

SHORT ANSWER TYPE

Solve for x , the inequalities in Exercises 1 to 12.

1. Solve for x , the inequalities in

$$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, (x > 0)$$

Solution:

According to the question,

$$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}$$

Multiplying each term by $(x+1)$

$$\Rightarrow 4 \leq 3(x+1) \leq 6$$

$$\Rightarrow 4 \leq 3x + 3 \leq 6$$

Subtracting each term by 3, we get,

$$\Rightarrow 1 \leq 3x \leq 3$$

Dividing each term by 3, we get,

$$\Rightarrow (1/3) \leq x \leq 1$$

2. Solve for x , the inequalities in

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

Solution:

According to the question,

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

Let $y = |x-2|$, then

$$\Rightarrow \frac{y-1}{y-2} \leq 0$$

Now, if $y < 1$, then

$$y-1 < 0 \text{ and } y-2 < 0$$

$$\frac{y-1}{y-2} > 0$$

and, $\frac{y-1}{y-2}$, which is not required

if $y > 2$, then

$$y-1 > 0 \text{ and } y-2 > 0$$

$$\frac{y-1}{y-2} > 0$$

and, $\frac{y-1}{y-2}$, which is not required

if $1 \leq y < 2$, then

$$y-1 \geq 0 \text{ and } y-2 < 0$$

and,

$$\frac{y-1}{y-2} < 0$$

, which is the required answer,

Hence,

$$1 \leq y < 2$$

$$\Rightarrow 1 \leq |x - 2| < 2$$

Here, there are two cases

$$\Rightarrow 1 \leq x - 2 < 2$$

$$\Rightarrow 3 \leq x < 4$$

And

$$\Rightarrow 1 \leq -(x - 2) < 2$$

$$\Rightarrow 1 \leq -x + 2 < 2$$

Multiplying each term by -1,

$$\Rightarrow -2 \leq x - 2 < -1$$

Adding 2 to each term,

$$\Rightarrow 0 \leq x < 1$$

\therefore Hence, solution is $[0, 1) \cup [3, 4)$

3. Solve for x, the inequalities in

$$\frac{1}{|x| - 3} \leq \frac{1}{2}$$

Solution:

According to the question,

$$\frac{1}{|x| - 3} \leq \frac{1}{2}$$

$$\Rightarrow \frac{1}{|x| - 3} - \frac{1}{2} \leq 0$$

$$\Rightarrow \frac{2 - |x| + 3}{2(|x| - 3)} \leq 0$$

$$\Rightarrow \frac{5 - |x|}{(|x| - 3)} \leq 0$$

$$\Rightarrow 5 - |x| \leq 0 \text{ and } |x| - 3 > 0 \text{ or } 5 - |x| \geq 0 \text{ and } |x| - 3 < 0$$

$$\Rightarrow |x| \geq 5 \text{ and } |x| > 3 \text{ or } |x| \leq 5 \text{ and } |x| < 3$$

$$\Rightarrow |x| \geq 5 \text{ or } |x| < 3$$

$$\Rightarrow x \in (-\infty, -5] \text{ or } [5, \infty) \text{ or } x \in (-3, 3)$$

$$\Rightarrow x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$$

4. Solve for x, the inequalities in $|x - 1| \leq 5$, $|x| \geq 2$

Solution:

$$|x - 1| \leq 5$$

There are two cases,

1:-

$$x - 1 \leq 5$$

Adding 1 to LHS and RHS

$$\Rightarrow x \leq 6$$

2:-

$$\Rightarrow -(x - 1) \leq 5$$

$$\Rightarrow -x + 1 \leq 5$$

Subtracting 1 from LHS and RHS,

$$\Rightarrow -x \leq 4$$

$$\Rightarrow x \geq -4$$

From cases 1 and 2, we have

$$\Rightarrow -4 \leq x \leq 6 \dots [i]$$

Also,

$$|x| \geq 2$$

$$\Rightarrow x \geq 2 \text{ and}$$

$$\Rightarrow -x \geq 2$$

$$\Rightarrow x \leq -2$$

$$\Rightarrow x \in (\infty, -2] \cup [2, \infty) \dots [ii]$$

Combining equation [i] and [ii], we get

$$x \in [-4, -2] \cup [2, 6]$$

5. Solve for x, the inequalities in

$$-5 \leq \frac{2-3x}{4} \leq 9$$

Solution:

