SHORT ANSWER TYPE

1. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

Solution:

We know that,

$^{n}P_{r} = \frac{n!}{(n-r)!}$

According to the question,

W_1 can occupy chairs marked 1 to 4 in 4 different way.

Chair	1	2	3	4	5	6	7	8
People	W_1	W_1,W_2	W_1,W_2	W_1,W_2				

W_2 can occupy 3 chairs marked 1 to 4 in 3 different ways.

So, total no of ways in which women can occupy the chairs,

$^4P_2 = 4!/(4-2)!$

$= (4 \times 3 \times 2 \times 1)/(2 \times 1)$

$^4P_2 = 12$

Now, 6 chairs will be remaining.

Chair	1	2	3	4	5	6	7	8
People	W_1	W_2						

M_1 can occupy any of the 6 chairs in 6 different ways,

M_2 can occupy any of the remaining 5 chairs in 5 different ways

M_3 can occupy any of the remaining 4 chairs in 4 different ways.

So, total no of ways in which men can occupy the chairs,

$^6P_3 = 6!/(6-3)!$

$=120$

Hence, total number of ways in which men and women can be seated

$^4P_2 x ^6P_3 = 120 \times 12$

$=1440$

2. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT?

[Hint: In each case number of words beginning with A, C, H, I is 5!]

Solution:

According to the question,

R	A	C	H	I	T

Arranging in alphabetical order, we get,

A C H I R T

Number of words that can start with A=5!

Number of words that can start with C=5!

Number of words that can start with H=5!

Number of words that can start with I=5!

Total = 5! +5!+5!+5!

= 120+120+120+120=480
Since, the word that can start with R is RACHIT
Number of words that can start with R = 1
Hence, we obtain that,
The rank of the word RACHIT = 480+1=481

3. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.
Solution:

We know that,
$^nC_r = \frac{n!}{r!(n-r)!}$
No of questions in group A=6
No of questions in group B=6
According to the question,
The different ways in which the questions can be attempted are,

Group A	2	3	4	5
Group B	5	4	3	2

Hence, the number of different ways of doing questions,
$= (^6C_2 x ^6C_5)+(^6C_3 x ^6C_4)+(^6C_4 x ^6C_3)+(^6C_5 x ^6C_2)$
$= (15\times6)+(20\times15)+(15\times20)+(6\times15)$
$=780$

4. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.
[Hint: Number of straight lines $= ^{18}C_2 - ^5C_2 + 1$]
Solution:

We know that,
$^nC_r = \frac{n!}{r!(n-r)!}$
According to the question,
Number of points = 18
Number of Collinear points = 5
Number of lines form by 18 points = $^{18}C_2$
For 5 points to be collinear = $^5C_2$
The number of lines that can be formed joining the point, = $^{18}C_2 - ^5C_2 + 1$

$$= \frac{18!}{2!\,6!} - \frac{5!}{2!\,3!} + 1$$
$=153-10+1$
$=144$

5. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?
Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

According to the question,

Case 1:

If both A and B are selected $=1 \times 1 \times {}^6C_4$

$$= \frac{6!}{4!\,(6-4)!} = \frac{6!}{4!\,2!} = 15$$

Case 2:

If neither A nor B are selected = $^6C_6 = 1$

If B is selected but A is not selected = $1 \times {}^6C_5$

$$= \frac{6!}{5!\,(6-4)!} = 6$$

Adding the results of both A and B being selected, neither A nor B being selected and B being selected but A not being selected,

We get,

15+1+6=22

6. How many committee of five persons with a chairperson can be selected from 12 persons? [Hint: Chairman can be selected in 12 ways and remaining in $^{11}C_4$.]

Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

Number of ways a chairperson can be selected =12

Selection of 4 other people $= {}^{11}C_4$

$$= \frac{11!}{4!7!} = 330$$

Selection of 5 people $= 330 \times 12$

=3960

7. How many automobile license plates can be made if each plate contains two different letters followed by three different digits?

Solution:

According to the question,

Number of letters in automobile license plates = 2

We know that,

There are 26 alphabets

So, Letter can be arranged without repetition in the following number of ways,

$= 26 \times 25$

=650

Number of digits in automobile license plates =3

We know that, there 10 digits

0	1	2	3	4	5	6	7	8	9

Hence the number of digits without repetitions

$=10\times9\times8=720$

Therefore, the total number of way automobile license plates

$=720\times650$

$=468000$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.

Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

According to the question,

Number of black balls = 5

Number of red balls = 6

Number of ways in which 2 black balls can be selected = $^5C_2$

$= \frac{5!}{2!3!} = 10$

Number of ways in which 3 red balls can be selected $= {}^5C_3$

$= \frac{6!}{3!3!} = 20$

Number of ways in which 2 black & 3 red ball can be selected

$= {}^5C_2 \times {}^5C_3$

$=10\times20$

$=200$

9. Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

Solution:

Permutations of n distinct things taken r together $= {}^nC_r$

And when 3 particular things must occur together, we get,

$= {}^{n-3}C_{r-3}$

$= {}^{n-3}C_{r-3} \times (r-2)! \times 3!$

10. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

Solution:

We know that,

$^nP_r = \frac{n!}{(n-r)!}$

According to the question,

Total number of vowels letter =3,

Total number of consonants letter =5

T	R	I	A	N	G	L	E

The vowels can be placed in

$^6P_3 = 6!/3! = 120$
The number of way consonants can be arranged placed =5! =120
Therefore, total number of ways it can be arranged $=5!\times{}^6P_3=120\times120=14400$

11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.
Solution:

(i)Thousand's Place can be fill with 6 alone.
Hence, number of way =1

6			

Unit place can be filled with either 0 or 5.
Hence, number of way =2

6			0 or 5

Hundred's place can be filled with the remaining 8 digits.
Hence, number of way =8

6	9, 8, 7, 4, 3, 2, 1 (0 or 5)		0 or 5

Ten's place can be filled with 7 digits.
Number of ways= 7.

6	9, 8, 7, 4, 3, 2, 1 (0 or 5)	Remaining 7	0 or 5

Thus, required number will be
=1 x 8 x 7 x 2
=112

12. There are 10 persons named $P_1, P_2, P_3, \ldots P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements. [Hint: Required number of arrangement $={}^7C_4\times 5!$]
Solution:

We know that,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,
There are 10 person named P1, P2, P3, ... P10.
Number of ways in which P_1 can be arranged =5! =120
Number of ways in which others can be arranged,
$^7C_4 = 7!/(4!3!) = 35$
Therefore, the required number of arrangement $=C_4^7\times 5$
=35 x 120
=4200

13. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.[Hint: Required number = 210 – 1].
Solution:

We know that,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

We also know that,

$\sum_{k=1}^{n} C_k^n = 2^n - 1$

According to the question,

Number of lamps in a hall =10

Given that,

One of the lamps can be switched on independently

Hence, the number of ways in which the hall can be illuminated is given by,

$C_1^{10} + C_2^{10} + C_3^{10} + C_4^{10} + C_5^{10} + C_6^{10} + C_7^{10} + C_8^{10} + C_9^{10} + C_{10}^{10}$

$=2^{10}-1$

$=1024-1$

$=1023$

14. A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw.
[Hint: Required number of ways $={}^3C_1\times{}^6C_2+{}^3C_2 x\ {}^6C_1+{}^3C_3$.]
Solution:

We know that,

${}^nC_r = \frac{n!}{r!(n-r)!}$

Drawing 1 black and 2 other ball = ${}^3C_1 \times {}^6C_2$

Drawing 2 black and 1 other ball = ${}^3C_2 \times {}^6C_1$

Drawing 3 black balls = ${}^3C_3$

Number of ways in which at least one black ball can be drawn =

=(1 black and 2 other)or(2 black and 1 other)or (3 black)

${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3$ $=3\times15+3\times6+1$

$= 45 + 18 + 1$

$= 64$

15. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.
[Hint: From equation using ${}^nC_r / {}^nC_{r+1}$ and ${}^nC_r / {}^nC_{r-1}$ to find the value of r.]
Solution:

We know that,

${}^nC_r = \frac{n!}{r!(n-r)!}$

According to the question,

${}^nC_{r-1} =36$,

${}^nC_r =84$,

${}^nC_{r+1}=126$

$$\frac{C_r^n}{C_{r+1}^n} = \frac{84}{126}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{84}{126} = \frac{2}{3}$$

2n-2r=3r+3

$\Rightarrow 2n - 3 = 5r$ … (i)

$$\frac{C_r^n}{C_{r-1}^n} = \frac{84}{36}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{7}{3}$$

=3n-3r+3=7r

3n+3=10r … (ii)

From (i) and (ii),

We get,

2(2n – 3) = 3n+3

4n – 3n – 6 – 3=0

n=9

And r=3

Now

$^rC_2 = {}^3C_2 = 3!/2!$

=3

16. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated. [Hint: Besides 4-digit integers greater than 7000, five digit integers are always greater than 7000.]

