Short Answer (S.A.)

1. Let A = {a, b, c} and the relation R be defined on A as follows:
R = {(a, a), (b, c), (a, b)}.
Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.
Solution:

Given relation, R = {(a, a), (b, c), (a, b)}
To make R as reflexive we should add (b, b) and (c, c) to R. Also, to make R as transitive we should add (a, c) to R.
Hence, the minimum number of ordered pairs to be added are (b, b), (c, c) and (a, c) i.e. 3.

2. Let D be the domain of the real valued function f defined by $f(x) = \sqrt{(25 - x^2)}$. Then, write D.
Solution:

Given, $f(x) = \sqrt{(25 - x^2)}$
The function is defined if $25 - x^2 \geq 0$
So, $x^2 \leq 25$
$-5 \leq x \leq 5$
Therefore, the domain of the given function is [-5, 5]

3. Let f, g: R → R be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, ∀ x ∈ R, respectively. Then, find g o f.
Solution:

Given,
$f(x) = 2x + 1$ and $g(x) = x^2 - 2$, ∀ x ∈ R
Thus, g o f = g (f (x))
= g (2x + 1)
$= (2x + 1)^2 - 2$
$= 4x^2 + 4x + 1 - 2$
$= 4x^2 + 4x - 1$

4. Let f: R → R be the function defined by f (x) = 2x – 3, ∀ x ∈ R. write f^{-1}.
Solution:

Given function,
f (x) = 2x - 3, ∀ x ∈ R
Let y = 2x – 3
x = (y + 3)/ 2
Thus,
$f^{-1}(x) = (x + 3)/2$

5. If A = {a, b, c, d} and the function f = {(a, b), (b, d), (c, a), (d, c)}, write f^{-1}.

Solution:

Given,
A = {a, b, c, d} and f = {(a, b), (b, d), (c, a), (d, c)}
So,
f^{-1} = {(b, a), (d, b), (a, c), (c, d)}

6. If f: R → R is defined by f (x) = $x^2 – 3x + 2$, write f (f (x)).
Solution:

Given, f (x) = $x^2 – 3x + 2$
Then,
$f(f(x)) = f(x^2 – 3x + 2)$
$= (x^2 – 3x + 2)^2 – 3(x^2 – 3x + 2) + 2,$
$= x^4 + 9x^2 + 4 – 6x^3 + 4x^2 – 12x – 3x^2 + 9x – 6 + 2$
$= x^4 – 6x^3 + 10x^2 – 3x$
Thus,
$f(f(x)) = x^4 – 6x^3 + 10x^2 – 3x$

7. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g (x) = αx + β, then what value should be assigned to α and β.
Solution:

Given, g = {(1, 1), (2, 3), (3, 5), (4, 7)}
It's seen that every element of domain has a unique image. So, g is function.
Now, also given that g (x) = αx + β
So, we have
g (1) = α(1) + β = 1
α + β = 1 …….. (i)
And, g (2) = α(2) + β = 3
2α + β = 3 …….. (ii)
Solving (i) and (ii), we have
α = 2 and β = -1
Therefore, g (x) = 2x - 1

8. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.
(i) {(x, y): x is a person, y is the mother of x}.
(ii){(a, b): a is a person, b is an ancestor of a}.
Solution:

(i) Given, {(x, y): x is a person, y is the mother of x}
It's clearly seen that each person 'x' has only one biological mother.
Hence, the above set of ordered pairs make a function.
Now more than one person may have same mother. Thus, the function is many-many one and surjective.

(ii) Given, {(a, b): a is a person, b is an ancestor of a}
It's clearly seen that any person 'a' has more than one ancestors.
Thus, it does not represent a function.

