) and (iv),

$$2\sin \theta (2\tan \theta) = (m + n)(m - n)$$

$$\Rightarrow 4 \sin \theta \tan \theta = m^2 - n^2$$

Hence,

$$m^2 - n^2 = 4 \sin \theta \tan \theta$$

**11. If  $\tan (A + B) = p$ ,  $\tan (A - B) = q$ , then show that  $\tan 2A = (p + q) / (1 - pq)$ .**

**[Hint: Use  $2A = (A + B) + (A - B)$ ]**

**Solution:**

We know that,

$$\tan 2A = \tan (A + B + A - B)$$

And also,

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$

$$\therefore \tan 2A = \frac{p + q}{1 - pq}$$

Substituting the values given in question,

$$\Rightarrow \tan 2A = \frac{p + q}{1 - pq}$$

$$\text{Hence, } \tan 2A = \frac{p + q}{1 - pq}$$

**12. If  $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$ , then prove that  $\cos 2\alpha + \cos 2\beta = -2\cos (\alpha + \beta)$ .**

**[Hint:  $(\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$ ]**

**Solution:**

According to the question,

$$\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta \dots(\text{i})$$

Since, LHS =  $\cos 2\alpha + \cos 2\beta$

We know that,

$$\cos 2x = \cos^2 x - \sin^2 x$$

Therefore,

$$\text{LHS} = \cos^2 \alpha - \sin^2 \alpha + (\cos^2 \beta - \sin^2 \beta)$$

$$\Rightarrow \text{LHS} = \cos^2 \alpha + \cos^2 \beta - (\sin^2 \alpha + \sin^2 \beta)$$

Also, since,

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow \text{LHS} = (\cos \alpha + \cos \beta)^2 - 2\cos \alpha \cos \beta - (\sin \alpha + \sin \beta)^2 + 2\sin \alpha \sin \beta$$

From equation (i),

$$\Rightarrow \text{LHS} = 0 - 2\cos \alpha \cos \beta - 0 + 2\sin \alpha \sin \beta$$

$$\Rightarrow \text{LHS} = -2(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$\because \cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\text{Therefore, } \text{LHS} = -2 \cos (\alpha + \beta) = \text{RHS}$$

$$\text{Hence, } \cos 2\alpha + \cos 2\beta = -2\cos (\alpha + \beta)$$

**13.**



If  $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$ , then show that  $\frac{\tan x}{\tan y} = \frac{a}{b}$ .

[Hint: Use componendo and Dividendo]

**Solution:**

According to the question,

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

Since,  $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$\therefore \frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$\frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$$

Applying componendo-dividendo rule,

We get,

$$\frac{(\sin x \cos y + \cos x \sin y) + (\sin x \cos y - \cos x \sin y)}{(\sin x \cos y + \cos x \sin y) - (\sin x \cos y - \cos x \sin y)} = \frac{(a+b) + (a-b)}{(a+b) - (a-b)}$$

$$\Rightarrow \frac{2 \sin x \cos y}{2 \cos x \sin y} = \frac{2a}{2b}$$

$$\Rightarrow \frac{2 \sin x \cos y}{2 \cos x \sin y} = \frac{2a}{2b}$$

$$\Rightarrow \left( \frac{\sin x}{\cos x} \right) \left( \frac{\cos y}{\sin y} \right) = \frac{a}{b}$$

Since,  $\tan A = (\sin A)/(\cos A)$

$$\Rightarrow \tan x \left( \frac{1}{\tan y} \right) = \frac{a}{b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

14.

$$\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha},$$

If  $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$ , then show that  $\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$ .

[Hint: Express  $\tan \theta = \tan(\alpha - \pi/2)$   $\theta = \alpha - \pi/4$ ]

**Solution:**

We know that,

$$\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$$

$$\Rightarrow \tan \theta = \frac{\cos \alpha \left( \frac{\sin \alpha}{\cos \alpha} - 1 \right)}{\cos \alpha \left( \frac{\sin \alpha}{\cos \alpha} + 1 \right)}$$

Since,  $\tan A = (\sin A)/(\cos A)$

$$\Rightarrow \tan \theta = (\tan \alpha - 1) / (\tan \alpha + 1)$$

Since,  $\tan \pi/4 = 1$

$$\Rightarrow \tan \theta = \frac{(\tan \alpha - \tan \frac{\pi}{4})}{(1 + \tan \frac{\pi}{4} \cdot \tan \alpha)}$$

