Short Answer (S.A.)

1. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?
Solution:

For a given matrix of order m x n, it has mn elements, where m and n are natural numbers.
Here we have, m x n = 28
(m, n) = {(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)}
So, the possible orders are 1 x 28, 2 x 14, 4 x 7, 7 x 4, 14 x 2, 28 x 1.
Also, if it has 13 elements, then m x n = 13
(m, n) = {(1, 13), (13, 1)}
Thus, the possible orders are 1 x 13, 13 x 1.

2. In the matrix A = $\begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$ **, write :**

(i) The order of the matrix A
(ii) The number of elements
(iii) Write elements a_{23}, a_{31}, a_{12}
Solution:

For the given matrix,
(i) The order of the matrix A is 3 x 3.
(ii) The number of elements of the matrix = 3 x 3 = 9
(iii) Elements: $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$

3. Construct $a_{2 \times 2}$ matrix where
(i) $a_{ij} = (i - 2j)^2/ 2$
(ii) $a_{ij} = |-2i + 3j|$
Solution:

We have,
$A = [a_{ij}]_{2x2}$

(i) Such that, $a_{ij} = (i - 2j)^2/ 2$; where $1 \leq i \leq 2$; $1 \leq j \leq 2$
So, the terms of the matrix are

$$a_{11} = \frac{(1-2)^2}{2} = \frac{1}{2} \qquad a_{12} = \frac{(1-2\times 2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2-2\times 1)^2}{2} = 0 \qquad a_{22} = \frac{(2-2\times 2)^2}{2} = 2$$

$$\text{Therefore, } A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(ii) Here, $a_{ij} = |-2i + 3j|$
So, the terms of the matrix are

$$a_{11} = |-2\times 1 + 3\times 1| = 1 \qquad a_{12} = |-2\times 1 + 3\times 2| = 4$$
$$a_{21} = |-2\times 2 + 3\times 1| = 1 \qquad a_{22} = |-2\times 2 + 3\times 2| = 2$$

$$\text{Therefore, } A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}$$

4. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = e^{i.x}\sin jx$
Solution:

Let A be a 3 x 2 matrix
Such that, $a_{ij} = e^{i.x}\sin jx$; where where $1 \le i \le 3$; $1 \le j \le 2$
So, the terms are given as

$$a_{11} = e^x \sin x \qquad a_{12} = e^x \sin 2x$$
$$a_{21} = e^{2x} \sin x \qquad a_{22} = e^{2x} \sin 2x$$
$$a_{31} = e^{3x} \sin x \qquad a_{32} = e^{3x} \sin 2x$$

$$\text{Therefore, } A = \begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}$$

5. Find values of a and b if A = B, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Solution:

Given, matrix A = matrix B
Then their corresponding elements are equal.

So, we have
$a_{11} = b_{11}$; $a + 4 = 2a + 2 \Rightarrow a = 2$
$a_{12} = b_{12}$; $3b = b^2 + 2 \Rightarrow b^2 - 3b + 2 = 0 \Rightarrow b = 1, 2$
$a_{22} = b_{22}$; $-6 = b^2 - 5b \Rightarrow b^2 - 5b + 6 = 0 \Rightarrow b = 2, 3$
Hence, a = 2 and b = 2 (common value)

6. If possible, find the sum of the matrices A and B, where A = $\begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}$ **and B =** $\begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}$
Solution:

The given two matrices A and B are of different orders. Two matrices can be added only if order of both the matrices is same. Thus, the sum of matrices A and B is not possible.

7. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, find

(i) X + Y
(ii) 2X – 3Y
(iii) A matrix Z such that X + Y + Z is a zero matrix.
Solution:

Given, $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}_{2\times3}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}_{2\times3}$

(i) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

(ii) $2X - 3Y = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix} - \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $X + Y = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

Also, $X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So, Z is the additive inverse of $(X + Y)$ or negative of $(X + Y)$.

Therefore, $Z = -(X+Y) = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}$

8. Find non-zero values of x satisfying the matrix equation:

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ (10) & 6x \end{bmatrix}.$$

Solution:

Given,

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

On comparing the corresponding elements, we get
$2x + 10x = 48$
$12x = 48$
Thus, $x = 4$
It's also seen that this value od x also satisfies the equation $3x + 8 = 20$ and $x^2 + 8x = 12x$.
Therefore, $x = 4$ (common) is the solution of the given matrix equation.

9. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, **show that $(A + B)(A - B) \neq A^2 - B^2$**

Solution:

Given, $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So, $(A+B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$

And, $(A-B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$

$$(A+B)\cdot(A-B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad \ldots (i)$$

Also, $A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

And, $B^2 = B \cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Therefore, $A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ ...(ii)

Hence, from (i) and (ii),
$(A + B)(A - B) \neq A^2 - B^2$

10. Find the value of x if

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O.$$

Solution:

Given, $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} 1+2x+15 & 3+5x+3 & 2+x+2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 16+2x & 5x+6 & x+4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$[16 + 2x + 10x + 12 + x^2 + 4x] = 0$
$[x^2 + 16x + 28] = 0$
$x^2 + 16x + 28 = 0$
$(x + 2)(x + 14) = 0$
Therefore, $x = -2, -14$

11. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ **satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.**
Solution:

Given,

$$A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$$

So, $A^2=A\cdot A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}\cdot\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}25-3 & 15-6\\ -5+2 & -3+4\end{bmatrix}=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}$

$$3A=3\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}$$

And, $7I=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

Hence, $A^2-3A-7I=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}-\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$$=\begin{bmatrix}22-15-7 & 9-9-0\\ -3+3-0 & 1+6-7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$$

Now,
$A^2-3A-7I=0$
Multiplying both sides with A^{-1}, we get
$A^{-1}[A^2-3A-7I]=A^{-1}\,0$
$A^{-1}.A.A-3A^{-1}.A-7A^{-1}.I=0$
$I.A-3I-7A^{-1}=0$ [As $A^{-1}.A=I$]
$A-3I-7A^{-1}=0$
$7A^{-1}=A-3I$

$$7A^{-1}=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}2 & 3\\ -1 & -5\end{bmatrix}$$

Therefore, $A^{-1}=\frac{1}{7}\begin{bmatrix}2 & 3\\ -1 & -5\end{bmatrix}$

12. Find the matrix A satisfying the matrix equation:

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Solution:

Given, $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Let $P=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$, $Q=\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now,

$P^{-1} PAQ = P^{-1} I$
So, $IAQ = P^{-1}$
$AQ = P^{-1}$
$AQQ^{-1} = P^{-1} Q^{-1}$
$AI = P^{-1} Q^{-1}$
$A = P^{-1} Q^{-1}$

Now adj. $P = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ and $|P| = 1$

Hence, $P^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Also adj. $Q = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$ and $|Q| = -1$

Hence, $Q^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

Thus, $A = P^{-1}Q^{-1}$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 6-5 & 4-3 \\ -9+10 & -6+6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

13. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

Solution:

Given, $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

Now, let $A = [x\ y\ z]$

So, $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}[xyz] = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

On comparing elements of both sides, we have
$4x = -4 \Rightarrow x = -1$
$4y = 8 \Rightarrow y = 2$
And, $4z = 4 \Rightarrow z = 1$
Therefore, A = [-1 2 1]

14. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, **then verify** $(BA)^2 \neq B^2 A^2$

Solution:

The given matrices A has order 3 x 2 and B has order 2 x 3.
So, BA is defined and will have order 3 x 3.
But, A^2 and B^2 are not defined as the orders don't satisfy the multiplication condition.
Hence, $(BA)^2 \neq B^2 A^2$

15. If possible, find BA and AB where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

Solution:

Given, $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times 2}$

So. AB and BA both are defined

Now, $$AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$$

And, $$BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8+1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$$

16. Show by an example that for A ≠ 0, B ≠ 0, AB = 0.
Solution:

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \neq O$ and $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \neq O$

So, the product $AB = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

- Hence Proved

17. Given $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$. **Is (AB)' = B' A' ?**
Solution:

Given, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3\times 2}$

So, their product is

$$AB = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

And, $(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix}$ ...(i)

Also, $B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2\times 3}$ and $A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3\times 2}$

Therefore, $B'A' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we have
(AB)' = B' A'

18. Solve for x and y:

$$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=O$$

Solution:

Given, $x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=O$

$\begin{bmatrix}2x\\x\end{bmatrix}+\begin{bmatrix}3y\\5y\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=O$ (Multiplying the variables with the matrices)

So, $\begin{bmatrix}2x+3y-8\\x+5y-11\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ (Addition of matrices)

Now, we have

2x + 3y – 8 = 0 ….. (1) and

x + 5y – 11 = 0 ….. (2)

On solving the equations (1) and (2), we get

x = 1 and y = 2

19. If X and Y are 2 x 2 matrices, then solve the following matrix equations for X and Y

$$2X+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix},\ 3X+2Y=\begin{bmatrix}-2&2\\1&-5\end{bmatrix}.$$

Solution:

Given, $2X+3Y=\begin{bmatrix}2&3\\4&0\end{bmatrix}$ …(i)

and $3X+2Y=\begin{bmatrix}-2&2\\1&-5\end{bmatrix}$ …(ii)

On subractiong equations (i) and (ii), we get

$$(3X+2Y)-(2X+3Y)=\begin{bmatrix}-2-2&2-3\\1-4&-5-0\end{bmatrix}$$

Thus, $X-Y=\begin{bmatrix}-4&-1\\-3&-5\end{bmatrix}$ …(iii)

On adding equations (i) and (ii), we get

$$5X+5Y=\begin{bmatrix}0&5\\5&-5\end{bmatrix}$$

$$X+Y=\frac{1}{5}\begin{bmatrix}0&5\\5&-5\end{bmatrix}=\begin{bmatrix}0&1\\1&-1\end{bmatrix} \quad \text{…(iv)}$$

On adding equations (iii) and (iv), we get

$$(X-Y)+(X+Y)=\begin{bmatrix}-4 & 0\\ -2 & -6\end{bmatrix}$$

$$2X=\begin{bmatrix}-4 & 0\\ -2 & -6\end{bmatrix}$$

Thus, $X=\begin{bmatrix}-2 & 0\\ -1 & -3\end{bmatrix}$

From equation (iv), we get

$$\begin{bmatrix}-2 & 0\\ -1 & -3\end{bmatrix}+Y=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$

$$Y=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}-\begin{bmatrix}-2 & 0\\ -1 & -3\end{bmatrix}$$

Thus, $Y=\begin{bmatrix}2 & 1\\ 2 & 2\end{bmatrix}$

20. If A = [3 5], B = [7 3], then find a non-zero matrix C such that AB = AC.
Solution:

Given, $A = [3\ \ 5]_{1\times2}$ and $B = [7\ \ 3]_{1\times2}$
For AC = BC
We have order of C = 2 x n

For $n = 1$, let $C=\begin{bmatrix}x\\ y\end{bmatrix}$

$\therefore AC=[3\ 5]\begin{bmatrix}x\\ y\end{bmatrix}=[3x+5y]$

And $BC=[7\ 3]\begin{bmatrix}x\\ y\end{bmatrix}=[7x+3y]$

For $AC = BC$,
$[3x+5y]=[7x+3y]$
$3x+5y=7x+3y$
$4x=2y$
$x=\frac{1}{2}y$
$y=2x$

Hence, $C=\begin{bmatrix}x\\ 2x\end{bmatrix}$

It's seen that on taking C of order 2 x 1, 2 x 2, 2 x 3, ... , we get

$$C=\begin{bmatrix}x\\2x\end{bmatrix},\begin{bmatrix}x&x\\2x&2x\end{bmatrix},\begin{bmatrix}x&x&x\\2x&2x&2x\end{bmatrix}\dots$$

In general,

$$C=\begin{bmatrix}k\\2k\end{bmatrix},\begin{bmatrix}k&k\\2k&2k\end{bmatrix}\text{etc}\dots$$

21. Give an example of matrices A, B and C such that AB = AC, where A is non-zero matrix, but B ≠ C.
Solution:

Let $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}, B=\begin{bmatrix}1&1\\1&0\end{bmatrix}$ and $C=\begin{bmatrix}1&1\\1&2\end{bmatrix}$ $[\because B\neq C]$

$$AB=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}1&1\\1&0\end{bmatrix}=\begin{bmatrix}1&1\\0&0\end{bmatrix} \quad \dots(i)$$

and $$AC=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}1&1\\1&2\end{bmatrix}=\begin{bmatrix}1&1\\0&0\end{bmatrix} \quad \dots(ii)$$

From (i) and (ii),
Hence, AB = AC but B ≠ C.

