

## Exercise 9.3

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**1. Find the solution of**  $\frac{dy}{dx} = 2^{y-x}$   
**Solution:**

Given differential equation,

$$\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x}$$

On separating the variables, we have

$$\frac{dy}{2^y} = \frac{dx}{2^x} \Rightarrow 2^{-y} dy = 2^{-x} dx$$

Now, integrating both the sides, we get

$$\begin{aligned} \int 2^{-y} dy &= \int 2^{-x} dx \\ \frac{-2^{-y}}{\log 2} &= \frac{-2^{-x}}{\log 2} + c \Rightarrow -2^{-y} = -2^{-x} + c \log 2 \\ -2^{-y} + 2^{-x} &= c \log 2 \\ 2^{-x} - 2^{-y} &= k \quad [\text{where } c \log 2 = k] \end{aligned}$$

Thus, the solution of the differential equation is  $2^{-x} - 2^{-y} = k$

**2. Find the differential equation of all non-vertical lines in a plane.**  
**Solution:**

We know that, the equation of all non-vertical lines are  $y = mx + c$

On differentiating w.r.t.  $x$ , we get

$$dy/dx = m$$

Again, on differentiating w.r.t.  $x$ , we have

$$d^2y/dx^2 = 0$$

Thus, the required equation is  $d^2y/dx^2 = 0$ .

**3. Given that**  $\frac{dy}{dx} = e^{-2y}$  **and**  $y = 0$  **when**  $x = 5$ .

**Find the value of**  $x$  **when**  $y = 3$ .

**Solution:**

Given equation,

$$\begin{aligned} \frac{dy}{dx} &= e^{-2y} \\ \frac{dy}{e^{-2y}} &= dx \Rightarrow e^{2y} \cdot dy = dx \end{aligned}$$

Integrating both sides, we get

$$\int e^{2y} dy = \int dx \Rightarrow \frac{1}{2} e^{2y} = x + c$$

Put  $y = 0$  and  $x = 5$

$$\Rightarrow \frac{1}{2} e^0 = 5 + c \Rightarrow c = \frac{1}{2} - 5 = -\frac{9}{2}$$

So, The equation becomes  $\frac{1}{2} e^{2y} = x - \frac{9}{2}$

Now putting  $y = 3$ , we get

$$\frac{1}{2} e^6 = x - \frac{9}{2} \Rightarrow x = \frac{1}{2} e^6 + \frac{9}{2}$$

Thus, the required value of  $x = \frac{1}{2} (e^6 + 9)$ .

**4. Solve the differential equation**  $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$   
**Solution:**

Given different equation,

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

On dividing by  $(x^2 - 1)$ , we have

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{1}{(x^2 - 1)^2}$$

Clearly, it is a linear differential equation of first order and first degree.

$$\text{Now, } P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

$$\text{Integrating factor I.E} = e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = (x^2 - 1).$$

Hence, the solution of the equation is

$$y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C.$$

**5. Solve the differential equation**  $\frac{dy}{dx} + 2xy = y$   
**Solution:**

Given differential equation,

$$dy/dx + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy \Rightarrow \frac{dy}{dx} = y(1 - 2x) \Rightarrow \frac{dy}{y} = (1 - 2x) dx$$

Integrating both sides, we have

$$\int \frac{dy}{y} = \int (1 - 2x) dx \Rightarrow \log y = x - 2 \cdot \frac{x^2}{2} + \log c$$

$$\log y = x - x^2 + \log c \Rightarrow \log y - \log c = x - x^2$$

$$\log \frac{y}{c} = x - x^2 \Rightarrow \frac{y}{c} = e^{x-x^2}$$

$$\therefore y = y = c \cdot e^{x-x^2}$$

Thus, the required solution is  $y = c \cdot e^{x-x^2}$ .

**6. Find the general solution of**  $\frac{dy}{dx} + ay = e^{mx}$   
**Solution:**

Given equation,  $dy/dx + ay = e^{mx}$

Solving for linear differential equation of first order, we have

$P = a$  and  $Q = e^{mx}$

So, I.F.  $= e^{\int P dx} = e^{\int a \cdot dx} = e^{ax}$

Solution of equation is  $y \times \text{I.F.} = \int Q \cdot \text{I.F.} dx + c$

$$y \cdot e^{ax} = \int e^{mx} \cdot e^{ax} dx + c \Rightarrow y \cdot e^{ax} = \int e^{(m+a)x} dx + c$$

$$y \cdot e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + c \Rightarrow y = \frac{e^{(m+a)x}}{(m+a)} \cdot e^{-ax} + c \cdot e^{-ax}$$

$$\therefore y = \frac{e^{mx}}{(m+a)} + c \cdot e^{-ax}$$

Thus, the required solution is  $y = \frac{e^{mx}}{(m+a)} + c \cdot e^{-ax}$ .

**7. Solve the differential equation**  $\frac{dy}{dx} + 1 = e^{x+y}$   
**Solution:**

Given differential equation,  $dy/dx + 1 = e^{x+y}$

Substituting  $x + y = t$  and differentiating w.r.t.  $x$ , we have

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} = e^t \Rightarrow \frac{dt}{e^t} = dx \Rightarrow e^{-t} dt = dx$$

Integrating both sides, we have

$$\int e^{-t} dt = \int dx \Rightarrow -e^{-t} = x + c$$

$$-e^{-(x+y)} = x + c \Rightarrow \frac{-1}{e^{x+y}} = x + c \Rightarrow (x + c) e^{x+y} = -1$$

Thus, the required solution is  $(x + c) \cdot e^{x+y} + 1 = 0$

**8. Solve:  $ydx - xdy = x^2 y dx$ .**

**Solution:**

Given equation,  $ydx - xdy = x^2ydx$

$$y dx - x^2y dx = xdy$$

$$y (1 - x^2) dx = xdy$$

$$\left( \frac{1 - x^2}{x} \right) dx = \frac{dy}{y} \Rightarrow \left( \frac{1}{x} - x \right) dx = \frac{dy}{y}$$