We know that,

$$\tan(x-y) = (\tan x - \tan y) / (1 + \tan x \cdot \tan y)$$

$$\text{Therefore, } \tan \theta = \tan (\alpha - \pi/4)$$

$$\Rightarrow \theta = \alpha - \pi/4$$

$$\Rightarrow \alpha = \theta + \pi/4 \dots (i)$$

To prove,

$$\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$$

$$\because \text{LHS} = \sin \alpha + \cos \alpha$$

From equation (i)

$$\Rightarrow \text{LHS} = \sin(\theta + \pi/4) + \cos(\theta + \pi/4)$$

$$\because \sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\text{And, } \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\text{Therefore, LHS} = \sin \theta \cos(\pi/4) + \sin(\pi/4) \cos \theta + \cos \theta \cos(\pi/4) - \sin(\pi/4) \sin \theta$$

$$\because \sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$$

$$\Rightarrow \text{LHS} = \sin \theta (1/\sqrt{2}) + (1/\sqrt{2}) \cos \theta + \cos \theta (1/\sqrt{2}) - \sin \theta (1/\sqrt{2})$$

$$\Rightarrow \text{LHS} = 2 \cos \theta (1/\sqrt{2})$$

$$\Rightarrow \text{LHS} = \sqrt{2} \cos \theta = \text{RHS}$$

$$\text{Therefore, } \sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$$

**15. If  $\sin \theta + \cos \theta = 1$ , then find the general value of  $\theta$ .**

**Solution:**

According to the question,

$$\sin \theta + \cos \theta = 1$$

$$\text{As, } \sin \theta + \cos \theta = 1$$

$$\Rightarrow \sqrt{2} \left( \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta \right) = 1$$

We know that,

$$\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$$

$$\Rightarrow \sqrt{2} \left( \sin \theta \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \theta \right) = 1$$

We know that,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + \theta \right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \left( \frac{\pi}{4} + \theta \right) = \sin \frac{\pi}{4}$$

Since we know,

$$\text{If } \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$$

We get,

$$\theta + \pi/4 = n\pi + (-1)^n (\pi/4)$$

$$\Rightarrow \theta = n\pi + (\pi/4)((-1)^n - 1)$$

**16. Find the most general value of  $\theta$  satisfying the equation  $\tan \theta = -1$  and  $\cos \theta = 1/\sqrt{2}$**

**Solution:**

According to the question,

We have,

$$\tan \theta = -1$$

$$\text{And } \cos \theta = 1/\sqrt{2}.$$

$$\Rightarrow \theta = -\pi/4$$

So, we know that,

$\theta$  lies in IV quadrant.

$$\theta = 2\pi - \pi/4 = 7\pi/4$$

So, general solution is  $\theta = 7\pi/4 + 2n\pi, n \in \mathbb{Z}$

**17. If  $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$ , then find the general value of  $\theta$ .**

**Solution:**

According to the question,

$$\Rightarrow \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = 2 \operatorname{cosec} \theta$$

Since,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = 2 \operatorname{cosec} \theta$$