22. If $A=\begin{bmatrix}1&2\\-2&1\end{bmatrix}$, $B=\begin{bmatrix}2&3\\3&-4\end{bmatrix}$ and $C=\begin{bmatrix}1&0\\-1&0\end{bmatrix}$,

(i) (AB) C = A (BC) **(ii) A (B + C) = AB + AC.**
Solution:

Given, $A=\begin{bmatrix}1&2\\-2&1\end{bmatrix}, B=\begin{bmatrix}2&3\\3&-4\end{bmatrix}$ and $C=\begin{bmatrix}1&0\\-1&0\end{bmatrix}$

(i) $$AB=\begin{bmatrix}1&2\\-2&1\end{bmatrix}\begin{bmatrix}2&3\\3&-4\end{bmatrix}=\begin{bmatrix}2+6&3-8\\-4+3&-6-4\end{bmatrix}=\begin{bmatrix}8&-5\\-1&-10\end{bmatrix}$$

and $$(AB)C=\begin{bmatrix}8&-5\\-1&-10\end{bmatrix}\begin{bmatrix}1&0\\-1&0\end{bmatrix}=\begin{bmatrix}8+5&0\\-1+10&0\end{bmatrix}=\begin{bmatrix}13&0\\9&0\end{bmatrix} \quad \dots(i)$$

Again, $$(BC)=\begin{bmatrix}2&3\\3&-4\end{bmatrix}\begin{bmatrix}1&0\\-1&0\end{bmatrix}=\begin{bmatrix}2-3&0\\3+4&0\end{bmatrix}=\begin{bmatrix}-1&0\\7&0\end{bmatrix}$$

And $$A(BC)=\begin{bmatrix}1&2\\-2&1\end{bmatrix}\begin{bmatrix}-1&0\\7&0\end{bmatrix}=\begin{bmatrix}-1+14&0\\2+7&0\end{bmatrix}=\begin{bmatrix}13&0\\9&0\end{bmatrix} \quad \dots(ii)$$

From (i) and (ii), we get
Hence, (AB)C = A(BC)

(ii) $B+C=\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}+\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}=\begin{bmatrix}3 & 3\\ 2 & -4\end{bmatrix}$

Now,

$$A\cdot(B+C)=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}3 & 3\\ 2 & -4\end{bmatrix}$$

$$=\begin{bmatrix}3+4 & 3-8\\ -6+2 & -6-4\end{bmatrix}$$

$$=\begin{bmatrix}7 & -5\\ -4 & -10\end{bmatrix} \quad \ldots\text{(iii)}$$

$$AB=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}=\begin{bmatrix}2+6 & 3-8\\ -4+3 & -6-4\end{bmatrix}=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}$$

and, $AC=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}=\begin{bmatrix}1-2 & 0\\ -2-1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ -3 & 0\end{bmatrix}$

Thus, $AB+AC=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}+\begin{bmatrix}-1 & 0\\ -3 & 0\end{bmatrix}=\begin{bmatrix}7 & -5\\ -4 & -10\end{bmatrix} \quad \ldots\text{(iv)}$

Hence from equation (iii) and (iv), we have

$A\cdot(B+C)=A\cdot B+A\cdot C$

23. If P = $\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$ and Q = $\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, prove that PQ = $\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}$ = QP.

Solution:

Given, $P=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}$ and $Q=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$

It's seen that both P and Q are diagonal matrices.
We know that, for diagonal matrices elements of product matrix are obtained by multiplying elements of matrices in the principal diagonal.