According to the question,

$$-5 \leq \frac{2-3x}{4} \leq 9$$

Multiplying each term by 4, we get

$$\Rightarrow -20 \leq 2 - 3x \leq 36$$

Adding -2 each term, we get

$$\Rightarrow -22 \leq -3x \leq 34$$

Dividing each term by 3, we get

$$\Rightarrow -22/3 \leq -x \leq 34/3$$

We know that,

Multiplication by -1 inverts the inequality.

So, multiplying each term by -1, we get

$$\Rightarrow -34/3 \leq x \leq 22/3$$

6. Solve for x, the inequalities in $4x + 3 \geq 2x + 17$, $3x - 5 < -2$.

Solution:

According to the question,

$$4x + 3 \geq 2x + 17$$

$$\Rightarrow 4x - 2x \geq 17 - 3$$

$$\Rightarrow 2x \geq 14$$

$$\Rightarrow x \geq 7 \dots (i)$$

Also,

$$3x - 5 < -2$$

$$\Rightarrow 3x < 3$$

$$\Rightarrow x < 1 \dots (2)$$

Since, equations [i] and [ii] cannot be possible simultaneously,

We conclude that x has no solution.

7. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26,000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

Solution:

We know that,

Profit = Revenue – cost

Requirement is, profit > 0

According to the question,

Revenue, $R(x) = 43x$

Cost, $C(x) = 26,000 + 30x$; where x is number of cassettes

\Rightarrow Profit = $43x - (26,000 + 30x) > 0$

$\Rightarrow 13x - 26,000 > 0$

$\Rightarrow 13x > 26,000$

$\Rightarrow x > 2000$

Therefore, the company should sell more than 2000 cassettes to realise profit.

8. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.

Solution:

According to the question,

First reading = 8.48

Second reading = 8.35

Now, let the third reading be ' x '

Average pH should be between 8.2 and 8.5

Average pH = $(8.48 + 8.35 + x)/3$

$$\Rightarrow 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

Multiplying each term by 3, we get

$$\Rightarrow 24.6 < 16.83 + x < 25.5$$

Subtracting 16.83 from each term, we get

$$\Rightarrow 7.77 < x < 8.67$$

Therefore, from the above equation,

We get that,

The third pH reading should be between 7.77 and 8.67

9. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

Solution:

According to the question,

Let x litres of 3% solution is to be added to 460 liters of the 9% of solution

Then, we get,

Total solution = $(460 + x)$ litres

Total acid content in resulting solution

$$= (460 \times 9/100 + x \times 3/100)$$

$$= (41.4 + 0.03x)\%$$

According to the question, we have,

Resulting mixture should be more than 5% acidic but less than 7% acidic

So we get,

$$\Rightarrow 5\% \text{ of } (460 + x) < 41.4 + 0.03x < 7\% \text{ of } (460 + x)$$

$$\Rightarrow 5/100 \times (460 + x) < 41.4 + 0.03x < 7/100 \times (460 + x)$$

$$\Rightarrow 23 + 0.05x < 41.4 + 0.03x < 32.2 + 0.07x$$

Now, we have

$$\Rightarrow 23 + 0.05x < 41.4 + 0.03x \text{ and } 41.4 + 0.03x < 32.2 + 0.07x$$

$$\text{i.e., } 0.02x < 18.4 \text{ and } 0.04x > 9.2$$

$$\Rightarrow 2x < 1840 \text{ and } 4x > 920$$

$$\Rightarrow x < 920 \text{ and } x > 230$$

$$\Rightarrow 230 < x < 920$$

Hence, solution between 230 l and 920 l should be added.

10. A solution is to be kept between 40°C and 45°C . What is the range of temperature in degree Fahrenheit, if the conversion formula is $F = 9/5 C + 32$?

