

Short Answer (S.A.)

1. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints: $2x + y \leq 6$, $x \leq 2$, $x \geq 0$, $y \geq 0$.

Solution:

Given: $Z = 11x + 7y$ and the constraints $2x + y \leq 6$, $x \leq 2$, $x \geq 0$, $y \geq 0$

Let $2x + y = 6$

x	0	3
y	6	0

Now, plotting all the constrain equations we see that the shaded area OABC is the feasible region determined by the constraints.

The feasible region is bounded. So, the maximum value will occur at a corner point of the feasible region.

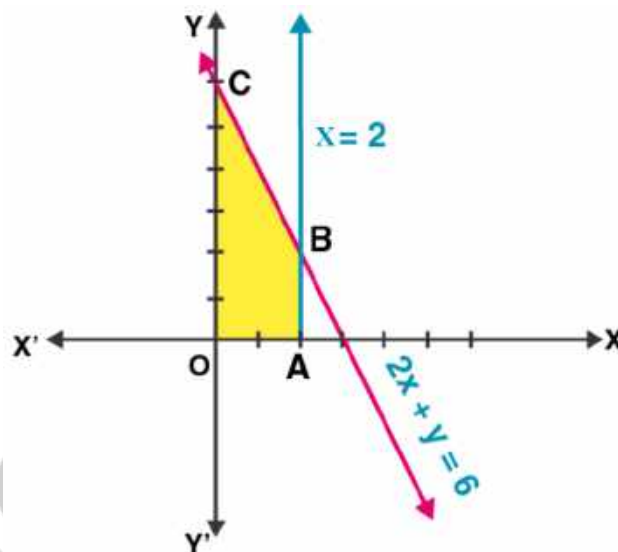
Corner points are $(0, 0)$, $(2, 0)$, $(2, 2)$ and $(0, 6)$.

On evaluating the value of Z , we get

Corner points	Value of Z
$O(0, 0)$	$11(0) + 7(0) = 0$
$A(2, 0)$	$11(2) + 7(0) = 22$
$B(2, 2)$	$11(2) + 7(2) = 36$
$C(0, 6)$	$11(0) + 7(6) = 42$

From the above table it's seen that the maximum value of Z is 42.

Therefore, the maximum value of Z is 42 at $(0, 6)$.



2. Maximize $Z = 3x + 4y$, subject to the constraints: $x + y \leq 1$, $x \geq 0$, $y \geq 0$.

Solution:

Given: $Z = 3x + 4y$ and the constraints $x + y \leq 1$, $x \geq 0$, $y \geq 0$

Taking $x + y = 1$, we have

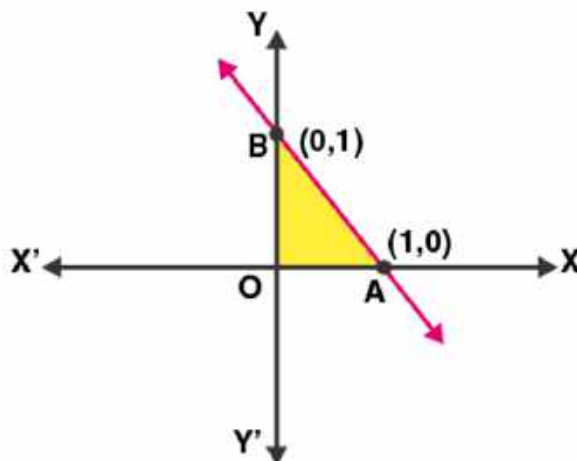
x	1	0
y	0	1

Now, plotting all the constrain equations we see that the shaded area OAB is the feasible region determined by the constraints.

The area is feasible. So, maximum value will occur at the corner points $O(0, 0)$, $A(1, 0)$, $B(0, 1)$.

On evaluating the value of Z , we get

Corner points	Value of Z
$O(0, 0)$	$3(0) + 4(0) = 0$
$A(1, 0)$	$3(1) + 4(0) = 3$



B(0, 1)	$3(0) + 4(1) = 4$
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From the above table it's seen that the maximum value of Z is 4.

Therefore, the maximum value of Z is 4 at (0, 1).

3. Maximize the function $Z = 11x + 7y$, subject to the constraints: $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

Solution:

Given: $Z = 11x + 7y$ and the constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

Plotting all the constrain equations we see that the shaded area OABC is the feasible region determined by the constraints.

The feasible region is bounded with four corner points O(0, 0), A(3, 0), B(3, 2) and C(0, 2).

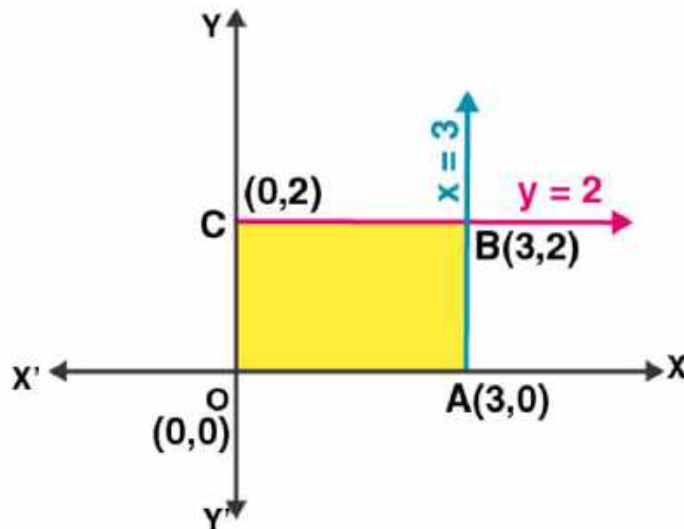
So, the maximum value can occur at any corner.

On evaluating the value of Z , we get

Corner points	Value of Z
O(0, 0)	$11(0) + 7(0) = 0$
A(3, 0)	$11(3) + 7(0) = 33$
B(3, 2)	$11(3) + 7(2) = 47$
C(0, 2)	$11(0) + 7(2) = 14$

From the above table it's seen that the maximum value of Z is 47.