Integrating both sides we get

$$\int \left( \frac{1}{x} - x \right) dx = \int \frac{dy}{y}$$

$$\log x - \frac{x^2}{2} = \log y + \log c$$

$$\log x - \frac{x^2}{2} = \log yc \Rightarrow \log x - \log c = \frac{x^2}{2} \Rightarrow \log \frac{x}{yc} = \frac{x^2}{2}$$

$$\Rightarrow \frac{x}{yc} = e^{x^2/2} \Rightarrow \frac{yc}{x} = e^{-x^2/2} \Rightarrow yc = xe^{-x^2/2}$$

$$\therefore y = \frac{1}{c} \cdot xe^{-x^2/2} \Rightarrow y = kxe^{-x^2/2} \quad \left[ \because k = \frac{1}{c} \right]$$

Thus, the required solution is  $y = kxe^{-x^2/2}$

**9. Solve the differential equation  $dy/dx = 1 + x + y^2 + xy^2$ , when  $y = 0, x = 0$ .**

**Solution:**

Given equation,  $dy/dx = 1 + x + y^2 + xy^2$

$$\frac{dy}{dx} = 1(1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2) \Rightarrow \frac{dy}{1+y^2} = (1+x) dx$$

On integrating both side, we get

$$\int \frac{dy}{1+y^2} = \int (1+x) dx \Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$$

Put  $x = 0$  and  $y = 0$ , we get  $\tan^{-1}(0) = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan \left( x + \frac{x^2}{2} \right)$$

Thus, the required solution is  $y = \tan \left( x + \frac{x^2}{2} \right)$ .

**10. Find the general solution of  $(x + 2y^3) dy/dx = y$ .**

**Solution:**

Given equation,  $(x + 2y^3) \frac{dy}{dx} = y$

$$\frac{dy}{dx} = \frac{y}{x + 2y^3} \Rightarrow \frac{dx}{dy} = \frac{x + 2y^3}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + \frac{2y^3}{y} \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

Here  $P = -\frac{1}{y}$  and  $Q = 2y^2$ .

So,

$$\text{Integrating factor I.F.} = e^{\int P dy} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = e^{\log \frac{1}{y}} = \frac{1}{y}$$

So the solution of the equation is

$$x \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dy + c$$

$$x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy + c$$

$$\frac{x}{y} = 2 \int y dy + c \Rightarrow \frac{x}{y} = 2 \cdot \frac{y^2}{2} + c \Rightarrow \frac{x}{y} = y^2 + c$$

$$x = y^3 + cy = y(y^2 + c)$$

Thus, the required solution is  $x = y(y^2 + c)$

**11. If  $y(x)$  is a solution of  $\left(\frac{2 + \sin x}{1 + y}\right) \frac{dy}{dx} = -\cos x$  and  $y(0) = 1$ , then find the value of  $y(\pi/2)$ .**  
**Solution:**

Given equation,

$$\left(\frac{2 + \sin x}{1 + y}\right) \frac{dy}{dx} = -\cos x$$

$$\left(\frac{2 + \sin x}{\cos x}\right) \frac{dy}{dx} = -(1 + y) \Rightarrow \frac{dy}{(1 + y)} = -\left(\frac{\cos x}{2 + \sin x}\right) dx$$

Integrating both sides, we get

$$\int \frac{dy}{1 + y} = - \int \frac{\cos x}{2 + \sin x} dx$$

$$\log |1 + y| = -\log |2 + \sin x| + \log c$$

$$\log |1 + y| + \log |2 + \sin x| = \log c$$

$$\log (1 + y)(2 + \sin x) = \log c \Rightarrow (1 + y)(2 + \sin x) = c$$

Put  $x = 0$  and  $y = 1$ , we get

$$(1 + 1)(2 + \sin 0) = c \Rightarrow 4 = c$$

So, equation is  $(1 + y)(2 + \sin x) = 4$

Now put  $x = \frac{\pi}{2}$



$$(1+y)\left(2+\sin\frac{\pi}{2}\right)=4$$

$$(1+y)(2+1)=4 \Rightarrow 1+y=\frac{4}{3} \Rightarrow y=\frac{4}{3}-1=\frac{1}{3}$$

$$\text{So, } y\left(\frac{\pi}{2}\right)=\frac{1}{3}$$

Thus, the required solution is  $y\left(\frac{\pi}{2}\right)=\frac{1}{3}$

**12. If  $y(t)$  is a solution of  $(1+t)\frac{dy}{dt}-ty=1$  and  $y(0)=-1$ , then show that  $y(1)=-1/2$ .**  
**Solution:**

Given equation,

$$(1+t)\frac{dy}{dt}-ty=1 \Rightarrow \frac{dy}{dt}-\left(\frac{t}{1+t}\right)y=\frac{1}{1+t}$$

$$\text{Here, } P=\frac{-t}{1+t} \text{ and } Q=\frac{1}{1+t}$$

Now,

$$\begin{aligned} \text{Integrating factor I.F.} &= e^{\int P dt} = e^{\int \frac{-t}{1+t} dt} = e^{-\int \frac{1+t-1}{1+t} dt} \\ &= e^{-\int \left(1-\frac{1}{1+t}\right) dt} = e^{-[t-\log(1+t)]} \\ &= e^{-t+\log(1+t)} = e^{-t} \cdot e^{\log(1+t)} \end{aligned}$$

$$\therefore \text{I.F.} = e^{-t} \cdot (1+t)$$

The required solution of the given different equation is,

$$y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dt + c$$

$$y \cdot e^{-t}(1+t) = \int \frac{1}{(1+t)} \cdot e^{-t} \cdot (1+t) dt + c$$

$$y \cdot e^{-t}(1+t) = \int e^{-t} dt + c$$

$$\Rightarrow y \cdot e^{-t}(1+t) = -e^{-t} + c$$

Putting  $t=0$  and  $y=-1$

$$[\because y(0)=-1]$$

$$-1 \cdot e^0 \cdot 1 = -e^0 + c$$

$$-1 = -1 + c \Rightarrow c = 0$$

So the equation becomes

$$ye^{-t}(1+t) = -e^{-t}$$

Now put  $t=1$

$$y \cdot e^{-1}(1+1) = -e^{-1}$$

$$2y = -1 \Rightarrow y = -\frac{1}{2}$$

Thus,  $y(1) = -\frac{1}{2}$  is verified.