$$\Rightarrow 1 = 2 \operatorname{cosec} \theta \sin \theta \cos \theta$$

We know that,

$$\sin \theta \operatorname{cosec} \theta = 1$$

$$\Rightarrow 1 = 2 \cos \theta$$

$$\Rightarrow \cos \theta = 1/2 = \cos(\pi/3)$$

Hence,

The solution of  $\cos x = \cos \alpha$  can be given by,

$$x = 2m\pi \pm \alpha \quad \forall m \in \mathbb{Z}$$

$$\Rightarrow \theta = 2n\pi \pm \pi/3, n \in \mathbb{Z}$$

**18. If  $2\sin^2 \theta = 3\cos \theta$ , where  $0 \leq \theta \leq 2\pi$ , then find the value of  $\theta$ .**

**Solution:**

According to the question,

$$2\sin^2 \theta = 3\cos \theta$$

We know that,

$$\sin^2 \theta = 1 - \cos^2 \theta$$

Given that,

$$2\sin^2 \theta = 3\cos \theta$$

$$2 - 2\cos^2 \theta = 3\cos \theta$$

$$2\cos^2 \theta + 3\cos \theta - 2 = 0$$

$$(\cos \theta + 2)(2\cos \theta - 1) = 0$$

Therefore,

$$\cos \theta = 1/2 = \cos \pi/3$$

$$\theta = \pi/3 \text{ or } 2\pi - \pi/3$$

$$\theta = \pi/3, 5\pi/3$$

Therefore,  $2(1 - \cos^2 \theta) = 3\cos \theta$

$$\Rightarrow 2 - 2\cos^2 \theta = 3\cos \theta$$

$$\Rightarrow 2\cos^2 \theta + 3\cos \theta - 2 = 0$$

$$\Rightarrow 2\cos^2 \theta + 4\cos \theta - \cos \theta - 2 = 0$$

$$\begin{aligned} &\Rightarrow 2\cos \theta (\cos \theta + 2) + 1 (\cos \theta + 2) = 0 \\ &\Rightarrow (2\cos \theta + 1)(\cos \theta + 2) = 0 \\ &\text{Since, } \cos \theta \in [-1, 1], \text{ for any value } \theta. \\ &\text{So, } \cos \theta \neq -2 \\ &\text{Therefore,} \\ &2\cos \theta + 1 = 0 \\ &\Rightarrow \cos \theta = -\frac{1}{2} \\ &= \frac{\pi}{3} \text{ or } 2\pi - \frac{\pi}{3} \\ &\theta = \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

**19. If  $\sec x \cos 5x + 1 = 0$ , where  $0 < x \leq \pi/2$ , then find the value of  $x$ .**

**Solution:**

According to the question,  
 $\sec x \cos 5x = -1$   
 $\Rightarrow \cos 5x = -1/\sec x$   
 We know that,  
 $\sec x = 1/\cos x$   
 $\Rightarrow \cos 5x + \cos x = 0$   
 By transformation formula of T-ratios,  
 We know that,

$$\begin{aligned} \cos A + \cos B &= 2\cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right) \\ \Rightarrow 2\cos \left(\frac{5x+x}{2}\right) \cos \left(\frac{5x-x}{2}\right) &= 0 \\ \Rightarrow 2\cos 3x \cos 2x &= 0 \\ \Rightarrow \cos 3x = 0 \text{ or } \cos 2x &= 0 \\ \because 0 < x \leq \pi/2 \\ \text{Therefore, } 0 < 2x \leq \pi \text{ or } 0 < 3x \leq 3\pi/2 \\ \text{Therefore, } 2x &= \pi/2 \\ \Rightarrow x &= \pi/4 \\ 3x &= \pi/2 \\ \Rightarrow x &= \pi/6 \\ \text{Or } 3x &= 3\pi/2 \\ \Rightarrow x &= \pi/2 \\ \text{Hence, } x &= \pi/6, \pi/4, \pi/2. \end{aligned}$$

**20. If  $\sin(\theta + \alpha) = a$  and  $\sin(\theta + \beta) = b$ , then prove that  $\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$**

**Solution:**

According to the question,  
 $\sin(\theta + \alpha) = a$  and  $\sin(\theta + \beta) = b$   
 $\text{LHS} = \cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta)$   
 Using  $\cos 2x = 2\cos^2 x - 1$ ,  
 Let us solve,  
 $\Rightarrow \text{LHS} = 2\cos^2(\alpha - \beta) - 1 - 4ab \cos(\alpha - \beta)$

$$\Rightarrow \text{LHS} = 2\cos(\alpha - \beta) \{ \cos(\alpha - \beta) - 2ab \} - 1$$

Since,

$$\cos(\alpha - \beta) = \cos \{ (\theta + \alpha) - (\theta + \beta) \}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\Rightarrow \cos(\alpha - \beta) = \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$$

Since,

$$\sin(\theta + \alpha) = a$$

$$\Rightarrow \cos(\theta + \alpha) = \sqrt{1 - \sin^2(\theta + \alpha)} = \sqrt{1 - a^2}$$

Similarly,

$$\cos(\theta + \beta) = \sqrt{1 - b^2}$$

Therefore,

$$\cos(\alpha - \beta) = \sqrt{1 - a^2}\sqrt{1 - b^2} + ab$$

Therefore,

$$\text{LHS} = 2 \{ ab + \sqrt{1 - a^2}(1 - b^2) \} \{ ab + \sqrt{1 - a^2}(1 - b^2) - 2ab \} - 1$$

$$\Rightarrow \text{LHS} = 2 \{ \sqrt{1 - a^2}(1 - b^2) + ab \} \{ \sqrt{1 - a^2}(1 - b^2) - ab \} - 1$$

$$\text{Using } (x + y)(x - y) = x^2 - y^2$$

$$\Rightarrow \text{LHS} = 2 \{ (1 - a^2)(1 - b^2) - a^2b^2 \} - 1$$

$$\Rightarrow \text{LHS} = 2 \{ 1 - a^2 - b^2 + a^2b^2 \} - 1$$

$$\Rightarrow \text{LHS} = 2 - 2a^2 - 2b^2 - 1$$

$$\Rightarrow \text{LHS} = 1 - 2a^2 - 2b^2 = \text{RHS}$$

Therefore,

We get,

$$\cos 2(\alpha - \beta) - 4ab \cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$$