Therefore, the maximum value of the function Z is 47 at (3, 2).



4. Minimize $Z = 13x - 15y$ subject to the constraints: $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.

Solution:

Given: $Z = 13x - 15y$ and the constraints $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.

Taking $x + y = 7$, we have

x	4	3
y	3	4

And, taking $2x - 3y + 6 = 0$ we have

x	1	-3
y	2	0

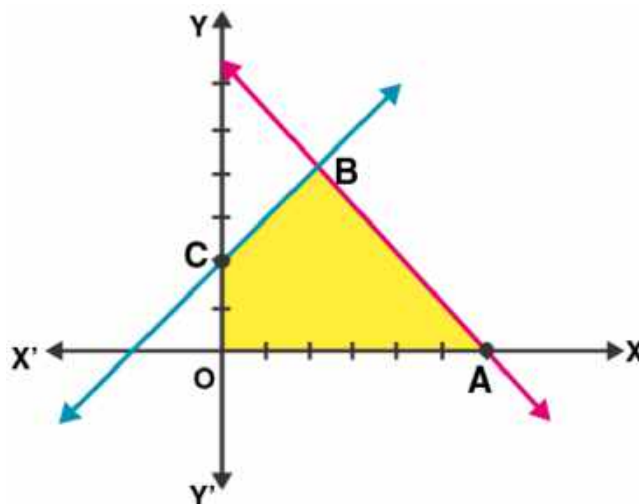
Now, plotting all the constrain equations we see that the shaded area OABC is the feasible region determined by the constraints.

The feasible region is bounded with four corners O(0, 0), A(7, 0), B(3, 4) and C(0, 2).

So, the maximum value can occur at any corner.

On evaluating the value of Z , we get

Corner points	Value of Z
O(0, 0)	$13(0) - 15(0) = 0$
A(7, 0)	$13(7) - 15(0) = 91$

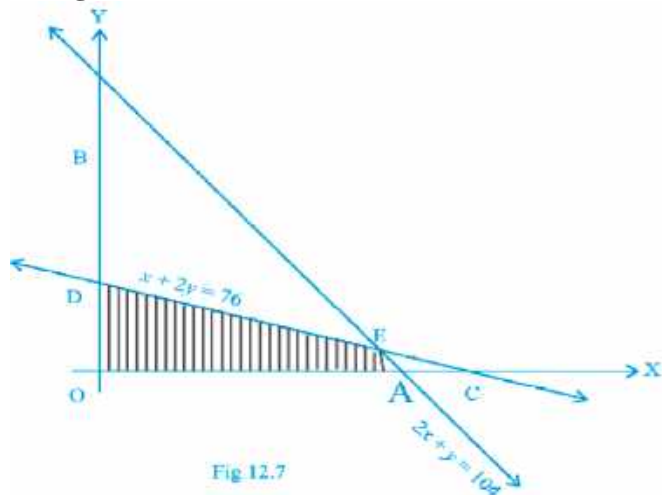


B(3, 4)	$13(3) - 15(4) = -21$
C(0, 2)	$13(0) - 15(2) = -30$

From the above table it's seen that the minimum value of Z is -30.

Therefore, the minimum value of the function Z is -30 at (0, 2).

5. Determine the maximum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in Fig. 12.7.



Solution:

As shown in the figure, OAED is the feasible region.

At A, $y = 0$ in equation $2x + y = 104$ we get,
 $x = 52$

This is a corner point A = (52, 0)

At D, $x = 0$ in equation $x + 2y = 76$ we get,
 $y = 38$

This is another corner point D = (0, 38)

Now, solving the given equations $x + 2y = 76$ and $2x + y = 104$ we have

$$\begin{array}{r} 2x + 4y = 152 \\ 2x + y = 104 \\ (-) \quad (-) \quad (-) \quad (-) \\ \hline 3y = 48 \Rightarrow y = 16 \end{array}$$

Using the value of y in the equation, we get

$$x + 2(16) = 76 \Rightarrow x = 76 - 32 = 44$$

So, the corner point E = (44, 16)

On evaluating the maximum value of Z, we get

Corner points	$Z = 3x + 4y$
O(0, 0)	$Z = 3(0) + 4(0) = 0$
A(52, 0)	$Z = 3(52) + 4(0) = 156$
B(44, 16)	$Z = 3(44) + 4(16) = 196$
D(0, 38)	$Z = 3(0) + 4(38) = 152$

From the above table it's seen that the maximum value of Z is 196.

Therefore, the maximum value of the function Z is 196 at (44, 16).

6. Feasible region (shaded) for a LPP is shown in Fig. 12.8. Maximize $Z = 5x + 7y$.

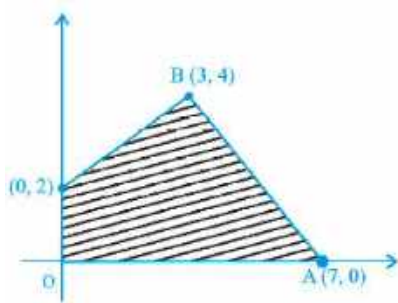


Fig. 12.8

Solution:

Given: $Z = 5x + 7y$ and feasible region OABC.

The corner points of the feasible region are $O(0, 0)$, $A(7, 0)$, $B(3, 4)$ and $C(0, 2)$.

On evaluating the value of Z , we get

Corner points	Value of Z
$O(0, 0)$	$Z = 5(0) + 7(0) = 0$
$A(7, 0)$	$Z = 5(7) + 7(0) = 35$
$B(3, 4)$	$Z = 5(3) + 7(4) = 43$
$C(0, 2)$	$Z = 5(0) + 7(2) = 14$

From the above table it's seen that the maximum value of Z is 43.

Therefore, the maximum value of the function Z is 43 at $(3, 4)$.

