

Multiple Choice Questions I

6.1 A square of side L meters lies in the x - y plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ T where B_0 is constant. The magnitude of flux passing through the square is

- (a) $2 B_0 L^2$ Wb.
- (b) $3 B_0 L^2$ Wb.
- (c) $4 B_0 L^2$ Wb.
- (d) $\sqrt{29} B_0 L^2$ Wb

Answer:

- (c) $4 B_0 L^2$ Wb.

6.2 A loop, made of straight edges has six corners at $A(0,0,0)$, $B(L,0,0)$, $C(L,L,0)$, $D(0,L,0)$, $E(0,L,L)$ and $F(0,0,L)$. A magnetic field $\mathbf{B} = B_0(\mathbf{i} + \mathbf{k})$ T is present in the region. The flux passing through the loop ABCDEFA (in that order) is

- (a) $B_0 L^2$ Wb.
- (b) $2 B_0 L^2$ Wb.
- (c) $2 B_0 L^2$ Wb.
- (d) $4 B_0 L^2$ Wb.

Answer:

- (b) $2 B_0 L^2$ Wb.

6.3 A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then

- (a) a direct current flows in the ammeter A.
- (b) no current flows through the ammeter A.
- (c) an alternating sinusoidal current flows through the ammeter A with a time period $T = 2\pi/\omega$.
- (d) a time varying non-sinusoidal current flows through the ammeter A.

Answer:

- (b) no current flows through the ammeter A.

6.4 There are two coils A and B. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counterclockwise. B is kept stationary when A moves. We can infer that

- (a) there is a constant current in the clockwise direction in A.
- (b) there is a varying current in A.
- (c) there is no current in A.
- (d) there is a constant current in the counterclockwise direction in A.

Answer:

- (d) there is a constant current in the counterclockwise direction in A.

6.5 Same as problem 4 except the coil A is made to rotate about a vertical axis. No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counterclockwise and the coil A is as shown at this instant, $t = 0$, is

- (a) constant current clockwise.
- (b) varying current clockwise.
- (c) varying current counterclockwise.
- (d) constant current counterclockwise.

Answer:

- (a) constant current clockwise.

6.6 The self inductance L of a solenoid of length l and area of cross-section A , with a fixed number of turns N increases as

- (a) l and A increase.
- (b) l decreases and A increases.
- (c) l increases and A decreases.
- (d) both l and A decrease.

Answer:

- (b) l decreases and A increases.

Multiple Choice Questions II

6.7 A metal plate is getting heated. It can be because

- (a) a direct current is passing through the plate.
- (b) it is placed in a time varying magnetic field.
- (c) it is placed in a space varying magnetic field, but does not vary with time.
- (d) a current (either direct or alternating) is passing through the plate.

Answer:

- (a) a direct current is passing through the plate.
- (b) it is placed in a time varying magnetic field.
- (d) a current (either direct or alternating) is passing through the plate.

6.8 An e.m.f is produced in a coil, which is not connected to an external voltage source. This can be due to

- (a) the coil being in a time varying magnetic field.
- (b) the coil moving in a time varying magnetic field.
- (c) the coil moving in a constant magnetic field.
- (d) the coil is stationary in external spatially varying magnetic field, which does not change with time.

Answer:

- (a) the coil being in a time varying magnetic field.
- (b) the coil moving in a time varying magnetic field.
- (c) the coil moving in a constant magnetic field.

6.9 The mutual inductance M_{12} of coil 1 with respect to coil 2

- (a) increases when they are brought nearer.
- (b) depends on the current passing through the coils.
- (c) increases when one of them is rotated about an axis.
- (d) is the same as M_{21} of coil 2 with respect to coil 1.

Answer:

- (a) increases when they are brought nearer.
- (d) is the same as M_{21} of coil 2 with respect to coil 1.

6.10 A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because

- (a) the magnetic field is constant.
- (b) the magnetic field is in the same plane as the circular coil and it may or may not vary.
- (c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.
- (d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction.

Answer:

- (b) the magnetic field is in the same plane as the circular coil and it may or may not vary.
- (c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is

decreasing suitably.