**21. If  $\cos(\theta + \phi) = m \cos(\theta - \phi)$ , then prove that  $\tan \theta = ((1 - m)/(1 + m)) \cot \phi$**

**[Hint: Express  $\cos(\theta + \phi)/\cos(\theta - \phi) = m/1$  and apply Componendo and Dividendo]**

**Solution:**

According to the question,

$$\begin{aligned}\cos(\theta + \phi) &= m \cos(\theta - \phi) \\ \therefore \cos(\theta + \phi) &= m \cos(\theta - \phi) \\ \frac{\cos(\theta - \phi)}{\cos(\theta + \phi)} &= \frac{1}{m}\end{aligned}$$

Applying componendo – dividend, we get,

$$\begin{aligned}\frac{\cos(\theta - \phi) + \cos(\theta + \phi)}{\cos(\theta - \phi) - \cos(\theta + \phi)} &= \frac{1+m}{1-m}\end{aligned}$$

From transformation formula, we know that,

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$$

$$\begin{aligned}\frac{2\cos\theta\cos\phi}{2\sin\theta\sin\phi} &= \frac{1+m}{1-m}\end{aligned}$$

Since,  $(\cos\theta)/(\sin\theta) = \cot\theta$

$$\begin{aligned}\Rightarrow \cot\theta \cot\phi &= \frac{1+m}{1-m} \\ \Rightarrow \left(\frac{1-m}{1+m}\right) \cot\phi &= \frac{1}{\cot\theta} \\ \Rightarrow \tan\theta &= \left(\frac{1-m}{1+m}\right) \cot\phi\end{aligned}$$

## 22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$$

**Solution:**

According to the question,

$$\text{Let, } y = 3[\sin^4(3\pi/2 - \alpha) + \sin^4(3\pi + \alpha)] - 2[\sin^6(\pi/2 + \alpha) + \sin^6(5\pi - \alpha)]$$

We know that,

$$\sin(3\pi/2 - \alpha) = -\cos\alpha$$

$$\sin(3\pi + \alpha) = -\sin\alpha$$

$$\sin(\pi/2 + \alpha) = \cos\alpha$$

$$\sin(5\pi - \alpha) = \sin\alpha$$

Therefore,

$$y = 3[(-\cos\alpha)^4 + (-\sin\alpha)^4] - 2[\cos^6\alpha + \sin^6\alpha]$$

$$\Rightarrow y = 3[\cos^4\alpha + \sin^4\alpha] - 2[\sin^6\alpha + \cos^6\alpha]$$

$$\Rightarrow y = 3[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha \cos^2\alpha] - 2[(\sin^2\alpha)^3 + (\cos^2\alpha)^3]$$

Since, we know that,

$$\sin^2\alpha + \cos^2\alpha = 1$$

Also, we know that,

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\Rightarrow y = 3[1 - 2\sin^2\alpha \cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)(\cos^4\alpha + \sin^4\alpha - \sin^2\alpha \cos^2\alpha)]$$

$$\Rightarrow y = 3[1 - 2\sin^2\alpha \cos^2\alpha] - 2[\cos^4\alpha + \sin^4\alpha - \sin^2\alpha \cos^2\alpha]$$

$$\Rightarrow y = 3[1 - 2\sin^2\alpha \cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha \cos^2\alpha - \sin^2\alpha \cos^2\alpha]$$

$$\Rightarrow y = 3[1 - 2\sin^2\alpha \cos^2\alpha] - 2[1 - 3\sin^2\alpha \cos^2\alpha]$$

$$\Rightarrow y = 3 - 6\sin^2\alpha \cos^2\alpha - 2 + 6\sin^2\alpha \cos^2\alpha$$

$$\Rightarrow y = 1$$

**23. If  $a \cos 2\theta + b \sin 2\theta = c$  has  $\alpha$  and  $\beta$  as its roots, then prove that  $\tan \alpha + \tan \beta = 2b/(a + c)$   
[Hint: Use the identities  $\cos 2\theta = ((1 - \tan^2 \theta)/(1 + \tan^2 \theta))$  and  $\sin 2\theta = 2 \tan \theta / (1 + \tan^2 \theta)$ ]**

**Solution:**

According to the question,

$$a \cos 2\theta + b \sin 2\theta = c$$

$\alpha$  and  $\beta$  are the roots of the equation.