**13. Form the differential equation having  $y = (\sin^{-1}x)^2 + A \cos^{-1}x + B$ , where A and B are arbitrary constants, as its general solution.**

**Solution:**

Given equation is  $y = (\sin^{-1}x)^2 + A \cos^{-1}x + B$

$$\frac{dy}{dx} = 2 \sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} + A \cdot \left( \frac{-1}{\sqrt{1-x^2}} \right)$$

On multiplying both sides by  $\sqrt{1-x^2}$ , we get

$$\sqrt{1-x^2} \frac{dy}{dx} = 2 \sin^{-1}x - A$$

Again differentiating w.r.t x, we get

$$\begin{aligned} \sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1 \times (-2x)}{2\sqrt{1-x^2}} &= \frac{2}{\sqrt{1-x^2}} \\ \Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \frac{dy}{dx} &= \frac{2}{\sqrt{1-x^2}} \end{aligned}$$

On multiplying both sides by  $\sqrt{1-x^2}$ , we get

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$$

Thus, the above is the required differential equation.

**14. Form the differential equation of all circles which pass through origin and whose centres lie on y-axis.**

**Solution:**

The equation of circles which pass through the origin and whose centre lies on the y-axis is given by,

$(x-0)^2 + (y-a)^2 = a^2$ , where (0, a) is the centre

$$x^2 + y^2 + a^2 - 2ay = a^2$$

$$x^2 + y^2 - 2ay = 0 \dots\dots\dots (i)$$

Differentiating both sides w.r.t. x, we get

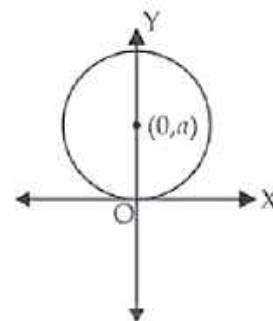
$$2x + 2y \cdot \frac{dy}{dx} - 2a \cdot \frac{dy}{dx} = 0$$

$$x + y \frac{dy}{dx} - a \cdot \frac{dy}{dx} = 0 \Rightarrow x + (y-a) \cdot \frac{dy}{dx} = 0$$

$$y-a = \frac{-x}{\frac{dy}{dx}}$$

$$\therefore a = y + \frac{x}{\frac{dy}{dx}}$$

Putting the value of a in eq. (i), we get



$$x^2 + y^2 - 2 \left( y + \frac{x}{\frac{dy}{dx}} \right) y = 0$$

$$\Rightarrow x^2 + y^2 - 2y^2 - \frac{2xy}{\frac{dy}{dx}} = 0 \Rightarrow x^2 - y^2 = \frac{2xy}{\frac{dy}{dx}}$$

$$\therefore (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

Thus, the required differential equation is

$$(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

**15. Find the equation of a curve passing through origin and satisfying the differential equation**

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$$

**Solution:**

Given equation,

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

Here,  $P = \frac{2x}{1+x^2}$  and  $Q = \frac{4x^2}{1+x^2}$

Integrating factor I.F. =  $e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$

So, the solution is

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + c$$

$$y(1+x^2) = \int \frac{4x^2}{1+x^2} \times (1+x^2) dx + c$$

$$y(1+x^2) = \int 4x^2 dx + c$$

$$\Rightarrow y(1+x^2) = \frac{4}{3} x^3 + c \quad \dots(i)$$

Since the curve is passing through origin i.e., (0, 0)

Putting  $y = 0$  and  $x = 0$  in eq. (i)

$$0(1+0) = \frac{4}{3} (0)^3 + c \Rightarrow c = 0$$

Now,

$$\text{Equation is } y(1+x^2) = \frac{4}{3} x^3 \Rightarrow y = \frac{4x^3}{3(1+x^2)}$$

Thus, the required solution is  $y = \frac{4x^3}{3(1+x^2)}$



**16. Solve:**  $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ .  
**Solution:**

Given equation,

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Now, taking  $y = vx$  [As it is a homogeneous differential equation]

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

So,

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + vx^2 + v^2x^2}{x^2}$$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2(1 + v + v^2)}{x^2}$$

$$v + x \cdot \frac{dv}{dx} = 1 + v + v^2 \Rightarrow x \cdot \frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow x \cdot \frac{dv}{dx} = 1 + v^2 \Rightarrow \frac{dv}{1 + v^2} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{1 + v^2} = \int \frac{dx}{x}$$

$$\Rightarrow \tan^{-1}v = \log x + c \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + c$$

Thus, the required solution is  $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

**17. Find the general solution of the differential equation**  $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$ .  
**Solution:**

Given differential equation,

$$(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$$

$$(x - e^{\tan^{-1}y}) \frac{dy}{dx} = -(1 + y^2) \Rightarrow \frac{dy}{dx} = \frac{-(1 + y^2)}{x - e^{\tan^{-1}y}}$$

$$\frac{dx}{dy} = \frac{x - e^{\tan^{-1}y}}{-(1 + y^2)} \Rightarrow \frac{dx}{dy} = -\frac{x}{(1 + y^2)} + \frac{e^{\tan^{-1}y}}{1 + y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1 + y^2)} = \frac{e^{\tan^{-1}y}}{1 + y^2}$$