7. The feasible region for a LPP is shown in Fig. 12.9. Find the minimum value of $Z = 11x + 7y$.

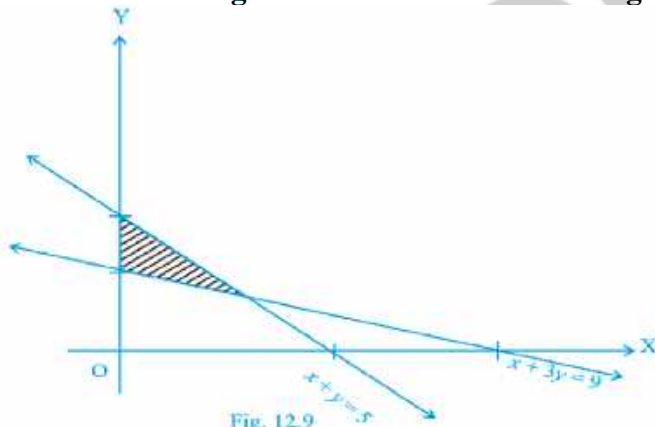


Fig. 12.9

Solution:

In the given figure, it's seen that the feasible region is ABCA. The corner points are $C(0, 3)$, $B(0, 5)$ and for A, we have to solve equations

$$x + 3y = 9 \text{ and}$$

$$x + y = 5$$

$$(-) \quad (-) \quad (-)$$

$$2y = 4 \Rightarrow y = 2$$

And, putting value of y in the equation we get $x = 3$

So, the corner point is A(3, 2).

Now, evaluating the value of Z we get

Corner points	Value of Z
A(3, 2)	$Z = 11(3) + 7(2) = 47$
B(0, 5)	$Z = 11(0) + 7(5) = 35$
C(0, 3)	$Z = 11(0) + 7(3) = 21$

From the above table it's seen that the minimum value of Z is 21.

Therefore, the minimum value of the function Z is 21 at (0, 3).

8. Refer to Exercise 7 above. Find the maximum value of Z.

Solution:

In the evaluating table for the value of Z, it's clearly seen that the maximum value of Z is 47 at (3, 2)

9. The feasible region for a LPP is shown in Fig. 12.10. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists.

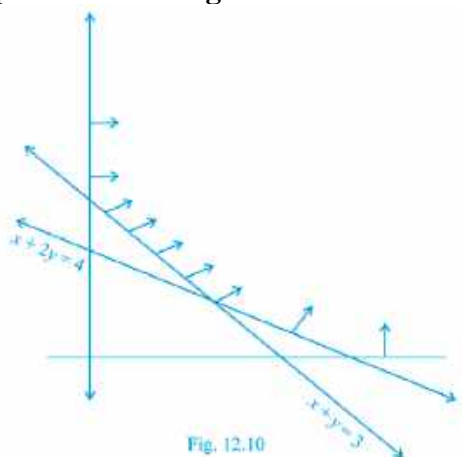


Fig. 12.10

Solution:

Given: $Z = 4x + y$

In the given figure, ABC is the feasible region which is open unbounded.

Here, we have

$$x + y = 3 \quad \dots (i)$$

$$\text{and } x + 2y = 4 \quad \dots (ii)$$

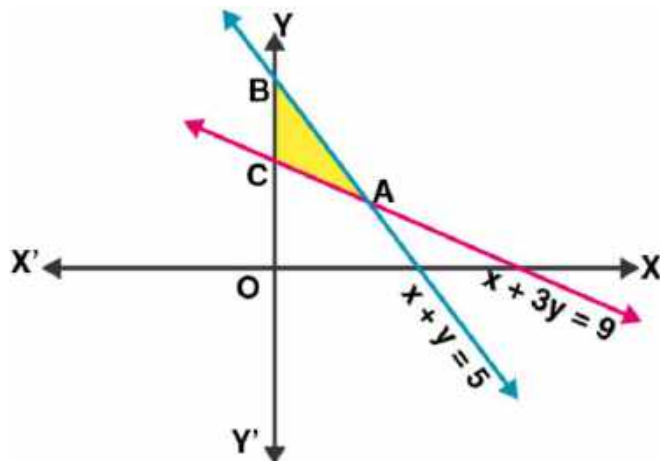
On solving equations (i) and (ii), we get

$$x = 2 \text{ and } y = 1$$

So, the corner points are A(4, 0), B(2, 1) and C(0, 3)

Now on evaluating the value of Z, we have

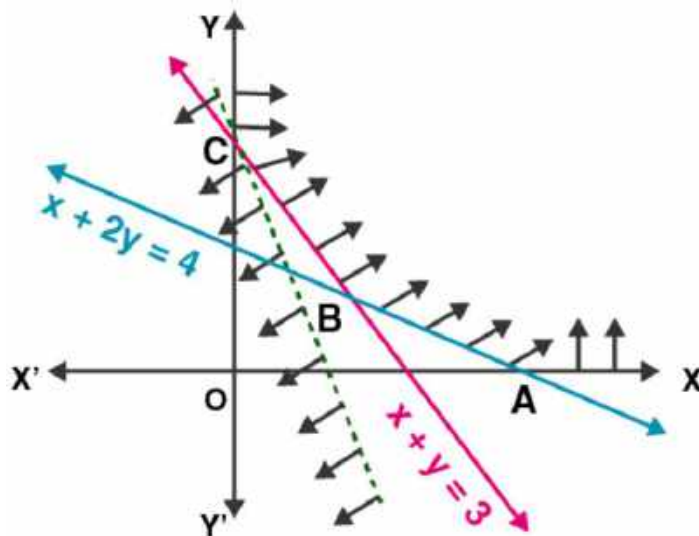
Corner points	$Z = 4x + y$
A(4, 0)	$Z = 4(4) + (0) = 16$



B(2, 1)	$Z = 4(2) + (1) = 9$
C(0, 3)	$Z = 4(0) + (3) = 3$

Now, the minimum value of Z is 3 at (0, 3) but as, the feasible region is open bounded so it may or may not be the minimum value of Z . Hence, in order to face such a situation, we usually draw a graph of $4x + y < 3$ and check whether the resulting open half plane has no point in common with feasible region. Otherwise Z will have no minimum value. So, from the graph, we can conclude that there is no common point with the feasible region.

Therefore, the function Z has the minimum value at (0, 3).