9. If the mappings f and g are given by f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}, write f o g.
Solution:

Given,
f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}
Now,
fog (2) = f(g(2)) = f(3) = 5
fog (5) = f(g(5)) = f(1) = 2
fog (1) = f(g(1)) = f(3) = 5
Thus,
fog = {(2, 5), (5, 2), (1, 5)}

10. Let C be the set of complex numbers. Prove that the mapping f: C → R given by f (z) = |z|, ∀ z ∈ C, is neither one-one nor onto.
Solution:

Given, f: C → R such that f (z) = |z|, ∀ z ∈ C
Now, let take z = 6 + 8*i*
Then,
f (6 + 8*i*) = |6 + 8*i*| = $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$
And, for z = 6 - 8*i*
f (6 - 8*i*) = |6 - 8*i*| = $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$
Hence, f (z) is many-one.
Also, |z| ≥ 0, ∀ z ∈ C
But the co-domain given is 'R'
Therefore, f(z) is not onto.

11. Let the function f: R → R be defined by f (x) = cos x, ∀ x ∈ R. Show that f is neither one-one nor onto.
Solution:

We have,
f: R → R, f(x) = cos x
Now,
$f(x_1) = f(x_2)$
$\cos x_1 = \cos x_2$
$x_1 = 2n\pi \pm x_2$, n ∈ Z
It's seen that the above equation has infinite solutions for x_1 and x_2
Hence, f(x) is many one function.
Also the range of cos x is [-1, 1], which is subset of given co-domain R.

Therefore, the given function is not onto.

12. Let X = {1, 2, 3} and Y = {4, 5}. Find whether the following subsets of X × Y are functions from X to Y or not.
(i) f = {(1, 4), (1, 5), (2, 4), (3, 5)} **(ii) g = {(1, 4), (2, 4), (3, 4)}**
(iii) h = {(1,4), (2, 5), (3, 5)} **(iv) k = {(1,4), (2, 5)}.**
Solution:

Given, X = {1, 2, 3} and Y = {4, 5}
So, X × Y = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}
(i) f = {(1, 4), (1, 5), (2, 4), (3, 5)}
f is not a function as f(1) = 4 and f(1) = 5
Hence, pre-image '1' has not unique image.

(ii) g = {(1, 4), (2, 4), (3, 4)}
It's seen clearly that g is a function in which each element of the domain has unique image.

(iii) h = {(1,4), (2, 5), (3, 5)}
It's seen clearly that h is a function as each pre-image with a unique image.
And, function h is many-one as h(2) = h(3) = 5

(iv) k = {(1, 4),(2, 5)}
Function k is not a function as '3' has not any image under the mapping.

13. If functions f: A → B and g: B → A satisfy g o f = I_A, then show that f is one-one and g is onto.
Solution:

Given,
f: A → B and g: B → A satisfy g o f = I_A
It's clearly seen that function 'g' is inverse of 'f'.
So, 'f' has to be one-one and onto.
Hence, 'g' is also one-one and onto.

14. Let f: R → R be the function defined by f(x) = 1/(2 – cos x) ∀ x ∈ R. Then, find the range of f.
Solution:

Given,
f(x) = 1/(2 – cos x) ∀ x ∈ R
Let y = 1/(2 – cos x)
2y – ycos x = 1
cos x = (2y - 1)/ y
cos x = 2 – 1/y
Now, we know that -1 ≤ cos x ≤ 1
So,
-1 ≤ 2 – 1/y ≤ 1
-3 ≤ – 1/y ≤ -1

$1 \leq -1/y \leq 3$
$1/3 \leq y \leq 1$
Thus, the range of the given function is [1/3, 1].

15. Let n be a fixed positive integer. Define a relation R in Z as follows: ∀ a, b ∈ Z, aRb if and only if a – b is divisible by n. Show that R is an equivalance relation.
Solution:

Given ∀ a, b ∈ Z, aRb if and only if a – b is divisible by n.
Now, for
aRa ⇒ (a - a) is divisible by n, which is true for any integer a as '0' is divisible by n.
Thus, R is reflective.
Now, aRb
So, (a - b) is divisible by n.
⇒ - (b - a) is divisible by n.
⇒ (b – a) is divisible by n
⇒ bRa
Thus, R is symmetric.
Let aRb and bRc
Then, (a - b) is divisible by n and (b - c) is divisible by n.
So, (a - b) + (b - c) is divisible by n.
⇒ (a - c) is divisible by n.
⇒ aRc
Thus, R is transitive.
So, R is an equivalence relation.