Using the formula of multiple angles,

We know that,

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \text{ and } \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\therefore a \left( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) - c = 0$$

$$\Rightarrow a(1 - \tan^2 \theta) + 2b \tan \theta - c(1 + \tan^2 \theta) = 0$$

$$\Rightarrow (-c - a)\tan^2 \theta + 2b \tan \theta - c + a = 0 \dots (i)$$

We know that,

The sum of roots of a quadratic equation,  $ax^2 + bx + c = 0$  is given by  $(-b/a)$

Therefore,

$$\tan \alpha + \tan \beta = -2b/(-c + a) = 2b/(c + a)$$

$$\text{Hence, } \tan \alpha + \tan \beta = 2b/(c + a)$$

**24. If  $x = \sec \phi - \tan \phi$  and  $y = \operatorname{cosec} \phi + \cot \phi$ , then show that  $xy + x - y + 1 = 0$ .**

**[Hint: Find  $xy + 1$  and then show  $\tan x - y = -(xy + 1)$ ]**

**Solution:**

According to the question,

$$x = \sec \phi - \tan \phi \text{ and } y = \operatorname{cosec} \phi + \cot \phi$$

Given that, LHS =  $xy + x - y + 1$

$$= (\sec \phi - \tan \phi)(\operatorname{cosec} \phi + \cot \phi) + (\sec \phi - \tan \phi) - (\operatorname{cosec} \phi + \cot \phi) + 1$$

$$= \sec \phi \operatorname{cosec} \phi + \cot \phi \sec \phi - \tan \phi \cot \phi - \tan \phi \operatorname{cosec} \phi$$

$$+ \sec \phi - \tan \phi - (\operatorname{cosec} \phi + \cot \phi) + 1$$

$$= \frac{1}{\sin \phi \cos \phi} + \frac{1}{\sin \phi} - 1 - \sec \phi + \sec \phi - \tan \phi - \left( \frac{1}{\sin \phi} + \frac{\cos \phi}{\sin \phi} \right) + 1$$

$$= \frac{1}{\sin \phi \cos \phi} + \frac{1}{\sin \phi} - \tan \phi - \left( \frac{1}{\sin \phi} + \frac{\cos \phi}{\sin \phi} \right)$$

$$= \frac{1}{\sin \phi \cos \phi} - \frac{\sin \phi}{\cos \phi} - \frac{\cos \phi}{\sin \phi}$$

$$= \frac{1}{\sin \phi \cos \phi} - \left( \frac{\cos \phi}{\sin \phi} + \frac{\sin \phi}{\cos \phi} \right)$$

$$= \frac{1}{\sin \phi \cos \phi} - \left( \frac{\cos^2 \phi + \sin^2 \phi}{\sin \phi \cos \phi} \right)$$

$$\text{Since, } \sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{1}{\sin \phi \cos \phi} - \left( \frac{1}{\sin \phi \cos \phi} \right) = 0$$

$$\text{Thus, LHS} = xy + x - y + 1 = 0$$

**25. If  $\theta$  lies in the first quadrant and  $\cos \theta = 8/17$ , then find the value of  $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$**

**Solution:**



According to the question,

$$\cos \theta = 8/17$$

$$\sin \theta = \pm\sqrt{1 - \cos^2 \theta}$$

Since,  $\theta$  lies in first quadrant, only positive sign can be considered.

$$\Rightarrow \sin \theta = \sqrt{1 - 64/289} = 15/17$$

$$\text{Let, } y = \cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$$

We know that,

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Therefore,

$$y = \cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta + \cos 45^\circ \cos \theta + \sin 45^\circ \sin \theta + \cos 120^\circ \cos \theta + \sin 120^\circ \sin \theta$$

Substituting values of  $\cos 30^\circ$ ,  $\sin 30^\circ$ ,  $\cos 120^\circ$ ,  $\sin 120^\circ$  and  $\cos 45^\circ$

$$\begin{aligned} \Rightarrow y &= \frac{\sqrt{3}}{2} \cdot \frac{8}{17} - \frac{1}{2} \cdot \frac{15}{17} + \frac{1}{\sqrt{2}} \cdot \frac{8}{17} + \frac{1}{\sqrt{2}} \cdot \frac{15}{17} + \left(-\frac{1}{2}\right) \left(\frac{8}{17}\right) + \left(\frac{\sqrt{3}}{2}\right) \left(\frac{15}{17}\right) \\ &= \frac{8\sqrt{3}}{34} - \frac{15}{34} + \frac{8 + 15}{17\sqrt{2}} - \frac{8}{34} + \frac{15\sqrt{3}}{34} \\ &= \frac{23\sqrt{3}}{34} + \frac{23}{17\sqrt{2}} - \frac{23}{34} \\ \Rightarrow y &= \frac{23}{34} (\sqrt{3} + \sqrt{2} - 1) \end{aligned}$$