10. In Fig. 12.11, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x + 2y$

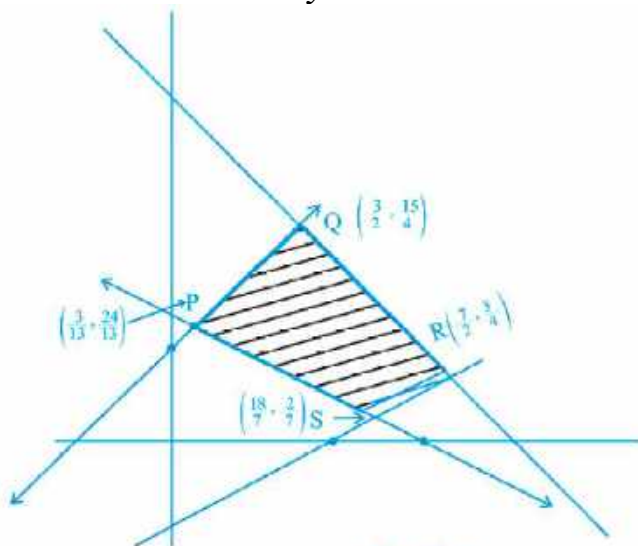


Fig. 12.11

Solution:

From the given figure, it's seen that the corner points are as follows:

$R(7/2, 3/4)$, $Q(3/2, 15/4)$, $P(3/13, 24/13)$ and $S(18/7, 2/7)$.

Now, on evaluating the value of Z for the feasible region RQPS.

Corner points	Value of $Z = x + 2y$
$R(7/2, 3/4)$	$Z = 7/2 + 2(3/4) = 5$
$Q(3/2, 15/4)$	$Z = 3/2 + 2(15/4) = 9$
$P(3/13, 24/13)$	$Z = 3/13 + 2(24/13) = 51/13$
$S(18/7, 2/7)$	$Z = 18/7 + 2(2/7) = 22/7$

From the above table it's seen that the minimum value of Z is $22/7$ and maximum value of Z is 9.

Therefore, the maximum value of Z is 9 at $(3/2, 15/4)$ and the minimum value of Z is $22/7$ at $(18/7, 2/7)$.

11. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximize his profit.

Solution:

Let x units of type A and y units of type B electric circuits be produced by the manufacturer.

From the given information the below table is constructed:

Items	Type A (x)	Type B (y)	Maximum stock
Resistors	20	10	200
Transistors	10	20	120
Capacitors	10	30	150
Profit	Rs 50	Rs 60	$Z = 50x + 60y$

Now, the total profit function in rupees $Z = 50x + 60y$ is to be maximized with subject to the constraints

$$20x + 10y \leq 200 \dots (i); \quad 10x + 20y \leq 120 \dots (ii)$$

$$10x + 30y \leq 150 \dots (iii); \quad x \geq 0, y \geq 0 \dots (iv)$$

Therefore, the required LPP is

Maximize $Z = 50x + 60y$ subject to the constraints

$$20x + 10y \leq 200 \quad 2x + y \leq 20;$$

$$10x + 20y \leq 120 \quad x + 2y \leq 12 \text{ and}$$

$$10x + 30y \leq 150 \quad x + 3y \leq 15, x \geq 0, y \geq 0.$$

12. A firm has to transport 1200 packages using large vans which can carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

Solution:

Let's consider x and y to be the number of large and small vans respectively.

From the given information the below constrains table is constructed:

Items	Large vans (x)	Small vans (y)	Maximum/Minimum
Packages	200	80	1200
Cost	400	200	3000

Now, the objective function for minimum cost is

$$Z = 400x + 200y$$

Subject to the constrains;

$$200x + 80y \geq 1200 \Rightarrow 5x + 2y \geq 30 \dots (i)$$

$$400x + 200y \leq 3000 \Rightarrow 2x + y \leq 15 \dots (ii)$$

$$x \leq y \dots (iii)$$

and $x \geq 0, y \geq 0$ (non-negative constraints)

Therefore, the required LPP is to minimize $Z = 400x + 200y$

Subject to the constraints $5x + 2y \geq 30$, $2x + y \leq 15$, $x \leq y$ and $x \geq 0$, $y \geq 0$.

13. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of Type A screws requires 2 minutes on the threading machine and 3 minutes on the slotting machine. A box of type B screws requires 8 minutes of threading on the threading machine and 2 minutes on the slotting machine. In a week, each machine is available for 60 hours.

On selling these screws, the company gets a profit of Rs 100 per box on type A screws and Rs 170 per box on type B screws.

Formulate this problem as a LPP given that the objective is to maximize profit.

Solution:

Let's consider that the company manufactures x boxes of type A screws and y boxes of type B screws. From the given information the below table is constructed:

Items	Type A (x)	Type B (y)	Minimum time available on each machine in a week
Time required on threading machine	2	8	$60 \times 60 = 3600$ minutes
Time required on slotting machine	3	2	$60 \times 60 = 3600$ minutes
Profit	Rs 100	Rs 170	

From the data in the above table, the objective function for maximum profit $Z = 100x + 170y$

Subject to the constraints

$$2x + 8y \leq 3600 \Rightarrow x + 4y \leq 1800 \dots (i)$$

$$3x + 2y \leq 3600 \dots (ii)$$

$$x \geq 0, y \geq 0 \quad (\text{non-negative constraints})$$

Therefore, the required LPP is

$$\text{Maximize: } Z = 100x + 170y$$

Subject to constraints,

$$x + 4y \leq 1800, 3x + 2y \leq 3600, x \geq 0, y \geq 0.$$

14. A company manufactures two types of sweaters: type A and type B. It costs Rs 360 to make a type A sweater and Rs 120 to make a type B sweater. The company can make at most 300 sweaters and spend at most Rs 72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs 200 for each sweater of type A and Rs 120 for every sweater of type B.

Formulate this problem as a LPP to maximize the profit to the company.

Solution:

Let's assume x and y to be the number of sweaters of type A and type B respectively.