Long Answer (L.A.)
16. If A = {1, 2, 3, 4 }, define relations on A which have properties of being:
(a) reflexive, transitive but not symmetric
(b) symmetric but neither reflexive nor transitive
(c) reflexive, symmetric and transitive.
Solution:

Given that, A = {1, 2, 3}.
(i) Let R_1 = {(1, 1), (1, 2), (1, 3), (2, 3), (2, 2), (1, 3), (3, 3)}
R_1 is reflexive as (1, 1), (2, 2) and (3, 3) lie is R_1.
R_1 is transitive as (1, 2) ∈ R_1, (2, 3) ∈ R_1 ⇒ (1, 3) ∈ R_1
Now, (1, 2) ∈ R_1 ⇒ (2, 1) ∉ R_1.

(ii) Let R_2 = {(1, 2), (2, 1)}
Now, (1, 2) ∈ R_2, (2, 1) ∈ R_2
So, it is symmetric,
And, clearly R_2 is not reflexive as (1, 1) ∉ R_2
Also, R_2 is not transitive as (1, 2) ∈ R_2, (2, 1) ∈ R_2 but (1, 1) ∉ R_2

(iii) Let R_3 = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

R_3 is reflexive as (1, 1) (2, 2) and (3, 3) ∈ R_1
R_3 is symmetric as (1, 2), (1, 3), (2, 3) ∈ R_1 ⇒ (2, 1), (3, 1), (3, 2) ∈ R_1
Therefore, R_3 is reflexive, symmetric and transitive.

17. Let R be relation defined on the set of natural number N as follows:
R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.
Solution:

Given function: R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}.
So, the domain = {1, 2, 3, ….., 20} [Since, y ∈ N]
Finding the range, we have
R = {(1, 39), (2, 37), (3, 35), …., (19, 3), (20, 1)}
Thus, Range of the function = {1, 3, 5, ….., 39}
R is not reflexive as (2, 2) ∉ R as 2 x 2 + 2 ≠ 41
Also, R is not symmetric as (1, 39) ∈ R but (39, 1) ∉ R
Further R is not transitive as (11, 19) ∉ R, (19, 3) ∉ R; but (11, 3) ∉ R.
Thus, R is neither reflexive nor symmetric and nor transitive.

18. Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:
(a) an injective mapping from A to B
(b) a mapping from A to B which is not injective
(c) a mapping from B to A.
Solution:

Given, A = {2, 3, 4}, B = {2, 5, 6, 7}
(i) Let f: A → B denote a mapping
f = {(x, y): y = x + 3} or
f = {(2, 5), (3, 6), (4, 7)}, which is an injective mapping.
(ii) Let g: A → B denote a mapping such that g = {(2, 2), (3, 2), (4, 5)}, which is not an injective mapping.
(iii) Let h: B → A denote a mapping such that h = {(2, 2), (5, 3), (6, 4), (7, 4)}, which is one of the mapping from B to A.

19. Give an example of a map
(i) which is one-one but not onto
(ii) which is not one-one but onto
(iii) which is neither one-one nor onto.
Solution:

(i) Let f: N → N, be a mapping defined by f (x) = x^2
For f (x_1) = f (x_2)
Then, $x_1^2 = x_2^2$
$x_1 = x_2$ (Since $x_1 + x_2 = 0$ is not possible)
Further 'f' is not onto, as for 1 ∈ N, there does not exist any x in N such that f (x) = 2x + 1.

(ii) Let f: R → [0, ∞), be a mapping defined by f(x) = |x|
Then, it's clearly seen that f (x) is not one-one as f (2) = f (-2).
But |x| ≥ 0, so range is [0, ∞].
Therefore, f (x) is onto.

(iii) Let f: R → R, be a mapping defined by f (x) = x^2
Then clearly f (x) is not one-one as f (1) = f (-1). Also range of f (x) is [0, ∞).
Therefore, f (x) is neither one-one nor onto.