**26. Find the value of the expression  $\cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8)$ .**

**[Hint: Simplify the expression to**

$$2 \left( \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} \right) = 2 \left[ \left( \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} \right)^2 - 2 \cos^2 \frac{\pi}{8} \cos^2 \frac{3\pi}{8} \right]$$

**Solution:**

According to the question,

$$\text{Let } y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8).$$

$$\Rightarrow y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(\pi - 3\pi/8) + \cos^4(\pi - \pi/8).$$

Since we know that,  $\cos(\pi - x) = -\cos x$ , we get,



$$\begin{aligned}
 &= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \left( \frac{3\pi}{8} \right) + \cos^4 \left( \frac{\pi}{8} \right) \\
 &= 2 \left( \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} \right) \\
 &= 2 \left( \cos^4 \frac{\pi}{8} + \cos^4 \left( \frac{\pi}{2} - \frac{\pi}{8} \right) \right) \\
 &= 2 \left( \cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right) \\
 &= 2 \left[ \left( \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)^2 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\
 &= 2 \left[ 1 - 2 \cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8} \right] \\
 &= 2 - \left( 2 \cos \frac{\pi}{8} \cdot \sin \frac{\pi}{8} \right)^2 \\
 &= 2 - \left( \sin \frac{2\pi}{8} \right)^2 \\
 &= 2 - (1/\sqrt{2})^2 \\
 &= 2 - 1/2 \\
 &= 3/2
 \end{aligned}$$

**27. Find the general solution of the equation**

$$5\cos^2\theta + 7\sin^2\theta - 6 = 0$$

**Solution:**

According to the question,

$$5\cos^2\theta + 7\sin^2\theta - 6 = 0$$

We know that,

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\text{Therefore, } 5\cos^2\theta + 7(1 - \cos^2\theta) - 6 = 0$$

$$\Rightarrow 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0$$

$$\Rightarrow -2\cos^2\theta + 1 = 0$$

$$\Rightarrow \cos^2\theta = 1/2$$

$$\text{Therefore, } \cos \theta = \pm 1/\sqrt{2}$$

$$\text{Therefore, } \cos \theta = \cos \pi/4 \text{ or } \cos \theta = \cos 3\pi/4$$

Since, solution of  $\cos x = \cos \alpha$  is given by

$$x = 2m\pi \pm \alpha \quad \forall m \in \mathbb{Z}$$

$$\theta = n\pi \pm \pi/4, n \in \mathbb{Z}$$

**28. Find the general solution of the equation  $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$**

**Solution:**

According to the question,

$$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$$

Grouping  $\sin x$  and  $\sin 3x$  in LHS and,  $\cos x$  and  $\cos 3x$  in RHS,

We get,

$$\sin x + \sin 3x - 3\sin 2x = \cos x + \cos 3x - 3\cos 2x$$

Applying transformation formula,

$$\cos A + \cos B = 2\cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\begin{aligned} \sin A + \sin B &= 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \\ \Rightarrow 2 \sin \left( \frac{3x+x}{2} \right) \cos \left( \frac{3x-x}{2} \right) - 3 \sin 2x &= 2 \cos \left( \frac{3x+x}{2} \right) \cos \left( \frac{3x-x}{2} \right) - 3 \cos 2x \\ \Rightarrow 2 \sin 2x \cos x - 3 \sin 2x &= 2 \cos 2x \cos x - 3 \cos 2x \\ \Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x &= 0 \\ \Rightarrow 2 \cos x (\sin 2x - \cos 2x) - 3(\sin 2x - \cos 2x) &= 0 \\ \Rightarrow (\sin 2x - \cos 2x)(2 \cos x - 3) &= 0 \\ \Rightarrow \cos x = 3/2 \text{ or } \sin 2x = \cos 2x \\ \text{As } \cos x \in [-1, 1] \\ \text{Hence, no value of } x \text{ exists for which } \cos x = 3/2 \\ \text{Therefore, } \sin 2x = \cos 2x \\ \Rightarrow \tan 2x = 1 = \tan \pi/4 \\ \text{We know solution of } \tan x = \tan \alpha \text{ is given by,} \\ x = n\pi \pm \alpha, n \in \mathbb{Z} \\ \text{Therefore, } 2x = n\pi \pm (\pi/4) \\ \Rightarrow x = n\pi/2 \pm (\pi/8), n \in \mathbb{Z} \end{aligned}$$