From the question, the following constraints are:

$$360x + 120y \leq 72000 \Rightarrow 3x + y \leq 600 \dots (i)$$

$$x + y \leq 300 \dots (ii)$$

$$x + 100 \geq y \Rightarrow y \leq x + 100 \dots (iii)$$

$$\text{Profit: } Z = 200x + 120y$$

Therefore, the required LPP to maximize the profit is

Maximize $Z = 200x + 120y$ subject to constraints
 $3x + y \leq 600, x + y \leq 300, y \leq x + 100, x \geq 0, y \geq 0$.

15. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel.

Express this problem as a linear programming problem.

Solution:

Let's assume the man covers x km on his motorcycle at the speed of 50 km/hr and covers y km at the speed of 80 km/hr.

So, cost of petrol = $2x + 3y$

The man has to spend Rs 120 atmost on petrol

$$\Rightarrow 2x + 3y \leq 120 \dots (i)$$

Now, the man has only 1 hr time

$$\text{So, } x/50 + y/80 \leq 1 \Rightarrow 8x + 5y \leq 400 \dots (ii)$$

$$\text{And, } x \geq 0, y \geq 0$$

To have maximum distance $Z = x + y$.

Therefore, the required LPP to travel maximum distance is maximize $Z = x + y$, subject to the constraints

$$2x + 3y \leq 120, 8x + 5y \leq 400, x \geq 0, y \geq 0.$$

Long Answer (L.A.)

16. Refer to Exercise 11. How many of circuits of Type A and of Type B, should be produced by the manufacturer so as to maximize his profit? Determine the maximum profit.

Solution:

As per the solution of exercise 11, we have
Maximize $Z = 50x + 60y$ subject to the constraints

$$20x + 10y \leq 200 \quad 2x + y \leq 20 \dots (i)$$

$$10x + 20y \leq 120 \quad x + 2y \leq 12 \dots (ii)$$

$$10x + 30y \leq 150 \quad x + 3y \leq 15 \dots (iv)$$

$$x \geq 0, y \geq 0 \dots (iv)$$

Now, let's construct a constrain table for the above

Table for (i)

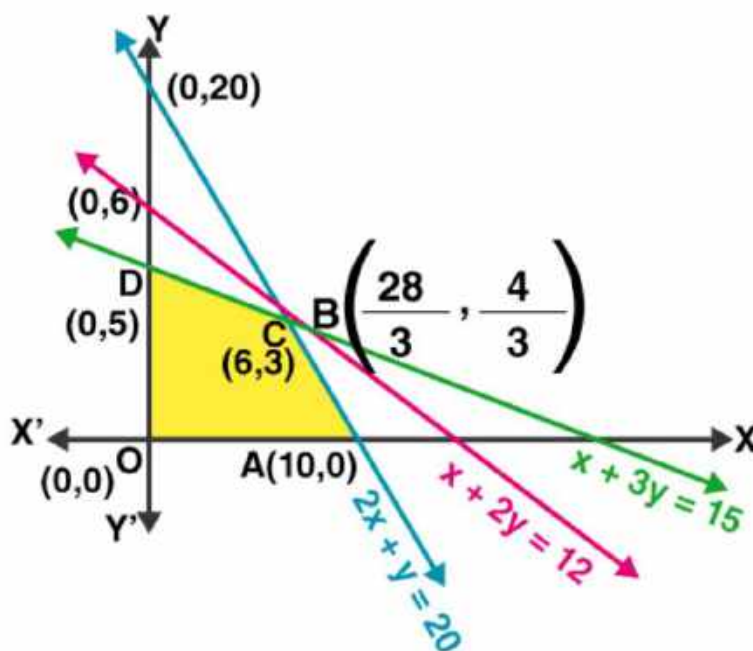
x	10	0
y	0	20

Table for (ii)

x	12	0
y	0	6

Table for (iii)

x	15	0
y	0	5



Next, solving equations (i) and (ii) we get,

$$x = 28/3, y = 4/3$$

So, the corner point is $B(28/3, 4/3)$.

Solving equations (ii) and (iii) we get,

$$x = 6, y = 3 \text{ and the corner point is } C(6, 3)$$

Lastly, solving equations (i) and (iii) we get,

$$x = 9, y = 2 \text{ (not included in the feasible region)}$$

Here, OABCD is the feasible region.

Hence, the corner points are $O(0, 0)$, $A(10, 0)$, $B(28/3, 4/3)$, $C(6, 3)$ and $D(0, 5)$.

Let us evaluate the value of Z

Corner points	Corresponding value of $Z = 50x + 60y$
$O(0, 0)$	$Z = 50(0) + 60(0) = 0$
$A(10, 0)$	$Z = 50(10) + 60(0) = 500$
$B(28/3, 4/3)$	$Z = 50(28/3) + 60(4/3) = 1400/3 + 240/3$ $= 1640/3 = 546.6$
$C(6, 3)$	$Z = 50(6) + 60(3) = 480$
$D(0, 5)$	$Z = 50(0) + 60(5) = 300$

So here, the maximum profit is Rs 546.6 which is not possible for number of items in fraction.

Therefore, the maximum profit for the manufacture is Rs 480 at $(6, 3)$ i.e. Type A = 6 and Type B = 3.