20. Let A = R – {3}, B = R – {1}. Let f : A → B be defined by f (x) = x – 2/ x – 3 ∀ x ∈ A . Then show that f is bijective.
Solution:

Given,
A = R – {3}, B = R – {1}
And,
f : A → B be defined by f (x) = x – 2/ x – 3 ∀ x ∈ A
Hence, f (x) = (x – 3 + 1)/ (x – 3) = 1 + 1/ (x – 3)
Let f(x_1) = f (x_2)

$$1+\frac{1}{x_1-3}=1+\frac{1}{x_2-3}$$

$$\frac{1}{x_1-3}=\frac{1}{x_2-3}$$

$$x_1=x_2$$

So, f (x) is an injective function.
Now let y = (x - 2)/ (x -3)
x – 2 = xy – 3y
x(1 - y) = 2 – 3y
x = (3y – 2)/ (y - 1)
y ∈ R – {1} = B
Thus, f (x) is onto or subjective.
Therefore, f(x) is a bijective function.

21. Let A = [–1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:
(i) f(x) = x/2 **(ii) g(x) = |x|**
(iii) h(x) = x|x| **(iv) k(x) = x^2**
Solution:

Given, A = [–1, 1]
(i) f: [-1, 1] → [-1, 1], f (x) = x/2
Let f (x_1) = f(x_2)

$x_1/2 = x_2$
So, f (x) is one-one.
Also $x \in [-1, 1]$
$x/2 = f(x) = [-1/2, 1/2]$
Hence, the range is a subset of co-domain 'A'
So, f (x) is not onto.
Therefore, f (x) is not bijective.

(ii) $g(x) = |x|$
Let $g(x_1) = g(x_2)$
$|x_1| = |x_2|$
$x_1 = \pm x_2$
So, g (x) is not one-one
Also $g(x) = |x| \geq 0$, for all real x
Hence, the range is [0, 1], which is subset of co-domain 'A'
So, f (x) is not onto.
Therefore, f (x) is not bijective.

(iii) $h(x) = x|x|$
Let $h(x_1) = h(x_2)$
$x_1|x_1| = x_2|x_2|$
If $x_1, x_2 > 0$
$x_1^2 = x_2^2$
$x_1^2 - x_2^2 = 0$
$(x_1 - x_2)(x_1 + x_2) = 0$
$x_1 = x_2$ (as $x_1 + x_2 \neq 0$)
Similarly for $x_1, x_2 < 0$, we have $x_1 = x_2$
It's clearly seen that for x_1 and x_2 of opposite sign, $x_1 \neq x_2$.
Hence, f (x) is one-one.
For $x \in [0, 1]$, $f(x) = x^2 \in [0, 1]$
For $x < 0$, $f(x) = -x^2 \in [-1, 0)$
Hence, the range is [-1, 1].
So, h (x) is onto.
Therefore, h (x) is bijective.

(iv) $k(x) = x^2$
Let $k(x_1) = k(x_2)$
$x_1^2 = x_2^2$
$x_1 = \pm x_2$
Therefore, k (x) is not one-one.

22. Each of the following defines a relation on N:
(i) x is greater than y, x, y ∈ N
(ii) x + y = 10, x, y ∈ N
(iii) x y is square of an integer x, y ∈ N
(iv) x + 4y = 10 x, y ∈ N.

Determine which of the above relations are reflexive, symmetric and transitive.
Solution:

(i) Given, x is greater than y; $x, y \in N$
If $(x, x) \in R$, then $x > x$, which is not true for any $x \in N$.
Thus, R is not reflexive.
Let $(x, y) \in R$
$\Rightarrow xRy$
$\Rightarrow x > y$
So, $y > x$ is not true for any $x, y \in N$
Hence, R is not symmetric.
Let xRy and yRz
$\Rightarrow x > y$ and $y > z$
$\Rightarrow x > z$
$\Rightarrow xRz$
Hence, R is transitive.

(ii) $x + y = 10$; $x, y \in N$
Thus,
$R = \{(x, y); x + y = 10, x, y \in N\}$
$R = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$
It's clear $(1, 1) \notin R$
So, R is not reflexive.
$(x, y) \in R \Rightarrow (y, x) \in R$
Therefore, R is symmetric.
Now $(1, 9) \in R$, $(9, 1) \in R$, but $(1, 1) \notin R$
Therefore, R is not transitive.