**29. Find the general solution of the equation  $(\sqrt{3}-1) \cos \theta + (\sqrt{3}+1) \sin \theta = 2$**   
**[Hint: Put  $\sqrt{3}-1 = r \sin \alpha$ ,  $\sqrt{3}+1 = r \cos \alpha$  which gives  $\tan \alpha = \tan((\pi/4) - (\pi/6))$   $\alpha = \pi/12$ ]**  
**Solution:**

$$\begin{aligned} \text{Let, } r \sin \alpha &= \sqrt{3}-1 \text{ and } r \cos \alpha = \sqrt{3}+1 \\ \text{Therefore, } r &= \sqrt{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2} = \sqrt{8} = 2\sqrt{2} \\ \text{And, } \tan \alpha &= (\sqrt{3}-1) / (\sqrt{3}+1) \\ \text{Therefore, } r(\sin \alpha \cos \theta + \cos \alpha \sin \theta) &= 2 \\ \Rightarrow r \sin (\theta + \alpha) &= 2 \\ \Rightarrow \sin (\theta + \alpha) &= 1/\sqrt{2} \\ \Rightarrow \sin (\theta + \alpha) &= \sin (\pi/4) \\ \Rightarrow \theta + \alpha &= n\pi + (-1)^n (\pi/4), n \in \mathbb{Z} \\ \Rightarrow \theta &= n\pi + (-1)^n (\pi/4) - (\pi/12), n \in \mathbb{Z} \end{aligned}$$

## OBJECTIVE TYPE QUESTIONS

**30. If  $\sin \theta + \operatorname{cosec} \theta = 2$ , then  $\sin^2 \theta + \operatorname{cosec}^2 \theta$  is equal to**

- A. 1
- B. 4
- C. 2
- D. None of these

**Solution:**

C. 2

Explanation:

According to the question,

$$\sin \theta + \operatorname{cosec} \theta = 2$$

Squaring LHS and RHS,

We get,

$$\Rightarrow (\sin \theta + \operatorname{cosec} \theta)^2 = 2^2$$

$$\Rightarrow \sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta = 4$$

$$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2 \sin \theta (1/\sin)\theta = 4$$

$$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2 = 4$$

$$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 2$$

Thus, option (C) 2 is the correct answer.

31. If  $f(x) = \cos^2x + \sec^2x$ , then

A.  $f(x) < 1$

B.  $f(x) = 1$

C.  $2 < f(x) < 1$

D.  $f(x) \geq 2$

[Hint: A.M  $\geq$  G.M.]

Solution:

D.  $f(x) \geq 2$

Explanation:

According to the question,

We have,  $f(x) = \cos^2x + \sec^2x$

We know that, A.M  $\geq$  G.M.

$$\Rightarrow \frac{\cos^2x + \sec^2x}{2} \geq \sqrt{\cos^2x \sec^2x}$$

$$\Rightarrow \frac{\cos^2x + \sec^2x}{2} \geq \sqrt{\cos^2x \frac{1}{\cos^2x}}$$

$$\Rightarrow \frac{\cos^2x + \sec^2x}{2} \geq 1$$

$$\Rightarrow \cos^2x + \sec^2x \geq 2$$

$$\Rightarrow f(x) \geq 2$$

Thus, option (D)  $f(x) \geq 2$  is the correct answer.

32. If  $\tan \theta = 1/2$  and  $\tan \phi = 1/3$ , then the value of  $\theta + \phi$  is

A.  $\pi/6$

B.  $\pi$

C. 0

D.  $\pi/4$

Solution:

D.  $\pi/4$

Explanation:

According to the question,

$$\tan \theta = \frac{1}{2} \text{ and } \tan \phi = \frac{1}{3}$$

We know that,

$$\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow \tan(\theta + \phi) = \tan \frac{\pi}{4}$$

Thus, option (D)  $\pi/4$  is the correct answer.

**33. Which of the following is not correct?**

A.  $\sin \theta = -1/5$

B.  $\cos \theta = 1$

C.  $\sec \theta = 1/2$

D.  $\tan \theta = 20$

**Solution:**

C.  $\sec \theta = 1/2$

Explanation:

According to the question,

We know that,

a)  $\sin \theta = -1/5$  is correct since  $\sin \theta \in [-1, 1]$

b)  $\cos \theta = 1$  is correct since  $\cos \theta \in [-1, 1]$

c)  $\sec \theta = 1/2$

$$\Rightarrow (1/\cos \theta) = 1/2$$

$$\Rightarrow \cos \theta = 2 \text{ is incorrect since } \cos \theta \in [-1, 1]$$

d)  $\tan \theta = 20$  is correct since  $\tan \theta \in \mathbb{R}$ .