Solution:

Let temperature in Celsius be C

Let temperature in Fahrenheit be F

According to the question,

Solution should be kept between 40°C and 45°C

$$\Rightarrow 40 < C < 45$$

Multiplying each term by $9/5$, we get

$$\Rightarrow 72 < 9/5 C < 81$$

Adding 32 to each term, we get

$$\Rightarrow 104 < 9/5 C + 32 < 113$$

$$\Rightarrow 104 < F < 113$$

Hence, the range of temperature in Fahrenheit should be between 104°F and 113°F .

11. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

Solution:

Let the length of shortest side = 'x' cm

According to the question,

The longest side of a triangle is twice the shortest side

$$\Rightarrow \text{Length of largest side} = 2x$$

Also, the third side is 2 cm longer than the shortest side

$$\Rightarrow \text{Length of third side} = (x + 2) \text{ cm}$$

Perimeter of triangle = sum of three sides

$$= x + 2x + x + 2$$

$$= 4x + 2 \text{ cm}$$

Now, we know that,

Perimeter is more than 166 cm

$$\Rightarrow 4x + 2 \geq 166$$

$$\Rightarrow 4x \geq 164$$

$$\Rightarrow x \geq 41$$

Hence, minimum length of the shortest side should be = 41 cm.

12. In drilling world's deepest hole it was found that the temperature T in degree Celsius, x km below the earth's surface was given by $T = 30 + 25(x - 3)$, $3 \leq x \leq 15$. At what depth will the temperature be between 155°C and 205°C ?

Solution:

According to the question,

$T = 30 + 25(x - 3)$, $3 \leq x \leq 15$; where, T = temperature and x = depth inside the earth

The Temperature should be between 155°C and 205°C ,

So, we get,

$$\Rightarrow 155 < T < 205$$

$$\Rightarrow 155 < 30 + 25(x - 3) < 205$$

$$\Rightarrow 155 < 30 + 25x - 75 < 205$$

$$\Rightarrow 155 < 25x - 45 < 205$$

Adding 45 to each term, we get

$$\Rightarrow 200 < 25x < 250$$

Dividing each term by 25, we get

$$\Rightarrow 8 < x < 10$$

Hence, temperature varies from 155°C to 205°C at a depth of 8 km to 10 km.

LONG ANSWER TYPE

13. Solve the following system of inequalities

$$\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$$

Solution:

According to the question,

$$\frac{2x+1}{7x-1} > 5$$

Subtracting 5 both side, we get

$$\frac{2x+1}{7x-1} - 5 > 0$$

$$\Rightarrow \frac{2x+1-35x+5}{7x-1} > 0$$

$$\Rightarrow \frac{6-33x}{7x-1} > 0$$

For above fraction be greater than 0, either both denominator and numerator should be greater than 0 or both should be less than 0.

$$\Rightarrow 6 - 33x > 0 \text{ and } 7x - 1 > 0$$

$$\Rightarrow 33x < 6 \text{ and } 7x > 1$$

$$\Rightarrow x < 2/11 \text{ and } x > 1/7$$

$$\Rightarrow 1/7 < x < 2/11 \dots(i)$$

Or

$$\Rightarrow 6 - 33x < 0 \text{ and } 7x - 1 < 0$$

$$\Rightarrow 33x > 6 \text{ and } 7x < 1$$

$$\Rightarrow x > 2/11 \text{ and } x < 1/7$$

$$\Rightarrow 2/11 < x < 1/7 \dots(\text{which is not possible since } 1/7 > 2/11)$$

Also,

$$\frac{x+7}{x-8} > 2$$

Subtracting 2 both sides, we get

$$\Rightarrow \frac{x+7}{x-8} - 2 > 0$$

$$\Rightarrow \frac{x+7-2x+16}{x-8} > 0$$

$$\Rightarrow \frac{23-x}{x-8} > 0$$

For above fraction to be greater than 0, either both denominator and numerator should be greater than 0 or both should be less than 0.