[First order linear differential equation in y]

Here,  $P = \frac{1}{1+y^2}$  and  $Q = \frac{e^{\tan^{-1}y}}{1+y^2}$

So, Integrating factor I.F. =  $e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1}y}$

And, the solution is

$$x \cdot \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dy + c$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \cdot e^{\tan^{-1}y} dy + c$$

Put  $e^{\tan^{-1}y} = t$ , we have

$$e^{\tan^{-1}y} \cdot \frac{1}{1+y^2} dy = dt$$

$$x \cdot e^{\tan^{-1}y} = \int t \cdot dt + c$$

$$x \cdot e^{\tan^{-1}y} = \frac{1}{2} t^2 + c$$

$$x \cdot e^{\tan^{-1}y} = \frac{1}{2} (e^{\tan^{-1}y})^2 + c \Rightarrow x = \frac{1}{2} (e^{\tan^{-1}y}) + \frac{c}{e^{\tan^{-1}y}}$$

$$2x = e^{\tan^{-1}y} + \frac{2c}{e^{\tan^{-1}y}}$$

$$\Rightarrow 2x \cdot e^{\tan^{-1}y} = (e^{\tan^{-1}y})^2 + 2c$$

Thus, the above is the required solution of the given differential equation.

**18. Find the general solution of  $y^2 dx + (x^2 - xy + y^2) dy = 0$ .**

**Solution:**

Given equation,  $y^2 dx + (x^2 - xy + y^2) dy = 0$

$$y^2 dx = -(x^2 - xy + y^2) dy$$

$$\Rightarrow \frac{dx}{dy} = -\frac{x^2 - xy + y^2}{y^2}$$

Since it is a homogeneous differential equation

Putting,  $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$

$$\text{So, } v + y \cdot \frac{dv}{dy} = -\left(\frac{v^2 y^2 - vy^2 + y^2}{y^2}\right)$$

$$v + y \cdot \frac{dv}{dy} = -\frac{y^2(v^2 - v + 1)}{y^2}$$

$$v + y \cdot \frac{dv}{dy} = -(v^2 + v - 1) \Rightarrow y \cdot \frac{dv}{dy} = -v^2 + v - 1 - v$$

$$y \cdot \frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{(v^2 + 1)} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\int \frac{dv}{(v^2 + 1)} = -\int \frac{dy}{y} \Rightarrow \tan^{-1} v = -\log y + c$$

$$\Rightarrow \tan^{-1} \left( \frac{x}{y} \right) + \log y = c$$

Thus, the required solution is  $\tan^{-1} \left( \frac{x}{y} \right) + \log y = c$ .

**19. Solve :**  $(x + y)(dx - dy) = dx + dy$ . [Hint: Substitute  $x + y = z$  after separating  $dx$  and  $dy$ ]

**Solution:**

Given differential equation,  $(x + y)(dx - dy) = dx + dy$

$$(x + y)dx - (x - y)dy = dx + dy$$

$$-(x + y)dy - dy = dx - (x + y)dx$$

$$-(x + y + 1)dy = -(x + y - 1)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y - 1}{x + y + 1}$$

Putting  $x + y = z$ , we have

$$\text{So, } 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\text{Now, } \frac{dz}{dx} - 1 = \frac{z - 1}{z + 1}$$

$$\frac{dz}{dx} = \frac{z - 1}{z + 1} + 1 \Rightarrow \frac{dz}{dx} = \frac{z - 1 + z + 1}{z + 1}$$

$$\frac{dz}{dx} = \frac{2z}{z + 1} \Rightarrow \frac{z + 1}{z} dz = 2 \cdot dx$$

Integrating both sides, we get

$$\int \frac{z + 1}{z} dz = 2 \int dx$$

$$\int \left( 1 + \frac{1}{z} \right) dz = 2 \int dx$$

$$z + \log |z| = 2x + \log |c|$$

$$x + y + \log |x + y| = 2x + \log |c|$$

$$y + \log |x + y| = x + \log |c|$$

$$\log |x + y| = x - y + \log |c|$$

$$\log |x + y| - \log |c| = (x - y)$$

$$\Rightarrow \log \left| \frac{x+y}{c} \right| = (x-y) \Rightarrow \frac{x+y}{c} = e^{x-y}$$

$$\therefore x+y = c \cdot e^{x-y}$$

Thus, the required solution is  $x+y = c \cdot e^{x-y}$

**20. Solve :  $2(y+3) - xy \, dy/dx = 0$ , given that  $y(1) = -2$ .**

**Solution:**

Given differential equation,  $2(y+3) - xy \, dy/dx = 0$

$$2(y+3) - xy \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{dy}{dx} = 2y+6$$

$$\left( \frac{y}{2y+6} \right) dy = \frac{dx}{x} \Rightarrow \frac{1}{2} \left( \frac{y}{y+3} \right) dy = \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{1}{2} \int \frac{y}{y+3} \cdot dy = \int \frac{dx}{x} \Rightarrow \frac{1}{2} \int \frac{y+3-3}{y+3} dy = \int \frac{dx}{x}$$

$$\frac{1}{2} \int \left( 1 - \frac{3}{y+3} \right) dy = \int \frac{dx}{x}$$

$$\frac{1}{2} \int 1 \cdot dy - \frac{3}{2} \int \frac{1}{y+3} dy = \int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} y - \frac{3}{2} \log |y+3| = \log x + c$$

Putting  $x = 1$  and  $y = -2$ , we have

$$\Rightarrow \frac{1}{2} (-2) - \frac{3}{2} \log |-2+3| = \log (1) + c$$

$$-1 - \frac{3}{2} \log (1) = \log (1) + c$$

$$-1 - 0 = 0 + c \quad [\because \log (1) = 0]$$

$$\therefore c = -1$$

Now, the equation is

$$\frac{1}{2} y - \frac{3}{2} \log |y+3| = \log x - 1$$

$$y - 3 \log |y+3| = 2 \log x - 2$$

$$y - \log |(y+3)^3| = \log x^2 - 2$$

$$\log |(y+3)^3| + \log x^2 = y + 2$$

$$\log |x^2 (y+3)^3| = y + 2 \Rightarrow x^2 (y+3)^3 = e^{y+2}$$

Thus, the required solution is  $x^2 (y+3)^3 = e^{y+2}$ .