Very Short Answers

6.11 Consider a magnet surrounded by a wire with an on/off switch S. If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit? Explain.

Answer:

No current will be induced as there is no change in any of the magnet or in the area of the circuit. Also, there is no change in the angle.

6.12 A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

Answer:

When the coil is stretched, there are gaps between successive elements of the spiral coil then the current will increase. For current to increase, reactance must decrease.

6.13 A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

Answer:

When an iron core is inserted into the solenoid, the magnetic flux increases. According to Lenz's law, when the flux increases, there is a decrease in the current flow through the coil.

6.14 Consider a metal ring kept on top of a fixed solenoid such that the centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.

Answer:

When the current is suddenly switched on, the metal ring jumps up because the magnetic flux increases across the ring.

6.15 Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I. The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?

Answer:

We know that the current was already flowing through the solenoid making it behave like a magnet such that the S pole is on the upper side. Therefore, there is no induced current in the ring. When the current is turned off, there is decrease in the magnetic flux such that the N pole is on the lower side and is attracted by the solenoid.

6.16 Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

Answer:

When a cylindrical bar magnet with radius 0.8 cm is dropped through the pipe, the magnetic flux across the pipe changes and there is a production of eddy currents. The presence of eddy current opposes the motion of the magnet.

Short Answers

6.17 A magnetic field in a certain region is given by $B = B_0 \cos \omega t \hat{k}$ and a coil of radius a with resistance R is placed in the x - y plane with its centre at the origin in the magnetic field. Find the magnitude and the

direction of the current at $(a, 0, 0)$ at $t = \pi/\omega$, $t = \pi/2\omega$ and $t = 3\pi/\omega$

Answer:

The direction of the magnetic field is along the z-axis.

$$\Phi = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$

Using Faraday's law of electromagnetic induction,

$$I = B_0 \pi a^2 \omega / R \sin \omega t$$

$$\text{Therefore, current at } t = \pi/\omega = I = B_0 \pi a^2 \omega / R \sin \omega t = 1$$

$$\text{Current at } t = \pi/2\omega = I = B_0 \pi a^2 \omega / R \sin \omega t = -B_0 \pi a^2 \omega / R$$

$$\text{Current } t = 3\pi/\omega = I = B_0 \pi a^2 \omega / R \sin \omega t = I = -B_0 \pi a^2 \omega / R$$

6.18 Consider a closed loop C in a magnetic field such that the flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = B_1 dA_1 + B_2 dA_2 + \dots$. Now if we chose two different surfaces S1 and S2 having C as their edge, would we get the same answer for flux. Justify your answer.

Answer:

The magnetic flux lines passing through is same as the magnetic flux lines passing through the surface. $\phi = B_1 dA_1 + B_2 dA_2$ represents the magnetic field lines in an area A with magnetic flux B. Therefore, the number of lines passing through S1 is same as the lines passing through S2.

6.19 Find the current in the wire for the configuration. Wire PQ has negligible resistance. B, the magnetic field is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by a distance d.

Answer:

F is the force on the free charge particle of PQ.

The motional emf is given as the product of E along the PQ and the effective length PQ.

Therefore, the induced current will be vBd/R which is independent of q .

6.20 A (current vs time) graph of the current passing through a solenoid is shown in Fig 6.9. For which time is the back electromotive force (u) a maximum. If the back emf at $t = 3s$ is e, find the back emf at $t = 7s$, $15s$ and $40s$. OA, AB and BC are straight line segments.

Answer:

From the graph we can say that when there is a maximum rate of change of magnetic flux, the electromagnetic force will be maximum which is proportional to the rate of change of current. The rate of current will be maximum when the time axis makes maximum angle at AB.

Therefore, when $t = 3$ seconds, $s = \text{slope of OA}$

Therefore, the rate of change of current at $t = 3 = 1/4 \text{ A/s}$

Therefore, the electromotive force is $L/5$.