$$\Rightarrow 23 - x > 0 \text{ and } x - 8 > 0$$

$$\Rightarrow x < 23 \text{ and } x > 8$$

$$\Rightarrow 8 < x < 23 \dots(ii)$$

Or

$$23 - x < 0 \text{ and } x - 8 < 0$$

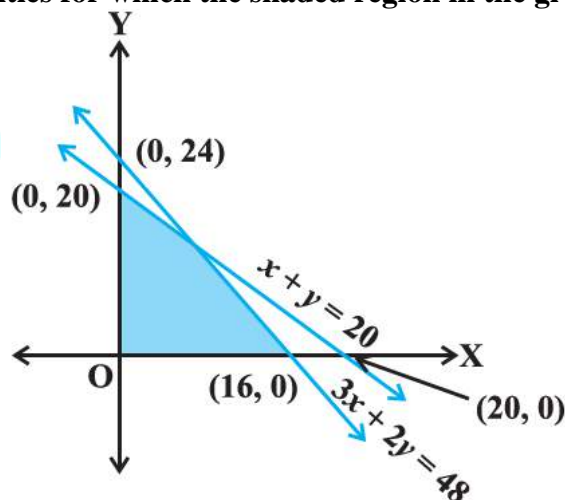
$$\Rightarrow x > 23 \text{ and } x < 8$$

$$\Rightarrow 23 < x < 8 \dots(\text{which is not possible, as } 23 > 8)$$

Therefore, from equations (i) and (ii), we infer that there is no solution satisfying both inequalities.

Hence, the given system has no solution.

14. Find the linear inequalities for which the shaded region in the given figure is the solution set.



Solution:

According to the question,

Considering $3x + 2y = 48$,

The shaded region and the origin both are on the same side of the graph of the line and $(0, 0)$ satisfy the constraint $3x + 2y \leq 48$.

Considering $x + y = 20$,

The shaded region and the origin both are on the same side of the graph of the line and $(0, 0)$ satisfy the constraint $x + y \leq 20$.

We also know that,

Shaded region is in the first quadrant i.e. $x \geq 0$ and $y \geq 0$,

Hence, the linear inequalities are

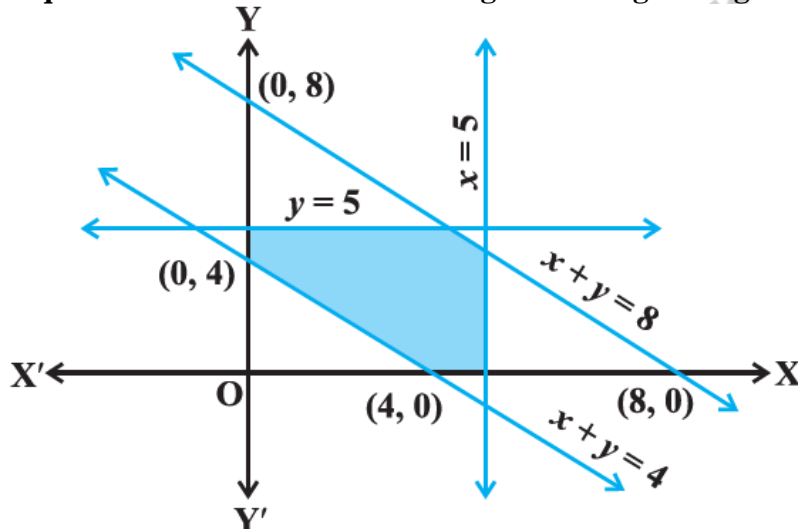
$$3x + 2y \leq 48$$

$$x + y \leq 20$$

$$x \geq 0$$

$$y \geq 0$$

15. Find the linear inequalities for which the shaded region in the given figure is the solution set.



Solution:

According to the question,

Considering $x + y = 8$,

The shaded region and the origin both are on the same side of the graph of the line and $(0, 0)$ satisfy the constraint $x + y \leq 8$.