17. Refer to Exercise 12. What will be the minimum cost?

Solution:

As per the solution of exercise 12, we have

The objective function for minimum cost is $Z = 400x + 200y$

Subject to the constraints;

$$5x + 2y \geq 30 \text{ (i)}$$

$$2x + y \leq 15 \text{ (ii)}$$

$$x \leq y \text{ (iii)}$$

and $x \geq 0, y \geq 0$ (non-negative constraints)

Now, let's construct a constrain

table for the above

Table for (i)

x	0	6
y	15	0

Table for (ii)

x	0	7.5
y	15	0

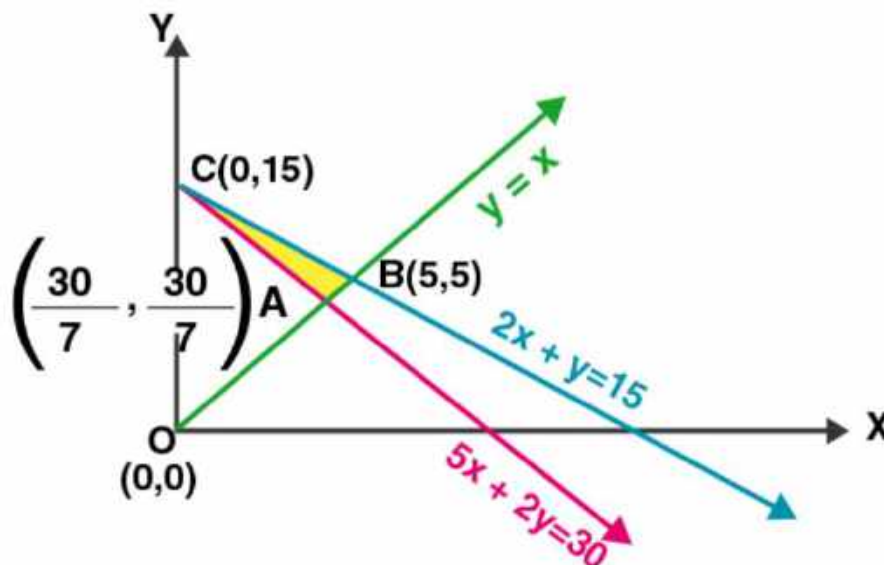
Table for (iii)

x	1	0
y	1	0

Next, solving equations (i) and (iii), we get

$$x = 30/7 \text{ and } y = 30/7, \text{ so the corner point is } A(30/7, 30/7)$$

On solving equations (ii) and (iii), we get



$x = 5$ and $y = 5$, so the corner point is $B(5, 5)$

Here, ABC is the shaded feasible region whose corner points are $A(30/7, 30/7)$, $B(5, 5)$ and $C(0, 15)$

On evaluating the value of Z , we have

Corner point	Value of $Z = 400x + 200y$
$A(30/7, 30/7)$	$Z = 400(30/7) + 200(30/7) = 18000/7 = 2571.4$
$B(5, 5)$	$Z = 400(5) + 200(5) = 3000$
$C(0, 15)$	$Z = 400(0) + 200(15) = 3000$

From the table it's seen that the minimum value is 2571.4

Therefore, the required minimum cost is Rs 2571.4 at $(30/7, 30/7)$

18. Refer to Exercise 13. Solve the linear programming problem and determine the maximum profit to the manufacturer.

Solution:

As per the solution of exercise 13, we have

The objective function for maximum profit

$$Z = 100x + 170y$$

Subject to constraints,

$$x + 4y \leq 1800 \dots (i)$$

$$3x + 2y \leq 3600 \dots (ii)$$

$$x \geq 0, y \geq 0$$

Now, let's construct a constrain table for the above

Table for (i)

x	0	1800
y	450	0

Table for (ii)

x	0	1200
y	1800	0

Next, solving equations (i) and (ii), we get

$$x = 1080 \text{ and } y = 180$$

It's seen that OABC is the feasible region

whose corner points are $O(0, 0)$, $A(1200, 0)$, $B(1080, 180)$ and $C(0, 450)$.

On evaluating the value of Z , we have

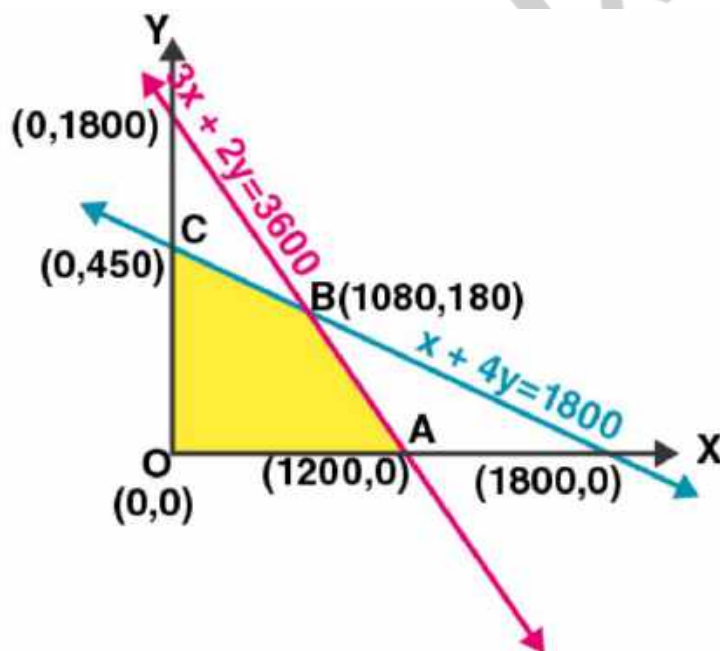
Corner points	Value of $Z = 100x + 170y$
$O(0, 0)$	$Z = 100(0) + 170(0) = 0$
$A(1200, 0)$	$Z = 100(1200) + 170(0) = 120000$
$B(1080, 180)$	$Z = 100(1080) + 170(180) = 138600$
$C(0, 450)$	$Z = 100(0) + 170(450) = 76500$

Form the table it's seen that the maximum value of Z is 138600.

Therefore, the maximum profit of the function Z is 138600 at $(1080, 180)$.

19. Refer to Exercise 14. How many sweaters of each type should the company make in a day to get a maximum profit? What is the maximum profit?