Solution:

According to the question,

Digits that can be used = 3,5,7,8,9

Since, no digits can be repeated,

The number of integers is $^5P_5 = 5! = 120$

For a four-digit integer to be greater than 7000,

The four-digit integer should begin with 7,8 or 9.

The number of such integer = $3 \times {}^4P_3 = 3 \times {}^4P_3 = 3(24) = 72$

Therefore, the total no of ways = 120+72 = 192

17. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?

Solution:

Let the number of intersection point of first line =0

Let the number of intersection point of 2nd line = 1

Let the number of intersection point of 3rd line = 2+1

Let the number of intersection point of 4th line = 3+2+1

. . .

Let the number of intersection point of nth line =(n-1) +(n-2).....(3)(2)(1), where n=20

S = (n – 1) × n/2

=19×10

=190

18. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all si x digits distinct?

Solution:

According to the question,
All telephone numbers have six digits
Given that,
The first two digits = 41 or 42 or 46 or 62 or 64
Hence, the number of 2 digits that the telephone number begins with =5
First two digits can be filled in 5 ways,
The remaining four-digits can be filled in $^8P_4$ ways,
$^8P_4 = 8!/(8 – 4)! = 1680$
Therefore, number of telephone numbers having six distinct digits =5×1680
=8400

19. In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

Solution:

We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,
Total number of questions =5
Number of questions to be answered =4
Compulsory questions are question number 1 and 2
Hence, the number of ways in which the student can make the choice = $^3C_2$
$^3C_2 = 3!/(2!1!)$ =3 ways

20. A convex polygon has 44 diagonals. Find the number of its sides. [Hint: Polygon of n sides has ($^nC_2$ – n) number of diagonals.]

Solution:

We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Let the number of sides the given polygon have = n
Now,
The number of line segments obtained by joining n vertices = $^nC_2$
So, number of diagonals of the polygon = $^nC_2 – n = 44$

$$\frac{n(n-1)}{2} - n = 44$$

$n^2 – 3n – 88 = 0$
$(n - 11) (n + 8) = 0$
$n = 11$ or $n = - 8$
The polygon has 11sides.

LONG ANSWER TYPE

21. 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

Solution:

According to the question,
Number of mice =18,
Number of groups = 3
Since the groups are equally large,
The number of mice in each group can be = 6 mice
The number of ways of placement of mice =18!
For each group the placement of mice = 6!
Hence, the required number of ways = 18!/(6!6!6!)
= $18!/(6!)^3$

22. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if
(a) they can be of any colour
(b) two must be white and two red and
(c) they must all be of the same colour.

Solution:

We know that,

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,
Number of white marbles = 6,
Number of red marbles = 5
Total number of marbles = 6 white + 5 red = 11 marbles

(a)If they can be of any colour
Then, any 4 marbles out of 11 can be selected
Therefore, the required number of ways $= {}^{11}C_4$

(b) two must be white and two red
Number of ways of choosing two white and two red $= {}^6C_2 \times {}^5C_2$

(c) they must all be of the same colour
Then, four white marbles out of 6 can be selected $= {}^6C_4$
Or, 4 red marbles out of 5 can be selected $= {}^5C_4$
Therefore, the required number of ways $= {}^6C_4 + {}^5C_4$

23. In how many ways can a football team of 11 players be selected from 16 players? How many of them will
(i) include 2 particular players?
(ii) exclude 2 particular players?

Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

According to the question,

11 players can be selected out of 16 = $^{16}C_{11}$

(i) include 2 particular players = $^{14}C_9$

(ii) exclude 2 particular players = $^{14}C_{11}$

24. A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class X II. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

A team of 11 students can be constituted in the following two ways

(i) 5 students from class XI and 6 students from XII = $^{20}C_5 .\ ^{20}C_6$

(ii) 6 students from class XI and 5 students from XII = $^{20}C_6 .\ ^{20}C_5$

The number ways the team can be constituted = Case(i)+case(ii)

= $^{20}C_5 .\ ^{20}C_6 + {}^{20}C_6 .\ ^{20}C_5$

= $2(^{20}C_5 .\ ^{20}C_6)$

25. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has

(i) no girls

(ii) at least one boy and one girl

(iii) at least three girls.

Solution:

We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

(i) No girls

Total number of ways the team can have no girls = $^4C_0 .\ ^7C_5 = 21$

(ii) at least one boy and one girl

Case(A)1 boy and 4 girls

$= {}^7C_1 .\ ^4C_4$

=7

Case(B)2 boys and3 girls

$= {}^7C_2 .\ ^4C_3$

=84

Case(C) 3boys and 2girls

$= {}^7C_3 .\ ^4C_2$

=210

Case(D)4 boys and 1 girls
$={}^7C_4.\ {}^4C_1$
=140

Total number of ways the team can have at least one boy and one girl,
= Case(A) + Case(B) + Case(C) + Case(D)
=7 + 84 + 210 + 140
=441

(iii) At least three girls
Total number of ways the team can have at least three girls $={}^4C_3.\ {}^7C_2+{}^4C_4.{}^7C_1$
= 4 × 21 + 7
= 84 + 7
= 91

OBJECTIVE TYPE QUESTIONS

Choose the correct answer out of the given four options against each of the Exercises from 26 to 40 (M.C.Q.).

26. If $^nC_{12} = {}^nC_8$, then n is equal to
A. 20
B. 12
C. 6
D. 30
Solution:
A. 20
Explanation:
According to the question,
$^nC_{12} = {}^nC_8$
We know that,
$^nC_r = \frac{n!}{r!(n-r)!}$
n – 8 = 12
n = 20
Hence, Option (A) 20 is the correct answer.

27. The number of possible outcomes when a coin is tossed 6 times is
A. 36
B. 64
C. 12
D. 32
Solution:
B. 64

Explanation:
The coin is tossed 6 times.
Number of possible outcomes = 2
The number of possible outcomes when a coin is tossed 6 times is $2^6 = 64$
Hence, Option (B) 64 is the correct answer.

28. The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is
A. 120
B. 96
C. 24
D. 100
Solution:
C. 24
Explanation:
We know that,
$^nP_r = \frac{n!}{(n-r)!}$
Four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once $= {}^4P_4 = 4! = 24$
Hence, Option (C) 24 is the correct answer.

29. The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is
A. 432
B. 108
C. 36
D. 18
Solution:
B. 108
Explanation:
The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time $=(3+4+5+6)3! =108$
Hence, Option (B) 108 is the correct answer.

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
A. 60
B. 120
C. 7200
D. 720
Solution:
C. 7200
Explanation:
We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

Total number of given vowels=4
Total number of given consonant=5
Total number of words that can be formed by 2 vowels and 3 consonants $= {}^4C_2 . {}^5C_3 = 4!/2!2!$
2 vowel and 3 consonant =5!
Total number of word = 5! ×4!/2!2! = 7200
Hence, Option (C) 7200 is the correct answer.

31. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is
A. 216
B. 600
C. 240
D. 3125
[Hint:5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]
Solution:

A. 216
Explanation:
5-digit numbers that can be formed using digits 0, 1, 2, 4, 5

4	4	3	2	1

4×4×3×2×1=96
5-digit numbers can be formed using digits 1, 2, 3, 4, 5 = 5!
Total number of ways = 5! +96
=216
Hence, Option (A) 216 is the correct answer.

32. Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is
A. 11
B. 12
C. 13
D. 14
Solution:

B. 12
Explanation:
We know that,

$^nC_r = \frac{n!}{r!(n-r)!}$

Let total time of handshakes=$^nC_2 = 66$

$$\frac{n!}{2!\,(n-2)!} = 66$$

$$\Rightarrow \frac{n(n-1)}{2} = 66$$

$\Rightarrow n^2 - n = 132$
$\Rightarrow (n-12)(n+11) = 0$
$n=12$ or $n = -11$
Therefore, $n = 12$
Hence, Option (B) 12 is the correct answer.