Hence,

$$PQ=\begin{bmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{bmatrix}\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$$

$$=\begin{bmatrix}xa & 0 & 0\\ 0 & yb & 0\\ 0 & 0 & zc\end{bmatrix}=\begin{bmatrix}ax & 0 & 0\\ 0 & by & 0\\ 0 & 0 & zc\end{bmatrix}=QP$$

Therefore, PQ = QP

24. If: $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ **= A, find A.**

Solution:

$$\text{Given, } [2\ 1\ 3]_{1\times 3} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}_{3\times 1} = A$$

$$\text{So, } [2\ 1\ 3]_{1\times 3} \begin{bmatrix} -1+0+1 \\ -1+0+0 \\ 0+0-1 \end{bmatrix}_{3\times 1} = A$$

$$[2\ 1\ 3]_{1\times 3} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}_{3\times 1} = A$$

$$[0-1-3] = A$$

$$\text{Thus, } A = [-4]$$

25. If $A = \begin{bmatrix} 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, **verify that A (B + C) = (AB + AC).**

Solution:

$$\text{Given, } A = [2\ 1],\ B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Now,

$$A(B+C) = [2\ 1] \begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix}$$

$$= [2\ 1] \begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix}$$

$$= [8+9\ \ 10+7\ \ 10+8]$$

$$= [17\ 17\ 18] \qquad \ldots(i)$$

And,

$$AB = [2\ 1] \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix} = [10+8\ \ 6+7\ \ 8+6] = [18\ 13\ 14]$$

And,

$$AC = [2\ 1]\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = [-2+1\ \ 4+0\ \ 2+2] = [-1\ 4\ 4]$$

So, $AB + AC = [18\ 13\ 14] + [-1\ 4\ 4]$

$$= [17\ 17\ 18] \quad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$A(B + C) = (AB + AC)$$

Hence, verified

26. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, **then verify that** $A^2 + A = A(A + I)$**, where I is 3 × 3 unit matrix.**

Solution:

Given, $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

So, $A^2 = A \cdot A$

Thus, $= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+0+0 & 0+0-1 & -1+0-1 \\ 2+2+0 & 0+1+3 & -2+3+3 \\ 0+2+0 & 0+1+1 & 0+3+1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \ldots\text{(i)}$$

Now,

$$A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

So,

$$A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & 0+0-1 & -1+0-2 \\ 4+2+0 & 0+2+3 & -2+3+6 \\ 0+2+0 & 0+2+1 & 0+3+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get

$A^2 + A = A(A + I)$

27. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, **then verify that:**

(i) $(A')' = A$
(ii) $(AB)' = B'A'$
(iii) $(kA)' = (kA')$.
Solution:

Given, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(i) We have to verify that, $(A')' = A$

So,

$$A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

And, $(A')' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$

(ii) We have to verify that, $(AB)' = B'A'$

So,

$$AB = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

and, $B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix} = (AB)'$

Hence proved.

(iii) We have to verify that, $(kA)' = (kA')$

Now, $(kA) = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$

And, $(kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

Also, $kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix} = (kA)'$

Hence proved.

28. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, **then verify that:**

(i) (2A + B)′ = 2A′ + B′
(ii) (A – B)′ = A′ – B′.
Solution:

Given, $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$

(i) $(2A + B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$

And, $(2A + B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$

Also, $2A' + B'$

$= \begin{bmatrix} 2 & 8 & 10 \\ 4 & 2 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix} = (2A + B)'$

Hence, $2A' + B' = (2A + B)'$

(ii) $A - B = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$

And, $(A - B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

Also, $A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix} = (A - B)'$

Thus, $A' - B' = (A - B)'$

- Hence proved

29. Show that A′A and AA′ are both symmetric matrices for any matrix A.
Solution:

Let P = A′A
So, P′ = (A′A) ′
= A′(A′)′ [As (AB) ′ = B′A′]
Hence, A′A is symmetric matrix for any matrix A.
Now, let Q = AA′
So, Q′ = (AA′)′ = (A) ′ = AA′ = Q
Hence, AA′ is symmetric matrix for any matrix A.

30. Let A and B be square matrices of the order 3 × 3. Is $(AB)^2 = A^2 B^2$? Give reasons.
Solution:

As, A and B be square matrices of order 3 x 3.
We have, $(AB)^2 = AB . AB$
$= A(BA)B$
$= A(AB)B$ [If AB = BA]
$= AABB$
$= A^2B^2$
Thus, $(AB)^2 = A^2B^2$ is true only if AB = BA.