**21. Solve the differential equation  $dy = \cos x (2 - y \operatorname{cosec} x) dx$  given that  $y = 2$  when  $x = \pi/2$ .**

**Solution:**

Given differential equation,  $dy = \cos x (2 - y \operatorname{cosec} x) dx$

$$\frac{dy}{dx} = \cos x (2 - y \operatorname{cosec} x) \Rightarrow \frac{dy}{dx} = 2 \cos x - y \cos x \cdot \operatorname{cosec} x$$

$$\frac{dy}{dx} = 2 \cos x - y \cot x \Rightarrow \frac{dy}{dx} + y \cot x = 2 \cos x$$

Here,  $P = \cot x$  and  $Q = 2 \cos x$ .

So, Integrating factor I.F. =  $e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

And, Required solution is  $y \times \text{I.F.} = \int Q \times \text{I.F.} dx + c$

$$y \cdot \sin x = \int 2 \cos x \cdot \sin x dx + c$$

$$y \cdot \sin x = \int \sin 2x dx + c \Rightarrow y \cdot \sin x = -\frac{1}{2} \cos 2x + c$$

Put  $x = \frac{\pi}{2}$  and  $y = 2$ , we get

$$2 \sin \frac{\pi}{2} = -\frac{1}{2} \cos \pi + c$$

$$2(1) = -\frac{1}{2}(-1) + c \Rightarrow 2 = \frac{1}{2} + c \Rightarrow c = 2 - \frac{1}{2} = \frac{3}{2}$$

Thus, the equation is  $y \sin x = -\frac{1}{2} \cos 2x + \frac{3}{2}$ .

**22. Form the differential equation by eliminating A and B in  $Ax^2 + By^2 = 1$ .**

**Solution:**

Given,  $Ax^2 + By^2 = 1$

Differentiating w.r.t. x, we get

$$2A \cdot x + 2By \frac{dy}{dx} = 0$$

$$Ax + By \cdot \frac{dy}{dx} = 0 \Rightarrow By \cdot \frac{dy}{dx} = -Ax$$

$$\Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

Differentiating both sides again w.r.t. x, we have

$$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \left( \frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2} \right) = 0$$

$$\frac{yx^2}{x} \cdot \frac{d^2y}{dx^2} + x \cdot \left( \frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{d^2y}{dx^2} + x \cdot \left( \frac{dy}{dx} \right)^2 - y \cdot \frac{dy}{dx} = 0 \Rightarrow xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$



Thus, the required equation is

$$xy \cdot y'' + x \cdot (y')^2 - y \cdot y' = 0$$

**23. Solve the differential equation  $(1 + y^2) \tan^{-1}x \, dx + 2y (1 + x^2) \, dy = 0$ .**

**Solution:**

Given differential equation,  $(1 + y^2) \tan^{-1}x \, dx + 2y (1 + x^2) \, dy = 0$

$$2y (1 + x^2) \, dy = -(1 + y^2) \tan^{-1}x \, dx$$

$$2y (1 + x^2) \, dy = -(1 + y^2) \cdot \tan^{-1}x \cdot dx$$

$$\frac{2y}{1 + y^2} \, dy = -\frac{\tan^{-1}x}{1 + x^2} \cdot dx$$

Integrating both sides, we get

$$\int \frac{2y}{1 + y^2} \, dy = -\int \frac{\tan^{-1}x}{1 + x^2} \cdot dx$$

$$\log|1 + y^2| = -\frac{1}{2}(\tan^{-1}x)^2 + c$$

$$\Rightarrow \frac{1}{2}(\tan^{-1}x)^2 + \log|1 + y^2| = c$$

Thus, the above equation is the required solution.

**24. Find the differential equation of system of concentric circles with centre (1, 2).**

**Solution:**

The family of concentric circles with centre (1, 2) and radius 'r' is given by

$$(x - 1)^2 + (y - 2)^2 = r^2$$

Differentiating both sides w.r.t, x we get

$$2(x - 1) + 2(y - 2) \frac{dy}{dx} = 0 \Rightarrow (x - 1) + (y - 2) \frac{dy}{dx} = 0$$

Thus, the above is the required equation.

**Long Answer (L.A.)**

**25. Solve :**  $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

**Solution:**

Given differential equation,

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$y + x \cdot \frac{dy}{dx} + y = x(\sin x + \log x)$$

$$x \frac{dy}{dx} = x(\sin x + \log x) - 2y$$

$$\Rightarrow \frac{dy}{dx} = (\sin x + \log x) - \frac{2y}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x}y = (\sin x + \log x)$$

Here,  $P = \frac{2}{x}$  and  $Q = (\sin x + \log x)$

Integrating factor I.F. =  $e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$