Considering $x + y = 4$,

The origin is on the opposite side of the shaded region and $(0, 0)$, hence, doesn't satisfy the constraint $x + y \geq 4$, therefore required constraint is $x + y \geq 4$

We see that,

The shaded region is in the first quadrant i.e. $x \geq 0$ and $y \geq 0$,

Also, shaded region is below the line $y = 5$ and left to the line $x = 5$

$$\Rightarrow y \leq 5 \text{ and } x \leq 5$$

Hence, the linear inequalities are

$$x + y \leq 8$$

$$x + y \geq 4$$

$$x \geq 0$$

$$y \geq 0$$

$$x \leq 5$$

$$y \leq 5$$

16. Show that the following system of linear inequalities has no solution $x + 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

Solution:

According to the question,

$$x + 2y \leq 3$$

$$\text{Line: } x + 2y = 3$$

x	3	1
y	0	1

Also, (0, 0) satisfies the $x + 2y \leq 3$, hence region is towards the origin

$$3x + 4y \leq 12$$

$$\text{Line: } 3x + 4y = 12$$

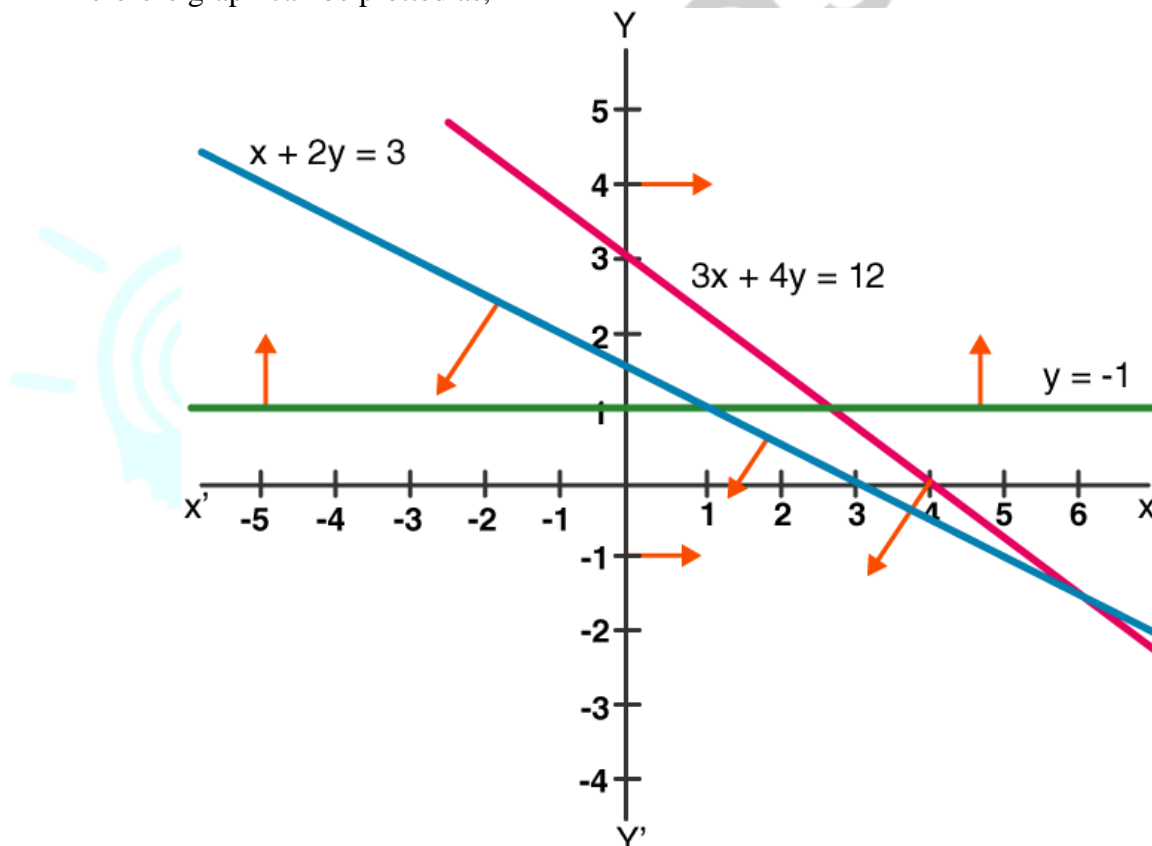
x	0	4
y	3	0

Also, (0, 0) satisfies the $3x + 4y \leq 12$, hence region is towards the origin

$x \geq 0 \Rightarrow$ region is to the right of the y-axis

And, $y \geq 1 \Rightarrow$ region is up above the line $y = 1$,

Therefore graph can be plotted as,



Hence, we can conclude from the graph that the above system has no common region as solution