31. Show that if A and B are square matrices such that AB = BA, then $(A + B)^2 = A^2 + 2AB + B^2$.
Solution:

Given, A and B are square matrices such that AB = BA.
So, $(A + B)^2 = (A + B) . (A + B)$
$= A^2 + AB + BA + B^2$
$= A^2 + AB + AB + B^2$ [Since, AB = BA]
$= A^2 + 2AB + B^2$

32. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$ **and** $a = 4$, $b = -2$.

Show that:
(a) $A + (B + C) = (A + B) + C$
(b) $A(BC) = (AB)C$
(c) $(a + b)B = aB + bB$
(d) $a(C - A) = aC - aA$
(e) $(A^T)^T = A$
(f) $(bA)^T = b A^T$
(g) $(AB)^T = B^T A^T$
(h) $(A - B)C = AC - BC$
(i) $(A - B)^T = A^T - B^T$
Solution:

(a) $A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

And,

$(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$

(b) $(BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$

And, $A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix} = \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$

Also, $AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

Thus, $(AB)C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix} = A(BC)$

Hence proved.

(c) $(a + b)B = (4 - 2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ [$\because$ given $a = 4$, $b = -2$]

$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

Also,

$aB + bB = 4B - 2B = \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix} = (a + b)B$

Hence proved.

(d) $C - A = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$

And, $a(C - A) = 4(C - A)$

$$= \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$

Also, $aC - aA = 4C - 4A$

$$= \begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix} = a(C - A)$$

Hence proved.

(e) $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Thus, $(A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A$

Hence proved.

(f) $(bA)^T = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T \qquad [\because b = -2]$

$$= \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$$

And, $A^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Thus, $bA^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix} = (bA)^T$

Hence proved.

(g) $AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$

So, $(AB)^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$

Now, $B^T A^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4+2 & -4+3 \\ 0+10 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix} = (AB)^T$

Hence proved.

(h) $(A-B)=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}-\begin{bmatrix}4 & 0\\ 1 & 5\end{bmatrix}=\begin{bmatrix}1-4 & 2-0\\ -1-1 & 3-5\end{bmatrix}=\begin{bmatrix}-3 & 2\\ -2 & -2\end{bmatrix}$

Thus, $(A-B)C=\begin{bmatrix}-3 & 2\\ -2 & -2\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}-4 & -4\\ -6 & 4\end{bmatrix}$

Now,

$$AC=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}4 & -4\\ 1 & -6\end{bmatrix}$$

And,

$$BC=\begin{bmatrix}4 & 0\\ 1 & 5\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -2\end{bmatrix}=\begin{bmatrix}8 & 0\\ 7 & -10\end{bmatrix}$$

Therefore,

$$AC-BC=\begin{bmatrix}4-8 & -4-0\\ 1-7 & -6+10\end{bmatrix}=\begin{bmatrix}-4 & -4\\ -6 & 4\end{bmatrix}=(A-B)C$$

Hence proved.

(i) $(A-B)^T=\begin{bmatrix}1-4 & 2-0\\ -1-1 & 3-5\end{bmatrix}^T=\begin{bmatrix}-3 & 2\\ -2 & -2\end{bmatrix}^T=\begin{bmatrix}-3 & -2\\ 2 & -2\end{bmatrix}$

Now, $A^T-B^T=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}-\begin{bmatrix}4 & 1\\ 0 & 5\end{bmatrix}=\begin{bmatrix}-3 & -2\\ 2 & -2\end{bmatrix}=(A-B)^T$

Hence proved.

33. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, **then show that** $A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$.

Solution:

Given, $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

Now, $A^2=A\cdot A$

$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & \cos\theta\sin\theta+\sin\theta\cos\theta\\ -\sin\theta\cos\theta-\cos\theta\sin\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\theta & 2\sin\theta\cos\theta\\ 2\sin\theta\cos\theta & \cos 2\theta\end{bmatrix}=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$$

Hence proved.

34. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **and $x^2 = -1$, then show that $(A + B)^2 = A^2 + B^2$**

Solution:

Given, $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$

So, $(A + B) = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$

And, $(A + B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$ ...(i)

Also, $A^2 = A \cdot A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix}$

And, $B^2 = B \cdot B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Thus,

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \quad \text{...(ii)}$$

Hence, from equations (i) and (ii), we have

$$(A + B)^2 = A^2 + B^2$$

35. Verify that $A^2 = I$ when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

Solution:

Given, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

So, $A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 0+4-3 & 0-3+3 & 0+4-4 \\ 0-12+12 & 4+9-12 & -4-12+16 \\ 0-12+12 & 3+9-12 & -3-12+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence proved.

36. Prove by Mathematical Induction that $(A')^n = (A^n)'$, where $n \in N$ for any square matrix A.
Solution:

Let P(n): $(A')^n = (A^n)'$
So, P(1): $(A') = (A)'$
$A' = A'$
Hence, P(1) is true.
Now, let $P(k) = (A')^k = (A^k)'$, where $k \in N$
And,
$P(k + 1)$: $(A')^{k+1} = (A')^k A'$
$= (A^k)' A'$
$= (AA^k)'$
$= (A^{k+1})'$
Hence, P(1) is true and whenever P(k) is true P(k + 1) is true.
Therefore, P(n) is true for all $n \in N$.

37. Find inverse, by elementary row operations (if possible), of the following matrices

(i) $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$.

Solution:

(i) Let $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

Now,

$$\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 + 5R_1]$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A \qquad \left[\because R_2 \to \frac{1}{22} R_2\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A \qquad [\because R_1 \to R_1 - 3R_2]$$

I = BA, where B is the inverse of A.
Hence,

$$B = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

Now, $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A$ $[\because R_2 \to R_2 + 2R_1]$

As we obtain all the zeroes in a row of the matrix A on the L.H.S., A^{-1} does not exist.

38. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, **then find values of x, y, z and w.**

Solution:

In the given matrix equation,
Comparing the corresponding elements, we get
$x + y = 6$,
$xy = 8$,
$z + 6 = 0$ and
$w = 4$
From the first two equations, we have
$(6 - y) . y = 8$
$y^2 - 6y + 8 = 0$
$(y - 2) (y - 4) = 0$
$y = 2$ or $y = 4$
Hence, $x = 4$ and $x = 2$
Also, $z + 6 = 0$
$z = -6$ and $w = 4$
Therefore,
$x = 2, y = 4$ or $x = 4, y = 2, z = -6$ and $w = 4$

39. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, **find a matrix C such that 3A + 5B + 2C is a null matrix.**

Solution:

Let's consider a matrix C, such that

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$3A + 5B + 2C = O$$

$$\begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the terms,

$2a + 48 = 0 \Rightarrow a = -24$

$20 + 2b = 0 \Rightarrow b = -10$

$56 + 2c = 0 \Rightarrow c = -28$

And,

$76 + 2d = 0 \Rightarrow d = -38$

Therefore, the matrix C is

$$C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

40. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Solution:

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ ...(i)

Now,

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$A^2 - 5A - 14I = O$$

$$A \cdot A^2 - 5A \cdot A = 14A\, I = O$$

$$A^3 - 5A^2 - 14A = O$$

$$A^3 = 5A^2 + 14A$$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

Hence,

$$A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

41. Find the value of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}.$$

Solution:

Given,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ -1+c+d & 2d+3 \end{bmatrix}$$

Now,
3a = a + 4 ⇒ a = 2
3b = 6 + a + b
3b – b = 8
⇒ b = 4
And,
3d = 3 + 2d
⇒ d = 3
And,
3c = c + d – 1
2c = 3 – 1 = 2
⇒ c = 1
Hence,
a = 2, b = 4, c = 1 and d = 3

42. Find the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}.$$

Solution:

Let, $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Now, by equality of matrices, we get
a = 1, b = -2, c = -5
And,
2a – d = -1 ⇒ d = 2a + 1 = 3;
2b – e = -8 ⇒ e = 2(-2) + 8 = 4
2c – f = -10 ⇒ f = 2c + 10 = 0
Thus,

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

43. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, find $A^2 + 2A + 7I$.

Solution:

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

44. If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, and $A^{-1} = A'$, find value of α.
Solution:

Given,

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Also,

$$A^{-1} = A'$$
$$AA^{-1} = AA'$$
$$I = AA'$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

By using equality of matrices, we get
$\cos^2 \alpha + \sin^2 \alpha = 1$, which is true for all real values of α.

45. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, find the values of a, b and c.

Solution:

Let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$

As, A is skew - symmetric matrix.
So, we have

$$A' = -A$$

Then,

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & 1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get
$a = -2$, $c = -3$ and $b = -b \Rightarrow b = 0$
Hence,
$a = -2$, $b = 0$ and $c = -3$

46. If P (x) = $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that

P (x) . P (y) = P (x + y) = P (y) . P (x).
Solution:

Given,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

So, $$P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

Now,

$$P(x)\cdot P(y)=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cdot\cos y-\sin x\cdot\sin y & \cos x\cdot\sin y+\sin x\cdot\cos y\\ -\sin x\cdot\cos y-\cos x\cdot\sin y & -\sin x\cdot\sin y+\cos x\cdot\cos y\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix}$$

$$=P(x+y) \quad \ldots\text{(i)}$$

Also,

$$P(y)\cdot P(x)=\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$$

$$=\begin{bmatrix}\cos y\cdot\cos x-\sin y\cdot\sin x & \cos y\cdot\sin x+\sin y\cdot\cos x\\ -\sin y\cdot\cos x-\sin x\cdot\cos y & -\sin y\cdot\sin x+\cos y\cdot\cos x\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix} \quad \ldots\text{(ii)}$$

Therefore, form (i) and (ii), we get

$$P(x)\cdot P(y)=P(x+y)=P(y)\cdot P(x)$$

47. If A is square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.
Solution:

We know that,
A . I = I . A
So, A and I are commutative.
Thus, we can expand $(I + A)^3$ like real numbers expansion.
So, $(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$
$= I + 3IA + 3A^2 + AA^2$ (As $I^n = I$, $n \in N$)
$= I + 3A + 3A + AA$
$= I + 3A + 3A + A^2 = I + 3A + 3A + A = I + 7A$

48. If A, B are square matrices of same order and B is a skew-symmetric matrix, show that A′BA is skew symmetric.
Solution:

Given, A and B are square matrices such that B is a skew-symmetric matrix
So, $B' = -B$
Now, we have to prove that A′BA is a skew-symmetric matrix.

$(A'BA)' = A'B'(A')'$ [Since, $(AB)' = B'A'$]
$= A'(-B)A$
$= -A'BA$
Hence, $A'BA$ is a skew-symmetric matrix.

Long Answer (L.A)
49. If AB = BA for any two square matrices, prove by mathematical induction that $(AB)^n = A^n B^n$.
Solution:

Let $P(n) : (AB)^n = A^nB^n$
So, $P(1) : (AB)^1 = A^1B^1$
$AB = AB$
So, P(1) is true.

Let P(n) is true for some $k \in N$
Now,

$$\begin{aligned}(AB)^{k+1} &= (AB)^k(AB) \quad \text{(using (i))}\\ &= A^kB^k(AB)\\ &= A^kB^{k-1}(BA)B\\ &= A^kB^{k-1}(AB)B \quad (\text{as given } AB = BA)\\ &= A^kB^{k-1}AB^2\\ &= A^kB^{k-2}(BA)B^2\\ &= A^kB^{k-2}ABB^2\\ &= A^kB^{k-2}AB^3\\ &\;\;\cdots\cdots\\ &\;\;\cdots\cdots\\ &= A^{k+1}B^{k+1}\end{aligned}$$

Hence, P(1) is true and whenever P(k) is true P(k + 1) is true.
Thus, P(n) is true for all $n \in N$.

50. Find x, y, z if A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfies $A' = A^{-1}$.**
Solution:

Matrix A is such that $A' = A^{-1}$
$AA' = I$

$$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2+z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$4y^2 + z^2 = 1$

$2y^2 - z^2 = 0$

$x^2 + y^2 + z^2 = 1$

$x^2 - y^2 - z^2 = 0$

$y^2 = 1/6, z^2 = 1/3, x^2 = 1/2$

So, the roots are:

$x = \pm\frac{1}{\sqrt{2}}$

$y = \pm\frac{1}{\sqrt{6}}$

And,

$z = \pm\frac{1}{\sqrt{3}}$