Thus, option (C)  $\sec \theta = 1/2$  is the correct answer.

**34. The value of  $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$  is**

A. 0

B. 1

C.  $1/2$

D. Not defined

**Solution:**

B. 1

Explanation:

According to the question,

$$\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$$

$$= \tan 1^\circ \tan 2^\circ \dots \tan 45^\circ \tan (90-44^\circ) \tan (90-43^\circ) \dots \tan (90-1^\circ)$$

$$= \tan 1^\circ \tan 2^\circ \dots \tan 45^\circ \cot 44^\circ \cot 43^\circ \dots \cot 1^\circ [\because \tan (90-\theta) = \cot \theta]$$

$$= \tan 1^\circ \cot 1^\circ \tan 2^\circ \cot 2^\circ \dots \tan 45^\circ \dots \tan 89^\circ \cot 89^\circ$$

$$= 1.1 \dots 1 = 1$$

Thus, option (B) 1 is the correct answer.

35. The value of  $(1 - \tan^2 15^\circ)/(1 + \tan^2 15^\circ)$  is

- A. 1
- B.  $\sqrt{3}$
- C.  $\sqrt{3}/2$
- D. 2

**Solution:**

C.  $\sqrt{3}/2$

Explanation:

According to the question,

Let  $\theta = 15^\circ \Rightarrow 2\theta = 30^\circ$

Now, since we know that,

$$\begin{aligned}\cos 2\theta &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \\ \Rightarrow \cos 30^\circ &= \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} \\ \Rightarrow \frac{\sqrt{3}}{2} &= \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}\end{aligned}$$

Thus, option (C)  $\sqrt{3}/2$  is the correct answer.

36. The value of  $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ$  is

- A.  $1/\sqrt{2}$
- B. 0
- C. 1
- D. -1

**Solution:**

B. 0

Explanation:

According to the question,

Since  $\cos 90^\circ = 0$

We get,

$$\Rightarrow \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 90^\circ \dots \cos 179^\circ = 0$$

Thus, option (B) 0 is the correct answer.

37. If  $\tan \theta = 3$  and  $\theta$  lies in third quadrant, then the value of  $\sin \theta$  is

- A.  $1/\sqrt{10}$
- B.  $-1/\sqrt{10}$
- C.  $-3/\sqrt{10}$
- D.  $3/\sqrt{10}$

**Solution:**

C.  $-3/\sqrt{10}$

Explanation:

According to the question,

Given that,  $\tan \theta = 3$  and  $\theta$  lies in third quadrant

$$\Rightarrow \cot \theta = 1/3$$

We know that,

$$\operatorname{Cosec}^2 \theta = 1 + \cot^2 \theta$$

$$= 1 + \left(\frac{1}{3}\right)^2 = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\Rightarrow \sin^2 \theta = \frac{9}{10}$$

$$\Rightarrow \sin \theta = \pm \frac{3}{\sqrt{10}}$$

$$\Rightarrow \sin \theta = -\frac{3}{\sqrt{10}}, \text{ since } \theta \text{ lies in third quadrant.}$$

Thus, option (C)  $-3/\sqrt{10}$  is the correct answer.

**38. The value of  $\tan 75^\circ - \cot 75^\circ$  is equal to**

A.  $2\sqrt{3}$

B.  $2 + \sqrt{3}$

C.  $2 - \sqrt{3}$

D. 1

**Solution:**

A.  $2\sqrt{3}$

Explanation:

According to the question,

We have,

$$\tan 75^\circ - \cot 75^\circ$$

$$= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ}$$

$$= \frac{\cos 75^\circ \sin 75^\circ - \sin 75^\circ \cos 75^\circ}{\sin^2 75^\circ - \cos^2 75^\circ}$$

$$= \frac{2 \cos 75^\circ \sin 75^\circ}{2(\sin^2 75^\circ - \cos^2 75^\circ)}$$

$$= \frac{2 \cos 75^\circ \sin 75^\circ}{\sin 150^\circ}$$

$$= -2 \cot 150^\circ$$

$$= -2 \cot (180^\circ - 30^\circ)$$

$$= 2 \cot 30^\circ$$

$$= 2\sqrt{3}$$

Thus, option (A)  $2\sqrt{3}$  is the correct answer.