(iii) Given, xy is square of an integer $x, y \in N$
$R = \{(x, y) : xy$ is a square of an integer $x, y \in N\}$
It's clearly $(x, x) \in R$, $\forall x \in N$
As x^2 is square of an integer for any $x \in N$
Thus, R is reflexive.
If $(x, y) \in R \Rightarrow (y, x) \in R$
So, R is symmetric.
Now, if xy is square of an integer and yz is square of an integer.
Then, let $xy = m^2$ and $yz = n^2$ for some $m, n \in Z$
$x = m^2/y$ and $z = x^2/y$
$xz = m^2n^2/ y^2$, which is square of an integer.
Thus, R is transitive.

(iv) $x + 4y = 10$; $x, y \in N$
$R = \{(x, y): x + 4y = 10; x, y \in N\}$
$R = \{(2, 2), (6, 1)\}$
It's clearly seen $(1, 1) \notin R$

Hence, R is not symmetric.
$(x, y) \in R \Rightarrow x + 4y = 10$
And $(y, z) \in R \Rightarrow y + 4z = 10$
$\Rightarrow x - 16z = -30$
$\Rightarrow (x, z) \notin R$
Therefore, R is not transitive.

23. Let A = {1, 2, 3, ... 9} and R be the relation in A ×A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A ×A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].
Solution:

Given, $A = \{1, 2, 3, \ldots 9\}$ and $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d) \in A \times A$.
Let (a, b) R(a, b)
So, $a + b = b + a$, $\forall$ a, b $\in$ A which is true for any a, b $\in$ A.
Thus, R is reflexive.
Let (a, b) R(c, d)
Then,
$a + d = b + c$
$c + b = d + a$
(c, d) R(a, b)
Thus, R is symmetric.
Let (a, b) R(c, d) and (c, d) R(e, f)
$a + d = b + c$ and $c + f = d + e$
$a + d = b + c$ and $d + e = c + f$
$(a + d) – (d + e = (b + c) – (c + f)$
$a – e = b – f$
$a + f = b + e$
(a, b) R(e, f)
So, R is transitive.
The equivalence class $[(2, 5)] = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$
Therefore, R is an equivalence relation.

24. Using the definition, prove that the function f : A→ B is invertible if and only if f is both one-one and onto.
Solution:

Let f: A → B be many-one function.
Let f(a) = p and f(b) = p
So, for inverse function we will have $f^{-1}(p) = a$ and $f^{-1}(p) = b$
Thus, in this case inverse function is not defined as we have two images ‘a and b’ for one pre-image ‘p’. But for f to be invertible it must be one-one.
Now, let f: A → B is not onto function.
Let B = {p, q, r} and range of f be {p, q}.
Here image ‘r’ has not any pre-image, which will have no image in set A.
And for f to be invertible it must be onto.
Thus, ‘f’ is invertible if and only if ‘f’ is both one-one and onto.

A function $f = X \to Y$ is invertible iff f is a bijective function.

25. Functions f , g : R → R are defined, respectively, by $f(x) = x^2 + 3x + 1$, $g(x) = 2x – 3$, find
(i) *f o g*　(ii) *g o f*　(iii) *f o f*　(iv) *g o g*
Solution:

Given, $f(x) = x^2 + 3x + 1$, $g(x) = 2x – 3$

(i) $fog = f(g(x))$
$= f(2x - 3)$
$= (2x - 3)^2 + 3(2x - 3) + 1$
$= 4x^2 + 9 – 12x + 6x – 9 + 1$
$= 4x^2 – 6x + 1$

(ii) $gof = g(f(x))$
$= g(x^2 + 3x + 1)$
$= 2(x^2 + 3x + 1) – 3$
$= 2x^2 + 6x – 1$

(iii) $fof = f(f(x))$
$= f(x^2 + 3x + 1)$
$= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$
$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$
$= x^4 + 6x^3 + 14x^2 + 15x + 5$