Solution:



As per the solution of exercise 14, we have

Maximize $Z = 200x + 120y$ subject to
constraints

$$3x + y \leq 600 \dots (i)$$

$$x + y \leq 300 \dots (ii)$$

$$x - y \leq -100 \dots (iii)$$

$$x \geq 0, y \geq 0$$

Now, let's construct a constrain table for the above

Table for (i)

x	0	200
y	600	0

Table for (ii)

x	0	300
y	300	0

Table for (iii)

x	-100	0
y	0	100

Next, solving equation (i) and (iii) we get

$$x = 100 \text{ and } y = 200$$

On solving equation (i) and (ii), we get

$$x = 150 \text{ and } y = 150$$

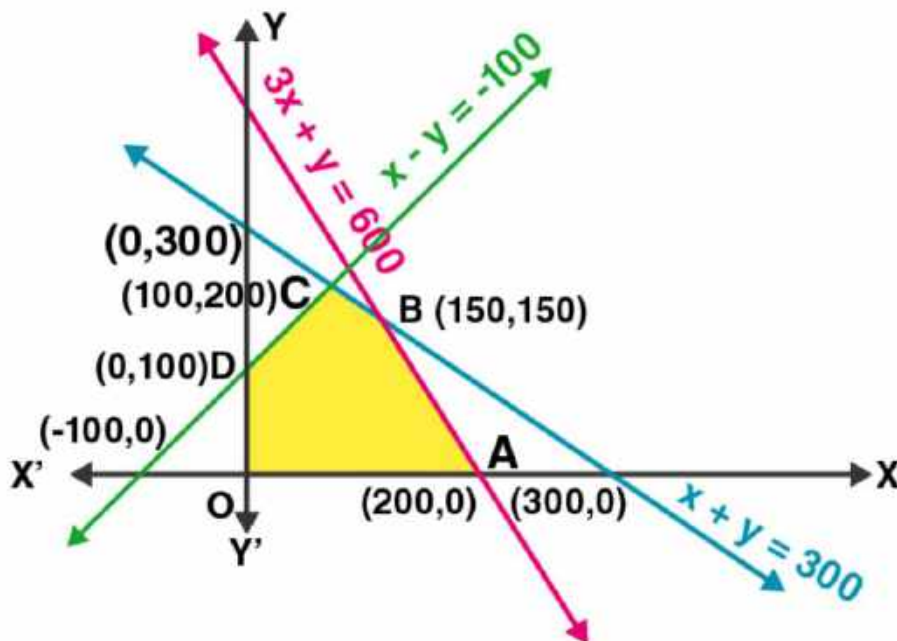
It's seen that the shaded region is the feasible region whose corner points are $O(0, 0)$, $A(200, 0)$, $B(150, 150)$, $D(0, 100)$.

Evaluating the value of Z , we have

Corner points	Value of $Z = 200x + 120y$
$O(0, 0)$	$Z = 200(0) + 120(0) = 0$
$A(200, 0)$	$Z = 200(200) + 120(0) = 40000$
$B(150, 150)$	$Z = 200(150) + 120(150) = 48000$
$C(100, 200)$	$Z = 200(100) + 120(200) = 44000$
$D(0, 100)$	$Z = 200(0) + 120(100) = 12000$

From the above table it's seen that the maximum value is 48000.

Therefore, the maximum value of Z is 48000 at $(150, 150)$ which means 150 sweaters of each type.



20. Refer to Exercise 15. Determine the maximum distance that the man can travel.

Solution:

As per the solution of exercise 15, we have

Maximize $Z = x + y$, subject to the constraints

$$2x + 3y \leq 120 \dots (i)$$

$$8x + 5y \leq 400 \dots (ii)$$

$$x \geq 0, y \geq 0$$

Now, let's construct a constrain table for the above

Table for (i)

x	0	60
y	40	0

Table for (ii)

x	0	50
y	80	0

Next, solving equation (i) and (iii) we get

$$x = 300/7 \text{ and } y = 80/7$$

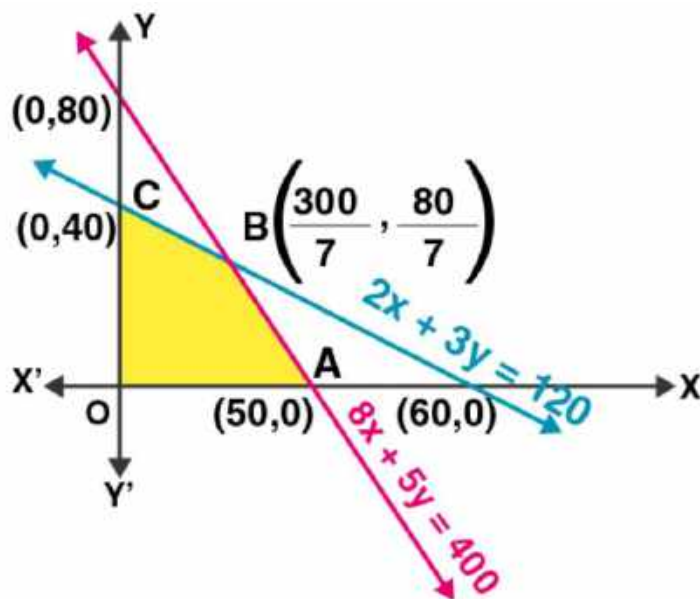
It's seen that the feasible region whose corner points are $O(0, 0)$, $A(50, 0)$, $B(300/7, 80/7)$ and $C(0, 40)$.

Let's evaluate the value of Z

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(50, 0)$	$Z = 50 + 0 = 50$
$B(300/7, 80/7)$	$Z = 300/7 + 80/7 = 380/7 = 54.3$
$C(0, 40)$	$Z = 0 + 40 = 40$

From above table the maximum value of Z is 54.3

Therefore, the maximum distance that the man can travel is 54.3 km at $(300/7, 80/7)$.