Now, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \, dx + c$$

$$\Rightarrow y \cdot x^2 = \int (\sin x + \log x) x^2 dx + c \quad \dots(1)$$

$$\begin{aligned} \text{Let } I &= \int (\sin x + \log x) x^2 dx \\ &= \int x^2 \sin x \, dx + \int x^2 \log x \, dx \\ &= \left[ x^2 \cdot \int \sin x \, dx - \int \left( D(x^2) \cdot \int \sin x \, dx \right) dx \right] + \\ &\quad \left[ \log x \cdot \int x^2 \, dx - \int \left( D(\log x) \cdot \int x^2 \, dx \right) dx \right] \\ &= \left[ x^2 (-\cos x) - 2 \int -x \cos x \, dx \right] + \left[ \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \right] \\ &= \left[ -x^2 \cos x + 2 \left( x \sin x - \int 1 \cdot \sin x \, dx \right) \right] + \left[ \frac{x^3}{3} \log x - \frac{1}{3} \int x^2 dx \right] \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 \end{aligned}$$

Now from eq (1) we get,

$$y \cdot x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 + c$$

$$\therefore y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9} x + c \cdot x^{-2}$$

Thus, the required solution is

$$y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9} x + c \cdot x^{-2}$$

**26. Find the general solution of  $(1 + \tan y)(dx - dy) + 2xdy = 0$ .**

**Solution:**

Given,  $(1 + \tan y)(dx - dy) + 2xdy = 0$

$$(1 + \tan y) dx - (1 + \tan y) dy + 2xdy = 0$$

$$(1 + \tan y) dx - (1 + \tan y - 2x) dy = 0$$

$$(1 + \tan y) \frac{dx}{dy} = (1 + \tan y - 2x) \Rightarrow \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$$

$$\frac{dx}{dy} = 1 - \frac{2x}{1 + \tan y} \Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Here,  $P = \frac{2}{1 + \tan y}$  and  $Q = 1$

Integrating factor I.F.

$$\begin{aligned}
 &= e^{\int \frac{2}{1+\tan y} dy} = e^{\int \frac{2 \cos y}{\sin y + \cos y} dy} \\
 &= e^{\int \frac{\sin y + \cos y - \sin y + \cos y}{(\sin y + \cos y)} dy} = e^{\int \left(1 + \frac{\cos y - \sin y}{\sin y + \cos y}\right) dy} \\
 &= e^{\int 1 dy} \cdot e^{\int \frac{\cos y - \sin y}{\sin y + \cos y} dy} \\
 &= e^y \cdot e^{\log(\sin y + \cos y)} = e^y \cdot (\sin y + \cos y)
 \end{aligned}$$

Now, the solution is  $x \times \text{I.F.} = \int Q \times \text{I.F.} dy + c$

$$\Rightarrow x \cdot e^y (\sin y + \cos y) = \int 1 \cdot e^y (\sin y + \cos y) dy + c$$

$$x \cdot e^y (\sin y + \cos y) = e^y \cdot \sin y + c$$

$$\left[ \because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c \right]$$

$$x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$$

Thus, the required solution is  $x(\sin y + \cos y) = \sin y + c \cdot e^{-y}$ .

**27. Solve:**  $dy/dx = \cos(x+y) + \sin(x+y)$ .

[Hint: Substitute  $x+y=z$ ]

**Solution:**

Putting  $x+y=v$  and on differentiating w.r.t.  $x$ , we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\frac{dv}{dx} = \cos v + \sin v + 1$$

So,

$$\frac{dv}{\cos v + \sin v + 1} = dx$$

Integrating both sides, we have

$$\int \frac{dv}{\cos v + \sin v + 1} = \int 1 \cdot dx$$

$$\int \frac{dv}{\left( \frac{1 - \tan^2 \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + \frac{2 \tan \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + 1 \right)} = \int 1 \cdot dx$$

$$\int \frac{\left( 1 + \tan^2 \frac{v}{2} \right)}{1 - \tan^2 \frac{v}{2} + 2 \tan \frac{v}{2} + 1 + \tan^2 \frac{v}{2}} dv = \int 1 \cdot dx$$

$$\int \frac{\sec^2 \frac{v}{2}}{2 + 2 \tan \frac{v}{2}} dv = \int 1 \cdot dx$$

Now, putting  $2 + 2 \tan \frac{v}{2} = t$

Differentiating,  $2 \cdot \frac{1}{2} \sec^2 \frac{v}{2} dv = dt \Rightarrow \sec^2 \frac{v}{2} dv = dt$

$$\int \frac{dt}{t} = \int 1 \cdot dx$$

$$\log |t| = x + c$$

$$\log \left| 2 + 2 \tan \frac{v}{2} \right| = x + c$$

$$\log \left| 2 + 2 \tan \left( \frac{x+y}{2} \right) \right| = x + c \Rightarrow \log 2 \left[ 1 + \tan \left( \frac{x+y}{2} \right) \right] = x + c$$

$$\log 2 + \log \left[ 1 + \tan \left( \frac{x+y}{2} \right) \right] = x + c$$

$$\Rightarrow \log \left[ 1 + \tan \left( \frac{x+y}{2} \right) \right] = x + c - \log 2$$

Thus, the required solution is

$$\log \left[ 1 + \tan \left( \frac{x+y}{2} \right) \right] = x + K \quad [c - \log 2 = K]$$

**28. Find the general solution of  $dy/dx - 3y = \sin 2x$ .**

**Solution:**

Given equation,  $dy/dx - 3y = \sin 2x$

It's a first order linear differential equation

Here  $P = -3$  and  $Q = \sin 2x$

So,

Integrating factor I.F. =  $e^{\int P dx} = e^{\int -3 dx} = e^{-3x}$

And, the solution is

$$y \times \text{I.F.} = \int Q \cdot \text{I.F.} \cdot dx + c$$

$$\Rightarrow y \cdot e^{-3x} = \int \sin 2x \cdot e^{-3x} dx + c$$

Now let,  $I = \int \sin 2x \cdot e^{-3x} dx$

$$I = \sin 2x \cdot \int e^{-3x} dx - \int (D(\sin 2x) \cdot \int e^{-3x} dx) dx$$