When the emf is between 5 to 10 seconds, it is $-3e$

When the emf is between 10 to 30 seconds, it is $+1/2e$

When the emf is at 40 seconds, $dI/dt = 0$

6.21 There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?

Answer:

I_a is the current passing through the coil

M_{ab} is the mutual induction between A and B

N_a is the number of turns in coil A

N_b is the number of turns in coil B
 ϕ_a is the flux linked to coil A due to coil B
 ϕ_b is the flux linked to coil B due to coil A
 Total flux is $B = M_2\phi_2 = M_{ab}I_1 = 5mH$
 Total flux through A = $M_a\phi_a = M_{ab}I_1 = 5mWb$

Long Answers

6.22 A magnetic field $B = B_0 \sin \omega t \hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d . The wires are in the x - y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v , what is the current in the circuit. What is the force needed to keep the wire moving at constant velocity?

Answer:

Let wire AB at $t = 0$ move with velocity v .

At t , $x(t) = vt$

Motional emf across AB = $e_1 = Blv$

$e_1 = (B_0 \sin \omega t)vd(-j)$

$e_2 = d(\phi_B)/dt$

$e_2 = -B_0 \omega \cos \omega t x(t)d$

Total emf in the circuit = emf due to change in field + motional emf across AB

Electric current in the clockwise direction is given as $= B_0 d/R (\omega x \cos \omega t + v \sin \omega t)$

Therefore, external force is given as, $F_{ext} = B_0^2 d^2/R (\omega x \cos \omega t + v \sin \omega t)(\sin \omega t) \hat{i}$

6.23 A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires. The closed circuit has a resistance R due to AC, AB and CD are perfect conductors. There is a magnetic field $B = B_0 t \hat{k}$.

(i) Write down equation for the acceleration of the wire XY.

(ii) If B is independent of time, obtain $v(t)$, assuming $v(0) = u_0$.

(iii) For (b), show that the decrease in kinetic energy of XY equals the heat lost in R .

Answer:

The magnetic flux linked with the loop is given as $\phi_m = B.A = BA \cos \theta$

Where A is the area vector and B is the magnetic field vector.

Emf induced due to change in magnetic field = $e_1 = -dB(t)/dt \times l \times (t)$

Emf induced due to motion = $e_2 = B(t)lv(t)(-j)$

Total emf in the circuit = emf due to change in field + the motional emf across XY

$E = -dB(t)/dt \times l \times (t) - B(t)lv(t)$

We know current, $I = E/R$

Force = $IB(t)/R[-dB(t)/dt \times l \times (t) - I^2 B^2(t)/R dx/dt]$

B is independent of the time

Power consumption is given as $P = I^2 R$

Therefore, energy consumed in time interval dt is given as $= m/2 u_0^2 - m/2 v^2(t)$

The above equation shows that there is decrease in the kinetic energy.

6.24 OADBAC is a fixed rectangular conductor of negligible resistance (CO is not connected) and OP is a conductor which rotates clockwise with an angular velocity ω . The entire system is in a uniform magnetic field B whose direction is along the normal to the surface of the rectangular conductor ABDC. The conductor OP is in electric contact with ABDC. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor, as it rotates by 180° .

Answer:

i) When the position of the rotating conductor is assumed to be at the time interval $t = 0$ to $t = \pi/4\omega$
We get current $I = 1/2 B l^2 \omega / \lambda \sec^2 \omega t \cos \omega t = B l \omega / 2 \lambda \cos \omega t$

ii) When the position of the rotating conductor is at time interval $\pi/4\omega < t < 3\pi/4\omega$
We get current $I = 1/2 B l \omega / \lambda \sin \omega t$

iii) When the position of the rotating conductor is at time interval $3\pi/4\omega < t < \pi/\omega$
We get current $I = 1/2 B l \omega / \lambda \sin \omega t$

6.25 Consider an infinitely long wire carrying a current $I(t)$, with $dI/dt = \lambda = \text{constant}$. Find the current produced in the rectangular loop of wire ABCD if its resistance is R .