(iv) $gog = g(g(x))$
$= g(2x - 3)$
$= 2(2x - 3) – 3$
$= 4x – 6 – 3$
$= 4x - 9$

26. Let * be the binary operation defined on Q. Find which of the following binary operations are commutative
(i) $a * b = a – b \ \forall\ a, b \in Q$
(ii) $a * b = a^2 + b^2 \ \forall\ a, b \in Q$
(iii) $a * b = a + ab \ \forall\ a, b \in Q$
(iv) $a * b = (a – b)^2 \ \forall\ a, b \in Q$
Solution:

Given that * is a binary operation defined on Q.

(i) $a * b = a – b$, $\forall\ a, b \in Q$ and $b * a = b - a$
So, $a * b \neq b * a$
Thus, * is not commutative.

(ii) $a * b = a^2 + b^2$
$b * a = b^2 + a^2$
Thus, * is commutative.

(iii) $a * b = a + ab$
$b * a = b + ab$
So clearly, $a + ab \neq b + ab$
Thus, * is not commutative.

(iv) $a * b = (a - b)^2$, $\forall\ a, b \in Q$
$b * a = (b –a)^2$

Since, $(a - b)^2 = (b - a)^2$
Thus, * is commutative.

27. Let * be binary operation defined on R by a * b = 1 + ab, ∀ a, b ∈ R. Then the operation * is
(i) commutative but not associative
(ii) associative but not commutative
(iii) neither commutative nor associative
(iv) both commutative and associative
Solution:

(i) Given that * is a binary operation defined on R by a * b = 1 + ab, ∀ a, b ∈ R
So, we have a * b = ab + 1 = b * a
So, * is a commutative binary operation.
Now, a * (b * c) = a * (1 + bc) = 1 + a (1 + bc) = 1 + a + abc
Also,
(a * b) * c = (1 + ab) * c = 1 + (1 + ab) c = 1 + c + abc
Thus, a * (b * c) ≠ (a * b) * c
Hence, * is not associative.
Therefore, * is commutative but not associative.

Objective Type Questions
Choose the correct answer out of the given four options in each of the Exercises from 28 to 47 (M.C.Q.)
28. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b ∀ a, b ∈ T. Then R is
(A) reflexive but not transitive
(B) transitive but not symmetric
(C) equivalence
(D) none of these
Solution:

(C) equivalence
Given aRb, if a is congruent to b, ∀ a, b ∈ T.
Then, we have aRa ⇒ a is congruent to a; which is always true.
So, R is reflexive.
Let aRb ⇒ a ~ b
b ~ a
bRa
So, R is symmetric.
Let aRb and bRc
a ~ b and b ~ c
a ~ c
aRc
So, R is transitive.
Therefore, R is equivalence relation.

29. Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is

(A) symmetric but not transitive **(B) transitive but not symmetric**
(C) neither symmetric nor transitive **(D) both symmetric and transitive**
Solution:

(B) transitive but not symmetric
aRb ⇒ a is brother of *b*.
This does not mean *b* is also a brother of a as *b* can be a sister of *a*.
Thus, R is not symmetric.
aRb ⇒ a is brother of *b*.
and bRc ⇒ *b* is brother of *c*.
So, a is brother of c.
Therefore, R is transitive.

30. The maximum number of equivalence relations on the set A = {1, 2, 3} are
(A) 1 **(B) 2**
(C) 3 **(D) 5**
Solution:

(D) 5
Given, set A = {1, 2, 3}
Now, the number of equivalence relations as follows
$R_1 = \{(1, 1), (2, 2), (3, 3)\}$
$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$
$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$
$R_5 = \{(1, 2, 3) \Leftrightarrow A \times A = A^2\}$
Thus, maximum number of equivalence relation is '5'.

31. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is
(A) reflexive **(B) transitive**
(C) symmetric **(D) none of these**
Solution:

(D) none of these
R on the set {1, 2, 3} be defined by R = {(1, 2)}
Hence, its clear that R is not reflexive, transitive and symmetric.