21. Maximize $Z = x + y$ subject to $x + 4y \leq 8$, $2x + 3y \leq 12$, $3x + y \leq 9$, $x \geq 0$, $y \geq 0$.

Solution:

Given: $Z = x + y$ subject to constraints, $x + 4y \leq 8$,

$2x + 3y \leq 12$, $3x + y \leq 9$, $x \geq 0$, $y \geq 0$

Constructing a constrain table for the above, we have

Table for $x + 4y = 8$

x	0	8
y	2	0

Table for $2x + 3y = 12$

x	0	6
y	4	0

Table for $3x + y = 9$

x	3	0
y	0	9

On solving equations $x + 4y \leq 8$ and $3x + y \leq 9$,

we get

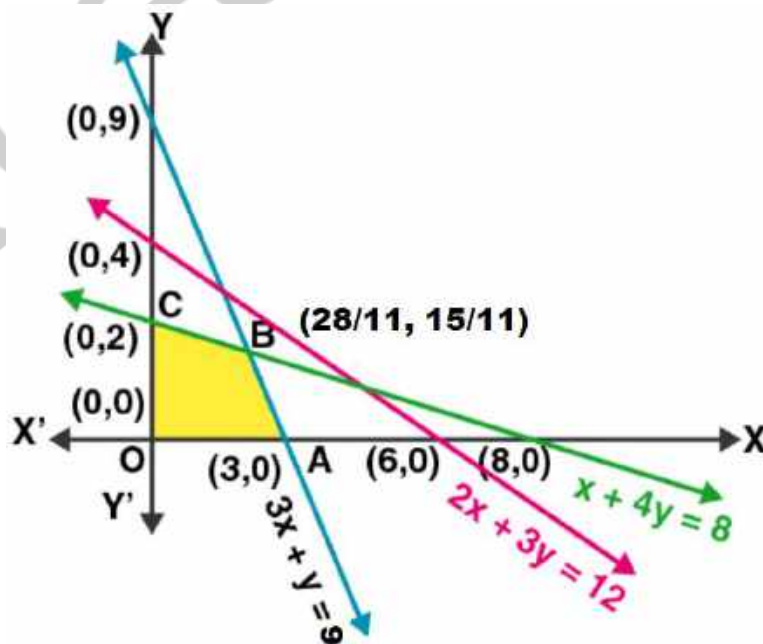
$$x = 28/11 \text{ and } y = 15/11$$

Here, it's seen that $OABC$ is the feasible region

whose corner points are $O(0, 0)$, $A(3, 0)$, $B(28/11, 15/11)$ and $C(0, 2)$.

Now, let's evaluate the value of Z

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(3, 0)$	$Z = 3 + 0 = 3$
$B(28/11, 15/11)$	$Z = 28/11 + 15/11 = 43/11 = 3.9$



$C(0, 2)$	$Z = 0 + 2 = 2$
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From the above table it's noticed that the maximum value of Z is 3.9
Therefore, the maximum value of Z is 3.9 at $(28/11, 15/11)$.

22. A manufacturer produces two Models of bikes - Model X and Model Y. Model X takes 6 man-hours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hour available per week. Handling and Marketing costs are Rs 2000 and Rs 1000 per unit for Models X and Y respectively. The total funds available for these purposes are Rs 80,000 per week. Profits per unit for Models X and Y are Rs 1000 and Rs 500, respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit.

Solution:

Let's take x and y to be the number of models of bike produced by the manufacturer.

From the question we have,

Model x takes 6 man-hours to make per unit

Model y takes 10 man-hours to make per unit

Total man-hours available = 450

So, $6x + 10y \leq 450 \Rightarrow 3x + 5y \leq 225$ (i)

The handling and marketing cost of model x and y are Rs 2000 and Rs 1000 respectively.

And, the total funds available is Rs 80,000 per week

So, $2000x + 1000y \leq 80000 \Rightarrow 2x + y \leq 80$... (ii)

And, $x \geq 0, y \geq 0$

Now, the total profit (Z) per unit of models x and y are Rs 1000 and Rs 500 respectively

$\Rightarrow Z = 1000x + 500y$

Hence, the required LPP is

Maximize $Z = 1000x + 500y$ subject to the constraints

$3x + 5y \leq 225, 2x + y \leq 80$ and $x \geq 0, y \geq 0$

Now, let's construct a constraint table for the above:

Table for (i)

x	75	0
y	0	45

Table for (ii)

x	0	40
y	80	0

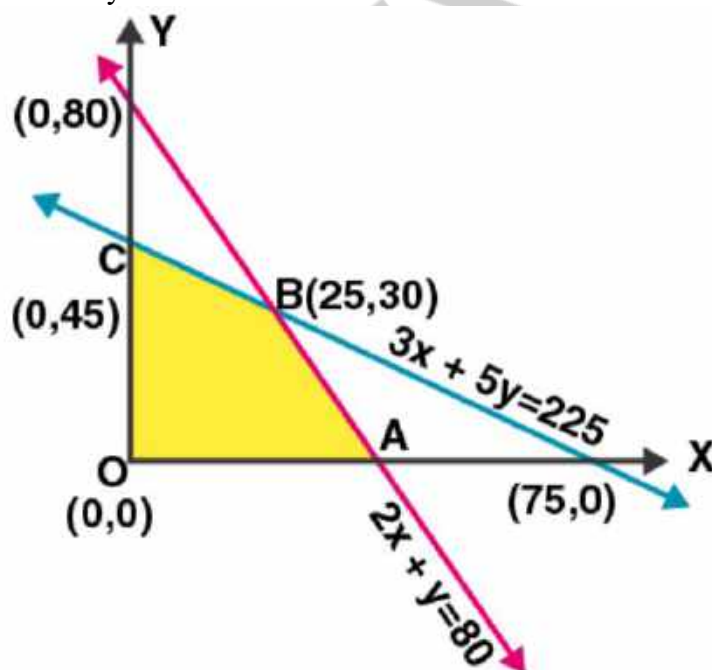
Next, on solving equation (i) and (ii) we get

$x = 25$ and $y = 30$

After plotting all the constraint equations, we observe that the feasible region is OABC, whose corner points are $O(0, 0)$, $A(40, 0)$, $B(25, 30)$ and $C(0, 45)$.

On evaluating the value of Z , we get

Corner points	Value of $Z = 1000x + 500y$
$O(0, 0)$	$Z = 1000(0) + 500(0) = 0$
$A(40, 0)$	$Z = 1000(40) + 500(0) = 40,000$



B(25, 30)	$Z = 1000(25) + 500(30) = 40,000$
C(0, 45)	$Z = 1000(0) + 500(45) = 22,500$

Therefore, from the above table it's seen that the maximum profit is Rs 40,000.

The maximum profit can be achieved by producing 25 bikes of model x and 30 bikes of model Y or by producing 40 bikes of model x.