$$I = \sin 2x \cdot \frac{e^{-3x}}{-3} - \int 2 \cos 2x \cdot \frac{e^{-3x}}{-3} dx$$

$$I = \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \int \cos 2x \cdot e^{-3x} dx$$

$$\begin{aligned}
 I &= \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[ \cos 2x \cdot \int e^{-3x} dx - \int [D \cos 2x \cdot \int e^{-3x} dx] dx \right] \\
 I &= \frac{e^{-3x}}{-3} \sin 2x + \frac{2}{3} \left[ \cos 2x \cdot \frac{e^{-3x}}{-3} - 2 \sin 2x \cdot \frac{e^{-3x}}{-3} \right] dx \\
 I &= \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} \cos 2x \cdot e^{-3x} - \frac{4}{9} \int \sin 2x \cdot e^{-3x} dx \\
 &= \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x - \frac{4}{9} I \\
 I + \frac{4}{9} I &= \frac{e^{-3x}}{-3} \sin 2x - \frac{2}{9} e^{-3x} \cos 2x \\
 \frac{13}{9} I &= -\frac{1}{9} [3 e^{-3x} \sin 2x + 2 e^{-3x} \cos 2x] \\
 I &= -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x]
 \end{aligned}$$

Hence, the equation becomes

$$\begin{aligned}
 y \cdot e^{-3x} &= -\frac{1}{13} e^{-3x} [3 \sin 2x + 2 \cos 2x] + c \\
 \therefore y &= -\frac{1}{13} [3 \sin 2x + 2 \cos 2x] + c \cdot e^{3x}
 \end{aligned}$$

Thus, the required solution is

$$y = -\left[ \frac{3 \sin 2x + 2 \cos 2x}{13} \right] + c \cdot e^{3x}$$

**29. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is  $(x^2 + y^2)/2xy$ .**

**Solution:**

Given that the slope of tangent to a curve at (x, y) is  $(x^2 + y^2)/2xy$ .

It's a homogeneous differential function

$$\text{So, putting } y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2x \cdot vx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{1 + v^2}{2v}$$



$$x \cdot \frac{dv}{dx} = \frac{1+v^2}{2v} - v \Rightarrow x \cdot \frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \frac{2v}{1-v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v}{1-v^2} dv = \int \frac{dx}{x} \Rightarrow -\log |1-v^2| = \log x + \log c$$

$$-\log \left| 1 - \frac{y^2}{x^2} \right| = \log x + \log c \Rightarrow -\log \left| \frac{x^2 - y^2}{x^2} \right| = \log x + \log c$$

$$\log \left| \frac{x^2}{x^2 - y^2} \right| = \log |xc| \Rightarrow \frac{x^2}{x^2 - y^2} = xc$$

As, the curve is passing through the point (2, 1)

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = 2c \Rightarrow \frac{4}{3} = 2c \Rightarrow c = \frac{2}{3}$$

Thus, the required equation is

$$\frac{x^2}{x^2 - y^2} = \frac{2}{3}x \Rightarrow 2(x^2 - y^2) = 3x$$

**30. Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (x, y) is (y - 1)/(x<sup>2</sup> + x)**

**Solution:**

Given that the slope of the tangent to the curve at (x, y) is

$$\frac{dy}{dx} = \frac{y-1}{x^2+x} \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

Integrating both sides, we have

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}} \text{ [making perfect square]}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\log |y-1| = \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}} \right| + \log c$$

$$\log |y - 1| = \log \left| \frac{x}{x+1} \right| + \log c$$

$$\log |y - 1| = \log \left| c \left( \frac{x}{x+1} \right) \right|$$

$$y - 1 = \frac{cx}{x+1} \Rightarrow (y - 1)(x + 1) = cx$$

As, the line is passing through the point (1, 0), then  $(0 - 1)(1 + 1) = c(1) \Rightarrow c = 2$

Thus, the required solution is  $(y - 1)(x + 1) = 2x$ .

**31. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abscissa and ordinate of the point.**

**Solution:**

We know that,

The slope of the tangent of the curve =  $dy/dx$

And the difference between the abscissa and ordinate =  $x - y$

So, as given in the question we have

$$dy/dx = (x - y)^2$$

Taking,  $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So, the equation becomes

$$1 - \frac{dv}{dx} = v^2 \Rightarrow \frac{dv}{dx} = 1 - v^2 \Rightarrow \frac{dv}{1 - v^2} = dx$$

Integrating both sides, we get

$$\int \frac{dv}{1 - v^2} = \int dx$$

$$\frac{1}{2} \log \left| \frac{1+v}{1-v} \right| = x + c \Rightarrow \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| = x + c \quad \dots(1)$$

As, the curve is passing through (0, 0)

$$\text{then } \frac{1}{2} \log \left| \frac{1+0-0}{1-0+0} \right| = 0 + c \Rightarrow c = 0$$

$\therefore$  On putting  $c = 0$  in eq. (1) we get

$$\frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| = x \Rightarrow \log \left| \frac{1+x-y}{1-x+y} \right| = 2x$$

$$\frac{1+x-y}{1-x+y} = e^{2x}$$

$$\Rightarrow (1+x-y) = e^{2x} (1-x+y)$$

Thus, the required equation is  $(1+x-y) = e^{2x} (1-x+y)$

**32. Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P (x, y) on the curve meets the co-ordinate axes at A and B such that P is the mid-point of AB.**

**Solution:**

Let's take P(x, y) be any point on the curve and AB be the tangent to the given curve at P.

Also given, P is the mid-point of AB

So, the coordinates of A and B are (2x, 0) and (0, 2y) respectively.