Answer:

The width of the strip is dr and the length is l which is inside the rectangular box at a distance r from the surface of the current carrying conductor. The magnetic field across the strip length is given as

$$B(r) = \mu_0 I / 2\pi r$$

$B(r)$ is perpendicular to the paper upward.

$$\text{Flux in the strip} = \phi = \mu_0 I l / 2\pi [\log_e r]$$

$$\epsilon = -d\phi/dt$$

$$\text{Therefore, } I = \mu_0 \lambda l / 2\pi R \log_e x/x_0$$

6.26 A rectangular loop of wire ABCD is kept close to an infinitely long wire carrying a current $I(t)$ $t = 0$ ($1 - t/T$) for $0 \leq t \leq T$ and $I(0) = 0$ for $t > T$. Find the total charge passing through a given point in the loop, in time T . The resistance of the loop is R .

Answer:

If t is the instantaneous current then,

$$I(t) = 1/R \, d\phi/dt$$

If q is the charge passing in time t

$$I(t) = dQ/dt$$

$$dQ/dt = 1/R \, d\phi/dt$$

Integrating the equation we get,

$$Q = \mu_0 L I L / 2\pi R \log (L/2 + x/x_0)$$

6.27 A magnetic field B is confined to a region $r \leq a$ and points out of the paper (the z -axis), $r = 0$ being the centre of the circular region. A charged ring (charge = Q) of radius b , $b > a$ and mass m lies in the x - y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt . Find the angular velocity ω of the ring after the field vanishes.

Answer:

The magnetic flux across the conducting ring reduces to zero from maximum when the magnetic field is reduced in Δt .

$$\text{Induced emf} = E = 2\pi b$$

From Faraday's law of emf,

$$\text{The induced emf} = \text{rate of change of magnetic flux} = B\pi a^2 / \Delta t$$

Using the above equations, we get

$$E = 2\pi b = B\pi a^2 / \Delta t$$

QE is the electric force experienced by the ring

$$\text{Torque on the ring} = Q.Ba^2 / 2\Delta t$$

Change in angular momentum = torque $\times \Delta t$

The initial momentum is zero and the final momentum is $Q.Ba^2/2$

$$\omega = Q.Ba^2/2mb^2$$

6.28 A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to the horizontal. The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.

Answer:

The angle between B and $PQ = 90^\circ$

$$d\phi = B \cdot dA$$

$$d\phi = B v d \cos \theta$$

$$-\varepsilon = B v d \cos \theta$$

$$I = -Bvd/R \cos \theta$$

Solving the above equation using Newton's second law, we get, v as

$$v = \alpha g \sin \theta [1 - e^{-t/\omega}]$$

6.29 Find the current in the sliding rod AB (resistance = R). B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.

Answer:

The current induced in the loop is $I_t = \varepsilon/R$

$$I_t = 1/R \cdot d/dt BA$$

$$I_t = vBd/R$$

The angle between B and A is zero.

As the switch S is closed at $t = 0$,

Charge through the capacitor, is $Q(t) = Cv$

Current through the capacitor, $I(c) = Q(t)/RC$

Using the above information, we can calculate the current through the circuit as $I = BvdC/RC e^{-t/RC}$.

6.30 Find the current in the sliding rod AB (resistance = R). B is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch S is closed at time $t = 0$.

Answer:

The angle between A and $B = 0^\circ$

Therefore, the emf is Bvd .

$$-LdI(t)/dt + Bvd = IR$$

$$LdI(t)/dt + RI(t) = Bvd$$

Solving the equation, we get

$$I = Bvd/R [1 - e^{-Rt/2L}]$$

6.31 A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z -component of magnetic field is $B_z = B_0 (1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v , find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m , B , λ and acceleration due to gravity g .

Answer:

The required relation is given as:

$$v = mgR/B_0^2 \pi^2 \lambda^2 l^4$$

6.32 A long solenoid 'S' has 'n' turns per meter, with diameter 'a'. At the centre of this coil we place a smaller coil of 'N' turns and diameter 'b' (where $b < a$). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as a function of $mt^2 + C$.

Answer:

The varying magnetic field in the solenoid is given as:

$$B_1(t) = \mu_0 n I(t)$$

Magnetic flux in the second coil is

$$\Phi_2 = \mu_0 n I(t) \cdot \pi b^2$$

Therefore, the induced emf in second coil due to solenoid's varying magnetic field is $-\mu_0 N n \pi b^2 \frac{dI}{dt}$



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