The slope of the tangent AB =  $(2y - 0) / (0 - 2x) = -y/x$

$$\text{So, } \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\log y + \log x = \log c \Rightarrow \log yx = \log c$$

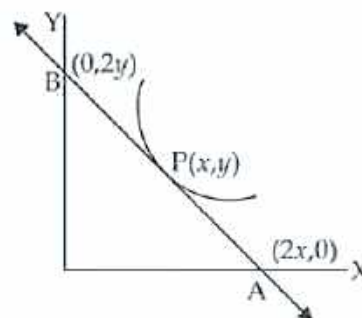
$$\therefore yx = c$$

As, the curve passes through (1, 1)

$$1 \times 1 = c \therefore c = 1$$

$$\Rightarrow yx = 1$$

Thus, the required equation is  $xy = 1$



**33. Solve :**  $x \frac{dy}{dx} = y (\log y - \log x + 1)$

**Solution:**

Given that:  $x \frac{dy}{dx} = y (\log y - \log x + 1)$

$$\Rightarrow x \frac{dy}{dx} = y \left[ \log \left( \frac{y}{x} \right) + 1 \right] \Rightarrow \frac{dy}{dx} = \frac{y}{x} \left[ \log \left( \frac{y}{x} \right) + 1 \right]$$

As its a homogeneous different equation.

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{vx}{x} \left[ \log \left( \frac{vx}{x} \right) + 1 \right]$$

$$v + x \cdot \frac{dv}{dx} = v [\log v + 1]$$

$$x \cdot \frac{dv}{dx} = v [\log v + 1] - v \Rightarrow x \cdot \frac{dv}{dx} = v [\log v + 1 - 1]$$

$$\Rightarrow x \cdot \frac{dv}{dx} = v \cdot \log v \Rightarrow \frac{dv}{v \log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

Now, put  $\log v = t$  on L.H.S.

$$\begin{aligned}\frac{1}{v} dv &= dt \\ \int \frac{dt}{t} &= \int \frac{dx}{x} \\ \log |t| &= \log |x| + \log c \\ \log |\log v| &= \log xc \Rightarrow \log v = xc \\ \Rightarrow \log \left( \frac{y}{x} \right) &= xc\end{aligned}$$

Thus, the required solution is  $\log \left( \frac{y}{x} \right) = xc$ .

### Objective Type

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q)

**34. The degree of the differential equation**  $\left( \frac{d^2 y}{dx^2} \right)^2 + \left( \frac{dy}{dx} \right)^2 = x \sin \left( \frac{dy}{dx} \right)$  **is:**  
(A) 1 (B) 2 (C) 3 (D) not defined

**Solution:**

Correct option is (D) not defined.

Since the value of  $\sin (dy/dx)$  on expansion will be in increasing power of  $dy/dx$ , the degree of the given differential equation is not defined.

**35. The degree of the differential equation**  $\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = \frac{d^2 y}{dx^2}$  **is**  
(A) 4 (B) 3/2 (C) not defined (D) 2

**Solution:**

Correct option is (D) 2.

Given differential equation is

$$\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = \left( \frac{d^2 y}{dx^2} \right)$$

Squaring both sides, we have

$$\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = \left( \frac{d^2 y}{dx^2} \right)^2$$

Thus, the degree of the given differential equation is 2.

**36. The order and degree of the differential equation**  $\frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$ , **respectively, are**

(A) 2 and not defined (B) 2 and 2 (C) 2 and 3 (D) 3 and 3

**Solution:**

**Correct option is (A) 2 and not defined.**

Given differential equation is

$$\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$$

$$\Rightarrow \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} = -x^{\frac{1}{5}}$$

As the degree of  $dy/dx$  is in fraction its undefined and the degree is 2.

**37. If  $y = e^{-x}$  ( $A \cos x + B \sin x$ ), then  $y$  is a solution of**

- (A)  $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} = 0$                       (B)  $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
- (C)  $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$                       (D)  $\frac{d^2 y}{dx^2} + 2y = 0$

**Solution:**

Correct option is (C).

Given equation,  $y = e^{-x} (A \cos x + B \sin x)$

Differentiating on both the sides, w.r.t.  $x$ , we get

$$\frac{dy}{dx} = e^{-x} (-A \sin x + B \cos x) - e^{-x} (A \cos x + B \sin x)$$

$$\frac{dy}{dx} = e^{-x} (-A \sin x + B \cos x) - y$$

Again differentiating w.r.t.  $x$ , we get

$$\frac{d^2 y}{dx^2} = e^{-x} (-A \cos x - B \sin x) - e^{-x} (-A \sin x + B \cos x) - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -e^{-x} (A \cos x + B \sin x) - \left[ \frac{dy}{dx} + y \right] - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -y - \frac{dy}{dx} - y - \frac{dy}{dx}$$

$$\frac{d^2 y}{dx^2} = -2\frac{dy}{dx} - 2y \Rightarrow \frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

**38. The differential equation for  $y = A \cos \alpha x + B \sin \alpha x$ , where  $A$  and  $B$  are arbitrary constants is**

- (A)  $\frac{d^2 y}{dx^2} - \alpha^2 y = 0$                       (B)  $\frac{d^2 y}{dx^2} + \alpha^2 y = 0$
- (C)  $\frac{d^2 y}{dx^2} + \alpha y = 0$                       (D)  $\frac{d^2 y}{dx^2} - \alpha y = 0$

**Solution:**

Correct option is (B).



Given equation is  $y = A \cos ax + B \sin ax$

Differentiating both sides w.r.t.  $x$ , we have

$$\begin{aligned}\frac{dy}{dx} &= -A \sin ax \cdot a + B \cos ax \cdot a \\ &= -Aa \sin ax + Ba \cos ax\end{aligned}$$

Again differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = -Aa^2 \cos ax - Ba^2 \sin ax$$

$$\frac{d^2y}{dx^2} = -a^2 (A \cos ax + B \sin ax)$$

$$\frac{d^2y}{dx^2} = -a^2 y \Rightarrow \frac{d^2y}{dx^2} + a^2 y = 0$$